VSP Report

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This is to certify that this report submitted by **K.M. Devansh**, to Raman Research Institute, Bengaluru, is a record of the original work carried out by him under my supervision. The contents of this report, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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CONTENTS

I. INTRODUCTION

The realization of quantum phenomena when particles are brought to temperatures fractionally above absolute zero push forth the boundaries of what is known in the classical world. Trapping and further studying them opens a realm no longer dictated by Maxwell-Boltzmann statistics, and it is this that is at the heart of the QuMix laboratory - in particular, sodium and potassium atoms are employed to elucidate said exotic states. To be able to confine atoms and isolate them in a separate system has, in recent decades, propelled phenomena such as the Bose-Einstein condensates. Matter waves happen to be very small when calculated for atoms in room temperature where their average velocity is known to be high. When temperatures reach close to absolute zero, the characteristic wavelength increases and parallels the average spacing between the atoms - specifically for Bosons, the atoms start overlapping and a condensate forms.

One way to cool these atoms requires the tapping into of optical molasses through Doppler cooling methods. A single frequency laser is used to dampen atomic motion by having it tuned slightly below atomic resonance of the atoms so as to allow the Doppler effect to induce a dissipative light force that is directed opposite to the motion of the atoms. Counter propagating lasers act to create the so-called optical molasses. Coupled with this in the MOT (Magneto-optical trap) apparatus, a spatially varying magnetic field in the form of two coils in an anti-Helmholtz configuration induces a trap that retains the atomic cloud for an extended period of time, allowing the study and investigation of their properties to take place

II. OPTICAL DIPOLE TRAP IN 1-D

Most experiments use alkali atoms when trapping and cooling as they provide a optimal spectral range and their optical transitions. The following equation and subsequent derivations outline the premise of the mechanics of trapping and cooling atoms.

The derived potential of a ground state with total angular momentum F and magnetic quantum number m_F, one of the four quantum numbers that help distinguish between orbitals in a given sub-shell, is given by the following:

$$V(x,y,z) = \frac{\pi c^2 \Gamma}{2\omega_0^3}(\frac{2 + P g_F m_F}{\Delta_{2,F}} + \frac{1 - P g_F m_F}{\Delta_{1,F}})\frac{2P}{\pi w_0^2} I(x,y,z) \tag{1}$$

where P as the laser polarization, g_F relates to the Lande factor, Gamma is the natural line width/decay rate, w_0 is the beam waist, and ω_0 is the angular laser frequency.

If the beam is propagating in one direction, say z, then we have the following expression as the one-dimensional dipole potential.

$$V_{1D-dip}(x,y,z) = V_{dip}\frac{1}{1 + \frac{z^2}{z_0^2}} e^{-2(x^2+y^2)/w^2(z)} \tag{2}$$

and when $x, y = 0$ since the beam is propagating only in the z-direction, equation 1 becomes:

$$V_{dip} = \frac{\pi c^2 \Gamma}{2\omega_0^3}(\frac{2 + P g_F m_F}{\Delta_{2,F}} + \frac{1 - P g_F m_F}{\Delta_{1,F}})\frac{2P}{\pi w_0^2} \tag{3}$$

A. Taylor Approximation of Dipole Potential

By condensing equation 2 with the Taylor expansion, the subsequent approximation can easily become applicable in experimental calculations. To do so, I separately Taylor expanded the first (quadratic) term in the expression using the general expansion formula:

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Let us find the first two derivatives of the quadratic expression from our equation, and substitute zero into them :

$$f(z) = \frac{1}{1 + \frac{z^2}{z_0^2}}$$

$$f(0) = \frac{1}{1+0} = 1$$

$$f'(z) = \frac{(z_0^2 + z^2)(0) - z_0^2(2z)}{(z_0^2 + z^2)^2}$$

$$f'(0) = 0$$

$$f''(z) = \frac{8z_0 z^2}{(z_0^2 + z^2)^3} - \frac{-2z_0^2}{(z_0^2 + z^2)^2}$$
$$f''(0) = -\frac{2}{z_0^2}$$

