# Moduli Stabilisation and Applications in IIB String Theory

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Per varios casus, per tot discrimina rerum tendimus in Latium; sedes ubi fata quietas ostendunt; illic fas regna resurgere Troiae. Durate, et vosmet rebus servate secundis.

Aeneid I:204-7

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#### Declaration

This dissertation is the result of my own work and includes nothing which is the outcome of work done in collaboration except where specifically indicated in the text. No part of this thesis has previously been submitted for a degree or other qualification at this or any other university.

This thesis is based on the research presented in the following papers:

- 1. J. P. Conlon and F. Quevedo, On the Explicit Construction and Statistics of Calabi-Yau Flux Vacua, JHEP 0410:039 (2004), arXiv:hep-th/0409215.
- 2. V. Balasubramanian, P. Berglund, J. P. Conlon and F. Quevedo, *Systematics of Moduli Stabilisation in Calabi-Yau Flux Compactifications*, JHEP 0503:007 (2005), arXiv:hep-th/0502058.
- 3. J. P. Conlon, F. Quevedo and K. Suruliz, Large Volume Flux Compactifications: Moduli Spectrum and D3/D7 Soft Supersymmetry Breaking, JHEP 0508:007 (2005), arXiv:hep-th/0505076.
- 4. J. P. Conlon and F. Quevedo, *Kähler Moduli Inflation*, JHEP 0601:146 (2006), arXiv:hep-th/0509012.
- 5. J. P. Conlon, *The QCD Axion and Moduli Stabilisation*, JHEP 0605:078 (2006), arXiv:hep-th/0602233.
- 6. J. P. Conlon and F. Quevedo, *Gaugino and Scalar Masses in the Landscape*, JHEP 0606:029 (2006), arXiv:hep-th/0605141.

These papers are references [2, 3, 4, 5, 6, 7] in the bibliography.

Sections 6.1, 6.2 and A.3 of this thesis are based on parts of the third paper above that were primarily written by Kerim Suruliz. They are included here to ensure completeness of the argument.

JOSEPH CONLON Cambridge, Feast of Ss. Peter and Paul 29th June 2006

### Summary

String compactifications represent the most promising approach towards unifying general relativity with particle physics. However, naive compactifications give rise to massless particles (moduli) which would mediate unobserved long-range forces, and it is therefore necessary to generate a potential for the moduli.

In the introductory chapters I review this problem and recall how in IIB compactifications the dilaton and complex structure moduli can be stabilised by 3-form fluxes. There exist very many possible discrete flux choices which motivates the use of statistical techniques to analyse this discretuum of choices. Such approaches generate formulae predicting the distribution of vacua and I describe numerical tests of these formulae on the Calabi-Yau  $\mathbb{P}^4_{[1,1,2,2,6]}$ . Stabilising the Kähler moduli requires nonperturbative superpotential effects. I review the KKLT construction and explain why this must in general be supplemented with perturbative Kähler corrections. I show how the incorporation of such corrections generically leads to non-supersymmetric minima at exponentially large volumes, giving a detailed account of the  $\alpha'$  expansion and its relation to Kähler corrections. I illustrate this with explicit computations for the Calabi-Yau  $\mathbb{P}^4_{[1,1,1,6,9]}$ .

The next part of the thesis examines phenomenological applications of this construction. I first describe how the magnitude of the soft supersymmetry parameters may be computed. In the large-volume models the gravitino mass and soft terms are volume-suppressed. As we naturally have  $\mathcal{V}\gg 1$ , this gives a dynamical solution of the hierarchy problem. I also demonstrate the existence of a fine structure in the soft terms, with gaugino masses naturally lighter than the gravitino mass by a factor  $\ln\left(\frac{M_P}{m_{3/2}}\right)$ . A second chapter gives a detailed analysis of the relationship of moduli stabilisation to the QCD axions relevant to the strong CP problem, proving a no-go theorem on the compatibility of a QCD axion with supersymmetric moduli stabilisation. I describe how QCD axions can coexist with nonsupersymmetric perturbative stabilisation and how the large-volume models naturally contain axions with decay constants that are phenomenologically allowed and satisfy the appealing relationship  $f_a^2 \sim M_P M_{susy}$ . A further chapter describe how a simple and predictive inflationary model can be built in the context of the above large-volume construction, using the no-scale Kähler potential to avoid the  $\eta$  problem.

I finally conclude, summarising the phenomenological scenario and outlining the prospects for future work.

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# Part I Introduction

# Chapter 1

# Introduction and Motivation

'If string theory is the answer, what is the question?'

The basic assumption of this thesis is that the appropriate question is 'What set of ideas underlies, unifies and explains the physics of both general relativity and the Standard Model?', and the basic aim of this thesis is to address how one can connect the formal structure of string theory with the particle physics and cosmology that can be tested experimentally.

General relativity and quantum mechanics are two of the best ideas of all time and were the two major achievements of twentieth century physics. The former originates in classical electrodynamics and the special theory of relativity. This was soon generalised by Einstein to its mature formulation as general relativity. In this theory, space and time are themselves the dynamical quantities. Particles follow geodesics in spacetime determined by the spacetime metric, which is in turn determined by the distribution of matter. The force of gravity is the manifestation of space-time curvature: the attraction of particles corresponds to the approach of geodesics. The action governing general relativity is the Einstein-Hilbert action

$$S_{EH} = \int d^4x \sqrt{-g} \left( \frac{\mathcal{R}}{16\pi G} + 2\Lambda + \mathcal{L}_M \right). \tag{1.1}$$

This action describes the physics of black holes, the expansion of the universe and the precession of the perihelion of Mercury. It is tested everyday in the use of GPS receivers, whose accuracy depends on the correct inclusion of general relativistic effects. It is a precise relativistic account of gravity, the highpoint of classical physics, and is consistent with all experimental tests to date [8].

The other cornerstone of modern physics is quantum mechanics. While general relativity is a theory of the big - tested on solar system and cosmological scales - quantum mechanics is a theory of the small. It originated in the study of physics at atomic length scales. The measurement of atomic emission and absorption spectra led to the first rudimentary quantisation laws of Bohr and

Einstein. In the 1920's these were developed into quantum theories of particle mechanics through the Schrödinger and Dirac equations,

$$\mathcal{H}|\psi\rangle = i\hbar \frac{\partial}{\partial t}|\psi\rangle \qquad (1.2)$$
$$(i\Gamma^{\mu}\partial_{\mu} + m)\psi = 0. \qquad (1.3)$$

$$(i\Gamma^{\mu}\partial_{\mu} + m)\psi = 0. (1.3)$$

The precision with which these equations worked - for example in the spectrum of the Hydrogen atom - confirmed quantum mechanics as a necessary part of the correct description of the world. However, many years elapsed before quantum theories of particles, such as electrons, were successfully extended to quantum theories of fields, such as electromagnetism.

This advance was famously conceived at the 1947 Shelter Island conference. The recent measurement of the Lamb shift in the hydrogen spectrum provoked the theorists present into developing the quantum field theory of electromagentism, quantum electrodynamics (QED). Particles were no longer understood as fundamental objects but rather as elementary excitations of an underlying field: for example the photon is the elementary excitation of the electromagnetic field. In modern language, QED is a U(1) gauge theory: a field theory with a local U(1)symmetry at every point in space time. In addition to electromagentism, two other short distance forces are known: the strong force, responsible for binding together atomic nuclei, and the weak force, responsible for radioactive  $\beta$  decay. The former is described by an SU(3) gauge theory and the latter by a (spontaneously broken) SU(2) gauge theory. It is not understood why we observe these forces and these forces only.

In the first development of quantum field theories, a major problem was encountered with infinities: naive attempts to calculate quantum corrections to classical effects gave infinite answers. This problem was addressed by the development of renormalisation: just because a quantity was infinite did not mean that quantity was zero. Specifically, by including a finite number of infinite counterterms, the infinities could be cancelled and physical predictions in agreement with experiment could be extracted. Originally this procedure was rather ad hoc, and it was thought that the only sensible quantum field theories were renormalisable ones. In the modern Wilsonian understanding, the infinities requiring renormalisation come from integrating out high energy degrees of freedom. Renormalisation is the procedure of doing low-energy quantum physics without knowing the high-energy completion. In non-renormalisable theories, the leading physics is suppressed high-energy physics, and in order to do quantum physics we must first understanding the underlying theory.

For example, we expect any quantum field theory to fail at the Planck scale,  $M_P = 2.4 \times 10^{18} \text{GeV}$ . Excepting the quadratic Higgs divergence, all the divergences encountered in particle physics are logarithmic. If a Planck-scale cutoff is introduced, the 'divergences' become finite,

$$\int_{p_0}^{\infty} \frac{d^4 p}{p^4} \to \int_{p_0}^{M_P} \frac{d^4 p}{p^4} \sim \ln\left(\frac{M_P}{p_0}\right). \tag{1.4}$$

The divergences are therefore not real divergences, but merely a consequence of our lack of knowledge of the fundamental high energy theory. The need to renormalise implies the existence of a more fundamental theory in which the divergent quantities are actually finite. The low-energy approximate theory carries the imprint of the fundamental theory in the texture of low-energy parameters such as masses and coupling constants.

The particular approximate theory that accurately describes the physics seen at particle accelerators is known as the Standard Model. It consists of an  $SU(3) \times SU(2) \times U(1)$  gauge theory coupled to three generations of chiral matter and has been rigorously tested in many different settings [9]. There is clearly structure sitting behind the Standard Model: for example, the matter content is replicated three times - this is not explained. The masses of the particles in each generation show a hierarchical structure - this is not explained. The strong interactions do not violate CP, and the weak interactions do - this is not explained. In the minimal Standard Model, the Higgs mass is unstable against radiative corrections: this may be explained by supersymmetry, but this is not known. There is clearly a structure behind the Standard Model, but it is not known what this structure is.

One further piece of physics definitely in nature and definitely not in the Standard Model is the gravitational force. General relativity does an excellent job of describing gravity, but it is an excellent classical job. As quantum mechanics is integral to nature, there must exist a quantum theory of the gravitational force not included in the Standard Model. One may hope that the correct formulation of this theory will help explain some of the structure visible and not understood in the Standard Model. Another hope is that the quantum theory of gravity will allow a description of gravity common with the other known forces of nature. This is not because of logical necessity, but rather an educated guess based on the historical trend towards unification in physics.

The natural approach to finding a quantum theory of gravity is to use the same methods that worked so effectively for the other forces. Unfortunately this method fails irrevocably. The reason conventional quantisation works for gauge theories is that in four dimensions gauge theories are renormalisable. This tells us that at low energy we do not need to know the high-energy theory in order to do quantum, as opposed to classical, computations. This is not true for general relativity, which is an irrevocably non-renormalisable theory. General relativity is an effective theory, valid at energies  $E \ll M_P$ . In addition to the interactions of equation (1.1), there will also be non-renormalisable interactions suppressed

by higher powers of  $M_P$ ,

$$S = \frac{M_P^2}{2} \int \sqrt{g} \left( \mathcal{R} + \frac{1}{M_P^2} \left( c_1 \mathcal{R}^2 + c_2 \mathcal{R}_{\alpha\beta} \mathcal{R}^{\alpha\beta} + c_3 \mathcal{R}_{\alpha\beta\gamma\delta} \mathcal{R}^{\alpha\beta\gamma\delta} \right) + \dots \right). \quad (1.5)$$

While unimportant at low energies, for scattering energies  $E \gtrsim M_P$  the higher-derivative interactions of equation (1.5) are as equally important as the Einstein-Hilbert term. Finding a quantum theory of gravity requires a way to calculate the constants  $c_1, c_2, \ldots$  in order that scattering at energies  $E \gtrsim M_P$  be predictive. The quantum effects are encoded in the values of these terms. Unfortunately, at high energies an infinite number of such terms are required and so it is not possible to be predictive with any finite number of measurements. Clearly, we expect new degrees of freedom to appear at the Planck scale - however as the action (1.1) describes an effective theory it gives no clue as to what these degrees of freedom should be.

String theory is a hypothetical account of physics at the Planck scale and in particular of the new degrees of freedom that are present there. The subject is well named - the fundamental perturbative excitations are one-dimensional extended objects, strings. The new degrees of freedom for energies above the Planck scale are a tower of excited string states. The strings contain both gauge and gravitational degrees of freedom in the oscillations of open and closed strings respectively. This explains the principal appeal of string theory, in that it offers a framework in which both gauge and gravitational physics can be simultaneously addressed. In particular, as the higher-derivative corrections to the classical gravitational action can be computed and are finite, string theory represents a quantum theory of gravity.

However, this is not as first advertised, for string theory was originally formulated in the late 1960's as a theory of the strong interactions. In this context it encountered three serious problems. First, the theory required more than four dimensions. Secondly, there always existed a massless spin two state present in the theory, while no such state was observed in the strong interactions. Finally, in string theory scattering amplitudes were exponentially soft at high energy while the observed (QCD) amplitudes were power-law soft. The interest in string theory therefore faded as quantum chromodynamics (QCD) was realised to be the correct description of the strong interactions. It was subsequently proposed by Scherk and Schwarz in 1974 [10] to turn the vices into virtues by re-interpreting the massless spin two particle as a graviton and treating string theory as a theory of quantum gravity. As the critical spacetime dimension for the superstring is ten, string theory gives rise to a ten-dimensional theory of quantum gravity.

Despite its current pre-eminence, this approach was for a long time unpopular. As a highly supersymmetric ten-dimensional theory, the low energy

<sup>&</sup>lt;sup>1</sup>assuming the Planck and string scales to be equivalent.

limit of string theory is ten dimensional supergravity, and it was 'known' that supergravity theories with charged chiral fermions suffered from both gauge and gravitational anomalies. However in 1984 it was shown by Green and Schwarz in a celebrated paper [11] that these anomalies exactly cancelled for gauge groups SO(32) and  $E_8 \times E_8$  with the addition of extra terms arising from the low-energy limit of string theory. This discovery provoked an intense study of the subject which has continued to the present day.

For string theory - a ten-dimensional theory - to represent a candidate description of nature, the extra dimensions have to be compactified. The possibility that extra-dimensional geometry underlies the observed forces and matter content is an old idea dating back to Kaluza and Klein [12, 13]. The original Kaluza-Klein observation was that five-dimensional general relativity compactified on a circle gives a four-dimensional Einstein-Maxwell-scalar theory. String compactifications involve six extra dimensions and the compactification spaces - typically Calabi-Yau threefolds - are far more complicated than a simple  $S^1$ . However while the details differ considerably, the essence is nonetheless the same - the particle content and forces in four dimensions are determined by the geometry of the extra, compactified dimensions.

In addition to the graviton, ten-dimensional supergravity theories also contain gauge fields and p-form potentials. On dimensional reduction, the particles observed in four dimensions come from the zero modes of these fields (there being no experimental evidence for Kaluza-Klein particle towers). Zero modes correspond to zero eigenvalues of a particular differential operator: for the reduction of scalar fields, this is the ordinary Laplacian  $\nabla^2$ . It is a well-known mathematical fact that there is a correspondence between zero modes of differential operators on a given space and topological data of that space such as the dimension of appropriate cohomology groups. For example, in the simplest heterotic string compactifications the number of generations (fermions minus antifermions) is determined by the Euler number of the Calabi-Yau used for the compactification,

$$N_{gen} = \frac{1}{2} |\chi(M)|.$$
 (1.6)

Yukawa couplings are another low-energy quantity that may be geometrically determined in string compactifications. Recall that the masses of Standard Model fermions are given by the fermion Yukawa coupling to the Higgs. In heterotic compactifications, the structure of Yukawa couplings may be determined at tree-level by the triple intersection form

$$\kappa_{ABC} = \int_{X_6} e_A \wedge e_B \wedge e_C,$$

where  $e_A$  are a basis of 2-forms on the Calabi-Yau.

The above explains the reason why string theory has been studied so intensely: it appears to represent a theory of quantum gravity which also naturally

includes the elements of particle physics. Furthermore, in string theory there are no fundamental constants: all statements about masses or couplings are statements about the vacuum state and thus about the dynamics of the theory. This intense study is further driven by the lack of any serious rival: the problem of finding consistent quantum theories unifying gauge and gravitational interactions is just hard, and may even have a unique solution.

The study of string theory has led to many profound results, some with deep connections to apparently unrelated fields. Among these are the discovery of mirror symmetry [14, 15], an exact microscopic calculation of the Bekenstein-Hawking black hole entropy [16], a smooth description of spacetime topology change [17, 18, 19, 20] and the AdS/CFT correspondence [21]. There exist fully pedagogical accounts of the subject in textbooks such as [22, 23]. Despite these successes, progress in phenomenology has been much more limited than had been hoped in 1985: the origin of the structure of the Standard Model is no better understood now than it was then. Advances in this area have been mostly internal and a decisive low-energy test of string theory does not seem possible.

There are two broad approaches to the study of the subject. The first primarily seeks a deeper understanding of the fundamental theory. Such a physicist feels either that the most interesting aspects of string theory are the mathematical ones or that the understanding of string theory is too poor to make contact with experiment: the royal road to experiment lies squarely through the centre of the M-theory duality web. He hopes that apparent problems, such as the multiplicity of vacua, may disappear once the theory is fully understood. A physicist following the second approach does not disagree about the existence of important conceptual problems in understanding and defining the theory. However he feels that we should try and come as close as possible to observed physics. This task sharpens our understanding of the underlying physics, and anyhow it is surely not the case that the deep structure and complexity of the Standard Model will just pop out, however well we understand the theory. These approaches are of course complementary; the possibilities for stringy model-building were greatly enriched by the 1995 discovery of D-branes [24], whereas the discovery of mirror symmetry followed scatter plots of the Hodge numbers of known Calabi-Yaus[14, 15]. However, in this thesis I shall align myself more with the second approach to the subject.

The ultimate objective of string model-building is to reproduce and explain the entire structure of the Standard Model: scales, particle content, masses, charges and interactions. As indicated above, explaining even a very limited subset of these would constitute a great success. This enterprise has one main conceptual problem and several important technical problems. The conceptual problem is that of vacuum selection. String theory seems to admit a very large number of vacuum solutions and there does not seem to be a good way of choosing between these. It is relatively easy to find vacua qualitatively similar to the

Standard Model, but extremely hard to find models quantitatively the same. This conceptual problem is partly philosophical and I shall not discuss it here: recent articles include [25, 26, 27].

The particular technical problems encountered are to

- 1. Obtain the correct gauge group and chiral spectrum.
- 2. Ensure the absence of exotic particles, either charged or uncharged.
- 3. Understand the physics of supersymmetry breaking.
- 4. Explain why the observed couplings take the values they do and predict new effects.
- 5. Describe the cosmology of the universe from the Planck epoch to nucleosynthesis.

These problems are separate but inter-related. In this thesis I shall discuss all problems in the above list except the first. Part II of this thesis will focus on the second problem, and in particular on a class of uncharged scalar particles known as moduli. The failure to observe fifth forces implies such particles should be given masses and hence potentials. This is the problem of moduli stabilisation. Solutions to the problem of moduli stabilisation are generally prerequisites for solutions of the third and fourth problems, and in part III of this thesis we shall apply the results of part II to study these problems, discussing soft supersymmetry breaking, inflation and the strong CP problem.

The work contained in this thesis is based on the papers [2] (chapter 3), [3] (chapter 5), [4] (chapters 4, 5 and 6), [5] (chapter 8), [6] (chapter 7) and [7] (chapter 6), As indicated in the preface I am grateful to my collaborators Vijay Balasubramanian, Per Berglund, Fernando Quevedo and Kerim Suruliz.

I have a final note on references. I have tried to cite relevant work where appropriate, but it is inevitable that there are lapses. As this is a thesis rather than a review article I have focused primarily on my own work and the results most directly relevant to it, with the consequence that I have failed to cite many interesting and important articles. I apologise in advance to the authors of these papers.

# Chapter 2

# Moduli and Fluxes: A Brief Review

The purpose of this chapter is to review background material on moduli and flux compactifications. An inexhaustive list of useful references extending this discussion is [28, 29, 30, 31, 32], and in particular the review of flux compactifications [33].

# 2.1 String Compactifications

The most straightforward formulation of string theory is in flat ten-dimensional Minkowksi space. There are then five string theories: the IIA and IIB closed string theories, the type I (type IIB orientifold) SO(32) theory, and the heterotic  $E_8 \times E_8$  and SO(32) theories. These are all unsatisfactory in (at least) two ways: the gauge group is much larger than that of the Standard Model and the number of dimensions is six too many.

The standard resolution of these problems is compactification, in which the ten dimensional space becomes a (possibly warped) product of our 4-dimensional world with an internal 6-dimensional space X. The most general metric reproducing 4-dimensional Poincare invariance is

$$ds_{10}^2 = e^{-2A(y)}ds_4^2 + e^{2A(y)}g_{mn}dy^m dy^n. (2.1)$$

The  $y^m$  parametrise the internal space X and the factor of  $e^{-2A(y)}$  allows for the possibility of warping in the ansatz, Large variations in the warp factor A(y) may generate interesting physical effects as in the Randall-Sundrum scenario [34, 35]. However it will not be important for our purposes, and so unless explicitly stated we shall not regard warping as significant. The characteristic signals of

extra dimensions are either the production of Kaluza-Klein (KK) copies of the Standard Model or the modification of gravity at short distances. Neither have been observed, and so consistency requires X to be compact and sufficiently small that the resulting physics below the Kaluza-Klein scale is effectively four-dimensional.

Compactification clearly reduces the number of large dimensions, but it also permits a reduction in the rank of the gauge group. In heterotic compactifications, this is achieved by giving vevs to extra-dimensional components of the gauge potential or field strength. To be unbroken, a gauge symmetry must leave such vevs unaffected, and so the surviving gauge group is the commutant of the turned-on fields in the ten-dimensional gauge group. In the type II orientifold compactifications we will be most interested in, the orientifold tadpole must be cancelled by the D-branes present. The loci and intersections of these branes determine the low-energy gauge group. The rank of this is model-dependent but may naturally be small.

This philosophy of compactification says very little about the internal space X. Fundamentally, X should be determined by the cosmology of the very early (Planckian) universe. As spacetime topology change has a smooth description in string theory [17, 18, 19, 20], such transitions ought to occur in the early universe. The choice between compactification manifolds  $X_1$  and  $X_2$  is then determined by cosmological dynamics. However, as Planck-scale cosmology is extremely speculative we instead look to particle physics for a guiding principle.

This guiding principle is taken to be the existence of  $\mathcal{N}=1$  supersymmetry at low energy. There are a variety of reasons why this is phenomenologically desirable. The most well known is the quadratic sensitivity of the Standard Model Higgs mass to the cutoff  $\Lambda$  used in loop integrals such as in figure 2.1. In the Standard Model unitarity bounds require the Higgs mass to be  $m_H \lesssim 1$ TeV. The diagram in figure 2.1 gives

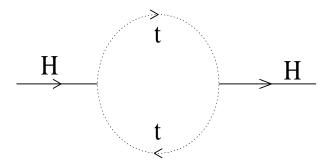


Figure 2.1: The Higgs mass is radiatively unstable in the Standard Model due to quadratic divergences from top loops.

$$m_H^2 \sim \int^{\Lambda} d^4 p \frac{1}{p^2} \sim m_{H,bare}^2 + \Lambda^2.$$
 (2.2)

Physically, the cutoff represents the scale of new physics. The Standard Model is renormalisable and thus in principle valid up to the Planck scale. However, if we take  $\Lambda = M_P$  we must fine-tune  $m_{H,bare}^2$  to an unacceptable degree of precision (around one part in  $10^{30}$ ) in order to have  $m_H$  around the electroweak scale - this is the electroweak hierarchy problem. This suggests that the cutoff  $\Lambda$  should instead appear at the TeV scale. A good candidate for the cutoff physics is supersymmetry - a symmetry relating bosons and fermions. This tends to cancel divergences in loop integrals, as particles and their superpartners give exactly opposite contributions when running around the loop. Of course, supersymmetry must be a broken symmetry, as no superpartners have currently been observed.

There are other attractions of low energy supersymmetry. Through the tachyonic running of the up-type Higgs mass term, it can explain why the scale of electroweak symmetry breaking and the scale of the physics stabilising the Higgs mass should be comparable. TeV-scale supersymmetry with the Minimal Supersymmetric Standard Model (MSSM) matter content also leads to the unification of the MSSM gauge coupling constants at the GUT scale  $10^{16}$  GeV. Finally, supersymmetry also provides a natural dark matter candidate in the lightest neutralino, whose stability is protected by R-parity. We shall not dwell further on low energy  $\mathcal{N}=1$  supersymmetry as an attractive phenomenological scenario. We shall return to supersymmetric phenomenology in chapter 6, but for now mention [36] as an extensive and detailed account with thorough references.

We add one cautionary note. The justification for Calabi-Yau compactification, on which most string model building (including this thesis) has been built, is closely tied to low energy supersymmetry. If the Large Hadron Collider (LHC) finds that physics other than supersymmetry is responsible for stabilising the Higgs mass, this would remove the prime motivation for low-energy supersymmetry. There would then be little theoretical reason to find supersymmetry below the Planck scale and the relevance of Calabi-Yau compactification to nature would be questionable.

The central topic of this thesis is moduli stabilisation. To frame this as a problem, we first review the *fons et origo* of string compactifications [28]. Although involving the heterotic string rather than the IIB case we are primarily interested in, both results and techniques are general.

The heterotic string, either with SO(32) or  $E_8 \times E_8$  gauge group, is one of the five consistent string theories in ten flat dimensions. At low energy, string theory reduces to supergravity and we only focus on the (massless) supergravity degrees of freedom. The massless fields are the graviton  $g_{MN}$ , dilaton  $\Phi$ , antisymmetric tensor field  $B_{MN}$  and a gauge field  $A_M$ . These last have 3- and 2-form field strengths,  $H_3 = dB_2$  and F = dA.

We ask when it is possible to preserve a global  $\mathcal{N}=1$  supersymmetry in compactifications of the heterotic string. If we do not turn on  $H_3$ , it was shown in [28] that in order to preserve  $\mathcal{N}=1$  supersymmetry the fields must satisfy

$$ds_{10}^2 = \eta_{\mu\nu} + g_{mn}dy^m dy^n, (2.3)$$

$$H_{mnp} = 0, (2.4)$$

$$\Phi = \Phi_0, \tag{2.5}$$

$$F_{ij} = F_{\bar{i}\bar{j}} = G^{i\bar{j}}F_{i\bar{j}} = 0,$$
 (2.6)

$$Tr(\mathcal{R}_2 \wedge \mathcal{R}_2) - Tr_v(F_2 \wedge F_2) = 0, \tag{2.7}$$

where the trace is taken in the vector representation of the gauge group.  $g_{mn}$  is the metric on the internal space, which must be a Ricci-flat Kähler manifold. This does not seem a strenuous requirement - surely *some* Ricci-flat metric must exist on any given Kähler manifold. However, in general this is true only locally, and there is a topological obstruction to the global existence of such a metric. It was conjectured by Calabi and proved by Yau that

**Theorem 1 (Yau)** Any Kähler manifold X with vanishing first Chern class admits a Ricci-flat metric, unique up to the complex structure of X and the cohomology class of the Kähler form.

Such manifolds are called Calabi-Yau manifolds. Compactification of either heterotic (as above) or type I strings on a Calabi-Yau gives rise to  $\mathcal{N}=1$  supersymmetry in four dimensions (compactifying type II strings preserves  $\mathcal{N}=2$  supersymmetry). There are consistency conditions (2.6) and (2.7) on the gauge bundle, which can most simply be satisfied by identifying F and  $\mathcal{R}$ . The resulting gauge group is the commutant of F in  $E_8 \times E_8$  (or SO(32)). For the standard embedding  $F = \mathcal{R}$ , the tangent and gauge bundles are SU(3) bundles. The commutant of SU(3) in  $E_8$  is  $E_6$  and so this gives the low-energy gauge group  $E_6 \times E_8$ .

It is also possible to look for compactifications with non-vanishing 3-form flux, with  $dH_{mnp} = 0$  but  $H_{mnp} \neq 0$ . In this case supersymmetry requires that X be an SU(3) structure manifold with nontrivial torsion [37, 33]. However, despite recent interest in this area we shall not consider heterotic flux compactifications further in this thesis.

## 2.2 Moduli

Given a particular compactification, the principal desiderata are the spectrum and interactions of the light particles. 'Light' implies the dropping of stringy 2.2. MODULI 27

states with characteristic masses of  $\mathcal{O}(\frac{1}{\sqrt{\alpha'}})$ , and Kaluza-Klein states with characteristic masses  $\mathcal{O}\left(\frac{1}{R\sqrt{\alpha'}}\right)$  for R the compactification radius in string units. This leaves us with the states that are present in the supergravity spectrum and survive dimensional reduction to four dimensions.

Any compactification leads to a particular particle spectrum and gauge group. In one sense, the most interesting feature of this will be the gauge sector of particles, as we can compare this to that present in the Standard Model or its various extensions. We may define this as consisting of all particles charged under the various gauge groups present. In heterotic compactifications these are determined by the gauge bundle over the Calabi-Yau, whereas in orientifold compactifications these come from open strings ending on D-branes. However, we shall not discuss this interesting and important issue further (see [22, 38, 39, 40] for reviews), and instead focus on the moduli sector.

Moduli are senso stricto massless scalars  $\phi_i$  that parametrise continuous families of vacua. They are characteristic of supersymmetric compactifications which typically come with large moduli spaces. They may or may not be charged. For example, the process of Higgsing a supersymmetric D-brane stack may be described as giving a vev to a charged modulus. However, we shall use the word mainly to refer to uncharged scalars whose only interactions are gravitational, and shall also broaden the use to include scalar fields after a potential has been generated for them.

We would like to enumerate the different kinds of moduli. Moduli parametrise continuous families of nearby vacua, and so for Calabi-Yau compactifications the most obvious moduli are those parametrising the space of Calabi-Yau manifolds. As a continuous deformation cannot alter toplogical properties, we are restricted to the space of topologically equivalent Calabi-Yaus. The mathematical question asked is, given that  $g_{mn}$  is the metric for a Calabi-Yau space Y, under what circumstances is  $g_{mn} + \delta g_{mn}$  a metric for a neighbouring Calabi-Yau space? Ricci-flatness implies

$$R_{mn}(g) = R_{mn}(g + \delta g) = 0.$$

This implies that  $\delta g$  satisfy the Lichnerowicz equation,

$$\nabla^l \nabla_l \delta g_{mn} + 2R^l_{m}{}^r{}_n \delta g_{lr} = 0. \tag{2.8}$$

For Kähler manifolds the solutions to this equation are independent and associated with either pure  $(\delta g = \delta g_{\mu\bar{\nu}})$  or mixed  $(\delta g = \delta g_{\mu\bar{\nu}})$  deformations. These are in one-to-one correspondence with harmonic (2,1)- and (1,1)-forms respectively. For mixed solutions this correspondence is given by

$$\delta g_{\mu\bar{\nu}} \leftrightarrow \delta g_{\mu\bar{\nu}} dx^{\mu} \wedge dx^{\bar{\nu}} \in H^{1,1}(M),$$
 (2.9)

 $<sup>^{1}</sup>$ We shall not discuss extremal transitions and paths in moduli space connecting topologically distinct Calabi-Yaus.

whereas for pure solutions  $\delta g_{\bar{\mu}\bar{\nu}}$  we have

$$\delta g_{\bar{\mu}\bar{\nu}} \leftrightarrow \Omega^{\bar{\nu}}_{\kappa\lambda} \delta g_{\bar{\mu}\bar{\nu}} dx^{\kappa} \wedge dx^{\lambda} \wedge dx^{\bar{\mu}} \in H^{2,1}(M),$$
 (2.10)

and likewise for  $\delta g_{\mu\nu}$  and (1,2)-forms. As the structure of harmonic differential forms is well known to be isomorphic to that of tangent bundle cohomology classes, the number of geometric moduli in compactifications on Y is determined by the cohomology of Y.

We note this is a general feature of string compactifications - the light particle spectrum is determined by topological considerations and the number of particles of given type is equivalent to the dimension of appropriate (sheaf) cohomologies.

The moduli associated with (1,1)-forms are called Kähler moduli and the moduli associated with (2,1)-forms are called complex structure moduli. This nomenclature is because the former modify the Kähler class of the manifold whereas the latter alter the complex structure. The first claim is manifest: under  $g \to g + \delta g$ , the Kähler form behaves as

$$J = ig_{\mu\bar{\nu}}dx^{\mu} \wedge dx^{\bar{\nu}} \to i(g_{\mu\bar{\nu}} + \delta g_{\mu\bar{\nu}})dx^{\mu} \wedge dx^{\bar{\nu}}.$$

To verify the latter, we first note that  $g + \delta g$  is a Kähler metric. This follows because the Kähler form is unaltered so long as  $\delta g$  is a pure deformation. Thus there exists a coordinate system in which all pure components of the metric vanish. However, under general coordinate transformations  $x^m \to x^m + f^m(x^n)$ ,  $\delta g$  transforms as

$$\delta g_{mn} \to \delta g_{mn} - \frac{\partial f^r}{\partial x^m} g_{rn} - \frac{\partial f^r}{\partial x^n} g_{mr}.$$
 (2.11)

Consequently holomorphic transformations  $f = f(x^{\mu})$  cannot alter the pure components  $g_{\mu\nu}$  of the metric. A coordinate system in which the metric is Kähler can only be obtained by a non-holomorphic coordinate change, which by definition will change the complex structure. This can also be understood more succinctly from Yau's theorem: as the Ricci-flat metric is unique up to the complex structure and the cohomology class of the Kähler form, the physical moduli should parametrise the possible modifications of these.

The above are geometric moduli. In addition to the geometric fields string theory contains antisymmetric form fields, which also give rise to moduli. For example, heterotic compactifications contain a 2-form  $B_2$  field, with a field strength

$$H_3 = dB_2 + CS(F) - CS(\mathcal{R}),$$

where CS refers to the Chern-Simons 3-form. This field strength is unaltered by a transformation

$$B_2 \to B_2 + b^i e_i$$
,

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where  $e_i \in H^2(Y,\mathbb{R})$ . The fields  $b_i(x)$  associated with internal 2-cycles thus do not affect the field strength and are also moduli. This feature is general across string compactifications - for example, in type IIB compactifications, the  $B_2$ ,  $C_2$  and  $C_4$  potentials all also give rise to moduli.

There are in addition other sources of moduli such as brane and bundle moduli. Brane moduli are moduli associated with the positions of branes in the compactification manifold. These may be charged and can act as Higgs fields for the gauge group on the branes. Bundle moduli occur when nontrivial gauge backgrounds exist in the theory and describe the possible deformations of the gauge bundle that are consistent with the equations of motion.

We shall mostly focus on the bulk moduli in this thesis. These are more generic: the gauge sector in string compactifications is heavily model-dependent, but the bulk sector is very similar across different string theories and compactifications. As mentioned above, in the simplest compactifications such as [28] the moduli represent massless uncharged scalar particles. The existence of such massless scalars is inconsistent with experiment. Moduli couple gravitationally to ordinary matter and so can generate forces due to particle exchange. For a modulus of mass  $m_{\phi}$ , the characteristic range of such forces is  $R \sim \mathcal{O}(\frac{1}{m_{\phi}})$ . As fifth force experiments have probed gravity to submillimetre distances, this requires that  $m_{\phi} > \mathcal{O}(10^{-3})$  eV [41]. Consequently experiment requires the existence of potentials giving masses to the moduli.

We note as an aside that the above does not apply for one particular class of particle, axions. Axions have topological couplings to gauge groups and derivative couplings to matter. Derivative couplings vanish for long-distance interactions (i.e. small momentum transfer) and so do not give rise to fifth forces. We shall discuss axions further in chapter 7.

As creating and controlling moduli potentials is a hard task, we may prefer to look for compactifications without moduli. However while massless scalars are inconsistent with experiment, the dynamics of massive scalars is very important. The physics of inflation - the dominant theory of structure formation - is determined by a scalar field potential. Scalar potentials also play an important role in theories of supersymmetry breaking, a topic we will return to in chapter 6.

Our aim is now clear. Given that massless moduli are both a generic feature of string compactifications and experimentally disallowed, we need techniques that will create a potential for these moduli and give them masses. Fluxes are a powerful example of such a technique: it is to these we now turn.

### 2.3 Fluxes and Moduli Stabilisation

The above establishes the need to give potentials to moduli. A useful way of doing this is through flux compactifications. These are distinguished from ordinary compactifications by having antisymmetric tensor fields that are nonvanishing in the compact space. They have a long history in string theory[37, 42, 43, 44, 45, 31, 46]. Our main interest will be 3-form fluxes in IIB orientifold compactifications. However, we first want to develop a general intuition for why fluxes can stabilise moduli. As a simple example, we take the geometry of the principal avatar of the AdS/CFT correspondence[21], namely type IIB string theory on  $AdS_5 \times S^5$ . This has 5-form flux present on the  $S^5$ . The ten-dimensional supergravity can be solved exactly to give

$$ds^{2} = h(r)^{-\frac{1}{2}} \left( -dt^{2} + dx^{2} + dy^{2} + dz^{2} \right) + h(r)^{\frac{1}{2}} \left( dr^{2} + r^{2} d\Omega_{5}^{2} \right),$$

$$\Phi = \Phi_{0},$$

$$F_{5} = \mathcal{F}_{5} + \mathcal{F}_{5}^{*}, \quad \text{with } \mathcal{F}_{5} = 16\pi\alpha'^{2}N\mathbf{vol}(S^{5}).$$
(2.12)

Here  $h(r) = 1 + \frac{L^4}{r^4}$ , where  $L = (4\pi g_s N \alpha'^2)^{\frac{1}{4}}$  is the radius of both  $AdS_5$  and  $S^5$  in the  $AdS_5 \times S^5$  near-horizon limit of this geometry. However, our interest here is less in the full ten-dimensional solution than in understanding why the  $S^5$  radius L obtains the value it does. To this end, we reduce this geometry to a 5d effective theory and evaluate the potential energy as a function of L.

The 10-dimensional type IIB supergravity action is given in Einstein frame by [22, 23]

$$S_{IIB} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left[ \mathcal{R} - \frac{\partial_M \tau \partial^M \bar{\tau}}{2(\text{Im}\tau)^2} - \frac{G_3 \cdot \bar{G}_3}{12\text{Im}\tau} - \frac{\tilde{F}_5^2}{4 \cdot 5!} \right] + S_{CS}, \quad (2.13)$$

where  $G_3 = F_3 - \tau H_3$  and  $S_{CS}$  is the Chern-Simons term. Here  $\kappa_{10}^2 \sim l_s^8$ , with  $l_s = 2\pi\sqrt{\alpha'}$ .  $\tau$  is the dilaton-axion and  $G_3$  and  $F_5$  are 3-form and 5-form fluxes.

We now suppress the dilaton-axion, 3-form fluxes and numerical coefficients as not relevant for our purposes. Compactifying on an  $S^5$  of radius  $L = Rl_s$  with N units of  $F_5$  flux on the  $S^5$ , we obtain schematically

$$S_{compactified} = \frac{1}{l_s^8} \int d^{10}x \sqrt{-g} \left( \mathcal{R}_{5D} + \mathcal{R}_{S^5} - F_5^2 \right)$$
$$= \frac{1}{l_s^8} \int d^5x \sqrt{-g} \left( \mathcal{R}_{5D} (R^5 l_s^5) + \frac{(R^5 l_s^5)}{R^2 l_s^2} - \frac{N^2 l_s^5}{R^5 l_s^2} \right). \quad (2.14)$$

We have used the quantisation condition for  $F_5$ ,

$$\int_{S^5} F_5 = (2\pi)^4 \alpha'^2 N \sim N l_s^4. \tag{2.15}$$

We now convert this to 5-dimensional Einstein frame. Under a metric rescaling,

$$g = \tilde{g}\Omega \Rightarrow \sqrt{-g}\mathcal{R} = \sqrt{-\tilde{g}}\tilde{\mathcal{R}}\Omega^{\frac{3}{2}}.$$

Therefore if we take

$$\Omega = \frac{l_s^2}{l_5^2} R^{-\frac{10}{3}},\tag{2.16}$$

where  $l_5$  is the 5D Planck mass, the resulting action for  $\tilde{g}$  is canonical and given by

$$S_{5D} = \frac{1}{l_5^3} \int d^5 x \sqrt{-\tilde{g}} \left( \tilde{\mathcal{R}} + \frac{1}{l_5^2} R^{-\frac{16}{3}} - \frac{1}{l_5^2} N^2 R^{-\frac{40}{3}} \right). \tag{2.17}$$

The effective potential for the radial modulus is then

$$V(R) = -R^{-\frac{16}{3}} + N^2 R^{-\frac{40}{3}}. (2.18)$$

The fluxes contribute positively to the potential energy and the curvature negatively. This potential has an AdS minimum at  $R \sim N^{\frac{1}{4}}$ , with scaling behaviour consistent with the full solution.

Physically, the intuition we derive is that fluxes wrapped on a cycle carry a potential energy which depends on the volume and geometry of the cycle. As moduli parametrise the geometry, this automatically generates a potential for the moduli describing these cycles. This potential can be analysed from an effective field theory viewpoint. There may be other contributions to the potential - here the  $S^5$  curvature is important - and these must also be included. Putting everything together gives a potential for the  $S^5$  radius. For completeness, we note that in the above example the  $S^5$  radius is not properly a modulus, because without the 5-form flux the solution is unstable rather than flat. However, this will not be the case in the more realistic flux compactifications we now study.

# 2.4 Flux Compactifications of IIB Orientifolds

Let us move from the above motivational example to the setup we shall use for the rest of this thesis. This is a class of flux compactifications of type IIB orientifolds with D3/D7 branes and O3/O7 planes [31] developed by Giddings, Kachru and Polchinski. Discrete NSNS and RR 3-form fluxes are present which generically give masses to dilaton, complex structure and D7 brane moduli. At this level the Kähler moduli are unstabilised and for these moduli the potential is in fact no-scale.

One useful feature of these compactifications is that they can be described in both ten- and four-dimensional language. While Kaluza-Klein reduction to deduce low energy actions is to some extent a necessity, given our lack of knowledge of, *inter alia*, Calabi-Yau metrics, it is not a completely ideal approach. This is

because it entails a truncation, rather than a Wilsonian integration out, of the heavy Kaluza-Klein modes. One can argue that the resulting low energy action is nonetheless correct, (see e.g. chapter 16 of [23]), but one would still prefer a ten-dimensional picture. It is an attractive feature of the compactifications of [31] that we can adopt either a four- or ten-dimensional viewpoint.

As these compactifications will play a fundamental role in the rest of the thesis we now review them in first ten- and then four-dimensional language.

#### 2.4.1 Ten-dimensional Perspective

The models of [31] are orientifold compactifications. That is, in addition to the purely geometric background there are non-dynamical negative tension orientifold planes. The cancellation of RR tadpoles then necessitates the inclusion of D-branes. As both O-planes and D-branes are localised sources, the appropriate action is ten dimensional IIB supergravity coupled to localised sources.

In Einstein frame this is

$$S = \frac{1}{(2\pi)^7 \alpha'^4} \int d^{10}x \sqrt{-g} \left[ \mathcal{R} - \frac{\partial_M \tau \partial^M \bar{\tau}}{2(\text{Im}\tau)^2} - \frac{G_3 \cdot \bar{G}_3}{12\text{Im}\tau} - \frac{\tilde{F}_5^2}{4 \cdot 5!} \right] + S_{cs} + S_{loc},$$
(2.19)

where  $G_3 = F_3 - \tau H_3$  with  $\tau = C_0 + ie^{-\phi}$  the axio-dilaton. The 5-form  $\tilde{F}_5$  is defined by

$$\tilde{F}_5 = F_5 - \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge F_3. \tag{2.20}$$

The Chern-Simons term is

$$S_{cs} = \frac{1}{4i(2\pi)^7 \alpha'^4} \int C_4 \wedge G_3 \wedge \bar{G}_3.$$
 (2.21)

Note that the presence of nontrivial 3-form flux  $G_3$  sources a tadpole for the  $C_4$  field, i.e. for D3-brane charge.

The localised sources are D3/7 branes and O3/7 planes. Dropping B and F fields, the action for a Dp-brane wrapping a cycle  $\Sigma$  is

$$S_{loc} = -T_p \int_{R^4 \times \Sigma} d^{p+1} \xi \sqrt{-g} + \mu_p \int_{R^4 \times \Sigma} C_{p+1}.$$
 (2.22)

In comparison to a D-brane, an orientifold plane has opposite tensions and RR charges.

In the above setup there are several discrete choices to be made, the most obvious of which is the compactification manifold. It is also necessary to specify the cohomology of the 3-form fluxes. These are integrally quantised with

$$\frac{1}{(2\pi)^2 \alpha'} \int_{\Sigma} F \in \mathbb{Z}, \quad \text{and} \quad \frac{1}{(2\pi)^2 \alpha'} \int_{\Sigma} H \in \mathbb{Z}, \tag{2.23}$$

where F and H are the RR and NSNS 3-form fluxes respectively. Given these choices, the equations of motion for the fields may be solved and the resulting solutions described as follows.

The metric takes the warped form

$$ds^{2} = e^{2A(y)} \eta_{\mu\nu} dx^{\mu} dx^{\nu} + e^{-2A(y)} \tilde{g}_{mn} dy^{m} dy^{n}, \qquad (2.24)$$

giving a warped product of Minkowski space with a conformally Calabi-Yau internal space. This latter property is key, as it allows much of the well-established technology of Calabi-Yau compactifications to be reused. The warp factor is related to the self-dual 5-form flux by

$$\tilde{F}_5 = (1+*) \left[ d\alpha \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \right], \tag{2.25}$$

with  $\alpha = e^{4A(y)}$ . The 3-form flux is imaginary self-dual (ISD), satisfying

$$G_3 = i *_6 G_3, (2.26)$$

where  $*_6$  is the Hodge dual on the internal space.

The resulting solution has  $\mathcal{N}=1$  supersymmetry, which may be spontaneously broken by the fluxes. The condition that supersymmetry be preserved is that the 3-form flux  $G_3$  is (2,1) and primitive. The primitivity condition  $G_3 \wedge J = 0$  holds automatically for a Calabi-Yau. The ISD condition requires that the (1,2) and (3,0) parts of  $G_3$  vanish, and so the residual requirement for the existence of supercharges leaving the vacuum invariant is that the (0,3) part of  $G_3$  vanish.

The ISD condition fixes the dilaton and complex structure moduli. Although naively a condition on  $G_3$ , we should instead regard it as a condition on the complex structure moduli, as the definition of  $*_6$  depends on the complex structure of the internal space. The complex structure adjusts to satisfy condition (2.26), thereby fixing all complex structure moduli. As the dilaton appears in the definition of  $G_3$ , it is also fixed by the fluxes.

The Kähler moduli are unfixed as there exists a continuous family of solutions (2.24) related by rescalings of the internal volume. The warp factor transforms nontrivially under these but vanishes at large volume as

$$\alpha = e^{4A} \sim 1 + \frac{1}{\mathcal{V}^{2/3}}. (2.27)$$

In the large-volume limit the warping therefore becomes negligible.

As mentioned above, the fluxes generate a tadpole for  $C_4$ . There are other sources for this tadpole; in particular, curvature couplings on wrapped D7-branes also generate negative D3 charge. In the language of F-theory, this charge is determined by the Euler number of the F-theory fourfold Z:

$$Q_3^{D7} = -\frac{\chi(Z)}{24}. (2.28)$$

The resulting tadpole equation is

$$\frac{1}{(2\pi)^4 \alpha'^2} \int H_3 \wedge F_3 + N_{D3} - N_{\bar{D}3} = \frac{\chi(Z)}{24}.$$
 (2.29)

(There may also be extra sources of D3-charge; for example, from gauge fields on D7 branes. We shall not concern ourselves with these.) The number of D3 branes that must be introduced varies with the discrete flux choices. If we wish to avoid the need to include D3 branes, we can always take the fluxes to saturate the tadpole condition.

The above is a consistent solution of 10D supergravity coupled to localised sources at leading order in the  $g_s$  and  $\alpha'$  expansions. We now analyse it from a four-dimensional perspective.

#### 2.4.2 Four-dimensional Perspective

On general grounds, the above compactifications ought to be described at low energy by a 4-dimensional  $\mathcal{N}=1$  supergravity theory. Here 'low-energy' refers to energies substantially below the Kaluza-Klein scale. Physically, this simply says that the ten-dimensional solution carries far more information than is needed. For probe energies much below the Kaluza-Klein scale, the precise geometry blurs and we can only resolve bulk properties such as the overall volumes, but not fine-grained structure.

A four-dimensional  $\mathcal{N}=1$  supergravity theory is specified at low energies (i.e. at two derivatives) by its matter content and by the Kähler potential  $\mathcal{K}$ , superpotential W and gauge kinetic functions  $f_i$ . The latter run over gauge groups and give the complexified gauge coupling for each gauge group. One advantage of working in 4-dimensional language is that nonrenormalisation results are available that would be very difficult to see directly in ten dimensions. Let us state these, using  $\phi_i$  to denote the chiral superfields present.

- 1. The Kähler potential  $\mathcal{K}(\phi_i, \bar{\phi}_i)$  is a real non-holomorphic function of the chiral superfields. It is renormalised at all orders in perturbation theory and nonperturbatively.
- 2. The superpotential  $W(\phi)$  is a complex holomorphic function of the chiral superfields. It is not renormalised in perturbation theory, but may receive nonperturbative corrections.
- 3. The gauge kinetic functions  $f_i(\phi)$  are complex holomorphic functions of the chiral superfields. They are perturbatively exact at one loop, but will receive nonperturbative corrections.

To specify the low energy theory, it is first necessary to specify the low energy fields. We are focusing on moduli, and so will neglect any charged matter that may be present. The reason why this is sensible is that moduli obtain large classical vevs, while the vevs of Standard Model matter are vanishing or hierarchically small (as with the Higgs). It is then consistent to neglect the dynamics of charged matter while studying the moduli potential.

We shall primarily be concerned with the closed string moduli which are more generic as they occur in all string compactifications. It is also possible to be more explicit when treating them. However, in the next section we shall also briefly discuss the open string moduli and how they mix with the closed string fields.

In the above IIB flux compactifications the dilaton superfield is defined by

$$S = e^{-\phi} + iC_0, (2.30)$$

where  $e^{\phi}$  is the string coupling and  $C_0$  the RR 0-form. Strictly this is only the scalar component of the full superfield, but we will generally overload notation and use the same symbol to denote both the superfield and its scalar component. Sometimes the alternative definition  $\tau = -C_0 + ie^{-\phi} \equiv iS$  is used.

