Predictive Control for the Lateral and Longitudinal Dynamics in Automated Vehicles

Ovidiu Pauca, Constantin F. Caruntu, Corneliu Lazar Department of Automatic Control and Applied Informatics Gheorghe Asachi Technical University of Iasi Iasi, Romania {pauca.ovidiu, caruntuc, clazar}@ac.tuiasi.ro

Abstract—The control of autonomous vehicles is a topic of great interest nowadays due to their inherent advantages. They can increase the safety, lower the fuel consumption and thus reduce the travel costs, improve the driving comfort and reduce the traffic congestion. Still the control of vehicle dynamics is an open subject, many researchers and companies wanting to improve the existing strategies or to develop novel ones. The developed solutions should help the driver and ensure a safer driving at the same time. Thus, this paper proposes a solution to control the lateral and longitudinal dynamics of a vehicle. The first step is the modeling of both dynamics by constructing a nonlinear model based on the laws of physics that considers the lateral and longitudinal motions of a vehicle. To design a linear model predictive control (MPC) strategy, the nonlinear lateral model is linearized. The aim is to design two control algorithms based on the linear model of the vehicle dynamics, one for the lateral dynamics and one for the longitudinal dynamics and apply them for the full nonlinear model of the system. Both controllers were tested together by simulating an overpassing. This is a maneuver in which the lateral position and the longitudinal velocity must be controlled. The MPC algorithm for the lateral dynamics uses a predefined reference trajectory and the states of the system to compute the command for the vehicle such that the vehicle follows the trajectory and satisfies the imposed constraints. The controller for the longitudinal velocity of the vehicle computes the command such that the velocity of the vehicle follows its reference. The results illustrate that the MPC algorithms manage to control the vehicle and to satisfy the defined requirements.

Keywords-vehicle dynamics, vehicle control, predictive control

Introduction

The study of vehicle dynamics has been a matter of interest for many researchers who were concerned to describe the motion of vehicles and to propose solutions for their control. The reasons to develop a new control strategy can be stated as follows: safety of the traffic participants, fuel consumption reduction and driving comfort improvement.

In the last years, this topic received even more consideration; many companies which have the activity in this domain are eager to develop systems which would allow the vehicles to be automated. In [1], a model that describes the dynamics of the vehicle is described. In [2], a model predictive control (MPC) algorithm is designed to control the motion of vehicles, while in [3], an adaptive and predictive controller is used to improve the lateral stability and the tracking performance of an autonomous vehicle. A comparison between the performances of a closed-loop system and a typical human driver along with the proof of efficiency and speed of the feedback controller has been done in [4]. In [5],

the focus is on trajectory and velocity tracking, while a combination of the longitudinal - lateral control method was studied in [6].

This paper proposes an architecture based on two predictive algorithms designed to control the lateral position and the longitudinal velocity of a vehicle. The controllers are developed using the linear model of the vehicle and they are applied for the nonlinear model of the vehicle. To involve both controllers, an overpassing is simulated; this scenario uses the nonlinear model of the vehicle which has the dynamics closer to the one of real vehicles.

The rest of paper is organized as follows. Section II deals with the vehicle dynamics modeling, in which the lateral and longitudinal motions are described. A nonlinear model is firstly determined and then is linearized to be used in the design phase of the predictive controller. The third section presents the MPC strategy, which uses the linear state space model of the system to predict its behavior and to compute a control command that minimizes the error between the imposed reference and the output of the system. In section IV, the controller is tested on the nonlinear model of the system and the results are analyzed. Section V presents the conclusions and future research directions.

VEHICLE MODELING

The modeling of vehicle dynamics, which is defined by the lateral and longitudinal motions, has been a concern of many researchers in the last years. In what follows, the vehicle dynamics modeling is briefly described.

Considering a coordinate system Oxyz, the longitudinal dynamics describes the motion of vehicle along axis x, the lateral dynamics describes the motion along axis y and the rotation around z axis is described by the yaw angle of the vehicle, ψ . In this paper, the lateral dynamics is described by a nonlinear model which is then linearized to be used in the design phase of the MPC algorithm. Because the nonlinear lateral model has a similar behavior with the real vehicle, the predictive controller is tested on it. Moreover, the longitudinal model is used to design a controller for the longitudinal velocity of the vehicle.