With the first two derivatives, we have the Taylor expansion for the first term as:

$$f(z) \approx 1 + 0 + \frac{\frac{-2}{z_0^2}}{2!} z^2$$
$$\approx 1 - \frac{z^2}{z_0^2}$$

Let us now expand the second term of the V_{dip} equation - this one is exponential now:

$$g(z) = e^{\frac{-2(x^2+y^2)}{\omega^2(z)}}$$

Where:

$$\omega(z) = \omega_0 \sqrt{1 + \frac{z^2}{z_0^2}}$$
$$g(z) = e^{\frac{-2(x^2+y^2)}{\omega_0^2 (1+\frac{z^2}{z_0^2})}}$$

We can recognize that $\frac{1}{1+\frac{z^2}{z_0^2}}$ was previously Taylor expanded, so let us instead rewrite the above expression as follows:

$$g(z) = e^{\frac{-2(x^2+y^2)}{\omega_0^2}(1-\frac{z^2}{z_0^2})}$$

Since the Taylor expansion for e^{-x} is given by $1 - x$, we can apply to the above expression:

$$g(z) \approx 1 - \frac{2(x^2 + y^2)}{\omega_0^2}(1 - \frac{z^2}{z_0^2})$$

Since we now have the expansion of both terms in the V_{1D-dip} expression, let us rewrite Eq 1

$$V_{1D-dip}(x, y, z) \approx V_{dip}[1 - \frac{z^2}{z_0^2}][1 - \frac{2(x^2 + y^2)}{\omega_0^2}(1 - \frac{z^2}{z_0^2})] \tag{4}$$

Evaluating the above gives us...

$$\approx 1 - \frac{z^2}{z_0^2} - \frac{2(x^2 + y^2)}{\omega_0^2} + \frac{2(x^2 + y^2)}{\omega_0^2}\frac{z^2}{z_0^2} + \frac{2(x^2 + y^2)}{\omega_0^2}\frac{z^2}{z_0^2}$$

$$\approx 1 - \frac{z^2}{z_0^2} - \frac{2(x^2+y^2)}{\omega_0^2} + \frac{4(x^2+y^2)}{\omega_0^2}\frac{z^2}{z_0^2}$$

Our constraints for the Taylor expansion are given by $z \ll z_0, x^2 + y^2 \ll \omega_0^2$, so we can make redundant the third term and discard it as the following is only an approximation.

$$V_{1D-dip-approx} = V_{dip}[1 - 2\frac{x^2+y^2}{\omega_0^2} - (\frac{z}{z_0})^2] \tag{5}$$

III. EXPERIMENTAL APPLICATIONS OF DIPOLE TRAPS

A. Calculation of Laser Beam Power

When placed in a dipole trap, it useful to know how much laser power from the incoming beam is required to trap atoms that are of a certain temperature, so as to adjust as necessary when proceeding with experimentation.

With equation 3, I used python to be able to calculate the laser power required to trap alkali atoms at a certain temperature. Since we know that potential is given by

$$V = k_B T$$

where $k_B T$ is the Boltzmann constant and T is the temperature in Kelvin, we can manipulate equation 3 to solve for beam power while having temperature as the independent variable.

$$k_B T = \frac{\pi c^2 \Gamma}{2\omega_0^3}(\frac{2 + Pg_F m_F}{\Delta_{2,F}} + \frac{1 - Pg_F m_F}{\Delta_{1,F}})\frac{2P}{\pi w_0^2}$$

We can have P, laser polarization, to be linear and therefore 0. Since the term $Pg_F m_F$ becomes zero, and the detunings are equal, the equation reduces to the following:

$$k_B T = \frac{\pi c^2 \Gamma}{\omega_0^3 w_0^2}(\frac{3}{\Delta})P$$

Rearranging for power gives us:

$$P = \frac{k_B T \omega_0^3 w_0^2}{c^2 \Gamma}(\frac{\Delta}{3})$$

If we have the values for sodium at a temperature at 2.2×10^{-6} kelvin, a beam waist (the one used in the lab) at $135 \times 10^{-6}/2$ meters, and 9.794 MHz as the value for gamma, and then similarly found values for potassium, they can be compared in the following manner.

