



UNIVERSITY GRANTS COMMISSION

NET BUREAU

SYLLABUS

Subject: GENERAL PAPER ON TEACHING & RESEARCH APTITUDE

Code No. : 00

PAPER-I

The main objective is to assess the teaching and research capabilities of the candidates. The test aims at assessing the teaching and research aptitude as well. Candidates are expected to possess and exhibit cognitive abilities, which include comprehension, analysis, evaluation, understanding the structure of arguments, deductive and inductive reasoning. The candidates are also expected to have a general awareness about teaching and learning processes in higher education system. Further, they should be aware of interaction between people, environment, natural resources and their impact on the quality of life.

The details of syllabi are as follows:

Unit-I Teaching Aptitude

- Teaching: Concept, Objectives, Levels of teaching (Memory, Understanding and Reflective), Characteristics and basic requirements.
- Learner's characteristics: Characteristics of adolescent and adult learners (Academic, Social, Emotional and Cognitive), Individual differences.
- Factors affecting teaching related to: Teacher, Learner, Support material, Instructional facilities, Learning environment and Institution.
- Methods of teaching in Institutions of higher learning: Teacher centred vs. Learner centred methods; Off-line vs. On-line methods (Swayam, Swayamprabha, MOOCs etc.).

- Teaching Support System: Traditional, Modern and ICT based.
- Evaluation Systems: Elements and Types of evaluation, Evaluation in Choice Based Credit System in Higher education, Computer based testing, Innovations in evaluation systems.

Unit-II Research Aptitude

- Research: Meaning, Types, and Characteristics, Positivism and Post-positivistic approach to research.
- Methods of Research: Experimental, Descriptive, Historical, Qualitative and Quantitative methods.
- Steps of Research.
- Thesis and Article writing: Format and styles of referencing.
- Application of ICT in research.
- Research ethics.

Unit-III Comprehension

- A passage of text be given. Questions be asked from the passage to be answered.

Unit-IV Communication

- Communication: Meaning, types and characteristics of communication.
- Effective communication: Verbal and Non-verbal, Inter-Cultural and group communications, Classroom communication.
- Barriers to effective communication.
- Mass-Media and Society.

Unit-V Mathematical Reasoning and Aptitude

- Types of reasoning.
- Number series, Letter series, Codes and Relationships.
- Mathematical Aptitude (Fraction, Time & Distance, Ratio, Proportion and Percentage, Profit and Loss, Interest and Discounting, Averages etc.).

Unit-VI Logical Reasoning

- Understanding the structure of arguments: argument forms, structure of categorical propositions, Mood and Figure, Formal and Informal fallacies, Uses of language, Connotations and denotations of terms, Classical square of opposition.
- Evaluating and distinguishing deductive and inductive reasoning.
- Analogies.
- Venn diagram: Simple and multiple use for establishing validity of arguments.
- Indian Logic: Means of knowledge.
- Pramanas: Pratyaksha (Perception), Anumana (Inference), Upamana (Comparison), Shabda (Verbal testimony), Arthapatti (Implication) and Anupalabddhi (Non-apprehension).
- Structure and kinds of Anumana (inference), Vyapti (invariable relation), Hetvabhasas (fallacies of inference).

Unit-VII Data Interpretation

- Sources, acquisition and classification of Data.
- Quantitative and Qualitative Data.
- Graphical representation (Bar-chart, Histograms, Pie-chart, Table-chart and Line-chart) and mapping of Data.
- Data Interpretation.
- Data and Governance.

Unit-VIII Information and Communication Technology (ICT)

- ICT: General abbreviations and terminology.
- Basics of Internet, Intranet, E-mail, Audio and Video-conferencing.
- Digital initiatives in higher education.
- ICT and Governance.

Unit-IX People, Development and Environment

- Development and environment: Millennium development and Sustainable development goals.
- Human and environment interaction: Anthropogenic activities and their impacts on environment.
- Environmental issues: Local, Regional and Global; Air pollution, Water pollution, Soil pollution, Noise pollution, Waste (solid, liquid, biomedical, hazardous, electronic), Climate change and its Socio-Economic and Political dimensions.
- Impacts of pollutants on human health.
- Natural and energy resources: Solar, Wind, Soil, Hydro, Geothermal, Biomass, Nuclear and Forests.
- Natural hazards and disasters: Mitigation strategies.
- Environmental Protection Act (1986), National Action Plan on Climate Change, International agreements/efforts -Montreal Protocol, Rio Summit, Convention on Biodiversity, Kyoto Protocol, Paris Agreement, International Solar Alliance.

Unit-X Higher Education System

- Institutions of higher learning and education in ancient India.
- Evolution of higher learning and research in Post Independence India.
- Oriental, Conventional and Non-conventional learning programmes in India.
- Professional, Technical and Skill Based education.
- Value education and environmental education.
- Policies, Governance, and Administration.

NOTE:

- (i) Five questions each carrying 2 marks are to be set from each Module.
- (ii) Whenever graphical/pictorial question(s) are set for sighted candidates, a passage followed by equal number of questions and weightage be set for visually impaired candidates.

SYLLABUS

Sub Unit – 1: Types of Reasoning

SL. NO	TOPICS
1	1. Meaning of Reasoning
2	2. Types of Reasoning
3	2.1. Deductive Reasoning
4	2.2. Inductive Reasoning
5	2.3. Abductive Reasoning
6	2.4. Backward Induction
7	2.5. Critical Thinking
8	2.6. Counterfactual Thinking
9	2.7. Intuition

Sub Unit – 2: Number Series, Letter Series, Codes and Relationships

SL. NO	TOPICS
10	3. Prime Number Series
11	4. Difference Series
12	5. Multiplication Series
13	6. Division Series
14	7. N^2 Series
15	8. (N^2-1) Series
16	9. (N^2+1) Series
17	10. (N^2+N) Series
18	11. (N^2-N) Series
19	12. N^3 Series
20	13. (N^3+1) Series
21	14. One Letter Series
22	15. Combined Two Letter Series
23	16. Three Letter Series
24	17. Alphabetical Coding
25	17.1. Simple Analogical Letter Coding
26	17.2. Specific Pattern Letter Coding
27	18. Letter Coding With Numerical Digits
28	18.1. Simple Analogical Letter Coding
29	18.2. Specific Pattern Letter Coding
30	19. Mixed Coding

Sub Unit – 3: Mathematical Aptitude

SL. NO	TOPICS
31	20. Fraction
32	21. Time And Distance
33	21.1. Formulae
34	22. Ratio, Proportion And Percentage
35	23. Profit and Loss
36	23.1.1. Some Important Formulae
37	24. Interest and Discounting
38	24.1. Important Concepts
39	24.2. Important Formulae
40	24.3. Simple and Compound Interest
41	25. Calender
42	25.1. Odd Days
43	25.2. Ordinary Year
44	25.3. Leap Year
45	25.4. Some Important Points
46	26. Arithmetic Mean
47	26.1. Formula
48	27. Direction Sense
49	28. Clock Related Test
50	28.1. Some Basic Concept
51	29. Permutations and Combinations
52	29.1. Factorial
53	29.2. Properties of Factorials
54	30. Mensuration Formulae
55	30.1. Mensuration Formulas for Cube
56	30.2. Mensuration Formulas for Cone
57	30.3. Mensuration Formulas for Cylinder
58	30.4. Mensuration Formulas for Sphere
59	30.5. Mensuration Formulas for Hemisphere
60	30.6. Mensuration Formulas for Hollow Cylinder
61	30.7. Mensuration Formulas for Frustum of a Right Circular Cone
62	30.8. Mensuration Formulas for Prism
63	30.9. Square
64	30.10. Rectangle:
65	30.11. Triangle:
66	30.12. Parallelogram:
67	30.13. Circle
68	31. Quantitative Aptitude Formulas
69	31.1. Ages:

Section – 1: At a Glance

Sub Unit – 1: Types of Reasoning

MEANING OF REASONING: Reasoning is the process of thinking about things in a logical or rational way. The process of reasoning is used to make decisions, solve problems and evaluate things.

DEDUCTIVE REASONING: Deductive reasoning is a formal method of top-down logic that seeks to find observations to prove a theory.

INDUCTIVE REASONING: Inductive reasoning is bottom-up logic that seeks theories to explain observations.

ABDUCTIVE REASONING: Like induction, abductive reasoning seeks theories to explain observations.

BACKWARD INDUCTION: Backward induction is a top-down approach that starts with theories and works backwards to explain them.

CRITICAL THINKING: Critical thinking is a process of rational thought that seeks to draw conclusions in an objective, thorough and informed manner.

COUNTERFACTUAL THINKING: Counterfactual thinking is considering things that are known to be impossible.

INTUITION: Intuition is judgements that are made by the mind that are made by the mind that are perceived by unconscious.

Sub Unit – 2: Number Series, Letter Series, Codes and Relationships

NUMBER SERIES: In this sub-section, candidates are asked to either insert a missing number or to find the odd one. There are 10 types of number series:

PRIME NUMBER SERIES: Different prime numbers are appeared serially and you are asked to find out the next prime number or the missing prime number.

DIFFERENCE SERIES: From a given number series you have to calculate the difference between each two terms i.e. the difference between second term and first term, the difference between fourth term and third term, and so on. Then you are asked to find out the next number or the missing number.

MULTIPLICATION SERIES: A number series is given in such a system that the second term is equal to the first term multiplied by a constant; the third term is equal to the second term multiplied by the same constant, and so on. Then you are asked to find out the next number or the missing number.

DIVISION SERIES: A number series is given in such a system that the second term is equal to the first term divided by a constant; the third term is equal to the second term divided by the same constant, and so on. Then you are asked to find out the next number or the missing number.

N^2 SERIES: A number series is given in such a system that the series is squares of each term of another series. You are asked to find out the appropriate answer.

(N^2-1) SERIES: A number series is given, $X_1, X_2, X_3, \dots, X_k$ in such a way that $X_1 = (a_1^2 - 1)$, $X_2 = (a_2^2 - 1)$, \dots , $X_k = (a_k^2 - 1)$, You are asked to find out the appropriate answer.

(N^2+1) SERIES: A number series is given, $X_1, X_2, X_3, \dots, X_k$ in such a way that $X_1 = (a_1^2 + 1)$, $X_2 = (a_2^2 + 1)$, \dots , $X_k = (a_k^2 + 1)$, You are asked to find out the appropriate answer.

(N^2+N) & (N^2-N) SERIES: A number series is given, $X_1, X_2, X_3, \dots, X_k$ in such a way that $X_1 = (a_1^2 \pm a_1)$, $X_2 = (a_2^2 \pm a_2)$, \dots , $X_k = (a_k^2 \pm a_k)$, You are asked to find out the appropriate answer.

N^3 SERIES: A number series is given, $X_1, X_2, X_3, \dots, X_k$ in such a way that this series is cubic of each term of another series $a_1, a_2, a_3, \dots, a_k$ i.e. $X_1 = a_1^3$, $X_2 = a_2^3$, \dots , $X_k = a_k^3$. You are asked to find out the appropriate answer.

