Classical Mechanics

June – 2014

(1) Consider a particle of mass m in simple harmonic oscillation about the origin with spring constant k; then for the Lagrangian L and the Hamiltonian H of the system

(a) $L(x, \dot{x}) = \frac{1}{2}\ m\ \dot{x}^2 - \frac{1}{2} k\ x^2$

$H(x, \dot{x}) = \frac{p^2}{2m} + \frac{1}{2} k\ x^2, p$ is generalized momentum.

(b) $(x, \dot{x}) = \frac{1}{2} m\ \dot{x}^2 + \frac{1}{2} k\ x^2\ ,\ p = m\dot{x}$

(c) $L(x, \dot{x}) = \frac{1}{2} m\ \dot{x}^2 - \frac{1}{2} k\ x^2, p = m\dot{x}$

(d) $L(x, \dot{x}) = \frac{1}{2} m\dot{x}^2 - \frac{1}{2}\ kx^2$

$H(x, \dot{x}) = \frac{m\dot{x}^2}{2} + \frac{1}{2} kx^2$

Answer: (a), (c) and (d)

Solution: $K.E\ = T = \frac{1}{2} m\ \dot{x}^2$

$F \propto x$

$F = -kx$

$F = -grad\ V = -\nabla\ \mathrm{V} = -\frac{\partial \mathrm{V}}{\partial \mathrm{x}}$

$V = -\int F\ dx\ = \int kxdx = \frac{kx^2}{2}$

$\therefore L = T - V = \frac{1}{2} m\ \dot{x}^2 - \frac{1}{2} kx^2$

$H = T + V = \frac{1}{2} m\ \dot{x}^2 + \frac{1}{2} kx^2$

$P = \frac{\partial L}{\partial \dot{x}} = m\ \dot{x}$

Also $H = \frac{1}{2} m\ \cdot \frac{p^2}{m^2} + \frac{1}{2} kx^2 = \frac{p^2}{2m} + \frac{1}{2} kx^2$

So, the options (a), (c) and (d) are correct.

December – 2014

(1) Consider a body of unit mass falling freely from rest under gravity with velocity v**.** If the air resistance retards the acceleration by cv where c is a constant then

(a) $v = \frac{g}{c}(1 + e^{ct})$

(b) $v = \frac{g}{c}(1 + e^{-ct})$

(c) $v = \frac{g}{c}(1 - e^{-ct})$

(d) $v = \frac{g}{c}(1 - e^{ct})$

Answer: (c)

Solution: Equation of motion is

$m \cdot \frac{dv}{dt} = mg - mc\ v$

$or, \frac{dv}{g\left(1-\frac{c}{g}v\right)} = dt$

Integrating, $\frac{\log\left(1-\frac{c}{g}v\right)}{g\times\left(-\frac{c}{g}\right)} = t + A$ (A being arbitrary constant)

$-\frac{1}{c}\log\left(1-\frac{c}{g}\ v\right) = t + A$

Initially, $t = 0, v = 0 \ \Rightarrow A = 0$

$\therefore \log\left(1-\frac{c}{g}\ v\right) = -ct$

$\Rightarrow 1 - \frac{c}{g}v = e^{-ct}$

$\Rightarrow v = \frac{g}{c}(1 - e^{-ct})$

So, the option (c) is correct.

June-2015

1. Consider two weightless, inextensible roots AB and BC; Suspended at A and joined by a flexible joint at B. Then the degrees of freedom of the system is

(a) 3

(b) 4

(c) 5

(d) 6

Answer: (a)

Solution:

Degrees of freedom is $= \frac{2(2+1)}{2} = 3$

So, the option (a) is correct.

2. A particle of mass m is constrained to move on the surface of a cylinder $x^2 + y^2 = a^2$ under the influence of a force directed towards the origin and proportional to the distance of the particle from the origin. Then

(a) The angular momentum about z-axis is constant.

(b) The angular momentum about z-axis is not constant.

(c) The motion is simple harmonic in z-direction.

(d) The motion is not simple harmonic in z-direction.

Answer: (a), (c)

Solution:

Here the motion is simple harmonic in z-direction and the angular momentum about z-axis is constant.

So, the options (a) and (c) are correct.