There are various equivalent ways to define the complex structure moduli. For a Calabi-Yau, they are uniquely specified by the periods

$$\Pi^i = \int_{\Sigma_i} \Omega,$$

where  $\Sigma_i$  is a 3-cycle,  $\Omega$  the holomorphic (3,0) form and  $\Pi^i$  the period vector.  $\Omega$  is only defined up to overall rescalings, and so one can use the period vector as projective coordinates for the complex structure moduli. Many Calabi-Yaus are hypersurfaces in weighted projective spaces and specified by a defining equation  $f(z_i) = 0$ . In this case we may instead define the complex structure moduli by the coefficients of the defining polynomial - this will be the approach adopted in section 3.2.

The Kähler moduli are defined by

$$T_i = \sigma_i + i\rho_i, \tag{2.31}$$

where  $\sigma_i$  is the Einstein-frame volume of a 4-cycle  $X_i$  and  $\rho_i = \int_{X_i} C_4$ .  $\sigma_i$  can be expressed in terms of 2-cycle volumes  $t^i$  by

$$\sigma_i = \frac{\partial \mathcal{V}}{\partial t^i} = \frac{1}{2} k_{ijk} t^j t^k. \tag{2.32}$$

We can now specify the Kähler and superpotentials for the compactifications of [31]. The superpotential is that of Gukov, Vafa and Witten and takes the form [47]

$$W = \frac{1}{(2\pi)^2 \alpha'} \int_M G_3 \wedge \Omega, \tag{2.33}$$

where  $G_3 = F_3 + iSH_3$ . As the periods specify the complex structure, it follows that the superpotential depends on all the complex structure moduli. However, note that (2.33) is independent of the Kähler moduli. In F-theory, (2.33) generalises to

$$W = \int_{M} G_4 \wedge \Omega. \tag{2.34}$$

In F-theory, the D7 position moduli correspond to complex structure moduli of the fourfold, and thus these are also fixed by the fluxes.

The Kähler potential is to leading order given by

$$\mathcal{K}_{no-scale} = -2\log\left[\mathcal{V}\right] - \log\left[-i\int_{M} \Omega \wedge \bar{\Omega}\right] - \log\left[S + \bar{S}\right]. \tag{2.35}$$

 $\mathcal{V} \equiv \frac{1}{6}k_{ijk}t^it^jt^k$  should be regarded as an implicit function of the Kähler moduli, as in general the relations (2.32) cannot be inverted. (2.35) possesses no-scale structure. That is, in the  $\mathcal{N} = 1$  F-term scalar potential,

$$V = e^{\mathcal{K}} \left[ \mathcal{K}^{i\bar{j}} D_i W \bar{D}_j \bar{W} - 3|W|^2 \right], \qquad (2.36)$$

with i, j running over all moduli, the sum over Kähler moduli cancels the  $3|W|^2$  term and the resulting potential is given by

$$V_{no-scale} = e^{\mathcal{K}} \mathcal{K}^{ab} D_a W \bar{D}_b \bar{W}, \qquad (2.37)$$

where a and b run over dilaton and complex-structure moduli only.

It follows that we can stabilise the dilaton and complex structure moduli at a minimum of the potential by solving

$$D_S W \equiv \partial_S W + (\partial_S \mathcal{K})W = 0, \qquad (2.38)$$

$$D_a W \equiv \partial_a W + (\partial_a \mathcal{K})W = 0. \tag{2.39}$$

We denote the resulting value of W as  $W_0 = \langle \int G_3 \wedge \Omega \rangle$ . It can be shown that for solutions of equations (2.39), the flux tadpole  $\int G_3 \wedge \bar{G}_3$  becomes positive definite. In this approximation, the Kähler moduli are unfixed due to the no-scale structure. As

$$D_T W = (\partial_T \mathcal{K}) W \propto \int G_3 \wedge \Omega,$$
 (2.40)

the F-terms corresponding to the Kähler moduli are nonvanishing unless W=0. This is the same condition to preserve supersymmetry as arose in the 10-dimensional picture: W=0 is equivalent to  $G_3$  having no (0,3) component. It can likewise be seen that requiring  $D_SW=D_\phi W=0$  eliminates the (3,0) and (1,2) components of W. Thus - as expected - we recover the same results in both four-dimensional and ten-dimensional approaches.

This construction will stabilise the dilaton and all complex structure moduli for a generic choice of fluxes. However, its use would be limited unless there were practical methods to compute the Kähler and superpotentials as a function of the moduli. In particular, in order to discuss complex structure moduli stabilisation in as explicit as fashion as possible, we need to be able to compute the periods of Calabi-Yaus. To this end, we now briefly review some standard material on complex geometry which will be needed in the next chapter.

For any Calabi-Yau 3-fold, the middle homology and cohomology are naturally expressed in terms of a symplectic basis. That is, there exists a basis of 3-cycles  $A^a$ ,  $B_b$  and a basis of 3-forms  $\alpha_a$ ,  $\beta^b$  (where  $a, b = 1, 2 \dots, h^{2,1} + 1$ ), such that in homology

$$A^{a} \cap B_{b} = -B_{b} \cap A^{a} = \delta_{b}^{a},$$
  

$$A^{a} \cap A^{b} = B_{a} \cap B_{b} = 0,$$
(2.41)

and

$$\int_{A^b} \alpha_a = -\int_{B_a} \beta^b = \delta_a^b, \tag{2.42}$$

$$\int_{\mathcal{M}} \alpha_a \wedge \beta^b = -\int_{\mathcal{M}} \beta^b \wedge \alpha_a = \delta_a^b. \tag{2.43}$$

Such a symplectic basis is only defined up to  $Sp(2n, \mathbb{Z})$  transformations, as these preserve the symplectic intersection form. The periods are defined as the integral of the holomorphic 3-form  $\Omega$  over this basis of cycles,

$$\int_{A_a} \Omega = z^a, \qquad \int_{B^b} \Omega = \mathcal{G}_a. \tag{2.44}$$

The periods are encapsulated in the period vector,  $\Pi = (\mathcal{G}_1, \dots, \mathcal{G}_n, z_1, \dots, z_n)$ , where  $n = h^{2,1} + 1$ . This inherits the holomorphic freedom of  $\Omega$  and is defined up to holomorphic rescalings  $\Omega \to f(\phi_i)\Omega$ . Then  $\Pi = \Pi(\phi_i)$ , where  $\phi_i$  are the complex structure moduli of the Calabi-Yau.

In terms of the periods, the Gukov-Vafa-Witten superpotential is

$$W = \int G_3 \wedge \Omega = (2\pi)^2 \alpha'(f + iSh) \cdot \Pi(\phi_i), \qquad (2.45)$$

where  $f = (f_1, ..., f_6)$  and  $h = (h_1, ..., h_6)$  are integral vectors of fluxes along the cycles. The quantity of D3-brane charge carried by the fluxes is

$$N_{flux} = \frac{1}{(2\pi)^4 \alpha'^2} \int H_3 \wedge F_3 = f^T \cdot \Sigma \cdot h. \tag{2.46}$$

Given the vector of periods  $\Pi(\phi_i)$ , the Kähler potential on complex structure moduli space is given by

$$\mathcal{K}_{cs}(S, \phi_i) = -\ln(S + \bar{S}) - \ln\left(-i\int\Omega \wedge \bar{\Omega}\right) 
= -\ln(S + \bar{S}) - \ln(-i\Pi^{\dagger} \cdot \Sigma \cdot \Pi) 
\equiv \mathcal{K}_S + \mathcal{K}_{\phi}.$$
(2.47)

where

$$\Sigma = \left( \begin{array}{cc} 0 & \mathbf{1}_n \\ -\mathbf{1}_n & 0 \end{array} \right).$$

We can then compute the metric on moduli space,

$$g_{\alpha\bar{\beta}} = \partial_{\alpha}\partial_{\bar{\beta}}\mathcal{K},$$
 (2.48)

and the Riemann and Ricci curvatures

$$R^{\lambda}_{\mu\bar{\nu}\rho} = -\partial_{\bar{\nu}}(g^{\lambda\bar{\alpha}}\partial_{\mu}g_{\rho\bar{\alpha}}),$$

$$R_{\mu\bar{\nu}} = R^{\lambda}_{\mu\bar{\nu}\lambda} = -\partial_{\mu}\partial_{\bar{\nu}}\log(\det g).$$
(2.49)

The Kähler metric (2.48) determines the moduli kinetic terms. The expressions (2.49) will be useful in chapter 3.

Thus in order to solve equations (2.38) and (2.39), the essential technical datum is an explicit expression for the period vector  $\Pi(\phi_i)$  in a symplectic basis. It is in general a highly non-trivial task to find this, but we shall see in section 3.2 that there are examples for which it is known.

However, before we do this we first review for completeness the definition of the chiral coordinates in the presence of open string fields.

## 2.5 Chiral Coordinates for $\mathcal{N}=1$ Supergravity

In any  $\mathcal{N}=1$  supersymmetric theory, we can always package the scalar fields into chiral superfields. It is only in terms of these that the action takes the canonical form used above and given in full detail in [48]. Thus if we wish to use the formulae of 4-dimensional supergravity with the conventions of [48], it is an important and necessary task to identify the chiral superfields.

For example, in heterotic compactifications the Kähler moduli are defined in terms of the 2-cycles of the internal space, and are given by

$$T_i = t_i + ib_i. (2.50)$$

 $t_i$  is the string-frame volume of a 2-cycle  $\Sigma_i$  and  $b_i$  is the component of the NSNS 2-form  $B_2$  along that cycle. The Kähler form J and NSNS 2-form B can be expressed in terms of a basis  $e_i$  of  $H^{1,1}(M,\mathbb{R})$ 

$$J = \sum t^i e_i$$
, and  $B = \sum b^i e_i$ .

In IIB orientifold compactifications, this definition is no longer appropriate. The reason for this is twofold. First, the orientifold action projects out fluctuations of B from the low energy spectrum and so these can no longer appear in chiral multiplets. Secondly, the correct definition of the Kähler moduli now involves 4-cycle rather than 2-cycle volumes.

As well as geometric moduli, there are also moduli corresponding to brane positions - for both D3 and D7 branes - and to Wilson lines on branes, and in general these mix. To determine the correct Kähler coordinates a careful dimensional reduction must be performed of the orientifold model with branes and fluxes. This problem has been studied in great detail by Louis and collaborators [49, 50, 51, 52, 53] and we shall simply state their results.

In D3/D7 compactifications the orientifold action  $\sigma$  must leave the Kähler form invariant. The 2-cycles can be written in a basis in which they are either even or odd under  $\sigma$ . We denote the  $(h^{1,1})^+$  even elements by  $e_{\alpha}$  and the  $(h^{1,1})^-$  elements by  $e_a$ . As the volume form  $J \wedge J \wedge J$  is even, it follows that the triple intersectons  $k_{\alpha\beta c}$  and  $k_{abc}$  vanish.

The general expressions for the Kähler coordinates are rather complicated, and as we do not need the fully general form we will leave these to the references. In particular, we will make the simplifying assumption that  $(h^{1,1})^- = 0$  - this will be the case for the explicit model considered in chapter 5. In this case, the Kähler coordinates in the presence of D3 and D7 branes are [51]

$$S = S_0 + \kappa_4^2 \mu_7 \mathcal{L}_{A\bar{B}} \zeta^A \bar{\zeta}^{\bar{B}}$$

$$T_\alpha = \sigma_\alpha + i\rho_\alpha + 2i\kappa_4^2 \mu_7 l^2 \mathcal{C}_\alpha^{I\bar{J}} a_I \bar{a}_{\bar{J}} + i\mu_3 l^2 (\omega_\alpha)_{i\bar{j}} \text{tr} \Phi^i (\bar{\Phi}^{\bar{J}} - \frac{i}{2} \bar{z}^{\tilde{a}} (\bar{\chi}_{\tilde{a}})_{\bar{l}}^{\bar{j}} \Phi^l).$$

$$(2.51)$$

Here  $S_0 = e^{-\phi} + iC_0$  is the dilaton-axion,  $\sigma_i$  is the Einstein frame volume of a 4-cycle,  $\rho_i$  is the component of  $C_4$  along this cycle,  $\zeta$  is a D7-brane position modulus,  $\Phi$  is a D3-brane position modulus and  $a_I$  are Wilson line moduli. The complex structure moduli surviving the orientifold projection are unchanged in definition.

To simplify these expressions, we assume first that no wandering D3 branes are present; this we may always achieve by saturating the tadpole with fluxes. We also assume that there are no Wilson line moduli on the D7-branes. A Calabi-Yau never has bulk one-cycles: what this assumption says is that there are also no one-cycles inside four-cycles. In this case the bulk moduli simplify to

$$S = S_0 + \kappa_4^2 \mu_7 \mathcal{L}_{A\bar{B}} \zeta^A \bar{\zeta}^{\bar{B}}$$

$$T_\alpha = \sigma_\alpha + i\rho_\alpha. \tag{2.52}$$

The Kähler potential is unaltered from (2.35). However, now  $S_0 + \bar{S}_0$  must be regarded as an implicit function of the dilaton-axion and D7-brane moduli. The Kähler moduli part of the Kähler potential,  $-2 \ln(\mathcal{V})$ , is unaffected by the brane moduli. This will be important later, as it implies the D7-brane moduli are fixed at tree-level by the fluxes and do not interfere with the stabilisation of Kähler moduli.<sup>2</sup>

The above expressions for the chiral coordinates have been derived by dimensional reduction of 10-dimensional actions. This process is rather laborious and we note there is a more intuitive way to understand some of the results, through noting that instantonic effects should be holomorphic in the chiral coordinates. For example, in heterotic theory the relevant instantons are worldsheet instantons. The worldsheet action is

$$S_{worldsheet} = \frac{1}{2\pi\alpha'} \int (\sqrt{g} + iB). \tag{2.53}$$

As worldsheet instantons correspond to calibrated embeddings of the worldsheet on spacetime 2-cycles, it follows that the natural chiral coordinates are  $t_i + ib_i$ . In the IIB case, it is D3-instantons that are present, with action

$$S_{D3} = \frac{1}{(2\pi)^3 \alpha'^2} \int e^{-\phi} \sqrt{g} + iC_4.$$
 (2.54)

The action (2.54) shows why chiral coordinates now involve combinations of Einstein frame 4-cycle volumes with  $C_4$  axions. This explanation become more cumbersome at one-loop level; for example the Wilson line contribution in (2.51) is loop-suppressed and the holomorphicity derivation requires a string loop calculation [54].

Having established conventions, notation and background we now move in part II to a more detailed study of moduli stabilisation.

<sup>&</sup>lt;sup>2</sup>If wandering D3s are present, dealing with the Kähler moduli becomes more complicated as these branes mix nontrivially in the definition of the Kähler moduli.

# Part II Moduli Stabilisation

# Chapter 3

# Statistics of Moduli Stabilisation

The second half of this chapter is based on the paper [2].

In the compactifications of [31] reviewed in chapter 2, the dilaton and complex structure moduli were stabilised by the fluxes, but the Kähler moduli still remain unfixed. It may seem natural to now discuss Kähler moduli stabilisation. However, we shall defer this to chapters 4 and 5 and shall instead give a detailed account of aspects of the flux-stabilisation. In particular, we discuss the question of where the complex structure moduli are stabilised and the extent to which there are preferred loci in moduli space.

At leading order in  $g_s$  and  $\alpha'$ , the calculational procedure is well-defined. We need to know the Calabi-Yau periods, which is a clean mathematical problem, and specify flux choices  $F_3$ ,  $H_3 \in H^3(M,\mathbb{Z})$ . These determine the F-term equations (2.38) and (2.39), which we solve to stabilise the dilaton and complex structure moduli. Throughout this chapter I shall abuse terminology and refer to such solutions as vacua. Clearly a genuine vacuum must also, at least, stabilise the Kähler moduli.

The total number of such flux vacua appears extremely large. There are  $h^3 = h^{3,0} + h^{2,1} + h^{1,2} + h^{0,3} = 2(h^{2,1} + 1) \equiv K$  cycles on which to wrap fluxes. For a typical Calabi-Yau,  $K \sim \mathcal{O}(100)$ . We specify  $F_3$  and  $H_3$  by choosing integers  $f_i$  and  $h_i$  for each 3-cycle, giving 2K integers in total. For  $\omega^i$  a symplectic basis of 3-forms,

$$F_3 = \sum_{i=1}^{K} f_i \omega^i, \qquad H_3 = \sum_{i=1}^{K} h_i \omega^i.$$
 (3.1)

We must also satisfy the tadpole cancellation condition (2.29). Although this appears to be of indefinite sign, it can be shown that for solutions of (2.38) and

(2.39) we have<sup>1</sup>

$$\int H_3 \wedge F_3 \ge 0. \tag{3.2}$$

The conditions (2.29) and (3.2) heuristically restrict the fluxes to a ball of radius  $\sqrt{L}$  in flux space, where  $L = \frac{\chi}{24}$ . An estimate for the number of consistent flux choices is then  $\sqrt{L}^{2K} = L^K$ . For F-theory compactifications and their IIB orientifold limits  $L \sim \mathcal{O}(1000)$ , and so  $L^K$  may easily take values of order  $\sim 1000^{200} \sim 10^{600}$ . The main area of physics in which such numbers are successfully encountered is that of statistical mechanics. This motivates the use of statistical techniques to tame and understand the multiplicity of vacua.

## 3.1 Derivation of Vacuum Distributions

In a series of papers [27, 55, 56, 57, 58, 59, 60, 61, 62] Douglas and collaborators have developed techniques to count solutions to equations (2.38) and (2.39) and to make rigorous statements about the ensemble of flux vacua. The purpose of this section is to give a technical account of the methodology and to derive the simplest formula thereby attainable. We shall also summarise other results attained in this fashion. In the subsequent section 3.2 we describe our tests of these formulae.

We aim to explain the derivation of the Ashok-Douglas formula [55] for the index density of vacua. This computes

$$d\mu_I(z) = \sum_{\text{vacua}} \operatorname{sign}(\det D^2 W)$$
 (3.3)

as a function of complex structure moduli space z, and evaluates it to be

$$\int_{\mathcal{F}} d\mu_I(z) = \frac{(2\pi L_{max}^K)(-1)^{\frac{K}{2}}}{\pi^{n+1}K!} \int_{\mathcal{F}} \det(-\mathcal{R} - \omega).$$
 (3.4)

Here  $L_{max}$  is the total available D3-brane charge from the orientifold, K the number of 3-cycles, and n the number of moduli.  $\mathcal{F}$  is a fundamental region in the combined dilaton-axion and complex structure moduli space, while  $\omega$  and  $\mathcal{R}$  represent the Kähler and curvature 2-forms on this space. These are both fully determined by the Kähler potential of the moduli space,

$$\mathcal{K} = -\ln(S + \bar{S}) - \ln\left(i \int \Omega \wedge \bar{\Omega}\right). \tag{3.5}$$

<sup>&</sup>lt;sup>1</sup>Intuitively, this is beacause supersymmetric solutions require the fluxes to be pseudo-BPS, carrying D3 tension and D3 (rather than anti-D3) charge.

We reemphasise that here 'vacuum' simply refers to a choice of fluxes satisfying the tadpole constraint and generating a superpotential  $W = \int G_3 \wedge \Omega$  such that, at the point in moduli space denoted by z,

$$D_S W(z) = D_{\phi_i} W(z) = 0.$$
 (3.6)

The phrase 'index density' refers to the fact that the vacua are counted with signs. As the index density is much easier to calculate, it is this quantity rather than the absolute density that is computed.

Nonetheless, the derivation of (3.4) is by no means obvious. The calculation is structured as follows. We first compute  $\langle d\mu_I(z)\rangle$  within a Gaussian ensemble of superpotentials, counting vacua with weights. Once this has been computed, it is relatively straightforward to convert this into a formula for the actual index density  $d\mu_I(z)$ , in which vacua are only weighted by a sign. This conversion is essentially an inverse Laplace transform on  $\langle d\mu_I(z)\rangle$ .

The ensemble considered is the ensemble of flux superpotentials. The flux superpotential is a linear combination of periods  $\int_{\Sigma_i} \Omega$  and so the ensemble consists of all superpotentials W(z) expressible as

$$W(z) = \sum_{\alpha=1}^{K} N^{\alpha} \Pi_{\alpha}(z), \tag{3.7}$$

where  $N^{\alpha}$  are complex numbers and  $\Pi_{\alpha}(z)$  are the Calabi-Yau periods. The main approximation made is to treat the  $N^{\alpha}$  as continuous, whereas flux quantisation renders them integral. In writing (3.7) we also assume a fixed dilaton; we will remedy this assumption later.

The expectation value of a quantity  $\mathcal{A}(W)$  in the ensemble is

$$\langle \mathcal{A}(W) \rangle = \frac{1}{\mathcal{N}} \int DW f(W) A(W),$$
 (3.8)

with the measure f(W) and normalisation  $\mathcal{N}$  defined to be

$$\mathcal{N} = \langle 1 \rangle = \int DW f(W),$$

$$f(W) = \int d^2 N^{\alpha} \exp(-Q_{\alpha\beta} N^{\alpha} \bar{N}^{\beta}) \delta(W - \sum N^{\alpha} \Pi_{\alpha}). \tag{3.9}$$

In the case at hand,

$$Q_{\alpha\beta} = \eta_{\alpha\beta} = \begin{pmatrix} 0 & \mathbf{1}_n \\ -\mathbf{1}_n & 0 \end{pmatrix}. \tag{3.10}$$

However it is useful to leave  $Q_{\alpha\beta}$  general.

The measure f(W) has no intrinsic significance: it is merely an efficacious stepping stone on the way to computing (3.3). The purpose of these definitions

is that computations in this ensemble reduce to integrals computable using the techniques of quantum field theory. Our aim is to compute

$$\langle d\mu_I(z_0)\rangle = \langle \delta^n(DW(z_0))\delta^n(\bar{D}W^*(\bar{z}_0))\det D^2W(z_0)\rangle. \tag{3.11}$$

Once (3.11) is computed, an inverse transform allows the derivation of  $d\mu_I(z)$ . The factor of det  $D^2W(z_0)$  accounts for the change of variables from z to DW(z) in defining the  $\delta$ -functions. We also recall that  $D^2W(z_0) = \partial DW(z_0)$  so long as  $DW(z_0) = 0$ . However, in order to compute (3.11), there are several necessary preliminary calculations. First,

$$\mathcal{N} = \int DW f(W) = \frac{\pi^K}{\det Q}.$$

This follows straightforwardly, as the integral reduces to

$$\int d^2 N^{\alpha} \exp\left(-Q_{\alpha\beta}N^{\alpha}\bar{N}^{\beta}\right) = \frac{\pi^K}{\det Q}.$$

The cornerstone of the computation is the 2-point function  $\langle W(z_1) W^*(\bar{z}_2) \rangle$ , which we denote by  $G(z_1, \bar{z}_2)$ . Through standard quantum field theory arguments this evaluates easily to

$$\langle W(z_1) W^*(\bar{z_2}) \rangle = \left( Q^{-1} \right)^{\delta \gamma} \Pi_{\gamma}(z_1) \Pi_{\delta}^*(\bar{z_2}). \tag{3.12}$$

For the specific ensemble of flux superpotentials,

$$\langle W(z_1) W^*(\bar{z_2}) \rangle = -\eta^{\delta \gamma} \Pi_{\gamma}(z_1) \Pi_{\delta}^*(\bar{z_2}) = \exp(-\mathcal{K}(z_1, \bar{z_2})),$$
 (3.13)

where K is the Kähler potential for the complex structure moduli. The importance of  $G(z_1, \bar{z}_2)$  is that, by Wick's theorem, all other quantities in the ensemble are expressible in terms of it.

A second important building block is

$$\mathcal{X} \equiv \langle \delta^{2n} \left( DW \left( z_0 \right) \right) \rangle$$
.

We compute this as follows

$$\langle \delta^{2n} \left( DW \left( z_{0} \right) \right) \rangle = \frac{1}{\mathcal{N}} \int DW f \left( W \right) \delta^{n} \left( D_{i}W \left( z_{0} \right) \right) \delta^{n} \left( \bar{D}_{j}W^{*} \left( \bar{z}_{0} \right) \right)$$

$$= \frac{1}{\mathcal{N}} \int dN^{\alpha} d\bar{N}^{\beta} \exp \left( -Q_{\alpha\beta}N^{\alpha}\bar{N}^{\beta} \right) \delta^{n} \left( N^{\gamma}D_{\mu}\Pi_{\gamma} \left( z_{0} \right) \right) \delta^{n} \left( \bar{N}^{\delta}\bar{D}_{\nu}\Pi_{\delta}^{*} \left( z_{0} \right) \right)$$

$$=\frac{1}{\mathcal{N}\times\pi^{2n}}\int d^{2}Nd\lambda_{\mu}d\bar{\lambda}_{\nu}\exp\left(-Q_{\alpha\beta}N^{\alpha}\bar{N}^{\beta}+i\lambda^{\mu}N^{\gamma}D_{\mu}\Pi_{\gamma}\left(z_{0}\right)+i\bar{\lambda}^{\nu}\bar{N}^{\delta}\bar{D}_{\nu}\Pi_{\delta}^{*}\left(z_{0}\right)\right),$$

where we have implemented the  $\delta$ -function constraints by introducing new integration variables  $\lambda_{\mu}$ . This integral may be performed by completing the square in  $N^{\alpha}$  and shifting

$$\begin{split} N^{\alpha} & \to & N^{\alpha} + i \left( Q^{-1} \right)^{\gamma \alpha} \bar{\lambda}^{\nu} \bar{D}_{\nu} \Pi_{\gamma}^{*} \left( z_{0} \right), \\ \bar{N}^{\beta} & \to & \bar{N}^{\beta} + i \left( Q^{-1} \right)^{\beta \delta} \lambda^{\mu} D_{\mu} \Pi_{\delta} \left( z_{0} \right). \end{split}$$

The integral becomes

$$\int dN^{\alpha} d\bar{N}^{\beta} d\lambda^{\mu} d\bar{\lambda}^{\nu} \exp\left(-Q_{\alpha\beta}N^{\alpha}\bar{N}^{\beta}\right) \exp\left(-\left(Q^{-1}\right)^{\gamma\delta} \lambda^{\mu}\bar{\lambda}^{\nu} D_{\mu} \Pi_{\delta}\left(z_{0}\right) \bar{D}_{\nu} \Pi_{\gamma}^{*}\left(z_{0}\right)\right). \tag{3.14}$$

The N and  $\lambda$  integrals have separated, and this evaluates to

$$\frac{1}{\pi^n} \times \frac{1}{\det_{\mu,\nu} \left( (Q^{-1})^{\delta \gamma} D_{\mu} \Pi_{\gamma} \left( z_0 \right) \bar{D}_{\nu} \Pi_{\delta}^* \left( z_0 \right) \right)} = \mathcal{X}.$$

If  $G(z_1, \bar{z}_2) = \exp(-\mathcal{K}(z_1, \bar{z}_2))$ , as is appropriate for the flux ensemble, this becomes

$$\frac{1}{\pi^n} \times \frac{e^{-n\mathcal{K}(z_0,\bar{z}_0)}}{(\det g)},$$

where g is the moduli space metric and n the number of  $\lambda\bar{\lambda}$  integrations performed, which equals the number of complex structure moduli. Therefore

$$\mathcal{X} = \frac{1}{\pi^n} \times \frac{e^{-n\mathcal{K}(z_0,\bar{z}_0)}}{(\det g)}.$$

The final prerequisite is  $G_{z_0}(z_1, \bar{z}_2) = \langle \delta^{2n} (DW(z_0)) W(z_1) W^*(\bar{z}_2) \rangle$ . This can be computed by identical techniques: we complete the square in N and perform the N and  $\lambda$  integrals separately. The calculation is messy but not difficult, and we obtain

$$G_{z_0}(z_1, \bar{z}_2) = \mathcal{X} \times \left( G(z_1, \bar{z}_2) - D_{z_0^{\mu}} G(z_0, \bar{z}_2) \left( D_{\mu} \bar{D}_{\nu} G(z_0, \bar{z}_0) \right)^{-1} \bar{D}_{\bar{z}_0^{\bar{\nu}}} G(z_1, \bar{z}_0) \right). \tag{3.15}$$

The reason for this computation is that  $\langle d\mu_I(z)\rangle$  is most simply expressed in terms of  $G_{z_0}(z_1,\bar{z}_2)$ .

We now have sufficient ingredients to compute the ensemble index density

$$\langle d\mu_I(z_0)\rangle = \langle \delta^{2n} \left(DW(z_0)\right) \det D^2W \rangle.$$
 (3.16)

The calculation is unenlightening and we shall not perform all steps explicitly, but shall instead merely show the structure. We first of all write

$$\det D^2 W = \int d^2 \psi^i \, d^2 \theta^i \, \exp \left[ \theta^a \psi^b \partial_a D_b W + \bar{\theta}^a \psi^b \bar{\partial}_a D_b W + \bar{\theta}^a \bar{\psi}^b \bar{\partial}_{\bar{a}} \bar{D}_{\bar{b}} W^* + \theta^a \bar{\psi}^b \partial_a \bar{D}_b W^* \right],$$

where  $\theta$  and  $\psi$  are Grassmann variables. We seek to compute

$$\frac{1}{N} \times \frac{1}{\pi^{2n}} \int d^2N \, d\lambda_{\mu} d\bar{\lambda}_{\nu} \, d^2\psi^i \, d^2\theta^i \, \exp\left(-Q_{\alpha\beta}N^{\alpha}\bar{N}^{\beta} + i\lambda^{\mu}N^{\gamma}D_{\mu}\Pi_{\gamma}(z_0) + i\bar{\lambda}^{\nu}\bar{N}^{\delta}\bar{D}_{\nu}\Pi_{\delta}^*(z_0)\right) \\
\exp\left[\theta^a\psi^b\partial_a D_b W + \bar{\theta}^a\psi^b\bar{\partial}_a D_b W + \bar{\theta}^a\bar{\psi}^b\bar{\partial}_{\bar{a}}\bar{D}_{\bar{b}}W^* + \theta^a\bar{\psi}^b\partial_a\bar{D}_bW^*\right].$$

We complete the square in N and integrate to obtain

$$\frac{1}{\pi^{2n}} \int d\lambda^{\mu} d\bar{\lambda}^{\nu} d^{2}\psi^{i} d^{2}\theta^{i} \exp\left(\left(Q^{-1}\right)^{\gamma\delta} \left(i\bar{\lambda}^{\nu}\bar{D}_{\nu}\Pi_{\gamma}^{*}(z_{0}) + \bar{\theta}^{a}\bar{\psi}^{b}\bar{\partial}_{a}\bar{D}_{b}\Pi_{\gamma}^{*}(z_{0}) + \theta^{a}\bar{\psi}^{b}\partial_{a}\bar{D}_{b}\Pi_{\gamma}^{*}(z_{0})\right) \times \left(i\lambda^{\mu}D_{\mu}\Pi_{\delta}(z_{0}) + \theta^{a}\psi^{b}\partial_{a}D_{b}\Pi_{\delta}(z_{0}) + \bar{\theta}^{a}\psi^{b}\bar{\partial}_{a}D_{b}\Pi_{\delta}(z_{0})\right)\right).$$

The integral is now quadratic in  $\lambda$ . We integrate over  $\lambda$  to obtain

$$\mathcal{X} \times \int d^2 \theta^i d^2 \psi^i$$

$$\exp \left[ \left( \theta^c \psi^d \partial_{1c} D_{1d} + \bar{\theta}^c \psi^d \bar{\partial}_{1c} D_{1d} \right) \left( \bar{\theta}^a \bar{\psi}^b \bar{\partial}_{2a} \bar{D}_{2b} + \theta^a \bar{\psi}^b \partial_{2a} \bar{D}_{2b} \right) G_{z_0} \left( z_1, \bar{z}_2 \right) \right] |_{z_1 = z_2 = z_0}.$$

We may write the terms inside the exponential as

$$\theta^c \psi^d \bar{\theta}^a \bar{\psi}^b \partial_{1c} D_{1d} \bar{\partial}_{2a} \bar{D}_{2b} G_{z_0}(z_1, \bar{z}_2) + \bar{\theta}^c \psi^d \theta^a \bar{\psi}^b \bar{\partial}_{1c} D_{1d} \partial_{2a} \bar{D}_{2b} G_{z_0}(z_1, \bar{z}_2),$$

with other terms vanishing. We recall that  $\mathcal{X} = \frac{1}{\pi^n} \times \frac{1}{(\det g)e^{n\mathcal{K}(z_0,\bar{z}_0)}}$  and bring the  $e^{n\mathcal{K}(z_0,\bar{z}_0)}$  into the Grassmann exponent. We then need to compute

$$F_{cd|\bar{a}\bar{b}} = e^{-K(z_0,\bar{z}_0)} D_{1c} D_{1d} \bar{D}_{2a} \bar{D}_{2b} G_{z_0} (z_1,\bar{z}_2)$$

and

$$F_{\bar{c}d|a\bar{b}} = e^{-K(z_0,\bar{z}_0)} \bar{D}_{1c} D_{1d} D_{2a} \bar{D}_{2b} G_{z_0} (z_1,\bar{z}_2).$$

These can be evaluated by brute force - as  $G_{z_0}(z_1, \bar{z}_2)$  is a combination of the Kahler potential and its derivatives, there is no difficulty of principle in the calculation. We find

$$F_{cd|\bar{a}\bar{b}} = R_{c\bar{a}d\bar{b}} + (g_{d\bar{a}}g_{c\bar{b}} + g_{c\bar{a}}g_{d\bar{b}}),$$

$$F_{\bar{c}d|a\bar{b}} = g_{a\bar{b}}g_{d\bar{c}}.$$
(3.17)

We then have

$$\langle d\mu_I(z)\rangle = \frac{1}{\pi^n \times \det g} \int d^2\psi^i d^2\theta^i \exp\left(-\theta^a \theta^{\bar{c}} \psi^b \psi^{\bar{d}} R_{a\bar{c}b\bar{d}} - g_{a\bar{c}} \theta^a \theta^{\bar{c}} g_{b\bar{d}} \psi^b \bar{\psi}^{\bar{d}}\right). \tag{3.18}$$

Performing the  $\theta$  integral, we get

$$\frac{1}{\pi^n \times \det g} \det \left( -\psi^b \psi^{\bar{d}} R_{a\bar{c}b\bar{d}} - g_{a\bar{c}} g_{b\bar{d}} \psi^b \bar{\psi}^{\bar{d}} \right) 
= \frac{1}{\pi^n} \det \left( (g^{-1}) \times \left( -\psi^b \psi^{\bar{d}} R_{a\bar{c}b\bar{d}} - g_{a\bar{c}} g_{b\bar{d}} \psi^b \bar{\psi}^{\bar{d}} \right) \right).$$

We now promote  $\psi^i$  and  $\bar{\psi}^j$  to differential forms - this is possible because forms have the same Grassmann properties as fermionic variables. We finally obtain

$$\langle d\mu_I(z)\rangle = \frac{1}{\pi^n} \det(-\mathcal{R} - \omega),$$
 (3.19)

where  $\mathcal{R}$  is the curvature two-form and  $\omega$  the volume form on moduli space.

This computes the index density within the Gaussian ensemble. However, we would like to count vacua without including a Gaussian weight. Let us state how to do this and then justify it. The claim is that if we can compute

$$N_{vac}(\alpha) = \sum_{vacua} e^{-2\alpha(\text{Im}\tau)\eta_{\alpha\beta}N_{RR}^{\alpha}N_{NS}^{\beta}},$$
(3.20)

then the total number of vacua, weighted by the index alone, is given by

$$N_{vac}\left(L \le L_{max}\right) = \frac{1}{2\pi i} \int_{C} \frac{d\alpha}{\alpha} e^{2\alpha(\operatorname{Im}\tau)L_{max}} N_{vac}\left(\alpha\right). \tag{3.21}$$

Why is this true? (3.20) contains a measure  $\exp\left(-2\alpha\left(\operatorname{Im}\tau\right)\eta_{\alpha\beta}N_{RR}^{\alpha}N_{NS}^{\beta}\right)$ . This is analogous to computing

$$\tilde{f}(\lambda) = \int f(x)e^{-\lambda x^2} dx \tag{3.22}$$

for some value of  $\lambda$ . However, given  $\tilde{f}(\lambda)$ , we may find f(x) by an appropriate inverse transform. For the flux ensemble, this is given by (3.21). We then evaluate

$$N_{vac}(\alpha) = (\operatorname{Im} \tau)^{-K} \int DW \int d^2 N^{\alpha} \exp(-i\alpha \eta_{\alpha\beta} N^{\alpha} \bar{N}^{\beta}) \delta^{2n}(DW(z_0)) \det D^2 W$$
$$= \frac{\pi^{K-n} (-1)^{K/2}}{(\alpha \operatorname{Im} \tau)^K} \det(-\mathcal{R} - \omega).$$

The  $(\operatorname{Im} \tau)^{-K}$  prefactor arises from converting  $\sum_{N_{RR},N_{NS}}$  to  $\int dN d\bar{N}$ . We can then take the inverse transform of this to get the total number of vacua.

As mentioned above, we must actually be a little more subtle. We have regarded the dilaton as fixed and have focused on the complex structure moduli, not requiring  $D_{\tau}W=0$ . We ought to repeat the entirety of the above analysis including the dilaton. If this is done, we finally obtain

$$d\mu_I(z) = \frac{(2\pi L_{max}^K)(-1)^{\frac{K}{2}}}{\pi^{n+1}K!} \det(-\mathcal{R} - \omega), \tag{3.23}$$

as above. This gives the index density of vacua stabilised at a point z in moduli space.  $\omega$  and  $\mathcal{R}$  represent the Kähler and curvature 2-forms on the combined dilaton-axion/complex structure moduli space.

(3.23) describes the distribution of flux vacua in complex structure moduli space. Although the final form is simple, it contains a lot of information about where the complex structure moduli tend to be stabilised. To a first approximation, the distribution is uniform in volume, being given by  $\det(-\omega)$ . However, the form of (3.23) also shows that vacua tend to cluster in regions of high moduli space curvature. An example of such a region is the vicinity of a conifold singularity, where the moduli space curvature diverges (although the integrated curvature is finite).

The statistical techniques above can be extended and applied to compute other quantities of interest in the ensemble of flux vacua. I leave full details to the references [55, 58, 61] and simply state the results that will be most relevant for the tests that are described in the next section. The first concerns the number of vacua with small tree-level superpotential. This is of interest as it is related to the gravitino mass and supersymmetry breaking scale. This is also the condition required for control of the approximations in the KKLT scenario.

$$N(\text{vacua s.t. } e^{\mathcal{K}}|W|^2 < \epsilon) \sim \epsilon.$$
 (3.24)

The second concerns the distribution of the stabilised string coupling constant. This is of interest if we wish to remain within weakly coupled string theory.

$$N(\text{vacua s.t. } \frac{1}{g_s} > \epsilon) \sim \epsilon.$$
 (3.25)

(The distribution (3.25) is actually implicit in (3.23)).

Let us expand on the relationship of  $e^{\mathcal{K}}|W|^2$  to the supersymmetry breaking scale. We work here in a truncation for which the Kähler moduli are excluded. If all moduli are included,  $m_{3/2}^2 = e^{\mathcal{K}}|W|^2$  and the gravitino mass  $m_{3/2}$  does give a true measure of the scale of supersymmetry breaking, either directly or as required to uplift a supersymmetric AdS minimum to Minkowski space. While the size of  $e^{\mathcal{K}}|W|^2$  with the truncation may allow us to say whether the gravitino mass is (relatively) 'larger' or 'smaller', in the absence of the vacuum values of the Kähler moduli we cannot make statements about the absolute magnitude of  $m_{3/2}$ .

Having described the derivation of the statistical formulae concerning the number and distribution of solutions to the dilaton and complex structure moduli F-term equations, let us now turn to our tests of these results.

## 3.2 Tests of the Statistical Predictions

This section is based on the paper [2].

For an idealised flux ensemble, (3.23) gives the distribution of vacua over moduli space. There are two ways in which (3.23) deviates from the actual

density of vacua. First, in its derivation the quantisation of the 3-form fluxes has been neglected and a continuum approximation used. Secondly, (3.23) does not count the absolute number of vacua but instead counts vacua with signs. It is therefore interesting to test whether the predictions of (3.23) actually work in real examples. The most direct way to test this is through Monte Carlo simulation: we randomly generate fluxes, study the distribution of stabilised moduli, and test for agreement with the predictions of equation (3.23).

This procedure was carried out for a 1-modulus Calabi-Yau in [63]. In this section we examine the distribution of the complex structure moduli, the dilaton-axion, and the 'susy breaking scale'  $e^{\mathcal{K}}|W_0|^2$  for a particular 2-modulus Calabi-Yau, the hypersurface in  $\mathbb{P}^4_{[1,1,2,2,6]}$ .

# **3.2.1** The Model Used: $\mathbb{P}^4_{[1,1,2,2,6]}$

There exist many Calabi-Yau manifolds that can be realised as hypersurfaces in weighted projective space. The weighted projective space  $\mathbb{P}^4_{[k_0,k_1,k_2,k_3,k_4]}$  is defined by the complex coordinates  $w_0$ ,  $w_1$ ,  $w_2$ ,  $w_3$ ,  $w_4$  subject to

$$(w_0, w_1, w_2, w_3, w_4) \equiv (\lambda^{k_0} w_0, \lambda^{k_1} w_1, \lambda^{k_2} w_2, \lambda^{k_3} w_3, \lambda^{k_4} w_4),$$

and has four complex dimensions. To obtain a space of three complex dimensions we must restrict to a hypersurface  $P(w_i) = 0$ , where P is polynomial in the  $w_i$ . We require the hypersurface to be Calabi-Yau, i.e. to have vanishing first Chern class. Using standard results from algebraic geometry (e.g. see chapter 5 of Hübsch [64]), this occurs provided

$$\deg(P) = \sum_{i=0}^{4} k_i.$$

We will make use of the Calabi-Yau hypersurface in  $\mathbb{P}^4_{[1,1,2,2,6]}$ , which requires a polynomial homogeneous of degree 12. The simplest form of this polynomial is

$$w_0^{12} + w_1^{12} + w_2^6 + w_3^6 + w_4^2 = 0. (3.26)$$

We shall denote this manifold by  $\mathcal{M}$ .  $\mathcal{M}$  has  $h^{1,1}=2$  and  $h^{2,1}=128$ . The number of complex structure moduli is determined by the number of monomial deformations of degree 12, modulo coordinate redefinitions, that can be added to (3.26). There are two complex structure moduli of particular relevance, which we shall denote by  $\psi$  and  $\phi$ . These perturb (3.26) to

$$P(w_i) = w_0^{12} + w_1^{12} + w_2^6 + w_3^6 + w_4^2 - 12\psi \left( w_0 w_1 w_2 w_3 w_4 \right) - 2\phi \left( w_0^6 w_1^6 \right) = 0. \quad (3.27)$$

Their significance is due to their survival in the mirror manifold W, obtained by identifying points in M related by the action of G, the maximal group of scaling

symmetries of (3.26). Here G is  $\mathbb{Z}_2 \times \mathbb{Z}_6^2$ , and its action is represented by

$$\mathbb{Z}_2 : (w_1, w_4) \to (\alpha w_1, \alpha w_4) \text{ with } \alpha^2 = 1,$$
  
 $\mathbb{Z}_6 : (w_1, w_3) \to (\beta^5 w_1, \beta w_3) \text{ with } \beta^6 = 1,$   
 $\mathbb{Z}_6' : (w_1, w_2) \to (\gamma^5 w_1, \gamma w_2) \text{ with } \gamma^6 = 1.$ 

These manifestly leave equation (3.27) invariant, whereas all other degree 12 deformations of equation (3.27) are not well defined on the mirror. As W has  $h^{1,1} = 128$  and  $h^{2,1} = 2$ , it is an example of a Calabi-Yau with two complex structure moduli. Both  $\mathcal{M}$  and W develop a conifold singularity when  $864\psi^6 + \phi = 1$ . This condition follows from requiring  $P(w_i) = \underline{\nabla} P(w_i) = 0$  with  $\underline{\nabla} \underline{\nabla} P$  non-singular.

To test the distribution (3.23) on the manifold  $\mathcal{M}$  we need its form for the case of two complex structure moduli. This is evaluated to be[65] <sup>2</sup>

$$d\mu = g_{\tau\bar{\tau}}d\tau \wedge d\bar{\tau} \wedge \left(4\pi^2 c_2 - \det(g_{a\bar{a}})d\psi^1 \wedge d\psi^{\bar{1}} \wedge d\psi^2 \wedge d\psi^{\bar{2}}\right), \tag{3.28}$$

where  $g_{a\bar{b}} = \partial_a \partial_{\bar{b}} \mathcal{K}$  and  $c_2$  is the second Chern class for the complex structure moduli space of  $\mathcal{M}$ , given by

$$c_2 = \frac{1}{8\pi^2} \left( \operatorname{tr}(\mathcal{R} \wedge \mathcal{R}) - \operatorname{tr}\mathcal{R} \wedge \operatorname{tr}\mathcal{R} \right). \tag{3.29}$$

(3.28) may be rewritten as

$$d\mu = g_{\tau\bar{\tau}}d\tau \wedge d\bar{\tau} \wedge d\psi^{1} \wedge d\psi^{\bar{1}} \wedge d\psi^{2} \wedge d\psi^{\bar{2}} \left[ \epsilon^{ab} \epsilon^{\bar{a}\bar{b}} \left( R^{1}_{a\bar{a}1} R^{2}_{b\bar{b}2} - R^{1}_{a\bar{a}2} R^{2}_{b\bar{b}1} \right) - \det g_{a\bar{a}} \right].$$

$$(3.30)$$

To make further progress we need a knowledge of the periods. This is in general a highly non-trivial task. However, the manifold defined by (3.27) is of a class that has been extensively studied. The relevant periods have been computed in the mirror symmetry literature [66, 67], following the classic treatment of the quintic [68], and we shall borrow these results. For the Calabi-Yau described by the hypersurface

$$P = \sum_{i=0}^{4} x_j^{d/k_j} - d\psi x_0 x_1 x_2 x_3 x_4 - \frac{d}{q_0} \phi x_0^{q_0} x_1^{q_1} x_2^{q_2} x_3^{q_3} x_4^{q_4} = 0,$$
 (3.31)

the fundamental period in the large  $\psi$  region is given by

$$\varpi_f(\psi,\phi) = \sum_{l=0}^{\infty} \frac{(q_0 l!)(d\psi)^{-q_0 l}(-1)^l}{l! \prod_{i=1}^4 \left(\frac{k_i}{d}(q_0 - q_i)l\right)!} u_l(\phi), \tag{3.32}$$

<sup>&</sup>lt;sup>2</sup>In this section we use  $\tau = C_0 + ie^{-\phi} = iS$  to denote the dilaton.

where

$$u_l(\phi) = (D\phi)^l \sum_{n=0}^{\left[\frac{l}{D}\right]} \frac{l! \prod_{i=1}^4 \left(\frac{k_i}{d} (q_0 - q_i)l\right)! (-D\phi)^{-Dn}}{(l - Dn)! n! \prod_{i=1}^4 \left(\frac{k_i q_i}{q_0} n + \frac{k_i}{d} (q_0 - q_i)l\right)!}.$$
 (3.33)

This is obtained by direct integration of  $\int \Omega$  and satisfies the Picard-Fuchs equation. There are other independent solutions to the Picard-Fuchs equation having a logarithmic dependence on  $\psi$ . In total there are six independent solutions, one for each 3-cycle, and the actual periods are a linear combination of these.

The two regions of moduli space that will most interest us are the vicinities of the Landau-Ginzburg point  $\psi=\phi=0$  and the conifold locus  $864\psi^6+\phi=1$ . To obtain a basis of periods in the small  $\psi$  region, (3.32) may be analytically continued to obtain

$$\varpi_{f}(\psi,\phi) = -\frac{2}{d} \sum_{n=1}^{\infty} \frac{\Gamma\left(\frac{2n}{d}\right) \left(-d\psi\right)^{n} u_{-\frac{2n}{d}}(\phi)}{\Gamma\left(n\right) \Gamma\left(1 - \frac{n}{d}\left(k_{1} - 1\right)\right) \Gamma\left(1 - \frac{k_{2}n}{d}\right) \Gamma\left(1 - \frac{k_{3}n}{d}\right) \Gamma\left(1 - \frac{k_{4}n}{d}\right)}.$$
(3.34)

Here  $u_{\nu}(\phi)$  is related to the hypergeometric functions and is defined through the contour integral

$$u_{\nu}(\phi) = \frac{2^{\nu}}{\pi} \int_{-1}^{1} \frac{d\zeta}{\sqrt{1-\zeta^2}} (\phi - \zeta)^{\nu}.$$
 (3.35)

The contour integral is initially defined for  $\text{Im}(\phi) > 0$  and then defined over the rest of the plane by deforming the integral contour. The branch cuts, which are unavoidable when  $\nu$  is non-integral, start at  $\pm 1$  and run to  $\pm \infty$ .

We may derive a basis of periods from the fundamental period in a simple manner. If we define

$$\varpi_j(\psi,\phi) = \frac{-(2\pi i)^3}{\psi} \varpi_f(\alpha^j \psi, \alpha^{jq_0} \phi)$$
 (3.36)

then  $\varpi(\psi, \phi) = (\varpi_0(\psi, \phi), \varpi_1(\psi, \phi), \varpi_2(\psi, \phi), \varpi_3(\psi, \phi), \varpi_4(\psi, \phi), \varpi_5(\psi, \phi))$  gives a basis of periods known as the Picard-Fuchs basis. Naively (3.36) would seem to give 12 independent periods, but there are interrelations discussed in [66]. The net result is that, as expected, there are six independent periods. However these periods do not correspond to a symplectic basis of 3-cycles. A symplectic basis is given by

$$\Pi(\psi,\phi) = m \cdot \varpi(\psi,\phi). \tag{3.37}$$

For  $\mathbb{P}^4_{[1,1,2,2,6]}$ , m was computed in [69, 70] and is given by

$$m = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & 0\\ \frac{3}{2} & \frac{3}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2}\\ 1 & 0 & 1 & 0 & 0 & 0\\ 1 & 0 & 0 & 0 & 0 & 0\\ -\frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0\\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}.$$

In principle, equation (3.37) completely determines the periods near  $\psi = 0$ . However, it involves the integral expression (3.35) for  $u_{\nu}(\phi)$  which is unsuitable for use in a computational treatment. This is facilitated by the power series expansion of  $u_{\nu}(\phi)$  in the small  $\phi$  region given in [66],

$$u_{\nu}\left(\phi\right) = \frac{e^{\frac{i\pi\nu}{2}}\Gamma\left(1 + \frac{\nu(k_{1}-1)}{2}\right)}{2\Gamma\left(-\nu\right)} \sum_{m=0}^{\infty} \frac{e^{i\pi m/2}\Gamma\left(\frac{m-\nu}{2}\right)\left(2\phi\right)^{m}}{m!\Gamma\left(1 - \frac{m-\nu k_{1}}{2}\right)}.$$
(3.38)

Let us briefly discuss the region of convergence of equations (3.34), (3.35), and (3.38), being specific to the particular Calabi-Yau  $\mathbb{P}^4_{[1,1,2,2,6]}$ . The Calabi-Yau develops a conifold singularity when  $\phi + 864\psi^6 = \pm 1$  and there is a further singularity at  $\phi = \pm 1$ . The singularities determine the regions of convergence; all three equations are only valid for  $\left|\frac{864\psi^6}{\phi\pm 1}\right| < 1$  and equation (3.38) has the additional restriction  $|\phi| < 1$ .