(N^3+1) SERIES: A number series is given, $X_1, X_2, X_3, \dots, X_k$ in such a way that $X_1 = (a_1^3 + a_1)$, $X_2 = (a_2^3 + a_2)$, \dots , $X_k = (a_k^3 + a_k)$, You are asked to find out the appropriate answer.

LETTER SERIES: Under this type of series completion some letters are given which follow a particular sequence or order. Examine the directions and the questions with care and try to detect the pattern that the order or sequence is following.

ONE LETTER SERIES: In this part you have to find out the next letter or missing letter to continue the series.

COMBINED TWO LETTER SERIES: In this, the first letters of the series follow one logic and the second letters of the series follow another logic, and then they pair with each other. You are asked to find out the appropriate answer.

THREE LETTER SERIES: This sequence consists of three letters in each term. The first letters of each pair of the series follow one logic, the second letters follow another logic, and the third letters follow some other logic or the same logic in all three cases.

ALPHABETICAL CODING: The letters of the alphabet are exclusively used and these letters do not stand for themselves but are allotted some artificial values based on some logical patterns or analogies. By applying these principles or observing the pattern involved, the candidates are required to decode a coded word or encode a word.

SIMPLE ANALOGICAL LETTER CODING: In this type of coding reasoning certain letters of alphabet stand for certain other letters of alphabet.

SPECIFIC PATTERN LETTER CODING: Candidates are required to observe the specific pattern involved in the question and then proceed with encoding and decoding as the case may be.

LETTER CODING WITH NUMERICAL DIGITS: The numerical values can be assigned to the letters. The values are distributed based on some specific pattern that has to be determined by the candidate in order to solve the problem.

SIMPLE ANALOGICAL LETTER CODING: The letters are assigned numerical values on the basis of analogy. There is no set of principles or pattern involved.

SPECIFIC PATTERN LETTER CODING: This is the pattern of coding that exhibits the general correlation between numbers with alphabetical letters.

Forward Pattern: Alphabets A to Z are assigned the numeric codes from 1 to 26, where each letter gets the assignment in the following pattern that $A = 1$, $B = 2$, $C = 3$, \dots , $Z = 26$.

Backward Pattern: Alphabets A to Z are assigned the numeric codes from 26 to 1, where each letter gets the assignment in the following pattern that $A = 26$, $B = 25$, $C = 24$, \dots , $Z = 1$.

Random Pattern: This pattern can be established in alternative ways, but in every case a particular pattern is involved, which has to be identified by the candidates carefully.

MIXED CODING: Mixed coding takes the pattern of coding with both letters of alphabets and numerical values.

RELATIONSHIP: This type of reasoning concerns the knowledge about human relationships. Usually a set of relations provided in the question and the candidates are asked to determine the relations of others by simply analysing the data provided.

Sub Unit – 3: Mathematical Aptitude

FRACTION: It represents a part of a whole or more generally, any number of equal parts. If a unit is divided into any number of equal parts, then one or more of these parts is termed as a fraction of the unit.

TIME AND DISTANCE:

RATIO, PROPORTION AND PERCENTAGE: Ratio can be represented as fractions. They represent the basic relationship between two quantities. Proportions are in comparison to the whole.

PROFIT AND LOSS: The price at which an article is purchased, is called its cost price (C.P.) and the price at which an article is sold, is called its selling price (S.P.)

INTEREST AND DISCOUNTING:

Principal:

The money borrowed or lent out for certain period is called the principal or the Sum.

Interest:

Extra money paid for using other money is called interest.

Simple Interest:

Simple Interest = Principal x Interest Rate x Term of the loan (Time of Loan)

$SI = P \times i \times n / 100$ when interest rate is taken in percent.

Compound Interest:

$CI = P [(1 + i)^n - 1]$; where P = Principal, i = annual interest rate in percentage terms, and n = number of compounding periods.

CALENDER: Calendars are very familiar in human life since ancient times for finding the days of the week on a particular given date. The Indian calendars are known as ‘**Saka Calendars**’ still used in complex astronomical calculations. Now a days, the most accepted calendars are ‘**Julian Calendars**’ or ‘**Roman Calendars**’.

ODD DAYS: Number of days more than the complete number of weeks in a given period is the number of odd days during that period. They can be calculated by dividing the total number of given days by 7. The remainder left is number of odd days.

ORDINARY YEAR: A year which has 365 days is called an ordinary year.

LEAP YEAR: A year which has 366 days is called leap year. A leap is always divisible by 4 e.g. 1984, 1992, 1996, 2004, 2008, 2012, 2016 etc. is leap years.

ARITHMETIC MEAN: The arithmetic mean (or mean or average) is the most commonly used and readily understood measure of central tendency. In statistics, the term average refers to any of the measures of central tendency. The arithmetic mean is defined as being equal to the sum of the numerical values of each and every observation divided by the total number of observations.

DIRECTION SENSE: There are four main directions: East, West, North and South. There are also four cardinal directions: North-East (NE), North-West (NW), South-East (SE) and South-West (SW).

CLOCK RELATED TEST: The face or dial of a watch is a circle whose circumference is divided into 60 equal parts, called minutes spaces. A clock has two hands, the smaller one is called the hour hand or short hand while the larger one is called the minute hand or long hand.

PERMUTATIONS AND COMBINATIONS:

Factorial: Let us consider a practical example for understanding the importance of the concept of factorial in arrangements.

MENSURATION FORMULAE: Mensuration Formulas for Cube, Mensuration Formulas for Cone, Mensuration Formulas for Cylinder, Mensuration Formulas for Sphere, Mensuration Formulas for Hemisphere, Mensuration Formulas for Hollow Cylinder, Mensuration Formulas for Frustum of a Right Circular Cone, Mensuration Formulas for Prism, Square, Rectangle, Triangle, Parallelogram, Circle

Section – 2: Key Statements

Every candidate appearing for NET/SET examination should follow these key (main) points those can help them a better understanding regarding this unit very quickly.

Key Statements:

Meaning of Reasoning (1), Deductive Reasoning (2.1), Inductive Reasoning (2.2), Abductive Reasoning (2.3), Backward Induction (2.4), Critical Thinking (2.5), Counterfactual Thinking (2.6), Intuition (2.7), Prime Number Series (3), Difference Series (4), Multiplication Series (5), Division Series (6), N^2 Series (7), (N^2-1) Series (8), (N^2+1) Series (9), (N^2+N) Series (10), (N^2-N) Series (11), N^3 Series (12), (N^3+1) Series (13), One Letter Series (14), Combined Two Letter Series (15), Three Letter Series (16), Alphabetical Coding (17): Simple Analogical Letter Coding (17.1), Specific Pattern Letter Coding (17.2), Letter Coding With Numerical Digits (18): Simple Analogical Letter Coding (18.1), Specific Pattern Letter Coding (18.2), Mixed Coding (19), Fraction (20), Time and Distance (21), Ratio, Proportion And Percentage (22), Profit and Loss (23), Interest and Discounting (24): **Simple and Compound Interest (24.3)**, Calender (25), Arithmetic Mean (26), Direction Sense (27), Clock Related Test (28), Permutations and Combinations (29): Factorial (29.1), Mensuration Formulae (30): Cube (30.1), Cone (30.2), Cylinder (30.3), Sphere (30.4), Hemisphere (30.5), Hollow Cylinder (30.6), Frustum of a Right Circular Cone (30.7), Prism (30.8), Square (30.9), Rectangle (30.10), Triangle (30.11), Parallelogram (30.12), Circle (30.13), Quantitative Aptitude Formulas (31), Ages (31.1)

[N.B. – Numbers in parenthesis are the reference number]

Section – 3: Key Facts and Figures

Sub Unit – 1 **Types of Reasoning**

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1. Meaning of Reasoning

Reasoning is the process of thinking about things in a logical or rational way. It is considered an innate human ability that has been formalized by fields such as logic, mathematics and artificial intelligence.

The process of reasoning is used to make decisions, solve problems and evaluate things. It can be formal or informal, top-down or bottom-up and differ in terms of handling of uncertainty and partial truth.

2. Types of Reasoning

The following are a few major types of reasoning:

2.1. Deductive Reasoning:

Deductive reasoning is a formal method of top-down logic that seeks to find observations to prove a theory. It uses formal logic and produces logically certain results.

2.2. Inductive Reasoning:

Inductive reasoning is bottom-up logic that seeks theories to explain observations. It is exploratory in nature and allows for uncertain but likely results.

2.3. Abductive Reasoning:

Like induction, abductive reasoning seeks theories to explain observations. It is less rigorous and allows for best guesses. Abductive reasoning is typically used in the context of uncertainty. It is associated with decision making and troubleshooting.

2.3. Backward Induction:

Backward induction is a top-down approach that starts with theories and works backwards to explain them. It allows for uncertainty and is commonly used in artificial intelligence. For example, it's a classic way for a computer to play chess by considering game end-states and working backwards to evaluate moves.

2.4. Critical Thinking:

Critical thinking is a process of rational thought that seeks to draw conclusions in an objective, thorough and informed manner. It is a product of human thought and is influenced by factors such as culture and language. Human thought is based on natural language that allows for a great range of ideas to be contemplated. For example, humans can easily process partial truth, commonly known as grey areas that tend to be a challenge in the field of logic. Critical thinking can also examine complexities such as emotion. For example, critical thinking can be used to critique a film or book.

2.5. Counterfactual Thinking:

Counterfactual thinking is considering things that are known to be impossible. The most common example of this is evaluating past decisions that were once possible but are now impossible as their time horizon has passed. Considering how past decisions might have worked out is a common human thought process that may improve decision making abilities.

2.6. Intuition:

Intuition is judgements that are made by the mind that are perceived by unconscious. Such judgements exhibit intelligence but the processes by which they are generated aren't well understood. Although intuition is sometimes taken highly, has played a significant role in scientific discovery.

Sub Unit – 2

Number Series, Letter Series, Codes and Relationships

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A. Number Series:

In number series questions, candidates are asked either to insert a missing number or to find out the odd one. By using various methods, a number of series can be formed. It is advisable for the students appearing for NET/SET examination to practice as much as possible.

3. Prime Number Series

In this part, different prime numbers are appeared serially and you are asked to find out the next prime number:

Example 1:

Let a number series be: 7, 11, 13, 17, 19, 21 ...

What is the next number?

Solution:

The given series is a prime number series. The next number is 23.

Example 2:

Let a number series be: 7, 13, 19, 31 ...

What is the next number?

Solution:

The given prime numbers in this range are 7, 11, 13, 17, 19, 23, 31, 37, 41 ...

Here, the prime numbers have been written one after another. Thus, after 31, the prime numbers are 37, 41 ... Ignoring 37, the next number is 41.

4. Difference Series

Let us suppose a number series be: $X_1, X_2, X_3, \dots, X_k$. You have to calculate the difference between $(X_2 - X_1), (X_3 - X_2), \dots, (X_k - X_{k-1})$, and to find out the appropriate answer:

Example 3:

Let a number series be: 3, 6, 9, 12, 15, ..., 21, 24

Choose the appropriate number.