December-2015

1. A force $5\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ acts on a particle with position vector $2\hat{\imath} + \hat{\jmath} - 2\hat{k}$. The torque of the force about the origin is

(a) $\hat{\imath} + 16\hat{\jmath} + 9\hat{k}$
(b) $-\hat{\imath} - 16\hat{\jmath} - 9\hat{k}$
(c) $\hat{\imath} + 16\hat{\jmath} - 9\hat{k}$
(d) $\hat{\imath} - 16\hat{\jmath} + 9\hat{k}$

Answer: (b)

Solution:

Torque is $\vec{r} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & -2 \\ 5 & -2 & 3 \end{vmatrix} = -\hat{\imath} - 16\hat{\jmath} - 9\hat{k}$

So, the option (b) is correct.

2. Consider a mass m moving in an inverse sequence central force with characteristic coefficient μ and described by the Lagrangian:

$L(r, \dot{r}, \theta, \dot{\theta}) = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{\mu m}{r}$ then

(a) The generalized momenta of the system are $p_r = m\dot{r}$ and $p_\theta = mr^2\dot{\theta}$
(b) The Hamiltonian of the system is $H = \frac{1}{2m}\left[{p_r}^2 + \frac{{p_\theta}^2}{r^2}\right] - \frac{1}{2}\frac{\mu m}{r}$
(c) The Hamiltonian of the system is $H = \frac{1}{2m}\left[{p_r}^2 + \frac{{p_\theta}^2}{r^2}\right] - \frac{\mu m}{r}$
(d) The generalized momenta of the system are $p_r = m\dot{r}$ and $p_\theta = -mr^2\dot{\theta}$

Answer: (a), (c)

Solution:

$p_r = \frac{rL}{r\dot{r}} = m\dot{r}, \quad p_\theta = \frac{rL}{r\dot{\theta}} mr^2\dot{\theta}$

Now, $\dot{r} = \frac{p_r}{m}$ and $\dot{\theta} = \frac{p_\theta}{mr^2}$

$$\begin{aligned} H = \sum p_i q_i - L &= p_r \cdot \dot{r} + p_\theta \cdot \dot{\theta} - L \\ &= p_r \cdot \dot{r} + p_\theta \cdot \dot{\theta} - \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - \frac{\mu m}{r} \\ &= p_r \cdot \frac{p_r}{m} + p_\theta \cdot \frac{p_\theta}{mr^2} - \frac{m}{2}\left(\frac{{p_r}^2}{m^2} + \frac{r^2 \cdot {p_\theta}^2}{m^2 r^4}\right) - \frac{\mu m}{r} \\ &= \frac{{p_r}^2}{m} + \frac{{p_\theta}^2}{mr^2} - \frac{{p_r}^2}{2m} - \frac{{p_\theta}^2}{2mr^2} - \frac{\mu m}{r} \\ &= \frac{1}{2m}\left[{p_r}^2 + \frac{{p_\theta}^2}{r^2}\right] - \frac{\mu m}{r} \end{aligned}$$

So, the options (a) and (c) are correct.

3. Consider the Hamiltonian (H) and the Lagrangian (L) for a free particle of mass m and velocity v. Then
(a) H and L are independent of each other.
(b) H and L are related but have different dependence on v.
(c) H and L are equal.
(d) Both H and L are quadratic in v.

Answer: (d)

Solution:
For free particle potential energy $V = 0$
$T = \frac{1}{2}mv^2$
$L = T - V,\ \ H = T + V$
$\Rightarrow H = L = \frac{1}{2}mv^2$ (quadratic in v)
So, the options (c) and (d) are correct.

June – 2016

(1) Consider the equations motion for a system

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0\ , i = 1, 2, 3, \cdots\cdots, n$$

Where $L = T - V$ [Where $T(t, q_i, \dot{q}_i)$ as kinetic energy and $V(t, q_i)$ as potential energy]
q_i be the generalized coordinates, $\dot{q}_i$ be the generalized velocities. Then the equations of motion in the form as above are
(a) Necessarily restricted to a conservative system but there is no unique choice of L.
(b) Not necessarily restricted to a conservative system and there is a unique choice of L.
(c) Necessarily restricted to a conservative system and there is a unique choice of L.
(d) Not Necessarily restricted to a conservative system and there is no unique choice of L.

