Chiral Bosons as solutions of the BV master equation for 2D chiral gauge theories

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## Abstract

We construct the chiral Wess-Zumino term as a solution for the Batalin-Vilkovisky master equation for anomalous two-dimensional gauge theories, working in an extended field-antifield space, where the gauge group elements are introduced as additional degrees of freedom. We analyze the Abelian and the non-Abelian cases, calculating in both cases the BRST generator in order to show the physical equivalence between this chiral solution for the master equation and the usual (non-chiral) one.

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The Batalin-Vilkovisky<sup>[1]</sup> scheme is a powerful method to build up a BRST invariant generating functional for a wide range of gauge theories. This scheme, also called fieldantifield formalism, works on a configuration space that includes, besides the original classical fields, a set of partners, called antifields, sharing the opposite statistics. The introduction of these antifields makes it possible to define a new operation, the antibracketing, in such a way that any BRST variation can be expressed in terms of the antibracket with an extended action  $W = S + \sum_{j=1}^{\infty} \hbar^{j} M_{j}$ . Where S is the classical part of the action and the  $M_i$  are the quantum corrections. This action is obtained as the solution of the so called master equation, which at the classical level (zero order in  $\hbar$ ) requires the vanishing of the antibracket of S with itself. The higher order terms of the master equation are related, at least formally, to the purely quantum effects reflecting the non gauge invariance of the path integral measure. In other words, quantum correction to the master equation take into account the anomalies. Moreover, the master equation is a systematic way to write down the anomalous BRST Ward identities. In fact, in a recent work, Troost, van Nieuwenhuizen and Van Proeyen<sup>[2]</sup> have deeply analyzed the problem of quantization of anomalous gauge theories in the field-antifield framework, showing that the presence of a genuine anomaly places an obstruction to the construction of a local solution for the master equation at the quantum level. This was observed in the quantization of the chiral Schwinger model, where we have explicitly constructed a non local solution for the first quantum contribution to the master equation, which turns local by the introduction of an auxiliary field $^{[3]}$ .

Following the proposal of Faddeev and Shatashvili<sup>[4]</sup>, and the mechanism to generate the Wess-Zumino term<sup>[5]</sup> developed by Babelon, Schaposnik and Viallet<sup>[6]</sup>, and by Harada and Tsutsui<sup>[7]</sup>, we have recently proposed a mechanism to get rid the obstruction in solving the master equation at the quantum level<sup>[8]</sup>. By considering that the quantum fluctuation may render the gauge group elements as dynamical variables, we extended the initial field-antifield configuration space also including them. The new extended action S, constructed on this extended field-antifield space, must also include in its Hessian matrix the generator of transformations of the gauge group elements. So, the action S contains an additional term involving the corresponding antifield. These term allows one to overcome the topo-

logical obstruction, giving rise to a local solution for the quantum master equation. By analyzing what happens from the canonical point of view, we have shown that the new term comes to restore the quantum nilpotency of the BRST generator, and to eliminate the Schwinger term of the Gauss law algebra<sup>[8,9]</sup>. Also, it was shown that the action of the BRST operator on the physical states annihilate a chiral sector of the bosonic field associated to the gauge group element. These idea was generalized by Gomis and Paris in reference [10], where by using the quasigroup structure of the gauge generators<sup>[11]</sup>, they find the general form of the transformation of the group elements only requiring that the initial quasigroup structure be preserved after the introduction of the new fields. They also build up the general form of the solution of the master equation.

In this letter, we analyze the master equation at the quantum level, for Abelian and non Abelian gauge field theories, in the extended field-antifield space, showing that it is possible to construct chiral Wess-Zumino terms as solutions for the master equation. This result is similar to ref. [12], where it was shown that the bosonised version of a chiral fermionic theory can be written in terms of a chiral scalar particle.