A. Nonlinear Bicycle Lateral Model

The vehicle front and rear axles are joined [1], so the control of the direction is done only by the front wheel.

The bicycle model is the most used model to describe the vehicle lateral dynamics. The lateral and longitudinal forces that act on the tires are represented in Fig. 1. These forces are represented by the cornering front and rear forces, F_{cf} , F_{cr} and

longitudinal front and rear force F_{lr} , F_{lr} . The components of these forces are represented by the front and rear forces acting along the lateral velocity axis F_{yf} , F_{yr} and front and rear forces acting along the longitudinal velocity axis, F_{xf} , F_{xr} .

The nonlinear model that describes the lateral and longitudinal motions of the vehicle has the form $\dot{\zeta} = f(\zeta, F_{yf}, F_{yr})$ where, $\zeta = [y\ \dot{y}\ \psi\ \dot{\psi}]^T$ in which y represents the lateral position, \dot{y} represents the lateral velocity, ψ represents the yaw angle of the vehicle in the coordinate system Oxyz that has the center in the center of gravity of the vehicle, $\dot{\psi}$ represents the yaw rate and F_{yf}, F_{yr} represent the front and rear forces acting along the lateral velocity axis.

The nonlinear bicycle model is described as [1]:

$$m\ddot{y} = -m\dot{x}\dot{\psi} + 2F_{vf} + 2F_{vr} \tag{1}$$

$$I\ddot{\psi} = 2l_{f}F_{vf} - 2l_{r}F_{vr} \tag{2}$$

where m represents the mass of the vehicle. The distance from the center of gravity of the vehicle to the front axle is represented by l_f and the distance from the center of gravity of the vehicle to the rear axle is represented by l_r . The lateral forces are described by [1]:

$$F_{vf} = C_f(\delta_f - \alpha_f) \tag{3}$$

$$F_{vr} = C_r(-\alpha_f) \tag{4}$$

where C_f and C_r represent the cornering stiffness coefficients of the tires. The steering angle of the front tire is represented by δ_f and steering angle of the rear tire is represented by δ_r which is considered equal to zero, i.e., $\delta_r = 0$. The angles of the tires are denoted by α_f and α_r and are described by [8]:

$$\alpha_r = \arctan\left(\frac{v_{cr}}{v_{lr}}\right) \tag{5}$$

$$\alpha_r = \arctan\left(\frac{v_{cr}}{v_{lr}}\right) \tag{6}$$

where v_{cf} , v_{cr} , v_{lf} , v_{lr} represent the lateral and longitudinal velocities of the tires and are described by [8]:

$$v_{cf} = v_{vf} \cos(\delta_f) - v_{vf} \sin(\delta_f) \tag{7}$$

$$v_{cr} = v_{vr}\cos(\delta_r) - v_{xr}\sin(\delta_r)$$
 (8)

$$v_{tf} = v_{xf} \cos(\delta_f) + v_{yf} \sin(\delta_f)$$
 (9)

$$v_{lr} = v_{xr}\cos(\delta_r) + v_{yf}\cos(\delta_r)$$
 (10)

The velocity components v_{xf} , v_{xr} , v_{yf} , v_{yr} can be computed using the relations [1]:

$$v_{vf} = \dot{y} + l_f \dot{\psi}, \ v_{xf} = \dot{x}$$
 (11)

$$v_{yy} = \dot{y} - l_y \dot{\psi}, \ v_{yy} = \dot{x}$$
 (12)

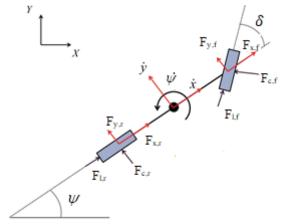


Fig. 1. Nonlinear bicycle lateral dynamics [7]