Solution:

The difference between the numbers is 3.

$$(X_2 - X_1) = (6 - 3) = 3$$

$$(X_3 - X_2) = (9 - 6) = 3$$

.

$$(X_k - X_{k-1}) = (24 - 21) = 3$$

Thus, the number will appear after 15 is $(15 + 3)$ equal to 18.

Example 4:

Let a number series be: 24, 20, ..., 12, 8, 4

Solution:

The difference between the consecutive numbers is 4.

$$(X_1 - X_2) = (24 - 20) = 4$$

.

$$(X_{k-2} - X_{k-1}) = (12 - 8) = 4$$

$$(X_k - X_{k-1}) = (8 - 4) = 4$$

Thus, the number will appear after 20 is equal to 16 $(20 - 4)$.

Example 5:

Find out the number which should come at the place of question mark which will complete the following series: 5, 4, 9, 17, 35, ? = 139

Solution:

$$5 + 4 + 9 + 17 + 35 = 70$$

$$\text{Thus, } 139 - 70 = 69$$

Therefore, the missing number is 69

Example 6:

Insert the missing number in the following: $\frac{2}{3}, \frac{4}{7}, ?, \frac{11}{21}, \frac{16}{31}$

Solution:

$$\frac{2}{3} = \frac{2}{2 \times 2 - 1}$$

$$\frac{4}{7} = \frac{4}{4 \times 2 - 1}$$

$$\frac{7}{7 \times 2 - 1} = \frac{7}{13}$$

$$\frac{11}{21} = \frac{11}{11 \times 2 - 1}$$

$$\frac{16}{31} = \frac{16}{16 \times 2 - 1}$$

5. Multiplication Series

Let us suppose a number series is $X_1, X_2, X_3, \dots, X_k$ in such a way that X_2 is equal to X_1 multiplied by a constant value (say, 2, 3, or 4 or whatever it may be), X_3 is equal to X_2 multiplied by that constant value and so on. You have to find out the appropriate answer:

Example 7:

Let a number series be: 3, 6, 12, 24, ..., 96, 192

Choose the appropriate number.

Solution:

The numbers are multiplied by 2 to get the next number.

$$3 \times 2 = 6$$

$$6 \times 2 = 12$$

$$12 \times 2 = 24$$

$$24 \times 2 = 48$$

$$48 \times 2 = 96$$

$$96 \times 2 = 192$$

Thus, the number will appear after 24 is equal to 48 ($24 \times 2 = 48$).

6. Division Series

Let us suppose a number series is $X_1, X_2, X_3, \dots, X_k$ in such a way that X_2 is equal to X_1 divided by a constant value (say, 2, 3, or 4 or whatever it may be), X_3 is equal to X_2 divided by that constant value and so on. You have to find out the appropriate answer:

Example 8:

Let a number series be: 16807, 2401, ..., 49, 7, 1

Choose the appropriate number.

Solution:

The numbers are divided by 7 to get the next number.

$$16807 \div 7 = 2401$$

$$2401 \div 7 = 343$$

$$48 \times 2 = 96$$

$$343 \div 7 = 49$$

$$49 \div 7 = 7$$

$$7 \div 7 = 1$$

Thus, the number will appear after 2401 is equal to 343 ($2401 \div 7 = 343$).

Example 9:

Let a number series be: 720, 120, 24, 6, ..., 1

Choose the appropriate number.

Solution:

$$720 \text{ divided by } 6 = 120$$

$$120 \text{ divided by } 5 = 24$$

$$24 \text{ divided by } 4 = 6$$

$$6 \text{ divided by } 3 = 2$$

$$2 \text{ divided by } 2 = 1$$

Thus, the number will appear after 6 is equal to 2 ($6 \div 3 = 2$).

Example 10:

Let a number series be: 32, 48, 72, ..., 162, 243

Choose the appropriate number.

Solution:

Each number is multiplied by $\frac{3}{2}$ to get the next number

$$32 \times \frac{3}{2} = 48$$

$$48 \times \frac{3}{2} = 72$$

$$72 \times \frac{3}{2} = 108$$

$$108 \times \frac{3}{2} = 162$$

$$162 \times \frac{3}{2} = 243$$

Thus, the number will appear after 72 is equal to 108 ($72 \times \frac{3}{2} = 108$).

7. N^2 Series

Let us suppose a number series is $X_1, X_2, X_3, \dots, X_k$ in such a way that this series is squares of each term of another series $a_1, a_2, a_3, \dots, a_k$ i.e. $X_1 = a_1^2, X_2 = a_2^2, \dots, X_k = a_k^2$. You have to find out the appropriate answer:

Example 11:

Let a number series be: 9, 16, 25, 36, ..., 64, 81

Choose the appropriate number.

Solution:

The series is squares of the series 3, 4, 5, 6 and so on.

$$9 = 3^2$$

$$16 = 4^2$$

$$25 = 5^2$$

$$36 = 6^2$$

$$7^2 = 49$$

$$64 = 8^2$$

$$81 = 9^2$$

Thus, the number will appear after 36 is equal to 49 ($7^2 = 49$).

Example 12:

Let a number series be: 4, 16, 36, 64, ..., 144

Choose the appropriate number.

Solution:

The series is squares of even numbers such as 2, 4, 6, 8, 10 and 12.

$$2^2 = 4$$

$$4^2 = 16$$

$$6^2 = 36$$

$$8^2 = 64$$

$$10^2 = 100$$

$$12^2 = 144$$

Thus, the number will appear after 64 is equal to 100 ($10^2 = 100$).

8. (N^2-1) Series

Let us suppose a number series is $X_1, X_2, X_3, \dots, X_k$ in such a way that $X_1 = (a_1^2 - 1)$, $X_2 = (a_2^2 - 1)$, \dots , $X_k = (a_k^2 - 1)$, You have to find out the appropriate answer:

Example 13:

Let a number series be: 0, 3, 8, 15, 24, 35, 48, 63, ...

Solution:

The series is:

$$1^2 - 1 = 0$$

$$2^2 - 1 = 3$$

$$3^2 - 1 = 8$$

$$4^2 - 1 = 15$$

$$5^2 - 1 = 24$$

$$6^2 - 1 = 35$$

$$7^2 - 1 = 48$$

$$8^2 - 1 = 63$$

$$9^2 - 1 = 80$$

Thus, the number will appear after 63 is equal to 80 ($9^2 - 1 = 80$).

9. (N^2+1) Series

Let us consider a number series is $X_1, X_2, X_3, \dots, X_k$ in such a way that $X_1 = (a_1^2 + 1)$, $X_2 = (a_2^2 + 1)$, \dots , $X_k = (a_k^2 + 1)$, You have to find out the appropriate answer:

Example 14:

Let a number series be: 2, 5, 10, 17, 26, 37, ..., 65, ...

Solution:

The series is:

$$1^2 + 1 = 2$$

$$2^2 + 1 = 5$$

$$3^2 + 1 = 10$$

$$4^2 + 1 = 17$$

$$5^2 + 1 = 26$$

$$6^2 + 1 = 37$$

$$7^2 + 1 = 50$$

$$8^2 + 1 = 65$$

Thus, the number will appear after 37 is equal to 50 ($7^2 + 1 = 50$).

10. $(N^2 + N)$ Series

Let us consider a number series is $X_1, X_2, X_3, \dots, X_k$ in such a way that $X_1 = (a_1^2 + a_1)$, $X_2 = (a_2^2 + a_2)$, \dots , $X_k = (a_k^2 + a_k)$, You have to find out the appropriate answer:

Example 15:

Let a number series be: 2, 6, 12, 20, 30, \dots , 56

Solution:

The series is:

$$1^2 + 1 = 2$$

$$2^2 + 2 = 6$$

$$3^2 + 3 = 12$$

$$4^2 + 4 = 20$$

$$5^2 + 5 = 30$$

$6^2 + 6 = 42$

$$7^2 + 7 = 56$$

Thus, the number will appear after 30 is equal to 42 ($6^2 + 6 = 42$).

11. $(N^2 - N)$ Series

Let us consider a number series is $X_1, X_2, X_3, \dots, X_k$ in such a way that $X_1 = (a_1^2 - a_1)$, $X_2 = (a_2^2 - a_2)$, \dots , $X_k = (a_k^2 - a_k)$, You have to find out the appropriate answer:

Example 16:

Let a number series be: 0, 2, 6, 12, 20, 30, \dots , 56

Solution:

The series is:

$$1^2 - 1 = 0$$

$$2^2 - 2 = 2$$

$$3^2 - 3 = 6$$

$$4^2 - 4 = 12$$

$$5^2 - 5 = 20$$

$$6^2 - 6 = 30$$

$$7^2 - 7 = 42$$

$$8^2 - 8 = 56$$

Thus, the number will appear after 30 is equal to 42 ($7^2 - 7 = 42$).

12. N^3 Series

Let us suppose a number series is $X_1, X_2, X_3, \dots, X_k$ in such a way that this series is cubic of each term of another series $a_1, a_2, a_3, \dots, a_k$ i.e. $X_1 = a_1^3, X_2 = a_2^3, \dots, X_k = a_k^3$. You have to find out the appropriate answer:

Example 17:

Let a number series be: 8, 27, 64, 125, 216 ...

Choose the appropriate number.

Solution:

The series is cubic of the series 2, 3, 4, 5, 6 and so on.

$$2^3 = 8$$

$$3^3 = 27$$

$$4^3 = 64$$

$$5^3 = 125$$

$$6^3 = 216$$

$$7^3 = 343$$

Thus, the number will appear after 216 is equal to 343 ($7^3 = 343$).

13. $(N^3 + 1)$ Series

Let us consider a number series is $X_1, X_2, X_3, \dots, X_k$ in such a way that $X_1 = (a_1^3 + a_1), X_2 = (a_2^3 + a_2), \dots, X_k = (a_k^3 + a_k)$, You have to find out the appropriate answer:

Example 18:

Let a number series be: 2, 9, 28, 65, 126, ... , 344

Solution:

The series is:

$$1^3 + 1 = 2$$

$$2^3 + 1 = 9$$

$$3^3 + 1 = 28$$

$$4^3 + 1 = 65$$

$$5^3 + 1 = 126$$

$$6^3 + 1 = 217$$

$$7^3 + 1 = 344$$

Thus, the number will appear after 126 is equal to 217 ($6^3 + 1 = 217$).

B. Letter Series

Under this type of series completion some letters are given which follow a particular sequence or order. Examine the directions and the questions with care and try to detect the pattern that the order or sequence is following. This will help you find the next letter or missing letter to continue the series.

To solve this type of problems, allocate numbers 1 to 26 to the English letters as given below:

A	B	C	D	E	F	G	H	I	J
1	2	3	4	5	6	7	8	9	10
K	L	M	N	O	P	Q	R	S	T
11	12	13	14	15	16	17	18	19	20
U	V	W	X	Y	Z				
21	22	23	24	25	26				

Suppose you have to find out the total value of a word ‘**LETTER**’

The candidates can determine the relative positions of various alphabets.