Let us start with the Abelian case, the Chiral Schwinger Model (CSM), which is described by the action:

$$S_o = \int d^2x \{ i\overline{\psi} \not\!\!\!\!D \, \frac{(1-\gamma_5)}{2} \, \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \}$$
 (1)

The standard application of the Batalin-Vilkovisky formalism leads to a classical action S including terms involving the antifields and the generators of the gauge symmetry:

$$S = S_o + \int d^2x \{ A_\mu^* \partial^\mu c + i\psi^* \psi c - i\overline{\psi} \,\overline{\psi^*} c \}$$
 (2)

This action satisfies the master equation at zero order in  $\hbar$ : (S,S)=0, where the antibracket is defined as  $(X,Y)=\frac{\partial_r X}{\partial \phi}\frac{\partial_l Y}{\partial \phi^*}-\frac{\partial_r X}{\partial \phi^*}\frac{\partial_l Y}{\partial \phi}$ 

The master equation to first order in  $\hbar$  reads

$$(M_1, S) = i\Delta S, \tag{3}$$

where  $\Delta \equiv \frac{\partial_r}{\partial \phi^A} \frac{\partial_l}{\partial \phi_A^*}$  the calculation of  $\Delta S$ , that represents the BRST anomaly, is explained in detail in references [3] and [8], resulting in:

$$\Delta S = \frac{i}{4\pi} \int d^2x \ c \left[ (1 - a)\partial_{\mu}A^{\mu} - \epsilon^{\mu\nu}\partial_{\mu}A_{\nu} \right] \tag{4}$$

As was explained in references [2] and [3], the master equation (3) does not admit local solutions so, in this context, it is not possible to construct a gauge independent generating functional for the CSM. Following the same reasoning of references [8],[9], and the fact that a local effective action for the CSM will include a dynamical extra field<sup>[13]</sup>, we enlarge the configuration space, adding the field  $\theta$  and its corresponding antifield  $\theta^*$ . So, we get the extended classical action:

$$S' = S_o + \int d^2x \{ A_\mu^* \partial^\mu c + i\psi^* \psi c - i\overline{\psi} \,\overline{\psi^*} c + \theta^* \, c \}$$
 (5)

where we have considered that the gauge group parameter  $\theta$  transforms as

$$\theta \to \theta + \lambda$$
 (6)

Observe that now the action S' depends on  $\theta^*$  through the inclusion of the term  $\theta^*c$ , and this extra term play a fundamental role modifying the master equation (3). Now, one is able to construct local solutions for it, depending also on the field  $\theta$ . The new master equation at  $\hbar$  order is

$$(M_1, S') = \int \left\{ \frac{\partial_r M_1}{\partial A^{\mu}} \frac{\partial_l S'}{\partial A^*_{\mu}} + \frac{\partial_r M_1}{\partial \theta} \frac{\partial_l S'}{\partial \theta^*} \right\} = i\Delta S \tag{7}$$

It is worth remarking that the inclusion of the  $\theta$  field does not modify  $\Delta S$ , which is still given by eq. (4).

It was shown in reference [9] that the usual Wess-Zumino term for the CSM:

$$\overline{M}_{1} = -\frac{1}{4\pi} \int d^{2}x \left\{ \frac{(a-1)}{2} \partial_{\mu}\theta \ \partial^{\mu}\theta + \theta \left[ (a-1)\partial_{\mu}A^{\mu} + \epsilon^{\mu\nu}\partial_{\mu}A_{\nu} \right] \right\}$$
(8)

is a solution to this equation. The canonical analysis of the theory described now by the action  $S' + \hbar \overline{M}_1$ , leads to the modified Gauss law:

$$\overline{\Omega} = \Pi_1' + J_0 - \frac{\hbar}{4\pi} [(a-1)A_0 + A_1] - \Pi_\theta + \frac{\hbar}{4\pi} \theta'$$
(9)

where  $J_0$  is the chiral current defined by:

$$J_0 = \overline{\psi}(x)\gamma_0 \frac{(1-\gamma_5)}{2}\psi(x)$$

The BRST operator can be obtained from the Gauss law (9) by the relation  $Q = \Omega c$ , and its nilpotency may be verified from the calculation of the anticonmutator  $[Q, Q]_+$ , by taking into account the non-trivial commutator<sup>[14]</sup>:

$$i[\Pi_1'(x), J_0(y)]_- = \frac{\hbar^2}{4\pi} \delta'(x_1 - y_1)$$
 (10)