B. Linear Bicycle Lateral Model

In this section some simplifications are considered to obtain a linear bicycle model from the nonlinear model described by (1)-(12). The linear model will be used in the design of the predictive algorithm that controls the lateral position of the vehicle. The first simplification is to consider the steering angle of the front tire, δ_f , smaller than 0.1745 rad. This leads to the following approximation: $\sin(\delta_f) \cong \delta_f$ and $\cos(\delta_f) \cong 1$. Considering small values for the velocity angles of the tires, α_f and α_r , one can obtain [1]:

$$\alpha_f = \arctan\left(\frac{v_{cf}}{v_{lf}}\right) \cong \frac{v_{cf}}{v_{lf}}$$
 (13)

$$\alpha_r = \arctan\left(\frac{v_{cr}}{v_{lr}}\right) \cong \frac{v_{cr}}{v_{lr}}$$
 (14)

Using these simplifications and the nonlinear model defined by (1) - (12), the linear bicycle lateral model can be represented in a state-space form as [1]:

in which the input of the system is represented by the steering angle of the front tire δ_f . To obtain the linear model, the longitudinal velocity was considered constant $\dot{x} = v_x = const.$

For the lateral position of the vehicle to follow the imposed reference it is necessary to add an integrator to the model (15):

$$\dot{\varsigma}_{vi} = y_{ref} - y \tag{16}$$

where y_{ref} is the reference of the lateral position of the vehicle.

C. Linear Longitudinal Model

The longitudinal model is used to describe the relation between the vehicle acceleration and the forces that influence the longitudinal motion of vehicle, that are illustrated in Fig. 2.

The rolling resistance forces that act on the tires are represented by R_{xf} and R_{xr} for the front and rear tires, respectively. The forces F_{xf} and F_{xr} represent the components of the longitudinal forces that act on the front and rear tires, respectively.

The longitudinal vehicle motion is described by [9]

$$m\frac{dv_x}{dt} = F_x - mg\sin\theta - fmg\cos\theta - 0.5\rho AC_d(v_x - v_w)^2$$
 (17)

where F_x is the longitudinal traction force, $v_x = \dot{x}$ is the longitudinal velocity of the vehicle, v_w is the wind speed, g is the gravitational acceleration, θ is the road slope, ρ is the air density, C_d is the drag coefficient, f is the rolling resistance coefficient and f is the vehicle frontal area. The friction force with the air is represented by f aero and represents the last term of the equation (17).

Linearizing (17) by considering a nominal operation point with $v_x = v_0$ and $\theta = \theta_0$, a second order model is obtained

$$\ddot{x} = -\frac{1}{\tau_{v_{x}}} \dot{x} + \frac{K_{v_{x}}}{\tau_{v_{x}}} (u + w)$$
 (18)

where x is longitudinal position and u is the longitudinal traction force and

$$\tau_{v_x} = \frac{m}{\rho A C_d (v_x - v_w)} \tag{19}$$

$$K_{v_x} = \frac{1}{\rho A C_d (v_x - v_w)}$$
 (20)

$$w = mg(f\sin\theta_0 - \cos\theta_0)\theta \tag{21}$$

For the longitudinal vehicle speed to follow the velocity reference, an extended model is constructed by adding an integrator:

$$\dot{\varsigma}_{v_x i} = v_{xref} - v_x \tag{22}$$

where v_{xref} is the reference of the longitudinal velocity of the vehicle. The extended longitudinal model will be used to design a predictive controller for the velocity of the vehicle.

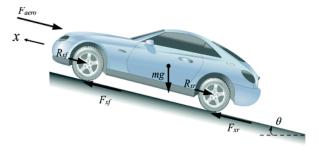


Fig. 2. Longitudinal dynamics of the vehicle

III. PREDICTIVE CONTROL FRAMEWORK

A. Problem formulation

The lateral dynamics model describes the lateral motion of the vehicle. The linear model of the lateral dynamics described in Subsection II.B is used to design the predictive controller and the nonlinear model of the lateral dynamics described in Subsection II.A is used to test the performances obtained by the controller. The output of the controller is represented by the steering angle of the front tire.

The linear longitudinal dynamics model described in Subsection II.C is used to design a predictive controller for the longitudinal velocity of the vehicle. The controller output is the longitudinal traction forces F_x . This force must be computed to ensure reference tracking and disturbance rejection.