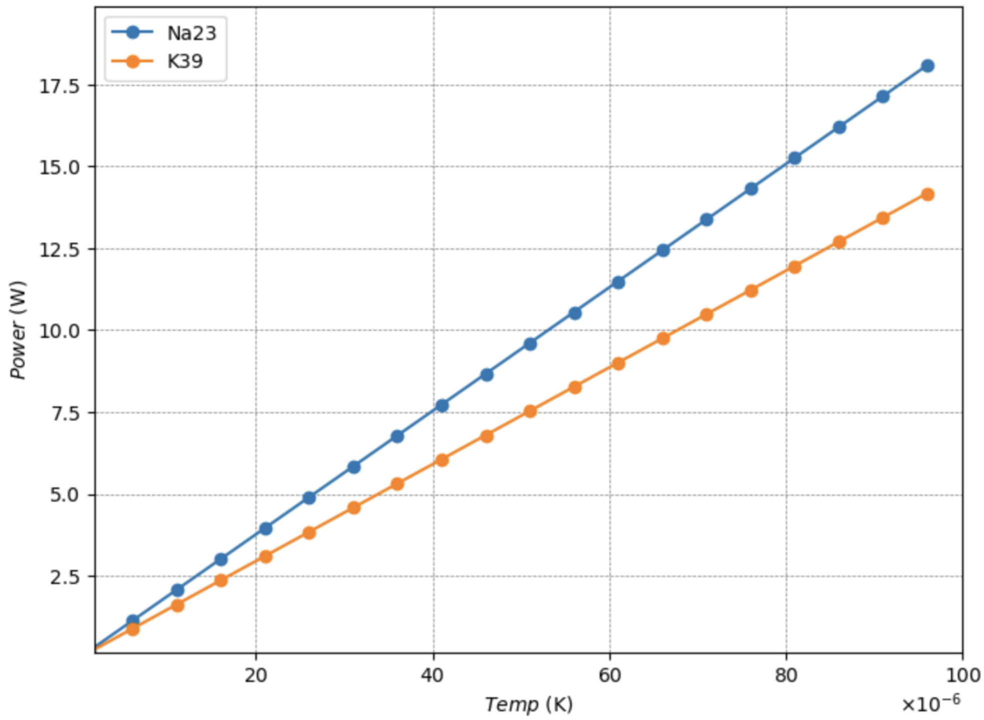

FIG. 1. Relationship between temperature of atoms and power of laser beam needed to trap them, corresponding to a 1D dipole trap.

As seen in the above graph, we have the heavier sodium atom requiring a lesser amount of laser power compared to the potassium atom. This can attributed to the heavier atom carrying more momentum and requiring lesser external force to remain in a trapped state, considering they are both at the same temperature at any given instant.

A further outlook in this investigation would be to try to find a relationship between the lifetime of an atom and the temperature that it is trapped in.

B. Calculation of Trapping frequency

If we continue from equation 5, we can use polar coordinates and have $r^2 = x^2 + y^2$. Since we are working in this coordinate system, we can drop the first and last terms in equation 5, giving us the following:

$$V_r = V_{dip} \cdot 2\frac{r^2}{w_0^2} = \frac{1}{2}k_r r^2$$

$$\implies \omega_r = \sqrt{\frac{k_r}{m}} = \sqrt{\frac{4V_{dip}}{mw_0^2}}$$

Having previously found v_{dip} and then rearranging for power of a laser beam, I used python to compute the trapping frequency at a given temperature.

```
#Sodium 23 trapping frequency at 2.2e-6 Kelvin

v_dip = (c**2 * gamma * (3 / delta) * 0.4144 ) / ((omega_n)**3 * (beam_waist)**2)
m = 3.8e-26
omega_r = math.sqrt((4 * v_dip) / (m * beam_waist))
print("Trapping frequency of a sodium atom is ", omega_r, "Hz")

Trapping frequency of a sodium atom is  6.882073999848898 Hz
```

