A MASTER FORMULA FOR NNLO SOFT AND VIRTUAL QCD CORRECTIONS *

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I present a master formula for the next-to-next-to-leading order (NNLO) soft and virtual QCD corrections for any process in hadron-hadron and lepton-hadron colliders. The formula is derived from a unified threshold resummation formalism. Applications to various QCD processes are discussed.

1 Introduction

Calculations of total and differential cross sections in perturbative QCD can be schematically represented by

$$\sigma = \sum_{f} \int \left[\prod_{i} dx_{i} \, \phi_{f/h_{i}}(x_{i}, \mu_{F}^{2}) \right] \, \hat{\sigma}(s, t_{i}, \mu_{F}, \mu_{R}) \tag{1}$$

with σ the physical cross section, $\hat{\sigma}$ the perturbatively calculable hard scattering factor, and ϕ_{f/h_i} the parton distribution for parton f in hadron h_i .

The partonic hard-scattering factors $\hat{\sigma}$ include soft and virtual corrections, arising from soft-gluon emission and loop diagrams, which manifest themselves as plus distributions and delta functions with respect to a kinematical variable x_{th} that measures distance from threshold. In single-particle-inclusive (1PI) kinematics, x_{th} is often called $s_4 = s + t + u - \sum m^2$, and the plus distributions are $\mathcal{D}_l(s_4) \equiv [\ln^l(s_4/M^2)/s_4]_+$, with $l \leq 2n-1$ at nth order in α_s beyond the leading order. In pair-inclusive (PIM) kinematics, the relevant distributions are $\mathcal{D}_l(z) \equiv [\ln^l(1-z)/(1-z)]_+$ with $z = Q^2/s$ (Q^2 is of the pair). These

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distributions can be formally resummed to all orders in α_s [1-5]. Here I present a unified approach and a master formula [1] for explicitly calculating these corrections at NNLO for total and differential cross sections for any process in

- hadron-hadron and lepton-hadron colliders 1PI and PIM kinematics
- \bullet simple and complex color flows \bullet $\overline{\rm MS}$ and DIS factorization schemes

2 NNLO master formula

I begin by presenting a unified formula for the threshold resummed cross section in moment space for an arbitrary process

$$\hat{\sigma}^{res}(N) = \exp\left[\sum_{i} E^{(f_{i})}(N_{i}) + E^{(f_{i})}_{scale}(\mu_{F}, \mu_{R})\right] \exp\left[\sum_{j} E'^{(f_{j})}(N_{j})\right]$$

$$\times \operatorname{Tr}\left\{H\left(\alpha_{s}(\mu_{R}^{2})\right) \bar{P} \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}_{j}} \frac{d\mu'}{\mu'} \Gamma'^{\dagger}_{S}\left(\alpha_{s}(\mu'^{2})\right)\right]\right\}$$

$$\times \tilde{S}\left(\alpha_{s}(s/\tilde{N}_{j}^{2})\right) P \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}_{j}} \frac{d\mu'}{\mu'} \Gamma'_{S}\left(\alpha_{s}(\mu'^{2})\right)\right]$$

with, in the $\overline{\rm MS}$ scheme,

$$E^{(f_i)}(N_i) = -\int_0^1 dz \frac{z^{N_i-1}-1}{1-z} \left\{ \int_{(1-z)^2 s}^{\mu_F^2} \frac{d\mu'^2}{\mu'^2} A^{(f_i)} \left(\alpha_s(\mu'^2)\right) + \nu_{f_i} \left[\alpha_s((1-z)^2 s)\right] \right\}. \tag{2}$$

 $E_{scale}^{(f_i)}$ describes the factorization and renormalization scale dependence of the cross section, while $E'^{(f_j)}$ appears if there are any massless final-state partons at lowest order. H and S are the hard and soft functions and Γ_S' the soft anomalous dimensions, all matrices in color space in general (but they reduce to simple functions for processes with simple color flows) and known at lowest order for most processes. For more details see Ref. [1].