B. Predictive Control Algorithm

Model based predictive control is an advanced methodology for process control that can handle constraints by including them in the design phase. The methodology offers several important advantages: the model of the system includes the dynamic and static interactions between inputs, outputs and disturbances, the constraints on the outputs and inputs are considered in a systematic manner. As such, accurate predictions of the system outputs can provide early warnings of potential problems.

Consider a system described by a discrete-time model:

$$\varsigma(k+1) = A\varsigma(k) + Bu(k) \tag{23}$$

$$z(k) = C\varsigma(k) \tag{24}$$

where the pair (A, B) is controllable and the pair (C, A) is detectable.

Then, the predictive control problem is defined as: given an initial state $\varsigma_0 = \varsigma(0)$, compute a finite horizon input sequence $\{u_0,u_1,\cdots,u_{_{N-1}}\}$ that minimizes the finite horizon cost function:

$$V(\varsigma, u_0, u_1, ..., u_{N-1}) = \varsigma_N^T P \varsigma_N + \sum_{i=0}^{N-1} (\varsigma_i^T Q \varsigma_i + u_i^T R u_i)$$
 (25)

where N is the prediction horizon, P and Q are the weight matrices of the states of system and R is the weight matrix of the control command. The performances of the system are determined by the proper choice of the parameters $Q \ge 0$ $R \ge 0$, P and N. The vector ς_i is the prediction of $\varsigma(k+i)$ given the current state $\varsigma(k)$ for all $i=1,\ldots,N$. The predictors of the system states are determined using the model of the system (23)-(24) and yields as:

$$\xi = \Phi_{\mathcal{G}} + \Gamma U \tag{26}$$

Thus, the cost function (25) can be rewritten in a matricial form

$$V(\varsigma, U) = \varsigma^{T} Q \varsigma + \xi^{T} \Omega \xi + U^{T} \Lambda U$$
 (27)

in which $size(\Omega) = [nN, nN], size(\Lambda) = [mN, mN], n$ is the number of states and m represents the number of inputs, $\Lambda = diag\{R, \dots, R\}, U = [u_0, u_1, \dots, u_{(N-1)}]^T, \xi = [\xi_1, \dots \xi_N]^T,$

$$\Omega = diag\{Q, \dots, Q, P\}, \ \Gamma = \begin{bmatrix} B & 0 & \dots & 0 \\ AB & B & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{N-1}B & A^{N-2}B & \dots & B \end{bmatrix}.$$

Because the real system has constraints on the inputs and outputs, $u_{low} \le u_i \le u_{high}$, $y_{low} \le y_i \le y_{high}$, these constraints must be considered when designing the controller. Thus, the constraints can be described using the following equations:

$$\Delta \zeta_0 + \Upsilon \xi + \varepsilon U \le c \tag{28}$$

where:
$$\Upsilon = \begin{bmatrix} 0 & \cdots & 0 \\ M_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & M_N \end{bmatrix}, \varepsilon = \begin{bmatrix} E_0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & E_{N-1} \\ 0 & \cdots & 0 \end{bmatrix}, c = [b_0 \cdots b_N]^T,$$

$$\begin{split} E_i &= [I-I \ 0 \ 0]^T, \Delta = [M_0...0]^T, i = \ 0,...,N-1, M_N = [-C \ C]^T, \\ M_i &= [0 \ 0-C-C]^T, b_N = [-y_{low} \ y_{high}]^T, b_i = [-u_{low} \ u_{high} \ -y_{low} \ y_{high}]^T, \\ size(M_i) &= [(m+p),n], \quad size(E_i) = [(m+p),1], \ p \ \text{represents} \\ \text{number of outputs.} \end{split}$$

Using these relations, the problem of constrained optimal control is given by:

$$\min_{U} \frac{1}{2} U^{T} G U + U^{T} F \varsigma \tag{29}$$

$$JU \le c + W\varsigma \tag{30}$$

where:

$$G = 2(\Lambda + \Gamma^T \Omega \Gamma) \succ 0$$
, $(\Omega \succ = 0 \text{ and } \Lambda \succ 0)$ (31)

$$F = 2\Gamma^T \Omega \Phi \tag{32}$$

$$J = \Upsilon \Gamma + \varepsilon \tag{33}$$

$$W = -\Delta - \Upsilon \Phi \tag{34}$$

IV. SIMULATION RESULTS

The predictive controller for the longitudinal velocity of the vehicle was designed using the linear model (18)-(22) and the controller for the lateral position of vehicle was designed using the linear model (15)-(16).