L	E	T	T	E	R
12	5	20	20	5	18

Total value of the word ‘**LETTER**’ is $12 + 5 + 20 + 20 + 5 + 18 = 80$

14. One Letter Series

In this part you have to find out the next letter or missing letter to continue the series.

Example 19:

Let a letter series be: A, C, E, G, ... , K

Choose the appropriate alphabet.

Solution:

The series is

$$A + 2 = C$$

$$C + 2 = E$$

$$E + 2 = G$$

$$G + 2 = I$$

$$I + 2 = K$$

Therefore, the alphabet will appear after G is I ($G + 2 = I$).

Example 20:

Let a letter series be: A, B, D, G, ...

Choose the appropriate alphabet.

Solution:

Examining the series you observe the letters A, B, D, G have a certain gap between each of them. As such there is no gap between A and B, there is a gap of one letter 'C' between B and D, this gap increases as the series progress – hence D and G has a gap of two letters E and F and so on. Should be of three letters viz. H, I, J and as logic says the next letter or missing letter must be 'K' The series is

$$B - A = 0$$

$$D - B = 1$$

$$G - D = 2$$

$$K - G = 3$$

Therefore, the alphabet will appear after G is K ($G + 3 = K$).

15. Combined Two Letter Series

In this, the first letters of the series follow one logic and the second letters of the series follow another logic, and then they pair with each other.

Example 21:

Let a letter series be: AR, BQ, CP, ... , EN

Write the appropriate pair of alphabet.

Solution:

The first letters of this series are A, B, C, D, and E.

The second letters of the series are in reverse direction i.e. R, Q, P, O, and N.

The missing pair is 'DO'

Example 22:

Let a letter series be: AB, DE, GH, ... , MN

Write the appropriate pair of alphabet.

Solution:

After every pair, one letter is skipped.

One letter 'C' is skipped from first two pairs 'AB & DE', one letter 'F' is skipped from second two pairs 'DE & GH'.

The letter 'I' should be skipped from third two pairs 'GH & JK'.

Thus the series is AB, DE, GH, JK MN

Example 23:

Let a letter series be: AA, CE, EI, GO, ...

Write the appropriate pair of alphabet.

Solution:

The first letters of all pairs in the question follow a sequence of $A + 2 = C$, $C + 2 = E$, $E + 2 = G$, $G + 2 = I$ and so on.

The second letters are vowels.

Thus, the pair will appear after GO is 'IU'.

16. Three Letter Series

This sequence consists of three letters in each term. The first letters of each pair of the series follow one logic, the second letters follow another logic, and the third letters follow some other logic or the same logic in all three cases.

Example 24:

Let a letter series be: MNP, OPR, ..., STV, UVX

Write the appropriate pair of alphabet.

Solution:

The first letter of each triplet follows a sequence of M, O, Q, S, U, and so on. The second letter of each triplet follows a sequence of N, P, R, T, V, and so on. The third letters forms a sequence of P, R, T, V, X, and so on.

Thus the correct pair will appear after OPR is QRT

Example 25:

Let a letter series be: DLZ, EMY, ..., GOW, HPV

Write the appropriate pair of alphabet.

Solution:

The first letter of each triplet follows a sequence of D, E, F, G, H, and so on. The second letter of each triplet follows a sequence of L, M, N, O, P, and so on. The third letters forms a sequence of Z, Y, X, W, V, and so on.

Thus the correct pair will appear after OPR is ENX

Example 26:

Let a letter series be: ABC, CBA, DEF, ..., GHI, IHG

Write the appropriate pair of alphabet.

Solution:

The second term is the reverse order of the first term; the fourth term is the reverse of the third term; the sixth term is also the reverse of the fifth term, and so on.

CBA = reverse of ABC

FID = reverse of DEF

IHG = reverse of GHI

And so on.

Thus the correct pair will appear after DEF is FID

C. Codes

In this type of reasoning secret messages are given in codes and they have to deciphered or decoded in order to understand. We can say that the letters of alphabets or the numbers do not stand for themselves but are used for some other letters or numbers or signs. These codes which are artificial in nature are given according to set principles and rules. These rules, principles or patterns are to be worked out by the students, so as to enable them to decode the messages or signs.

17. Alphabetical Coding

In this type of questions, the letters of the alphabet are exclusively used. These letters do not stand for themselves but are allotted some artificial values based on some logical patterns or analogies. By applying these principles or observing the pattern involved, the candidates are required to decode a coded word or encode a word.

17.1. Simple Analogical Letter Coding:

In this type of coding reasoning certain letters of alphabet stand for certain other letters of alphabet.

Example 27:

If 'BOY' is coded as 'CPZ' then what could be the code of 'MAN'?

Solution:

Assessing the pattern used in the coding 'BOY', we can observe that **C** is coded for **B**, **P** is coded for **O**, **Z** is coded for **Y** i.e. each letter is replaced by its following letter. Applying the same principle or pattern to 'MAN', M will be replaced by **N**, A with **B** and N with **O** – thus code for 'MAN' could be **NBO**

Example 28:

If 'MANUFACTURE' is coded as 'DGHVOGPXVJL', then how will you code 'NUACT'?

Solution:

The coding is done as follows:

Letters:	M	A	N	U	F	A	C	T	U	R	E
Codes:	D	G	H	V	O	G	P	X	V	J	L

Thus, the code for 'NUACT' is **HVG PX**

17.2. Specific Pattern Letter Coding:

At first the candidates are required to observe the specific pattern involved in the question and then proceed with encoding and decoding as the case may be.

Example 29:

If 'CLOUD' is coded as 'DUOLC', then how will you code 'PRIME'?

Solution:

After careful observation, we see that the letters have been written in reverse order.

Letters	C	L	O	U	D
Codes	D	U	O	L	C

Hence, 'PRIME' will be written as **EMIRP**

Example 30:

If 'SUPER' is coded as 'RTODQ', then what is the code of 'POWER'?

Solution:

Here, each letters of the word 'SUPER' is coded as just the preceding letters.

Letters	S	U	P	E	R
Codes	S-1 R	U-1 T	P-1 O	E-1 D	R-1 Q

Hence, 'POWER' will be written as **ONVDQ**

Letters	P	O	W	E	R
Codes	P-1 O	O-1 N	W-1 V	E-1 D	R-1 Q

18. Letter Coding with Numerical Digits

The numerical values can be assigned to the letters. The values are distributed based on some specific pattern that has to be determined by the candidate in order to solve the problem.

Example 31:

If 'AGAIN' is coded as 47589, how will you code 'GAIN'?

Solution:

Letters	A	G	A	I	N
Codes	4	7	5	8	9

These values have been assigned arbitrarily. The problem can be solved on the basis of the relationship established. For 'GAIN', the code is **7589**

18.1. Simple Analogical Letter Coding:

The letters are assigned numerical values on the basis of analogy. There is no set of principles or pattern involved.

Example 32:

If 'KINGDOM' is coded as '5786321', how will you code 'KING'?

Solution:

Letters	K	I	N	G	D	O	M
Codes	5	7	8	6	3	2	1

On the basis of analogical relationship established between the letters and the numbers, we can code 'KING' as 5786

18.2. Specific Pattern Letter Coding:

This is the pattern of coding that exhibits the general correlation between numbers with alphabetical letters. Alphabets A to Z are assigned the numeric codes from 1 to 26, where each letter gets the assignment in the following pattern that A = 1, B = 2, C = 3, ... , Z = 26.

Example 33: Forward Pattern:

If 'RIGHT' is coded as 18-9-7-8-20, how will you code 'MIGHT'?

Solution:

Letters	A	B	C	D	E	F	G	H	I
Codes	1	2	3	4	5	6	7	8	9
Letters	J	K	L	M	N	O	P	Q	R
Codes	10	11	12	13	14	15	16	17	18
Letters	S	T	U	V	W	X	Y	Z	
Codes	19	20	21	22	23	24	25	26	

RIGHT: 18-9-7-8-20

R = 18, I = 9, G = 7, H = 8, T = 20

Thus, 'MIGHT' will be coded as 13-9-7-8-20

M = 13, I = 9, G = 7, H = 8, T = 20

Example 34: Backward Pattern:

In this case, alphabets A to Z are assigned the numeric codes from 26 to 1, where each letter gets the assignment in the following pattern that A = 26, B = 25, C = 24, ... , Z = 1.

Letters	A	B	C	D	E	F	G	H	I
Codes	26	25	24	23	22	21	20	19	18
Letters	J	K	L	M	N	O	P	Q	R
Codes	17	16	15	14	13	12	11	10	9

Letters	S	T	U	V	W	X	Y	Z
Codes	8	7	6	5	4	3	2	1

If ‘SUN’ is coded as 8-6-13, how will you code ‘RUN’?

Solution:

SUN: 8-6-13

S = 8, U = 6, N = 13

Thus, ‘RUN’ will be coded as 9-6-13

R = 9, U = 6, N = 13

Example 35: Random Pattern:

This pattern can be established in alternative ways, but in every case a particular pattern is involved, which has to be identified by the candidates carefully.

Let us suppose a pattern is as follows: If A = 5, B = 7, C = 9, ..., Z = 30, or an another pattern, let, A = 4, B = 5, C = 6, ... Z = 29.

Example 36:

If ‘JAPAN’ is coded as 14-5-20-5-18, how will you code ‘ENGLAND’?

Solution:

Letters	A	B	C	D	E	F	G	H	I
Codes	5	6	7	8	9	10	11	12	13
Letters	J	K	L	M	N	O	P	Q	R
Codes	14	15	16	17	18	19	20	21	22
Letters	S	T	U	V	W	X	Y	Z	
Codes	23	24	25	26	27	28	29	30	

JAPAN = 14-5-20-5-18

J = 14, A = 5, P = 20, A = 5, N = 18

Thus, **ENGLAND** will be coded as, 9-18-11-16-5-18-8

E = 9, N = 18, G = 11, L = 16, A = 5, N = 18, D = 8

Example 37: Special Pattern:

In a certain code, ‘PAN’ is written as 31 and ‘PAR’ as 35, how will you code ‘PAT’?

Solution:

Letters	A	B	C	D	E	F	G	H	I
Codes	1	2	3	4	5	6	7	8	9
Letters	J	K	L	M	N	O	P	Q	R
Codes	10	11	12	13	14	15	16	17	18
Letters	S	T	U	V	W	X	Y	Z	

Codes	19	20	21	22	23	24	25	26
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PAN is coded as 31.

$P = 16, A = 1, N = 14.$

$PAN = 16 + 1 + 14 = 31.$

Similarly, **PAR** is coded as 35.

$PAR = 16 + 1 + 18 = 35$

Thus **PAT** will be coded as 37

$PAT = 16 + 1 + 20 = 37$

19. Mixed Coding

Mixed coding takes the pattern of coding with both letters of alphabets and numerical values.

Example 38:

If 'A3T15R' is coded as 'ACTOR' and 'D1T5' is coded as 'DATE', how will you code 'ROTATE'?