However, the solution (8) is not unique. Another interesting solution for the master equation (7) is:

$$M_{1} = \frac{1}{4\pi} \int d^{2}x \{ -\theta'\dot{\theta} + (a-1)(\theta')^{2} + 2\theta'[A_{o} - (a-1)A_{1}] - \frac{1}{2}(a-1)A_{o}^{2} - A_{1}A_{o} + \frac{3}{2}(a-1)A_{1}^{2} \}$$

$$(11)$$

that represents the action of a chiral boson coupled to the gauge field. The are no higher order contributions to the master equation, so the quantum action is just

$$W = S' + \hbar M_1$$

This Wess-Zumino term is the same as the action obtained in reference [12], after the so called chiral bosonization of the chiral Schwinger model. However, it should be emphasized that, in this reference, the chiral constraint is imposed to the functional generator by means of a delta functional in momentum space based on the reasoning that only one chirality of a scalar field should be necessary in order to represent a chiral fermion. In the present case, we obtain a Wess-Zumino term that involves a chiral scalar field, just choosing one of the solutions of the Batalin-Vilkovisky master equation.

Let us now investigate the physical space of states, by analyzing the consequences of this  $M_1$ , from the canonical point of view. Calculating the canonical momenta associated to action (11), we get the chiral constraint  $\Pi_{\theta} = -\frac{\hbar}{4\pi}\theta'$  and the other primary constraint:

$$\Pi_0 = 0 \tag{12}$$

and, building up the Hamiltonian, the time evolution of (12) leads to the secondary constraint:

$$\Omega = \Pi_1' + J_0 - \frac{\hbar}{4\pi} [(a-1)A_o - A_1] + \frac{\hbar}{2\pi} \theta'$$
 (13)

that, by using the  $\Pi_{\theta}$  definition, it acquires the same form as (9)

$$\Omega = \Pi_1' + J_0 - \frac{\hbar}{4\pi} [(a-1)A_o - A_1] - (\Pi_\theta - \frac{\hbar}{4\pi}\theta')$$

The canonical generator of the BRST transformations is thus:

$$Q = \int dx_1 \ \Omega(x) c(x) \tag{14}$$

In order to investigate the quantum nilpotency of Q, we will build up the anticommutator  $[Q,Q]_+=2Q^2.$ 

$$[Q,Q]_{+} = \int dx_{1} \int dy_{1} \left\{ c(x) \left[ \pi'_{1}(x) + J_{0}(x) + \frac{\hbar}{4\pi} A_{1}(x), \pi'_{1}(y) + J_{0}(y) + \frac{\hbar}{4\pi} A_{1}(y) \right]_{-} + \left[ \pi_{\phi}(x) - \frac{\hbar}{4\pi} \phi'(x), \pi'_{1}(y) - \frac{\hbar}{4\pi} \phi'(y) \right]_{-} c(y) \right\} \Big|_{x_{o} = y_{o}}$$

$$(15)$$

This anti-commutator depends on the equal time commutator of the chiral current that requires a careful management at the quantum level as it involves the product of operators at the same point . A regularization scheme is then needed in order to overcome this problem. As it is well known, the calculation of this commutator generates the Schwinger term, transforming the current algebra in a Kac-Moody one. By using the Bjorken, Jonhson, Low limit<sup>[15]</sup>, one is able to relate the vacuum expectation value of this current commutator to the second functional derivative of the effective action  $S_{eff}$ , which is equivalent

to the logarithm of the determinant of the fermionic operator  $[\partial + A \frac{(1-\gamma_5)}{2}]^{[16-19]}$ . Thus, following ref. [19], one gets the standard result:

$$i[J_0(x), J_0(y)]_- = \frac{\hbar^2}{2\pi} \,\delta'(x_1 - y_1) \tag{16}$$

Furthermore, the non-trivial Jacobian of the transformation generated by  $J_0$  gives rise to a Schwinger term also in the commutator between  $J_0$  and  $\Pi'_1$  [eq. (10)], as shown in ref. [14].