The control structure for the longitudinal dynamics of the vehicle is illustrated is Fig. 3, in which the output x represents the longitudinal position of the vehicle and v_x represents its longitudinal velocity, while the input is represented by longitudinal traction force. The input w from Fig. 3 represents a disturbance for the longitudinal dynamics of the vehicle computed using (21).

The predictive controller for the longitudinal dynamics was designed using the following parameters: N = 10 - prediction horizon, $Q = 75I_3$, $P = 100I_3$, $R = 2.3529*10^{-4}$, where I_3 represents the 3^{rd} order identity matrix.

The longitudinal velocity of the vehicle is illustrated in Fig. 4 and the traction force, F_x , is illustrated in Fig. 5. It can be observed from Fig. 4 that the controller succeeds to compensate the disturbance w and the control signal (the force F_x) satisfies the imposed constraint, $0N \le F \le 2000N$.

The small value of the parameter R is due to the high value of the control signal, i.e., longitudinal traction force. These small values reduce the influence of the input in the cost function (25), otherwise, the states of the system would have a negligible influence on the cost function value.

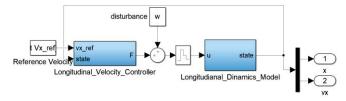


Fig. 3. Longitudinal control structure

The parameters of the vehicle used in simulations are described in Table I:

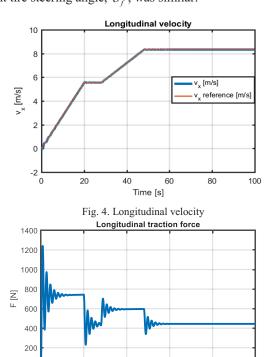
TABLE I. VEHICLE PARAMETERS

	Vehicle Parameters	
	Name	Value
m	Vehicle mass	1094 Kg
C_f	Cornering stiffness coefficient (front)	63291 N/rad
C_r	Cornering stiffness coefficient (rear)	50041 N/rad
l_f	Longitudinal distance from the center of gravity to the front tires	1.108 m
l_r	Longitudinal distance from the center of gravity to the rear tires	1.392 m
I	Rotational inertia of the vehicle	1608 kg. m ²
$\delta_{f ext{min}}$	Minimum value of front wheel steering angle	-0.1745 rad
$\delta_{f\max}$	Maximum value of front wheel steering angle	0.1745 rad
g	Gravitational acceleration	9.81 m/s ²
v_0	Initial velocity of vehicle	0 m/s
$ heta_0$	Road slope	0 rad
ρ	Air density	1.202 Kg/m ³
A	Vehicle frontal area	1.5 m ²
C_d	Drag coefficient	0.5
f	Rolling resistance	0.0015
v_w	Wind speed	2 m/s
F_{xmin}	Minimum value of the longitudinal traction force	0 N
$F_{x \max}$	Maximum value of the longitudinal traction force	2000 N

The predictive controller for the lateral dynamics was designed using the following parameters: N = 10 – prediction horizon, $Q = 8I_5$, $P = 10I_5$, R = 0.02, where I_5 represents the 5th order identity matrix. The controller parameters, i.e., Q, R, P, N, were chosen to obtain a response with good performances.