We will also be interested in the periods near the conifold locus. As will be further discussed in section 3.2.3, the periods here take a certain standard form. However, for their exact numerical determination, we will use a neolithic approach, parametrising them through direct evaluation of the (implicit) power series (3.37) near the conifold locus.

Finally, the Calabi-Yau we use is the original manifold  $\mathcal{M}$  defined by the locus of (3.27), and not its mirror  $\mathcal{W}$ .  $\mathcal{M}$  has a total of 128 complex structure moduli. Some will be removed by the orientifold projection, but there are still many not considered directly. The validity of this was explained in [70]. The group G is a symmetry of (3.27) and if we only turn on fluxes invariant under this symmetry then the superpotential can only have a higher-order dependence on the other moduli. It is thus consistent to set all other moduli equal to zero and focus only on the moduli in equation (3.27) and their associated fluxes. We will comment briefly on the general situation in section 3.2.4.

We will now test the Ashok-Douglas formula in the vicinities of the Landau-Ginzburg point  $\psi = \phi = 0$  and the conifold locus  $864\psi^6 + \phi = 1$ . We solve

$$D_{\tau}W = D_{\phi}W = D_{\psi}W = 0, \tag{3.39}$$

and study the distribution of vacua.

## **3.2.2** The Landau-Ginzburg Point $\psi = \phi = 0$

In equation (3.37) a symplectic basis for the periods was given. Let us untangle this in the vicinity of  $\psi = 0$ . We can expand  $\Pi(\psi, \phi)$  as

$$\Pi = \underline{a}(\phi) + \underline{b}(\phi)\psi^2 + \underline{c}(\phi)\psi^4 + \mathcal{O}(\psi^6). \tag{3.40}$$

Here  $\underline{a}, \underline{b}$  and  $\underline{c}$  are vector functions of  $\phi$  whose explicit form arises from the combination of equations (3.34), (3.36), (3.37) and (3.38). It can be checked that  $\underline{a}^{\dagger} \cdot \Sigma \cdot \underline{b} = \underline{a}^{\dagger} \cdot \Sigma \cdot \underline{c} = 0$ , implying

$$\Pi^{\dagger} \cdot \Sigma \cdot \Pi = (\underline{a}^{\dagger} \cdot \Sigma \cdot \underline{a}) + (\underline{b}^{\dagger} \cdot \Sigma \cdot \underline{b}) \psi^{2} \bar{\psi}^{2} + \mathcal{O}(|\psi|^{6}),$$

and consequently

$$\mathcal{K}_{\phi}(\psi,\phi) = -\ln\left(-i\underline{\Pi}^{\dagger} \cdot \Sigma \cdot \underline{\Pi}\right) 
= -\ln\left(-i\underline{a}^{\dagger} \cdot \Sigma \cdot \underline{a}\right) - \ln\left(1 + \frac{(\underline{b}^{\dagger} \cdot \Sigma \cdot \underline{b})}{(\underline{a}^{\dagger} \cdot \Sigma \cdot \underline{a})}\psi^{2}\overline{\psi}^{2} + \mathcal{O}\left(|\psi|^{6}\right)\right) 
= -\ln\left(-i\underline{a}^{\dagger} \cdot \Sigma \cdot \underline{a}\right) - \frac{(\underline{b}^{\dagger} \cdot \Sigma \cdot \underline{b})}{(a^{\dagger} \cdot \Sigma \cdot \underline{a})}\psi^{2}\overline{\psi}^{2} + \mathcal{O}\left(|\psi|^{6}\right).$$
(3.41)

Equations (3.39) then have the form

$$\frac{1}{\psi}D_{\psi}W = 0 \Rightarrow (f - \tau h) \cdot \left(\underline{\alpha}_{1}(\phi) + \underline{\alpha}_{2}(\phi)\psi^{2} + \underline{\alpha}_{3}(\phi)\bar{\psi}^{2}\right) = 0, \quad (3.42)$$

$$D_{\phi}W = 0 \Rightarrow \qquad (f - \tau h) \cdot \left(\underline{\beta}_{1}(\phi) + \underline{\beta}_{2}(\phi)\psi^{2}\right) = 0, \quad (3.43)$$

$$D_{\tau}W = 0 \Rightarrow \qquad (f - \bar{\tau}h) \cdot (\underline{a}(\phi) + \underline{b}(\phi)\psi^{2}) = 0, \quad (3.44)$$

$$D_{\tau}W = 0 \Rightarrow \qquad (f - \bar{\tau}h) \cdot (\underline{a}(\phi) + \underline{b}(\phi)\psi^2) \qquad = 0, \quad (3.44)$$

where we have dropped terms of  $\mathcal{O}(|\psi|^4)$ . Here  $\underline{\alpha}(\phi)$  and  $\beta(\phi)$  are complicated functions of  $\phi$  depending on the integral expressions for  $u_{\nu}(\phi)$ . However, when  $|\phi| < 1$  the use of the power series expansion in equation (3.38) converts  $\underline{\alpha}(\phi)$  and  $\beta(\phi)$  to a more tractable form. The leading behaviour of the metric  $g_{\alpha\bar{\beta}} = \partial_{\alpha}\partial_{\bar{\beta}}\mathcal{K}$ is given by

$$g_{\psi\bar{\psi}} \sim \psi\bar{\psi}, \qquad g_{\psi\bar{\phi}} \sim \psi\bar{\psi}^2\bar{\phi}, \qquad g_{\phi\bar{\psi}} \sim \psi^2\bar{\psi}\phi, \qquad g_{\phi\bar{\phi}} \sim 1.$$

This is consistent with expectation, as at the Landau-Ginzburg point  $\psi = \phi = 0$ the moduli space metric becomes singular. The curvature 2-form  $\mathcal{R}$  and the Chern class  $c_2$  may be calculated straightforwardly from the full expressions for the metric using a symbolic algebra program. Evaluating the Ashok-Douglas density, we find it has leading behaviour

$$d\mu \sim g_{\tau\bar{\tau}} d\tau \wedge d\bar{\tau} \wedge (\psi \bar{\psi} d\psi \wedge d\bar{\psi} \wedge d\phi \wedge d\bar{\phi}).$$

The regions on which we compare the Ashok-Douglas formula to our empirical results are balls in  $\psi$  and  $\phi$  space. We then expect as leading behaviour

$$N(\text{vacua s.t.} |\psi| < r_1) \sim r_1^4,$$
  
 $N(\text{vacua s.t.} |\phi| < r_2) \sim r_2^2.$ 

However, in the actual plots we evaluated the Ashok-Douglas density numerically rather than simply using the leading behaviour.

To test this expectation, we generated random fluxes and sought solutions of equations (3.42) to (3.44) using a numerical root finder. The range of fluxes used was (-20, 20). This is not as large as one might prefer. However, a larger range of fluxes resulted in solutions being produced insufficiently rapidly for our purposes. In order to be able to trust our truncation of the power series, we only kept solutions satisfying

$$\left| \frac{864\psi^6}{\phi \pm 1} \right| < 0.5 \text{ and } |\phi| < 0.75.$$
 (3.45)

While processing the numerical results, there is an important subtlety that must be accounted for<sup>3</sup>. It is well known that there is an exact  $SL(2,\mathbb{Z})$  symmetry of type IIB string theory,

$$\tau \to \frac{a\tau + b}{c\tau + d}, \qquad \left(\begin{array}{c} F_3 \\ H_3 \end{array}\right) \to \left(\begin{array}{cc} a & b \\ c & d \end{array}\right) \left(\begin{array}{c} F_3 \\ H_3 \end{array}\right).$$

where  $a, b, c, d \in \mathbb{Z}$  and ad-bc=1. Thus each vacuum found has many physically equivalent  $SL(2,\mathbb{Z})$  copies that we should not double-count. One way to deal with this would be to fix the gauge explicitly and then perform the Monte-Carlo analysis. Our approach is instead to weight each vacuum by the inverse of the number of copies it has within the sampled flux range. The purpose of this is to ensure that vacua with many  $SL(2,\mathbb{Z})$  copies are not unduly preferred.

As well as the  $SL(2,\mathbb{Z})$  symmetry, there is a monodromy near the Landau-Ginzburg point that needs similar treatment. It follows from the definition of the periods (3.36) that they have a monodromy under  $(\psi, \phi) \to (\alpha \psi, -\phi)$ , where  $\alpha^{12} = 1$ , of

$$\varpi(\psi,\phi) \to a \cdot \varpi(\psi,\phi),$$

where

$$a = \left(\begin{array}{ccccc} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{array}\right).$$

In the symplectic basis, the monodromy matrix A is given by  $m \cdot a \cdot m^{-1}$ . The effect of this monodromy is to generate a family of physically equivalent solutions related by

$$(\psi, \phi) \rightarrow (\alpha^{-1}\psi, -\phi),$$

$$f \rightarrow f \cdot A,$$

$$h \rightarrow h \cdot A.$$
(3.46)

<sup>&</sup>lt;sup>3</sup>We thank S. Kachru for bringing this to our attention.

When weighting vacua we need to find the total number of copies due to symmetries and monodromies lying within the sampled flux range. This has important systematic effects as vacua with smaller values of  $f_i$  and  $h_i$ , and thus smaller values of  $N_{flux}$ , have more copies. A naive counting that neglects the symmetries or monodromies that are present places undue emphasis on vacua with smaller values of  $N_{flux}$ .

We examined the distribution of vacua within fixed balls in  $\psi$  and  $\phi$  space. In figure 3.1 we plot the number of vacua satisfying (3.45) and having  $|\psi| < r$ . The results are seen to agree well with the theoretical prediction. Likewise, figure

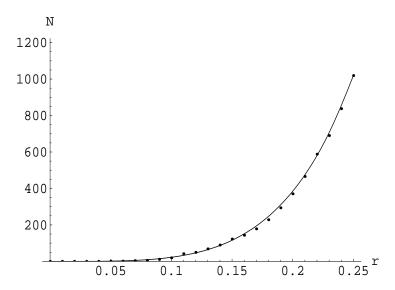


Figure 3.1: Number of vacua with  $|\psi| < r$ . The value of N plotted includes a weighting due to the  $SL(2,\mathbb{Z})$  copies of each vacuum within the range of fluxes sampled. The dots represent the numerical results, and the continuous line the (rescaled) numerical integration of  $\int_{|\psi| < r} d\mu$ , where  $d\mu$  is the index density. The flux range used was (-20,20).

3.2 shows the distribution of vacua for a ball  $|\phi| < r$  in  $\phi$  space. The continuous line again represents the cumulative number of vacua and the dots the rescaled numerical integration of  $\int_{|\phi| < r} d\mu$ . The empirical results are again well captured by the theoretical prediction. Finally, in figure 3.3 we examine the dependence of the number of vacua on the distance in flux space  $N_{\text{flux}} = f^T \cdot \Sigma \cdot h$ . The graph is fit by  $N \sim L^{4.3}$ . This is surprising, as the expected scaling is  $L^6$ . Furthermore, in the vicinity of the Landau-Ginzburg point for an analogous one-modulus example, the correct  $L^4$  scaling is found [71]. As discussed further in section 3.2.4, we believe our results are an artifact of the small flux range used, and that were a larger flux range used we would obtain the correct scaling. As we will shortly describe, in the vicinity of the conifold locus we do obtain the expected scaling with a flux range of (-40, 40).

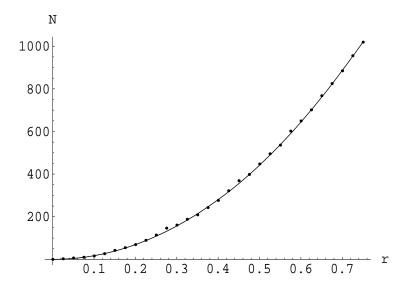


Figure 3.2: The weighted number of vacua, N, with  $|\phi| < r$ . The dots represent the numerical results and the continuous line the numerical integration of  $\int_{|\phi| < r} d\mu$ , rescaled by the same factor as in diagram 3.1.

It is also of interest to study the supersymmetry breaking scale, as measured by  $(2\pi\sqrt{\alpha'})^4e^{\mathcal{K}}|W|^2$ , for vacua in the vicinity of  $\psi=\phi=0$ . This is shown in figure 3.4. The most noticeable thing about this graph is that this distribution is uniform near the origin.

## 3.2.3 The Conifold Locus $864\psi^6 + \phi = 1$

The Calabi-Yau  $\mathcal{M}$  has a codimension one conifold degeneration along the moduli space locus  $864\psi^6 + \phi = 1$ . As the moduli space curvature diverges near a conifold point, the expectation from (3.23) is that vacua should cluster near the conifold locus. A conifold singularity is locally a cone over  $S^2 \times S^3$ . At the singularity, both the  $S^2$  and  $S^3$  cycles collapse and the (spatial) curvature diverges.<sup>4</sup> The periods measure the cycle of a size, and so as we approach the locus in moduli space at which the cycle collapses, the period of the collapsing cycle must vanish. Indeed the Calabi-Yau periods take a certain standard form in the vicinity of a conifold singularity in moduli space. We denote the period of the collapsing  $S^3$  cycle by  $\mathcal{G}_1$  and the period of its dual cycle by  $z_1$ . If  $\epsilon$  measures the moduli space

<sup>&</sup>lt;sup>4</sup>We note that two entirely separate curvatures are diverging. At the conifold locus in moduli space, the moduli space curvature diverges. At this same locus, a point-like conifold singularity appears in the Calabi-Yau, at which the spatial curvature of the Calabi-Yau also diverges.

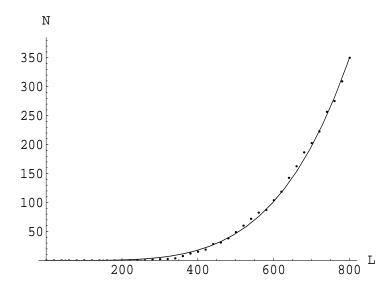


Figure 3.3: The weighted number of vacua N with  $N_{flux} < L$ .  $4.5 \times 10^6$  sets of flxues were generated, with values of L equally distributed between 0 and 800 and the range of fluxes being (-20, 20). The results are fit by  $N \sim L^{4.3}$ .

distance from the singular locus, we have

$$\mathcal{G}_1 = \epsilon,$$

$$z_1 = -\frac{1}{2\pi i} \mathcal{G}_1 \ln (\mathcal{G}_1) + \text{ analytic}, \qquad (3.47)$$

with all periods entirely regular in the vicinity of the singularity.

To study these vacua numerically, we must restrict attention to a small region near the conifold locus where we can compute the periods explicitly. We take this region to be the neighbourhood of the point  $\phi = 0, \psi = \psi_0 = 864^{-\frac{1}{6}}$ . If we write  $\psi = \psi_0 + \xi$  and truncate the periods at first order in  $\xi$  and  $\phi$ , then  $\Pi(\xi,\phi) = (\mathcal{G}_1,\mathcal{G}_2,\mathcal{G}_3,z_1,z_2,z_3)$  is approximated by

$$\mathcal{G}_{1} = 3202\xi + 171.8\phi + \mathcal{O}(\xi^{2}, \phi^{2}, \xi\phi) 
\mathcal{G}_{2} = 4323 - i(1553\xi + 107.4\phi) + \mathcal{O}(\xi^{2}, \phi^{2}, \xi\phi) 
\mathcal{G}_{3} = (-492.7 + 1976.8i) + (371.0 - 300.2i)\xi + (-259.0 - 59.0i)\phi + \mathcal{O}(\xi^{2}, \phi^{2}, \xi\phi) 
z_{1} = \frac{-1}{2\pi i}\mathcal{G}_{1}\ln(\mathcal{G}_{1}) + 784.8i - 2306i\xi - 44.35i\phi + \mathcal{O}(\xi^{2}, \phi^{2}, \xi\phi) 
z_{2} = (-994.6 - 184.8i) + (861.9 + 476.5i)\xi + (9.91 - 112.7i)\phi + \mathcal{O}(\xi^{2}, \phi^{2}, \xi\phi) 
z_{3} = i(369.5 - 953.0\xi + 225.4\phi) + \mathcal{O}(\xi^{2}, \phi^{2}, \xi\phi)$$
(3.48)

The numerical values were found by evaluating the series (3.34) up to one hundred terms in  $\psi$  and twenty-five terms in  $\phi$ . The values for the coefficients of the  $\mathcal{O}(\xi, \phi)$  terms were sensitive to the number of terms used in the power series at the level

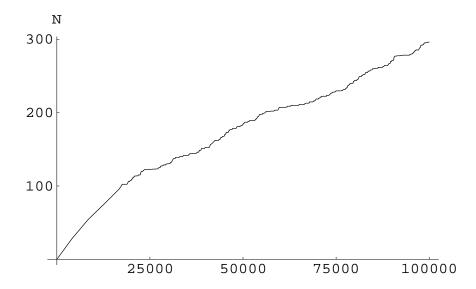


Figure 3.4: The value of  $(2\pi\sqrt{\alpha'})^4 e^{\mathcal{K}} |W|^2$  in units of  $(\alpha')^2$  for vacua in the vicinity of  $\psi = \phi = 0$ , for  $(2\pi)^4 e^{\mathcal{K}} |W|^2 < 100000$ . The flux range was (-20, 20), and the vacua satisfied the conditions (3.45).

of a couple of percentage points. We did not keep terms quadratic in  $\xi$  and  $\phi$  -inclusion of these would lead to  $\mathcal{O}(\xi)$  corrections to the results below.

The general form of the periods is as anticipated above. The cycles corresponding to  $\mathcal{G}_2$ ,  $\mathcal{G}_3$ ,  $z_2$  and  $z_3$  are all remote from the conifold singularity and the associated periods are both regular and non-vanishing near the conifold degeneration. The cycle corresponding to  $\mathcal{G}_1$  is the  $S^3$  that shrinks to zero size along the conifold locus. The period along this cycle is regular near the conifold locus and vanishing along it. Finally, the cycle corresponding to  $z_1$  is that dual to the  $S^3$ . It is not uniquely defined and in fact has a monodromy under a loop in moduli space around the conifold locus. As above, its period takes the form  $\left(-\frac{1}{2\pi i}\mathcal{G}_1\ln(\mathcal{G}_1) + \text{analytic terms}\right)$ .

It is convenient to define  $Z = \left(\frac{3202}{171.8}\right)\xi + \phi$  and rewrite the periods as

$$\mathcal{G}_{1} = 171.8 \, Z, 
\mathcal{G}_{2} = 4323 + 107.4i \, Z - 3554i \, \xi, 
\mathcal{G}_{3} = 784.8 - (259 + 59i) \, Z + (5198 + 799i) \, \xi, 
z_{1} = \frac{-1}{2\pi i} 171.8 \, Z \ln(Z) + 784.8 - 44.4i \, Z - 1479i \, \xi, 
z_{2} = (-994.6 - 184.8i) + (9.91 - 112.7i) \, Z + (677 + 2577i) \, \xi, 
z_{3} = i(369.5 + 225.4 \, Z - 5154 \, \xi).$$
(3.49)

Z is a measure of the distance from the conifold locus. We are interested in vacua extremely close to the conifold locus - typically  $\ln |Z| < -5$  - and thus

we will regard  $|Z| \ll |\xi| \ll 1$ . Having set up the periods (3.49), we can now compute the Ashok-Douglas expectation for the index density and compare this with numerical results.

Let us first solve equations (3.39). We have,

$$D_{\tau}W = 0 \Rightarrow (f - \bar{\tau}h) \cdot \Pi = 0 \Rightarrow \tau = \frac{f \cdot \Pi^{\dagger}}{h \cdot \Pi^{\dagger}}.$$
 (3.50)

This can be written as

$$\tau = \frac{a_0 + a_1 \bar{\xi}}{b_0 + b_1 \bar{\xi}} + \mathcal{O}(Z \ln Z), \tag{3.51}$$

where  $a_i$  and  $b_i$  are flux-dependent quantities. Next,

$$D_{\xi}W = 0 \Rightarrow (f - \tau h) \cdot (c_0 + c_1 \xi + c_2 \bar{\xi} + \mathcal{O}(\xi^2)) = 0.$$
 (3.52)

Using (3.51) this becomes a linear equation for  $\xi$ , easily solved to determine  $\xi$  and  $\tau$ . We finally need the value of  $\ln Z$ . This is obtained by considering

$$D_Z W = 0 \Rightarrow (f_4 - \tau h_4) \ln Z = (d_0 + d_1 \tau) + (d_2 + d_3 \tau) \xi + (d_4 + d_5 \tau) \bar{\xi}$$
 (3.53)

Substituting in for  $\tau$  and  $\xi$  from (3.51) and (3.52) gives the value of  $\ln Z$ .

When analysing the results we must account for the  $SL(2,\mathbb{Z})$  copies discussed in section 3.2.2. There is also a monodromy near the conifold. When solving for  $\ln Z$ , we impose no restriction on the imaginary part of  $\ln Z$ . There is a monodromy

$$\ln Z \rightarrow \ln Z + 2\pi i, 
(f_1, f_2, f_3, f_4, f_5, f_6) \rightarrow (f_1 + f_4, f_2, f_3, f_4, f_5, f_6), 
(h_1, h_2, h_3, h_4, h_5, h_6) \rightarrow (h_1 + h_4, h_2, h_3, h_4, h_5, h_6).$$
(3.54)

corresponding to a loop in moduli space around the conifold locus. This gives a further source of physically equivalent solutions that should not be double-counted. For each vacuum, we count the total number of copies within the specified flux range and weight by the inverse of this number.

Results are shown below. In figure 3.5 we show the clustering of vacua by plotting the distribution of vacua transverse to the conifold locus. A similar plot of vacua parallel to the conifold locus shows no such clustering, indicating that there is no preferred position along the conifold locus. In figure 3.6 we perform a quantitative comparison with the expected vacuum density, finding very good agreement over a large range of values of  $\ln Z$ . In figure 3.7 we examine the scaling with L of the number of vacua having  $N_{flux} < L$ . This reproduces the expected  $L^6$  scaling of (3.23). We also computed the 'susy breaking scale',  $(2\pi\sqrt{\alpha'})^4 e^K |W|^2$ . We reemphasise that the actual susy breaking scale can only be computed once the Kähler moduli have been fixed. This distribution is plotted in figure 3.8, and is again uniform near zero.

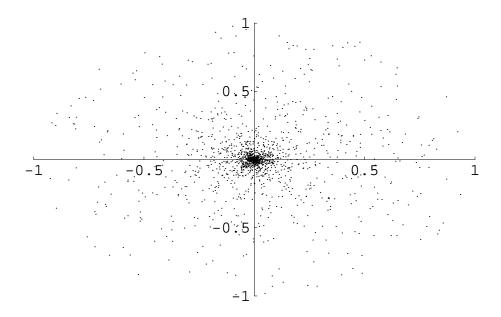


Figure 3.5: The value of Z for vacua near the conifold. We have restricted to |Z| < 0.0001 and have rescaled the above plot by  $10^4$ . The flux range used was (-40,40).

#### 3.2.4 Results and Limitations

Let us briefly summarise the results of these tests.

- 1. We constructed a large class of flux vacua for the two-moduli Calabi-Yau threefold  $\mathbb{P}^4_{[1,1,2,2,6]}$ . We independently computed the Ashok-Douglas density and compared with our results. We find good agreement which improves as the range of fluxes is increased. The number of vacua was limited mostly by working in two patches in moduli space: the region near  $\psi = \phi = 0$  and the region close to the conifold singularity  $\phi + 864\psi^6 = \pm 1$ .
- 2. We found a large concentration of vacua close to the conifold singularity as predicted, with the detailed distribution of the vacua being in close accordance with expectation.
- 3. In both regions, the values of  $e^{\mathcal{K}}|W_0|^2$  are uniformly distributed near zero. As a consequence, large values of the superpotential, corresponding to a large 'no-scale' supersymmetry breaking scale, are more abundant than small ones.
- 4. In both regions the number of models scaled as a power of the maximum permitted value of  $N_{flux}$ ,  $L_{max}$ . In the vicinity of the conifold we reproduced the expected  $L^6$  scaling. In the region close to the Landau-Ginzburg point

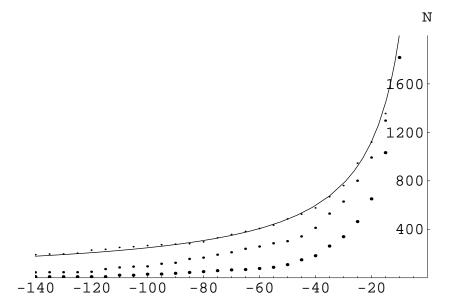


Figure 3.6: The distribution of vacua transverse to the conifold. We plot the number of vacua having  $\ln Z < D$  for  $D \in (-120, -5)$  against D, restricting to  $|\xi| < 0.05$ . The dots refer to the vacua found numerically and the smooth line to the Ashok-Douglas prediction. We include results for three flux ranges - (-20, 20), (-30, 30) and (-40, 40). The fit of the results to the expected distribution improves markedly as the flux range is increased.

we only found a scaling of  $L^{4.3}$ . We attribute the failure to achieve the expected scaling in this region to the smaller range of fluxes used there.

An important feature of our analysis is that the flux range used has a significant effect on the results. If the flux range is insufficiently large, then the distribution of vacua found numerically will not fit with the theoretical density. This is most strikingly illustrated for the case of the conifold in figure 3.6. Similar behaviour was seen for the scaling of the number of vacua with L - in the vicinity of the conifold locus, a reduction of the flux range to (-20,20) caused a reduction in the power of the scaling from  $\approx 6$  to  $\approx 5$ . Given this, we believe that the scaling of  $N(\text{vacua} | N_{flux} < L) \sim L^{4.3}$  found in the vicinity of the Landau-Ginzburg point is simply an artefact of the flux range used.

We find a similar dependence on the range of fluxes used in the distribution of the dilaton. After being transformed to the  $SL(2,\mathbb{Z})$  fundamental region, the expectation from equation (3.28) is that the number of vacua with  $\text{Im}(\tau) > \tau_0$  should scale like  $\frac{1}{\tau_0}$ . In figure 3.2.4 we compare this with numerical results arising from using flux ranges (-20, 20), (-40, 40) and (-60, 60).

We see that as the flux range increases the empirical distribution tends towards the theoretical one, and also that even with a flux range of (-60, 60) the two distributions do not yet fully match. In general terms these results

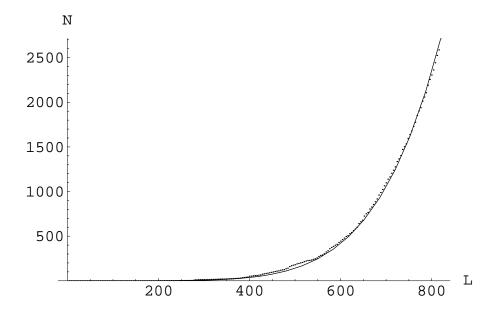


Figure 3.7: The weighted number of vacua with  $N_{\rm flux} < L$ . The curve is fit well by  $N \propto L^6$ . The range of fluxes used was (-40,40).

are reassuring in the sense that arbitrarily large values of the dilaton can be obtained, consistent with the weak coupling aproximation, even though they are not statistically preferred. We have only explored a small set of the flux vacua on the given Calabi-Yau. Nonetheless, the results are consistent with expectations and support the notion that relatively simple formulae can capture the distribution of flux vacua.

Let us finally make a general observation unrelated to the particular model above. The Calabi-Yau considered above has  $h^{1,1}=2$  and  $h^{2,1}=128$ . We have turned on fluxes only along cycles corresponding to two of the 128 complex structure moduli. We expect that some of the remaining moduli should be frozen out by the orientifold symmetry, but it would still be obviously impractical to attempt either to write down or to solve the moduli stabilisation equations with all fluxes turned on. However, if we assume that the Ashok-Douglas density remains valid then we can say something about the generic situation. Suppose we have K cycles supporting flux and that - as holds for this and many other F-theory models -

$$N_{D3} + N_{flux} = L_{max} \sim 1000. (3.55)$$

The Ashok-Douglas density (3.23) tells us that

$$N(\text{vacua } | N_{flux} < L_*) \sim L_*^K. \tag{3.56}$$

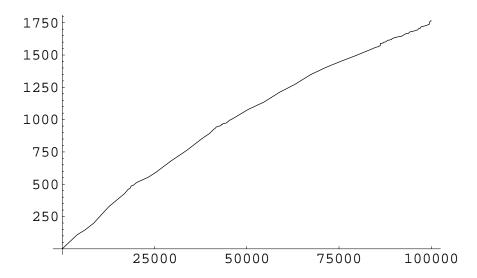


Figure 3.8: The value of  $(2\pi)^4 e^{\mathcal{K}} |W|^2$  in units of  $(\alpha')^2$  for vacua near the conifold, for  $(2\pi)^4 e^{\mathcal{K}} |W|^2 < 100000$ . The flux range was (-40, 40) and we restricted to vacua satisfying  $|\xi| < 0.05$ .

The fraction of vacua having  $N_{D3} \ge n$  is then estimated by

$$\frac{N(\text{vacua} \mid N_{flux} \le L_{max} - n)}{N(\text{vacua} \mid N_{flux} \le L_{max})} = \frac{(L_{max} - n)^K}{L_{max}^K}.$$
 (3.57)

For  $K \sim 200$ , then for n=5 this is approximately  $\frac{1}{e} \approx 0.36$ . We then see that despite the large amount of D3-brane charge we have to play with, generic choices of flux come close to saturating the D3-brane tadpole (a similar observation is made in [72]). This is appealing if the standard model were to live on D3 branes as in the models of [73, 74, 75, 76]. Clearly however actual numbers depend on the particular Calabi-Yau, the number of 3-cycles surviving the orientifold projection and the modifications to the Ashok-Douglas density when  $L \approx (a \text{ few})K$ . This also requires the Standard Model to be realised on D3 branes: we will argue in chapter 7 that to solve the strong CP problem it is instead preferable to realise the Standard Model on D7-branes.

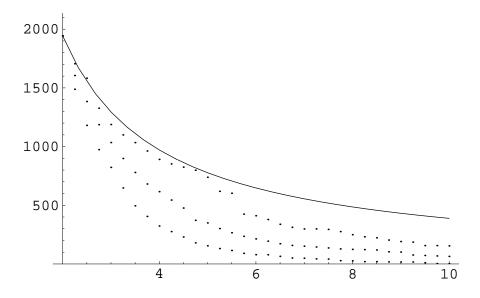


Figure 3.9: Distribution of vacua near the conifold locus with  $\text{Im}(\tau) > y$  for  $y \in (2,10)$ . Results have been brought to the same scale and plotted for three separate flux ranges: (-20,20), (-40,40) and (-60,60). We see that as the flux range increases the empirical plot moves closer to the expected result (represented by a smooth line).

# Chapter 4

## Kähler Moduli Stabilisation

This chapter is based on aspects of the paper [4].

#### 4.1 Recall of Definitions

We have discussed above techniques to stabilise complex structure moduli and described how statistical methods may be used to characterise the resulting solutions. However, these results are incomplete as it is necessary to stabilise all the compactification moduli and in particular the Kähler moduli. This problem is arguably more important, as these determine the overall size of the internal space, which in turn determines the ratio of the string and Planck scales. The value of the string scale is the most basic question in any attempt to do string phenomenology. In addition, the Kähler moduli also determine the coupling strength for gauge groups living on D7 branes.

As the Kähler moduli determine the size of the extra dimensions, there do exist limited direct experimental bounds. In the braneworld scenarios relevant here, the extra-dimensional radii must satisfy  $r \lesssim 10^{-5} \mathrm{m}$  [41]. Expressed as a mass, this also bounds the lightest moduli to have masses  $m \gtrsim 10^{-4}$  eV. While rather weak, these bounds should be kept in mind - in particular, the bound on moduli masses will be relevant later.

We want to discuss the moduli in an  $\mathcal{N}=1$  supergravity context. Therefore let us recall from section 2.5 the chiral coordinates for the Kähler moduli in D3/D7 IIB orientifold compactifications. We shall for simplicity assume the absence of 2-form fluxes on D7-branes and that no wandering D3 branes are present in the compactification; this latter can be achieved by taking the fluxes to saturate the  $C_4$  tadpole. We do not expect these assumptions to affect the physics substantially.

The Kähler moduli are defined by

$$T_i = \tau_i + ib_i, \tag{4.1}$$

where  $\tau_i$  is the Einstein frame volume of a 4-cycle  $\Sigma_i$ , measured in units of  $l_s = (2\pi)\sqrt{\alpha'}$ , and  $b_i$  is the component of the RR 4-form  $C_4$  along this cycle:  $\int_{\Sigma_i} C_4 = b_i$ . The 4-cycle volumes  $\tau_i$  may be related to 2-cycle volumes  $t_i$ . If the overall volume is

$$\mathcal{V} = \frac{1}{6} \int_{CV} J \wedge J \wedge J = \frac{1}{6} k_{ijk} t^i t^j t^k, \tag{4.2}$$

the 4-cycle moduli  $\tau_i$  are defined by

$$\sigma_i = \partial_i \mathcal{V} = \frac{1}{2} k_{ijk} t^j t^k. \tag{4.3}$$

The Kähler moduli do not appear in the tree-level superpotential. This follows because a tree-level presence of the Kähler moduli would violate the exact axionic shift symmetry  $b_i \to b_i + 2\pi$ . The Kähler potential is given by [50]

$$\mathcal{K} = -2\ln(\mathcal{V}) - \ln\left(-i\int\Omega\wedge\bar{\Omega}\right) - \ln(S+\bar{S}). \tag{4.4}$$

This can be derived by dimensional reduction.  $\mathcal{V}$  should be understood as an implicit function of the Kähler moduli, as the relations (4.3) cannot generally be inverted. This Kähler potential is no-scale and satisfies

$$\sum \mathcal{K}^{i\bar{j}} \partial_i \mathcal{K} \partial_{\bar{j}} \mathcal{K} = 3, \tag{4.5}$$

where the sum is over Kähler moduli.

There are two important results concerning the generation of potentials for the Kähler moduli. Although simple in themselves, these have profound consequences.

- 1. At tree level, the Kähler moduli are unfixed and do not appear in the superpotential.
- 2. In supersymmetric field theories, the superpotential is not renormalised at any order in perturbation theory.

The immediate consequence is that the Kähler moduli can only appear nonperturbatively in the superpotential.

In general, such nonperturbative effects both can and do appear. Terms of the form  $e^{-aT}$  are manifestly compatible with the shift symmetry given above. There are two known ways for the Kähler moduli to appear nonperturbatively in the superpotential. The first is through Euclidean D3-brane instantons[77].

In type IIB language, this corresponds to a Euclidean D3 brane wrapped on a 4-cycle in the Calabi-Yau. The instanton action is

$$S_{D3} = -\frac{1}{(2\pi)^3 \alpha'^2} \int \sqrt{g} + \frac{i}{(2\pi^3)\alpha'^2} \int C_4.$$
 (4.6)

A BPS brane is calibrated with respect to  $e^{J}$ . This implies that for a BPS instanton on a cycle  $\Sigma_{i}$ ,

$$\frac{1}{l_s^4} \int_{\Sigma_i} \sqrt{g} = \frac{1}{2} \int_{\Sigma_i} J \wedge J, \tag{4.7}$$

and the resulting instanton action is holomorphic in the Kähler moduli. An expansion about an instanton background gives a contribution to the path integral weighted as  $e^{-S_{D3}} = e^{-2\pi T}$ . To contribute to the superpotential, the instanton must have exactly two fermionic zero modes, in order that the integral over collective coordinates generates a term

$$\int d^4x d^2\theta(\ldots).$$

For the brane instantons relevant here, there are topological restrictions on when this can occur. In the absence of flux, a necessary condition is that [77]

$$\chi_g(D) = \sum_{i=0}^{3} (-1)^i h^{i,0}(D) = 1, \tag{4.8}$$

where D is the (effective) divisor wrapped by the brane. In the presence of 3-form flux, this topological constraint can be relaxed [78, 79, 80, 81, 82, 83].

The other source of nonperturbative contributions is gaugino condensation on wrapped D7-branes. Expanding the DBI and Chern-Simons terms, the D7-brane worldvolume action is

$$-\frac{1}{(2\pi)^{7}\alpha'^{4}} \int_{\mathbb{R}^{4}\times\Sigma} e^{-\phi} \sqrt{g} - \frac{1}{4(2\pi)^{5}\alpha'^{2}} \int_{\mathbb{R}^{4}\times\Sigma} \sqrt{g} e^{-\phi} F_{\mu\nu} F^{\mu\nu} + \frac{i}{(2\pi)^{7}\alpha'^{4}} \int_{\mathbb{R}^{4}\times\Sigma} C_{8} + \frac{i}{2(2\pi)^{5}\alpha'^{2}} \int_{\mathbb{R}^{4}\times\Sigma} F \wedge F \wedge C_{4}.$$

$$(4.9)$$

By examining the 4-dimensional  $F^2$  terms we see that the holomorphic gauge kinetic function is

$$f_i = \frac{T_i}{2\pi},$$

with  $\frac{1}{g^2}=\mathrm{Re}(f)$ . Gaugino condensation on such branes can produce effects described by a nonperturbative superpotential, which appears at order  $e^{-\frac{4\pi^2}{Ng_{YM}^2}}\sim e^{-\frac{2\pi T}{N}}$ , where N is the rank of the gauge group.

#### 4.2 The KKLT Scenario

The gist of the above discussion is that the Kähler moduli are not stabilised at tree level and only appear nonperturbatively in the superpotential. This is an apposite place to introduce an important scenario that is in many ways a reference point for work on moduli stabilisation. This is the now-famous paper [84] by Kachru, Kallosh, Linde and Trivedi (KKLT), outlining a procedure to construct de Sitter vacua by first stabilising Kähler moduli to create a supersymmetric AdS vacuum, and subsequently introducing effects to uplift this to a de Sitter vacuum.

This approach starts by observing that in the flux compactifications of [31] reviewed in chapter 2, the complex structure moduli receive tree-level masses of  $\mathcal{O}(m_s/\sqrt{\mathcal{V}})$ , whereas the Kähler moduli are massless. It should therefore be possible to integrate out the complex structure moduli by setting them equal to their vevs, and subsequently to consider a low energy effective field theory for the Kähler moduli.

If we assume there to be only one Kähler modulus T, the superpotential for the Kähler moduli effective theory is

$$W = W_0 + Ae^{-aT}. (4.10)$$

Here  $W_0 = \langle \int G_3 \wedge \Omega \rangle$  is the (now constant) tree-level flux superpotential,  $a = \frac{2\pi}{N}$  is the coefficient of the exponent, and A represents threshold effects. As A depends only on complex structure moduli it is here regarded as a constant.

The Kähler potential remains no-scale,

$$\mathcal{K} = \mathcal{K}_{cs} - 3\log(T + \bar{T}). \tag{4.11}$$

 $\mathcal{K}_{cs}$  is a constant from integrating out the dilaton and complex structure moduli. With these potentials, the scalar potential can be extremised by solving

$$D_T W \equiv \partial_T W + (\partial_T \mathcal{K})W = 0. \tag{4.12}$$

This equation gives

$$-aAe^{-aT} - \frac{3}{T + \bar{T}} \left( W_0 + Ae^{-aT} \right) = 0. \tag{4.13}$$

Solving this stabilises the Kähler modulus at  $\operatorname{Re}(T) \sim \frac{1}{a} \log W_0$ . As this is a supersymmetric AdS minimum, the vacuum energy is given by

$$V_{AdS} = -3e^{\mathcal{K}}|W|^2. {(4.14)}$$

The Kähler potential (4.11) will receive perturbative  $\alpha'$  and  $g_s$  corrections. In order to suppress these, it is necessary that the T modulus be stabilised at large values, which in turn requires that  $W_0$  be sufficiently small. However, as T depends logarithmically on  $W_0$ , rather small values of  $W_0$  are necessary to obtain large values of T. The value of  $W_0$  arises from the possible choices of integral fluxes. It is argued that, because of the vast range of possible flux choices, some will generate the necessary small values of  $W_0$ . The large discretuum of flux choices is thus essential to the KKLT construction, as it is the source of the claim that the necessary small values of  $W_0$  are possible.

As discussed in chapter 3, this last statement was quantified in [58, 61]. Specifically, if we adopt a measure in which each flux choice receives equal weight, then

$$N(e^{\mathcal{K}_{cs}}|W_0|^2 < \epsilon) \sim \epsilon \tag{4.15}$$

for small  $W_0$ .  $W_0$  is typically  $\mathcal{O}(1)$  and thus values for  $W_0$  of  $\mathcal{O}(10^{-5})$  require a tuning of discrete fluxes of  $\mathcal{O}(10^{-10})$ . Given the large numbers of discrete flux choices, there will be however, in absolute terms, many such choices that realise these values.

To uplift to a dS solution and break supersymmetry, [84] propose to add an anti-D3 brane living at the bottom of a warped throat. The warping red-shifts the energy allowing it to be tuned to sufficient precision to uplift to de Sitter space. There also exist other proposals for the uplift [85, 86]. However it is generally agreed that this step is the least tractable part of the construction and I shall not discuss it significantly.

## 4.3 Perturbative Effects in $\mathcal{N} = 1$ Supergravity

Despite the simplicity of the above scenario, some caveats naturally arise. The most obvious of these is that only nonperturbative corrections are included in the potential - the perturbative corrections appearing in the Kähler potential are absent. In order to justify this, it is necessary that the tree level superpotential  $W_0$  be extremely small, which necessitates a considerable degree of fine-tuning.

We shall now make this more precise and address the question of when  $\alpha'$  corrections may be neglected in IIB flux compactifications, obtaining the answer 'almost never'. However, it is helpful to view this as a particular example of more general behaviour.

#### The General Case

Let us start with an  $\mathcal{N}=1$  supergravity theory with tree-level Kähler potential  $\mathcal{K}$  and tree-level superpotential W. In general  $\mathcal{K}$  receives corrections at every order in perturbation theory,  $\mathcal{K}_p$ , and non-perturbative corrections  $\mathcal{K}_{np}$ , whereas W is not renormalised in perturbation theory and only receives non-perturbative

corrections  $W_{np}$ . Therefore we can write<sup>1</sup>

$$\mathcal{K} = \mathcal{K}_0 + \mathcal{K}_p + \mathcal{K}_{np} \approx \mathcal{K}_0 + J, \tag{4.16}$$

$$W = W_0 + W_{np} \approx W_0 + \Omega, \tag{4.17}$$

where J represents the leading (perturbative) correction to  $\mathcal{K}$  and  $\Omega$  the leading non-perturbative correction to W. We ask when it is safe to neglect the corrections J or  $\Omega$ .

The F-term scalar potential is

$$V_F = e^{\mathcal{K}} \left[ \mathcal{K}^{i\bar{k}} D_i W D_{\bar{k}} \bar{W} - 3|W|^2 \right]. \tag{4.18}$$

This can be expanded in powers of J and  $\Omega$ :

$$V_F = V_0 + V_J + V_\Omega + \cdots, \tag{4.19}$$

where

$$V_0 \sim W_0^2$$
,  $V_J \sim JW_0^2$ ,  $V_\Omega \sim \Omega^2 + W_0\Omega$ ,

and the ellipses refer to higher-order terms combining J and  $\Omega$ . The exact expressions for these quantities may be explicitly computed but are not relevant for our immediate purposes.

Normally, the structure of V is primarily determined by  $V_0$ , while the other terms provide small corrections in a weak coupling expansion. However if the tree-level potential has a flat direction along which  $V_0$  is constant, then the structure of the potential, and in particular its critical points, are determined by the corrections. Such behaviour is characteristic of the no-scale models common in string theory, with the Kähler moduli of IIB flux compactifications being just one example. No-scale models have a Kähler potential satisfying  $\mathcal{K}^{i\bar{k}}\mathcal{K}_i\mathcal{K}_{\bar{k}}=3$  and a constant superpotential  $W_0$ , implying  $V_0=0$ . The corrections then determine the structure of the potential. Although both  $V_J$  and  $V_\Omega$  will play a role, typically we expect  $V_J$  to dominate over  $V_\Omega$ , as the former is perturbative in the coupling and the latter non-perturbative. However, since  $\Omega$  is the only correction to  $W_0$  it cannot be totally neglected.

There are some special cases where the  $\mathcal{O}(J)$  corrections may be neglected. First, if  $W_0 = 0$  then  $V_J = 0$  automatically and the leading correction comes from  $V_{\Omega}$ . For example, this occurs in the heterotic string racetrack scenario. Likewise, if  $W_0 \ll 1$  in suitable units, then  $\Omega$  may be comparable to  $W_0$ . In this case, we have

$$\Omega \sim W_0 \Rightarrow V_\Omega \sim \Omega^2$$
 and  $V_J \sim J\Omega^2$ ,

and therefore

$$\frac{V_J}{V_{\Omega}} \ll 1. \tag{4.20}$$

<sup>&</sup>lt;sup>1</sup>The quantities J and  $\Omega$  introduced here should not be confused with the Calabi-Yau forms.

This is the relevant behaviour for the KKLT scenario. However, once  $W \gtrsim \frac{\Omega}{J}$  (which is  $\ll 1$  as  $\Omega \ll J$ ), the perturbative effects dominate and must be included.

It is worth noting that the limit  $W \sim \Omega$ , in which the tree level superpotential is comparable to its non-perturbative corrections, is very unnatural. There is furthermore no need to restrict to this limit if we have information on the perturbative corrections to  $\mathcal{K}$ , which is true in both type IIB and heterotic cases.<sup>2</sup>

#### Application to Type IIB Flux Compactifications

Let us illustrate these issues in the concrete setting of type IIB flux compactifications. As discussed above, the Kähler and superpotentials take the forms,

$$W = \int G_3 \wedge \Omega + \sum A_i e^{-a_i T_i},$$

$$\mathcal{K} = -2 \ln(\mathcal{V}_E) - \ln\left(-i \int \Omega \wedge \bar{\Omega}\right) - \ln(S + \bar{S}). \tag{4.21}$$

There is also a normalisation factor in front of W that is unimportant here but shall be made more precise later. The volume  $\mathcal{V}_E$  and moduli  $T_i$  are measured in Einstein frame  $(g_{\mu\nu,E} = e^{-\frac{\phi}{2}}g_{\mu\nu,s})$  and in units of  $l_s = 2\pi\sqrt{\alpha'}$ . The non-perturbative superpotential is generated by either D3-brane instantons  $(a_i = 2\pi)$  or gaugino condensation  $(a_i = \frac{2\pi}{N})$ . If the dilaton and complex structure moduli have been fixed by the fluxes, these potentials reduce to

$$W = W_0 + \sum A_i e^{-a_i T_i},$$

$$\mathcal{K} = \mathcal{K}_{cs} - 2 \ln(\mathcal{V}_E). \tag{4.22}$$

In the language of the previous section, the scalar potential derived from (4.22) includes  $V_{\Omega}$  but not  $V_{J}$ . We now show that for almost all flux choices and moduli values, the use of (4.22) without  $\alpha'$  corrections is inconsistent. Although the argument extends easily to any number of Kähler moduli, for illustration we shall consider one Kähler modulus and the geometry appropriate to the quintic. (4.22) then reduces to

$$W = W_0 + Ae^{-aT},$$
  

$$\mathcal{K} = \mathcal{K}_{cs} - 3\ln(T + \bar{T}).$$
(4.23)

<sup>&</sup>lt;sup>2</sup>Note that the original proposal for gaugino condensation in the heterotic string [87, 88, 89] included a constant term in the superpotential from the antisymmetric tensor  $H_{mnp}$  as well as the nonperturbative, gaugino condensation superpotential for the dilaton. This was abandoned because the constant was found to be quantised in string units and could not be of order the nonperturbative correction. It may be interesting to reinvestigate this problem including perturbative corrections to  $\mathcal{K}$ .

For the quintic,  $\mathcal{V}_E = \frac{5}{6}t^3 = \frac{1}{6\sqrt{5}}(T+\bar{T})^{\frac{3}{2}} = \frac{\sqrt{2}}{3\sqrt{5}}\sigma^{\frac{3}{2}}$ , where  $T=\sigma+ib$ . The leading  $\alpha'$  correction to the Kähler potential is [90]

$$\mathcal{K}_{\alpha'} = \mathcal{K}_{cs} - 2\ln\left(\mathcal{V}_E + \frac{\xi}{2g_s^{\frac{3}{2}}}\right),\tag{4.24}$$

where  $\xi = \frac{-\chi(M)\zeta(3)}{2(2\pi)^3} = 0.48$ . The factor of  $g_s^{-\frac{3}{2}}$  arises from our working in Einstein frame; it would be absent in string frame, in which  $\mathcal{V}_s = \mathcal{V}_E g_s^{\frac{3}{2}}$ . The resulting scalar potential is

$$V = e^{\mathcal{K}} \left( \frac{4\sigma^2 (Aa)^2 e^{-2a\sigma}}{3} - 4\sigma W_0 (Aa) e^{-a\sigma} + \frac{9\sqrt{5}\xi W_0^2}{4\sqrt{2}q_s^{\frac{3}{2}}\sigma^{\frac{3}{2}}} \right). \tag{4.25}$$

The  $\alpha'$  correction  $V_J$  dominates at both small and large volume. Although perhaps counter-intuitive, this is to be expected - the  $\alpha'$  correction is perturbative in volume whereas the competing terms are non-perturbative.

We may quantify when the neglect of  $\alpha'$  corrections is permissible. The allowed range of  $\sigma$  is

$$\operatorname{Max}\left(\mathcal{O}\left(\frac{1}{g_s}\right), \sigma_{min}\right) < \sigma < \sigma_{max},$$
 (4.26)

where  $\sigma_{min}$  and  $\sigma_{max}$  are taken to be the solutions of  $|V_J(\sigma)| = |V_{\Omega}(\sigma)|$ , that is

$$\frac{4\sigma^2|A^2 a^2|e^{-2a\sigma}}{3} - 4\sigma|W_0 A a|e^{-a\sigma} = \frac{9\sqrt{5}\xi|W_0|^2}{4\sqrt{2}q_s^{\frac{3}{2}}\sigma^{\frac{3}{2}}}.$$
 (4.27)

The  $\mathcal{O}(\frac{1}{g_s})$  bound on  $\sigma$  comes from requiring  $\mathcal{V}_s > 1$  in order to control the  $\alpha'$  expansion. In general this regime is rather limited. For very moderate values of  $W_0$ , equation (4.27) has no solutions and there is *no* region of moduli space in which it is permissible to neglect  $\alpha'$  corrections to (4.23).