The appearance of these Schwinger terms would means the breakdown of the gauge symmetry and would also destroy the nilpotency of the operator Q. However, the presence of the extra field  $\theta$  leads to the appearance of the two other commutators in (14). These commutators are trivial, in the sense that they are not anomalous. Introducing these results in (14) one gets

$$[Q,Q]_+=0$$

showing explicitly that the introduction of the field  $\theta$  leads to a theory that has a BRST generator that is nilpotent at the quantum level.

It is important to remark that, comparing the present results with those from reference [9], we see that the BRST generator is essentially the same in the chiral and the non-chiral solution for the master equation (7). As already pointed out in [9] Q involves the chiral constraint  $\dot{\theta} = \theta'$ . Therefore, the physical states, that are annihilated by Q involve just chiral scalars, in any formulation.

Now we will consider the non-Abelian gauge field theory in two dimensions, that means:  $QCD_2$ . The classical action, including gauge fixing terms, is<sup>[8]</sup>:

This equation satisfies the master equation at order zero in  $\hbar$ : (S, S) = 0. Now, to first order in  $\hbar$  the master equation involves  $\Delta S$ . One can see that  $\Delta$  affects all the terms in

 $S_o$  and a naive calculus yields  $\Delta S \propto c \, \delta(0)$ . The arising of this ill-defined expression corresponds to the well-known fact that one needs a regularization to define the measure<sup>[1,2]</sup>. This calculation, already explained in references [3,8], yields

$$\Delta S = \frac{i}{4\pi} \int d^2x \, tr\{c \left[\varepsilon^{\mu\nu}\partial_{\mu}A_{\nu} - \partial_{\mu}A^{\mu}\right]\}$$
 (18)

where one can identify the consistent gauge anomaly. The master equation to  $\hbar$  order is essentially the anomalous BRST Ward identity and, as it is well known, it does not admit local solutions.

As already explained, in order to build up local solutions to the master equation, we introduce the extra field h, associated to the gauge group. Following reference [8] we add to the classical action (17) the gauge fixing term associated with the new field, so that

$$S' = S + \int dx^2 \, h^* hc$$

Thus, the path integral measure also must include the measure Dh, which stands for the local product of Haar measures on G. The higher order contributions to the master equation take account of the variation of the path integral measure under the above symmetries, through the evaluation of  $\Delta S'$ . Observe that, because of the trivial gauge invariance of the Haar measure, the transformation  $s_o h = h T^a c_a$  leaves the measure Dh invariant, which means that the corresponding Jacobian admits a covariant regularization rendering it a trivial one. So, the only non-trivial Jacobian come again from the fermionic measure. Then, the  $\Delta S$  is still given by the expression (18).

In reference [8] we found a solution to  $(M_1, S') = i\Delta S'$  of the form

$$\overline{M}_{1}(A_{\mu}, h) = \Gamma[h] + \int d^{2}x \ tr\{\frac{1}{2}(\alpha + \frac{1}{4\pi}) \ \partial_{\mu}h^{-1}\partial^{\mu}h + \alpha h^{-1}\partial_{\mu}h \ A^{\mu} - \frac{1}{4\pi} \varepsilon^{\mu\nu} h^{-1}\partial_{\mu}h \ A_{\nu} - \frac{1}{2}(\alpha + \frac{1}{4\pi}) A_{\mu}A^{\mu}\}$$
(19)

where  $\alpha$  is an arbitrary parameter and  $\Gamma[h]$  is the Wess-Zumino-Witten action<sup>[4,9]</sup>:

$$\Gamma[h] = \frac{-1}{8\pi} \int d^2x \ tr \left(\partial_{\mu} h^{-1} \partial^{\mu} h\right) + \frac{1}{4\pi} \int_0^1 dr \int d^2x \ \varepsilon^{\mu\nu} \ tr(h_r^{-1} \partial_r h_r \ h_r^{-1} \partial_{\mu} h_r \ h_r^{-1} \partial_{\nu} h_r)$$
(21)

with  $h_r$  being some interpolation between  $h_0 = 1$  and  $h_1 = h$  This solution corresponds to the usual Wess-Zumino term for the chiral  $QCD_2$ . As in the Abelian case, it is possible to build up a chiral solution for the master equation, that corresponds to a non abelian chiral boson coupled to the gauge field<sup>[20]</sup>:

$$M_{1}(A_{\mu},h) = \int d^{2}x \, tr\{\frac{1}{4\pi} \left(h^{-1}\partial_{0}hh^{-1}\partial_{1}h + h^{-1}\partial_{1}hh^{-1}\partial_{1}h\right)$$

$$+ \frac{1}{4\pi} \left(-2h^{-1}\partial_{1}hA_{+} - \frac{1}{2}A_{0}^{2} + A_{0}A_{1} + \frac{3}{2}A_{1}^{2}\right)\}$$

$$+ \frac{1}{4\pi} \int_{0}^{1} dr \int d^{2}x \, \varepsilon^{\mu\nu} \, tr(h_{r}^{-1}\partial_{r}h_{r} \, h_{r}^{-1}\partial_{\mu}h_{r} \, h_{r}^{-1}\partial_{\nu}h_{r})$$

$$(22)$$

Again, it should be stressed that we did not impose the chiral constraint. We are just choosing one of the possible solutions of the master equation. As in the abelian case, the higher order contributions to the master equation can be taken as zero. So, the quantum action is just  $W = S' + \hbar M_1$ 

Now we analyze the action W from the canonical point of view, building up the BRST generator. The main consequence of the additional term  $M_1$  is to modify the Gauss Law so that now it reads:

$$\Omega^{a} = D_{1}^{ab}\Pi_{1}^{b} + J_{0}^{a} - \frac{\hbar}{8\pi}[A_{0}^{a} - A_{1}^{a}] - \frac{\hbar}{2\pi}tr(h^{-1}\partial_{1}hT^{a}) + f^{abc}\Pi_{gh}^{b}c_{d}$$
 (23)

with  $J^a_\mu$  being the chiral current

$$J_0^a = \overline{\psi}\gamma^\mu (\frac{1-\gamma^5}{2})T^a\psi$$

Then, following the standard recipe for constructing the BRST generator from the constraints, we get:

$$Q = Q_o + Q'$$

$$= \int dx_1 \{ J_0^a c_a - \frac{1}{2} [D^{\nu}, F_{o\nu} c] + \frac{1}{2} A_o^{*a} f^{abc} c^b c^c - \frac{\hbar}{2\pi} tr(h^{-1} \partial_1 h c) - \frac{\hbar}{2\pi} (A_0^a - A_1^a) c_a \}$$
(25)

where  $Q_o$  stands for the BRST generator arising from the action (17) and Q' is the contribution coming from (22). If the chiral constraint is explicitly implemented, the term

involving the field h can be written exactly as the one in reference 8 where the BRST charge associated to the non-chiral Wess-Zumino term was build up.

In checking the nilpotency of Q, we need the commutator of the chiral current for the non-Abelian case<sup>[17]</sup>:

$$[J_0^a(x), J_0^b(x')]_- = -f^{abc}J_0^c(x)\delta(x_1 - x_1') + \frac{i}{4\pi}\delta^{ab}\delta'(x_1 - x_1') + \frac{i}{8\pi}f^{abc}(A_o - A_1)^c\delta(x_1 - x_1')$$
(27)

together with the non-trivial commutator

$$[\Pi_1^{a\prime}(x), J_0^b(y)] = \frac{\hbar^2}{4\pi} \delta^{ab} \, \delta'(x_1 - y_1)$$

and the others commutator being the usual ones. In this way it is immediate to verify that

$$Q^2 = \frac{1}{2}[Q,Q]_+ = 0 (28)$$

In conclusion we have shown that when applying the Batalin-Vilkovisky quantization procedure to chiral gauge theories, one is able to choose, among the various possible solutions of the master equation, a solution involving a chiral Wess-Zumino field. Therefore chiral Wess-Zumino terms are generated without imposing the chiral constraint by hand, as usually done in the literature. We have also calculated the BRST generator for the Abelian and non-Abelian cases. The nilpotency of this operator at the quantum level was shown for both cases. This ensures that the physical states can be defined by the cohomology class of this operator. It was also pointed out that these generators are the same as in the case of the standard (non chiral) Wess-Zumino terms, leading us to the conclusion that the spaces of physical states are the same.

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