Expanding the resummed cross section in α_s and inverting back from moment space we can derive master formulas at fixed order for any process. The master formula for the next-to-leading-order (NLO) soft and virtual corrections for processes with simple color flow in the $\overline{\rm MS}$ scheme is

$$\hat{\sigma}^{(1)} = \sigma^B \frac{\alpha_s(\mu_R^2)}{\pi} \left\{ c_3 \mathcal{D}_1(x_{th}) + c_2 \mathcal{D}_0(x_{th}) + c_1 \delta(x_{th}) \right\}$$
(3)

where σ^B is the Born term, $c_3 = \sum_i 2C_{f_i} - \sum_j C_{f_j}$,

$$c_2 = 2 \operatorname{Re}\Gamma_S^{(1)} - \sum_i \left[C_{f_i} + 2C_{f_i} \, \delta_K \, \ln\left(\frac{-t_i}{M^2}\right) + C_{f_i} \ln\left(\frac{\mu_F^2}{s}\right) \right]$$

$$- \sum_j \left[B_j^{(1)} + C_{f_j} + C_{f_j} \, \delta_K \, \ln\left(\frac{M^2}{s}\right) \right] \equiv T_2 - \sum_i C_{f_i} \ln\left(\frac{\mu_F^2}{s}\right)$$
 (4)

and $c_1 = c_1^{\mu} + T_1$, with

$$c_1^{\mu} = \sum_i \left[C_{f_i} \, \delta_K \, \ln \left(\frac{-t_i}{M^2} \right) - \gamma_i^{(1)} \right] \ln \left(\frac{\mu_F^2}{s} \right) + d_{\alpha_s} \frac{\beta_0}{4} \ln \left(\frac{\mu_R^2}{s} \right) \,. \tag{5}$$

We note that the C_f 's are color factors, M is any appropriate hard scale, δ_K is 0 (1) for PIM (1PI) kinematics, and the sum over j is only relevant if we use 1PI kinematics and there are massless final-state partons at lowest-order. More details and definitions are given in [1] where results are also given in the DIS scheme.

The master formula for the NNLO soft and virtual corrections for processes with simple color flow in the $\overline{\rm MS}$ scheme is $\hat{\sigma}^{(2)} = \sigma^B \left(\alpha_s^2(\mu_R^2)/\pi^2\right) \hat{\sigma'}^{(2)}$ with

$$\hat{\sigma'}^{(2)} = \frac{1}{2}c_3^2 \mathcal{D}_3(x_{th}) + \left[\frac{3}{2}c_3 c_2 - \frac{\beta_0}{4}c_3 + \sum_j C_{f_j} \frac{\beta_0}{8} \right] \mathcal{D}_2(x_{th})
+ \left\{ c_3 c_1 + c_2^2 - \zeta_2 c_3^2 - \frac{\beta_0}{2} T_2 + \frac{\beta_0}{4} c_3 \ln \left(\frac{\mu_R^2}{s} \right) + \sum_i C_{f_i} K \right.
+ \sum_j C_{f_j} \left[-\frac{K}{2} + \frac{\beta_0}{4} \delta_K \ln \left(\frac{M^2}{s} \right) \right] - \sum_j \frac{\beta_0}{4} B_j^{(1)} \right\} \mathcal{D}_1(x_{th})
+ \left\{ c_2 c_1 - \zeta_2 c_2 c_3 + \zeta_3 c_3^2 - \frac{\beta_0}{2} T_1 + \frac{\beta_0}{4} c_2 \ln \left(\frac{\mu_R^2}{s} \right) + 2 \operatorname{Re} \Gamma_S^{(2)} - \sum_i \nu_{f_i}^{(2)} \right.
+ \sum_i C_{f_i} \left[\frac{\beta_0}{8} \ln^2 \left(\frac{\mu_F^2}{s} \right) - \frac{K}{2} \ln \left(\frac{\mu_F^2}{s} \right) - K \delta_K \ln \left(\frac{-t_i}{M^2} \right) \right]
- \sum_j \left(B_j^{(2)} + \nu_j^{(2)} \right) + \sum_j C_{f_j} \delta_K \left[\frac{\beta_0}{8} \ln^2 \left(\frac{M^2}{s} \right) - \frac{K}{2} \ln \left(\frac{M^2}{s} \right) \right]
- \sum_j \frac{\beta_0}{4} B_j^{(1)} \delta_K \ln \left(\frac{M^2}{s} \right) \right\} \mathcal{D}_0(x_{th}) + R_{\delta(x_{th})} \delta(x_{th}) .$$
(6)