Answer: (a)

Solution: Option (a) is correct.

(2) A particle of unit mass moves in the direction of $x - axis$ such that it has the Lagrangian $L = \frac{1}{12}\dot{x}^4 + \frac{1}{2}x\,\dot{x}^2 - x^2$. Let $Q = \dot{x}^2\ddot{x}$ represent a force (not arising from a potential) acting on the particle in the $x -$ direction. If $x(0) = 1$ and $\dot{x}(0) = 1$, then the value of $\dot{x}$ is
(a) Some non-zero finite value at $x = 0$.
(b) $1\ at\ x = 1$.
(c) $\sqrt{5}\ at\ x = \frac{1}{2}$.
(d) $0\ at\ x = \sqrt{\frac{3}{2}}$.

Answer:

<u>Solution:</u>

December – 2016

(1) A bead slides without friction on a frictionless wire in the shape of a cycloid with equation $x = a(\theta - \sin\theta), y = a(1 + \cos\theta), 0 \leq \theta \leq 2\pi$.
The Lagrangian function is
(a) $ma^2(1 + \cos\theta)\dot{\theta}^2 - mga\,(1 + \cos\theta)$
(b) $ma^2(1 - \cos\theta)\dot{\theta}^2 - mga\,(1 + \cos\theta)$
(c) $ma^2(1 - \cos\theta)\dot{\theta}^2 + mga\,(1 + \cos\theta)$
(d) $ma^2(1 + \cos\theta)\dot{\theta}^2 - mga\,(1 - \cos\theta)$

Answer: (b)

Solution: $L = T - V$
Where $T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2)$

$$= \frac{1}{2}m\left[\left\{a\left(\dot{\theta} - \cos\theta\,\dot{\theta}\right)\right\}^2 + \left\{-a\sin\theta\,\dot{\theta}\right\}^2\right]$$
$$= \frac{1}{2}ma^2\dot{\theta}^2[1 - 2\cos\theta + \cos^2\theta + \sin^2\theta]$$
$$= \frac{1}{2}ma^2\dot{\theta}^2 \cdot 2(1 - \cos\theta)$$
$$= ma^2(1 - \cos\theta)\dot{\theta}^2$$

$V = mg\,y = mga(1 + \cos\theta)$
$\therefore L = ma^2(1 - \cos\theta)\dot{\theta}^2 - mga\,(1 + \cos\theta)$
So, answer (b) is correct.

(2) Which of the following are canonical transformation? (where q, p represent generalized coordinate and generalized momentum respectively.
(a) $P = \log\sin p\ , Q = q\tan p$
(b) $P = qp^2\ , Q = \frac{1}{p}$
(c) $P = q\cot p\ , Q = \log\left(\frac{1}{q}\sin p\right)$
(d) $P = q^2\sin 2p\ \ , Q = q^2\cos 2p$

Answer: (a), (b), and (c)

Solution:
(i) $P = \log\sin p\ , Q = q\tan p$
Now, $p\,dq - P\,dQ$
$= p\,dq - \log\sin p\,.[q\sec^2 p\ \ dp + \tan p\,dq]$
$= (-q\log\sin p \cdot \sec^2 p)dp + (p - \log\sin p \cdot \tan p)dq$
$= d[q \cdot (p - \log\sin p \cdot \tan p)]$
Which is exact.

(ii) $P = qp^2, Q = \frac{1}{p}$
Now, $p\,dq - P\,dQ$
$= p\,dq - qp^2 \cdot \left(-\frac{1}{p^2}\right)dp = p\,dq + q\,dp = d(pq)$ Which is exact.

(iii) $P = q\cot p \ , Q = \log\left(\frac{1}{q}\sin p\right)$

Now, $p\ dq - P\ dQ$

$= p\ dq - q\cot p \cdot \left(\frac{\frac{1}{q}\cos p\ dp + \sin p \cdot \left(-\frac{1}{q^2}\right)dq}{\frac{1}{q}\sin p}\right)$

$= p\ dq - q \cdot \cot p \left[\cot p\ dp - \frac{1}{q}dq\right]$

$= p\ dq - q\cot^2 p\ dp + \cot p\ dq$

$= -q\cot^2 p\ dp + (p + \cot p)dq$

$= q(1 - \operatorname{cosec}^2 p)dp + (p + \cot p)dq$

$= d[q \cdot (p + \cot p)]$

Which is exact.