Solution:

While coding for **ACTOR**, first, third and fifth letters have been put directly. C and O are placed at 3rd and 15th position in the alphabetical series. Hence, **ACTOR** is coded as **A3T15R**. Similarly **DATE** is coded as **D1T5**.

Thus, 'ROTATE' will be coded as R15T1T5

D. Relationship

This type of reasoning concerns the knowledge about human relationships. Usually a set of relations provided in the question and the candidates are asked to determine the relations of others by simply analyzing the data provided.

Before starting the answer of questions on human relationships it is better to keep in mind various sets of relations which are given as under for your interest:

Blood Relations:

Relations of Paternal Side	
Father's father	Grand father
Father's mother	Grand mother
Father's brother	Uncle
Grandfather's Son	Father or Uncle
Grandfather's only Son	Father
Father's sister	Aunt
Children of uncle	Cousin

Wife of uncle	Aunt
Children of aunt	Cousin
Husband of aunt	Uncle
Sister's husband	Brother-in-law
Wife's brother	Brother-in-law
Brother's son	Nephew
Brother's wife	Sister-in-law
Brother's daughter	Niece
Grandson's or Granddaughter's daughter	Great granddaughter
Mother's or Father's Son/Daughter	Brother/Sister
Son's wife	Daughter-in-law
Relations of Maternal Side	
Mother's Father	Maternal Grandfather
Mother's Mother	Maternal Grandmother
Mother's Brother	Maternal Uncle
Mother's Sister	Aunt
Children of Maternal Uncle	Cousin
Wife of Maternal Uncle	Maternal Aunt
Others	
Children of same Parents	Siblings
Common term for Husband and Wife	Spouse

While analyzing the data given in the question you can use the following signs and symbols:

- (i) When person 'A' is male: (+A)
- (ii) When person 'B' is female: (-B)
- (iii) M and N are siblings: (M ↔ N)
- (iv) Upward relation: (↑) [e.g. Father, Mother, Uncle, Grandparents etc.]
- (v) Downward relation: (↓) [e.g. Son, Daughter, Brother's son or daughter, Sister's son or daughter, Grand children etc.]
- (vi) Parallel relation: (→) [e.g. Wife, Husband, Brother, Sister etc.]

Example 39:

Pointing towards a woman, a man said, 'Her mother is the only daughter of my mother'. How is the man related to the woman?

Solution:

The woman's mother is sister of the man. So, the man should be **Maternal Uncle** of the woman.

Example 40:

‘A’ and ‘B’ are brothers. ‘P’ and ‘Q’ are sisters. A’s son is Q’s brother. How is B related to P?

Solution:

B is the brother of A; A’s son is Q’s brother. This implies that Q is the daughter of A. As P and Q are sisters, P is also the daughter of A. Hence B is the **Uncle** of P.

Example 41:

A told B that C is A’s father’s nephew. D is A’s cousin but not the brother of C. How is D related to C?

Solution:

D and C are children of A’s father’s brother or sister. Since D is not brother of C, then D must be **Sister** of C because both are cousin of A.

Sub Unit – 3

Mathematical Aptitude

.....

20. Fraction

It represents a part of a whole or more generally, any number of equal parts. If a unit is divided into any number of equal parts, then one or more of these parts is termed as a fraction of the unit.

Example 42:

Solve this problem:

$$5\frac{1}{5} + 4\frac{1}{2} + 4\frac{1}{3} = ?$$

Solution:

$$\begin{aligned} 5\frac{1}{5} + 4\frac{1}{2} + 4\frac{1}{3} &= 5 + 4 + 4 + \frac{1}{5} + \frac{1}{2} + \frac{1}{3} = 13 + \frac{31}{30} \\ &= 13 + 1\frac{1}{30} = 13 + 1 + \frac{1}{30} = 14 + \frac{1}{30} = 14\frac{1}{30} \end{aligned}$$

Example 43:

In a class $\frac{3}{4}$ th of the students do not know either English or Hindi. But $\frac{1}{6}$ th of the students know English. How much students know both English and Hindi if students who know Hindi are $\frac{1}{8}$ th of total students in the class?

Solution:

$\frac{3}{4}$ th of the students do not know English or Hindi. So, $\frac{1}{4}$ th students know English or Hindi or both. $\frac{1}{6}$ th know English and $\frac{1}{8}$ th know Hindi.

$$\text{Now, } \frac{1}{4} - \frac{1}{6} - \frac{1}{8} = -\frac{1}{24}$$

Thus, the answer is $\frac{1}{24}$

[**Note:** We get negative value because of double counting $\frac{1}{24}$ i.e. students knowing both languages. Students knowing English includes the students who know both languages. Also, students knowing Hindi too includes the students who know both languages.]

21. Time and Distance

21.1. Formulae:

- (i) Distance = (Speed \times Time)
- (ii) Speed = $(\frac{\text{Distance}}{\text{Time}})$
- (iii) Time = $(\frac{\text{Distance}}{\text{Speed}})$
- (iv) 1 km / hour = $\frac{5}{18}$ m/sec
- (v) 1 m/sec = $\frac{18}{5}$ km / hour
- (vi) If the ratio of the speeds of A and B is a : b, then the ratio of the times taken by them to cover the same distance is $\frac{1}{a} : \frac{1}{b} = b : a$
- (vii) Suppose a man covers a certain distance at x km / hr. and an equal distance at y km / hr. Then the average speed during the whole journey is $(\frac{2xy}{x+y})$ km / hr.
- (viii) If a man travels for 't' minutes by car at x km/hr. and 't' minutes by train at y km/hr, the average speed of the whole distance is $(\frac{x+y}{2})$ km / hr.

Example 44:

A man covers 60 km in 4 hrs. Find the speed in m/s.

Solution:

Speed = Distance / Time = $60/4 = 15$ km/hr.

The km/hr. can be converted into m/s by multiplying by 5/18.

Thus, Speed = $15 \times \frac{5}{18}$ m/sec = $4\frac{3}{18}$ m/sec

Example 45:

A car completes a journey in 6 hours, the first half at speed of 50 km/hr. and the second half at 60 km/hr. Find the total distance covered.

Solution:

As the total journey is divided into equal parts, the average speed can be calculated by the formula $\frac{2xy}{x+y} = (2 \times 50 \times 60) / (50 + 60) = 54.54545$ km/hr.

Thus, total distance covered by the car is equal to 54.54545×6 km. = 327.2727 km.

Example 46:

A student walks from his house at a speed of 3 km/hr. and reaches at the school 10 minutes late. If he walks at a speed of 4 km/hr. he reaches the school 10 minutes earlier. What is the distance between his school and his house?

Solution:

Let the distance = x km.

Difference between timings of reaching the school at different speeds = 10 + 10 = 20 minutes
or $20 / 60 = 1/3$ hrs.

Now the difference between timings =

$$(x/3 - x/4) = 1/3$$

$$\text{Or, } \frac{4x-3x}{12} = \frac{1}{3}$$

$$\text{Or, } \frac{x}{12} = \frac{1}{3}$$

$$\text{Or, } \frac{x}{4} = 1$$

$$\text{Or, } x = 4 \text{ km}$$

22. Ratio, Proportion and Percentage

Ratio can be represented as fractions. They represent the basic relationship between two quantities. Proportions are in comparison to the whole.

In a matrix of 40 litre of milk and 60 litre of water, the ratio of milk and water is 4:6 or 2:3.

This can be converted to fraction of milk in the solution as $2/5$ or $2/5$ th.

As seen, $2/5$ is nothing but $2/5 \times 100 = 40\%$.

Example 47:

A and B together have Rs. 1210. If $4/5$ of A's amount is equal to $2/5$ of B's amount, how much amount does B have?

Solution:

$$\frac{4}{5} A = \frac{2}{5} B$$

$$\text{Or, } A = \left(\frac{2}{5} \times \frac{15}{4}\right) B$$

$$\text{Or, } A = \frac{3}{2} B$$

$$\text{Or, } \frac{A}{B} = \frac{3}{2}$$

$$\text{Or, } A : B = 3 : 2$$

Or, B's share = Rs. $(1210 \times \frac{2}{5})$ = Rs. 484

Example 48:

A sum of money is to be distributed among A, B, C, D in the proportion of 5: 2 : 4 : 3. If C gets Rs. 1000 more than D, what is B's share?

Solution:

Let the shares of A, B, C and D be Rs. 5x, Rs. 2x, Rs. 4x and Rs. 3x respectively.

Then, $4x - 3x = 1000$

Or, $x = 1000$

B's share = Rs. 2x = Rs. (2 x 1000) = Rs. 2000

Example 49:

Seats for Mathematics, Physics and Biology in a school are in the ratio 5 : 7 : 8. There is a proposal to increase these seats by 40%, 50% and 75% respectively. What will be the ratio of increased seats?

Solution:

Originally, let the number of seats for Mathematics, Physics and Biology be 5x, 7x and 8x respectively.

Number of increased seats are (140% of 5x), (150% of 7x) and (175% of 8x).

Or, $(\frac{140}{100} \times 5x)$, $(\frac{150}{100} \times 7x)$ and $(\frac{175}{100} \times 8x)$

Or, 7x, $21x/2$ and 14x

Thus the required ratio = $7x : \frac{21x}{2} : 14x$

Or, 14x: 21x: 28x

Or, 2: 3: 4

23. Profit and Loss

- (i) The price at which an article is purchased, is called its cost price (C.P.)
- (ii) The price at which an article is sold, is called its selling price (S.P.)

23.1. Some Important Formulae:

- (i) $\text{Gain} = (\text{S.P.}) - (\text{C.P.})$
- (ii) $\text{Gain \%} = (\frac{\text{Gain}}{\text{C.P.}}) \times 100$
- (iii) $\text{Loss} = (\text{C.P.}) - (\text{S.P.})$
- (iv) $\text{Loss \%} = (\frac{\text{Loss}}{\text{C.P.}}) \times 100$

- (v) $S.P. = \frac{(100 + \text{Gain \%})}{100} \times C.P.$
- (vi) $S.P. = \frac{(100 - \text{Loss \%})}{100} \times C.P.$
- (vii) $C.P. = \frac{100}{(100 + \text{Gain \%})} \times S.P.$
- (viii) $C.P. = \frac{100}{(100 - \text{Loss \%})} \times S.P.$
- (ix) If an article is sold at a gain of 45%, then S.P. = 145% of C.P.
- (x) If an article is sold at a loss of 45%, then S.P. = 55% of C.P.
- (xi) If x is the gain percent on C.P., then the gain percent on the S.P. = $\frac{x}{(100 + x)} \times 100\%$
- (xii) If x is the profit percent on S.P., then the profit percent on the C.P. = $\frac{x}{(100 - x)} \times 100\%$
- (xiii) If x is the loss percent on S.P., then the loss on the C.P. = $\frac{x}{(100 + x)} \times 100\%$
- (xiv) $\text{Cost price} = \frac{\text{More profit}}{\text{Difference in percent profit}} \times 100$

Example 50:

A dealer allows a discount of 15% on the marked price. How much above cost price must he mark his goods to gain 10%?