For concreteness let us consider superpotentials generated by D3-brane instantons and let us take<sup>3</sup> A=1 and  $g_s=\frac{1}{10}$ . Then we require  $W_0<\mathcal{O}(10^{-75})$  to have any solutions at all of (4.27) with  $\mathcal{V}_s>1$ . This can be improved somewhat by using gaugino condensation with very large gauge groups. Thus using  $W_0=10^{-5},\ g_s=\frac{1}{10}$  and  $N=50,\ \sigma_{max}\sim150,$  which corresponds to  $\mathcal{V}_s\sim12.$  However, as generic values of  $W_0$  are  $\mathcal{O}(\sqrt{\frac{\chi}{24}})\sim\mathcal{O}(10)$  and  $W_0^2$  is uniformly distributed [91],  $W_0=10^{-5}$  represents a tuning of one part in  $10^{12}$ , and even then the range of validity is rather limited.

<sup>&</sup>lt;sup>3</sup>Properly this should be  $e^{\mathcal{K}_{cs}/2}A$  to be Kähler covariant, but we drop the  $e^{\mathcal{K}_{cs}/2}$ .

We therefore conclude that for generic values of  $W_0$  there is no regime in which the perturbative corrections to the Kähler potential can be neglected; the inclusion solely of non-perturbative corrections is inconsistent. Furthermore, even for the small values of  $W_0$  for which there does exist a regime in which nonperturbative terms are the leading corrections, this is still only true for a small range of moduli values and in particular never holds at large volume.<sup>4</sup>

In the next chapter we shall study the effect of systematically including the perturbative Kähler corrections.

 $<sup>^4</sup>$ We do not consider the special case  $\chi(M)=0$ , when the  $\alpha'^3$  corrections considered above vanish. We do not know the status of higher  $\alpha'$  corrections in such models, but we would expect these to likewise dominate at large volume.

# Chapter 5

# Large Volume Flux Compactifications

This chapter is based on the papers [3] and [4].

The previous chapter has argued that, if we wish to study the scalar potential across the full range of flux choices and moduli values, it is essential to include perturbative corrections. The aim of this chapter is to describe how the inclusion of such corrections leads very generally to a non-supersymmetric minimum of the scalar potential at exponentially large volumes. This minimum was first identified in [3]. The rest of this thesis will be devoted to describing the construction, properties and phenomenological applications of this minimum.

The structure of this minimum depends crucially on the inclusion of  $\alpha'$  corrections. A natural worry is that it is not possible to only include a subset of the perturbative  $\alpha'$  corrections: if some are important, all must be. We shall address this point below; the fundamental point is that the  $\alpha'$  expansion is an expansion in inverse volume and thus at large volume can be controlled.

The leading corrections to the Kähler potential were computed in [90] and arise from the ten-dimensional  $\mathcal{O}(\alpha'^3)$   $\mathcal{R}^4$  term. Measuring dimensionful quantities in units of  $l_s = 2\pi\sqrt{\alpha'}$ , the resulting Kähler potential takes the form

$$\frac{\mathcal{K}}{M_P^2} = -2\ln\left(\mathcal{V}_s + \frac{\xi g_s^{3/2}}{2e^{3\phi/2}}\right) - \ln(S + \bar{S}) - \ln\left(-i\int\Omega\wedge\bar{\Omega}\right). \tag{5.1}$$

where  $\xi = -\frac{\chi(M)\zeta(3)}{2(2\pi)^3}$ .  $\mathcal{V}_s$  is the internal volume, measured with an Einstein frame metric defined by  $g_{\mu\nu,E} = e^{(\phi_0 - \phi)/2} g_{\mu\nu,s}$ . This is defined so that at the minimum the Einstein and string frame metrics coincide.<sup>1</sup>  $S = e^{-\phi} + iC_0$  is the dilaton-axion. We will require  $\xi > 0$ , which is equivalent to  $h^{2,1} > h^{1,1}$ : i.e., more complex

 $<sup>{}^{1}</sup>$ This is why the subscript  ${}_{s}$  is used.

structure than Kähler moduli. Here  $g_s = \langle e^{\phi} \rangle = e^{\phi_0}$  is the value of the stabilised dilaton. The superpotential receives non-perturbative corrections causing it to depend on the Kähler moduli. These arise either from D3-brane instantons [77] or gaugino condensation from wrapped D7-branes [87, 88, 89]. It takes the form

$$\hat{W} = \frac{g_s^{3/2} M_P^3}{\sqrt{4\pi} l_s^2} \left( \int G_3 \wedge \Omega + \sum A_i e^{-\frac{a_i T_i}{g_s}} \right).$$
 (5.2)

The factor of  $1/g_s$  in the exponent comes from the definition of Einstein frame used. The prefactor is derived in section A.1 of the Appendix by a careful dimensional reduction. The  $A_i$  represent threshold effects and depend on the positions of complex structure moduli and wandering D3-branes (if present). Here  $a_i = \frac{2\pi}{K}$  with  $K \in \mathbb{Z}_+$  and K = 1 for D3-instantons.

Although the internal volume is measured in units of  $l_s$ , the  $\mathcal{N}=1$  supergravity potential is in units of  $M_{pl}$ . This arises from the string theoretic starting point by dimensional reduction and Weyl rescaling to 4-d Einstein frame. The scalar potential is given by the  $\mathcal{N}=1$  supergravity formula

$$V = \int d^4x \sqrt{-g_E} e^{\mathcal{K}/M_P^2} \left[ \mathcal{K}^{a\bar{b}} D_a \hat{W} D_{\bar{b}} \bar{\hat{W}} - \frac{3}{M_P^2} \hat{W} \bar{\hat{W}} \right]. \tag{5.3}$$

Thus equations (5.1) and (5.2) completely specify the potential. However, trying to directly visualise this is not illuminating. We therefore follow KKLT [84] and first integrate out the complex structure moduli. Technically, we stabilise the dilaton and complex structure moduli through solving (2.39), and then regard their values as fixed. This leaves a theory only depending on Kähler moduli, which we then stabilise separately. It is important to ask whether the resulting critical point of the full potential, including the moduli that have been integrated out, is genuinely a minimum or merely a saddle point. For the simplest implementation of the KKLT scenario with one Kähler modulus and a rigid Calabi-Yau, the resulting potential has no minima [92]. However, in multi-modulus models true minima can indeed occur [93]. We shall show below that the vacua we find, whether AdS or uplifted dS, are automatically tachyon-free.

After integrating out the dilaton and complex structure moduli the Kähler and superpotential become

$$\frac{\mathcal{K}}{M_P^2} = \mathcal{K}_{cs} - 2\log\left[\mathcal{V} + \frac{\xi}{2}\right],\tag{5.4}$$

$$\hat{W} = \frac{g_s^2 M_P^3}{\sqrt{4\pi}} \left( W_0 + \sum_n A_n e^{-\frac{a_n T_n}{g_s}} \right). \tag{5.5}$$

The extra factor of  $g_s^{\frac{1}{2}}$  comes from absorbing the dilaton-dependent part of  $e^{\mathcal{K}_{cs}/2}$  into  $\hat{W}$ . Although they are not shown explicitly above, the dilaton-dependent

parts in (5.4) should be used when determining the precise form of the inverse metric, and in particular the cross terms between dilaton and Kähler moduli. This will be relevant when computing D3 soft terms in chapter 6. We note for subsequent use that as  $\mathcal{V} \to \infty$  the Kähler potential behaves as

$$e^{\mathcal{K}} \sim \frac{e^{\mathcal{K}_{cs}}}{\mathcal{V}^2} + \mathcal{O}\left(\frac{1}{\mathcal{V}^3}\right).$$
 (5.6)

If we substitute (5.4) and (5.5) into equation (5.3), we obtain the following potential [94]:

$$V = e^{\mathcal{K}} \left[ \mathcal{K}^{\rho_{j}\bar{\rho}_{k}} \left( a_{j} A_{j} a_{k} \bar{A}_{k} e^{-\left( a_{j} T_{j} + a_{k} \bar{T}_{k} \right)} - \left( a_{j} A_{j} e^{-a_{j} T_{j}} \hat{\bar{W}} \partial_{\bar{T}_{k}} \mathcal{K} + a_{k} \bar{A}_{k} e^{-a_{k} \bar{T}_{k}} \hat{W} \partial_{T_{j}} \mathcal{K} \right) \right)$$

$$+ 3\xi \frac{\left( \xi^{2} + 7\xi \mathcal{V} + \mathcal{V}^{2} \right)}{\left( \mathcal{V} - \xi \right) \left( 2\mathcal{V} + \xi \right)^{2}} |\hat{W}|^{2} \right]$$

$$\equiv V_{np1} + V_{np2} + V_{\alpha'}. \tag{5.7}$$

To simplify notation we shall now absorb factors of  $\frac{1}{g_s}$  into the  $a_i$ : the possible values for  $a_i$  are now  $\frac{2\pi}{Ng_s}$ . We shall also drop factors of  $M_P$  and the prefactors in  $\hat{W}$ , returning to these later when we discuss scales.

# 5.1 Large Volume Limit

We now study the large-volume limit of the potential (5.7) for a general Calabi-Yau manifold. We then illustrate these ideas through explicit computations on a particular orientifold model,  $\mathbb{P}^4_{[1,1,1,6,9]}$ . The aim is to establish the existence of a non-supersymmetric minimum at exponentially large volume.

## 5.1.1 General Analysis

The argument for a large-volume AdS minimum of the potential (5.7) has two stages. We first show that there will in general be a decompactification direction in moduli space along which

- 1. The divisor volumes  $\tau_i \equiv \text{Im}(\rho_i) \to \infty$ .
- 2. V < 0 for large V, and thus the potential approaches zero from below.

This leads to an argument that there must exist a large volume AdS vacuum. Naively we would expect that the positive  $(\alpha')^3$  term, scaling as  $+\frac{1}{\nu^3}$ , will dominate at large volume over the non-perturbative terms which are exponentially suppressed. However, care is needed: the  $(\alpha')^3$  term is perturbative in the volume

of the entire Calabi-Yau, whereas the naively suppressed terms are exponential in the divisor volumes separately. Hence, in a large volume limit in which some of the divisors are relatively small the non-perturbative terms can compete with the peturbative ones.

The  $(\alpha')^3$  term in (5.7) denoted by  $V_{\alpha'}$  is easiest to analyse in the large  $\mathcal{V}$  limit. Owing to the large volume behaviour (5.6) of the Kähler potential, this scales as

$$V_{\alpha'} \sim +\frac{3\xi}{4\mathcal{V}^3} e^{\mathcal{K}_{cs}} |\hat{W}|^2 + \mathcal{O}\left(\frac{1}{\mathcal{V}^4}\right). \tag{5.8}$$

We observe that  $V_{\alpha'}$  is always positive and depends purely on the overall volume  $\mathcal{V}$ 

However,  $V_{np1}$  and  $V_{np2}$  both depend explicitly on the Kähler moduli and we must be more precise in specifying the decompactification limit. We first consider the  $\mathcal{V} \to \infty$  limit in moduli space where  $\tau_i \to \infty$  for all moduli except one, which we denote by  $\tau_s$ . There are two conditions on  $\tau_s$ . The first is that this limit be well-defined; for example, the volume should not become formally negative in this limit. The second is that  $\tau_s$  must appear non-perturbatively in W. It is however not essential for this purpose that all Kähler moduli appear non-perturbatively in the superpotential.

Let us study  $V_{np1}$  in this limit. From (5.7),

$$V_{np1} = e^{\mathcal{K}} \mathcal{K}^{T_j \bar{T}_k} \left( a_j A_j a_k \bar{A}_k e^{-\left( a_j T_j + a_k \bar{T}_k \right)} \right). \tag{5.9}$$

This is seen to be positive definite. As we have taken  $\tau_i$  large for  $i \neq s$ , the only term not exponentially suppressed in (5.9) is that involving  $T_s$  alone.  $V_{np1}$  then reduces to

$$V_{np1} = e^{\mathcal{K}} \mathcal{K}^{T_s \bar{T}_s} a_s^2 |A_s|^2 e^{-2a_s \tau_s}.$$
 (5.10)

We need to determine the inverse metric  $\mathcal{K}^{T_i\bar{T}_j}$ . With  $\alpha'$  corrections included, this is given by (e.g. see [95])<sup>2</sup>

$$\mathcal{K}^{T_i \bar{T}_j} = -\frac{2}{9} (2\mathcal{V} + \xi) k_{ijk} t^k + \frac{4\mathcal{V} - \xi}{\mathcal{V} - \xi} \tau_i \tau_j.$$
 (5.11)

In the large  $\mathcal{V}$  limit this becomes

$$\mathcal{K}^{T_i\bar{T}_j} = -\frac{4}{9}\mathcal{V}k_{ijk}t^k + 4\tau_i\tau_j + \text{ (terms subleading in }\mathcal{V}). \tag{5.12}$$

Thus in the limit described above we have

$$\mathcal{K}^{T_s\bar{T}_s} = -\frac{4}{9} \mathcal{V} k_{ssk} t^k + \mathcal{O}(1), \tag{5.13}$$

<sup>&</sup>lt;sup>2</sup>The conventions used for the Kähler moduli in [95] differ slightly from ours; as we are in this section only interested in the overall sign of the potential these are not important.

with

$$V_{np1} \sim \frac{(-k_{ssk}t^k)a_s^2|A_s|^2e^{-2a_s\tau_s}e^{\mathcal{K}_{cs}}}{\mathcal{V}} + \mathcal{O}\left(\frac{e^{-2a_s\tau_s}}{\mathcal{V}^2}\right). \tag{5.14}$$

We have dropped numerical prefactors. Despite the minus sign in (5.13) this component of the inverse metric will be positive since the Kähler metric as a whole is positive definite and this component computes the length squared of the (dual) vector  $\partial_{T_s}W$ . In the limit we consider, so long as we remain inside the Kähler cone, the leading term must be positive.

We can perform a similar analysis for  $V_{np2}$ , from whence negative contributions to the potential arise. We have

$$V_{np2} = -e^{\mathcal{K}} \left( \mathcal{K}^{T_j \bar{T}_k} a_j A_j e^{-a_j T_j} \bar{W} \partial_{\bar{T}_k} \mathcal{K} + \mathcal{K}^{T_k \bar{T}_j} a_k \bar{A}_k e^{-a_k \bar{T}_k} W \partial_{T_j} \mathcal{K} \right). \tag{5.15}$$

The only surviving exponential terms are again those involving  $\tau_s$ .  $V_{np2}$  thus reduces to

$$V_{np2} = -e^{\mathcal{K}} \left[ \mathcal{K}^{T_s \bar{T}_k} \left( a_s A_s e^{-a_s T_s} \bar{W} \partial_{\bar{T}_k} \mathcal{K} \right) + \mathcal{K}^{T_k \bar{T}_s} \left( a_s \bar{A}_s e^{-a_s \bar{T}_s} W \partial_k \mathcal{K} \right) \right]. \quad (5.16)$$

The form of the inverse metric (5.11) implies  $\mathcal{K}^{T_s\bar{T}_k} = \mathcal{K}^{T_k\bar{T}_s}$ . The sign of  $V_{np2}$  is determined by the value of the axionic field  $b_s = \operatorname{Im}(T_s)$ , which will adjust to make  $V_{np2}$  negative. To see this, note that at leading order at large volume  $W = W_0 + O(1/\mathcal{V})$ , and the only dependence on the axion  $b_s$  is in  $V_{np2}$ . Now write  $V_{np2} = e^{ia_sb_s}X + e^{-ia_sb_s}\bar{X}$ , where we have collected all factors in (5.16) except for the axion into X and  $\bar{X}$ . Extremizing the potential with respect to  $b_s$ , it is easy to see that at a minimum the axion will arrange its value to cancel the overall phase from the prefactors and make  $V_{np2}$  negative.

Thus we may without loss of generality simplify the calculation by replacing  $T_s$  by  $\tau_s$  and assuming  $A_s$  and W to be both real. Recall that

$$\partial_{T_k}\mathcal{K} = rac{t^k}{2\mathcal{V} + \xi} 
ightarrow rac{t^k}{2\mathcal{V}} + \mathcal{O}\left(rac{1}{\mathcal{V}^2}
ight).$$

Therefore

$$V_{np2} \sim -e^{\mathcal{K}} a_s A_s W e^{-a_s \tau_s} \mathcal{K}^{T_s \bar{T}_j} \frac{t^j}{\mathcal{V}} + \mathcal{O}\left(\frac{e^{-a_s \tau_s}}{\mathcal{V}^2}\right). \tag{5.17}$$

We have introduced a minus sign as a reminder that this term will be negative. Substituting in for  $\mathcal{K}^{T_s\bar{T}_k}$  then gives

$$V_{np2} \sim -e^{\mathcal{K}_{cs}} a_s A_s W e^{-a_s \tau_s} \frac{-\frac{4}{9} \mathcal{V} k_{sjk} t^j t^k + 4 \tau_s \tau_j t^j}{\mathcal{V}^3} + \mathcal{O}\left(\frac{e^{-a_s \tau_s}}{\mathcal{V}^3}\right)$$
$$\sim -e^{\mathcal{K}_{cs}} a_s A_s W e^{-a_s \tau_s} \frac{-\frac{8}{9} \mathcal{V} \tau_s + 4 \tau_s \tau_j t^j}{\mathcal{V}^3} + \mathcal{O}\left(\frac{e^{-a_s \tau_s}}{\mathcal{V}^3}\right).$$

As from the definition  $\tau_i t^j \propto \mathcal{V}$ , we conclude that in the limit described above,

$$V_{np2} \sim -\frac{a_s \tau_s e^{-a_s \tau_s}}{\mathcal{V}^2} |A_s W_0| e^{\mathcal{K}_{cs}} + \mathcal{O}\left(\frac{e^{-a_s \tau_s}}{\mathcal{V}^3}\right). \tag{5.18}$$

We may now study the full potential by combining equations (5.8), (5.14) and (5.18).

$$V \sim \left[ \frac{1}{\mathcal{V}} a_s^2 |A_s|^2 (-k_{ssk} t^k) e^{-2a_s \tau_s} - \frac{1}{\mathcal{V}^2} a_s \tau_s e^{-a_s \tau_s} |A_s W| + \frac{\xi}{\mathcal{V}^3} |W|^2 \right].$$
 (5.19)

We have absorbed factors of  $e^{\mathcal{K}_{cs}}$  into the values of W and  $A_s$ . We can now demonstrate the existence of a decompactification limit in which this potential approaches zero from below. This limit is given by

$$V \to \infty$$
 with  $a_s \tau_s = \ln(V)$ . (5.20)

In this limit the potential takes the following form

$$V \sim \left[ a_s^2 A_s^2 \frac{(-k_{ssj}t^j)}{\mathcal{V}^3} - |A_s W_0| \frac{(a_s k_{sjk}t^j t^k)}{\mathcal{V}^3} + \frac{\xi}{\mathcal{V}^3} |W_0|^2 \right] + \mathcal{O}\left(\frac{1}{\mathcal{V}^4}\right). \tag{5.21}$$

As in this limit the non-perturbative corrections to W are subleading by a power of  $\mathcal{V}$ , we have replaced W by  $W_0$ . We have also written out  $\tau_s = \frac{\ln \mathcal{V}}{a_s}$  in terms of 2-cycle volumes  $t^i$ .

All terms in equation (5.21) have the same volume dependence and it is not immediately obvious which is dominant at large volume. However, the numerator of the second term of equation (5.21) is quadratic in the 2-cycle volumes, whereas the others have at most a linear dependence. As  $\tau_i \to \infty$  for all i, all 2-cycles must blow up to infinite volume. The numerator of the second term is proportional to  $\tau_s$  and thus scales as  $\ln \mathcal{V}$ . Thus, this term scales as  $\frac{\ln \mathcal{V}}{\mathcal{V}^3}$ , and overcomes the first and third terms which scale schematically as  $\frac{\sqrt{\ln \mathcal{V}}}{\mathcal{V}^3}$  and  $\frac{1}{\mathcal{V}^3}$ . In the limit (5.20) the potential eventually behaves as

$$V \sim -e^{\mathcal{K}_{cs}} |A_s W_0| \frac{\ln \mathcal{V}}{\mathcal{V}^3},\tag{5.22}$$

and approaches zero from below.<sup>3</sup>

<sup>&</sup>lt;sup>3</sup> One concern in the above analysis may be our treatment of  $A_s$ , which we have treated as a constant. If the  $A_s$  were to depend on the Kähler moduli, this may invalidate the argument. However, as a correction to the superpotential  $A_s$  must be holomorphic in the Kähler moduli and must also respect the axion shift symmetry. This implies that  $A_s$  cannot have a perturbative dependence on the Kähler moduli. It may depend on complex structure moduli, but these have been fixed by the fluxes. I thank Liam McAllister for discussions on this point.

Given this, it is straightforward to argue that there must exist a large volume AdS minimum. At smaller volumes, the dominant term in the potential (5.19) is either the non-perturbative term  $V_{np1}$  or the  $(\alpha')^3$  term  $V_{\alpha'}$ , depending on the value of  $\tau_s$ . Both are positive; the former because the metric on moduli space is positive definite and the latter because we have required  $h^{2,1} > h^{1,1}$ . Thus at small volumes the potential is positive, and so since the potential must go to zero at infinity and is known to have negative values at finite volume, there must exist a local AdS minimum along the direction in Kähler moduli space where the volume changes.

One may worry that this argument involves the behaviour of the potential at small values of the volume where the  $\alpha'$  expansion cannot be trusted. However, for minima at very large volume, the 'small' volumes required in the above argument are extremely large in string units, and so we can self-consistently neglect terms of higher order in  $\alpha'$ . The relative strength of the  $\alpha'$  correction varies between Calabi-Yaus depending on the precise details of the geometry. For the explicit example studied in the next section, the 'small' volumes used in the above argument to establish the positivity of the potential may easily be  $\mathcal{O}(10^8)l_s^6$ .

It remains to argue that the potential also has a minimum in the remaining directions of the moduli space. Imagine moving along the surfaces in the moduli space that are of fixed Calabi-Yau volume,  $\mathcal{V}$ . Then, as one approaches the walls of the Kähler cone the first term in (5.19) dominates, since it has the fewest powers of volume in the denominator and the exponential contributions of the moduli that are becoming small cannot be neglected. Only the exponential contribution of  $\tau_s$  is given in (5.19) because of the assumed limit, but it is easy to convince oneself that a similar term will appear for any modulus that is small while the overall volume is large. Thus at large overall volume we expect the potential to grow in the positive direction towards the walls of the Kähler cone, provided all the moduli appear in the non-perturbative superpotential. All told, the potential is negative along the special direction in moduli space that we have identified and eventually rises to be positive or to vanish in all other directions, giving an AdS minimum. Since  $V \sim O(1/\mathcal{V}^3)$  at the minimum, while  $-3e^{\mathcal{K}}|W|^2 \sim O(1/\mathcal{V}^2)$ , it is clear that this minimum is non-supersymmetric

We can heuristically see why the minimum we are arguing for can be at exponentially large volume. The naive measure of the location of the minimum is the value of the volume at which the second term of equation (5.21) becomes dominant. As this only occurs when  $\ln \mathcal{V}$  is large, we expect to be able to find vacua at large values of  $\ln \mathcal{V}$ . We will see this explicitly in the example studied below.

We can also see that the gravitino mass for these vacua will, for fixed  $g_s$ , be independent of the flux choice. If the minimum exists at large volume, it is found by playing off the three terms in equation (5.19) against each other. If we

write  $\tilde{\mathcal{V}} = \frac{A_s \mathcal{V}}{W_0}$ , then (5.19) becomes

$$V \sim \left(\frac{A_s^3}{W_0}\right) \left[ \frac{1}{\tilde{\nu}} a_s^2 (-k_{ssk} t^k) e^{-2a_s \tau_s} - \frac{1}{\tilde{\nu}^2} a_s \tau_s e^{-a_s \tau_s} + \frac{\xi}{\tilde{\nu}^3} \right].$$
 (5.23)

The minimum of this potential as a function of  $\tilde{\mathcal{V}}$  is independent of  $A_s$  and  $W_0$  and, given  $a_s$ , depends only on the Calabi-Yau. We therefore have

$$\mathcal{V} \sim \frac{W_0}{A_s} f(a_s, \mathcal{M}) + \text{(subleading corrections)},$$
 (5.24)

where f is a function of the geometry. The gravitino mass is then given by

$$m_{\frac{3}{2}} = e^{\mathcal{K}/2}|W| \approx \frac{A_s}{2f(a_s, \mathcal{M})}.$$
 (5.25)

Although we have just considered the Kähler moduli in finding this minimum, it is straightforward to see that it must actually be a minimum of the full potential. Reinstating the dilaton-axion and complex structure moduli, the full potential can be written[90]

$$V = e^{\mathcal{K}} (\mathcal{K}^{a\bar{b}} D_a W \bar{D}_b \bar{W} + \mathcal{K}^{\tau\bar{\tau}} D_\tau W \bar{D}_\tau \bar{W}) + e^{\mathcal{K}} \frac{\xi}{2\mathcal{V}} (W \bar{D}_\tau \bar{W} + \bar{W} D_\tau W)$$

$$+ V_{\alpha'} + V_{np1} + V_{np2}.$$

$$(5.26)$$

Recall that the moduli values found above give rise to a negative value of the potential of  $\mathcal{O}\left(\frac{1}{\mathcal{V}^3}\right)$ . The first term in (5.26) is positive definite and of  $\mathcal{O}\left(\frac{1}{\mathcal{V}^2}\right)$ . This vanishes iff  $D_{\tau}W = D_{\phi_i}W = 0$ . Therefore, any movement of either the dilaton or complex structure moduli away from their stabilised values would create a positive term of  $\mathcal{O}\left(\frac{1}{\mathcal{V}^2}\right)$ , which the negative term cannot compete with. Thus this must increase the potential, and so the solution above automatically represents a minimum of the full potential.

It is instructive to compare this with the behaviour of KKLT solutions. The scalar potential here is

$$V = e^{\mathcal{K}_{cs}} \left( \frac{\mathcal{K}^{i\bar{j}} D_j W \bar{D}_j \bar{W}}{\mathcal{V}^2} - \frac{3|W|^2}{\mathcal{V}^2} \right). \tag{5.27}$$

If  $D_iW=0$  for all moduli, the potential is negative and of magnitude  $\mathcal{O}\left(\frac{1}{\mathcal{V}^2}\right)$ . However, if we move one modulus, for concreteness the dilaton, away from its stabilised value, the resulting positive definite contribution  $e^{\mathcal{K}_{cs}} \frac{\mathcal{K}^{\tau\bar{\tau}} D_{\tau} W \bar{D}_{\tau} \bar{W}}{\mathcal{V}^2}$  is only of the same order as the minimum. Moving the dilaton alters the value of  $e^{\mathcal{K}_{cs}}$  and thus may increase the numerator of the negative term. As the positive and negative contributions are of the same order, we see that depending on the magnitude of  $\mathcal{K}^{\tau\bar{\tau}}D_{\tau}W\bar{D}_{\tau}\bar{W}$ , this may in general decrease the overall value of the

potential. Therefore it is necessary to check explicitly for each choice of fluxes that the resulting potential has no minimum.

The above solution can be uplifted to a de Sitter vacuum through the usual mechanisms of adding anti-D3 branes [84] or turning on magnetic fluxes on D7-branes [85]. For concreteness we take the uplift potential to be

$$V_{uplift} = +\frac{\epsilon}{\mathcal{V}^2}. (5.28)$$

When  $\epsilon = 0$ , the above minimum still exists and there are many values of the moduli for which V < 0. For  $\epsilon$  sufficiently large, the minimum is entirely wiped out and the potential is positive for all values of the moduli. At a critical value of  $\epsilon$  the minimum will pass through zero. By construction, this must still represent a minimum of the full potential. After adding the uplift terms, the total potential will once again go to zero from above at large volumes because the scaling of (5.28) will overwhelm the  $O(1/\mathcal{V}^3)$  negative terms even in the special limit that we have been studying. This then leads to the metastable de Sitter vacua popular in the attempts to incorporate accelerating universes into string theory.

# 5.1.2 Explicit Calculations for $\mathbb{P}^4_{[1,1,1,6,9]}$

We now illustrate the above ideas through explicit calculations for flux compactifications on an orientifold of the Calabi-Yau manifold given by the degree 18 hypersurface in  $\mathbb{P}^4_{[1,1,1,6,9]}$ . This has been studied by Denef, Douglas and Florea[91] following earlier work in [96]. The defining equation is

$$z_1^{18} + z_2^{18} + z_3^{18} + z_4^{18} + z_5^{18} - 18\psi z_1 z_2 z_3 z_4 z_5 - 3\phi z_1^6 z_2^6 z_3^6 = 0,$$
 (5.29)

with  $h^{1,1}=2$  and  $h^{2,1}=272$ . The complex structure moduli  $\psi$  and  $\phi$  that have been written in (5.29) are the two moduli invariant under the  $\Gamma=\mathbb{Z}_6\times\mathbb{Z}_{18}$  action whose quotient gives the mirror manifold [15]. There are another 270 terms not invariant under  $\Gamma$  which have not been written explicitly, although some will be projected out by the orientifold action.

We first stabilise the complex structure moduli through an explicit choice of fluxes. If  $W_{cs}$  denotes the flux superpotential, we must solve

$$D_{\tau}W_{cs} = 0$$
 and  $D_{\phi_i}W_{cs} = 0,$  (5.30)

for the dilaton and complex structure moduli. There are two possibilities. First, we may of course turn on fluxes along all relevant three-cycles and solve (5.30) for all moduli. As we would need to know all 200-odd periods this is impractical. We however know of no theoretical reasons not to do it. The easier approach is to turn on fluxes only along cycles corresponding to  $\psi$  and  $\phi$ , and then solve

$$D_{\tau}W_{cs} = D_{\psi}W_{cs} = D_{\phi}W_{cs} = 0. \tag{5.31}$$

As first described in [70] and reviewed in chapter 3, the invariance of (5.29) under  $\Gamma$  ensures that, at  $\phi_k = 0$ ,  $D_{\phi_k}W = 0$  for all other moduli  $\phi_k$ . The necessary periods have been computed in [96] and appropriate fluxes and solutions to (5.31) could be found straightforwardly as in chapter 3 along the lines of [70, 63, 2]. This is not our focus here and we henceforth assume this to have been done.

We now return to the Kähler moduli. Using the notation of [91], the Kähler geometry is specified by

$$\mathcal{V} = \frac{1}{9\sqrt{2}} \left( \tau_5^{\frac{3}{2}} - \tau_4^{\frac{3}{2}} \right), \tag{5.32}$$

with 
$$\tau_4 = \frac{t_1^2}{2}$$
 and  $\tau_5 = \frac{(t_1 + 6t_5)^2}{2}$ .

Here  $\tau_4$  and  $\tau_5$  are volumes of the divisors  $D_4$  and  $D_5$ , corresponding to a particular set of 4-cycles, and  $t_1$  and  $t_5$  2-cycle volumes. Generally the volume is only an implicit function of  $\tau_i$ , but here we are fortunate and have an explicit expression. As shown in [91], both  $D_4$  and  $D_5$  correspond to divisors which appear non-perturbatively in the superpotential. We write this superpotential as

$$W = W_0 + A_4 e^{-a_4 T_4} + A_5 e^{-a_5 T_5}. (5.33)$$

We now take the limit described above, in which  $\mathcal{V} \to \infty$  (and hence  $\tau_5 \to \infty$ ) and  $\tau_4 \sim \log \mathcal{V}$ . Note that the alternative limit  $\tau_4 \to \infty$  with  $\tau_5 \sim \log \mathcal{V}$  would not be well-defined, as the volume of the Calabi-Yau becomes formally negative.

The  $\alpha'$  correction is given by equation (5.8). For  $V_{np1}$  and  $V_{np2}$  we must compute the inverse metric, which in this limit is given by

$$\mathcal{K}^{T_4\bar{T}_4} = 24\sqrt{2}\sqrt{\tau_4}\mathcal{V} \sim \sqrt{\tau_4}\mathcal{V}, \tag{5.34}$$

$$\mathcal{K}^{T_4\bar{T}_5} = \mathcal{K}^{T_5\bar{T}_4} = 4\tau_4\tau_5 \sim \tau_4 \mathcal{V}^{\frac{2}{3}}, \tag{5.35}$$

$$\mathcal{K}^{T_5\bar{T}_5} = \frac{4}{3}\tau_5^2 \sim \mathcal{V}^{\frac{4}{3}}. \tag{5.36}$$

We can then compute  $V_{np1}$  and  $V_{np2}$  with the result that the full potential takes the schematic form

$$V \sim \left[ \frac{1}{\mathcal{V}} a_4^2 |A_4|^2 \sqrt{\tau_4} e^{-2a_4\tau_4} - \frac{1}{\mathcal{V}^2} a_4 \tau_4 e^{-a_4\tau_4} |A_4W| + \frac{\xi}{\mathcal{V}^3} |W|^2 \right]. \tag{5.37}$$

where numerical coefficients have been dropped. We have implicitly extremised with respect to the axion  $b_4$  to get a negative sign in front of the second term, as described below equation (5.16). It is obvious that in the limit

$$\tau_5 \to \infty \quad \text{with} \quad a_4 \tau_4 = \ln \mathcal{V}, \tag{5.38}$$

the potential approaches zero from below as the middle term of equation (5.37) dominates. This is illustrated in figure 5.1 where we plot the numerical values of  $\ln(V)$ .

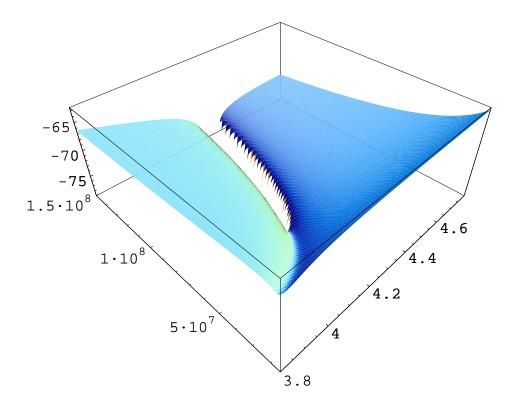


Figure 5.1:  $\ln(V)$  for  $\mathbb{P}^4_{[1,1,1,6,9]}$  in the large volume limit, as a function of the divisors  $\tau_4$  and  $\tau_5$ . The void channel corresponds to the region where V becomes negative and  $\ln(V)$  undefined. As  $V \to 0$  at infinite volume, this immediately shows that a large-volume minimum must exist. Here the values  $W_0 = 20$ ,  $A_4 = 1$  and  $a_4 = 2\pi$  have been used.

The location and properties of the AdS minimum may be found analytically. To capture the form of equation (5.37), we write

$$V = \frac{\lambda \sqrt{\tau_4} e^{-2a_4 \tau_4}}{\mathcal{V}} - \frac{\mu}{\mathcal{V}^2} \tau_4 e^{-a_4 \tau_4} + \frac{\nu}{\mathcal{V}^3}.$$
 (5.39)

The axion field  $b_5$  has been ignored as terms in which it appears are exponentially suppressed.

We regard  $V = V(\mathcal{V}, \tau_4)$ , and solve

$$\frac{\partial V}{\partial \mathcal{V}} = \frac{\partial V}{\partial \tau_4} = 0.$$

One may easily check that the first of these equations may be rearranged into a quadratic and solved for V to give

$$\mathcal{V} = \frac{\mu}{\lambda} \sqrt{\tau_4} e^{a_4 \tau_4} \left( 1 \pm \sqrt{1 - \frac{3\nu\lambda}{\mu^2 \tau_4^{\frac{3}{2}}}} \right)$$
 (5.40)

We also have

$$\frac{\partial V}{\partial \tau_4} = 0 \Rightarrow \frac{\lambda \mathcal{V} e^{-a_4 \tau_4}}{\tau_4^{\frac{1}{2}}} \left( \frac{1}{2} - 2a_4 \tau_4 \right) - \mu \left( 1 - a_4 \tau_4 \right) = 0. \tag{5.41}$$

We then use (5.40) to obtain an implicit equation for  $\tau_4$ ,

$$\left(1 \pm \sqrt{1 - \frac{3\nu\lambda}{\mu\tau_4^{\frac{3}{2}}}}\right) \left(\frac{1}{2} - 2a_4\tau_4\right) = (1 - a_4\tau_4).$$
(5.42)

We do not need to solve this fully; as we require  $a_4\tau_4 \gg 1$  to be able to ignore higher instanton corrections, we can use this to simplify (5.42) and solve for  $\tau_4$  and  $\mathcal{V}$ , obtaining

$$\tau_4 = \left(\frac{4\nu\lambda}{\mu^2}\right)^{\frac{2}{3}},$$

$$\mathcal{V} = \frac{\mu}{2\lambda} \left(\frac{4\nu\lambda}{\mu^2}\right)^{\frac{1}{3}} e^{a_4 \left(\frac{4\nu\lambda}{\mu^2}\right)^{\frac{2}{3}}}.$$
(5.43)

For the potential of (5.39),

$$\lambda \sim a_4^2 |A_4|^2$$
,  $\mu \sim a_4 |A_4 W_0|$ , and  $\nu \sim \xi |W_0|^2$ .

We then have

$$\tau_4 \sim (4\xi)^{\frac{2}{3}} \quad \text{and } \mathcal{V} \sim \frac{\xi^{\frac{1}{3}}|W_0|}{a_4 A_4} e^{a_4 \tau_4}.$$
(5.44)

This formula justifies our earlier claim that these vacua can generically be at exponentially large volume.

For the  $\mathbb{P}^4_{[1,1,1,6,9]}$  example,

$$\xi = 1.31, \quad \lambda = 3\sqrt{2}a_4^2|A_4|^2, \quad \mu = \frac{1}{2}a_4|A_4W_0|, \quad \nu = 0.123|W_0|^2.$$

The analytic results derived here agree well with the exact locations of the minima found numerically, with the small error almost entirely due to the approximation made in solving equation (5.42). As discussed above, the values of the Kähler moduli found above combine with the flux-stabilised complex structure moduli to give a minimum of the full potential (2.36).

We note that both the overall and divisor volumes are clearly larger than the string scale. The 'large' modulus  $\tau_5 \gg 1$ , while for the 'small' modulus  $a_4\tau_4 \sim \ln \mathcal{V} \gtrsim 1$ . Putting back the factors of  $g_s$  and N, we have  $a_4 = \frac{2\pi}{g_s N}$ . We shall discuss scales and phenomenological features in more detail below, but shall now simply observe that small adjustments in  $g_s$  and N allow a very wide variety of scales to be realised.

Let us summarise our results so far. The argument above shows that there exists a decompactification direction in moduli space along which nonperturbative effects dominate over the perturbative  $\alpha'^3$  corrections, leading to an exponentially large volume minimum with all geometric moduli stabilised in a very general class of compactifications (for which  $h_{12} > h_{11} > 1$ ).<sup>4</sup>

There are several very interesting features about this limit. The mechanism described here stabilises all moduli and results in internal spaces that are exponentially large in string units, which is the first time this has been achieved in a string construction. While large internal volumes are useful from the point of view of control, they also give the interesting possibility of lowering the string scale, as the string and Planck scales are related by  $M_s \sim \frac{M_P}{\sqrt{\mathcal{V}}}$ . Furthermore, no tuning has been required to obtain this minimum. In particular, the volume scales as  $\mathcal{V} \propto W_0$  and there is no requirement that  $W_0$  be small. This is in contrast to the behaviour encountered in the KKLT scenario, where very small values of  $W_0$  are necessary for consistency.

As the volume is exponentially sensitive to (for example) the stabilised dilaton, there is no fine-tuning involved in adjusting the volume to give a TeV-scale gravitino mass. The above moduli stabilisation mechanism therefore gives a genuine solution, as opposed to simply a reinterpretation, of the supersymmetric hierarchy problem.

# 5.2 The $g_s$ and $\alpha'$ Expansions

In chapter 4, we argued that under almost all circumstances the neglect of  $\alpha'$  corrections in IIB flux compactifications is inconsistent. In the above section we have described how the inclusion of the  $\alpha'^3$  corrections of [90] gives a minimum of the potential at exponentially large volumes. The natural worry is that this is inconsistent: it is often felt that if some  $\alpha'$  corrections are important all will be, and it is inconsistent to include only a subset. Fortunately this is wrong and the inclusion of some  $\alpha'$  corrections does not necessarily require full knowledge of the string theory. At small volumes, the  $\alpha'$  corrections do indeed appear democratically, and it would be difficult to extract reliable results. However, the  $\alpha'$  expansion is at heart an expansion in inverse volume. At very large volumes, the expansion parameter for  $\alpha'$  effects is  $\frac{1}{\nu}$ , and a systematic inclusion of such effects will give a controlled expansion. In discussing this, it will be useful to consider the origins of perturbative effects from both ten- and four-dimensional perspectives.

<sup>&</sup>lt;sup>4</sup>If  $h_{12} < h_{11}$ , the  $\alpha'$  corrections cause the fields to roll in towards small volume. In this limit it is very difficult to maintain control as all  $\alpha'$  corrections become equally important.

#### Perturbative corrections in 10 dimensions

It will be useful in this section to keep in mind the dimensional reduction scalings analysed in chapter 2. We will analyse perturbative corrections to the scalar potential by studying the volume scaling of terms arising from dimensional reduction of the  $\alpha'$ -corrected type IIB supergravity action. Of course this action is not known fully, but our arguments will only depend on the general form of the terms rather than the specific details of the tensor structure. The supergravity action consists of bulk and localised terms and is

$$S_{IIB} = S_{b,0} + \alpha'^3 S_{b,3} + \alpha'^4 S_{b,4} + \alpha'^5 S_{b,5} + \dots + S_{cs} + S_{l,0} + \alpha'^2 S_{l,2} + \dots$$
 (5.45)

The localised sources present are D3/D7 or O3/O7 planes. In string frame, we have [23]

$$S_{b,0} = \frac{1}{(2\pi)^7 \alpha'^4} \int d^{10}x \sqrt{-g} \left\{ e^{-2\phi} \left[ \mathcal{R} + 4(\nabla \phi)^2 \right] - \frac{F_1^2}{2} - \frac{1}{2 \cdot 3!} G_3 \cdot \bar{G}_3 - \frac{\tilde{F}_5^2}{4 \cdot 5!} \right\},$$

$$S_{cs} = \frac{1}{4i(2\pi)^7 \alpha'^4} \int e^{\phi} C_4 \wedge G_3 \wedge \bar{G}_3,$$

$$S_{l,0} = \sum_{sources} \left( -\int d^{p+1} \xi \, T_p \, e^{-\phi} \sqrt{-g} + \mu_p \int C_{p+1} \right). \tag{5.46}$$

We may avoid the need to include D3-branes by taking the fluxes to saturate the  $C_4$  tadpole. We shall work throughout in the F-theory orientifold limit, in which the dilaton is constant:  $\phi = \phi(y) = \phi_0$ .

For flux compactifications with ISD 3-form fluxes, as reviewed in chapter 2, the metric and fluxes take the form

$$ds_{10}^{2} = e^{2A(y)} \eta_{\mu\nu} dx^{\mu} dx^{\nu} + e^{-2A(y)} \tilde{g}_{mn} dy^{m} dy^{n},$$
  

$$\tilde{F}_{5} = (1+*) \left[ d\alpha \wedge dx^{0} \wedge dx^{1} \wedge dx^{2} \wedge dx^{3} \right],$$
  

$$F_{3}, H_{3} \in H^{3}(M, \mathbb{Z}),$$
(5.47)

where  $\alpha=e^{4A}$  parametrises both the magnitude of the warping and the size of the 5-form flux. Here  $\tilde{g}$  is a Calabi-Yau metric; the flux back-reacts to render the compact space only conformally Calabi-Yau. The warp-factor transforms non-trivially under internal rescalings, with warping effects suppressed at large volume. Specifically, under  $\mathcal{V} \to \lambda^6 \mathcal{V}$ , where  $\lambda \gg 1$ ,  $\alpha = 1 + \mathcal{O}(\frac{1}{\lambda^4}) + \ldots$ 

We first consider contributions tree level in  $\alpha'$ . The leading contribution to the 4-d scalar potential arises from the flux term  $\frac{1}{(2\pi)^7\alpha'^4}\int G_3\cdot \bar{G}_3$  [31]. This has a topological part and a moduli-dependent component. The former is pseudo-BPS and cancels against the tension of localised sources. Dimensional reduction of the latter gives

$$V_{flux} \sim \mathcal{K}_{cs}^{a\bar{b}} \frac{D_a W D_{\bar{b}} W}{\mathcal{V}^2},$$
 (5.48)

where the sum is over dilaton and complex structure moduli. This term is positive semi-definite and vanishes at its minimum. As we will work in a limit  $\mathcal{V} \gg 1$ , the most important element of (5.48) is its volume scaling. This is understood as follows:

conversion to 4d Einstein frame internal integral 
$$V_{flux} \sim V^{-2} \times V^{-2} \times V^{-2} \times V^{-2}$$
.

(5.49)

This volume scaling is valid for both topological and moduli-dependent parts.

In the absence of warping, (5.48) is the only  $\mathcal{O}(\alpha'^0)$  contribution to the potential energy, as  $\tilde{F}_5 = \mathcal{R} = 0$  and dilaton gradients vanish. However, there is also a warping contribution. At large volume,  $\alpha \sim 1 + \frac{1}{\nu^{\frac{2}{3}}}$  and thus  $V_{F_5} = \int d^6x \sqrt{g} F_5^2 \sim \int d^6x \sqrt{g} (\mathrm{d}\alpha)^2$  contributes

conversion to Einstein frame internal integral 
$$V_{F_5} \sim V^{-2} \times V^{-\frac{5}{3}} \sim V^{-8/3}$$
. (5.50)

As now  $\mathcal{R} \neq 0$ , the Einstein-Hilbert term  $\int_{Y_6} \sqrt{-g} \mathcal{R}$  is also important and in fact contributes identically with  $V_{F_5}$ . These terms may be related to the tree-level flux term and in fact serve as an additional prefactor. The net result is [97]

$$V_{0,unwarped} = \frac{1}{2\kappa_{10}^2 \text{Im } \tau} \int_M G_3^+ \wedge *_6 \bar{G}_3^+ \to V_{0,warped} = \frac{1}{2\kappa_{10}^2 \text{Im } \tau} \int_M e^{4A} G_3^+ \wedge *_6 \bar{G}_3^+.$$

It is important that this potential remains no-scale, with no potential generated for the Kähler moduli [97].

The IIB bulk effective action receives higher-derivative corrections starting at  $\mathcal{O}(\alpha'^3)$ , which is also the order at which string loop corrections first appear; the tree level action is already  $SL(2,\mathbb{Z})$  invariant and receives no  $g_s$  corrections. The discussion of loop corrections to  $S_b$  is thus subsumed into the discussion of higher-derivative corrections.

The bosonic fields are the metric, dilaton-axion, 3-form field strength  $G_3$  and self-dual five form field strength  $F_5$ . While its precise form is unknown,  $S_{b,3}$  is expected to include all combinations of these consistent with the required dimensionality. At  $\mathcal{O}(\alpha'^3)$ , the bosonic action takes the schematic form

$$S_{b,3} \sim \frac{\alpha'^3}{\alpha'^4} \int d^{10}x \sqrt{-g} \Big[ \Big( \mathcal{R}^4 + \mathcal{R}^3 \left( G_3 G_3 + \bar{G}_3 \bar{G}_3 + G_3 \bar{G}_3 + F_5^2 + \partial \tau \cdot \partial \tau + \nabla^2 \tau \right) \\ + \mathcal{R}^2 ((DG_3)^2 + (DF_5)^2 + G^4 + \ldots) + \mathcal{R}(G_3^6 + \ldots) + (G_3^8 + \ldots) \Big) \Big].$$

Terms linear in the fluxes (e.g.  $\mathcal{R}^3DG_3$ ) are forbidden as the action must be invariant under world-sheet parity. The tensor structure and modular behaviour

of the majority of these terms is unknown. A notable exception is the  $\mathcal{R}^4$  term, the coefficient of which is known exactly to be an Eisenstein series in the dilaton [98]. As stressed above however, our interest is in the volume scaling, which can be extracted on merely dimensional grounds.

Let us first consider terms independent of warping. The  $\mathcal{R}^3(G^2 + c.c.)$  and  $\mathcal{R}^3G\bar{G}$  terms are most easily understood. These are similar to the  $\mathcal{O}(\alpha'^0)$   $G_3\bar{G}_3$  term but with three extra powers of curvature. Then

$$V_{\mathcal{R}^3G^2} \sim \overbrace{\mathcal{V}^{-2}}^{\text{Einstein frame internal integral}} \times \overbrace{\mathcal{V}}^{\mathcal{R}^3} \times \overbrace{\mathcal{V}^{-1}}^{\mathcal{G}_3^2} \times \overbrace{\mathcal{V}^{-1}}^{\mathcal{G}_3^2} \sim \mathcal{V}^{-3}.$$
 (5.51)

The same argument tells us that a similar scaling applies for  $\mathcal{R}^2(DG_3)^2$  terms, whereas  $\mathcal{R}^2G^4$  terms contribute as  $\sim \frac{1}{V^{11/3}}$ ,  $\mathcal{R}G^6$  terms as  $\sim \frac{1}{V^{13/3}}$  and  $G^8$  terms as  $\sim \frac{1}{V^5}$ . In the absence of warping, the  $\mathcal{R}^4$  term does not contribute to the potential energy; integrated over a Calabi-Yau, it vanishes. This geometric result can be understood macroscopically; were this not to vanish, it would generate a potential for the volume even in flux-less  $\mathcal{N}=2$  IIB compactifications. However, it is known there that tree level moduli remain moduli to all orders in the  $\alpha'$  and  $g_s$  expansions, and thus higher derivative terms must not be able to generate a potential for them. This term contributes indirectly by modifying the dilaton equations of motion at  $\mathcal{O}(\alpha'^3)$ ; we will return to this later.