(iv) $P = q^2 \sin 2p\, , Q = q^2\cos 2p$

Now, $p\ dq - P\ dQ$

$= p\ dq - q^2\sin 2p \cdot (-q^2 . 2\sin 2p\ dp + 2q\cos 2p\ dq)$

$= p\ dq + 2q^4 \cdot \sin^2 2p\ dp - 2q^3 sin\ 2p\cos 2p\ dq$

$= 2q^4\sin^2 2p\ dp + (p - 2q^3\sin 2p\cos 2p)dq$

$= q^4(1 - \cos 4\,p)dp + (p - 2q^3\sin 4p)dq$

$= M\ dp + N\ dq(say)$

$\frac{\partial M}{\partial q} = 4q^3(1 - \cos 4p), \frac{\partial N}{\partial p} = 1 - 4q^3\cos p$

$\therefore \frac{\partial M}{\partial q} \neq \frac{\partial N}{\partial p}$

So, it is not exact.

Hence the options (a), (b), and (c) are correct.

June – 2017

(1) A rigid body having one point 0 fixed and no external torque about 0 has equal principal moments of inertia. Then the body must rotate with
(a) Angular velocity of variable magnitude.
(b) Angular velocity with constant magnitude.
(c) Constant angular momentum but varying angular velocity.
(d) Varying angular momentum with varying angular velocity.
Answer: (b)

Solution: Option (b) is correct

(2) Consider a spherical pendulum consisting of a particle of mass m which moves under gravity on a smooth sphere of radius a. In terms of spherical polar angles $\theta, \phi, with\ \theta$ measured up from the downward vertical, the Lagragian is given by
(a) $ma\left[\frac{a}{2}\left(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta\right) - g\cos\theta\right]$
(b) $ma\left[\frac{a}{2}\left(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta\right) + g\cos\theta\right]$
(c)$ma\left[\frac{a}{2}\left(\dot{\theta}^2 + \dot{\phi}^2 \cos^2\theta\right) + g\sin\theta\right]$
(d) $ma\left[\frac{a}{2}\left(\dot{\theta}^2 + \dot{\phi}^2 \cos^2\theta\right) - g\sin\theta\right]$

Answer: (a)

Solution: $x = a\sin\theta\cos\phi\,, y = a\sin\theta\sin\phi\,, z = a\cos\theta$
Now $L = T - V$
$= \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - mg\,z$
$\dot{x} = a\left[\cos\theta\cos\phi\,\dot{\theta} - \sin\theta\sin\phi\,\dot{\phi}\right]$
$\dot{y} = a\left[\cos\theta\sin\phi \oplus + \sin\theta\cos\phi\,\dot{\phi}\right]$
$\dot{z} = -\mathrm{a}\sin\theta\,\dot{\theta}$
$\therefore L = \frac{1}{2}m\,.\,a^2\left[\cos^2\theta\cos^2\phi\,\dot{\theta}^2 + \sin^2\theta\sin^2\phi\,\dot{\phi}^2 - 2\sin\theta\cos\theta\cos\phi\sin\phi\,\dot{\theta}\dot{\phi}\ + \right.$
$\left.\cos^2\theta\sin^2\phi \oplus^2 + \sin^2\theta\cos^2\phi\,\dot{\phi}^2 + 2\cos\theta\sin\theta\sin\phi\cos\phi\,\dot{\theta}\dot{\phi} + \sin^2\theta\,\dot{\theta}^2\right] - mga\cos\theta$
$= \frac{1}{2}ma^2\left[\cos^2\theta\,\dot{\theta}^2 + \sin^2\theta\,\dot{\phi}^2 + \sin^2\theta\,\dot{\theta}^2\right] - mga\cos\theta$
$= \frac{1}{2}ma^2\left[\dot{\theta}^2 + \sin^2\theta\,\dot{\phi}^2\right] - mga\cos\theta$
$= ma\left[\frac{a}{2}\left(\dot{\theta}^2 + \sin^2\theta\,\dot{\phi}^2\right) - g\cos\theta\right]$
So, the option (a) is correct.