FIG. 2. Python code to calculate trapping frequency of a sodium atom. The trapping frequency as shown for a sodium atom at 2.2e-6 K is shown to be 6.88 Hz

IV. CHARACTERIZATION OF SPATIAL PROFILE OF LASER

I used the lab's 1064nm Azurlight laser to investigate its characteristics and efficiency. In order to do so, I accurately measured the waist of the laser beam in the x and y directions at its point of minima. Such characterization is necessary to ensure that it is accurately carrying out experimentation when used, and that a systemic error does not persistent results. To do so, we used ThorCam (CS2100M), a running Matlab code, screw gauge to make precise distance measurements.

A. Experimental Setup and Data Analysis

With the camera positioned ahead of the minima of the laser which was running at 3W, I used the screw gauge (shown in FIG 4 as 'position adjustable ThorCam) to bring the camera within increments of 0.1 mm (or 100 microns) continually moving it closer to the minima. At each position, I captured its image matrix with a running matlab code and recorded its subsequent .csv file, with the intention of capturing the minima of the laser beam at some point along the measurements.

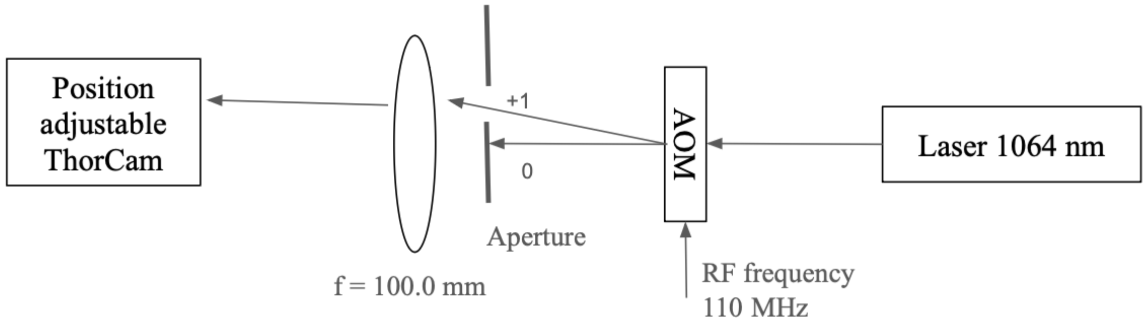

FIG. 3. Schematic diagram of experimental setup - The ThorCam (CS2100M) is mounted on a screw gauge and can move along the direction of the beam propagation. The lens helps focus the beam further and is positioned accordingly based on its focal length. An aperture was used to ensure that none of the other orders of diffracted light would interfere with the readings. The laser was running at 3W.

An AOM (Acousto-optic modulator) works on the basis of the acousto-optic effect, where a radio frequency wave is fed and due to the periodic variations in the compression of the wave that is triggered by a piezoelectric transducer attached to a material such as glass, it creates a moving diffraction grating. Through this, the input laser beam is split into its diffraction pattern, of which we used the 1st order to focus into the camera. The AOM was positioned such that the first order passes through the aperture and is collimated by the lens before striking the ThorCam.