The control structure of the lateral dynamics is represented in Fig. 6. In this structure the first output represents the lateral position of the nonlinear model of the vehicle and the second output represents the lateral position of the linear lateral model. The nonlinear lateral model of the vehicle was implemented using (1) - (12). The control signal is represented by the steering angle of the front tire. The results

for vehicle lateral control are illustrated in Fig. 7 and Fig. 8 for the linear and nonlinear models of the lateral dynamics, respectively. Form Fig. 7 and Fig. 8 it can be observed that both dynamics, linear and nonlinear, have responses that follow the reference. This means that the controller designed with the linear model has good results for the nonlinear model. The lateral position follows the desired trajectory and the commands (front wheel steering angle) illustrated in Fig. 9 and Fig. 10 satisfy the constrains. The predictive controller was designed using the linear model considering the longitudinal velocity constant, i.e., $v_x = 5.55 m/s$. In the simulation, the same controller was used for both models, linear and nonlinear. The nonlinear model of the lateral dynamics was linearized considering the front tire steering angle, δ_f , smaller than 0.1745 rad, then the predictive controller was designed considering that $-0.1745rad \le \delta_f \le 0.1745rad$. From Fig. 8 it is observed that this constraint is satisfied. For the nonlinear lateral dynamics model, the constraint on the front tire steering angle, δ_f , was similar.



An overpassing is a maneuver in which both controllers, one for the longitudinal velocity and one for the lateral position, must work together. A vehicle has to changes it lateral position in order to overpass another vehicle and in the same time it has to increase or maintain its longitudinal velocity. The aim is to simulate a case as close to reality as possible.

Time [s] Fig. 5. Longitudinal traction force

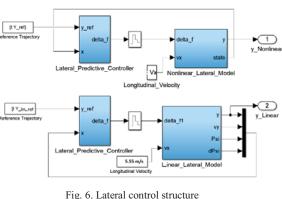
100

As such the imposed reference is a trajectory to be followed by the ego vehicle, which was generated using [10]:

$$y_{ref} = \frac{d_{y_1}}{2} (1 + \tanh(z_1)) - \frac{d_{y_2}}{2} (1 + \tanh(z_2))$$
 (35)

where
$$z_1 = a_1(x - x_1) - 1.2$$
, $z_2 = a_2(x - x_2) - 1.2$, $d_{y1} = 3.5m$, $d_{y2} = d_{y1}$, $x_1 = 170.19m$, $x_2 = 320.46m$, $a_1 = 0.096$, $a_2 = a_1$.

Equation (35) models a curvature like the trajectory illustrated in Fig. 11. The parameter d_{y1} represents the maximum value of y_{ref} and the parameter d_{y2} represents the difference between d_{y1} and the minimum value of y_{ref} . The longitudinal position of the ego vehicle when it starts to overpass the vehicle in front of it is x_1 and the longitudinal position of the vehicle when it arrives on the initial lane is x_2 . The parameters a_1 and a_2 determine the curvature steep.



Response of linear lateral model

3.5
3
2.5
2
1.5
1
0.5

Fig. 7. Lateral position of linear model

Response of nonlinear lateral model

y - reference [m]

y [m]

y - reference [m]

y [m]

2.5

1.5

0.5

0

10.5

0

Time [s]

Fig. 8. Lateral position of nonlinear model

Consider two vehicles which have the d_{12} distance between them. The first vehicle has the velocity v_1 equal to $4.166 \, m/s$ and is in front of the second vehicle (ego vehicle) which has the velocity v_2 described in Fig. 4 in the first 28 seconds. Both are on the same lane and have the same direction of movement. When the distance between the vehicles is $d_{change} = 25m$ which is smaller the d_{12} , $d_{change} < d_{12}$, then the second (ego) vehicle will change the lane to overpass the vehicle in front of it. Before this maneuver, the ego vehicle will increase its velocity. When it changes the lane it will have velocity, v_3 , described in Fig. 4

after second 28, $v_3 > v_2 > v_1$. After overpassing the first vehicle, it will return to the initial lane. During the overpassing, the first vehicle will maintain its velocity constant. The width of a lane is equal to 3.5m and the lines y = 0 and y = 3.5m from Fig. 11 represent the middle of the lanes. The simulation results are illustrated in Fig. 11 and Fig. 12. The trajectory of the vehicle in front is represented by a red line and the trajectory of the ego vehicle is represented with a blue line. It can be noticed that the ego vehicle is capable of overpassing the other vehicle. Also, it can be observed that when the ego vehicle arrives again on the first lane, the distance between the vehicles is approximately 25 m; this means that a collision between them is not possible. In Fig. 11 and Fig. 12 the points emphasized on the blue line marked with green color represent the position in which the vehicle starts the overpassing, the position in which it decides to return on the initial lane and the position in which it arrives on the middle of the initial lane. The points marked with black on the red line correspond to the positions of the first vehicle at the equivalent moments of time in which the ego vehicle takes the actions described above. For this simulation the nonlinear lateral model (1) -(12) was used. This model allows the vehicle to have a varying longitudinal velocity over time, which is described in Fig. 4.