(3) Let q_α and $p_\alpha(\alpha = 1, 2, \cdots\cdots\cdots, n)$ be the generalized coordinates and the generalized momenta respectively. If H denotes the Hamiltonian and q_α (for some $\alpha = \alpha_0$) is an ignorable coordinate, then which of the following equations are satisfied?

(a) $\dot{p}_\alpha = -\frac{\partial H}{\partial q_\alpha}, \dot{q}_\alpha = \frac{\partial H}{\partial p_\alpha}, \forall\, \alpha$

(b) $\dot{p}_\alpha = \frac{\partial H}{\partial q_\alpha}, \dot{q}_\alpha = -\frac{\partial H}{\partial p_\alpha}\ \forall\, \alpha$

(c) $\dot{p}_{\alpha_0} = 0, \dot{q}_{\alpha_0} = \frac{\partial H}{\partial p_{\alpha_0}}$

(d)$\dot{p}_{\alpha_0} = \frac{-\partial H}{\partial q_{\alpha_0}}, \dot{q}_{\alpha_0} = 0.$

Answer: (a), (c)

Solution: q_{α_0} is ignorable $\Rightarrow \frac{\partial L}{\partial q_{\alpha_0}} = 0$

Lagrange's equation is

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_{\alpha_0}}\right) - \frac{\partial L}{\partial q_{\alpha_0}} = 0 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_{\alpha_0}}\right) = 0$

$\therefore \frac{d}{dt}\left(\bar{p}_{\alpha_0}\right) = 0 \Rightarrow \dot{p}_{\alpha_0} = 0$

Also, $\dot{q}_{\alpha_0} = \frac{\partial H}{\partial p_{\alpha_0}}$

Hamilton's equations are

$\dot{p}_\alpha = \frac{-\partial H}{\partial q_\alpha}$ and $\dot{q}_\alpha = \frac{\partial H}{\partial p_\alpha}\ \forall\, \alpha.$

So, the option (a) and (c) are correct.

(4) For a conservative system, the end configurations are fixed and the velocity in the varied motion is such that $T + V = E$. Here T, V and E represent, respectively the kinetic energy, the potential energy and the total energy. If $\delta(A)$ denotes the infinitesimal change in a variable $\mathbb{A}$, and p_α and $q_\alpha(\alpha = 1,2, \cdots\cdots\cdots, n)$ represent the generalized momenta and generalized coordinates, respectively, then

(a) $\delta \int T\, dt = 0$

(b) $\delta \int \sum_{\alpha=1}^{n} p_\alpha\, dq_\alpha = 0$

(c) $\delta \int \sum_{\alpha=1}^{n} q_\alpha\, dp_\alpha = 0$

(d) $\delta \int \sum_{\alpha=1}^{n} (p_\alpha\, dq_\alpha + q_\alpha dp_\alpha) = 0$

Answer: (a), (b)

Solution: For conservative system,

$\delta \int T\, dt = 0$

Also, $\delta H = 0$

$\delta(\sum_{\alpha=1}^{n} p_\alpha\, \dot{q}_\alpha - L) = 0$

$\Rightarrow \delta \int \sum_{\alpha=1}^{n} p_\alpha\, dq_\alpha = 0$

So, the options (a) and (b) are correct.

December – 2017

(1) Let $I(m)$ denote the moment of inertia of a regular solid tetrahedron about an axis m passing through its centre of gravity. Which of the following is true?
(a) If the axis l passes through a vertex and the axis l' does not pass through a vertex then $I(l) > I(l')$.
(b) If the axis l passes through the midpoint of an edge and l' is any other axis then $I(l) < I(l')$.
(c) $I(l)$ is the same for all axes l.
(d) If the axis l passes through a vertex and the axis l' does not pass through a vertex then $I(l) < I(l')$.

Answer: (c)

Solution: Moment of inertia is same for all axes.
So, the option (c) is correct.