B. Gaussian Fitting of Collected Data

The following code performs the Gaussian fit needed on the arrays of the sum of rows and columns, separately.

```python
def gauss_fit(x,mu,sigma,C,A):
    y = C+(A*(np.exp((-1*(x-mu)**2)/(2*sigma**2))))
    return y
```

FIG. 4. Gaussian function in python

In one set of readings (out of three), I made 200 measurements which gave me 200 .csv files. I proceeded to fine tune the initial conditions of the Gaussian function on python to accurately capture all 200 measurements. The arrays of sums of rows and columns were initially normalized, but I noticed that the Gaussian fit did not greatly differ from before, so I continued to use the original fits. Shown below is an example of how I would fit such data, as well as an example of the initial conditions given to the *popt* function:

```
popt, pcov = curve_fit(gauss_fit, x, row_sum_partial,p0=[250,100,2.2e6,0],maxfev=100000)
```

FIG. 5. Gaussian fitting plotted in python - the initial conditions are given in the argument p0

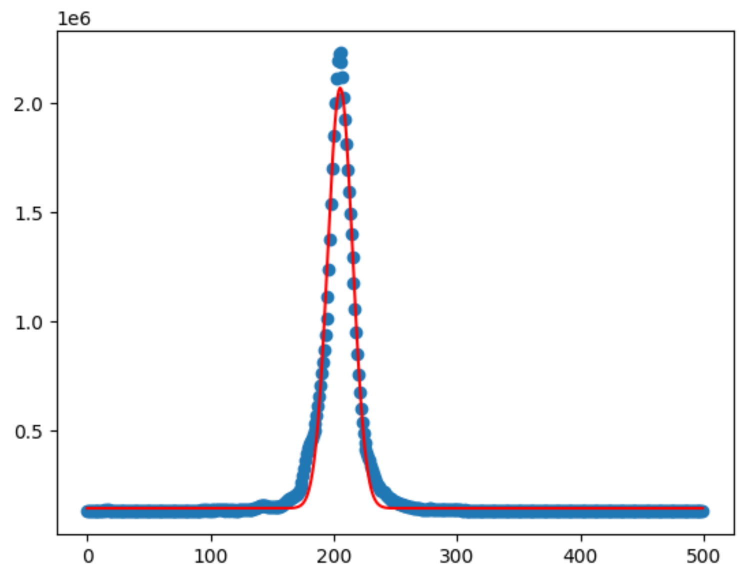

FIG. 6. Gaussian fitting plotted in python with initial conditions that I thought to be accurate for this particular set of values. In this way, I would change the file name in the code that accesses the files (each corresponding to a different position of the camera) to verify for a host of random measurements and their ability to adjust to the initial conditions that I provided as input for the Gaussian fit. This is so that when I run the loop to fit all 200 files, all the standard deviations would be accurate

The output of the function *popt* for the example fitting was given as $[2.05130416e+021.01193425e+011.45191781e+051.92684176e+06]$. In this case, the second value, 1.0119 corresponds to the standard deviation of this Gaussian fit. This value was found across the array of sums of rows and collected into a separate array. The same was done with the sum of columns from all 200 readings.

These were then graphed on Origin to produce a visual of the spatial profiling of the beam. In the graph, the sigma_row and sigma_column refer to the standard deviation of the respective Gaussian fits. Upon graphing it initially, I noticed that my movement of the camera did not sufficiently capture the full extent of values needed to accurately characterize the spatial width of the laser beam. So I repeated the measurement three times, each time starting at a different position so as to capture values that I perhaps did not cover the previous time. Each of the three attempts are shown in the following graphs.