For more details and explicit expressions for the full $\delta(x_{th})$ terms see Ref. [1]. For the more general case of complex color flow, where H, S, and Γ'_{S} are matrices, the $\overline{\rm MS}$ scheme NLO master formula generalizes to

$$\hat{\sigma}^{(1)} = \hat{\sigma}_{\text{simple}}^{(1)} + \frac{\alpha_s^{d_{\alpha_s} + 1}(\mu_R^2)}{\pi} \left[A^c \mathcal{D}_0(x_{th}) + T_1^c \delta(x_{th}) \right], \tag{7}$$

where $\hat{\sigma}_{\text{simple}}^{(1)}$ denotes the result in Eq. (3) but without the $2\text{Re}\Gamma_S^{(1)}$ term in c_2 , d_{α_s} denotes the power of α_s in the Born term, and

$$A^{c} = \text{Tr}\left(H^{(0)}\Gamma_{S}^{\prime(1)\dagger}S^{(0)} + H^{(0)}S^{(0)}\Gamma_{S}^{\prime(1)}\right). \tag{8}$$

At NNLO for processes with complex color flow the $\overline{\rm MS}$ scheme master formula generalizes to

$$\hat{\sigma}^{(2)} = \hat{\sigma}_{simple}^{(2)} + \frac{\alpha_s^{d_{\alpha_s} + 2}(\mu_R^2)}{\pi^2} \left\{ \frac{3}{2} c_3 A^c \mathcal{D}_2(x_{th}) + \left[\left(2c_2 - \frac{\beta_0}{2} \right) A^c + c_3 T_1^c + F \right] \mathcal{D}_1(x_{th}) + \left[\left(c_1 - \zeta_2 c_3 + \frac{\beta_0}{4} \ln \left(\frac{\mu_R^2}{s} \right) \right) A^c + \left(c_2 - \frac{\beta_0}{2} \right) T_1^c + F \delta_K \ln \left(\frac{M^2}{s} \right) + G \right] \mathcal{D}_0(x_{th}) + R_{\delta(x_{th})}^c \delta(x_{th}) \right\}, \quad (9)$$

with $\hat{\sigma}_{\text{simple}}^{(2)}$ denoting Eq. (6) times $\alpha_s^2(\mu_R^2)/\pi^2$ (but without any Γ_s' terms), and

$$F = \text{Tr}\left[H^{(0)}\left(\Gamma_S^{\prime(1)\dagger}\right)^2 S^{(0)} + H^{(0)}S^{(0)}\left(\Gamma_S^{\prime(1)}\right)^2 + 2H^{(0)}\Gamma_S^{\prime(1)\dagger}S^{(0)}\Gamma_S^{\prime(1)}\right]. \tag{10}$$

Again, for more details and the full explicit $\delta(x_{th})$ terms see Ref. [1]. Eq. (9) serves as the most general master formula for the NNLO soft and virtual corrections for any process in hadron-hadron and lepton-hadron collisions.

3 Applications to QCD processes

Using the NNLO master formula I have rederived known NNLO results for Drell-Yan and related processes and for deep inelastic-scattering (DIS), and produced new results for many other processes [1]. Here I give a few examples.

For deep inelastic scattering, $\gamma^* q \to q$, in the $\overline{\rm MS}$ scheme, we have $c_3 = C_F$, $c_2 = -3C_F/4 - C_F \ln(\mu_F^2/Q^2)$, and $c_1 = (-3/4)C_F \ln(\mu_F^2/Q^2) - C_F\zeta_2 - (9/4)C_F$. The NNLO corrections from the master formula then agree with the results for the coefficient functions in Ref. [6], and we also derive the two-loop function $B_q^{\prime(2)}$ which is needed in NNLL resummation [1]. In the DIS scheme, the NLO and NNLO corrections vanish, as expected.

For heavy quark hadroproduction in the channel $q\bar{q} \to Q\bar{Q}$ in 1PI kinematics, we have $c_3 = 4C_F$, $c_2 = -2C_F - 2C_F \ln(t_1u_1/m^4) - 2C_F \ln(\mu_F^2/s)$, $c_1^\mu = C_F[\ln(t_1u_1/m^4) - 3/2] \ln(\mu_F^2/s) + (\beta_0/2) \ln(\mu_R^2/s)$, $A^c = (\sigma^B/\alpha_s^2) 2 \operatorname{Re}\Gamma_{S,22}^{(1)}$. The NNLO corrections are in agreement with and extend earlier NNLO-NNLL calculations [7]. Results have also been obtained in PIM kinematics and in the DIS scheme, and also for the $qq \to Q\bar{Q}$ channel.

A final application is jet production, where many processes are involved. For example, for $gg \to gg$ we have $c_3 = 2C_A$ and $c_2 = -2C_A \ln(\mu_F^2/s) - \beta_0/2 - 4C_A$. The NNLO corrections extend earlier NNLO-NLL calculations [8].

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