There are also higher-derivative terms dependent on the warp factor. Examples are

$$\frac{\alpha'^3}{\alpha'^4} \int \sqrt{g} \mathcal{R}^4, \qquad \frac{\alpha'^3}{\alpha'^4} \int \sqrt{g} \mathcal{R}^3 F_5^2, \qquad \frac{\alpha'^3}{\alpha'^4} \int \sqrt{g} \mathcal{R}^2 (DF_5)^2.$$

These are similar to the corresponding tree-level terms but with three extra powers of curvature. As  $\mathcal{R}^3 \sim \frac{1}{\mathcal{V}}$ , the contribution of such terms is no larger than  $\mathcal{O}(\frac{1}{\mathcal{V}^{11/3}})$ .

There are also potential contributions from internal dilaton gradients. As we have worked in the orientifold limit of F-theory, we have thus far regarded such terms as vanishing. However, this is not quite true. In the presence of higher derivative corrections, a constant dilaton no longer solves the equations of motion. Instead, we have

$$\phi(y) = \phi_0 + \frac{\zeta(3)}{16}Q(y). \tag{5.52}$$

An explicit expression for Q(y) may be found in [90], but for our purposes it is sufficient to note that  $Q \sim \mathcal{R}^3 \sim \frac{1}{\mathcal{V}}$ . It is then easy to see that terms such as

$$\int (\partial \tau \cdot \partial \bar{\tau}) \mathcal{R}^2 G^2 \qquad \text{or} \qquad \int (\nabla^2 \phi) (DF_5)^2 \mathcal{R}$$

are suppressed compared to the terms considered above. Note that dilaton-curvature terms such as

 $\int (\nabla^2 \phi) \mathcal{R}^3$ 

will not contribute to the potential energy; these exist in  $\mathcal{N}=2$  Calabi-Yau compactifications and so must vanish either directly or by cancellation. Similar comments apply for the fact that, even without warping, the internal space ceases to be Calabi-Yau at  $\mathcal{O}(\alpha'^3)$ .

There is one further effect associated with (5.52). As the dilaton is no longer constant, under dimensional reduction the four-dimensional Einstein-Hilbert term is renormalised. Rescaling this to canonical form introduces a term  $\frac{V_{tree}}{\mathcal{V}}$  of  $\mathcal{O}(\frac{1}{\mathcal{V}^3})$ . However, as  $V_{tree}$  is no-scale this correction does not break the no-scale structure and in particular vanishes at the minimum.

String loop corrections first appear at  $\mathcal{O}(\alpha'^3)$  and are thus subsumed into the above analysis. While it may be difficult to derive anything explicitly, we may conjecture their effect in the large volume limit. The corrections to the Kähler potential arise from dimensional reduction of the 10-dimensional  $\mathcal{R}^4$  term. In Einstein frame, the string tree-level  $\alpha'^3$  corrected Kähler potential derived in [90] takes the form

$$\mathcal{K} = \mathcal{K}_{cs} - 2\ln\left(\mathcal{V} + \frac{\xi}{2e^{3\phi/2}}\right). \tag{5.53}$$

Here  $\xi = -\frac{\chi(M)\zeta(3)}{2(2\pi)^3}$ .  $\zeta(3)$  is distinctive as the tree-level coefficient of the tendimensional  $\mathcal{R}^4$  term, the coefficient of which is known exactly to be

$$f_{\frac{3}{2}}^{(0,0)}(\tau,\bar{\tau}) = \sum_{(m,n)\neq(0,0)} \frac{e^{-\frac{3\phi}{2}}}{|m+n\tau|^3}.$$
 (5.54)

This has the expansion

$$f_{\frac{3}{2}}^{(0,0)}(\tau,\bar{\tau}) = \frac{2\zeta(3)}{e^{\frac{3\phi}{2}}} + \frac{2\pi^2}{3}e^{\frac{\phi}{2}} + \text{ instanton terms}.$$
 (5.55)

Therefore, to incorporate  $\mathcal{O}(\alpha'^3)$  string loop corrections to  $S_{IIB}$ , the natural conjecture is that we should modify (5.53) to

$$\mathcal{K} = \mathcal{K}_{cs} - 2\ln\left(\mathcal{V} - \frac{\chi(M)}{8(2\pi)^3} f_{\frac{3}{2}}^{(0,0)}(\tau,\bar{\tau})\right). \tag{5.56}$$

Let us finally mention further higher derivative corrections at  $\mathcal{O}(\alpha'^4)$  and above. At large volume these are all subleading, with any terms generated being subdominant to the  $\frac{1}{\mathcal{V}^3}$  terms present at  $\mathcal{O}(\alpha'^3)$ . For example, an  $\mathcal{O}(\alpha'^4)$  term  $G^2\mathcal{R}^4$  would give a  $\mathcal{V}^{-\frac{10}{3}}$  contribution to the potential. There are other terms

that would naively give a  $\frac{1}{\mathcal{V}^3}$  contribution, such as a possible  $\mathcal{O}(\alpha'^6)$  term  $\mathcal{R}^6$ . However, on a Calabi-Yau such a term vanishes, either explicitly or through cancellation. This is for the reasons discussed above: terms present even in pure  $\mathcal{N}=2$  Calabi-Yau compactification cannot generate potentials for the moduli.

Our conclusion is therefore that the leading  $\alpha'$  corrections breaking the noscale structure appear at  $\mathcal{O}(\frac{1}{\nu^3})$ , coming from  $\mathcal{R}^3G^2$  and  $\mathcal{R}^2(DG)^2$  terms in ten dimensions.<sup>5</sup> This is consistent with the result of [90], where the corresponding correction (5.53) to the Kähler potential was computed and the resulting scalar potential interpreted as descending from such ten-dimensional terms. Although the scaling argument used above cannot reproduce the coefficient of the correction, it makes it easier to see that other terms are subleading; in particular, the effects associated with non-vanishing  $F_5$  (which were not considered in [90]) do not compete.

Let us now consider higher derivative corrections to localised sources. The D3-brane action is

$$S_{D3} = -T_3 \int d^4x \sqrt{-g} e^{-\phi} + \mu_3 \int C_4.$$
 (5.57)

The D3-brane world-volume is space-filling. Therefore, so long as higher derivative corrections arise from pull-backs onto the brane world volume then they necessarily involve space-time derivatives and so cannot give a potential energy. This is not so for D7-branes; fortunately in that case such effects were already included in the analysis of [31]. We briefly review the relevant results. The leading  $\alpha'$  correction to the D7-brane Chern-Simons action is (we do not turn on internal D7-brane fluxes)

$$S_{D7,\alpha'^2} = \frac{\mu_7}{96} (2\pi\alpha')^2 \int_{\mathbb{M}^4 \times \Sigma} C_4 \wedge \text{Tr}(\mathcal{R}_2 \wedge \mathcal{R}_2), \tag{5.58}$$

whereas the leading correction to the wrapped D7-brane tension arises from a term

$$\frac{-\mu_7}{96} (2\pi\alpha')^2 \int_{\mathbb{M}^4 \times \Sigma} \sqrt{-g} \operatorname{Tr}(\mathcal{R}_2 \wedge *\mathcal{R}_2). \tag{5.59}$$

These contribute effective D3-brane charge and tension. In F-theory, the D3-brane charge from the wrapped D7-branes is

$$Q_{D3} = -\frac{\chi(X)}{24},\tag{5.60}$$

where X is the Calabi-Yau fourfold. As the D7 branes are BPS, (5.60) also gives the resulting effective D3 tension.

<sup>&</sup>lt;sup>5</sup>One can indeed show that if  $\mathcal{V} \gg 1$  any effect that is string scale and localised in the extra dimensions can only contribute to the F-term potential V at  $\mathcal{O}\left(\frac{1}{\mathcal{V}^3}\right)$ .

We can then make a similar argument that higher  $\alpha'$  corrections to the D7-brane action need not be considered. As the branes are BPS,  $\alpha'$  corrections to the tension are related to  $\alpha'$  corrections to the charges. However, these involve spacetime curvature, and so the corresponding correction to the DBI action will not give rise to a 4d potential energy.

The arguments above have considered contributions to the 4d effective potential coming from dimension reduction of the 10d effective action. This style of argument knows nothing about the low-energy physics. It is well-known that the existence of 4-dimensional supersymmetry places considerable constraints on the type of effective action allowed. We now analyse allowed corrections from the viewpoint of 4-dimensional supergravity. The combination of 4d and 10d intuition leads to the most stringent constraints on the possible perturbative corrections that may appear.

#### Perturbative corrections in 4 dimensions

We now use the fact that the effective theory admits a 4d supergravity description to constrain the perturbative  $g_s$  and  $\alpha'$  corrections. We will confirm the self-consistency of the effective field theory description below in section 5.4. The supergravity structure implies all perturbative corrections to the potential should manifest themselves through corrections to the Kähler potential, as the superpotential is not renormalised in perturbation theory. To be specific we shall illustrate the discussion with the Kähler potential appropriate to the  $\mathbb{P}^4_{[1,1,1,6,9]}$  model studied above. At tree-level this is given by

$$\mathcal{K} = -2\ln\left((T_1 + \bar{T}_1)^{3/2} - (T_2 + \bar{T}_2)^{3/2}\right). \tag{5.61}$$

We recall the geometry has one overall Kähler mode  $(T_1)$  and one blow-up mode  $(T_2)$ . We specialise to our limit of interest  $\tau_1 \gg \tau_2 > 1$ , with  $\tau_i = \text{Re}(T_i)$ . The resulting Kähler metric has the form (neglecting terms subleading in  $\mathcal{V}$ )

$$\mathcal{K}_{i\bar{j}} = \begin{pmatrix} \frac{3}{\mathcal{V}^{4/3}} & -\frac{9\sqrt{\tau_2}}{2\mathcal{V}^{5/3}} \\ -\frac{9\sqrt{\tau_2}}{2\mathcal{V}^{5/3}} & \frac{3}{2\sqrt{2\tau_2}\mathcal{V}} \end{pmatrix}$$
(5.62)

The central point in this analysis is that valid perturbative corrections to the Kähler potentials should also give perturbative corrections to the Kähler metric. If a hypothetical correction term  $\delta K$  generates a correction in the metric that dominates the tree-level metric in an appropriate classical limit (weak coupling and large volume), then it is unphysical and should not be allowed.

We can apply this condition to restrict the form of potential corrections to  $\mathcal{K}$ . For example, consider the possible correction

$$\mathcal{K} + \delta \mathcal{K} = -2\ln(\mathcal{V}) + \frac{\epsilon \sqrt{2\tau_2}}{\mathcal{V}^{\alpha}},\tag{5.63}$$

with  $0 < \alpha < 1$ . The  $2\bar{2}$  component of the corrected Kähler metric is

$$\mathcal{K}_{2\bar{2}} + \delta \mathcal{K}_{2\bar{2}} = \frac{3}{2\sqrt{2\tau_2}\mathcal{V}} - \frac{\epsilon}{8\sqrt{2}\tau_2^{3/2}\mathcal{V}^{\alpha}}.$$
 (5.64)

As  $\alpha < 1$ , the correction to the kinetic term would always dominate the tree-level term in the limit  $\mathcal{V} \gg 1$ . This seems implausible as a large-volume limit ought to make the correction less, rather than more, important.

A similar comment applies to a correction

$$\mathcal{K} + \delta \mathcal{K} = -2\ln(\mathcal{V}) + \frac{\epsilon \tau_2^2}{\mathcal{V}},\tag{5.65}$$

which leads to

$$\mathcal{K}_{2\bar{2}} + \delta \mathcal{K}_{2\bar{2}} = \frac{3}{2\sqrt{2\tau_2}\mathcal{V}} + \frac{\epsilon}{4\mathcal{V}}.$$
 (5.66)

In this case, as we take  $\tau_2$  large, the correction becomes increasingly dominant over the tree-level term. Given that large  $\tau_2$  reduces both the curvature of this cycle and the gauge coupling on any brane wrapping it, we would again expect exactly opposite behaviour to occur.

We could also consider the correction

$$\mathcal{K} + \delta \mathcal{K} = -2\ln(\mathcal{V}) + \frac{2\epsilon \tau_2}{\mathcal{V}^{\alpha}},$$
 (5.67)

with  $0 < \alpha < 1$ . In this case  $\delta K_{2\bar{2}}$  is subleading to  $\mathcal{K}_{2\bar{2}}$  in the classical limit. However, if we consider the  $1\bar{2}$  component we now have

$$\mathcal{K}_{1\bar{2}} + \delta \mathcal{K}_{1\bar{2}} = -\frac{9\sqrt{\tau_2}}{2\mathcal{V}^{5/3}} + \frac{3\epsilon}{2\mathcal{V}^{\alpha+2/3}},\tag{5.68}$$

and the correction again dominates in the large-volume limit.

Generally, what the above tells us is that, for  $\alpha < 1$  and nontrivial  $f(\tau_s)$ , corrections of the form

$$\delta K = -2\ln(\mathcal{V}) + \frac{f(\tau_s)}{\mathcal{V}^{\alpha}}$$

are excluded. Furthermore, for a correction

$$\delta K = -2\ln(\mathcal{V}) + \frac{f(\tau_s)}{\mathcal{V}}$$

to be permissible, we must have

$$\lim_{\tau_s \to \infty} \frac{f''(\tau_s)}{\sqrt{\tau_s}} \to 0. \tag{5.69}$$

We note a correction

$$\mathcal{K} + \delta \mathcal{K} = -2\ln(\mathcal{V}) + \frac{\epsilon \sqrt{\tau_2}}{\mathcal{V}}$$
 (5.70)

is not excluded by the above arguments. As a correction to the Kähler metric, this would give

$$\mathcal{K}_{2\bar{2}} + \delta \mathcal{K}_{2\bar{2}} = \frac{3}{2\sqrt{2\tau_2}\mathcal{V}} - \frac{\epsilon}{16\tau_2^{3/2}\mathcal{V}}.$$
 (5.71)

This correction is suppressed compared to the tree-level term by a factor  $\tau^{-1}$ , i.e.  $g^2$  of the field theory on the brane. It is well-behaved (i.e. subdominant) in the classical limit, as there does not exist a 'bad' scaling limit in which it dominates the tree-level term. We shall consider such a possible correction further in chapter 7.

The above arguments show that for corrections which have a nontrivial functional dependence on the 'small' moduli, the correction to  $\mathcal{K}$  - and hence the correction to V - must be suppressed by a factor of  $\mathcal{V}$ . This implies they can contribute to the scalar potential at a maximal order of  $1/\mathcal{V}^3$  in the volume expansion. This is a similar order to the  $\alpha'^3$  correction. Consequently, while these corrections might modify the exact locus of the exponentially large volume minimum, they cannot affect the main feature, namely the exponentially large volume.

The above arguments do not allow us to say anything about corrections to  $\mathcal{K}$  that depend purely on the volume, such as

$$\delta \mathcal{K} = -2\ln(\mathcal{V}) + \frac{1}{\mathcal{V}^{\alpha}},\tag{5.72}$$

with  $0 < \alpha < 1$ . No evidence exists for such corrections arising from dimensional reduction of local ten-dimensional terms. They may still be generated by non-local effects. For example, there exists a string loop calculation of corrections to the Kähler potential in IIB orientifold backgrounds [99]. This found the existence of an effective correction

$$\delta \mathcal{K} = -2 \ln(\mathcal{V}) + \frac{1}{\mathcal{V}^{2/3}} + \mathcal{O}(\mathcal{V}^{-4/3}).$$
 (5.73)

Naively such a correction will at large volume dominate in the scalar potential over the  $1/\mathcal{V}$   $\alpha'^3$  correction. However, in fact this is not the case. Explicit calculation shows that for the correction (5.73)

$$(\mathcal{K}^{i\bar{j}}\mathcal{K}_i\mathcal{K}_j - 3) = 0 + \frac{\alpha(2/3 - 2/3)}{\mathcal{V}^{2/3}} + \frac{\beta}{\mathcal{V}^{4/3}},\tag{5.74}$$

and so the expected contribution at  $\mathcal{O}(\mathcal{V}^{-2/3})$  is absent. It is amusing that this cancellation occurs only for  $\mathcal{V}^{-2/3}$  corrections, which is precisely the case that is relevant.

We have used the above arguments to constrain perturbative corrections to  $\mathcal{K}$ . A similar argument shows that the non-perturbative correction

$$\delta K = -2\ln(\mathcal{V}) + e^{-2\pi(T_s + \bar{T}_s)},\tag{5.75}$$

which naively would appear relevant, must also be absent as one can again find a classical limit in which the 'corrections' to the Kähler metric would dominate the tree-level term.

We note that for the case where Kähler corrections have been computed [99], the corrections do fit into the form above. We only focus on the Kähler moduli dependence: the full expressions can be found in [99]. The correction gives

$$\mathcal{K} + \delta \mathcal{K} = -\ln(T_1 + \bar{T}_1) - \ln(T_2 + \bar{T}_2) - \ln(T_2 + \bar{T}_3) + \sum_{i=1}^{3} \frac{\epsilon_i}{T_i + \bar{T}_i}, \quad (5.76)$$

with for example

$$\mathcal{K}_{1\bar{1}} + \delta \mathcal{K}_{1\bar{1}} = \frac{1}{4\tau_1^2} + \frac{\epsilon_1}{4\tau_1^3}.$$
 (5.77)

The loop-corrected Kähler metric is suppressed by a factor  $\tau_1^{-1} = g^2$ .

The point of this discussion is that the form of possible corrections to the Kähler potential can be heavily constrained by the reasonable requirement that in a classical (large volume, weak coupling) limit, corrections to the metric become increasingly subdominant to tree-level terms: simply because Kähler corrections are very hard to calculate does not make us entirely ignorant of their form. In particular, denoting the 'small', blow-up moduli by  $\tau_i$ , these considerations exclude corrections of the form

$$\mathcal{K} + \delta K = -2\ln(\mathcal{V}) + \frac{f(\tau_i)}{\mathcal{V}^{\alpha}}, \tag{5.78}$$

with  $\alpha < 1$ , as in a classical, large volume limit there will be metric components whose correction dominates the tree-level term. Corrections that are allowed (with  $\alpha > 1$ ) contribute terms at most of order  $\frac{1}{\nu^3}$  to the scalar potential.

It is often said that, because the Kähler potential is non-holomorphic, there is no control over its form. The arguments above show that in certain cases we may restrict the form of possible corrections without being able to do a quantum computation. This is due to the existence of a classical limit in which quantum corrections should be sub-dominant to the tree-level terms.

This justifies the neglect of higher perturbative and non-perturbative corrections to the Kähler potential. The underlying reason why these can be neglected is the exponentially large volume: the  $\alpha'$  expansion is an inverse volume, and  $\mathcal{V} \sim 10^{15}$  (for example) implies that this expansion can be controlled.

# 5.3 Moduli Spectroscopy

We have shown above both that  $\alpha'$  corrections must generally be included to study Kähler moduli stabilisation, and that at large volume we only need include the

leading corrections of [90]. We now commence our study of the phenomenological properties of the models developed above by computing the moduli and modulino spectra. It is easy (by small adjustments of  $g_sN$ ) to stabilise the overall volume at essentially any value. Our primary interest is therefore in the scaling behaviour of the masses with the internal volume.

We focus on the 2-modulus  $\mathbb{P}^4_{[1,1,1,6,9]}$  model. As described above, in the limit where  $\tau_5 \gg \tau_4 > 1$ , the scalar potential takes the form

$$V = \frac{\lambda\sqrt{\tau_4}(a_4A_4)^2 e^{-2\frac{a_4\tau_4}{g_s}}}{\mathcal{V}} - \frac{\mu W_0(a_4A_4)\tau_4 e^{-\frac{a_4\tau_4}{g_s}}}{\mathcal{V}^2} + \frac{\nu\xi W_0^2}{\mathcal{V}^3},\tag{5.79}$$

where  $\lambda$ ,  $\mu$  and  $\nu$  are model-dependent numerical constants.

#### 5.3.1 Bosonic Fields

We set  $\hbar = c = 1$  but will otherwise be pedantic on frames and factors of  $2\pi$  and  $\alpha'$ . Our basic length will be  $l_s = 2\pi\sqrt{\alpha'}$  and our basic mass  $m_s = \frac{1}{l_s}$ . These represent the only dimensionful scales, and unless specified otherwise volumes are measured in units of  $l_s$ . We will furthermore require that at the minimum the 4-dimensional metric is the metric the string worldsheet couples to.

We give notice here that this section unavoidably contains one messy point. This involves the relations between string and Einstein frame volumes. The origin of the messiness is that it is the string frame volume that is the physical volume as seen by the string and which determines the validity of string perturbation theory. However, it is the Einstein frame volume - which includes factors of  $g_s$  - which appears in the supergravity Kähler potential and in defining the moduli.

We now study the mass spectrum. Stringy excitations have

$$m_S^2 = \frac{n}{\alpha'} \Rightarrow m_S \sim 2\pi m_s. \tag{5.80}$$

To estimate Kaluza-Klein masses we recall toroidal compactifications. A stringy ground state of Kaluza-Klein and winding integers n and w has mass

$$m_{KK}^2 = \frac{n^2}{R^2} + \frac{w^2 R^2}{\alpha'^2},\tag{5.81}$$

where R is the dimensionful Kaluza-Klein radius. Strictly (5.81) only holds for toroidal compactifications, but it should suffice to estimate the relevant mass scale. If we write  $R = R_s l_s$  and assume  $R_s \gg 1$ , we have

$$m_{KK} \sim \frac{m_s}{R_s}$$
 and  $m_W \sim (2\pi)^2 R_s m_s$ . (5.82)

It is conceivable that the geometry of the internal space is elongated such that the Kaluza-Klein radius  $R_s$  is uncorrelated with the overall volume, giving KK

masses of order  $m_{KK}^4 \sim 1/\tau_i$  for the different cycles. However, in absence of evidence to the contrary we assume the simplest scenario in which  $(2\pi R_s)^6 = \mathcal{V}_s$ . Then

 $m_{KK} \sim \frac{2\pi m_s}{\mathcal{V}_s^{\frac{1}{6}}}.\tag{5.83}$ 

Here  $m_{KK}$  refers only to the lightest KK mode, as in our situation the overall volume is large but there are relatively small internal cycles. Therefore, while there may be many KK modes, there is a hierarchy with the others being naturally heavier than the scale of (5.83).

We next want to determine the masses of the complex structure and Kähler moduli. This requires an analytic expression for the potential in terms of canonically normalised fields. The dimensional reduction of the 10-dimensional action into this framework is carried out in more detail in section A.1 of the Appendix; here we shall just state results.

An  $\mathcal{N}=1$  supergravity is completely specified by a Kähler potential, superpotential and gauge kinetic function. Neglecting the gauge sector to focus on moduli dynamics, the action is

$$S_{\mathcal{N}=1} = \int d^4x \sqrt{-G} \left[ \frac{M_P^2}{2} \mathcal{R} - \mathcal{K}_{i\bar{j}} D_\mu \phi^i D^\mu \bar{\phi}^j - V(\phi, \bar{\phi}) \right], \tag{5.84}$$

where

$$V(\phi, \bar{\phi}) = e^{\mathcal{K}/M_P^2} \left( \mathcal{K}^{i\bar{j}} D_i \hat{W} D_{\bar{j}} \bar{\hat{W}} - \frac{3}{M_P^2} \hat{W} \bar{\hat{W}} \right) + \text{ D-terms.}$$
 (5.85)

 $\mathcal{K}$  is the Kähler potential, which has mass dimension 2, and  $\hat{W}$  the superpotential, with mass dimension 3.  $M_P$  is the reduced Planck mass  $M_P = \frac{1}{(8\pi G)^{\frac{1}{2}}} = 2.4 \times 10^{18} \text{GeV}$  and the Planck and string scales are related by

$$M_P^2 = \frac{4\pi V_s^0}{g_s^2 l_s^2}$$
 or  $m_s = \frac{g_s}{\sqrt{4\pi V_s^0}} M_P$ . (5.86)

Here  $\mathcal{V}_s^0 = \langle \mathcal{V}_s \rangle$  is the string-frame volume at the minimum.  $\mathcal{V}_s$  is defined as

$$\mathcal{V}_s = \int_X d^6 x \sqrt{\tilde{g}}.$$
 (5.87)

It measures the volume of the internal space using the metric  $\tilde{g}$  defined by

$$\tilde{g}_{MN} = e^{(\phi_0 - \phi)/2} g_{MN},$$
(5.88)

where  $g_{MN}$  defines the ten-dimensional string frame metric. The factor of  $e^{-\phi/2}$  in  $\tilde{q}$  ensures it is an Einstein-frame metric, but the factor  $e^{\phi_0/2}$  also ensures that

for the vacuum solution volumes measured with  $\tilde{g}_{MN}$  agree with those measured with  $g_{MN}$ .

As described in the Appendix, including the  $\alpha'$  and non-perturbative corrections, the Kähler and superpotentials are

$$\frac{\mathcal{K}}{M_P^2} = -2\ln\left(\mathcal{V}_s + \frac{\xi g_s^{\frac{3}{2}}}{2e^{\frac{3\phi}{2}}}\right) - \ln(S + \bar{S}) - \ln\left(-i\int_{CY} \Omega \wedge \bar{\Omega}\right),$$

$$\hat{W} = \frac{g_s^{\frac{3}{2}} M_P^3}{\sqrt{4\pi} l_s^2} \left(\int_{CY} G_3 \wedge \Omega + \sum A_i e^{\frac{-2\pi}{g_s} T_i}\right) \equiv \frac{g_s^{\frac{3}{2}} M_P^3}{\sqrt{4\pi}} W.$$
(5.89)

Here  $\xi = -\frac{\zeta(3)\chi(M)}{2(2\pi)^3}$  and  $T_i = \tau_i + ib_i$ , where  $\tau_i = \int_{\Sigma_i} d^4x \sqrt{\tilde{g}}$  is a 4-cycle volume and  $b_i$  its axionic partner arising from the RR 4-form; these are good Kähler coordinates for IIB orientifold compactifications.

The gravitino mass can be read off immediately from (5.89):

$$m_{\frac{3}{2}} = e^{\mathcal{K}/2} |\hat{W}| = \frac{g_s^2 e^{\frac{\mathcal{K}_{cs}}{2}} |W_0|}{\mathcal{V}_s^0 \sqrt{4\pi}} M_P.$$
 (5.90)

It will be useful to relate the scale of the bosonic masses to (5.90).

To calculate scalar masses, we must express the potential (5.85) in terms of canonically normalised fields. The Kähler metric for the complex structure moduli is given by

$$\mathcal{K}_{i\bar{j}} = \partial_i \partial_{\bar{j}} \mathcal{K}_{cs} = \partial_i \partial_{\bar{j}} \ln \left( -i \int_{CV} \Omega \wedge \bar{\Omega} \right). \tag{5.91}$$

In the no-scale approximation, which holds to leading order, the potential for the complex structure moduli is

$$V = \frac{g_s^4 M_P^4}{8\pi (\mathcal{V}_s^0)^2} \int d^4 x \sqrt{-g_E} e^{\mathcal{K}_{cs}} \left[ G^{a\bar{b}} D_a W D_{\bar{b}} \bar{W} \right], \tag{5.92}$$

where the sum runs over complex structure moduli only. The inverse of (5.91) is hard to make explicit, as to do so would require knowledge of all the Calabi-Yau periods. However, as (5.91) is independent of dilaton and Kähler moduli, this process will not introduce extra factors of  $\mathcal{V}_s$  or  $g_s$ . Thus if we assume numerical factors to be  $\mathcal{O}(1)$ , we find

$$m_{cs}^2 = \mathcal{O}(1) \frac{g_s^4 N^2 M_P^2}{4\pi (\mathcal{V}_s^0)^2},$$
 (5.93)

where  $N \sim \mathcal{O}(\sqrt{\frac{\chi}{24}})$  is a measure of the typical number of flux quanta and arises from the  $D_aW$  terms. We therefore have

$$m_{cs} = \mathcal{O}(1) \frac{g_s N m_s}{\sqrt{\mathcal{V}_s^0}}. (5.94)$$

As emphasised in [100], one requires a clear separation between Kaluza-Klein and complex structure masses to trust the supergravity analysis. We have

$$\frac{m_{cs}}{m_{KK}} \sim \frac{g_s N}{2\pi (\mathcal{V}_s^0)^{\frac{1}{3}}}.$$
 (5.95)

At large volumes, this ratio is much less than one, which is reassuring.

For the concrete  $\mathbb{P}^4_{[1,1,1,6,9]}$  example, the Kähler moduli may be treated more explicitly. For this model we shall give a completely explicit analysis in chapter 7. Here we focus on scaling properties, as these generalise to multi-modulus models in a way that numerical factors will not.

It is nonetheless hard to normalise the fields canonically across the entirety of moduli space. However, to compute the spectrum we only need normalise the moduli at the physical minimum. It turns out (see section A.2 of the Appendix) that the appropriately normalised fields are

$$\tau_5^c = \sqrt{\frac{3}{2}} \frac{\tau_5}{\tau_5^0} M_P, \qquad b_5^c = \sqrt{\frac{3}{2}} \frac{b_5}{\tau_5^0} M_P,$$

$$\tau_4^c = \sqrt{\frac{3}{4}} \frac{\tau_4}{(\tau_5^0)^{\frac{3}{4}} (\tau_4^0)^{\frac{1}{4}}} M_P, \qquad b_4^c = \sqrt{\frac{3}{4}} \frac{b_4}{(\tau_5^0)^{\frac{3}{4}} (\tau_4^0)^{\frac{1}{4}}} M_P. \tag{5.96}$$

Here  $\tau_5^0 = \langle \tau_5 \rangle$ , etc. The bosonic mass matrix follows by taking the second derivatives of the scalar potential with respect to  $\tau_i^c$  and  $b_i^c$ . In the vicinity of the large volume minimum, the scalar potential takes the form

$$V = g_s^4 M_P^4 \left( \frac{\lambda' \sqrt{\tau_4} e^{-2\frac{a_4 \tau_4}{g_s}}}{\tau_5^{\frac{3}{2}}} + \frac{\mu'}{\tau_5^3} \tau_4 e^{-\frac{a_4 \tau_4}{g_s}} \cos\left(\frac{a_4 b_4}{g_s}\right) + \frac{\nu'}{\tau_5^{\frac{9}{2}}} \right). \tag{5.97}$$

As discussed above we have

$$\lambda' \sim \frac{a_4^2 |A_4|^2}{g_s^2}, \quad \mu' \sim \frac{a_4 |A_4 W_0|}{g_s}, \text{ and } \nu' \sim \xi |W_0|^2.$$

The  $b_5$  axion appears only in terms suppressed by  $e^{-a_5\tau_5}$  and has not been written explicitly. As  $\tau_5^0 \gg 1$ , it follows that this field is essentially massless. In terms of the canonical fields, the scalar potential (5.97) becomes

$$V = \frac{\lambda \sqrt{\tau_4^c \beta} e^{-2a_4 \beta \tau_4^c}}{(\tau_5^0)^{\frac{3}{2}} (\tau_5^c)^{\frac{3}{2}}} + \frac{\mu \tau_4^c \beta e^{-a_4 \beta \tau_4^c} \cos(a_4 \beta b_4)}{(\tau_5^0)^3 (\tau_5^c)^3} + \frac{\nu}{(\tau_5^0)^{\frac{9}{2}} (\tau_5^c)^{\frac{9}{2}}},$$
 (5.98)

where  $\tau_4 = \beta g_s \tau_4^c$ . The mass matrix is

$$d^{2}V = \begin{pmatrix} \frac{\partial^{2}V}{\partial \tau_{5}^{c}\partial \tau_{5}^{c}} & \frac{\partial^{2}V}{\partial \tau_{5}^{c}\partial \tau_{4}^{c}} & \frac{\partial^{2}V}{\partial \tau_{5}^{c}\partial b_{4}^{c}} \\ \frac{\partial^{2}V}{\partial \tau_{4}^{c}\partial \tau_{5}^{c}} & \frac{\partial^{2}V}{\partial \tau_{4}^{c}\partial \tau_{4}^{c}} & \frac{\partial^{2}V}{\partial \tau_{4}^{c}\partial b_{4}^{c}} \\ \frac{\partial^{2}V}{\partial b_{4}^{c}\partial \tau_{5}^{c}} & \frac{\partial^{2}V}{\partial b_{4}^{c}\partial \tau_{4}^{c}} & \frac{\partial^{2}V}{\partial b_{4}^{c}\partial b_{4}^{c}} \\ \frac{\partial^{2}V}{\partial b_{4}^{c}\partial \tau_{5}^{c}} & \frac{\partial^{2}V}{\partial b_{4}^{c}\partial \tau_{4}^{c}} & \frac{\partial^{2}V}{\partial b_{4}^{c}\partial b_{4}^{c}} \end{pmatrix}.$$
 (5.99)

At the minimum this mass matrix takes the schematic form

$$\frac{M_P^2 g_s^4}{(\mathcal{V}_s^0)^2} \begin{pmatrix} \frac{\frac{a}{\mathcal{V}_s^0}}{\frac{b}{(\mathcal{V}_s^0)^{\frac{1}{2}} g_s}} & 0\\ \frac{b}{(\mathcal{V}_s^0)^{\frac{1}{2}} g_s} & \frac{c}{g_s^2} & 0\\ 0 & 0 & d \end{pmatrix}.$$

Cross terms involving the axion decouple and we obtain

$$m_{\tau_5^c} = \mathcal{O}(1) \frac{g_s^2 W_0}{\sqrt{4\pi} (\mathcal{V}_s^0)^{\frac{3}{2}}} M_P, \qquad m_{b_5^c} \sim \exp(-\tau_5^0) M_P,$$

$$m_{\tau_4^c} = \mathcal{O}(1) \frac{a_4 g_s W_0}{\sqrt{4\pi} \mathcal{V}_s^0} M_P, \qquad m_{b_4^c} = \mathcal{O}(1) \frac{a_4 g_s W_0}{\sqrt{4\pi} \mathcal{V}_s^0} M_P. \tag{5.100}$$

This division of scales between the large modulus ( $\tau_5$ ) and the small modulus ( $\tau_4$ ) is a general feature of these models. The  $\mathcal{O}(1)$  factors depend on the detailed geometry of the particular Calabi-Yau and are therefore not written explicitly, although given the Kähler potential their numerical computation is straightforward.

Comparing formulae (5.100) with (5.90) suggests the existence of a small hierarchy between the small modulus  $\tau_4$  and the gravitino mass  $m_{3/2}$  set by  $\frac{a_4}{g_s}$ . The above analysis has focused on the scaling of masses with volume  $\mathcal{V}_s^0$ . We will see in chapter 7 that a precise treatment indeed gives a small hierarchy between  $m_{\tau_4}$  and  $m_{3/2}$ , which is in fact set by  $\left(\frac{a_4\tau_4}{g_s}\right) \sim \ln\left(\frac{M_P}{m_{3/2}}\right)$ .

$$\frac{m_{\tau_4}}{m_{3/2}} \sim \frac{a_4 \tau_4}{g_s} \sim \ln\left(\frac{M_P}{m_{3/2}}\right).$$

The  $\tau_4$  and  $b_4$  moduli have masses similar to - indeed marginally heavier than - the dilaton and complex structure moduli. We may worry that our above treatment was inconsistent, as we first integrated out complex structure moduli and only then considered Kähler moduli. However, as argued in section 5.1, we see from the form of the full potential that the solution derived by first integrating out the complex structure moduli remains a minimum of the full potential. Physically, the reason why we can get away with 'integrating out' light fields is that the two sectors - Kähler and complex structure moduli - decouple, as is evident from the Kähler potential (5.89).

#### 5.3.2 Fermionic Fields

The fermions divide into the gravitino and the fermionic partners of the chiral superfields. The gravitino mass is given by

$$m_{\frac{3}{2}} = e^{\mathcal{K}/2} |\hat{W}| = \frac{g_s^2 e^{\frac{\mathcal{K}_{cs}}{2}} |W_0|}{\mathcal{V}_s^0 \sqrt{4\pi}} M_P.$$
 (5.101)

with K and  $\hat{W}$  given by (5.89).

We use expressions appropriate for V=0 and so assume we have included lifting terms to restore a Minkowski minimum. The mass matrix for the other fermions is then  $[M_{\psi}]_{ij}\bar{\psi}_{Li}\psi_{Lj}$ , where  $[M_{\psi}]_{ij}=\sum_{n=1}^{4}[M_{\psi}^{n}]_{ij}$ , with

$$\begin{aligned}
\left[M_{\psi}^{1}\right]_{ij} &= -e^{\mathcal{K}/2}|\hat{W}|\left\{\mathcal{K}_{ij} + \frac{1}{3}\mathcal{K}_{i}\mathcal{K}_{j}\right\}, \\
\left[M_{\psi}^{2}\right]_{ij} &= -e^{\mathcal{K}/2}|\hat{W}|\left\{\frac{\mathcal{K}_{i}\hat{W}_{j} + \mathcal{K}_{j}\hat{W}_{i}}{3\hat{W}} - 2\frac{\hat{W}_{i}\hat{W}_{j}}{3\hat{W}^{2}}\right\}, \\
\left[M_{\psi}^{3}\right]_{ij} &= -e^{\mathcal{K}/2}\sqrt{\frac{\hat{W}}{\hat{W}}}\hat{W}_{ij}, \\
\left[M_{\psi}^{4}\right]_{ij} &= e^{\mathcal{G}/2}\mathcal{G}_{l}(\mathcal{G}^{-1})_{k}^{l}\mathcal{G}_{ij}^{k}, \\
\end{aligned} (5.102)$$

where  $\mathcal{G} = \mathcal{K} + \ln(\hat{W}) + \ln(\hat{W})$ ,  $\hat{W}_i = \partial_i \hat{W}$ ,  $\mathcal{K}_i = \partial_i \mathcal{K}$ ,  $\mathcal{G}^l = \partial_{\bar{l}} \mathcal{G}$ , etc. Here derivatives are with respect to the canonically normalised fields (5.96).

There is one massless fermion corresponding to the goldstino, eaten by the gravitino in the superHiggs effect. This corresponds to the fermionic partner of the field breaking supersymmetry, which is essentially  $\tilde{\tau}_5$ , although there is some small mixing with  $\tilde{\tau}_4$ . The mass of  $\tilde{\tau}_4$  can be estimated from (5.102); we find

$$m_{\tilde{\tau}_4} \approx \frac{g_s^2 a_4 W_0}{\mathcal{V}_s} M_P \approx m_{\tau_4} \approx m_{3/2}.$$
 (5.103)

As with the bosonic spectrum, it is hard to obtain explicit expressions for modulino masses for the complex structure moduli. However, there is no explicit volume dependence in  $\mathcal{K}_{cs}$  or  $W_{flux}$ , and so the volume dependence of  $m_{\tilde{\phi}}$  is determined by the  $e^{\frac{\mathcal{K}}{2}}$  terms. Therefore

$$m_{\tilde{\phi}} \sim \frac{g_s^2 W_0}{\mathcal{V}_s} M_P, \qquad m_{\tilde{\tau}} \sim \frac{g_s^2 W_0}{\mathcal{V}_s} M_P.$$
 (5.104)

and modulino masses have a scale set by the gravitino mass. Thus, as expected,  $m_{3/2}$  determines the scale of Bose-Fermi splitting. As at large volume  $m_{3/2} \ll m_s, m_{KK}$ , the moduli and modulino physics should decouple from that associated with stringy or Kaluza-Klein modes.

### 5.3.3 Properties of the Spectrum

At large volume, the single most important factor in determining the moduli masses is the stabilised internal volume. The different scales are suppressed compared to the 4-dimensional Planck scale by various powers of the internal

Scale	Mass	
4-dimensional Planck mass	$\frac{4\pi \mathcal{V}_s^0}{g_s} m_s = M_P$	
String scale $m_s$	$m_s = \frac{g_s}{\sqrt{4\pi\mathcal{V}_s^0}} M_P$	
Stringy modes $m_S$	$2\pi m_s = \frac{g_s \sqrt{\pi}}{\sqrt{\mathcal{V}_s^0}} M_P$	
Kaluza-Klein modes $m_{KK}$	$\frac{2\pi}{(\mathcal{V}_s^0)^{\frac{1}{6}}} m_s = \frac{g_s \sqrt{\pi}}{(\mathcal{V}_s^0)^{\frac{2}{3}}} M_P$	
Gravitino $m_{3/2}$	$\frac{g_s W_0}{\sqrt{\mathcal{V}_s^0}} m_s = \frac{g_s^2 W_0}{\sqrt{4\pi} \mathcal{V}_s^0} M_P$	
Dilaton-axion $m_{\tau}$	$\frac{g_s N}{\sqrt{\mathcal{V}_s^0}} m_s = \frac{g_s^2 N}{\sqrt{4\pi} \mathcal{V}_s^0} M_P$	
Complex structure moduli $m_{\phi}$	$\frac{g_s N}{\sqrt{\mathcal{V}_s^0}} m_s = \frac{g_s^2 N}{\sqrt{4\pi} \mathcal{V}_s^0} M_P$	
'Small' Kähler modulus $m_{ au_4}, m_{ ilde{ au}_4}$	$\frac{a_4 \tau_4 W_0}{\sqrt{v_s^0}} m_s = \frac{a_4 \tau_4 g_s W_0}{\sqrt{4\pi} v_s^0} M_P$	
Modulinos $m_{\tilde{\tau}}, m_{\tilde{\phi}}$	$\frac{g_s W_0}{\sqrt{\mathcal{V}_s^0}} m_s = \frac{g_s^2 W_0}{\sqrt{4\pi} \mathcal{V}_s^0} M_P$	
'Large' volume modulus $m_{ au_5}$	$\frac{g_s \dot{W}_0}{\mathcal{V}_s^0} m_s = \frac{g_s^2 W_0}{\sqrt{4\pi} (\mathcal{V}_s^0)^{\frac{3}{2}}} M_P$	
Volume axion $m_{b_5}$	$\exp(-(\mathcal{V}_s^0)^{\frac{2}{3}})M_P \sim 0$	

Table 5.1: Moduli spectrum for  $\mathbb{P}^4_{[1,1,1,6,9]}$  in terms of  $\mathcal{V}^0_s = \langle \mathcal{V}_s \rangle$ 

volume. In table 5.1 we show this scaling explicitly for the various moduli. There are also model-dependent  $\mathcal{O}(1)$  factors, which we do not show explicitly.

This spectrum has several characteristic features. First, the string scale is hierarchically smaller than the Planck scale. The internal volume depends exponentially on the inverse string coupling and thus very small changes in the stabilised dilaton lead to large effects in the compact space. The above construction therefore represents the first stringy construction of large extra dimensions.<sup>6</sup> We note that in this framework the extra dimensions are isotropic, with all six dimensions having comparable radii.

The majority of moduli masses are stabilised at a high scale  $\mathcal{O}\left(\frac{M_P}{\mathcal{V}_v^0}\right)$ , comparable to  $m_{\frac{3}{2}}$  but below the scale of stringy and Kaluza-Klein modes. There are two light moduli; the radion and its associated axion. The latter has an extremely small mass that is light in any units. As an axion, one would like to use this as a solution to the strong CP problem. Unfortunately this axion corresponds precisely to the D7 gauge theory with gauge coupling determined by the inverse size of the large 4-cycle. As this is extremely small, we do not expect the Standard Model to live on such branes. It may be possible to use axions associated with

<sup>&</sup>lt;sup>6</sup>It is good to keep in mind that 'large' does not mean macroscopic. With six extra dimensions and  $m_s \sim 1 \text{TeV}$ , the radii of the extra dimensions are given by  $r \sim \mathcal{V}^{\frac{1}{6}} m_s^{-1} \sim 10^{-14} \text{m}$ .

Scale	Mass	GUT	Intermediate	${ m TeV}$
$M_P$	$M_P$	$2.4 \times 10^{18} \text{ GeV}$	$2.4 \times 10^{18} \; \mathrm{GeV}$	$2.4 \times 10^{18} \; \mathrm{GeV}$
$m_s$	$\frac{g_s}{\sqrt{4\pi\mathcal{V}_s^0}}M_P$	$1.0 \times 10^{15} \text{ GeV}$	$1.0 \times 10^{12} \text{ GeV}$	$1.0 \times 10^3 \text{ GeV}$
$m_S$	$2\pi m_s = \frac{g_s \sqrt{\pi}}{\sqrt{\mathcal{V}_s^0}} M_P$	$6 \times 10^{15} \text{ GeV}$	$6 \times 10^{12} \text{ GeV}$	$6 \times 10^3 \text{ GeV}$
$m_{KK}$	$\frac{2\pi m_s}{(\mathcal{V}_s^0)^{\frac{1}{6}}} = \frac{g_s\sqrt{\pi}}{(\mathcal{V}_s^0)^{\frac{2}{3}}} M_P$	$1.5 \times 10^{15} \text{ GeV}$	$1.5 \times 10^{11} \text{ GeV}$	$0.15~{\rm GeV}$
$m_{3/2}$	$\frac{g_s^2 W_0}{\sqrt{4\pi} \mathcal{V}_s^0} M_P$	$1.5 \times 10^{12} \text{ GeV}$	$1.5 \times 10^6 \mathrm{GeV}$	$1.5\times 10^{-12} \mathrm{GeV}$
$m_{ au}$	$\frac{g_s N m_s}{\sqrt{\mathcal{V}_s^0}} = \frac{g_s^2 N}{\sqrt{4\pi} \mathcal{V}_s^0} M_P$	$1.5 \times 10^{12} \text{ GeV}$	$1.5 \times 10^6 \text{GeV}$	$1.5 \times 10^{-12} \mathrm{GeV}$
$m_{cs}$	$\frac{g_s N m_s}{\sqrt{\mathcal{V}_s^0}} = \frac{g_s^2 N}{\sqrt{4\pi} \mathcal{V}_s^0} M_P$	$1.5 \times 10^{12} \text{ GeV}$	$1.5 \times 10^6 \text{GeV}$	$1.5 \times 10^{-12} \mathrm{GeV}$
$m_{ au_4}, m_{b_4}$	$\frac{a_4 g_s W_0}{\sqrt{4\pi} \mathcal{V}_o^0} M_P$	$7.5 \times 10^{12} \; { m GeV}$	$1.5 \times 10^7 \text{GeV}$	$1.5 \times 10^{-10} \mathrm{GeV}$
$m_{ au_5}$	$\frac{g_s^2 W_0}{\sqrt{4\pi}(\mathcal{V}_s^0)^{\frac{3}{2}}} M_P$	$2.2 \times 10^{10} \text{ GeV}$	$22 \; \mathrm{GeV}$	$2.2 \times 10^{-26} \text{ GeV}$
$m_{b_5}$	$\exp(-a_5\tau_5)M_P \sim 0$	$\sim 10^{-300} \; {\rm GeV}$	$\exp(-10^6) \text{ GeV}$	$\exp(-10^{18}) \text{ GeV}$

Table 5.2: Moduli spectra for typical GUT, intermediate and TeV string scales

a small cycle as a QCD axion: we shall develop this idea further in chapter 7. The radion mass is hierarchically lighter than the gravitino mass: this may have cosmological implications which it would also be interesting to explore further.

The principal factor entering the scales is the internal volume, and we present in table 5.2 possible spectra arising from various choices of the internal volume. We reemphasise here that the reason we can talk about 'choices' of the volume is that its stabilised value is exponentially sensitive to  $\mathcal{O}(1)$  parameters such as the string coupling, and thus it may be dialled freely from the Planck to TeV scales. The moduli spectra are shown for GUT, intermediate and TeV string scales. If the fundamental scale is at the GUT scale, all moduli are heavy with the exception of the light  $b_5$  axion. For the intermediate scale, the volume modulus  $\tau_5$  is relatively light. Its mass does not present a problem with fifth force experiments but may be problematic in a cosmological context [101, 102]. Finally, for TeV scale gravity all moduli are very light. In particular,  $\tau_5$  is now so light  $(10^{-17} \text{ eV})$  as to be in conflict with fifth force experiments. It would then be difficult to realise this scenario unless either observable matter did not couple to  $\tau_5$  or its mass received extra corrections.

Overall, the clearest feature of the spectra in tables 5.1 and 5.2 is the presence of significant hierarchies governed by the stabilised volume. It is a very interesting that a single quantity (the overall volume) is capable of generating such large hierarchies. In addition to the large hierarchies, there is also a small hierarchy between  $m_{\tau_4}$  and  $m_{3/2}$ . We shall discuss this small hierarchy in more

detail in chapter 6.

## 5.4 On The Validity of Effective Field Theory

Given these results, we can return and check that the four-dimensional effective field theory is self-consistent. Our use of an  $\mathcal{N}=1$  supergravity framework should be valid so long as there is a separation of scales in which the 4-dimensional physics decouples from the high-energy physics. Let us enumerate the consistency conditions that a candidate minimum must satisfy in order to trust the formalism. First,

$$\langle \mathcal{V}_s \rangle = \langle \mathcal{V}_E g_s^{\frac{3}{2}} \rangle \gg 1.$$
 (5.105)

To control the  $\alpha'$  expansion, the string-frame compactification volume must be much greater than unity. It is important to keep track of frames here; this condition is often incorrectly stated with an Einstein frame  $(g_{\mu\nu,s} = e^{\phi/2}g_{\mu\nu,E})$  volume. However, in Einstein frame the  $\alpha'$  corrections come with inverse powers of  $g_s$  (as in (4.24)) and thus the string frame volume is the correct measure of the reliability of the perturbative expansion.

Secondly, we require

$$\langle V \rangle \ll m_s^4,$$
  
 $\langle V \rangle \ll m_{KK}^4.$  (5.106)

The vacuum energy must be suppressed compared to the string and compactification scales. Otherwise, the neglect of stringy and Kaluza-Klein modes in the analysis is untrustworthy. To trust the moduli potential we likewise require the decoupling of the moduli masses, namely

$$m_{\frac{3}{2}}, m_{\phi}, m_{\tilde{\phi}} \ll m_s, m_{KK},$$
 (5.107)

where  $\phi$  and  $\tilde{\phi}$  are generic moduli and modulini.