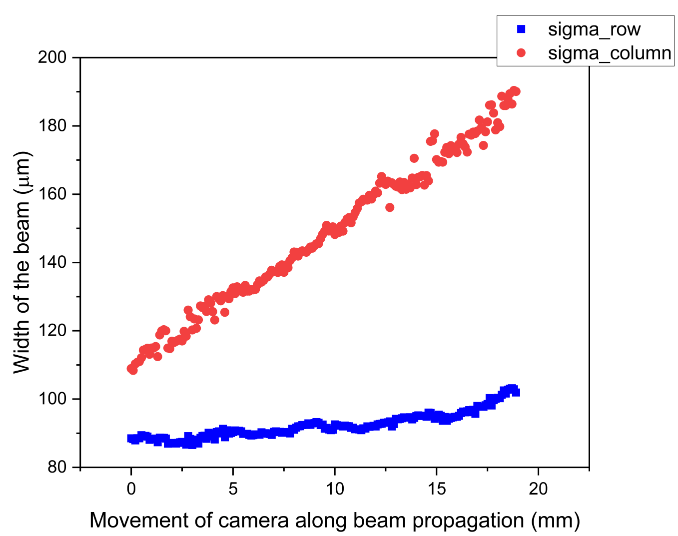

FIG. 7. First set of measurements. Since the data points in this set convey that the camera was positioned slightly off the expected minima of the laser beam, I moved the initial position of the camera in the other direction for the next day.

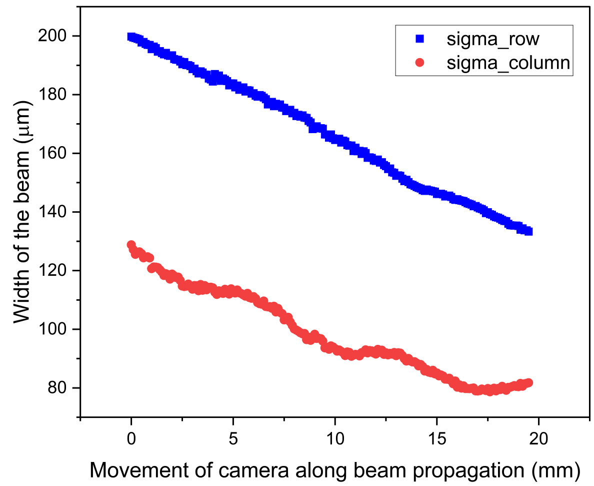

FIG. 8. Second set of measurements. This time, I noticed that the camera was perhaps pushed too far in the other direction, and so I adjusted the initial position again for a third attempt.

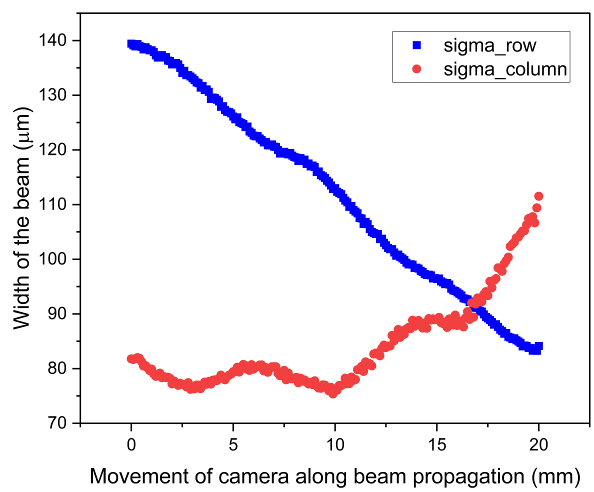

FIG. 9. Third set of measurements. Since 1 pixel from the camera is 5 microns, I multiplied the values on the y-axis by 2 since the standard deviation only provides half the width of the laser beam. Similarly, the x-axis was changed to reflect values in mm of the camera movement along the propagation of the beam.

The investigation of the laser beam reveals that there are discrepancies that need to be addressed. Ideally, the width of the beam should be equal throughout the measurement, or at least at the point of minima (where the width of the beam is the smallest).

Another point of interest would be to investigate why it would differ so largely on the scale of millimeters - the erroneous result would lead to incorrect experimental results if used externally. One possibility is that there might be a problem with the camera, considering the degree to which the discrepancy exists, but further characterization of different setups is required to verify the root of the problem.

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[3] T. Salez, *Towards quantum degenerate atomic Fermi mixtures*, Ph.D. thesis, Ecole Normale Supérieure de Paris (2011).