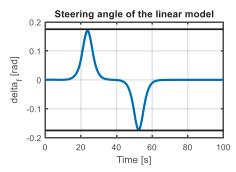


Fig. 9. Command for the linear model

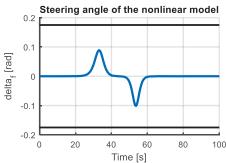


Fig. 10. Command for the nonlinear model

V. CONCLUSIONS

This paper proposes the design of two model based predictive controllers for the longitudinal and lateral dynamics of a vehicle. Because the longitudinal dynamics influences the lateral dynamics, the simulations were done combining the two dynamics. A predictive controller for the longitudinal velocity of vehicle was designed so that the velocity follows the reference and the constraints on the longitudinal traction force are satisfied. The second predictive controller was designed for the trajectory control of a vehicle. This algorithm is designed based on the linear model of the vehicle lateral dynamics, but its final role was to control the lateral position of a nonlinear lateral model, which has a more similar dynamics with a real vehicle. The control algorithm succeeded

to control the lateral position so that the reference trajectory was followed and the constraints of the command signal were satisfied. The simulations were performed in Matlab/Simulink and the results illustrate good performances, obtained by the proposed control architecture. Future work will focus on improving the control strategy and on designing an appropriate trajectory planner.

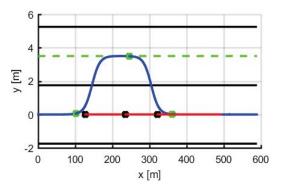


Fig. 11. Simulation of a passing

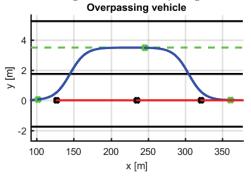


Fig. 12. Detail of overpassing

REFERENCES

- [1] R. Rajamani, "Vehicle Dynamics and Control, Springer", USA, 2006.
- [2] T. Keviczky, P. Falcone, F. Borrelli, J. Asgari, D. Hrovat, "Predictive control approach to autonomous vehicle steering", American Control Conference Minneapolis, pp. 4670-4675, USA, June 2006.
- [3] P. Falcone, F. Borrelli, H. E. Tseng, J. Asgari, D. Hrovat, "A Hierarchical Model Predictive Control Framework for Autonomous Ground Vehicles", American Control Conference, pp. 3719-3724, USA, June 2008.
- [4] Z. Ercan, M. Gokasan, F. Borrelli, "An Adaptive and Predictive Controller Design for Lateral Control of an Autonomous Vehicle", IEEE International Conference on Vehicular Electronics and Safety, pp. 5090-5676, Austria, June 2017.
- [5] J. Jiang and A. Astolfi, "Lateral control of an autonomous vehicle", IEEE Transactions on Intelligent Vehicles, 3(2), pp. 228-237, June 2018.
- [6] F. Lin, Y. Zhang, Y. Zhao, G. Yin, H. Zhang, K. Wang, "Trajectory Tracking of Autonomous Vehicles with the Fusion of DYC and Longitudinal–Lateral Control", Chinese Journal of Mechanical Engineering, February 2019.
- [7] R. Attia, R. Orjuela, M. Basset, "Combined longitudinal and lateral control for automated vehicle guidance", Vehicle System Dynamics, 52 (2), pp. 261-379, July 2014.
- [8] D. Margolis, J. Asgari, "Multipurpose Models of Vehicle Dynamics for Controller Design", SAE Tehnical Paper, 1991.
- [9] A. G. Ulsoy, H. Peng, M. Çakmakc, "Automotive Control Systems", Cambridge University Pres, USA, 2012.
- [10] Y. Gao, "Model Predictive Control for Autonomous and Semiautonomous Vehicles", University of California, Berkeley, 2014.