June – 2018

(1) Given that the Lagraugian for the motion of a simple pendulum is
$L = \frac{1}{2}me^2\dot{\theta}^2 + mgl\cos\theta$, where m is the mass of the pendulum bob suspended by a string of length l, g is the acceleration due to gravity and θ is the amplitude of the pendulum from the mean position, then a Hamiltonian corresponding to L is
(a) $H(p,\theta) = \frac{p^2}{2ml^2} + mgl\cos\theta$
(b) $H(p,\theta) = \frac{p^2}{2ml^2} - mgl\cos\theta$
(c) $H(p,\theta) = \frac{p^2}{ml^2} - mgl\cos\theta$
(d) $H(p,\theta) = \frac{3p^2}{2ml^2} + mgl\cos\theta$

Answer: (b)

Solution: $H(p,\theta) = p\cdot\dot{\theta} - L$
Where $p = \frac{\partial L}{\partial\dot{\theta}} = ml^2\dot{\theta}$
$\therefore H(p,\theta) = ml^2\dot{\theta}^2 - \frac{1}{2}ml^2\dot{\theta}^2 - mgl\cos\theta$
$= \frac{ml^2\dot{\theta}^2}{2} - mgl\cos\theta = \frac{ml^2}{2}\cdot\frac{p^2}{m^2l^4} - mgl\cos\theta \ [\because \dot{\theta} = \frac{p}{ml^2}] = \frac{p^2}{2ml^2} - mgl\cos\theta$
So, the option (b) is correct.

(2) The Hamiltonian for a simple harmonic oscillator is $H(p,q) = \frac{p^2}{2m} + \frac{k}{2}q^2$. Then a possible Lagrangian corresponding to H can be
(a) $L = \frac{1}{2}m\,\dot{q}^2 - \frac{k}{2}q^2$
(b) $L = \frac{1}{2}\,m\,\dot{q}^2 - \frac{k}{2}(q^2 + 3q^2\dot{q})$
(c) $L = \frac{1}{2}m\,\dot{q}^2 + \frac{k}{2}q^2$
(d) $L = \frac{1}{2}m\,\dot{q}^2 + \frac{k}{2}(q^2 + 3q^2 + \dot{q})$

Answer: (a)

Solution: $H = p\,\dot{q} - L$

or, $L = p\dot{q} - \frac{p^2}{2m} - \frac{k}{2}q^2$

$= m\dot{q}^2 - \frac{1}{2}m\dot{q}^2 - \frac{k}{2}q^2$

$= \frac{1}{2}m\dot{q}^2 - \frac{k}{2}q^2$

$$\dot{q} = \frac{\partial H}{\partial p} = \frac{p}{m}$$

$$or, p = m\dot{q}$$

This is option (a)

For option (b) $L = \frac{1}{2}m\dot{q}^2 - \frac{k}{2}(q^2 + 3q^2\dot{q})$

$p = \frac{\partial L}{\partial \dot{q}} = m\dot{q} - \frac{k}{2}3q^2$

$or, \dot{q} = \frac{1}{m}\left(p + \frac{3}{2}kq^2\right)$

Now, $H = p\dot{q} - L$

$= \frac{p^2}{m} + \frac{3pkq^2}{2m} - \frac{1}{2}m \cdot \frac{1}{m^2}\left(p + \frac{3}{2}kq^2\right)^2 + \frac{k}{2}(q^2 + 3q^2\dot{q})$

$= \frac{p^2}{m} + \frac{3pkq^2}{2m} - \frac{p^2}{2m} - \frac{3pkq^2}{2m} - \frac{9}{8}\frac{k^2q^4}{m} + \frac{k}{2}q^2 + \frac{3kq^2}{2m}\left(p + \frac{3}{2}kq^2\right)$

$= \frac{p^2}{2m} + \frac{k}{2}q^2 + \left(\frac{9}{8m}k^2q^4 + \frac{3}{2m}pkq^2\right)$

It is not true.

So, the option (a) is true.

December – 2018

(1) Consider the two-dimensional motion of a mass m attached to one end of a spring whose other end is fixed. Let k be the spring constant. The kinetic energy T and the potential energy V of the system are given by

$T = \frac{1}{2}m\left(\dot{r}^2 + \left(r\dot{\theta}\right)^2\right)$ $and\ V = \frac{1}{2}kr^2$ where $\dot{r} = \frac{dr}{dt}$ $and\ \dot{\theta} = \frac{d\theta}{dt}$ $with\ t\ as$ time. Then which of the following statement is correct?

(a) r is an ignorable coordinate.