There is one further potential constraint we wish to discuss. The  $\mathcal{N}=1$  SUGRA potential can be written as

$$V_{\mathcal{N}=1} = \underbrace{e^{\mathcal{K}} \left[ \mathcal{K}^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} \right]}_{\text{F-term energy}} - \underbrace{3e^{\mathcal{K}} |W|^2}_{\text{gravitational energy}}, \qquad (5.108)$$

and we may thus define a susy breaking energy  $m_{susy}$ , by  $m_{susy}^4 = e^{\mathcal{K}} \left[ \mathcal{K}^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} \right]$ . In no-scale models,  $m_{susy}^4$  cancels against  $3e^{\mathcal{K}} |W|^2$  to give a vanishing cosmological constant. Is it necessary to require  $m_{susy} \ll m_{KK}, m_s$ ? As  $m_{susy} \sim W_0 m_s$ , imposing this would lead to the constraints

$$W_0 \ll 1$$

$$W_0 \langle \mathcal{V}_s \rangle^{\frac{1}{3}} \ll 1. \tag{5.109}$$

Normally, there is a twofold reason for requiring  $m_{susy} \ll m_s$ . First, once susy is broken the vacuum energy is of  $\mathcal{O}(m_{susy}^4)$  and secondly, the boson-fermion mass splittings are  $\mathcal{O}(m_{susy})$ . However, neither reasons are valid here. In a no-scale model, irrespective of the value of  $m_{susy}$ , the vacuum energy vanishes. Furthermore, the boson associated with supersymmetry breaking is massless in the no-scale approximation (as the potential is flat), whereas the associated fermion is the goldstino that is eaten by the gravitino. It is the gravitino that sets the scale of Bose-Fermi splitting, but this has mass  $m_{3/2} = e^{\frac{K}{2}}|W| \sim \frac{m_{susy}}{\sqrt{\mathcal{V}}}$  at large volumes. Requiring  $m_{3/2} \ll m_{KK}, m_s$  leads to the much weaker constraint

$$W_0 \ll \langle \mathcal{V}_s \rangle^{\frac{1}{3}}.\tag{5.110}$$

Thus  $m_{susy}$  as defined above is an imaginary scale; it sets neither the scale of Bose-Fermi splitting nor the vacuum energy. Thus we shall only consider (5.105), (5.106) and (5.107) as relevant constraints. We should note that this distinction only arises in a large extra dimensions scenario. If  $\langle \mathcal{V}_s \rangle \sim \mathcal{O}(1)$ , the conditions (5.109) and (5.110) coalesce.

There is finally a consistency condition peculiar to IIB flux compactifications. If  $\tau_i$  are the divisor volumes, we require

$$\frac{a_i \tau_{i,s}}{q_s} = a_i \tau_{i,E} \gg 1. \tag{5.111}$$

This allows us to neglect multi-instanton contributions.

Let us now consider these constraints as applied to our model. As the stabilised volume is exponentially large, (5.105) is trivially satisfied. The conditions (5.106) depend on the vacuum energy at the minimum. After the de Sitter uplift, these are satisfied by construction. Before the uplift, we recall that the vacuum energy at the minimum is  $\mathcal{O}(\frac{W_0^2 M_P^4 g_s^4}{4\pi \mathcal{V}^3})$ , whereas  $m_{KK}^4 \sim \frac{\pi^2 g_s^4 M_P^4}{\mathcal{V}^3}$ . This gives a restriction

$$W_0 \ll \pi^{\frac{3}{2}} \langle \mathcal{V}_s \rangle^{\frac{1}{6}}. \tag{5.112}$$

At large volumes this is not an onerous condition to satisfy.

The particle mass constraints are likewise satisfied. The most dangerous of these is the requirement  $m_{3/2}, m_{\phi} \ll m_{KK}$ . From (5.95) we require

$$\frac{g_s N}{2\pi} \ll \langle \mathcal{V} \rangle^{\frac{1}{3}},\tag{5.113}$$

were N is a measure of the typical number of flux quanta<sup>7</sup> and can be taken to be  $\mathcal{O}(\frac{\chi}{24}) \sim 30$ . At the large volumes we work at this constraint is satisfied

<sup>&</sup>lt;sup>7</sup>Not to be confused with N measuring the rank of the hidden sector group that enters in the coefficients  $a_i$  for gaugino condensation superpotentials.

comfortably. The constraints on the divisor volumes are also satisfied at large volume, as for the 'small' divisor  $\tau_4$ ,  $a_4\tau_4 \sim \ln \mathcal{V}$  at the minimum.

Thus all consistency conditions are satisfied and we see no reason to regard the use of  $\mathcal{N}=1$  supergravity as inconsistent. An important point is that we obtain no strong constraints on the value of  $W_0$ ; this is in contrast to KKLT-type solutions, for which very small values of  $W_0$  are essential. As large values of  $W_0$ are preferred by the statistical results of Douglas and collaborators [55, 58, 61], we expect a typical solution to have large  $W_0$ . The maximum value of  $W_0$  is determined by the fluxes satisfying the tadpole conditions and can in general be  $\mathcal{O}(10-100)$ .

In KKLT constructions the constraint (5.105) leads to the requirement  $W_0 \ll 1$  and  $g_s$  not too small. To be more precise, as  $A_i e^{-\frac{a_i \tau_i}{g_s}} \sim W_0$ , this gives

$$-\frac{g_s \ln(W_0)}{a_i} \gg 1.$$

If this is satisfied then satisfying (5.106) is automatic. As in such compactifications  $\langle \mathcal{V}_s \rangle^{\frac{1}{3}} \sim \mathcal{O}(1)$ , (5.107) gives

$$\frac{g_s N}{2\pi} \ll \mathcal{O}(1).$$

This is in general hard to satisfy (as also pointed out in [100]).

# 5.5 The Many-Moduli Case

Our discussion of cycle sizes and moduli masses has so far focused on a particular model,  $\mathbb{P}^4_{[1,1,1,6,9]}$ . We now argue that the above framework will extend to more general Calabi-Yaus with  $h^{2,1} > h^{1,1}$ , so long as we assume the existence of appropriate non-perturbative superpotentials.

In the  $\mathbb{P}^4_{[1,1,1,6,9]}$  example, we found that of the two Kähler moduli one  $(\tau_5)$  was large, whereas the other  $(\tau_4)$  was stabilised at a small value. Our claim is that this behaviour will persist in the general case, with one Kähler modulus being large and all others being small. It is the one overall modulus that is responsible for the large volumes obtained.

To show this, let us write the large-volume expression for the scalar potential (5.7) as:

$$V = \sum_{i,j} \frac{C_1 e^{-a_i \tau_i - a_j \tau_j}}{\mathcal{V}} - \sum_i \frac{C_2 e^{-a_i \tau_i}}{\mathcal{V}^2} + \frac{C_3}{\mathcal{V}^3}, \tag{5.114}$$

with  $C_1 \sim \sqrt{\tau}$ ,  $C_2 \sim \tau$  and  $C_3 \sim -\chi(M)$ .  $C_3$  is therefore positive so long as  $h^{2,1} > h^{1,1}$ . We shall not quibble here over frames or the factors of  $g_s$  in the exponent as they do not affect the argument.

Let us start in a position where  $\mathcal{V} \gg 1$ , V < 0 and there are many large moduli  $\tau_i \gg 1$ . This is the limit identified above as leading to a decompactification direction with V < 0. We may investigate the behaviour of the potential as one of the  $\tau_i$  fields change. Originally, all terms non-perturbative in the large  $\tau_i$  can be neglected. As the second term is the only negative term, it wants to increase its magnitude in order to minimise the value of the potential. This will naturally reduce the value of the corresponding  $\tau_i$  to make the exponential more relevant. As long as the other large moduli are adjusted to keep  $\mathcal{V}$  constant, this reduces the value of the potential. This continues until  $e^{-a_i\tau_i} \sim \frac{1}{\mathcal{V}}$ , when the (positive) double exponential term in (5.114) becomes important and the modulus will be stabilised.

We may carry on doing this with all the large moduli, but since we are holding  $\mathcal{V}$  constant, one (combination) of the fields must remain large. This finally leaves us with a picture of a manifold with one large 4-cycle (and corresponding 2-cycle), but all other 4-cycles of size close to the string scale.

We can now identify the locus of the minimum. In the limit  $\tau_b \to \infty$  with  $\tau_{s,i}$  small, the scalar potential takes the form

$$V = e^{\mathcal{K}} \left[ \mathcal{K}^{i\bar{\jmath}} \partial_i W \partial_{\bar{\jmath}} \bar{W} + \mathcal{K}^{i\bar{\jmath}} ((\partial_i \mathcal{K}) W \partial_{\bar{\jmath}} \bar{W} + c.c.) + \frac{3\xi}{4\mathcal{V}} \right]$$
 (5.115)

We take W to be

$$W = W_0 + \sum_{s,i} A_i e^{-a_i T_i}. (5.116)$$

We may include an exponential dependence on  $\tau_b$  in W; as  $\tau_b \gg 1$  this is in any case insignificant. Now,

$$\mathcal{K}^{i\bar{\jmath}}\partial_i \mathcal{K} \propto \tau_j,$$
 (5.117)

and we may also verify that, so long as i and j both correspond to small moduli,

$$\mathcal{K}^{i\bar{\jmath}} \sim \mathcal{V}\sqrt{\tau_s'} \tag{5.118}$$

where  $\tau_s'$  is a complicated function of the  $\tau_{s,i}$  that scales linearly under  $\tau_{s,i} \to \lambda \tau_{s,i}$ . The scalar potential then takes the form

$$V = \frac{\sqrt{\tau_s'}\partial_i W \cdot \partial_j W}{\mathcal{V}} - \frac{\tau_i \cdot \partial_i W}{\mathcal{V}^2} + \frac{3\xi}{4\mathcal{V}^3}.$$
 (5.119)

If we then take the decompactification limit  $\tau_b \to \infty$  with  $\partial_i W = \frac{1}{\mathcal{V}}$  and  $\tau_i \sim \mathcal{O}(\ln \mathcal{V})$ , then the potential goes to zero from below. As this result is independent of the strength of the non-perturbative corrections, which always eventually come to dominate the positive  $\alpha'^3$  terms, there is an associated minimum at large volume.

In order to get a clearer picture let us make a brief geometric digression. The volume of a Calabi-Yau can be expressed either in terms of 2-cycles,  $\mathcal{V} = \frac{1}{6}k_{ijk}t^it^jt^k$ , or 4-cycles  $\mathcal{V} = \mathcal{V}(\tau_i)$ . It is a fact that the matrix

$$M_{ij} = \frac{\partial^2 \mathcal{V}}{\partial \tau_i \partial \tau_j} \tag{5.120}$$

has signature  $(1, h^{1,1} - 1)$  (one plus, the rest minus). This follows from the result that  $\tau_i = \tau_i(t^j)$  is simply a coordinate change on Kähler moduli space, and it is a standard result [29] that

$$M'_{ij} = \frac{\partial^2 \mathcal{V}}{\partial t^i \partial t^j} \tag{5.121}$$

has signature  $(1, h^{1,1} - 1)$ . The coordinate change cannot alter the signature of the metric.

This signature manifests itself in explicit models. In both the  $\mathbb{P}^4_{[1,1,1,6,9]}$  example studied above and an  $\mathcal{F}_{11}$  model also studied in [91], the volume may in fact be written explicitly in terms of the divisor volumes. With the  $\tau_i$  as defined in [91]<sup>8</sup>, we have

$$\mathbb{P}^{4}_{[1,1,1,6,9]} \qquad \mathcal{V} = \frac{1}{9\sqrt{2}} \left(\tau_{5}^{\frac{3}{2}} - \tau_{4}^{\frac{3}{2}}\right), 
\tau_{4} = \frac{t_{1}^{2}}{2} \text{ and } \tau_{5} = \frac{(t_{1} + 6t_{5})^{2}}{2}. 
\mathcal{F}_{11} \qquad \mathcal{V} = 2 \left(\tau_{1} + \tau_{2} + 2\tau_{3}\right)^{3/2} - \left(\tau_{2} + 2\tau_{3}\right)^{3/2} - \tau_{2}^{3/2}, 
\tau_{1} = \frac{t_{2}}{2} \left(2t_{1} + t_{2} + 4t_{3}\right), \ \tau_{2} = \frac{t_{1}^{2}}{2}, \ \text{and} \ \tau_{3} = t_{3} \left(t_{1} + t_{3}\right). (5.122)$$

For the  $\mathcal{F}_{11}$  model, we see from the expressions for  $\tau_i$  in terms of 2-cycles that it is consistent to have  $\tau_1$  large and  $\tau_2, \tau_3$  small but not otherwise. The signature of  $\mathrm{d}^2 V$  is manifest in (5.122); each expression contains  $h^{1,1}-1$  minus signs. There is another important point. In each case, there is a well-defined limit in which the overall volume goes to infinity and all but one divisors remain small. These limits are given by  $(\tau_5 \to \infty, \tau_4 \text{ constant})$  and  $(\tau_1 \to \infty, \tau_2, \tau_3 \text{ constant})$  respectively. Furthermore, in each case this limit is unique: e.g. the alternative limit  $(\tau_2 \to \infty, \tau_1, \tau_3 \text{ constant})$  is not well-defined.

This motivates a 'Swiss-cheese' picture of the Calabi-Yau, illustrated in figure 5.2. A Swiss cheese is a 3-manifold with 2-cycles. Of these 2-cycles, one  $(t_b)$  is large and controls the size of the cheese, while the others  $(t_{s,i})$  are small and control the size of the holes.<sup>9</sup> The volume of the cheese can be written

$$\mathcal{V} = t_b^{3/2} - \sum_i t_{s,i}^{3/2},\tag{5.123}$$

<sup>&</sup>lt;sup>8</sup>Note that in the notation of [91] we do not have  $\tau_i \neq \frac{\partial \mathcal{V}}{\partial t_i}$ , but rather linear combinations thereof.

<sup>&</sup>lt;sup>9</sup>This distinction between large  $t_b$  and small  $t_{s,i}$  only holds for reputable cheesemongers!

and  $\frac{\partial^2 \mathcal{V}}{\partial t_i \partial t_j}$  has signature  $(1, h^2 - 1)$ . The small cycles are internal; increasing their volume decreases the overall volume of the manifold. There is one distinguished cycle that controls the overall volume; this cycle may be made arbitrarily large while holding all other cycles small, and controls the overall volume. For all other cycles, an arbitrary increase in their volume decreases the overall volume and eventually leads to an inconsistency. The small cycles may be thought of as local blow-up effects; if the bulk cycle is large, the overall volume is largely insensitive to the size of the small cycles.

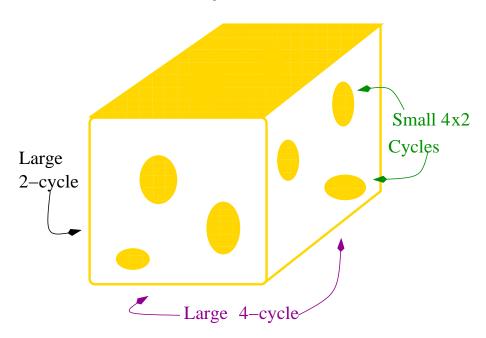


Figure 5.2: A Swiss cheese picture of a Calabi-Yau. There is one large 4-cycles - increasing this cycle volume increases the overall volume. The other cycles are such that increasing the cycle volume decreases the overall volume.

To capture the above, let us consider a Calabi-Yau with divisors  $\tau_b, \tau_{s,i}$  such that the volume can be written

$$\mathcal{V} = \left(\tau_b + \sum a_i \tau_{s,i}\right)^{\frac{3}{2}} - \left(\sum b_i \tau_{s,i}\right)^{\frac{3}{2}} - \dots - \left(\sum k_i \tau_{s,i}\right)^{\frac{3}{2}}.$$
 (5.124)

We assume that a limit  $\tau_b \gg \tau_{s,i}$  is well-defined. The minus signs are necessary to ensure (5.120) is satisfied. The form given above is valid globally for both  $\mathbb{P}^4_{[1,1,1,6,9]}$  and  $\mathcal{F}_{11}$  models. The form (5.124) is illustrative and it is not important for our argument that it hold generally; the important assumption is that there exists a well-defined limit  $\tau_b \gg \tau_{s,i}$ . The effect of the  $\alpha'$ -corrected potential is then to drive the moduli to a limit where  $\tau_b \gg 1$  and  $\tau_s \sim \ln(\tau_b)$ .

The geometric interpretation of this is that the 'external' cycle controlling the overall volume may be very large, whereas the small, 'internal' cycles are always stabilised at small volumes. As in section 5.3, the masses associated with the moduli parametrising the small cycles have a naive scaling  $m_{\tau_i} \sim \mathcal{O}(\frac{g_s^2 W_0}{\mathcal{V}})$ , comparable to the masses of the dilaton and complex structure moduli. Thus the features of the resulting spectrum are largely model-independent - in particular, the scales of moduli masses are set by the overall volume irrespective of the detailed model. As before the 'volume' modulus, which is distinguished by its large size, is relatively light, being suppressed by a factor of  $\mathcal{V}^{3/2}$ .

We caution here that the 'Swiss Cheese' picture is only a picture. The signature (5.120) is a feature of the second derivatives of the volume rather than the first derivatives. Nonetheless, it captures the outcome of the moduli stabilisation results and is a useful way to visualise and understand them.

We also comment here that in order for the above moduli stabilisation mechanism to work, we do need at least one 'blow-up' modulus that can sensibly be kept small while the overall volume is made arbitrarily large. For example, this approach cannot be made to work on tori with  $h^{1,1}=3$ . This is because the limit in which one 4-cycle is kept small while the overall volume is made extremely large is very anisotropic, with two large dimensions and four small ones. The assumptions made above are then no longer valid and the resulting structure of the scalar potential is quite different from that analysed above.

# 5.6 Comparison with KKLT Vacua

This chapter has described the construction of new large-volume non-supersymmetric minima of the scalar potential. The third part of this thesis will study phenomenological applications of this construction. Before moving on to this, let us first compare this construction with the more standard KKLT vacua

1. First, there is a sense in which the above construction completes KKLT across the entire range of moduli space. As shown in chapter 4, the regime of validity of the KKLT potential is extremely limited, being restricted both in the values of  $W_0$  allowed and the regime of moduli space over which the potential can be trusted. In particular, the KKLT potential is always invalid in the asymptotically large volume limit. This can be traced to the neglect of the perturbative corrections which dominate at large volume. By including these corrections, the resulting potential includes both the large-volume minimum discussed above and, for very small values of  $W_0$ , the KKLT minimum. Unlike KKLT, the large-volume minimum has no restrictions on the permissible values of  $W_0$ . It is a property of the large-volume minimum that  $\mathcal{V} \propto W_0$ . When  $W_0 \ll 1$  the two minima coexist,

and gradually decreasing  $W_0$  causes the two minima to approach each other and to eventually merge. This behaviour is illustrated in figures 5.3 and 5.4.

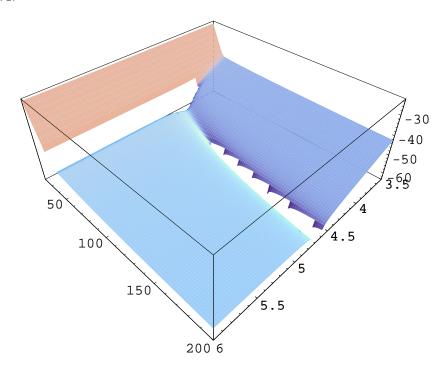


Figure 5.3: A plot of  $\ln V$ , showing the region of the scalar potential in which the large volume minimum coexists with a KKLT minimum at smaller volume. This picture is valid for very small  $W_0$  ( $\sim 10^{-10}$ )

- 2. The most obvious difference is in the volume of the extra dimensions. In KKLT the stabilised volume is always relatively small, going as  $\ln(W_0)$ , and the string and Planck scales are comparable. In the scenario constructed above, the volume is naturally exponentially large and is in fact exponentially sensitive to the stabilised dilaton. As  $m_{3/2} \sim \frac{M_P W_0}{\mathcal{V}}$ , this large volume allows a natural generation of hierarchies. In particular it has the distinct advantage that the hierarchy between the weak and Planck scales may be explained without the fine-tuning of  $W_0$  that is necessary in KKLT.
- 3. A second obvious difference is in supersymmetry breaking. In KKLT, the AdS minimum is supersymmetric. and the principal source of supersymmetry breaking terms comes from the uplift mechanism. The lifting term dominates the soft supersymmetry breaking terms in the sense that all F-terms vanish if this term is absent. In the above scenario the original minimum is already non-supersymmetric AdS, and the sources of supersymmetry breaking are primarily the Kähler moduli F-terms. This arises

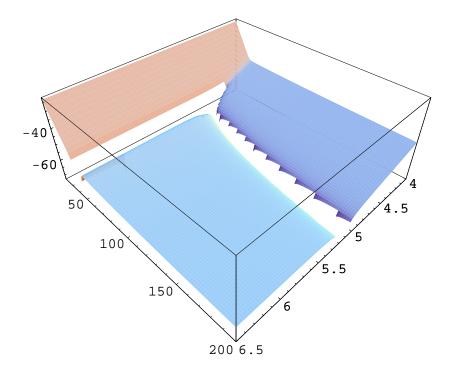


Figure 5.4: A plot of  $\ln V$ , showing how as  $W_0$  is further decreased the large volume minimum merges with the KKLT minimum.

from the underlying no-scale structure. These F-terms give the dominant contribution to soft terms irrespective of the uplift mechanism (as we shall see in chapter 6).

4. In the KKLT scenario the potential may develop tachyonic directions after fixing the Kähler moduli. This is not so for the large-volume minimum, for which the minimum of the potential is at order  $-\frac{e^{\mathcal{K}_{cs}}|W|^2(S,U)}{\mathcal{V}^3}$ . As the contribution from dilaton and complex structure F-terms is always positive and  $\mathcal{O}(\frac{1}{V^2})$ , any displacement of these moduli from  $D_iW=0$  increases the potential due to the different volume scalings present. Thus in this scenario there are no tachyonic directions in the geometric moduli. This is the main reason these models are far simpler to analyse regarding soft supersymmetry breaking. In the KKLT scenario, it is a hard problem to minimise the full potential without following the two step procedure in which dilaton and complex structure moduli are fixed by the fluxes and then integrated out. This procedure fails in many cases giving rise to tachyonic directions. This was explicitly seen in [92, 103] for the simplest cases with no complex structure moduli. For more complicated examples with several complex structure moduli minima can be found, but this is model-dependent [58, 61, 93].

The above construction of the non-supersymmetric large-volume minimum concludes Part II of this thesis. In Part III we will investigate the phenomenological properties of this scenario.

# Part III

# Applications: Towards String Phenomenology

# Chapter 6

# Soft Supersymmetry Breaking

This chapter is based on part of the paper [4] and the paper [7]. Sections 6.1 and 6.2 are based on a part of the paper [4] primarily due to Kerim Suruliz.

The second part of this thesis described the construction of a moduli potential with a minimum at exponentially large values of the internal volume. The minimum found was an AdS minimum, non-supersymmetric even before uplifting. As the minimum is non-supersymmetric, the supersymmetry is spontaneously broken. The moduli whose potential we studied are gravitationally coupled to matter fields. As matter fields are assumed to have vevs that are either vanishing or hierarchically suppressed (as with the Higgs), it was consistent to neglect them when considering the moduli potential.

If supersymmetry is realised in nature, it is realised as a broken symmetry. The breaking of supersymmetry by the moduli potential is transmitted to the (observable) matter sector through Planck-suppressed interactions - this is the 'gravity mediation' scenario. For phenemomenological purposes, we are interested in the matter Lagrangian and in particular the terms arising from supersymmetry breaking. In this context it is a well-known result that *spontaneous* breaking in the locally supersymmetric supergravity Lagrangian gives rise to *explicit* soft breaking in the globally supersymmetric matter Lagrangian. If the LHC sees low-energy supersymmetry, it is these soft breaking terms that phenomenologists will seek to extract from the data.

The problem of how to go from a spontaneously broken supergravity theory to an explicitly but softly broken supersymmetric field theory is a classic one and the general formalism has existed for some time [104, 105, 106]. While the formalism is general, the matter Lagrangian is model-dependent. In particular there are various choices as to where to embed the Standard Model and how to realise the matter content. In the context of the IIB flux compactifications considered in this thesis, the main choice is whether the Standard Model gauge

groups should be supported on D3 or D7 branes.<sup>1</sup>

We start by reviewing the general formalism before estimating the magnitude of the soft parameters for the specific large-volume models studied in the previous chapter.

## 6.1 General Formulae

## Matter-Moduli Couplings

In order to calculate soft scalar masses we need to know the matter Kähler potential. A full account of matter-moduli couplings would require a full embedding of the Standard Model and is not feasible. In general, computing matter Kähler potentials on Calabi-Yau spaces is a notoriously hard problem, the difficulty stemming from the fact that the Kähler potential is non-holomorphic and so not protected. Standard Model matter belongs to bifundamental representations and corresponds to strings stretching between brane stacks. It is not known how to compute the Kähler potential for such strings on arbitrary backgrounds even at tree-level. There do exist explicit orbifold computations [107], but these are restricted to one very special locus in moduli space.

Instead of using bifundamental matter to estimate soft terms we therefore use adjoint matter, corresponding physically to the motion of branes in the internal space or mathematically to deformations of the calibrated cycle wrapped by the brane world-volume. In this case progress has been made in a Calabi-Yau context and formulae exist for the combined matter-moduli Kähler potential for both D3 and D7 adjoint matter. As reviewed in section 2.5, in [49] the general Kähler potential for Calabi-Yau orientifolds with D3 branes was derived by a dimensional reduction. The result is

$$K(S, T, U, \phi) = -\log(S + \bar{S}) - \log\left(-i\int \Omega \wedge \bar{\Omega}\right) - 2\log(\mathcal{V}(T, U, \phi)). \tag{6.1}$$

Here U are the complex structure moduli and  $\phi^i$ , i = 1, 2, 3, are scalar fields corresponding to the positions of D3 branes on the Calabi-Yau. The Calabi-Yau volume  $\mathcal{V}$  is to be understood as a function of the complexified Kähler moduli  $T^i$ , the expression for which is:

$$T_{\alpha} = \tau_{\alpha} + i\rho_{\alpha} + i\mu_{3}l^{2}(\omega_{\alpha})_{i\bar{j}}\operatorname{Tr}\phi^{i}\left(\bar{\phi}^{\bar{j}} - \frac{i}{2}\bar{U}^{\hat{a}}(\bar{\chi}_{\hat{a}})_{l}^{\bar{j}}\phi^{l}\right).$$
 (6.2)

<sup>&</sup>lt;sup>1</sup>A detailed realisation of the Standard Model may involve more choices, but this is the most fundamental one.

 $\omega_{\alpha}$  are a basis for (1,1) forms on the Calabi-Yau surviving the orientifold projection, while the  $\chi_{\hat{a}}$  are a basis of (2,1) forms with negative sign under the orientifold projection - these correspond to the complex structure moduli  $U^{\hat{a}}$  surviving the orientifold projection. Also

$$\mu_3 = \frac{1}{(2\pi)^3 \alpha'^2}, \qquad l = 2\pi \alpha' \qquad \text{and} \qquad \mu_3 l^2 = \frac{1}{2\pi}.$$

If both D3 and D7 branes are present, the Kähler potential was derived in [51] and has the form

$$\mathcal{K}(S, T, U, \zeta, \phi) = -\log\left(S + \bar{S} + 2i\mu_7 \mathcal{L}_{AB}\zeta^A \bar{\zeta}^{\bar{B}}\right) - \log\left(-i\int\Omega \wedge \bar{\Omega}\right) - 2\log\left(\mathcal{V}\right),$$
(6.3)

where  $\mathcal{L}_{AB}$  are certain geometric quantities,  $\zeta^A$  are the moduli describing the position of the D7 brane and  $\mathcal{V}$  is understood as a function of T, U and  $\phi$ . In a similar fashion to (6.2) the dilaton modulus S undergoes a redefinition, such that the Kähler potential remains

$$\mathcal{K}(S, T, U, \zeta, \phi) = -\ln\left(\frac{2}{g_s}\right) - \ln\left(-i\int \Omega \wedge \bar{\Omega}\right) - 2\ln\left(\mathcal{V}\right). \tag{6.4}$$

We neglect in the above the possibility of Wilson line moduli.

The main point we take from these expressions is not so much the detailed form as how the matter fields appear. For D3 branes, the matter fields couple to the Kähler moduli, leading to a redefinition of  $T_{\alpha}$ . For D7 branes, the matter fields couple to the dilaton leading to a redefinition of S. This can be understood naturally from F-theory, where both the dilaton and the D7 moduli are complex structure moduli of the F-theory fourfold.

#### F-terms

We use the standard formalism for calculating soft supersymmetry breaking terms, as described for example in [108]. We proceed by expanding the Kähler potential and superpotential in terms of the visible sector fields  $\varphi$ 

$$\mathcal{K} = \hat{\mathcal{K}} + (\tilde{\mathcal{K}}_{i\bar{j}})\varphi^i\bar{\varphi}^{\bar{j}} + Z_{ij}\varphi^i\varphi^j + \cdots,$$

$$W = \hat{W} + \mu_{ij}\varphi^i\varphi^j + Y_{ijk}\varphi^i\varphi^j\varphi^k + \cdots,$$
(6.5)

where  $\hat{\mathcal{K}}$ ,  $\tilde{\mathcal{K}}_{i\bar{\jmath}}$ ,  $Z_{ij}$ ,  $\mu_{ij}$  and  $Y_{ijk}$  depend on the hidden moduli only. Hidden sector F-term supersymmetry breaking is characterised by nonvanishing expectation values for the auxiliary fields of the hidden sector chiral superfields. These F-terms are fundamental for the calculation of soft masses and may be written as

$$\bar{F}^{\bar{m}} = e^{\hat{\mathcal{K}}/2M_P^2} \hat{\mathcal{K}}^{\bar{m}n} \frac{D_n \hat{W}}{M_P^2}.$$
 (6.6)

In this and all subsequent formulae, m and n range over the hidden moduli - the dilaton, complex structure and Kähler moduli in our case. These moduli are called 'hidden' because, being uncharged, they have no gauge couplings and so cannot be seen in collider experiments. We henceforth work in Planck mass units and do not include explicit factors of  $M_P$ .

Given the F-terms, the various soft parameters can be calculated. Perhaps the simplest of all quantities to compute are the canonically normalised gaugino masses  $M_a$ . For a sector with gauge kinetic function  $f_a$ , these are

$$M_a = \frac{1}{2} (\operatorname{Re} f_a)^{-1} F^m \partial_m f_a. \tag{6.7}$$

Once we know  $f_a$  the gaugino masses can be computed directly from the moduli vevs. There is also a general formula for scalar masses. Assuming a diagonal matter field metric  $\tilde{\mathcal{K}}_{i\bar{\jmath}} = \tilde{\mathcal{K}}_i \delta_{i\bar{\jmath}}$ , the squared soft masses of canonically normalised matter fields  $\varphi^i$  can be written as

$$m_i^2 = m_{3/2}^2 + V_0 - F^m \bar{F}^{\bar{n}} \partial_m \partial_{\bar{n}} \log \tilde{\mathcal{K}}_i, \tag{6.8}$$

where  $V_0$  denotes the value of the cosmological constant. The A-terms of the normalised matter fields  $\hat{\varphi}^i$  are defined by  $A_{ijk}\hat{Y}_{ijk}\hat{\varphi}^i\hat{\varphi}^j\hat{\varphi}^k$  with

$$A_{ijk} = F^{m}(\hat{\mathcal{K}}_{m} + \partial_{m} \log Y_{ijk} - \partial_{m} \log(\tilde{\mathcal{K}}_{i}\tilde{\mathcal{K}}_{j}\tilde{\mathcal{K}}_{k})),$$

$$\hat{Y}_{ijk} = Y_{ijk} \frac{\hat{W}^{*}}{|\hat{W}|} e^{\hat{\mathcal{K}}/2} (\tilde{\mathcal{K}}_{i}\tilde{\mathcal{K}}_{j}\tilde{\mathcal{K}}_{k})^{-1/2}.$$
(6.9)

Finally, if  $Z_{ij} = \delta_{ij}Z$  and  $\mu_{ij} = \mu \delta_{ij}$ , the B-term  $\hat{\mu}B\hat{\varphi}^i\hat{\varphi}^i$  for the field  $\varphi^i$  can be written as

$$\hat{\mu}B = (\tilde{\mathcal{K}}_{i})^{-1} \left\{ \frac{\hat{W}^{*}}{|\hat{W}|} e^{\hat{\mathcal{K}}/2} \mu \left[ F^{m} (\hat{\mathcal{K}}_{m} + \partial_{m} \log \mu - 2\partial_{m} \log(\tilde{\mathcal{K}}_{i})) \right] - m_{3/2} \right] + (2m_{3/2}^{2} + V_{0})Z - m_{3/2} \bar{F}^{\bar{m}} \partial_{\bar{m}} Z + m_{3/2} F^{m} (\partial_{m} Z - 2Z\partial_{m} \log(\tilde{\mathcal{K}}_{i})) - \bar{F}^{\bar{m}} F^{n} (\partial_{\bar{m}} \partial_{n} Z - 2\partial_{\bar{m}} Z\partial_{n} \log(\tilde{\mathcal{K}}_{i})) \right\}.$$

$$(6.10)$$

where the effective  $\mu$ -term is given by

$$\hat{\mu} = \left(\frac{\hat{W}^*}{|\hat{W}|} e^{\hat{\mathcal{K}}/2} \mu + m_{3/2} Z - F^{\bar{m}} \partial_{\bar{m}} Z\right) (\tilde{\mathcal{K}}_i)^{-1}.$$
 (6.11)

<sup>&</sup>lt;sup>2</sup>Note that there is no sum over i. Furthermore,  $\tilde{\mathcal{K}}_i$  is not to be confused with the derivative of the Kähler potential with respect to modulus i.

We shall set  $V_0 = 0$  since soft masses are to be evaluated after lifting the vacuum energy.

It is important to have an idea of the approximate sizes of the various F-terms used in the computation of soft terms. We continue to work in the context of the  $\mathbb{P}^4_{[1,1,1,6,9]}$  model with two Kähler moduli  $T_4, T_5$ , although we expect our results to extend to other models in this framework (but not to KKLT-style models).

At large volume, the volume scaling of the relevant parts of the inverse metric are

$$\mathcal{K}^{\bar{T}_4 T_4} \sim \mathcal{V}, 
\mathcal{K}^{\bar{T}_4 T_5} \sim \mathcal{V}^{2/3}, 
\mathcal{K}^{\bar{T}_5 T_5} \sim \mathcal{V}^{4/3}, 
\mathcal{K}^{\bar{S} T_4} \sim \frac{1}{\mathcal{V}}, 
\mathcal{K}^{\bar{S} T_5} \sim \frac{1}{\mathcal{V}^{1/3}}.$$
(6.12)

The derivatives of the Kähler potential are

$$\partial_4 \mathcal{K} \equiv \partial_{T_4} \mathcal{K} \sim \frac{1}{\mathcal{V}}$$
 $\partial_5 \mathcal{K} \equiv \partial_{T_5} \mathcal{K} \sim \frac{\tau_5^{1/2}}{\mathcal{V}} \sim \frac{1}{\mathcal{V}^{2/3}}.$ 

Since at the minimum of the potential  $D_iW = 0$  for the dilaton and complex structure moduli, the volume dependence of the relevant F-terms is given by

$$F^{4} \sim \frac{1}{\mathcal{V}} \left( \frac{\mathcal{V}}{\mathcal{V}} + \frac{\mathcal{V}^{2/3}}{\mathcal{V}^{2/3}} \right) \sim \frac{1}{\mathcal{V}}$$

$$F^{5} \sim \frac{1}{\mathcal{V}} \left( \frac{\mathcal{V}^{4/3}}{\mathcal{V}^{2/3}} + \frac{\mathcal{V}^{2/3}}{\mathcal{V}} \right) \sim \frac{1}{\mathcal{V}^{1/3}}$$

$$F^{S} \sim \frac{1}{\mathcal{V}} \left( \frac{1}{\mathcal{V}} + \frac{1}{\mathcal{V}^{2}} \right) \sim \frac{1}{\mathcal{V}^{2}}.$$
(6.13)

We see that at large volume,

$$\mathcal{V}^{-1/3} \sim F^5 \gg F^4 \sim \mathcal{V}^{-1} \gg F^S \sim \mathcal{V}^{-2}$$
.

The F-terms corresponding to complex structure moduli vanish since for i ranging over dilaton and Kähler moduli  $\mathcal{K}^{\bar{U}i}=0$ , even after the inclusion of the  $\alpha'$  corrections of [90]. Furthermore,  $D_UW=0$  at the minimum of the scalar potential so long as we only turn on ISD fluxes. If we use IASD fluxes for the uplift, the complex structure F-terms will be nonvanishing but still suppressed, and we do not anticipate any significant modifications to the results below.

# 6.2 Soft Parameters: Basic Estimates

We now use the above formalism to estimate the magnitude of the soft parameters for matter fields living on either D3 or D7 branes.

### Soft Parameters for D3 branes

To calculate the masses of D3 moduli it is sufficient to work with a low energy theory only containing the Kähler moduli and the D3 fields. This is because ISD fluxes do not give masses to D3 moduli but do give large masses  $m \sim \mathcal{O}(m_{3/2})$  to dilaton, complex structure moduli and D7 brane moduli [31, 49, 109]. We also restrict to a single D3-brane rather than a stack, and assume for simplicity that the D3 moduli metric is diagonal. Of course all these assumptions are naive, but since we are primarily concerned with the volume scaling of the soft parameters, we hope the features we obtain survive this approximation.

Equations (6.1) and (6.2) describe how D3 moduli enter the Kähler potential. Concentrating on a single D3 modulus  $\phi$ , the Kähler potential becomes

$$\mathcal{K} = -2\log\left[ (T_5 + \bar{T}_5 - c|\phi|^2)^{3/2} - (T_4 + \bar{T}_4 - d|\phi|^2)^{3/2} + \frac{\xi'}{2} \right] + \mathcal{K}_{cs} + 2\log(36). \quad (6.14)$$

This is obtained from the original Kähler potential  $\mathcal{K} = -2\log\left(\mathcal{V} + \frac{\xi}{2}\right) + \mathcal{K}_{cs}$ . Here  $\xi' = 36\xi$  and  $c, d \sim \mathcal{O}(1)$  parametrise our ignorance of the forms  $\omega_{\alpha}$ . The factors of 36 come from  $9\sqrt{2} \times 2\sqrt{2}$  (recall  $\mathcal{V} = \frac{1}{9\sqrt{2}} \left(\tau_5^{3/2} - \tau_4^{3/2}\right)$ ).

The superpotential is

$$W = \hat{W} = \frac{g_s^2}{\sqrt{4\pi}} \left( W_0 + A_4 e^{-i\frac{a_4}{g_s}T_4} + A_5 e^{-i\frac{a_5}{g_s}T_5} \right), \tag{6.15}$$

there being no supersymmetric  $\mu$ -term for D3 brane scalars. In another simplifying assumption we also assume that W is real.

After expanding  $\mathcal{K}$  around  $\phi = 0$  we obtain the following expressions for  $\tilde{\mathcal{K}}_i$  and  $\hat{\mathcal{K}}$ :

$$\tilde{\mathcal{K}}_{i} = 3 \frac{c(T_{5} + \bar{T}_{5})^{1/2} - d(T_{4} + \bar{T}_{4})^{1/2}}{(T_{5} + \bar{T}_{5})^{3/2} - (T_{4} + \bar{T}_{4})^{3/2} + \frac{\xi'}{2}},$$

$$\hat{\mathcal{K}} = -2 \log \left[ (T_{5} + \bar{T}_{5})^{3/2} - (T_{4} + \bar{T}_{4})^{3/2} + \frac{\xi'}{2} \right].$$
(6.16)

We now introduce the variables  $X = (T_4 + \bar{T}_4)^{1/2} = \sqrt{2\tau_4}, Y = (T_5 + \bar{T}_5)^{1/2} = \sqrt{2\tau_5}$  to simplify various expressions appearing in the rest of this section.

It is important to note that in the no-scale approximation (obtained by setting  $\xi = 0$  in  $\mathcal{K}$  and  $A_4 = A_5 = 0$  in W), the nonvanishing F-terms are

 $F^4 = -e^{\hat{\mathcal{K}}/2}X^2W$ ,  $F^5 = -e^{\hat{\mathcal{K}}/2}Y^2W$  (these are derived in the Appendix). We also have, after a redefinition of c and d,

$$\log(\tilde{\mathcal{K}}_i) = \log(cX + dY) - \log(\mathcal{V}) + const. \tag{6.17}$$

It is easy to check that  $F^m \bar{F}^{\bar{n}} \partial_m \partial_{\bar{n}} \log(cX + dY) = -e^{\hat{\mathcal{K}}} |W|^2 / 2$ . Also

$$F^{m}\bar{F}^{\bar{n}}\partial_{m}\partial_{\bar{n}}\log(\mathcal{V}) = -\frac{1}{2}\hat{\mathcal{K}}_{m\bar{n}}F^{m}\bar{F}^{\bar{n}} = -\frac{3}{2}e^{\hat{\mathcal{K}}}|W|^{2}$$

$$(6.18)$$

in this approximation, so that

$$F^m \bar{F}^{\bar{n}} \partial_m \partial_{\bar{n}} \log \tilde{\mathcal{K}}_i = e^{\hat{\mathcal{K}}} |W|^2. \tag{6.19}$$

This cancels against  $m_{3/2}^2 = e^{\hat{\mathcal{K}}}|W|^2$  in the expression for soft masses (6.8) giving the no-scale result  $m_i^2 = 0$ .

Let us now estimate the size of soft scalar masses in the  $\mathbb{P}^4_{[1,1,1,6,9]}$  model. Consider the expression (6.8). The inclusion of  $\alpha'$  and nonperturbative effects alters the F-terms (through  $\hat{\mathcal{K}}$ ,  $\hat{\mathcal{K}}^{\bar{T}_i T_j}$  and  $\partial_i W$ ) and the expression for  $\tilde{\mathcal{K}}_i$ . After including nonperturbative contributions (but temporarily neglecting  $\alpha'$  corrections) the F-terms are

$$F^{4} \sim \frac{1}{\mathcal{V}} + \frac{1}{\mathcal{V}} \mathcal{K}^{T_{4}\bar{T}_{4}}(\partial_{4}W)$$

$$F^{5} \sim \frac{1}{\mathcal{V}^{1/3}} + \frac{1}{\mathcal{V}} \mathcal{K}^{T_{4}\bar{T}_{5}}(\partial_{4}W). \tag{6.20}$$

By construction, the modulus  $\tau_4$  is small at the minimum, while  $\tau_5$  is exponentially large, so we are justified in including only the nonperturbative contribution from  $\tau_4$ . Moreover,  $\tau_4$  is such that at the (AdS) minimum we have

$$-\partial_{\tau_4} W = \frac{a_4 A_4}{g_s} e^{-\frac{a_4}{g_s} \tau_4} \sim \frac{\xi^{\frac{1}{3}} |W_0|}{\langle \mathcal{V}_s \rangle}, \tag{6.21}$$

and so  $(\partial_4 W) \sim 1/\mathcal{V}$ . Therefore, we may use the expressions for the inverse metric  $\mathcal{K}^{\bar{T}_i T_j}$  given in (6.12) to write

$$F^{4} \sim \frac{1}{\mathcal{V}} + \frac{1}{\mathcal{V}},$$
  
 $F^{5} \sim \frac{1}{\mathcal{V}^{1/3}} + \frac{1}{\mathcal{V}^{4/3}}.$  (6.22)

In section 6.3 we shall see that there is actually a cancellation in  $F^4$  and the above overestimates this F-term by a factor  $\ln(\mathcal{V}) \sim \ln(m_{3/2})$ . In the expressions (6.22) the second term corresponds to the modification coming from the nonperturbative addition to the superpotential. The dominant contributions to scalar masses will

come from terms of the form  $F_{np}^m \bar{F}^{\bar{n}} \partial_m \partial_{\bar{n}} \log(\tilde{\mathcal{K}}_i)$  where  $F_{np}^m$  is the nonperturbative contribution to the F-term. Finally we have to see how  $\partial_m \partial_{\bar{n}} \log \tilde{\mathcal{K}}_i$  scales with the volume, for  $m, n \in \{4, 5\}$ . Using the explicit expressions (A.24) derived in the appendix we see that

$$\partial_4 \partial_4 \log(cX + dY) \sim \mathcal{V}^{-1/3},$$

$$\partial_4 \partial_5 \log(cX + dY) \sim \frac{1}{\mathcal{V}},$$

$$\partial_5 \partial_5 \log(cX + dY) \sim \mathcal{V}^{-4/3}.$$
(6.23)

Using the above expressions for F-terms and (6.23), we have

$$F_{np}^{4}\bar{F}^{\bar{4}}\partial_{4}\partial_{\bar{4}}\log(cX+dY) \sim \frac{1}{\nu}\frac{1}{\nu}\nu^{-1/3} = \nu^{-7/3},$$

$$F_{np}^{4}\bar{F}^{\bar{5}}\partial_{4}\partial_{\bar{5}}\log(cX+dY) \sim \frac{1}{\nu}\frac{1}{\nu^{4/3}}\nu^{-1} = \nu^{-10/3},$$

$$F_{np}^{5}\bar{F}^{\bar{5}}\partial_{5}\partial_{\bar{5}}\log(cX+dY) \sim \frac{1}{\nu^{4/3}}\frac{1}{\nu^{1/3}}\nu^{-4/3} = \nu^{-3}.$$
(6.24)

We therefore expect the masses squared to be  $\mathcal{O}(\mathcal{V}^{-7/3})$ . A similar analysis can be done for the terms of type  $F_{np}^{m}\bar{F}^{\bar{n}}\hat{K}_{m\bar{n}}$  and also including  $\alpha'$  corrections - these turn out to be subleading compared to the contribution from nonperturbative corrections to the superpotential. Explicit expressions can be found in appendix C.

Our conclusion is that the masses of D3 moduli are suppressed compared to the gravitino mass by a factor  $\mathcal{V}^{-1/6}$  (the factors of  $g_s$  and  $W_0$  also present are derived in appendix C):

$$m_i = \mathcal{O}(1) \frac{g_s^2 W_0}{\sqrt{4\pi} (\mathcal{V}_s^0)^{7/6}} \sim \frac{m_{3/2}}{\mathcal{V}^{1/6}}.$$
 (6.25)

We next examine D3 gaugino masses. The gauge kinetic function is  $f=\mu_3 l^2 S=\frac{S}{2\pi}$  and the normalised gaugino masses are

$$M_{D3} = \frac{2\pi}{2} (\text{Re}S)^{-1} F^S = \mathcal{O}\left(\frac{1}{\mathcal{V}^2}\right).$$
 (6.26)

The dilaton and Kähler moduli mix in the metric through the  $\alpha'^3$  correction.  $F^S$  must therefore be calculated using the full Kähler potential, before integrating out the dilaton and complex structure moduli.  $M_{D3}$  turns out to be proportional to  $g_s^2$  and  $W_0$  in the same way scalar masses are—this can be deduced from the  $1/g_s$  prefactor in the inverse metric  $\mathcal{K}^{\bar{S}T_i}$  (as shown in [95]) and the factor of  $g_s^{3/2}$  in W, which can be observed in (5.89).

We can also estimate the magnitude of the A-terms, which again vanish in the no-scale approximation. If we use (6.16) in the A-term expression (6.9), with constant  $Y_{ijk}$ , we obtain

$$A_{\phi\phi\phi} = e^{\hat{\mathcal{K}}/2} \left\{ -\frac{3\xi}{4\mathcal{V}} W + (\partial_4 W) \left[ \frac{X^2}{36\mathcal{V}} (2Y^3 + X^3 - \frac{\xi'}{2}) - \frac{3Y}{\mathcal{V}} X^2 Y^2 + \frac{3}{2(cX + dY)} \left( \frac{c}{3} (2Y^3 + X^3 - \xi'/2) + dX^2 Y \right) \right] \right\}.$$
(6.27)

As  $\partial_4 W \sim \mathcal{V}^{-1}$ ,  $A \sim Y^2/\mathcal{V}^2 \sim \mathcal{V}^{-4/3}$ . As with the scalar and gaugino masses, the dependence of A on  $g_s$  and  $W_0$  is given by by  $A \propto g_s^2 W_0$ .

Let us finally consider  $\mu$  and B-terms. For D3 branes, the supersymmetric  $\mu$  term vanishes, but there is an effective  $\mu$ -term generated by the Giudice-Masiero mechanism [110]. This is due to the appearance of a bilinear in the Kähler potential dependent on complex structure moduli, as follows from formulae (6.1) and (6.2). The prefactor of the bilinear  $\phi^i\phi^j$  in the expansion of the Kähler potential is

$$Z_{ij} = \frac{3\mu_3 l^2}{\mathcal{V} + \xi/2} t^{\alpha}(\omega_{\alpha})_{(j|\bar{s}}(\bar{\chi}_{\hat{a}})_{|i\rangle}^{\bar{s}} \bar{U}^{\hat{a}}.$$
 (6.28)

For simplicity we consider only one Z with  $Z = (c'X + d'Y)(a_iU^i)/(\mathcal{V} + \xi/2)$ . The complex structure moduli dependence of Z is in fact unimportant since the F-terms corresponding to these moduli are vanishing. The calculation of the effective  $\mu$ -term will be very similar to the computation of A-terms; we do not need to differentiate by U, and can absorb  $a_iU^i$  into c' and d' to write  $Z = (c'X + d'Y)/(\mathcal{V} + \xi/2)$ . Then

$$\partial_4 Z \sim \frac{1}{\mathcal{V}},$$

$$\partial_5 Z \sim \frac{1}{\mathcal{V}^{4/3}},$$
(6.29)

so that  $F^m \partial_m Z$  gives rise to terms

$$e^{\mathcal{K}/2}\mathcal{K}^{44}(\partial_4 W)\partial_4 Z \sim \mathcal{V}^{-2},$$
  
 $e^{\mathcal{K}/2}\mathcal{K}^{54}\partial_4 W\partial_5 Z \sim \mathcal{V}^{-8/3}.$  (6.30)

Assuming the complex structure moduli to be fixed at  $\mathcal{O}(1)$  values, it is easy to confirm that the  $\hat{\mu}$  term scales as  $\mathcal{O}(\mathcal{V}^{-4/3})$ .

As Z can be treated as  $(c'X + d'Y)/(\mathcal{V} + \xi/2)$  and by analogy with the calculation of the masses squared, it is easy to see that the expression for  $\hat{\mu}B$  behaves like  $\mathcal{O}(\mathcal{V}^{-7/3})$ . If we do not include anomaly mediated contributions, the dependence of D3 brane soft terms on  $\mathcal{V}, W_0$  and  $g_s$  is summarised in table 6.2 for GUT, intermediate and TeV string scales.