Solution:

Let, C.P. = Rs. 100

Then, S.P. = Rs. 110

Also let, marked price be Rs. X

Thus, 85% of x = 110

$$\text{Or, } x = (110 \times \frac{100}{85})$$

$$\text{Or, } x = 129.41$$

Mark up = $129.41 - 100 = 29.41$ i.e. marked price should be 29.41% more than cost price.

Example 51:

A reduction of 25% in the price of oil enables a purchaser to obtain 4 kg more for Rs. 150. Find the original rate and the reduced price per kg.

Solution:

Let original rate = Rs. X per kg

$$\text{New rate} = 75\% \text{ of } x = \text{Rs. } (\frac{75}{100} x) = \text{Rs. } \frac{3x}{4}$$

$$\text{Original quantity for Rs. 150} = \frac{150}{x}$$

$$\text{New quantity} = (150 \times \frac{4}{3x}) = \frac{600}{3x} = \frac{200}{x}$$

$$\text{Thus, } \frac{200}{x} - \frac{150}{x} = 4$$

$$\text{Or, } \frac{50}{x} = 4$$

$$\text{Or, } 4x = 50$$

$$\text{Or, } x = 12.5$$

Thus, original rate = Rs. 12.5 per kg.

Reduced rate = 75% of Rs. 12.5 = 9.38 per kg.

24. Interest and Discounting

24.1. Important Concepts:

Suppose a man has to pay Rs. 156 after 4 years and the rate of interest is 14% per annum. Clearly, Rs. 100 at 14% will amount to Rs. 156 in 4 years. So, the payment of Rs. now will clear off the debt of Rs. 156 due 4 years hence. We say that:

Sum Due = Rs. 156 due 4 years hence;

Present Worth (P.W.) = Rs. 100;

True Discount (T.D.) = Rs. (156 - 100) = Rs. 56 = (Sum due) - (P.W.)

We define: T.D. = Interest on P.W.; Amount = (P.W.) + (T.D.)

Interest is reckoned on P.W. and true discount is reckoned on the amount.

24.2. Important Formulae:

Let rate = R% per annum and Time = T years. Then,

$$1. \text{ P.W.} = \frac{100 \times \text{Amount}}{100 + (R \times T)} = \frac{100 \times \text{T.D.}}{R \times T}$$

$$2. \text{ T.D.} = \frac{(\text{P.W.}) \times R \times T}{100} = \frac{\text{Amount} \times R \times T}{100 + (R \times T)}$$

$$3. \text{ Sum} = \frac{(\text{S.I.}) \times (\text{T.D.})}{(\text{S.I.}) - (\text{T.D.})}$$

$$4. (\text{S.I.}) - (\text{T.D.}) = \text{S.I. on T.D.}$$

When the sum is put at compound interest, then $\text{P.W.} = \frac{\text{Amount}}{(1 + \frac{R}{100})^T}$

24.3. Simple and Compound Interest:

Principal:

The money borrowed or lent out for certain period is called the principal or the Sum.

Interest:

Extra money paid for using other money is called interest

The cost of borrowing money is defined as Simple Interest. It is of two types – simple interest or compound interest. Simple interest(SI) is calculated only on the principal (P) whereas Compound interest(CI) is calculated on the principal and also on the accumulated interest of previous periods i.e. “interest on interest.” This compounding effect makes a big difference in the amount of interest payable on the principal.

Simple Interest:

Simple Interest = Principal x Interest Rate x Term of the loan (Time of Loan)

$SI = P \times i \times n/100$ when interest rate is taken in percent.

Compound Interest:

$CI = P [(1 + i)^n - 1]$; where P = Principal, i = annual interest rate in percentage terms, and n = number of compounding periods.

Compounding periods:

When calculating compound interest, the number of compounding periods makes a significant difference. The basic rule is that the higher the number of compounding periods, the greater the amount of compound interest. So for every INR 100 principal over a certain period of time, the amount of interest accrued at 10% annually will be lower than interest accrued at 5% semi-annually, which will in turn be lower than interest accrued at 2.5% quarterly.

25. Calendar

Calendars are very familiar in human life since ancient times for finding the days of the week on a particular given date. The Indian calendars are known as ‘**Saka Calendars**’ still used in complex astronomical calculations. Now a days, the most accepted calendars are ‘**Julian Calendars**’ or ‘**Roman Calendars**’.

25.1. Odd Days:

Number of days more than the complete number of weeks in a given period is the number of odd days during that period. They can be calculated by dividing the total number of given days by 7. The remainder left is number of odd days.

25.2. Ordinary Year:

A year which has 365 days is called an ordinary year.

25.3. Leap Year:

A year which has 366 days is called leap year. A leap is always divisible by 4 e.g. 1984, 1992, 1996, 2004, 2008, 2012, 2016 etc. is leap years.

25.4. Some Important Points:

- (i) Numbers of odd days play an important role in finding the particular day of a particular year.
- (ii) Every 4th century is a leap year but no other century is a leap year, e.g. 400, 800, 1200, 2000 etc. is a leap year but 1100, 2100, 2700 etc. is not a leap year.
- (iii) For making calculation easy the number of days in a year is given by

Months	No of days in ordinary year	No of days in Leap year
1 st Jan. to 31 st March	90	91
1 st April to 30 th June	91	91
1 st July to 30 th Sep	92	92
1 st Oct to 30 th Dec	92	92
1 st Jan. to 31 st Dec	365	366

- (iv) In a leap year February has 29 days otherwise in ordinary year it has 28 days.

- (v) Odd days can be counted as follows:

(a). 1 ordinary year = 365 days

= (52 weeks + 1 day)

Thus, an ordinary year has 1 odd day.

(b). 1 leap year = 366 days

= (52 weeks + 2 days)

Thus, an ordinary year has 2 odd days.

(c). 100 years have 5 odd days.

100 years = 76 ordinary year + 24 leap year

$[(76 \times 52) \text{ week} + 76 \text{ days}] + [(24 \times 52) \text{ week} + 48 \text{ days}]$

= 3952 weeks + 76 days + 1248 weeks + 48 days

= 5200 weeks + 124 days

= 5200 weeks + (119+5) days

= 5200 weeks + 17 weeks + 5 days

Thus every 100 years have 5 odd days.

Similarly, 200 years have 3 odd days.

300 years have 1 odd day and 400, 800, 1200, 1600 etc. have zero odd day.

(vi) The first day of the century must be a Monday, Tuesday, Thursday or Saturday.

(vii) The last day of the century must be Sunday, Monday, Wednesday or Friday.

(viii) We must count days from Sunday as 1st Jan. 1 A.D. was Monday, i.e. Sunday for 0 odd days, Monday for 1 odd day, Tuesday for 2 odd days and so on.

Example 52:

Today is Sunday. What will be the day after 50 days?

Solution:

Today is Sunday. Each day of the week is repeated after 7 days.

After 49 days, it will be Sunday.

Thus, after 50 days, it will be Monday

Example 53:

If 9th December, 2001 happens to be Sunday, then what will be the day on 9th December, 2005?

Solution:

Number of years from 9/12/2001 to 9/12/2005 = (1 leap year + 3 ordinary year)

= (2 + 3) odd days

= 5 odd days

5 odd days after Sunday, i.e. Friday

Thus it will be Friday on 9/ 12/2005.

Example 54:

What will be the day of the week on 29th Nov. 2009?

Solution:

29th Nov. 2009

2000 year have 0 odd day

8 years = 2 leap years + 6 ordinary years

= 4 odd days + 6 odd days

= 10 odd days

= 3 odd days

Number of days from 1st Jan. 2009 to 29th Nov. 2009

= (91 + 91 + 92 + 60) days

= 334 days = 47 weeks + 5 odd days

Total number of odd days = (0 + 3 + 5) odd days = 1 odd day.

Thus it will be Monday on 29th Nov. 2009.

26. Arithmetic Mean

The arithmetic mean (or mean or average) is the most commonly used and readily understood measure of central tendency. In statistics, the term average refers to any of the measures of central tendency. The arithmetic mean is defined as being equal to the sum of the numerical values of each and every observation divided by the total number of observations.

26.1. Formula:

Simple Arithmetic Mean: Symbolically, if we have a data set containing the values X_1, X_2, \dots, X_n , the arithmetic mean 'AM' is defined by the formula,

$$AM = \frac{1}{n} \sum_{i=1}^n X_i$$

Weighted Arithmetic Mean: If we have a data set containing the values X_1, X_2, \dots, X_n have frequencies f_1, f_2, \dots, f_n then, $AM = \frac{1}{N} \sum_{i=1}^n f_i X_i$; where, $N = \sum_{i=1}^n f_i$

Composite Mean: If two groups contain n_1 and n_2 observations with means \bar{x}_1 and \bar{x}_2 respectively, then the mean (\bar{x}) of the composite group ($n_1 + n_2$) observations is given by the relation, $N\bar{x} = n_1 \bar{x}_1 + n_2 \bar{x}_2$; where, $N = n_1 + n_2$

Characteristics	Groups		Composite Group
	I	II	

No. of observations	n_1	n_2	N
Mean	\bar{x}_1	\bar{x}_2	\bar{x}

Example 55: Calculation of Simple Arithmetic Mean:

Let us consider the monthly salary of 10 employees of a firm: 2500, 2700, 2400, 2300, 2550, 2650, 2750, 2450, 2600, 2400

The Simple Arithmetic Mean (\bar{x}) is

$$\frac{2500+2700+2400+2300+2550+2650+2750+2450+2600+2400}{10} = 2530$$

Example 56: Calculation of Weighted Arithmetic Mean:

Calculation for Weighted Arithmetic Mean

Price (Rs) per table	Number of table sold(f)	fx
36	14	504
40	11	440
44	9	396
48	6	288
Total	40	1628

$$\text{Weighted Arithmetic Mean } (\bar{x}) = \frac{\sum fx}{N} = 1628/40 = 40.70$$

Example 57:

Calculate the Arithmetic Mean from the following frequency distribution of earners by monthly income:

Income (Rs.):	Below 200	200-399	400-599	600-799	800-999	1000-1199
No. of earners:	25	72	47	22	13	7

Calculation for Weighted Arithmetic mean

Class Interval	Frequency	Mid Value (x)	$y = \frac{x-499.5}{200}$	f.y
0 – 199	25	99.5	-2	-50
200 – 399	72	299.5	-1	-72
400 – 599	47	499.5	0	0
600 – 799	22	699.5	1	22
800 – 999	13	899.5	2	26
1000 – 1199	7	1099.5	3	21
Total	186			-53

If $y = (x - c) / d$, then $\bar{x} = c + d\bar{y}$

Here, $c = 499.5$ and $d = 200$

Thus, $\bar{x} = 499.5 + 200 \left(\frac{-53}{186} \right) = 499.5 - 56.99 = 442.51 \text{Rs.}$

Example 58: Calculation of Composite Mean:

There are two branches of an establishment employing 100 and 80 persons respectively. If the arithmetic means of the monthly salaries paid by the two branches are Rs. 275 and Rs. 225 respectively, find the A.M. of the salaries of the employees of the establishment as a whole.