(b) θ is not an ignorable coordinate.

(c) $r^2\dot{\theta}$ remains constant throughout the motion.

(d) $r\dot{\theta}$ remains constant throughout the motion.

Answer:

Solution: r is not ignorable coordinate.

θ is ignorable coordinate.

$L = T - V = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - \frac{1}{2}kr^2$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0$

$or, \frac{d}{dt}\left(mr^2\dot{\theta}\right) - 0 = 0$

$\Rightarrow mr^2\dot{\theta} = constant$

$r^2\dot{\theta} = constant$

So, the option (c) is correct.

(2) Consider a point mass of mass m which is attached to a massless rigid rod of length $'a'$. The other end of the rod is made to move vertically such that its downward displacement from the origin at time t is given by $z = (t) = z_0 \cos(wt)$. The mass is moving in a fixed plane and its position vector at time t is given by $\vec{r}(t) = \left(a \sin\theta\,(t), z(t) + a\cos\theta\,(t)\right)$. Then the equation of motion of the point mass is

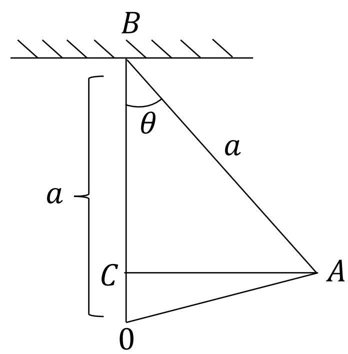
(a) $a\frac{d^2\theta}{dt^2} + (g + z_0 w^2 \cos(wt))\sin\theta = 0$

(b) $a\frac{d^2\theta}{dt^2} + (g - z_0 w \cos(wt))\sin\theta = 0$

(c) $a\frac{d^2\theta}{dt^2} + (g + z_0^2 w^2 \cos(wt))\cos\theta = 0$

(d) $a\frac{d^2\theta}{dt^2} + (g - z_0 w^2 \cos(wt))\cos\theta = 0$

Answer: (a)



Solution: $K.E = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2)$

$x = a\sin\theta,$

$\dot{x} = a\cos\theta\,\dot{\theta}$

$y = z(t) + a\cos\theta = z_0\cos wt + a\cos\theta$

$\dot{y} = -z_0 w \sin wt - a\sin\theta\,\dot{\theta}$

$\therefore T = k.E = \frac{1}{2}m\begin{bmatrix} a^2\cos^2\theta\,\dot{\theta}^2 + z_0^2 w^2 \sin^2 wt + a^2\sin^2\theta\,\dot{\theta}^2 \\ +2z_0\, w\, a\sin wt\sin\theta\,\dot{\theta} \end{bmatrix}$

$= \frac{1}{2}m\left[a^2\dot{\theta}^2 + z_0^2 w^2\sin^2 wt + 2az_0\, w\sin wt\sin\theta\,\dot{\theta}\right]$

$V = P.E$ at the $pt\ A\ is = mg.0c$

$= mg(oB - Bc)$

$= mg(a - a\cos\theta)$

$= mga(1 - \cos\theta)$

$\therefore L = T - V$

$= \frac{1}{2}m\left[a^2\dot{\theta}^2 + z_0^2 w^2\sin^2 wt + 2a\, z_0 w\sin wt\sin\theta\,\dot{\theta}\right] - mga(1 - \cos\theta)$

Lagrange's equations are

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}}\right) - \frac{\partial L}{\partial\theta} = 0$$

$$\frac{d}{dt}\left[\frac{1}{2}m\left(2a^2\dot{\theta} + 2a\, z_0\, w\sin wt\sin\theta\right)\right] - \left[\frac{1}{2}m\left(2a\, z_0\, w\sin wt\ .\cos\theta\,\dot{\theta}\right) - mga\sin\theta\right] = 0$$

$$or, a\frac{d^2\theta}{dt^2} + z_0 w^2\cos wt\cdot\sin\theta + z_0 w\sin wt\cos\theta\,\dot{\theta} - z_0 w\sin wt\cos\theta\,\dot{\theta} + g\sin\theta = 0$$

$or, a\frac{d^2\theta}{dt^2} + (g + z_0 w^2\cos wt)\sin\theta = 0$

So, the option (a) is correct.