Scale	Mass	GUT	Intermediate	${ m TeV}$
Scalars $m_i$	$\frac{g_s^2}{(\mathcal{V}_s^0)^{7/6}} W_0 M_P$	$3.6 \times 10^{11} \text{ GeV}$	$3.6 \times 10^4 \text{GeV}$	$3.6\times10^{-17}{\rm GeV}$
Gauginos $M_{D3}$	$\frac{g_s^2}{(\mathcal{V}_s^0)^2} W_0 M_P$	$3.6 \times 10^9 \text{GeV}$	$3.6 \times 10^{-3} \text{GeV}$	$3.6 \times 10^{-39} \text{ GeV}$
A-term $A$	$\frac{g_s^2}{(\mathcal{V}_s^0)^{4/3}} W_0 M_P$	$3.2 \times 10^{11} \text{GeV}$	$3.2 \times 10^3 \text{ GeV}$	$3.2\times10^{-21}{\rm GeV}$
$\mu$ -term $\hat{\mu}$	$\frac{g_s^2}{(\mathcal{V}_s^0)^{4/3}} W_0 M_P$	$3.2 \times 10^{11} \text{GeV}$	$3.2 \times 10^3 \text{ GeV}$	$3.2 \times 10^{-21} \mathrm{GeV}$
B term $\hat{\mu}B$	$\frac{g_s^2}{(\mathcal{V}_s^0)^{7/6}} W_0 M_P$	$3.6 \times 10^{11} \text{ GeV}$	$3.6 \times 10^4 \text{GeV}$	$3.6\times10^{-17}{\rm GeV}$

Table 6.1: Soft terms for D3 branes (AMSB contributions not included)

#### Soft Parameters for D7 Branes

Standard Model matter may also be supported on D7 branes. The open string sector can give rise to several different types of moduli in the low energy theory. There are geometric moduli corresponding to deformations of the internal 4-cycle  $\Sigma$  that the D7 brane wraps. As discussed in [51], the number of such moduli is related to the (2,0) cohomology of the cycle  $\Sigma$ . There are also Wilson line moduli  $a_I$ , which are present if the cycle  $\Sigma$  possesses harmonic (1,0) forms. There are no harmonic (1,0) forms on Calabi-Yaus as  $h^{1,0} = 0$ , but they may exist when we restrict to the submanifold  $\Sigma$ . If present, these enter the Kähler potential through the complexified Kähler moduli, which are further redefined from (6.2) to

$$T_{\alpha} = \tau_{\alpha} + i\rho_{\alpha} + i\mu_{3}l^{2}(\omega_{\alpha})_{i\bar{\jmath}}\operatorname{Tr}\phi^{i}\left(\bar{\phi}^{\bar{\jmath}} - \frac{i}{2}\bar{U}^{\hat{a}}(\bar{\chi}_{\hat{a}})_{l}^{\bar{\jmath}}\phi^{l}\right) + \mu_{7}l^{2}C^{I\bar{J}}a_{I}\bar{a}_{\bar{J}}, \qquad (6.31)$$

for geometry dependent coefficients  $C^{I\bar{J}}$ . The D7 geometric moduli generically obtain large,  $\mathcal{O}(m_{3/2})$  masses after turning on fluxes. This is most easily seen from the F-theory perspective, where the D7 moduli are among the complex structure moduli of the Calabi-Yau 4-fold  $M_8$ . The relevant Gukov-Vafa-Witten superpotential is then

$$W = \int_{M_8} G_4 \wedge \Omega, \tag{6.32}$$

which generically induces a nontrivial potential for the D7 moduli. These are analogous to dilaton and complex structure moduli and we expect the  $\alpha'$  and non-perturbative effects to only have small effects.

For the case of a single geometric D7 brane modulus, the Kähler potential is

$$\mathcal{K} = -\log(S + \bar{S} - \mathcal{L}|\zeta|^2), \tag{6.33}$$

giving  $\tilde{\mathcal{K}} = \mathcal{L}/(S+\bar{S})$ . As  $F^S$  vanishes before breaking the no-scale structure, for D7 branes  $F^m \bar{F}^{\bar{n}} \partial_m \partial_{\bar{n}} \log \tilde{\mathcal{K}}$  vanishes and the flux-induced soft mass is  $\mathcal{O}(m_{3/2})$ .

Including  $\alpha'$  corrections,  $F^S$  is no longer zero and

$$F^{S}\bar{F}^{\bar{S}}\partial_{S}\partial_{\bar{S}}(-\log(S+\bar{S})) = \frac{1}{(S+\bar{S})^{2}}F^{S}\bar{F}^{\bar{S}} = \mathcal{O}\left(\frac{1}{\mathcal{V}^{4}}\right). \tag{6.34}$$

This is a manifestly tiny correction to  $m_{\zeta}$ , and so

$$m_{\zeta} \approx m_{3/2} = \mathcal{O}(1) \frac{g_s^2 W_0}{\sqrt{4\pi} \mathcal{V}_o^0} M_P.$$
 (6.35)

Note that D7 moduli receive both supersymmetric and soft-breaking masses.

The masses of Wilson line moduli for D7 branes may be found through the modified Kähler coordinates (6.31). These moduli appear in the Kähler potential in a similar fashion to D3 moduli and the calculation of their masses squared will be parallel. We therefore expect the Wilson line moduli to obtain  $\mathcal{O}(\mathcal{V}^{-7/6})$  masses due to nonperturbative effects:

$$m_{Wilson} = \mathcal{O}(1) \frac{g_s^2 W_0}{\sqrt{4\pi} (\mathcal{V}_s^0)^{7/6}} M_P.$$
 (6.36)

For D7 branes, the gauge kinetic function  $f_a$  is given by the Kähler modulus,  $T_a$ , of the cycle the D-branes wrap:

$$f_a = \frac{T_a}{2\pi}.$$

In general  $F^{T^a} \neq 0$  and the gaugino masses are nonvanishing. For the  $\mathbb{P}^4_{[1,1,1,6,9]}$  example, we have

$$M_4 = \frac{F^4}{2\tau_A} \sim \mathcal{V}^{-1},$$
 (6.37)

$$M_5 = \frac{F^5}{2\tau_5} \sim \mathcal{V}^{-1/3}/\mathcal{V}^{2/3} = \mathcal{V}^{-1}.$$
 (6.38)

In either case

$$M_{D7} = \mathcal{O}(1) \frac{g_s^2 W_0}{\sqrt{4\pi} \mathcal{V}_s^0} \sim m_{3/2}.$$
 (6.39)

As noted, in fact there is a suppression in  $F^4$ . We shall see shortly in section 6.3 that actually  $M_4 \sim \frac{m_{3/2}}{\ln(m_{3/2})}$ .

We note a crucial difference between D3 and D7 branes is that a supersymmetric  $\mu$ -term is induced for geometric D7 moduli with only ISD fluxes; in fact, it was shown in [111] that for vanishing magnetic fluxes on the D7 brane, the  $\mu$ -term corresponds to the (2,1) component of flux  $G_3$ .  $\mu$  terms do not exist for Standard Model fermion multiplets, and so in estimating the soft masses we only use the soft breaking contributions.

Scale	Mass	GUT	Intermediate	TeV
Scalars $m_{\zeta}$	$m_{3/2}$	$1.5 \times 10^{12} \text{ GeV}$	$1.5 \times 10^3 \text{GeV}$	$1.5 \times 10^{-12} \mathrm{GeV}$
Gauginos $M_4, M_5$	$m_{3/2}$	$1.5 \times 10^{12} \text{ GeV}$	$1.5 \times 10^3 \text{GeV}$	$1.5 \times 10^{-12} \mathrm{GeV}$
A-term $A$	$m_{3/2}$	$1.5 \times 10^{12} \text{ GeV}$	$1.5 \times 10^3 \text{GeV}$	$1.5 \times 10^{-12} \mathrm{GeV}$
$\mu$ -term $\hat{\mu}$	$m_{3/2}$	$1.5 \times 10^{12} \text{ GeV}$	$1.5 \times 10^3 \text{GeV}$	$1.5 \times 10^{-12} \text{GeV}$
B term $\hat{\mu}B$	$m_{3/2}$	$1.5 \times 10^{12} \text{ GeV}$	$1.5 \times 10^3 \text{GeV}$	$1.5 \times 10^{-12} \text{GeV}$

Table 6.2: Soft terms for D7 branes (AMSB not included)

Let us now consider the A-term corresponding to a D7 scalar field  $\phi$  with  $\tilde{\mathcal{K}}_i = \tilde{\mathcal{K}}_j = \tilde{\mathcal{K}}_k = \frac{\mathcal{L}}{S+S}$ . We have

$$A_{\phi\phi\phi} = F^4(\partial_4\hat{\mathcal{K}}) + F^5(\partial_5\hat{\mathcal{K}}) + F^S(\partial_S\hat{\mathcal{K}}) + \frac{3F^S}{S + \bar{S}}.$$
 (6.40)

As  $\partial_S \hat{\mathcal{K}} = -1/(S + \bar{S}) + \mathcal{O}(\mathcal{V}^{-1})$  and  $F^S \sim 1/\mathcal{V}^2$ , the largest contribution to the D7 A-term comes from  $F^5(\partial_5 \mathcal{K})$ , and so

$$A \sim F^5(\partial_5 K) \sim \frac{1}{\mathcal{V}^{1/3}} \cdot \frac{1}{\mathcal{V}^{2/3}} = \frac{1}{\mathcal{V}}.$$
 (6.41)

Thus the A-term is  $\mathcal{O}(m_{3/2})$ .

If we try and realise the Standard Model on D7 branes, there is one potential worry. The Yang-Mills gauge coupling is determined by  $g_{YM,a}^{-2} = \text{Re } f_a$ , and thus if  $f_a = T^5$  then  $g_{YM}$  would be unacceptably small. However, as shown above, in general we have  $T_b \gg 1$  but  $T_{s,i}$  relatively small. Thus so long as the Standard Model is realised on branes wrapping the smaller cycles, the resulting gauge kinetic function will have phenomenologically acceptable values. We show in table 6.2 soft masses for GUT, intermediate and TeV string scales.

#### D3-D7 States

The intersections of D3 and D7 branes can also give rise to massless open string states. Unfortunately their appearance in the Kähler potential of the low energy theory cannot be deduced from the dimensional reduction of Dirac-Born-Infeld and Chern-Simons actions for stacks for branes, and thus their soft terms are difficult to analyse from the four-dimensional point of view. In [111], the soft masses for 3-7 scalar fields due to a general flux background were computed using various symmetry arguments. It was found that no scalar or fermion masses are generated. In some particular examples where it is known how the 3-7 scalars

enter the 4D Kähler potential this can be seen from the four dimensional perspective. For example, in [112, 113] the Kähler potential was obtained for D3/D7 compactifications on  $T^2 \times T^2 \times T^2$ . For a D7-brane wrapping the first two tori, the dependence on 3-7 fields  $\phi_{37}$  is

$$\mathcal{K} = \frac{|\phi_{37}|^2}{(T_1 + T_1^*)^{1/2} (T_2 + T_2^*)^{1/2}} + \cdots$$
 (6.42)

If there is only one overall Kähler modulus  $T = T_1 = T_2 = T_3$  this becomes

$$\mathcal{K} = \frac{|\phi_{37}|^2}{T + \bar{T}} + \cdots, \tag{6.43}$$

and we obtain the same no-scale cancellation argument for  $\phi_{37}$  that applied previously for D3 matter. We therefore likewise expect that after including no-scale breaking effects,

$$m_{37} \sim m_{D3} = \mathcal{O}(1) \frac{g_s^2 W_0}{\sqrt{4\pi} (\mathcal{V}_s^0)^{7/6}}.$$
 (6.44)

## D-terms and de Sitter Uplifting

To perform a semirealistic computation of the soft supersymmetry breaking terms, we must uplift the nonsupersymmetric AdS vacuum obtained by fixing the moduli using  $\alpha'$  corrections and nonperturbative effects. This can be done in several ways. The original option, proposed in [84], involves the use of an anti-D3 brane at the bottom of a highly warped Klebanov-Strassler throat. Although the resulting uplifting term is reminiscent of a D-term in the low energy theory, supersymmetry is broken explicitly, albeit by a small amount.

An alternative method of uplift was proposed in [85]: turning on magnetic fluxes on a D7 brane wrapping a compact 4-cycle in the Calabi-Yau. The advantage of this is that the uplifting term generated can indeed be interpreted as a D-term in the low energy theory. For fine-tuning the cosmological constant, one again needs the brane to be in a highly warped region. A further mechanism was proposed in [86]: instead of using a strongly warped region, one can look for local minima of the no-scale potential with  $V_0 \neq 0$ . This will happen for certain non-ISD choices of fluxes; for example, in a model with only one Kähler modulus T, setting  $W = \int G_3 \wedge \Omega$  gives a source term for T

$$V_0 = \frac{1}{(S + \bar{S})(T + \bar{T})^3} \left| \int G_3^* \wedge \Omega \right|^2.$$
 (6.45)

If  $G_3$  contains a (3,0) component  $V_0$  will be non-vanishing. The large number of flux choices generically present in Calabi-Yau compactifications suggests there

should exist non-supersymmetric minima with sufficiently small  $\int G_3^* \wedge \Omega$ . The existence of non-supersymmetric minima of the flux ensemble has been investigated statistically in [61].

Irrespective of the details of the uplift mechanism, there will be extra contributions to scalar masses. For concreteness let us use an uplift appropriate to IASD fluxes

$$V_{uplift} = \frac{\epsilon}{\mathcal{V}^2} = \frac{\epsilon}{[(T_5 + \bar{T}_5 - c|\phi|^2)^{3/2} - (T_4 + \bar{T}_4 - d|\phi|^2)^{3/2}]^2},$$
 (6.46)

expressed in terms of the complexified Kähler moduli  $T_4$  and  $T_5$ . This may be expanded around  $\phi = 0$  as

$$V_{uplift} = \frac{\epsilon}{\left[ (T_5 + \bar{T}_5)^{3/2} - (T_4 + \bar{T}_4)^{3/2} \right]^2} \left( 1 + \frac{3|\phi|^2 (c(T_5 + \bar{T}_5)^{3/2} - d(T_4 + \bar{T}_4)^{3/2})}{(T_5 + \bar{T}_5)^{3/2} - (T_4 + \bar{T}_4)^{3/2}} \right). \tag{6.47}$$

Ignoring  $\alpha'$  corrections, the coefficient of  $|\phi|^2$  in the brackets can be identified with the kinetic term prefactor for the  $\phi$  fields (6.16). The uplift gives a contribution  $\epsilon/\mathcal{V}^2$  to scalar masses squared. To uplift to Minkowski space we require  $\langle V_{uplift} \rangle = \langle V_{min} \rangle \sim \mathcal{O}(1/\mathcal{V}^3)$  and thus  $\epsilon \sim 1/\mathcal{V}$ . Consequently the uplift contribution to scalar masses squared is  $\mathcal{O}(1/\mathcal{V}^3)$ , which is much smaller than the  $\mathcal{O}(\mathcal{V}^{-7/3})$  contribution obtained from nonperturbative and  $\alpha'$  effects.

If IASD fluxes are present, we no longer have  $F^U = 0$ . However, if the IASD fluxes are to be used as a lifting term, we can estimate the magnitude of the complex structure F-terms. We require

$$\frac{e^{\mathcal{K}_{cs}}\mathcal{K}_{cs}^{\bar{\imath}\jmath}(D_{\bar{\imath}}W)(D_{j}W)}{\mathcal{V}^{2}} \sim \frac{1}{\mathcal{V}^{3}},$$

and so  $D_iW \sim \mathcal{V}^{-1/2}$ . It then follows that  $F^U \sim \mathcal{V}^{-\frac{3}{2}}$  and the resulting effect on the masses is subleading to the F-terms associated with the Kähler moduli.

The conclusion is that the uplift has negligible effect on the soft terms. This result is in fact intuitive. Because of the underlying no-scale structure of the scalar potential, the magnitude of the potential is far smaller than would be naively expected from the size of the F-terms. In particular,  $V \sim \mathcal{V}^{-3}$  while  $|F|^2 \sim \mathcal{V}^{-2}$ . Thus the 'extra' supersymmetry breaking that must be introduced to uplift to de Sitter space is much smaller than the susy breaking already present in the AdS vacuum, and so does not alter the soft terms already computed for the AdS minimum.

# Comparison with Anomaly-Mediated Contributions

In a general model with hidden sector supersymmetry breaking, scalar masses, gaugino masses and A-terms are generated through loop effects as a consequence

of the super-Weyl anomaly [114, 115]. Assuming that soft terms are generated solely through anomaly mediation, their values are

$$M = \frac{\beta_g}{g} m_{3/2},$$

$$m_i^2 = -\frac{1}{4} \left( \frac{\partial \gamma_i}{\partial g} \beta_g + \frac{\partial \gamma_i}{\partial y} \beta_y \right) m_{3/2}^2,$$

$$A_y = -\frac{\beta_y}{y} m_{3/2},$$
(6.48)

for gaugino mass M and scalar masses  $m_i$ . Here  $\beta$  are the relevant  $\beta$  functions,  $\gamma_i$  the anomalous dimensions of the chiral superfields, and g and y respectively denote the gauge and Yukawa couplings, respectively. One can alternatively use the compensator field formalism to write

$$M = \frac{bg^{2}}{8\pi^{2}} \frac{F^{C}}{C_{0}},$$

$$A_{ijk} = -\frac{\gamma_{i} + \gamma_{j} + \gamma_{k}}{16\pi^{2}} \frac{F^{C}}{C_{0}},$$

$$m_{i}^{2} = \frac{1}{32\pi^{2}} \frac{d\gamma_{i}}{d\log\mu} \left| \frac{F^{C}}{C_{0}} \right|^{2} + \frac{1}{16\pi^{2}} \left( \gamma_{m}^{i} F^{m} \left( \frac{F^{C}}{C_{0}} \right)^{*} \right), \qquad (6.49)$$

where C is the compensator field and  $\gamma_m^i \equiv \partial_m \gamma^i$ . In this case  $F^C/C_0 \sim m_{3/2}$ . The contribution to scalar masses from anomaly mediation is then (see [97])

$$m \sim b_0 \left(\frac{g^2}{16\pi^2}\right) m_{3/2},$$
 (6.50)

where  $b_0$  is the one loop beta function coefficient. Note that (6.50) is suppressed with respect to the gravitino mass by a factor  $1/(16\pi^2)$ . In a similar fashion, the gaugino masses are suppressed compared to  $m_{3/2}$  by  $1/(8\pi^2)$ . For D3 branes, the large volume suppression in the gaugino mass formula (6.26) implies that anomaly mediation ought to dominate gaugino mass generation. As the D3 scalar masses are also suppressed compared to the gravitino mass in (6.25), the anomaly-mediated contribution will also be important at large volume. A full phenomenological analysis of the D3-brane scenario should therefore include anomaly-mediated contributions.

For D7 branes, a naive application of the above results suggests that anomaly mediation will be irrelevant as all soft masses are comparable to the gravitino mass. However we shall now see that this is not the case as gaugino masses are in fact suppressed compared to the gravitino mass. In this case anomaly mediation contributes to gaugino masses but not scalar masses (which will have  $m_i^2 \sim m_{3/2}^2$ ).

We now justify these claims by analysing the fine structure of the soft terms.

# 6.3 Soft Parameters: Fine Structure

This thesis has mostly focused on the large-volume models developed in the previus chapter. However we now show a more general result, namely that whenever a modulus  $T_a$  in the IIB landscape is stabilised by nonperturbative effects there is a small hierarchy between the masses of the gravitino  $m_{3/2}$  and the associated D7 gaugino  $M_a$ ,

$$M_a^2 \sim \frac{m_{3/2}^2}{\ln(m_{3/2})^2}.$$
 (6.51)

The large-volume models of chapter 5 are a particular case of this. This small hierarchy was first identified in single-modulus KKLT models [92] (with a two-modulus example also studied in [116]). For the KKLT case this suppression of  $M_a$  leads to mixed modulus-anomaly mediation and the phenomenology of this scenario has been analysed in [117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127].<sup>3</sup> Here we show this relation to be a truly generic feature of the landscape, by showing it to hold for arbitrary multi-modulus models and to be independent of the precise details of the scenario used to stabilise the moduli.

In section 6.4.3 I also demonstrate soft scalar mass universality in the large volume scenario and estimate the fractional non-universality to be  $1/\ln(m_{3/2})^2 \sim 1/1000$ . This is an interesting result as flavour non-universality is usually a serious problem for gravity-mediated models. I briefly discuss the phenomenology but do not anticipate the detailed discussion that will appear in [128].

# Gaugino Masses

For IIB flux compactifications the appropriate Kähler potential and superpotential are given by

$$\mathcal{K} = -2\ln\left(\mathcal{V} + \frac{\hat{\xi}}{2}\right),$$

$$W = W_0 + \sum A_i e^{-a_i T_i}.$$
(6.52)

Here  $\mathcal{V} = \frac{1}{6}k_{ijk}t^it^jt^k$  is the Calabi-Yau volume,  $\hat{\xi} = -\frac{\chi(M)\zeta(3)}{2(2\pi)^3g_s^{3/2}}$  and  $T_i = \tau_i + ib_i$  are the Kähler moduli, corresponding to the volume  $\tau_i = \frac{\partial \mathcal{V}}{\partial t^i}$  of a 4-cycle  $\Sigma_i$ , complexified by the axion  $b_i = \int_{\Sigma_i} C_4$ .  $W_0 = \langle \int G_3 \wedge \Omega \rangle$  is the flux-induced superpotential that is constant after integrating out dilaton and complex structure moduli. In general there does not exist an explicit expression for  $\mathcal{V}$  in terms of the  $T_i$ . We

<sup>&</sup>lt;sup>3</sup>In these references this hierarchy is present for all soft supersymmetry breaking terms. We can establish it in generality only for gaugino masses. As discussed below, in the large volume scenario scalar masses are generically  $\mathcal{O}(m_{3/2})$ .

have included the perturbative Kähler corrections of [90]. These are crucial in the exponentially large volume scenario but are not important for KKLT. We use single exponents in the superpotential and do not consider racetrack scenarios.

For a D7-brane wrapped on a cycle  $\Sigma_i$ , the gauge kinetic function is given by

$$f_i = \frac{T_i}{2\pi}. (6.53)$$

As above, the gaugino masses are computed through the general expression (6.7)

$$M_a = \frac{1}{2} \frac{1}{\text{Re } f_a} \sum_{\alpha} F^{\alpha} \partial_{\alpha} f_a, \tag{6.54}$$

where a runs over gauge group factors and  $\alpha$  over the moduli fields. The F-term  $F^{\alpha}$  is

$$F^{\alpha} = e^{\mathcal{K}/2} \sum_{\bar{\beta}} \mathcal{K}^{\alpha\bar{\beta}} D_{\bar{\beta}} \bar{W}$$
$$= e^{\mathcal{K}/2} \sum_{\bar{\beta}} \mathcal{K}^{\alpha\bar{\beta}} \left( \partial_{\bar{\beta}} \bar{W} + (\partial_{\bar{\beta}} \mathcal{K}) \bar{W} \right), \tag{6.55}$$

where we have expanded the covariant derivative  $D_{\bar{\beta}}\bar{W} = \partial_{\bar{\beta}}\bar{W} + (\partial_{\bar{\beta}}\mathcal{K})\bar{W}$ .

Thus for a brane wrapping cycle k we have

$$M_k = \frac{1}{2} \frac{F^k}{\tau_k}. (6.56)$$

It is a property of the Kähler potential  $\mathcal{K} = -2\ln(\mathcal{V} + \frac{\hat{\xi}}{2})$  that

$$\sum_{\bar{j}} \mathcal{K}^{k\bar{j}} \partial_{\bar{j}} \mathcal{K} = -2\tau_k \left( 1 + \frac{\hat{\xi}}{4\mathcal{V}} \right) \equiv -2\hat{\tau}_k. \tag{6.57}$$

We therefore obtain

$$F^{k} = e^{\mathcal{K}/2} \left( \sum_{\bar{j}} \mathcal{K}^{k\bar{j}} \partial_{\bar{j}} \bar{W} - (2\hat{\tau}_{k}) \bar{W} \right). \tag{6.58}$$

From the superpotential (6.52), we see that  $\partial_{\bar{j}}\bar{W} = -a_j\bar{A}_je^{-a_j\bar{T}_j}$ , and so

$$F^{k} = e^{\mathcal{K}/2} \left( \sum_{\bar{j}} -\mathcal{K}^{k\bar{j}} a_{j} \bar{A}_{j} e^{-a_{j}\bar{T}_{j}} - 2\hat{\tau}_{k} \bar{W} \right). \tag{6.59}$$

We now show that if the modulus  $T_k$  is stabilised by nonperturbative effects, the two terms in equation (6.59) cancel to leading order. To see this, we start with the full F-term supergravity potential,

$$V_{F} = e^{\mathcal{K}} \mathcal{K}^{i\bar{j}} \partial_{i} W \partial_{\bar{j}} \bar{W} + e^{\mathcal{K}} \mathcal{K}^{i\bar{j}} \left( (\partial_{i} K) W \partial_{\bar{j}} \bar{W} + (\partial_{\bar{j}} \mathcal{K}) \bar{W} \partial_{i} W \right) + e^{\mathcal{K}} (\mathcal{K}^{i\bar{j}} \mathcal{K}_{i} \mathcal{K}_{\bar{j}} - 3) |W|^{2}.$$

$$(6.60)$$

As in chapter 5, this becomes

$$V = \sum_{i\bar{j}} \frac{\mathcal{K}^{i\bar{j}}(a_i A_i)(a_j \bar{A}_j) e^{-a_i T_i - a_j \bar{T}_j}}{\mathcal{V}^2} + \sum_j \frac{2\hat{\tau}_j \left(a_j \bar{A}_j e^{-a_j \bar{T}_j} W + a_j A_j \bar{W} e^{-a_j T_j}\right)}{\mathcal{V}^2} + \frac{3\hat{\xi}|W|^2}{4\mathcal{V}^3}.$$
(6.61)

The perturbative Kähler corrections of (6.52) break no-scale, giving the third term of (6.61). We have only displayed the leading large-volume behaviour of these corrections. This is reasonable as in KKLT such corrections are not important, while in the exponentially large volume scenario  $\mathcal{V} \gg 1$  and the higher volume-suppressed terms are negligible. We shall also assume throughout that  $m_{3/2} \ll M_P$ . This is motivated by phenomenological applications and is any case necessary to make sense of a small hierarchy governed by  $\ln(M_P/m_{3/2})$ .

We assume the modulus  $T_k$  is stabilised by effects non-perturbative in  $T_k$  and locate the stationary locus by extremising with respect to its real and imaginary parts. We first perform the calculation keeping the dominant terms to demonstrate the cancellation in (6.59). We subsequently show that the subleading terms are indeed subleading and estimate their magnitude.

#### Leading Terms

We first solve for the axionic component,  $\partial V/\partial b_k = 0$ . The axion only appears in the superpotential and we have

$$\frac{\partial V}{\partial b_{k}} = \frac{i}{V^{2}} \left[ \sum_{i\bar{j}} \mathcal{K}^{i\bar{j}}(a_{i}A_{i})(a_{j}\bar{A}_{j}) \left[ -a_{i}\delta_{ik} + a_{j}\delta_{jk} \right] e^{-(a_{i}T_{i} + a_{j}\bar{T}_{j})} + \right. \\
+ \sum_{j} 2a_{j}\hat{\tau}_{j} \left( \bar{A}_{j}Wa_{j}\delta_{jk}e^{-a_{j}\bar{T}_{j}} - A_{j}\bar{W}a_{j}\delta_{jk}e^{-a_{j}T_{j}} \right) \right] + \\
\sum_{j} \frac{2\hat{\tau}_{j} \left( a_{j}\bar{A}_{j}e^{-a_{j}\bar{T}_{j}}(\partial_{b_{k}}W) + a_{j}A_{j}(\partial_{b_{k}}\bar{W})e^{-a_{j}T_{j}} \right)}{V^{2}} + \frac{3\hat{\xi}((\partial_{b_{k}}W)\bar{W} + W(\partial_{b_{k}}\bar{W}))}{4V^{3}} \\
= \frac{i}{V^{2}} \left[ -\sum_{j} \mathcal{K}^{k\bar{j}}(a_{k}^{2}A_{k})(a_{j}\bar{A}_{j})e^{-(a_{k}T_{k} + a_{j}\bar{T}_{j})} + \sum_{i} \mathcal{K}^{i\bar{k}}(a_{i}A_{i})(a_{k}^{2}\bar{A}_{k})e^{-(a_{i}T_{i} + a_{k}\bar{T}_{k})} + \\
+2a_{k}^{2}\hat{\tau}_{k} \left( \bar{A}_{k}We^{-a_{k}\bar{T}_{k}} - A_{k}\bar{W}e^{-a_{k}T_{k}} \right) \right]. \tag{6.63}$$

In going from (6.62) to (6.63) we have dropped the third line of (6.62) as subleading. We will estimate the magnitude of these subleading terms below.

We now change the dummy index in (6.63) from j to i, and use  $\mathcal{K}^{k\bar{i}} = \mathcal{K}^{i\bar{k}}$  together with  $\frac{\partial V}{\partial b_k} = 0$  to obtain

$$2\hat{\tau}_{k}(\bar{A}_{k}We^{-a_{k}\bar{T}_{k}} - A_{k}\bar{W}e^{-a_{k}T_{k}}) = \sum_{i} \mathcal{K}^{k\bar{i}}\left((a_{i}\bar{A}_{i})A_{k}e^{-(a_{k}T_{k} + a_{i}\bar{T}_{i})} - (a_{i}A_{i})\bar{A}_{k}e^{-(a_{k}\bar{T}_{k} + a_{i}T_{i})}\right) + (\text{subleading terms}).$$
(6.64)

As the axion does not appear (at least in perturbation theory) in the Kähler potential, its stabilisation is always entirely due to nonperturbative superpotential effects.

We next consider the stabilisation of  $\tau_k = \text{Re}(T_k)$ . As stated above, our main assumption is that  $T_k$  is stabilised by superpotential effects nonperturbative in  $T_k$ . Another way of stating this is to say that, when computing  $\frac{\partial V}{\partial \tau_k}$ , the dominant contribution must arise from the superpotential term  $A_k e^{-a_k T_k}$ : if this were not the case, our assumption about how  $T_k$  is stabilised is invalid. In evaluating  $\frac{\partial V}{\partial \tau_k}$ , we therefore focus on such terms as dominant and neglect terms arising from e.g.  $\frac{\partial}{\partial \tau_k} \left( \frac{\mathcal{K}^{i\bar{j}}}{\mathcal{V}^2} \right)$  as subdominant. We show below that the magnitude of the subdominant terms is suppressed by factors of  $\ln \left( m_{3/2} \right)$ .

If we only consider superpotential terms, the calculation of  $\frac{\partial V}{\partial \tau_k}$  exactly parallels that of  $\frac{\partial V}{\partial b_k}$  above. The only differences lie in the signs, as

$$\frac{\partial T_k}{\partial \tau_k} = \frac{\partial \bar{T}_k}{\partial \tau_k} = 1$$
, whereas  $\frac{\partial T_k}{\partial b_k} = -\frac{\partial \bar{T}_k}{\partial b_k} = i$ .

In a similar fashion to (6.63) we therefore obtain

$$\frac{\partial V}{\partial \tau_k} = \frac{a_k^2}{\mathcal{V}^2} \left[ \sum_i \mathcal{K}^{k\bar{i}} \left( -(a_i \bar{A}_i) A_k e^{-(a_k T_k + a_i \bar{T}_i)} - (a_i A_i) \bar{A}_k e^{-(a_k \bar{T}_k + a_i T_i)} \right) -2\hat{\tau}_k \left( \bar{A}_k W e^{-a_k \bar{T}_k} + A_k \bar{W} e^{-a_k T_k} \right) \right] + \text{(subleading terms)}.$$
(6.65)

Setting  $\frac{\partial V}{\partial \tau_k} = 0$  then implies

$$2\hat{\tau}_{k}\left(\bar{A}_{k}We^{-a_{k}\bar{T}_{k}} + A_{k}\bar{W}e^{-a_{k}T_{k}}\right) = -\sum_{i} \mathcal{K}^{k\bar{i}}\left(a_{i}\bar{A}_{i}A_{k}e^{-(a_{k}T_{k} + a_{i}\bar{T}_{i})} + a_{i}A_{i}\bar{A}_{k}e^{-(a_{k}\bar{T}_{k} + a_{i}T_{i})}\right) + (\text{subleading terms}).$$

$$(6.66)$$

We now sum (6.64) and (6.66) to obtain

$$4\hat{\tau}_{k}\bar{A}_{k}We^{-a_{k}\bar{T}_{k}} = -2\sum_{i}\mathcal{K}^{k\bar{i}}a_{i}A_{i}\bar{A}_{k}e^{-(a_{k}\bar{T}_{k}+a_{i}T_{i})}$$

$$\Rightarrow -2\hat{\tau}_k W = \sum_i \mathcal{K}^{k\bar{i}} a_i A_i e^{-a_i T_i}$$

$$\Rightarrow -2\hat{\tau}_k \bar{W} = \sum_i \mathcal{K}^{\bar{k}i} a_i \bar{A}_i e^{-a_i \bar{T}_i}.$$
(6.67)

Comparison with equations (6.56) and (6.59) shows that there exists a leading-order cancellation in the computation of the gaugino mass. This cancellation has followed purely from the assumption that the modulus  $\tau_k$  was stabilised by non-perturbative effects: we have only required  $\frac{\partial V}{\partial \tau_k} = 0$  and not  $D_{T_k}W = 0$ .

In deriving equations (6.63) and (6.65) we dropped subleading terms suppressed by  $\ln\left(\frac{M_P}{m_{3/2}}\right)$ . We then expect the cancellation from (6.67) and (6.59) to fail at this order, giving

$$F^k \sim -2\frac{\hat{\tau}_k e^{K/2}\bar{W}}{\ln(m_{3/2})}.$$
 (6.68)

Equation (6.56) then gives

$$M_k \sim \frac{e^{\mathcal{K}/2}\bar{W}}{\ln(m_{3/2})} = \frac{m_{3/2}}{\ln(m_{3/2})},$$
 (6.69)

the relation we sought.

The above argument is general and model-independent. We have used the Kähler potential appropriate to an arbitrary compactification, making no assumptions about the number of moduli. Furthermore, as the result comes from directly extremising the scalar potential it is independent of whether the moduli stabilisation is approximately supersymmetric or not. Indeed, the argument above has not depended on finding a global minimum of the potential, or even on extremising the potential with respect to any of the moduli except  $T_k$ . This result shows that the small hierarchy of (6.51) will exist in both KKLT and exponentially large volume approaches to moduli stabilisation. In the latter case this is possibly surprising as the minimum is in no sense approximately susy: each contribution to the sum in (6.59) individually gives  $M_a \sim m_{3/2}$ : it is only when summed the mass suppression is obtained.

We note, as an aside, that if all moduli are stabilised by non-perturbative effects then by contracting (6.67) with  $\mathcal{K}_{i\bar{k}}$  we obtain

$$-2\sum_{k} \mathcal{K}_{j\bar{k}}\hat{\tau}_{k}\bar{W} = a_{j}\bar{A}_{j}e^{-a_{j}\bar{T}_{j}}.$$
(6.70)

Now,  $\mathcal{K}_j$  is homogeneous of degree -1 in  $\tau_k$ , so recalling that  $\frac{\partial}{\partial \tau_k} = 2 \frac{\partial}{\partial T_k}$ ,

$$\sum_{k} -2\mathcal{K}_{j\bar{k}}\tau_{k} = \sum_{k} -\frac{\partial \mathcal{K}_{j}}{\partial \tau_{k}}\tau_{k} = \mathcal{K}_{j},$$

and therefore to leading order

$$\partial_i W + (\partial_i \mathcal{K}) \bar{W} = 0. \tag{6.71}$$

Consequently if *all* moduli are stabilised by nonperturbative effects then the stabilisation is always approximately supersymmetric.

#### Subleading terms

We now want to show that the terms neglected in computing  $\frac{\partial V}{\partial \tau_k}$  are all suppressed, under the assumption that the modulus is solely stabilised by nonperturbative effects. For concreteness we focus on the term in the potential

$$\sum_{i,\bar{j}} \left( \frac{\mathcal{K}^{i\bar{j}}}{\mathcal{V}^2} \right) (a_i A_i) (a_j \bar{A}_j) e^{-(a_i T_i + a_j \bar{T}_j)}. \tag{6.72}$$

The argument used for this term will also apply to the other terms of (6.61).  $\frac{\mathcal{K}^{i\bar{j}}}{\mathcal{V}^2}$  is homogeneous in the  $\tau_k$  of degree -1. To see this, we note that as by dimensional analysis  $\mathcal{V}$  is homogeneous in the  $\tau_i$  of degree 3/2,  $\mathcal{K}_{i\bar{j}} = \frac{\partial}{\partial T_i} \frac{\partial}{\partial T_j} (-2 \ln(\mathcal{V}))$  is homogeneous in the  $\tau_i$  of degree -2, and so  $\mathcal{K}^{i\bar{j}}$  is homogeneous in the  $\tau_i$  of degree 2. Therefore, summing over k,

$$\sum_{k} \tau_{k} \frac{\partial}{\partial \tau_{k}} \left( \frac{\mathcal{K}^{i\bar{j}}}{\mathcal{V}^{2}} \right) = -\frac{\mathcal{K}^{i\bar{j}}}{\mathcal{V}^{2}}, \tag{6.73}$$

and so

$$\frac{\partial}{\partial \tau_k} \left( \frac{\mathcal{K}^{i\bar{j}}}{\mathcal{V}^2} \right) \lesssim \frac{\mathcal{K}^{i\bar{j}}}{\tau_k \mathcal{V}^2}. \tag{6.74}$$

Cosequently, differentiating (6.72) w.r.t  $\tau_k$  gives

$$\mathcal{O}\left(\frac{1}{\tau_k}\sum_{i,j}\frac{\left(\mathcal{K}^{i\bar{j}}(a_iA_i)(a_j\bar{A}_j)e^{-(a_iT_i+a_j\bar{T}_j)}\right)}{\mathcal{V}^2}\right) + a_k\sum_j\frac{\left(\mathcal{K}^{k\bar{j}}(a_kA_k)(a_j\bar{A}_j)e^{-(a_kT_k+a_j\bar{T}_j)} + c.c\right)}{\mathcal{V}^2}.$$

The basic assumption we make is that the location of the minimum for  $\tau_k$  is dominantly determined by the effects nonperturbative in  $\tau_k$ . Therefore in the first sum we should only include the terms which depend nonperturbatively on  $a_k T_k$ . This gives

$$\mathcal{O}\left(\frac{1}{\tau_{k}}\sum_{j}\frac{\left(\mathcal{K}^{k\bar{j}}a_{j}a_{k}\bar{A}_{j}A_{k}e^{-(a_{k}T_{k}+a_{j}\bar{T}_{j})}+c.c\right)}{\mathcal{V}^{2}}\right)+\sum_{j}\frac{\left(\mathcal{K}^{k\bar{j}}(a_{k}^{2}A_{k})(a_{j}\bar{A}_{j})e^{-(a_{k}T_{k}+a_{j}\bar{T}_{j})}+c.c\right)}{\mathcal{V}^{2}}.$$
(6.75)

We then see that the first term of (6.75) is suppressed compared to the second by a factor  $a_k \tau_k$ .

We note that there can exist moduli  $\tau_k$  not stabilised by effects nonperturbative in  $\tau_k$  - this certainly holds for the volume modulus for the large volume models. Furthermore, one can argue that in order to avoid generating a potential for the QCD axion, the modulus  $\tau_k$  associated with the QCD cycle should be stabilised without using effects nonperturbative in  $\tau_k$ . This point will be discussed further in chapter 7. The argument above is restricted to the case where the modulus  $\tau_k$  is stabilised by effects nonperturbative in  $\tau_k$ .

An identical analysis applies to the other two terms of equation (6.61). As the Kähler dependent terms are polynomials in  $\tau_k$ , derivatives of these with respect to  $\tau_k$  also give a suppression factor of  $\tau_k$ , while the derivatives of superpotential exponents are enhanced by a factor  $a_k$ . The latter (which we keep) are therefore larger by a factor  $a_k \tau_k$  than the terms discarded.

In passing from (6.62) to (6.63) we dropped the last line of (6.62). This is self-consistent so long as  $\sum_j A_j e^{-a_j T_j}$  is suppressed compared to W. In the models of chapter 5 this is trivial as  $e^{-a_k \tau_k} \sim \frac{1}{\mathcal{V}}$  while  $W \sim \mathcal{O}(1)$ . In KKLT models, as

$$\partial_{T_i} \mathcal{K} = -2 \frac{\partial_{T_i} \mathcal{V}}{\mathcal{V}} \lesssim \frac{2}{\tau_i},$$

we can use (6.71) to see that

$$\bar{W} \gtrsim (a_k \tau_k) A_k e^{-a_k \tau_k}, \tag{6.76}$$

and so the third line of (6.62) is suppressed compared to the second by a factor of (at least)  $a_k \tau_k$ .

The above arguments imply that the terms dropped in our evaluation of  $\frac{\partial V}{\partial \tau_k}$  are either suppressed by a factor of  $a_k \tau_k \sim \ln(m_{3/2})$  or have no non-perturbative dependence on  $\tau_k$ . Consequently the gaugino mass suppression found above will hold at leading order in  $\frac{1}{\ln(m_{3/2})}$ .

#### The uplift term

In order to attain almost vanishing but positive vacuum energy, an uplift term must also be included. For KKLT models this is in a sense responsible for the soft masses, as the original AdS minimum is supersymmetric. In the large volume models the AdS minimum is already non-supersymmetric and the contribution of the uplift to soft terms is less relevant.

We take as uplift

$$V_{uplift} = \frac{\epsilon}{\mathcal{V}^{\alpha}},\tag{6.77}$$

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where the power  $4/3 \le \alpha \le 2$  depends on the uplift mechanism [84, 85, 86]. Including this phenomenological uplift term, the full potential is

$$V_{full} = V_{SUGRA} + V_{uplift}. (6.78)$$

At the minimum  $\langle V_{full} \rangle = 0$  and so we must have

$$\langle V_{SUGRA} \rangle = -\langle V_{uplift} \rangle.$$

 $\mathcal{V}^{-\alpha}$  is homogeneous of degree  $-3\alpha/2$  in the  $\tau_i$  and so

$$\sum_{k} \tau_{k} \frac{\partial}{\partial \tau_{k}} \mathcal{V}^{-\alpha} = -\frac{3\alpha}{2} \mathcal{V}^{-\alpha}, \tag{6.79}$$

implying

$$\frac{\partial}{\partial \tau_k} \mathcal{V}^{-\alpha} \lesssim -\frac{3\alpha}{2\tau_k} \mathcal{V}^{-\alpha}. \tag{6.80}$$

Thus  $\frac{\partial V_{uplift}}{\partial \tau_k}$  is suppressed compared to  $V_{uplift}$  by a factor of  $\tau_k$ . In contrast, the derivatives of  $V_{SUGRA}$  involve an enhancement by  $a_k$  due to the exponentials. As the two terms of (6.78) are by definition equal at the minimum, we see that

$$\frac{\partial V_{SUGRA}}{\partial \tau_k} \gtrsim a_k \tau_k \frac{\partial V_{uplift}}{\partial \tau_k}.$$
 (6.81)

This fits in with our previous analysis: the terms giving rise to the cancellation in (6.59) are the leading ones, with subleading terms suppressed by  $a_k \tau_k$ . We do not control the subleading terms and they will generically be non-vanishing, giving further contributions to the gaugino masses at  $\mathcal{O}\left(\frac{m_{3/2}}{\ln(m_{3/2})}\right)$ .

As an aside, we note that for the large volume models

$$\frac{\partial V_{uplift}}{\partial \tau_k} \sim \frac{1}{\mathcal{V}} V_{uplift},\tag{6.82}$$

and so the presence of the uplift term does not significantly affect the stabilisation of  $\tau_k$ .

# **6.4** Explicit Calculations for $\mathbb{P}^4_{[1,1,1,6,9]}$

The above has established that a suppression of gaugino masses compared to the gravitino mass is generic in the landscape. We now illustrate the above with explicit calculations for the  $\mathbb{P}^4_{[1,1,1,6,9]}$  model analysed in chapter 5.

## 6.4.1 Gaugino Masses

We first briefly recall the relevant properties of this model from chapter 5. As the manifold has  $h^{1,1}=2$ , there are two moduli,  $T_b$  and  $T_s$ .  $T_b$  controls the overall volume and  $T_s$  is a blow-up mode. At the minimum  $\tau_b = \text{Re}(T_b) \gg \tau_s = \text{Re}(T_s) \sim \ln(\tau_b)$ . The Kähler and superpotential are given by

$$\mathcal{K} = -2\ln\left(\mathcal{V} + \frac{\hat{\xi}}{2}\right) \equiv -2\ln\left(\tau_b^{3/2} - \tau_s^{3/2} + \frac{\hat{\xi}}{2}\right),$$
 (6.83)

$$W = W_0 + A_s e^{-a_s T_s} + A_b e^{-a_b T_b}. (6.84)$$

The resulting Kähler metric is (dropping terms subleading in powers of  $\mathcal{V}$ )

$$\mathcal{K}_{i\bar{j}} = \begin{pmatrix} \mathcal{K}_{b\bar{b}} & \mathcal{K}_{b\bar{s}} \\ \mathcal{K}_{s\bar{b}} & \mathcal{K}_{s\bar{s}} \end{pmatrix} = \begin{pmatrix} \frac{3}{4\mathcal{V}^{4/3}} & -\frac{9\tau_s^{\frac{1}{2}}}{8\mathcal{V}^{5/3}} \\ -\frac{9\tau_s^{\frac{1}{2}}}{8\mathcal{V}^{5/3}} & \frac{3}{8\sqrt{\tau_s}\mathcal{V}} \end{pmatrix},$$
(6.85)

with inverse metric

$$\mathcal{K}^{i\bar{j}} = \begin{pmatrix} \mathcal{K}^{b\bar{b}} & \mathcal{K}^{b\bar{s}} \\ \mathcal{K}^{s\bar{b}} & \mathcal{K}^{s\bar{s}} \end{pmatrix} = \begin{pmatrix} \frac{4\mathcal{V}^{4/3}}{3} & 4\tau_s\tau_b \\ 4\tau_s\tau_b & \frac{8\sqrt{\tau_s}\mathcal{V}}{3} \end{pmatrix}.$$
(6.86)

In a limit  $\mathcal{V} \equiv \tau_b^{3/2} - \tau_s^{3/2} \gg 1$  with  $\tau_s \sim \mathcal{O}(1)$ , direct evaluation of the scalar potential gives (dropping terms subleading in  $\mathcal{V}$ ),

$$V = \frac{\lambda a_s^2 A_s^2 \sqrt{\tau_s} e^{-2a_s \tau_s}}{\mathcal{V}} - \frac{\mu a_s A_s \tau_s |W_0| e^{-a_s \tau_s}}{\mathcal{V}^2} + \frac{\nu |W_0|^2}{\mathcal{V}^3}.$$
 (6.87)

Explicitly,  $\lambda = \frac{8}{3}$  and  $\mu = 4$ . The minus sign in (6.87) comes from minimising the potential with respect to the axion  $b_s$ . As  $\tau_b \gg 1$  all terms nonperturbative in  $\tau_b$  vanish. Noting that  $\frac{\partial}{\partial \tau_s} (\mathcal{V}^{-1}) \sim \mathcal{O}\left(\frac{1}{\mathcal{V}^2}\right)$ , we obtain

$$\frac{\partial V}{\partial \tau_s} = \frac{\lambda a_s^2 A_s^2 \sqrt{\tau_s} e^{-2a_s \tau_s}}{\mathcal{V}} \left( -2a_s + \frac{1}{2\tau_s} \right) - \frac{\mu a_s A_s e^{-a_s \tau_s} |W_0|}{\mathcal{V}^2} \left( -a_s \tau_s + 1 \right) + \mathcal{O}\left(\frac{1}{\mathcal{V}^2}\right). \tag{6.88}$$

Imposing  $\frac{\partial V}{\partial \tau_s} = 0$  and rearranging (6.88) gives

$$e^{-a_s\tau_s} = \left(\frac{\mu}{2\lambda}\right) \frac{|W_0|}{\mathcal{V}a_s} \sqrt{\tau_s} \left(1 - \frac{3}{4a_s\tau_s}\right) + \mathcal{O}\left(\frac{1}{(a_s\tau_s)^2}\right). \tag{6.89}$$

If we also solve  $\frac{\partial V}{\partial \mathcal{V}} = 0$ , we obtain

$$\mathcal{V} \sim \left| \frac{W_0}{A_s} \right| e^{a_s \tau_s} \quad \text{with } \tau_s \sim \hat{\xi}^{\frac{2}{3}}.$$

<sup>&</sup>lt;sup>4</sup>For simplicity we drop a factor of  $\frac{1}{9\sqrt{2}}$  from  $\mathcal{V}$ : this does not affect the results.

 $\mathcal{V}$  is exponentially sensitive to  $\hat{\xi}$  (which includes  $g_s$ ) and  $a_4$  and so can take on essentially any value. A TeV scale gravitino mass requires  $\mathcal{V} \sim 10^{14}$ , which we assume.

There may be gaugini associated with each of the moduli  $\tau_b$  and  $\tau_s$ . From (6.56) these have masses  $\frac{F^b}{2\tau_b}$  and  $\frac{F^s}{2\tau_s}$  respectively. We calculate both masses: however we note that the small cycle is the only cycle appropriate for Standard Model matter, as a brane wrapped on the large cycle would have too small a gauge coupling. First,

$$F^{b} = e^{\mathcal{K}/2} \left( \mathcal{K}^{b\bar{b}} D_{\bar{b}} \bar{W} + \mathcal{K}^{b\bar{s}} D_{\bar{s}} \bar{W} \right) \tag{6.90}$$

$$= e^{\mathcal{K}/2} \left( -2\tau_b \bar{W} + \mathcal{K}^{b\bar{b}} \partial_{\bar{b}} \bar{W} + \mathcal{K}^{b\bar{s}} \partial_{\bar{s}} \bar{W} \right), \tag{6.91}$$

where we have used  $\mathcal{K}^{b\bar{b}}\partial_{\bar{b}}\mathcal{K} + \mathcal{K}^{b\bar{s}}\partial_{\bar{s}}\mathcal{K} = -2\tau_b$  from (6.57). However, as  $\tau_b \sim \mathcal{V}^{2/3} \gg 1$ ,  $\partial_{\bar{b}}\bar{W} \sim \exp(-a_b\tau_b) \sim 0$ . Furthermore,

$$\mathcal{K}^{b\bar{s}}\partial_{\bar{s}}\bar{W} \sim (4\tau_b\tau_s) \times (a_sA_s\exp(-a_s\tau_s)) \sim \mathcal{V}^{-1/3}$$

and so

$$F^{b} = \frac{1}{\mathcal{V}} \left( -2\tau_{b} W_{0} + \mathcal{O}\left(\mathcal{V}^{-1/3}\right) \right), \tag{6.92}$$

implying

$$|M_b| = \left| \frac{F^b}{2\tau_b} \right| = m_{3/2} + \mathcal{O}(\mathcal{V}^{-1/3}).$$
 (6.93)

This is an identical relation to that of the fluxed MSSM [129].