Mean of Composite Group

Characteristics	Groups		Composite Group
	I	II	
No. of observations	$n_1 = 100$	$n_2 = 80$	$N = 180$
Mean salary (Rs.)	$\bar{x}_1 = 275$ 225	$\bar{x}_2 =$	$\bar{x} = ?$

$$N\bar{x} = n_1 \bar{x}_1 + n_2 \bar{x}_2$$

$$\text{Or, } 180\bar{x} = 100 * 275 + 80 * 225 = 45,500$$

$$\text{Or, } \bar{x} = 45500 / 180 = 252.78 \text{ Rs.}$$

27. Direction Sense

- (i) There are four main directions: East, West, North and South
- (ii) There are four cardinal directions: North-East (NE), North-West (NW), South-East (SE) and South-West (SW).

27.1. Some Important Points:

- (i) At the time of sunrise, if a man stands facing the east, his shadow will be towards west.
- (ii) At the time of sunset, the shadow the shadow of an object is always in the east.
- (iii) If a man stands facing north, at the time of sunrise his shadow will towards his left, and at the time of sunset, it will be towards his right.
- (iv) At 12.00 noon, the rays of the sun are vertically downward; hence, there will be no shadow.

Example 59:

Rampur is on the west of Kasauli. Bharatpur is on the east of Rampur. Lakhanpur is on the west of Kasauli. Bharatpur is on the east of Chandpur. If Lakhanpur is on the west of Rampur, in which direction from Bharatpur is Lakhanpur?

Solution:

The position of the various towns is as follows:

Lakhanpur ↔ Rampur ↔ Kasauli ↔ Chandpur ↔ Bharatpur

Lakhanpur is to the west of Bharatpur

Thus the answer is West

Example 60:

Ramesh starts walking towards South. After walking 15 metres he turns towards North. After walking 20 metres, he turns towards East and walk 10 metres. He then turns towards South and walks 5 metres. How far is he from his original position and in which direction?

Solution:

Suppose Ramesh starts from P walks 15m towards South and reaches Q. From Q he walks 20m towards North and reaches R. At R, he turns 10m towards East and reaches S. At S, he turns towards South and after walking 5m reaches T.

Clearly,

PT = RS = 10m East

Thus, Ramesh is 10m East

28. Clock Related Test

The face or dial of a watch is a circle whose circumference is divided into 60 equal parts, called minutes spaces.

A clock has two hands, the smaller one is called the hour hand or short hand while the larger one is called the minute hand or long hand.

28.1. Some Basic Concept:

- (i) In 60 minutes, the minute hand gains 55 minutes on the hour hand.
- (ii) In every hour, both the hands coincide once.
- (iii) In every hour, the hands are once in opposite direction.
- (iv) In every hour, the hands are twice at right angles.

- (v) The hands are in the same straight line when they are coincident or opposite to each other.
- (vi) When the two hands are at right angles, they are 15 minute spaces apart.
- (vii) When the hands are in opposite directions, they are 30 minute spaces apart.
- (viii) The minute hand moves through 6 degree in each minute while the hour hand moves through $\frac{1}{2}$ degree in each minute.
- (ix) The hands coincide 11 times in every 12 hours.
- (x) The hands are 22 times at right angles in every 12 hours.
- (xi) Interchangeable positions occur when the interval between the hour hand and minute hand is $60/13$ minute divisions or some multiple of this.

Example 61:

A clock is set right at 8 a.m. The clock gains 10 minutes in 24 hours. What will be the right time when the clock indicates 1 p.m. on the following day?

Solution:

Time from 8 a.m. on a day to 1 p.m. on the following day = 29 hours.

24 hrs. 10 min of his clock = 24 hrs. of the correct clock

$145/6$ hrs. of this clock = 24 hrs. of the correct clock

29 hrs. of this clock = $(24 \times \frac{6}{145} \times 29)$ hrs. of the correct clock = 28 hrs. 48 min. of correct clock

The correct time is 28 hrs. 48 min. after 8 a.m.

Thus, the answer is 48 min. past 12.

Example 62:

At what time are the hands of a clock together between 5 and 6?

Solution:

At 5 o'clock the hands are 25 minutes apart. Therefore in order to coincide, the long hand must gain 25 minutes. As 55 minutes are gained by minute hand in 60 min.

Thus, 25 minutes are gained by minute hand in 60 minutes = $\frac{60 \times 25}{55} = \frac{300}{11} = 27\frac{3}{11}$ minutes

Hence the hands will coincide at $27\frac{3}{11}$ minutes past 5.

29. Permutations and Combinations

29.1. Factorial:

Let us consider a practical example for understanding the importance of the concept of factorial in arrangements.

Example 63:

How many different ways can arrange the letters of the word 'FACE'?

This is basically an arrangement context, but it is an arrangement without the repletion of any letter.

There are four different letters F A, C and E.

1st letter	2nd letter	3rd letter	4th letter
Anyone among the four	Anyone other than the first letter	Anyone other than the first two letters	Remaining one letter
4 ways	3 ways	2 ways	1 way

Therefore the total number of arrangements = $4 \times 3 \times 2 \times 1 = 24$

i.e. The product of first four natural numbers.

This is a frequently facing situation in the area of arrangements.

Here we can see the relevancy of the concept of factorial.

The product of first 4 natural numbers can be expressed as $4!$ (4 factorial)

i.e. $n!$ means the product of first n natural numbers

$$n! = 1 \times 2 \times 3 \times \dots \times (n - 2) \times (n - 1) \times n$$

Hence, the number of ways to arrange one object in one place = $1! = 1$ way

And the number ways to arrange '0' object (no object) in '0' place (no place) can be considered as ' $0!$ ', i.e. it can be assumed in only one way.

Hence $0! = 1$

Factorial	Value
"0!"	1
1!	1
2!	2
3!	6
4!	24
5!	120
6!	720
7!	5040
And so on	

29.2. Properties of Factorials:

- Factorial of any negative quantity is not valid.
- $n! = n \times (n - 1)!$

i.e. $6! = 6 \times 5! = 6 \times 5 \times 4!$ and so on

- $n!/n = (n - 1)!$
- $n!/(n - 1)! = n$

The numerical problems related to factorials explained in the topic 'Numbers'. Here we are considering the application of 'Factorials' in 'Arrangements'.

Here we have three different kinds of applications of factorials in regards of Non-circular Arrangements.

- Arrangement of 'n' distinct objects.
- Arrangement of 'n' objects in which some of them are repeated.
- Conditional arrangements of 'n' objects (distinct or non-distinct objects).

Example 64: For Type 1:

How many different ways can arrange the letters from the word RAINBOW?

Solution:

Here we have 7 distinct letters.

Hence the required number of arrangements = $7! = 5040$

Result:

The number of ways to arrange 'n' distinct letters in a row = $n!$

Example 65: For Type 2:

How many different ways can arrange the letters of the word INDIA?

Solution:

Here we have 5 letters and two among them are repeated.

i.e. two 'I's are repeated. These identical letters make duplication in the number of arrangements.

For avoiding the duplication in the counting, just divide the total number of arrangements (irrespective of repetition) by $2!$.

i.e. the required number of arrangements = $5! / 2! = 120/2 = 60$ ways

Result:

The number of ways to arrange 'n' objects in a row such that 'p' objects out of 'n' objects are identical and 'q' objects out of 'n' objects are identical is $n!/(p! q!)$.

Example 66: For Type 3:

How many different ways can re-arrange the letters from the word SONG so that the word should start with a vowel?

Solution:

Out of the four distinct letters, only one letter (O) is a vowel and the arrangement should start with 'O'.

i.e. the first letter can be arranged in only one way and the rest three places can be arranged in $3!$ ways.

Hence the total number of arrangements = $1 * 3! = 6$ ways.

Permutation (Non-Circular)

Permutation means the arrangement without repetition of distinct objects. There are two different types of permutations.

- Non-Circular Permutation.
- Circular Permutation.

In this first part, we are considering non circular permutations. Mainly this is applicable in the context of row wise arrangements.

Result:

The number of ways to arrange 'r' objects out of 'n' distinct objects is expressed as n_{Pr}

$$n_{Pr} = \frac{n!}{(n-r)!}, \text{ where } n > 0, r \geq 0 \text{ and } n \geq r.$$

Example 67:

How many different three digits numbers can be formed such that the digits are distinct prime numbers?

Single digit prime numbers are: 2, 3, 5 and 7.

i.e. there are 4 distinct single digit prime numbers.

For finding the required three digit numbers, we have to arrange any three out of the above four distinct prime numbers in a row.

This is an arrangement of three out of four digits in a row.

$$\text{i.e. } 4P_3 = 4! / (4 - 3)! = 24$$

Therefore, there are 24 such distinct three digit numbers are possible.

Note: In this arrangement:

Total number of objects = 4 (four distinct single digit prime numbers)

The number of places to be filled = 3 (three digit numbers)

As per the above example, in the result n_{P_r} ,

$n \rightarrow$ number of objects

$r \rightarrow$ number of places

Example 2: How many different ways 4 cars can be parked in 5 different parking slots?

Here the 4 cars are the four distinct objects and 5 available slots are the places.

Required number of arrangements = $5P_4 = 5!/(5 - 4)! = 120$ ways

As per the above example, in the result nP_r ;

$n \rightarrow$ number of objects

$r \rightarrow$ number of places

Important Note:

As per the above two examples, we can see that in a result of n_{P_r} , n and r can be either objects or places depends upon the situations of counting.

$$n_{P_0} = n!/(n-0)! = 1$$

$$n_{P_1} = n!/(n-1)! = n(n-1)!/(n-1)! = n$$

$$n_{P_n} = n!/(n-n)! = n!/1 = n!$$

There for, $0!=1$

Sum of the numbers formed by different arrangements.

This is an extended application of Linear permutation. All the B school entrance exams are frequently asking questions from this area. We are already familiar with the method of framing different numbers by the arrangements of different digits which are given. Here we are considering the method for finding the sum of all numbers formed by such arrangements.

Example 68:

Find the sum of all the three digit numbers formed by the digits 1, 2, and 3 without repetition.

Solution:

Without repetition of the digits we can frame $3!$ numbers in total, i.e. 6 numbers.

The possible numbers are given below.

123, 132, 213, 231, 312, 321

Column 1	Column 2	Column 3
1	2	3
1	3	2
2	1	3
2	3	1
3	1	2
3	2	1
Total = 12	Total = 12	Total = 12

From the above tabulation, it is easy to understand that, after the mentioned arrangements, each column has the given digits in equal frequency but different order. This frequency or repetition of a digit in each column is equal to $(n-1)!$, where n is the number of distinct digits in the arrangements. In the given question we have to arrange three digits, therefore $n = 3$. So $(n-1)! = (3 - 1)! = 2$

Therefore the individual sums of each column are equal and the sum of each column can be found in the following way.

