Stat261 - Spring 2018

Assignment #1 - due Thursday, January 11 in class

Neatly hand write your solutions - marks will be assigned for presentation

1. Let X be a random variable with mean $\mu = 4$ and variance $\sigma^2 = 8$. What is $E(2X^2)$?

2. In November of each year, a walk-in clinic allows people to walk in to get a flu shot. Let X be the number of people who come to the clinic for a flu shot on a randomly selected day (in November). Suppose X has the following distribution:

x	0	1	2	3	4	5
$P(X = x)$	0.3	0.2	0.2	0.1	0.1	0.1

   If at least 2 people walk in for a flu shot on a particular day, what is the probability that there are 4 or fewer who walk in for a flu shot that day?

3. You want to read a disk sector from a 7200rpm disk drive. Let T be the time you wait, in milliseconds, after the disk head is positioned over the correct track, until the desired sector rotates under the head. The random variable T has the following probability density function (*pdf*),

$$f(x) = \begin{cases} c(x^2 - 1) & 1 \le x < 3 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Compute the value of c in the *pdf*.

   (b) Compute the probability X is between 1.5 and 2.25 inclusive.

4. Assume a page in the book will be considered **defective** if there are more than 3 typos on the page. On average, on one page there is 1 typo. Let X denote the number of typos on one page and assume that it has a Poisson distribution. What is the probability that a randomly chosen page is defective?

5. The thickness measurements of glass sheets produced by a certain process are normally distributed with a mean of $\mu = 3.00$ mm and a standard deviation of $\sigma = 0.12$ mm. What is the value of c for which there is a 99% probability that a glass sheet has a thickness measurement within the interval (3.00-c, 3.00+c)?

6. Daily sales at a gas station are thought to be independent of one another with daily mean \$5250 and standard deviation \$700. Approximate the probability that the average daily sales over one year (i.e. 365 days) is greater than \$6,000.

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1. $\mu = 4, \ \sigma^2 = 8 \qquad E(2X^2)$

$E(X^2) = Var(X) + [E(X)]^2 = 8 + 4^2 = 24$

$E(2X^2) = 2E(X^2) = 2 \times 24 = 48.$

2. X = # people who come for a flu shot.

$P(X \le 4 \mid X \ge 2) = \dfrac{P(X \le 4 \text{ and } X \ge 2)}{P(X \ge 2)} = \dfrac{P(X = 4 \text{ or } 3 \text{ or } 2)}{P(X \ge 2)}$

$= \dfrac{.1 + .1 + .2}{.2 + .1 + .1 + .1} = \dfrac{.4}{.5} = .8.$

3. (a) $f(x) = \begin{cases} c(x^2 - 1) & 1 \le x < 3. \\ 0 & \text{else.} \end{cases}$

$1 = \int_1^3 c(x^2 - 1)\,dx = c\left[\dfrac{x^3}{3} - x\right]_1^3$

$= c\left[(9 - 3) - (\tfrac{1}{3} - 1)\right] = c(6\tfrac{2}{3}) = c\,\dfrac{20}{3}.$

$\therefore c = \dfrac{3}{20}.$

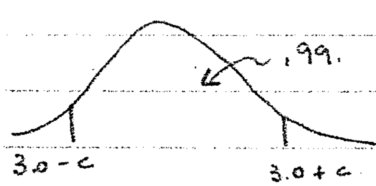
4. X = # typos on one page. ~ Poisson ($\mu = 1$).

$P(X > 3) = 1 - P(X \le 3).$

$= 1 - \dfrac{\mu^0 e^{-\mu}}{0!} - \dfrac{\mu^1 e^{-\mu}}{1!} - \dfrac{\mu^2 e^{-\mu}}{2!} - \dfrac{\mu^3 e^{-\mu}}{3!}.$

$= 1 - e^{-1}\left[1 + 1 + \tfrac{1}{2} + \tfrac{1}{6}\right] = 1 - e^{-1}\,\dfrac{8}{3} = .019.$

5. $\mu = 3.0$ mm $\quad \sigma = .12$ mm



$X \sim N(\mu = 3.0,\ \sigma^2 = .12^2)$

$P(3.0 - c < X < 3.0 + c) = .99$

$= P\left(\frac{3.0 - c - \mu}{\sigma} < Z < \frac{3.0 + c - \mu}{\sigma}\right) = .99$

$\frac{3.0 + c - \mu}{\sigma} = 2.576 \Rightarrow c = .30912$

6. $n = 365 \quad \mu = 5250. \quad \sigma = 700$

X_i = daily sales for day i. , $i = 1, \ldots, 365$

$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \approx N(0,1)$ by CLT.

$P(\bar{X} > 6000) = P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} > \frac{6000 - \mu}{\sigma/\sqrt{n}}\right)$

$\approx P(Z > 20.47) \simeq 0.$