Sum in each column = sum of distinct digits in arrangement $\times (n - 1)!$

$$= (1 + 2 + 3) \times (3 - 1)!$$

$$= 6 \times 2 = 12$$

Sum of all numbers = $1200 + 120 + 12$ (consider the place values)

$$= 1332$$

For finding the final sum, it is possible to apply another method, i.e. 12×111 (number of 1's is equal to the number of places)

As per the above explanation, we can conclude the method in the following manner.

Sum of all the numbers formed by the arrangements of 'n' non-zero digits = $(n - 1)! \times$ Sum of all digits in the arrangement $\times 1111...n$ times.

30. Mensuration Formulae

Mensuration Formulas for CUBOID In the following formulae, l = length, b = breadth and h = height

(i) Total surface area of cuboid = $2(lb + bh + lh)$

(ii) Length of diagonal of cuboid = $\sqrt{l^2 + b^2 + h^2}$

(iii) Volume of cuboid = $l \times b \times h$

30.1. Mensuration Formulas for Cube:

In the following formulae, a = side of a cube

- (i) Volume of cube = a^3
- (ii) Total surface area of cube = $6a^2$
- (iii) Length of Leading Diagonal of Cube = $a\sqrt{3}$

30.2. Mensuration Formulas for Cone:

In the following formulae, r = radius of base, l = slant height of cone and h = height of the cone (perpendicular to base)

Slant height of a cone = $l = \sqrt{h^2 + r^2}$

- (i) Curved surface area of a cone = $C = \pi \times r \times l$
- (ii) Total surface area of a cone = $\pi \times r \times (r + l)$
- (iii) Volume of right circular cone = $\frac{1}{3} \pi r^2 h$

30.3. Mensuration Formulas for Cylinder:

In the following formulae, r = radius of base, h = height of cylinder

- (i) Curved surface area of a cylinder = $2\pi rh$
- (ii) Total surface area of a cylinder = $2\pi r(r + h)$
- (iii) Volume of a cylinder = $\pi r^2 h$

30.4. Mensuration Formulas for Sphere:

In the following formulae, r = radius of sphere, d = diameter of sphere

- (i) Surface area of a sphere = $4\pi r^2 = \pi d^2$
- (ii) Volume of a sphere = $\frac{4}{3} \pi r^3 = \frac{1}{6} \pi d^3$

30.5. Mensuration Formulas for Hemisphere:

In the following formulae, r = radius of sphere

- (i) Volume of a hemisphere = $\frac{2}{3} \pi r^3$
- (ii) Curved surface area of a hemisphere = $2\pi r^2$
- (iii) Total surface area of a hemisphere = $3\pi r^2$

30.6. Mensuration Formulas for Hollow Cylinder:

Hollow cylinder made by cutting a smaller cylinder of same height and orientation out of a bigger cylinder.

Volume of hollow cylinder = $\pi h (R^2 - r^2)$; (Where, R = radius of cylinder, r = radius of cavity, h = height of cylinder) 15.

30.7. Mensuration Formulas for Frustum of a Right Circular Cone:

Frustum is created when a plane cuts a cone parallel to its base. In the following formulae, R = radius of the base of the frustum, r = radius of the top of the frustum, h = height of the frustum, l = slant height of the frustum

If a cone is cut by a plane parallel to the base of the cone, the lower part is called the frustum of the cone.

- (i) Slant height of the frustum = $l = \sqrt{h^2 + (R - r)^2}$
- (ii) Curved surface area of frustum = $\pi(R + r) l$
- (iii) Total surface area of frustum = $\pi(R + r) l + \pi(R^2 + r^2)$
- (iv) Volume of the frustum = $(1/3) \pi h (R^2 + r^2 + Rr)$

30.8. Mensuration Formulas for Prism:

Prism consists of two polygonal bases which are parallel to each other.

- (i) These bases are joined by lateral faces, which are perpendicular to the polygonal bases.
- (ii) The number of lateral faces is equal to the number of sides in the polygonal base. Thus, the base of a prism could be of various shapes, namely, triangular, quadrangular, pentagonal etc.
- (iii) Volume of prism = Base area \times height
- (iv) Lateral surface area of prism = perimeter of base \times height
- (v) Total surface area of prism = Lateral surface area + (2 \times base area)

30.9. Square:

- (i) A square is a plane figure with four equal straight sides and four right angles.
- (ii) Area of Square = $(\text{side})^2 = (\text{diagonal})^2$ divided by 2
- (iii) Perimeter of Square = 4 \times side

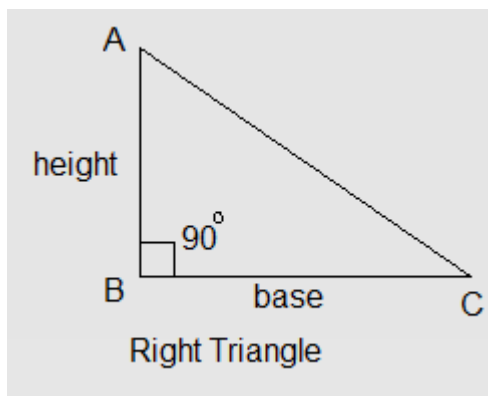
30.10. Rectangle:

- (i) A rectangle is a four-sided shape that is made up of two pairs of parallel lines and that has four right angles.
- (ii) Area of Rectangle = Length x Breadth
- (iii) Perimeter of Rectangle = 2(Length + Breadth)
- (iv) 3. Area of 4 walls of a room = 2(Length + Breadth) x Height

30.11. Triangle:

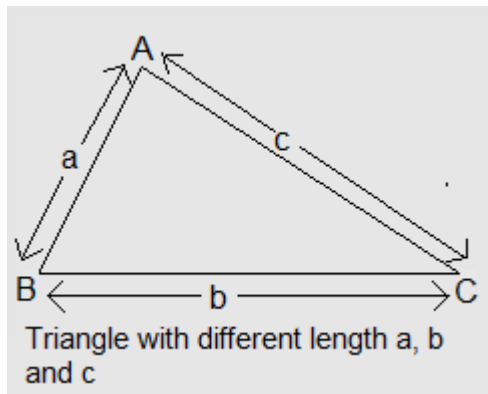
A triangle is a plane figure with three straight sides and three angles.

- (i) Right triangle with base and height given.



Area of a triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$

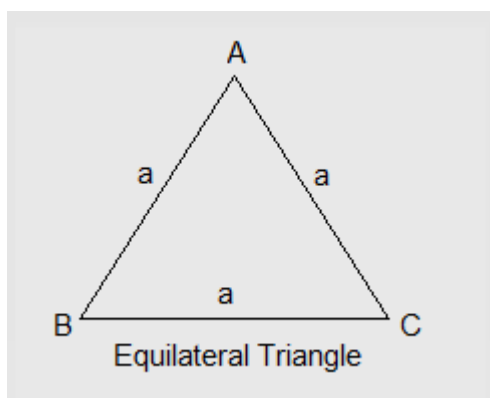
- (ii) Triangle with three different sides a, b and c.



Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$s = \frac{1}{2}(a + b + c)$

- (iii) Equilateral triangle - A triangle with all three sides of equal length.



Area of a equilateral triangle = $\frac{\sqrt{3}}{4} \times (\text{side})^2$

(iv) Radius of incircle of an equilateral triangle of side $a = \frac{a}{2\sqrt{3}}$

(v) Radius of circumcircle of an equilateral triangle of side $a = \frac{a}{\sqrt{3}}$

(vi) Radius of incircle of a triangle of area a and semi-perimeter $s = \frac{a}{s}$

30.12. Parallelogram:

A parallelogram is a 4-sided flat shape with straight sides where opposite sides are parallel.

(i) Area of Parallelogram = Base \times Height.

(ii) Area of Rhombus = $\frac{1}{2} \times (\text{Product of diagonals})$

(iii) Area of Trapezium = $\frac{1}{2} \times (\text{sum of parallel sides} \times \text{distance between them})$

30.13. Circle:

A circle is a round plane figure whose boundary (the circumference) consists of points equidistant from a fixed point (the centre).

(i) Area of a circle = πR^2 , where R is radius of the circle

(ii) Circumference of a circle = $2\pi R$

(iii) Length of an arc = $\frac{2\pi R\theta}{360}$ where θ is the central angle

(iv) Area of sector = $\frac{1}{2} (\text{arc} \times R) = \frac{\pi R^2 \theta}{360}$

(v) Area of a semicircle = $\pi R^2/2$

(vi) Circumference of a semicircle = πR

31. Quantitative Aptitude Formulas

31.1. Ages:

Problems on Ages are asked in majority of bank and competitive examinations.

They are generally simple to attempt if you have done practice and remember the formulae.

Important formulae to remember are:

- (i) If the current age is x , then n times the age is nx
- (ii) If the current age is x , then age n years later/hence $= x + n$
- (iii) If the current age is x , then age n years ago $= x - n$.
- (iv) The ages in a ratio $a : b$ will be ax and bx .
- (v) If current age is x , then $1/n$ of the age is x/n .

Example 69:

Father is four times the age of his daughter. If after 5 years, he would be three times of daughter's age, then further after 5 years, how many times he would be of his daughter's age?

Solution:

Let the daughter's age be x and father's age be $4x$.

So as per question, $4x + 5 = 3(x + 5)$

So $x = 10$

Hence present age of daughter is 10 years and present age of father is 40 years.

So after $5 + 5 = 10$ years, daughter age would be 20 years and father's age would be 50 years.

Hence father would be $50/20 = 2.5$ times of daughter's age.

Example 70:

What is Aman's present age, if after 20 years his age will be 10 times his age 10 years back?

Solution:

Let Aman's present age be x

Aman's age before 10 years $= x - 10$

Aman's age after 20 years $= (x + 20)$

We are given that, Aman's age after 20 years $(x + 20)$ is 10 times his age 10 years back $(x - 10)$

Therefore, $(x + 20) = 10(x - 10)$

Solving the equation, we get $x + 20 = 10x - 100$

$$9x = 120, x = 13.3 \text{ years}$$

Example 71:

One year ago, the ratio of Honey and Piyush ages was 2:3 respectively. After five years from now, the ratio becomes 4:5. How old is Piyush now?

Solution:

We are given that age ratio of Honey: Piyush = 2: 3

Honey's age = $2x$ and Piyush's age = $3x$

One year ago, their age was $2x$ and $3x$.

Hence at present, Honey's age = $2x + 1$ and Piyush's age = $3x + 1$

After 5 years, Honey's age = $(2x + 1) + 5 = (2x + 6)$

Piyush's age = $(3x + 1) + 5 = (3x + 6)$

After 5 years, this ratio becomes 4: 5.

Therefore, $(2x+6) / (3x+6) = 4/5$

$$10x + 30 = 12x + 24 \Rightarrow x = 3$$

Piyush's present age = $(3x + 1) = (3 \times 3 + 1) = 10$ years

Honey's present age = $(2x + 1) = (2 \times 3 + 1) = 7$ year