We now calculate  $M_s$ . Using  $\mathcal{K}^{i\bar{j}}\mathcal{K}_{\bar{j}} = -2\tau_i$  and  $\mathcal{K}^{s\bar{b}}\partial_{\bar{b}}W \sim 0$ , we get

$$F^{s} = e^{\mathcal{K}/2} \left( \mathcal{K}^{s\bar{s}} \partial_{s} \bar{W} - 2\tau_{s} \bar{W} \right) \tag{6.94}$$

$$= e^{\mathcal{K}/2} \left( \mathcal{K}^{s\bar{s}} \left( -a_s A_s e^{-a_s T_s} \right) - 2\tau_s \bar{W} \right). \tag{6.95}$$

From (6.86),  $\mathcal{K}^{s\bar{s}} = \frac{8\sqrt{\tau_s}\nu}{3}$ . Using (6.89) we then have

$$F^{s} = \frac{2\tau_{s}\bar{W}}{\mathcal{V}}\left(\left(1 - \frac{3}{4a_{s}\tau_{s}}\right) - 1\right). \tag{6.96}$$

We therefore obtain

$$|M_s| = \frac{3m_{3/2}}{4a_s\tau_s} \left( 1 + \mathcal{O}\left(\frac{1}{a_4\tau_4}\right) \right) = \frac{3m_{3/2}}{4\ln(m_{3/2})} \left( 1 + \mathcal{O}\left(\frac{1}{\ln(m_{3/2})}\right) \right), \quad (6.97)$$

with the expected small hierarchy. We therefore conclude that the gaugino mass associated to the exponentially large modulus  $\tau_b$  is equal to the gravitino mass, whereas the gaugino mass associated to the small modulus  $\tau_s$  is suppressed by  $\ln(m_{3/2})$ . While in the  $\mathbb{P}^4_{[1,1,1,6,9]}$  case there is only one small modulus, as shown above this result will extend to more realistic multi-modulus examples. This confirms the logarithmic suppression of gaugino masses compared to the naive estimates of section 6.2.

### 6.4.2 Moduli Masses

We also here prove the existence of an enhancement by  $\ln(m_{3/2})$  in the mass of the small modulus  $\tau_s$  compared to  $m_{3/2}$ . This is similar behaviour as found in KKLT solutions [117]. This again extends the simple volume scaling arguments of chapter 5 which gave  $m_s \sim \frac{M_P}{\mathcal{V}}$  and  $m_b \sim \frac{M_P}{\mathcal{V}^{3/2}}$ . Focusing purely on the field  $\tau_s$ , its Lagrangian is

$$\int d^4x \, \mathcal{K}_{s\bar{s}} \partial_\mu \tau_s \partial^\mu \tau_s + V(\tau_s). \tag{6.98}$$

As  $\tau_s$  is the heavier of the two moduli  $\tau_s$  and  $\tau_b$ , a lower bound on its mass is given by

$$m_s^2 \gtrsim \frac{1}{2\mathcal{K}_{s\bar{s}}} \left\langle \frac{\partial^2 V}{\partial \tau_s^2} \right\rangle.$$
 (6.99)

Now,  $\mathcal{K}_{s\bar{s}} = \frac{3}{8\sqrt{\tau_s}\mathcal{V}}$ . If we evaluate  $\frac{\partial^2 V}{\partial \tau_s^2}$ , we obtain

$$\frac{\partial^2 V}{\partial \tau_s^2} = \frac{4\lambda a_s^4 \sqrt{\tau_s} e^{-2a_s \tau_s}}{\mathcal{V}} - \frac{\mu a_s^3 \tau_s e^{-a_s \tau_s} |W_0|}{\mathcal{V}^2} + \left(\text{terms suppressed by } \frac{1}{a_s \tau_s}\right). \tag{6.100}$$

Substituting in our evaluation of  $\langle e^{-a_s\tau_s}\rangle$  from (6.89), we obtain

$$\frac{\partial^2 V}{\partial \tau_s^2} = \left(\frac{\mu^2}{2\lambda}\right) \frac{a_s^2 \tau_s^{3/2} |W_0|^2}{\mathcal{V}^3} \left(1 + \mathcal{O}\left(\frac{1}{a_s \tau_s}\right)\right) \tag{6.101}$$

Consequently

$$m_{\tau_s}^2 \gtrsim \left(\frac{4\sqrt{\tau_s}\mathcal{V}}{3}\right) \left(\frac{\mu^2}{2\lambda}\right) \frac{a_s^2 \tau_s^{3/2} |W_0|^2}{\mathcal{V}^3}$$

$$= \left(\frac{2\mu^2}{3\lambda}\right) \frac{a_s^2 \tau_s^2 |W_0|^2}{\mathcal{V}^2}.$$
(6.102)

This gives

$$m_{\tau_s} \gtrsim 2 \ln \left( \frac{M_P}{m_{3/2}} \right) m_{3/2} \left( 1 + \mathcal{O} \left( \frac{1}{\ln(m_{3/2})} \right) \right).$$
 (6.103)

While technically a lower bound, this is actually a very good estimate of  $m_{\tau_s}$ , as the canonically normalised heavy modulus has only a very small admixture of  $\tau_b$ . This is confirmed by explicit numerical evaluation, which shows the formulae (6.97) and (6.103) to be accurate to within a couple of per cent.

#### 6.4.3 Scalar Masses

The suppressed values for the gaugino masses are a direct consequence of a cancellation in the calculation for the F-terms for the 'small' moduli. Having suppressed

F-terms could naively lead to the conclusion that the other soft terms must also be suppressed. However, consider the expression for scalar masses<sup>5</sup>

$$m_{\varphi}^2 = m_{3/2}^2 + V_0 - F^m \bar{F}^{\bar{n}} \partial_m \partial_{\bar{n}} (\ln \tilde{\mathcal{K}}).$$
 (6.104)

In order to get suppressed values for  $m_{\varphi}^2$  there must be a contribution cancelling the leading  $m_{3/2}^2$  contribution (assuming a negligible vacuum energy  $V_0$ ). In the KKLT models, this is provided by the anti-D3 brane [117]. However, this cancellation is not generic.

For the large-volume models the uplift term is subdominant in susy breaking and so has no significant effect on (6.104). As the F-terms are suppressed by  $\ln(m_{3/2})$  for all small moduli, the only F-term that can cancel the gravitino mass contribution is that associated to the large volume modulus. The dependence of  $\tilde{K}$  on  $\mathcal{V}$  varies depending on the type of scalar field considered. To leading order we can write

$$\tilde{K} = h(\tau_s) \mathcal{V}^{-a}, \tag{6.105}$$

with the exponent  $a \ge 0$  taking different values for the different kinds of matter fields in the model. Here h is a flavour dependent function of the smaller moduli that in general will be very hard to compute.

One can show that the F-term contribution only cancels the leading  $m_{3/2}^2$ , giving scalars suppressed by  $\ln(m_{3/2})$ , if a=2/3. This applies to D3 brane adjoint scalars and D7 Wilson lines. For adjoint D7 matter, a=0 and the scalar masses are comparable to  $m_{3/2}$ . Of course, Standard Model matter fields are in bifundamental representations and should correspond to D3-D7 or D7-D7 matter. In this case the only calculations for  $\mathcal{K}$  are in the context of toroidal orbifolds, where  $0 \le a < 2/3$ . In this case there is no cancellation in (6.104) and the scalars are comparable to the gravitino mass, with positive mass squared, and heavier than the gauginos by the small hierarchy  $\ln(m_{3/2})$ .

This also allows us to say something about flavour universality. In the large volume scenario developed in this thesis, the physical picture is of Standard Model matter supported on almost-vanishing small cycles within a very large internal space ( $V \sim 10^{14}$ ). The physics of flavour is essentially local physics which is determined by the geometry of the small cycles and their intersections. Consequently, all flavours should see the large bulk in the same way, as the distinctive flavour physics is local not global. Therefore the power of a in (6.105) should be flavour-universal. The function  $h(\tau_s)$ , in contrast, is sensitive to the local geometry and so should not be flavour-universal.

In these circumstances we can both show universality for the soft scalar masses and also estimate the fractional level of non-universality. In the sum

<sup>&</sup>lt;sup>5</sup>This form assumes the matter metric is diagonal: the results below are unaffected if we use the fully general expressions [128].

(6.104) the leading  $m_{3/2}^2$  term and the terms involving  $F^b$  are flavour-universal and give a universal contribution of  $\mathcal{O}(m_{3/2}^2)$ . Universality fails due to the F-terms associated with the small moduli. We can then rewrite (6.104) as

$$m_i^2 \sim \underbrace{\left(m_{3/2}^2 + F^b \bar{F}^b \partial_b \partial_{\bar{b}} \ln \tilde{K}\right)}_{\text{universal}} + \underbrace{\left(\sum_s F^s \bar{F}^s \partial_s \partial_{\bar{s}} \ln \tilde{K}\right)}_{\text{non-universal}}.$$
 (6.106)

The  $F^bF^{\bar{s}}+F^{\bar{b}}F^s$  cross-terms vanish. As  $F^s\sim \frac{m_{3/2}}{\ln(m_{3/2})}$  we obtain

$$m_i^2 \sim m_{3/2}^2 (1 + \epsilon_i),$$
  
 $\Rightarrow m_i \sim m_{3/2} \left(1 + \frac{\epsilon_i}{2}\right),$  (6.107)

where non-universality is encoded in  $\epsilon_i \sim \frac{1}{\ln(m_{3/2})^2}$ . As we require  $m_{3/2} \sim 1 \text{TeV}$ , we estimate the fractional non-universality for soft masses as  $\sim 1/(\ln(10^{18}/10^3))^2 \sim 1/1000$ .

It is remarkable that these general results can be extracted despite our ignorance of the precise dependence of the Kähler potential on the matter fields. This is possible because in the above scenario flavour physics is local while supersymmetry breaking is global, and there exists a controlled expansion in  $\frac{1}{V}$ .

We also note that the large-volume scenario naturally addresses the  $\mu$  problem. This is because the natural scale for any mass term, susy or non-susy, is  $\mu \sim \frac{M_P}{\mathcal{V}} \sim 1 \text{TeV}$ . Indeed the dilaton and complex structure moduli, which are stabilised supersymmetrically by the fluxes, do acquire masses of this order. This arises because the scalar potential has a prefactor  $e^{\mathcal{K}} \sim \frac{1}{\mathcal{V}^2}$ . A superpotential term  $\mu\phi\phi$  generates a scalar potential

$$V_{\phi} \sim e^{\mathcal{K}}(DW)(DW) \sim \frac{\mu^2}{\mathcal{V}^2}\phi^2.$$

and so masses are naturally suppressed by  $\frac{1}{\nu}$  and at the gravitino mass scale.

The phenomenology of flux compactifications has been much studied recently. However the above suggests new well-motivated scenarios to consider. The most obvious case is that of an intermediate string scale and thus a TeV scale gravitino mass, with squarks and sleptons heavy and comparable to the gravitino mass, while gauginos are suppressed by a  $(\ln m_{3/2})$  factor. As the gaugino masses are suppressed, it is necessary to include anomaly mediated contributions in addition to the gravity-mediation expressions above. However as the scalar masses are heavy and comparable to the gravitino mass the contribution of anomaly mediation is in that case negligible - this is just as well given the notorious problem of tachyonic sleptons for pure anomaly mediation. It will also be interesting to

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analyse the phenomenology of the non-universality predicted in equation (6.107). A detailed investigation of these and related scenarios is in progress [128].

There is one final note of caution. In attempting to build realistic models, there are sound reasons to suppose that not all Kähler moduli are stabilised by nonperturbative effects. In particular, if all moduli were stabilised by nonperturbative effects then their axionic parts would all also be heavy and a QCD axion capable of solving the strong CP problem would not exist. If the Kähler modulus corresponding to the QCD cycle is partially stabilised through perturbative effects, it is possible that gluinos may be heavier than the remaining gauginos. It is therefore also interesting to analyse the phenomenology of a mixed scenario with an ordering  $m_{3/2} \sim m_i \gtrsim m_{\tilde{g}} > m_{\tilde{W}} \sim \frac{m_{3/2}}{\ln(m_{3/2})}$ .

# Chapter 7

## Axions and Moduli Stabilisation

This chapter is based on the paper [6].

In chapter 6 we analysed the structure of soft supersymmetry-breaking terms for the large-volume minimum constructed in chapter 5. Through judicious assumptions about the loci of Standard Model matter we were able to analyse the magnitude of soft terms without having a detailed construction of the Standard Model. This is as well, since obtaining the Standard Model is a hard problem typically involving substantial amounts of algebraic geometry. In this chapter we continue with this style of approach, focusing on the axionic solution to the strong CP problem.

This chapter is structured as follows. Section 7.1 reviews both the strong CP problem and the ways axions can arise in different string theory constructions. In section 7.2 I investigate how to stabilise moduli while keeping an axion sufficiently light to solve the strong CP problem. This section also contains a no-go theorem, showing that there exist no supersymmetric minima of the F-term potential consistent with stabilised moduli and unfixed axions. Section 7.3 addresses the axion decay constant and in it I show how the large volume compactifications described in chapter 5 can generate a phenomenologically acceptable value of  $f_a$  together with the relationship  $f_a \sim \sqrt{M_{SUSY} M_P}$ .

## 7.1 Axions

## 7.1.1 Axions and the Strong CP Problem

CP (charge conjugation and parity reversal) is a very good approximate symmetry of the Standard Model. It is violated in the electroweak sector through complex phases in the CKM matrix. CP violation was first observed experimentally in kaon decays [130]. Recently CP violation has also been observed in the decays of B mesons [131, 132].

Experimentally CP violation has only been observed in weak interactions. In principle the strong interactions could also violate CP. The QCD Lagrangian is

 $\mathcal{L}_{QCD} = -\frac{1}{4g_s^2} \int F_{\mu\nu}^a F^{a,\mu\nu} + \frac{i\theta}{16\pi^2} \int F_{\mu\nu}^a \tilde{F}^{a,\mu\nu} + \mathcal{L}_{matter}.$  (7.1)

The coupling  $F\tilde{F}$ , which violates CP, is instantonic in nature. Being topological, it vanishes in perturbation theory. However, the amplitude of QCD instantons are sensitive to the value of  $\theta$ , and if  $\theta \neq 0$  this coupling will lead to observable CP violation in strong interactions. In particular, if  $\theta \neq 0$  a nonzero neutron electric dipole moment is generated. As this is not observed the magnitude of  $\theta$  is highly constrained, with an experimental limit of  $|\theta| \lesssim 10^{-10}$  [133]. The 'strong CP problem' is the fact that  $\theta$ , a periodic constant whose natural range is  $-\pi < \theta < \pi$ , is so small.

The strong CP problem is of course a well-known problem with a well-known answer: a Peccei-Quinn axion [134, 135]. There do exist other resolutions. The measured value of  $\theta$  involves not just the  $\theta$  of equation (7.1) but also a contribution from the phases in the quark mass matrix,

$$\theta = \theta_0 + \arg \det(M_{q_i q_i}).$$

If a quark - say the u - is massless, this phase can be used to gauge away the  $\theta$ -angle, which becomes unphysical. However, this approach is disfavoured by lattice data, and  $m_u = 0$  does not seem possible [136]. A second approach - the Nelson-Barr mechanism - is to assume that CP is an exact symmetry of the high energy theory, which is spontaneously broken at low energies. The smallness of  $\theta$  is to be understood as a legacy of exact CP conservation at high scales. However in this thesis I assume the Peccei-Quinn solution to be correct. As I review, this entails promoting  $\theta$  to a field whose potential is dynamically minimised for  $\theta = 0$ .

Promoting  $\theta$  to a scalar field is natural in string theory, where all 'coupling constants' are vevs of dynamical fields. In this respect one essential feature of the Peccei-Quinn solution is always present in string theory, as  $\theta$  is the imaginary part of a complex field which is the scalar component of a modulus superfield. However, this then creates a modulus anti-stabilisation problem. There are many string theory axions that may in principle solve the strong CP problem. However, to do so an axion must remain massless throughout the thicket of moduli stabilisation effects and down to the QCD scale. This problem is clean and sharply posed, as effects very weak on the Planck scale may be very large on the QCD scale. Given the necessity of moduli stabilisation, this question is best analysed within the context of string constructions for which all moduli have been stabilised. Once we have reviewed the Peccei-Quinn solution we shall analyse this problem in section 7.2.

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The Peccei-Quinn approach is characterised by promoting  $\theta$  to a dynamical field  $\theta(x)$ , with Lagrangian

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2} f_a^2 \partial_\mu \theta(x) \partial^\mu \theta(x) + \frac{\theta(x)}{16\pi^2} F_{\mu\nu}^a \tilde{F}^{a,\mu\nu}. \tag{7.2}$$

In (7.2)  $f_a$  has dimensions of mass and is known as the axion decay constant. Conventionally a scalar has mass dimension one, and so we redefine  $a \equiv \theta f_a$ . This gives

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2} \partial_{\mu} a \partial^{\mu} a + \frac{a}{16\pi^2 f_a} F^a_{\mu\nu} \tilde{F}^{a,\mu\nu}. \tag{7.3}$$

In equation (7.3) there exists an anomalous global U(1) symmetry,  $a \to a + \epsilon$ . This symmetry is violated by QCD instanton effects, which break it to a discrete subgroup. These generate a potential for a,

$$V_{1-\text{instanton}} \sim \Lambda_{QCD}^4 \left( 1 - \cos \left( \frac{a}{f_a} \right) \right).$$
 (7.4)

In the absence of other effects, this potential is minimised at a=0. This sets the  $\theta$ -angle to zero and dynamically solves the strong CP problem. The mass scale for the axion a is

$$m_a \sim \frac{f_\pi^2 m_\pi^2}{f_a} = \left(\frac{10^{11} \text{GeV}}{f_a}\right) 10^{-4} \text{eV}.$$
 (7.5)

The replacement of the  $\Lambda_{QCD}^2$  of (7.4) by  $f_{\pi}m_{\pi}$ , where  $f_{\pi} \sim 90 \text{MeV}$  is the pion decay constant, comes about through a precise calculation.

By its nature, the Peccei-Quinn symmetry is anomalous - it is violated by QCD instanton effects that generate a potential for the axion. It is necessary that QCD instanton effects dominate the axion potential - the limit on  $|\theta|$  is so strong that even tiny additional contributions to the potential may displace the minimum of the axion potential sufficiently to be in conflict with experiment. In particular, higher-order Planck suppressed contributions such as  $a^5/M_P$  must be entirely absent. This may seem unnatural, but is easily accomplished if the U(1) Peccei-Quinn symmetry is only broken to a discrete subgroup. This is in fact what happens in string theory.

Even if an axion solves the strong CP problem, there are strong constraints on the allowed value of  $f_a$ .  $f_a$  gives the axion-matter coupling. The smaller the value of  $f_a$ , the more strongly the axion couples to matter. Assuming the axion to couple to QED as well as QCD, the axion may be sought through direct searches in a strong magnetic field, excluding the regime  $f_a \lesssim 10^6 \text{GeV}$ . Astrophysical studies of supernova cooling further exclude the regime  $10^6 \text{GeV} \lesssim f_a \lesssim 10^9 \text{GeV}$ . In this limit, axions are produced in the core of a supernova and are emitted without reabsorption. The energy carried away by the axions causes the supernova to cool more rapidly than is observed.

These lower bounds are hard. There is also a cosmological upper bound on  $f_a$ . The axion field presumably starts its cosmological evolution with  $\theta$  an arbitrary value between 0 and  $2\pi$ . From (7.4), there is a primoridal axion energy density of  $\rho_{\rm axion} \sim (0.1 {\rm GeV})^4$ . Cosmologically, a scalar field evolves according to

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial V}{\partial \phi}.\tag{7.6}$$

Once  $H \sim m_a$  at time  $t_0$  the axion field starts oscillating and the energy density subsequently dilutes as matter,

$$\rho_{\text{axion}}(t) \sim (0.1 \text{GeV})^4 \frac{a(t_0)^3}{a(t)^3}.$$
(7.7)

As  $m_a \propto 1/f_a$ , larger values of  $f_a$  imply the axion starts oscillating at later cosmological times. The bound comes from requiring the current energy density in the axion field to be below that of the dark matter density, which gives the approximate constraint  $f_a \lesssim 10^{12} \text{GeV}$ .

This upper bound assumes a standard cosmology. In non-standard cosmologies - for example including an intermediate inflationary stage to dilute the axion density - the upper bound may be avoided. String compactifications typically have long-lived moduli and so it can be argued that in string theory early universe cosmology should be non-standard. However, low-reheat temperatures generically encounter problems with baryogenesis and dark matter abundance, and so here we restrict to a standard cosmological evolution.

In this case detailed recent analyses of the astrophysical and cosmological constraints on the above 'invisible axion' are [137, 138]. These give a current bound for the axion window as

$$10^9 \text{GeV} \lesssim f_a \lesssim 3 \times 10^{11} \text{GeV}.$$

## 7.1.2 Axions in String Theory

String compactifications generically contain fields  $a_i$  which have  $a_i F \tilde{F}$  couplings and possess the anomalous global symmetry  $a \to a + \epsilon$  featuring in the axionic solution to the strong CP problem. We first enumerate possible axions, before considering their relation to the physics of moduli stabilisation.

### Axions in the Heterotic String

In heterotic compactifications, the axions are traditionally divided into the universal, or model-independent, axion and the model-dependent axions. The model-independent axion is the imaginary part of the dilaton multiplet,  $S = e^{-2\phi} \mathcal{V} + ia$ .

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It is the dual of the 2-form potential  $B_{2,\mu\nu}$  arising from the NS-NS 2-form field:  $da = e^{-2\phi} * dB_{\mu\nu}$ . The dilaton superfield is the tree-level gauge kinetic function for all gauge groups,

$$\mathcal{L} \sim \operatorname{Re}(S) \int F_{\mu\nu}^a F^{a\mu\nu} + \operatorname{Im}(S) \int F_{\mu\nu}^a \tilde{F}^{a\mu\nu}.$$

Consequently any realistic heterotic compactification always has an  $aF_{QCD}\tilde{F}_{QCD}$  coupling.

There are also the model independent axions,  $b_i$ , given by the imaginary parts of the Kähler moduli  $T_i$ . For a basis  $\Sigma_i$  of 2-cycles of the Calabi-Yau,  $T_i = t_i + ib_i$ , with  $t_i = \int_{\Sigma_i} J$  the string frame volume of the cycle  $\Sigma_i$  and  $b_i = \int_{\Sigma_i} B_2$ . Such axions have no tree-level couplings to QCD. However, such a coupling may be generated through the one loop correction to the gauge kinetic function. For the  $E_8 \times E_8$  heterotic string, this correction is

$$f_1 = S + \beta_i T_i, \tag{7.8}$$

$$f_2 = S - \beta_i T_i, \tag{7.9}$$

where 1 and 2 refer to the first and second  $E_8$  respectively. The factors  $\beta_i$  are determined by the gauge bundles on the compactification manifold X. For gauge bundles  $V_1$  and  $V_2$ ,

$$\beta_i = \frac{1}{8\pi^2} \int e_i \wedge (c_2(V_1) - c_2(V_2)), \qquad (7.10)$$

where  $e_i$  is the 2-form dual to the cycle  $\Sigma_i$ . This can be derived by dimensional reduction of the Green-Schwarz term  $\int B_2 \wedge X_8(F_1, F_2, \mathcal{R})$ . The axions associated to the Kähler moduli are called model-dependent as their appearance in f depends on the one-loop correction, which in turn depends on the specific properties of the compactification.

#### Axions in Intersecting Brane Worlds

The discovery of D-branes [24] led to the extension of string model building beyond the heterotic string. The type II string theories, or more properly orientifolds thereof, can give rise to 'intersecting brane worlds'. In these the Standard Model is localised on a stack of branes while gravity propagates in the bulk. Light bifundamental matter arises from strings located at intersection loci and stretching between brane stacks.

<sup>&</sup>lt;sup>1</sup>Strictly, this only gives the correction to Im(f). The corresponding correction to Re(f) is however implied by holomorphy.

The bosonic action of a single Dp-brane is the sum of DBI and Chern-Simons terms,  $\,$ 

$$S_p = \frac{-2\pi}{(2\pi\sqrt{\alpha'})^{p+1}} \left( \int_{\Sigma} d^{p+1}\xi e^{-\phi} \sqrt{\det(g+B+2\pi\alpha'F)} + i \int_{\Sigma} e^{B+2\pi\alpha'F} \wedge \sum_{q} C_q \right). \tag{7.11}$$

 $\Sigma$  is the cycle wrapped by the brane and the sum is a formal sum over all RR potentials in which only relevant terms are picked out.

The gauge kinetic term  $F_{\mu\nu}F^{\mu\nu}$  comes from the DBI action and the instanton action  $F \wedge F$  from the Chern-Simons term. The gauge coupling corresponds to the inverse volume of  $\Sigma$  and the  $\theta$  angle to the component of  $C_{p-3}$  along  $\Sigma$ . These fields pair up to become the scalar component of the chiral multiplet which is the gauge kinetic function of the resulting gauge theory.

As IIB compactifications are our main focus, we shall be more explicit here. In principle, IIB string theory allows, consistent with supersymmetry, space-filling D3, D5, D7 and D9-branes. However, in an orientifold setting we are restricted to either D3/D7 or D5/D9 pairs. In this thesis we have been concerned with the former case. The branes must wrap appropriate cycles to cancel the negative charge and tension carried by the orientifold planes; we assume this has been done.

In such compactifications, the relevant superfields are those of the dilaton and Kähler moduli multiplets. Their scalar components are<sup>2</sup>

$$S = e^{-\phi} + ic_0, (7.12)$$

$$T_i = \tau_i + ic_i. (7.13)$$

 $c_0$  is the Ramond-Ramond 0-form and  $e^\phi \equiv g_s$  the string coupling. For  $\Sigma_i$  a 4-cycle of the Calabi-Yau,

$$c_i = \frac{1}{l_s^4} \int_{\Sigma_i} C_4$$
 and  $\tau_i = \int_{\Sigma_i} \frac{e^{-\phi}}{2} J \wedge J$ ,

where  $l_s = 2\pi\sqrt{\alpha'}$  denotes the string length. Indeed, S is the universal gauge kinetic function for D3-branes, whereas  $T_i$  is the gauge kinetic function for the field theory on a D7-brane stack wrapping the 4-cycle  $\Sigma_i$ .

The axionic couplings arise from the Chern-Simons term in the action. For D3-branes, this gives

$$S_{F\tilde{F}} = \frac{c_0}{4\pi} \int F \wedge F, \tag{7.14}$$

<sup>&</sup>lt;sup>2</sup>As noted in chapter 2, technically this is only for the case that  $h_{-}^{1,1} = 0$ . This will not be important for the issues we discuss, and so we will use this simplifying assumption.

while for D7-branes

$$S_{F\tilde{F}} = \frac{c_i}{4\pi} \int F \wedge F. \tag{7.15}$$

By expanding the DBI action, we obtain the field theory couplings

$$\left. \frac{1}{g^2} \right|_{D3} = \frac{e^{-\phi}}{2\pi} \quad \text{and} \quad \left. \frac{1}{g^2} \right|_{D7} = \frac{\tau_i}{2\pi}.$$

There exists a similar story for IIA intersecting brane worlds, where the Standard Model must be realised on wrapped D6-branes (a Calabi-Yau has no 1- or 5-cycles to wrap D4- or D8-branes on). The gauge coupling now comes from the calibration form  $\text{Re}(\Omega)$  and the axion from the reduction of the 3-form potential  $C_3$ ,

$$\frac{1}{g^2}\Big|_{D6} = \int_{\Sigma_i} \operatorname{Re}(\Omega) \quad \text{and} \quad \theta\Big|_{D6} = \int_{\Sigma_i} C_3.$$

Our interest in this chapter is the interaction of axions with moduli stabilisation and supersymmetry breaking, to which we now turn.

### 7.2 Axions and Moduli Stabilisation

It is obvious from the above that potential axions are easily found in string compactifications; indeed, they are superabundant. For axions to solve the strong CP problem, they must also be light, with QCD instantons giving the dominant contribution to their potential. Light axions are not in themselves problematic. Pure type II Calabi-Yau compactifications have many axions, which remain exactly massless as a consequence of four-dimensional  $\mathcal{N}=2$  supersymmetry. However, the same  $\mathcal{N}=2$  supersymmetry that guarantees the axions remain massless also guarantees a non-chiral spectrum with the axions' scalar partners massless. These are modes of the graviton and will lead to unobserved fifth forces.

More realistic string constructions have  $\mathcal{N}=1$  supersymmetry in four dimensions, allowing a potential to be generated for the moduli. To avoid bounds from fifth force experiments, the size moduli - the saxions - must at a minimum receive masses at a scale  $m_T \gtrsim (100 \mu \text{m})^{-1} \sim 2 \times 10^{-3} \text{eV}$ . However, the expected scale is much larger and typical constructions give moduli masses comparable to the supersymmetry breaking scale. Much of this thesis has been concerned with recent progress made in moduli stabilisation, with fluxes and non-perturbative effects being used to lift the degeneracies associated with the geometric moduli. In the context of strong CP, this same progress creates a modulus anti-stabilisation problem. There are many stringy effects that can generate a potential for a putative QCD axion. These include worldsheet instantons, D-instantons and gaugino

condensation, the last two of which we have already made much use of in chapters 4 and 5. If any one of these effects is more important for a given axion than QCD instantons, that axion does not solve the strong CP problem.

Axions do not receive a potential in perturbation theory. In heterotic compactifications, worldsheet perturbation theory is an expansion about the trivial (point-like) embedding of the worldsheet in spacetime. However, from the definition of the heterotic axions, we see these can only contribute to the action, and thus the Feynman path integral, if the embedding of the worldsheet in spacetime is topologically non-trivial. Such an embdedding is a worldsheet instanton and thus a nonperturbative effect. In IIB compactifications, the axions are components of the RR fields. As the fundamental string is uncharged under these, string perturbation theory cannot generate a potential for them - brane instantons, objects nonperturbative in both  $g_s$  and  $\alpha'$ , are necessary.

Consequently axion potentials come from nonperturbative effects whose magnitude is exponentially sensitive to the values of the stabilised moduli. The natural place to analyse the importance of such effects is therefore within a context in which all moduli are stabilised. To this end we focus on the constructions described in chapters 4 and 5, but along the way we will prove a no-go theorem applicable to all string compactifications and relevant to some recent approaches to moduli stabilisation.

### 7.2.1 The Simplest Scenarios: Why Axions are Heavy

We shall examine both KKLT and large-volume scenarios. Our first point is that the simplest versions of these scenarios lack a QCD axion. All potential axions receive a high scale mass and thus cannot solve the strong CP problem. For simplicity we concentrate on the KKLT construction, but a very similar argument holds for the exponentially large volume compactifications.

We start by asking whether QCD is to be realised on D3 or D7 branes. If we were to use D3-branes, the QCD axion must be the imaginary component of the dilaton multiplet,  $S = e^{-\phi} + ic_0$ . However, as discussed in chapter 2 the dilaton is stabilised at tree-level by the fluxes, with a mass  $m_S \sim \frac{N}{R^6} M_P$ , with R the radius in units of  $l_s$ . We note this perturbative flux stabilisation may seem at odds with the axionic shift symmetry  $c_0 \to c_0 + 2\pi$  and our earlier statement that axions do not get a potential in perturbation theory. However, in this case the axion shift symmetry is a subgroup of the fundamental  $SL(2,\mathbb{Z})$  symmetry, under which the fluxes also transform. The fluxes appear in the low-energy theory as coupling constants through the flux superpotential. If the fluxes have vevs, then at low energy the  $SL(2,\mathbb{Z})$  symmetry is spontaneously broken and cannot prevent the axion acquiring a mass. As  $R \lesssim 5$  in KKLT, the axion obtains a mass  $m_{c_0} \gtrsim 10^{15} \text{GeV}$  and cannot be a QCD axion. For the exponentially large

volume compactifications, R is larger but the conclusion unchanged: QCD on a D3 brane stack is inconsistent with the existence of a Peccei-Quinn axion.

This implies that QCD ought to be realised on D7 branes. The axions are now the imaginary parts of the Kähler moduli, and the instanton effects used to stabilise these moduli will also give the axions a mass. To estimate the scale of this mass, it is simplest just to construct the potential explicitly.

We take the superpotential

$$W = W_0 + \sum_{i=1}^{h^{1,1}} A_i \exp(-a_i T_i), \tag{7.16}$$

and the Kähler potential

$$\mathcal{K} = -2\log \mathcal{V}.\tag{7.17}$$

Note that  $\mathcal{K} = \mathcal{K}(T_i + \bar{T}_i)$ , and so the axions appear neither in  $\mathcal{K}$  nor its derivatives. The supergravity F-term potential is

$$V = e^{\mathcal{K}} (\mathcal{K}^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2). \tag{7.18}$$

The no-scale property of the Kähler potential simplifies (7.18) to

$$V = e^{\mathcal{K}} \left( \mathcal{K}^{i\bar{j}} \partial_i W \partial_{\bar{j}} \bar{W} + \mathcal{K}^{i\bar{j}} \left( (\partial_i \mathcal{K}) W \partial_{\bar{j}} \bar{W} + (\partial_{\bar{i}} \mathcal{K}) \bar{W} \partial_j W \right) \right). \tag{7.19}$$

It is a property of the Kähler potential (7.17) that  $\mathcal{K}^{i\bar{j}}\partial_i\mathcal{K} = -2\tau_j$ . (7.19) becomes

$$V = e^{\mathcal{K}} \left( \mathcal{K}^{i\bar{j}} a_i a_j \left( A_i \bar{A}_j e^{-a_i T_i - a_j \bar{T}_j} + \bar{A}_i A_j e^{-a_i \bar{T}_i - a_j \bar{T}_j} \right) -2a_i \tau_i \left( W \bar{A}_i e^{-a_i \bar{T}_i} + \bar{W} A_i e^{-a_i T_i} \right) \right).$$

$$(7.20)$$

It is easy to extract the axionic dependence of the potential (7.20).  $\mathcal{K}$  and its derivatives are all real and phases only come from the superpotential. The potential becomes

$$V = e^{\mathcal{K}} \left( \mathcal{K}^{i\bar{j}} \left( 2a_i a_j | A_i A_j | e^{-a_i \tau_i - a_j \tau_j} \cos(a_i \theta_i + a_j \theta_j + \gamma_{ij}) \right) -4a_i \tau_i | W_0 A_i | e^{-a_i \tau_i} \cos(a_i \theta_i + \beta_i) - 4a_i \tau_i | A_i A_j | \cos(a_i \theta_i + a_j \theta_j + \gamma_{ij}) \right).$$

$$(7.21)$$

 $\theta_i$  denote the axions and the phases  $\gamma_{ij}$  and  $\beta_i$  come from the phases of  $A_i \bar{A}_j$  and  $\bar{A}_i W$  respectively. The axionic mass matrix is

$$M_{ij}^2 = \frac{\partial^2 V}{\partial \theta_i \partial \theta_j},\tag{7.22}$$

and we obtain $^3$ 

$$M_{ij}^2 \sim \mathcal{O}(a_i a_j) V_{min}, \tag{7.23}$$

with  $V_{min}$  the magnitude of the potential at the AdS minimum. In KKLT,  $D_{T_i}W = 0$  for all i and  $V_{min} \sim -3\frac{|W_0|^2}{\mathcal{V}^2}$ . As there are  $h^{1,1}$  independent phases in the superpotential, there is no reason for  $M_{ij}^2$  to be degenerate and we expect all eigenvalues to be  $\mathcal{O}(a^2V_{min})$ , where a is the typical magnitude of the  $a_i$ .

The determination of physical masses also requires the Kähler potential. In general there is no explicit expression for the overall volume  $\mathcal{V}$  in terms of 4-cycle volumes  $\tau_i$ . The Kähler metric may however be written as [50]

$$\mathcal{K}_{i\bar{j}} = \frac{G_{i\bar{j}}^{-1}}{\mathcal{V}^2}, \qquad G_{i\bar{j}} = -\frac{3}{2} \left( \frac{k_{ijk}v^k}{\mathcal{V}} - \frac{3}{2} \frac{k_{imn}t^m t^n k_{jpq}t^p t^q}{\mathcal{V}^2} \right).$$
(7.24)

If  $\mathcal{V} \sim (\text{a few})l_s^6$ ,  $\mathcal{K}_{i\bar{j}} \sim \mathcal{O}(1)$  and the mass matrix  $M_{ij}^2$  gives a good estimate of the scale of axion masses. If  $\mathcal{V} \gg l_s^6$ , then  $\mathcal{K}_{i\bar{j}} \ll \mathcal{O}(1)$ , and  $M_{ij}^2$  underestimates the axion masses.  $M_{ij}^2$  could only overestimate the axion masses if  $\mathcal{V} \ll 1$ . This realm of moduli space is not accessible in a controlled fashion and we do not concern ourselves with it.

In units where  $M_P = 1$ , we therefore have

$$m_{\tau_i} \sim m_{c_i} \sim a_i \sqrt{V_{min}} \sim \frac{a_i W_0}{V} \sim a_i m_{3/2}.$$
 (7.25)

The axion masses are consequently set by the value of the tree-level superpotential  $W_0$ . This also determines the vacuum energy, the gravitino mass and, implicitly, the energy scale of supersymmetry breaking required to cancel the vacuum energy. TeV-scale soft terms require hierarchically small  $W_0$ . For the (gravity-mediated) case studied in [117, 118, 119], this required  $W_0 \sim 10^{-13}$ , with

$$m_{\tau_i} \sim m_{a_i} \sim m_{3/2} \sim 10 \text{TeV}.$$
 (7.26)

This scale is vastly greater than that associated with QCD instanton effects, and thus the axions are incapable of solving the strong CP problem. We could insist on a QCD axion, and require that  $W_0$  be sufficiently small that QCD instantons dominate over the D-instanton effects of moduli stabilisation. This would require  $W_0 \sim 10^{-40}$ . However, this scenario is entirely excluded as the size moduli are light enough to violate fifth force experiments and the susy breaking scale would be  $\mathcal{O}(10^{-14}\text{eV})$ . Consequently, in the simplest KKLT scenario it is impossible to generate a QCD axion. The D3 axion receives a high scale mass from fluxes,

<sup>&</sup>lt;sup>3</sup>We emphasise this does not say that the mass matrix is a direct product  $a \otimes a$  - such a matrix is degenerate. The point is simply that if the coefficient of the exponent is large, the mass matrix may receive an enhancement.

whereas the instanton effects give the D7 axions large masses comparable to the size moduli.

A similar argument holds for the exponentially large volume compactifications. As in KKLT, the D3 axion receives a large flux-induced mass. The 'small' cycles are stabilised by instanton effects, and these give the corresponding axions masses of a similar scale to the size moduli,  $m_{a_i} \sim m_{\tau_i} \sim m_{3/2}$ . One difference is that there is a modulus, the 'large' modulus  $\tau_b$ , which need not appear in the superpotential. We recall this is stabilised through the Kähler potential, and while it is massive its axionic partner indeed remains massless. However, this cycle is exponentially large, and any gauge group supported on this cycle is far too weakly coupled to be QCD. The same conclusion holds: the simplest version of this scenario does not generate a QCD axion.

The above formulates the 'modulus anti-stabilisation problem': naive scenarios of moduli stabilisation are incompatible with a QCD axion.

We next examine an apparent solution to this problem with a subtle flaw. We require a method of stabilising moduli without stabilising the axions. Axions correspond to phases in the superpotential and do not appear in the Kähler potential. If we included a multi-exponential term  $e^{-\alpha^i T_i}$  in the superpotential, a massless axion would certainly survive, as at least one phase would be absent. As the size moduli all appear in the Kähler potential, by solving the F-term equations we may hope to stabilise the size moduli while keeping some axions massless.

To illustrate this idea, let us consider a toy KKLT model on a toroidal orientifold,

$$\mathcal{K} = -2\ln(\mathcal{V}) = -\ln(T_1 + \bar{T}_1) - \ln(T_2 + \bar{T}_2) - \ln(T_3 + \bar{T}_3), \quad (7.27)$$

$$W = W_0 + Ae^{-2\pi(T_1 + T_2 + T_3)}. \quad (7.28)$$

The Kähler potential (7.27) is that appropriate for toroidal orbifolds, with  $\mathcal{V} = t_1 t_2 t_3$ . To understand where the superpotential could arise from, we can hypothesise that the cycle (1+2+3) is the smallest cycle with only two fermionic zero modes, and that instantons wrapping (for example) the cycle (2+3) all have more than two zero modes and do not appear in the superpotential. However, we are not here really concerned with the microscopic origin of the superpotential: at this level we simply regard equations (7.27) and (7.28) as defining the model.

The F-term equations  $D_{T_1}W = D_{T_2}W = D_{T_3}W = 0$  give

$$-2\pi A e^{-2\pi(T_1+T_2+T_3)} - \frac{1}{T_1 + \bar{T}_1} \left( W_0 + A e^{-2\pi(T_1+T_2+T_3)} \right) = 0, \quad (7.29)$$

$$-2\pi A e^{-2\pi(T_1+T_2+T_3)} - \frac{1}{T_2 + \bar{T}_2} \left( W_0 + A e^{-2\pi(T_1+T_2+T_3)} \right) = 0, \quad (7.30)$$

$$-2\pi A e^{-2\pi(T_1+T_2+T_3)} - \frac{1}{T_3 + \bar{T}_3} \left( W_0 + A e^{-2\pi(T_1+T_2+T_3)} \right) = 0. \quad (7.31)$$

These immediately imply

$$\tau_1 = \tau_2 = \tau_3, \tag{7.32}$$

with equations (7.29) to (7.31) collapsing to

$$2\pi A e^{-6\pi\tau_1} e^{-2\pi i(\theta_1 + \theta_2 + \theta_3)} + \frac{1}{2\tau_1} \left( W_0 + A e^{-6\pi\tau_1} e^{-2\pi i(\theta_1 + \theta_2 + \theta_3)} \right) = 0.$$
 (7.33)

While the sum  $\theta_1 + \theta_2 + \theta_3$  is fixed, there are clearly two axionic directions not relevant for the solution of the F-term equations. On the other hand, the combination of (7.32) and (7.33) imply there is a unique value for the size moduli such that the F-term equations are solved. Except for the massless axionic directions, the scales of the masses are unaltered from above, and we would expect

$$m_{\tau_i} \sim m_{\theta_1 + \theta_2 + \theta_3} \sim \frac{W_0}{\mathcal{V}}, \qquad m_{\theta_1 - \theta_2} = m_{\theta_1 - \theta_3} = 0.$$
 (7.34)

As this is supergravity rather than rigid supersymmetry, there is no contradiction in having a mass splitting for the multiplet in the presence of unbroken supersymmetry.

While this is superficially promising, in fact the above has a serious problem. Even though all F-term equations can be solved, numerical investigation shows that at the supersymmetric locus the resulting scalar potential is tachyonic, with signature (+, -, -). Although supersymmetry ensures the moduli are Breitenlohner-Freedman stable [139], this notion of AdS stability ceases to be relevant after the (required) uplift.

We now show that these tachyons are in fact generic for any attempt to stabilise the moduli supersymmetrically while preserving one or more unfixed axions.

### 7.2.2 A No-Go Theorem

We start with an arbitrary  $\mathcal{N}=1$  supergravity theory with moduli fields,  $\Phi_{\alpha}$ ,  $T_{\beta}=\tau_{\beta}+ib_{\beta}$ , where the  $b_{\beta}$  are the axions. We write the superpotential and Kähler potential as

$$W = W(\Phi_{\alpha}, T_{\beta}), \tag{7.35}$$

$$\mathcal{K} = \mathcal{K}(\Phi_{\alpha}, T_{\beta} + \bar{T}_{\beta}). \tag{7.36}$$

The Peccei-Quinn symmetry  $b_{\beta} \to b_{\beta} + \epsilon_{\beta}$  implies the form of (7.36) should hold in perturbation theory.

We further suppose we have solved

$$D_{\Phi_{\alpha}}W = 0 \text{ and } D_{T_{\beta}}W = 0 \tag{7.37}$$

for all  $\alpha$  and  $\beta$ , but that at least one axion  $b_u = \sum_{\beta} \lambda_{\beta} b_{\beta}$  is unfixed: the solution to (7.37) is independent of  $\langle b_u \rangle$ .

We redefine the basis of chiral superfields so that there exists a superfield  $T_u$  with  $b_u = \text{Im}(T_u)$ ,

$$T_1 \rightarrow T_u,$$
 $T_2 \rightarrow T_2,$ 
 $T_n \rightarrow T_n.$ 

$$(7.38)$$

This is a good redefinition as it does not affect holomorphy properties.

As the solution to all F-term equations is independent of  $b_u$ ,  $b_u$  is a flat direction of the potential (7.18) at the supersymmetric locus.<sup>4</sup> The potential at the supersymmetric locus is given by

$$V = -3e^{\mathcal{K}}|W|^2. \tag{7.39}$$

As  $b_u$  does not appear in  $\mathcal{K}$ , it follows that if  $b_u$  is a flat direction |W| must be independent of  $b_u$ . Up to one exception this then implies that W is independent of  $b_u$ .

The sole exception is if  $b_u$  purely represents an overall phase, i.e.  $W = e^{-aT_u}$  with no constant term. This may arise if the flux superpotential vanishes exactly due to a discrete symmetry [71], while a combination of non-perturbative effects and Kähler corrections stabilise the Kähler moduli. While potentially interesting, this is an exceptional case for which moduli stabilisation is not well understood, and we do not analyse it further.

If W has no explicit dependence on  $b_u = \text{Im}(T_u)$ , it follows by holomorphy that it also has no explicit dependence on  $\tau_u = \text{Re}(T_u)$  and thus no explicit dependence on  $T_u$ . Therefore

$$\partial_{T_u} W \equiv 0. \tag{7.40}$$

However, as  $D_{T_u}W = 0$ , it follows that at the supersymmetric locus,

either 
$$(\partial_{T_n} \mathcal{K}) = 0$$
 or  $W = 0$ .

The latter is overdetermined and non-generic, so we first focus on  $(\partial_{T_u}\mathcal{K})=0$ .

Direct calculation now shows that the  $\tau_u$  direction is tachyonic at the supersymmetric locus. To see this, note that from the scalar potential (7.18)

$$\partial_{\tau_u} V = e^{\mathcal{K}} \mathcal{K}^{i\bar{j}} \left( \partial_{\tau_u} (D_i W) D_{\bar{j}} \bar{W} + D_i W \partial_{\tau_u} (D_{\bar{j}} \bar{W}) \right) - 3(\partial_{\tau_u} \mathcal{K}) e^{\mathcal{K}} W \bar{W}. \tag{7.41}$$

<sup>&</sup>lt;sup>4</sup>The requirement of *flatness* is stronger than the requirement that the axion simply be *massless*. Flatness is the right requirement, as if an axion is fixed in any way it does not solve strong CP.

We have used  $\partial_{\tau_u} W \equiv 0$  and have only kept terms that will give non-vanishing contributions to  $\partial_{\tau_u} \partial_{\tau_u} V$  at the supersymmetric locus. Expanding  $D_i W$  and again using  $\partial_{\tau_u} W \equiv 0$ , (7.41) simplifies to

$$\partial_{\tau_u} V = e^{\mathcal{K}} \mathcal{K}^{i\bar{j}} \left( \partial_{\tau_u} (\partial_i \mathcal{K}) W(D_{\bar{j}} \bar{W}) + D_i W \partial_{\tau_u} (\partial_{\bar{j}} \mathcal{K}) \bar{W} \right) - 3(\partial_{\tau_u} \mathcal{K}) e^{\mathcal{K}} W \bar{W}. \tag{7.42}$$

If we again only keep terms non-vanishing at the supersymmetric locus, the second derivative is

$$\partial_{\tau_n} \partial_{\tau_n} V = e^{\mathcal{K}} \mathcal{K}^{i\bar{j}} \left( 2 \partial_{\tau_n} (\partial_i \mathcal{K}) \partial_{\tau_n} (\partial_{\bar{j}} \mathcal{K}) W \bar{W} \right) - 3 (\partial_{\tau_n} \partial_{\tau_n} \mathcal{K}) e^{\mathcal{K}} W \bar{W}. \tag{7.43}$$

Now, as  $\tau_u = \frac{1}{2}(T_u + \bar{T}_u)$ ,

$$\partial_{\tau_u} \mathcal{K}(T + \bar{T}) = 2\partial_{T_u} \mathcal{K}(T + \bar{T}),$$

and we have

$$\partial_{\tau_u} \partial_{\tau_u} V = 4e^{\mathcal{K}} W \bar{W} (2\mathcal{K}^{i\bar{j}} \mathcal{K}_{iu} \mathcal{K}_{u\bar{j}} - 3\mathcal{K}_{u\bar{u}})$$
$$= -4e^{\mathcal{K}} W \bar{W} \mathcal{K}_{u\bar{u}}, \tag{7.44}$$

where we have used  $\mathcal{K}_{i\bar{j}} = \mathcal{K}_{ij}$ . As  $\mathcal{K}_{i\bar{j}}$  is a metric,  $\mathcal{K}_{u\bar{u}}$  is positive definite and it follows that the  $\tau_u$  direction is tachyonic.

Now consider the W=0 case. As indicated above, this is non-generic: even if W originally vanishes, it is expected to receive non-perturbative corrections which make it non-vanishing. Even so, it follows easily that in the case W=0

$$\partial_{\tau_u} \partial_{\tau_u} V = 0, \tag{7.45}$$

and so the  $\tau_u$  size modulus is massless, leading to unobserved fifth forces.

The above gives a no-go theorem: there does not exist any supersymmetric minimum of the F-term potential consistent with stabilised moduli and unfixed axions.

It is in the nature of no-go theorems that they admit loopholes, so let us discuss ways around this result. One point to consider is the form of the Kähler potential, as we have used in the above argument the fact that

$$\mathcal{K} = \mathcal{K}(T + \bar{T}). \tag{7.46}$$

While true in perturbation theory because of the axionic shift symmetry, this equation will break down nonperturbatively, and the argument showing that the  $\tau_u$  direction is tachyonic will no longer hold. However, this same breakdown will cause  $\mathcal{K}$ , and hence the potential V, to depend on the axion. As this lifts the required axionic flat direction, the no-go theorem will cease to apply.

A second loophole is that although the F-term potential might be tachyonic, the D-term potential might come to the rescue. For example, a Fayet-Iliopoulos term might have exactly the right structure to render the supersymmetric locus an actual minimum of the full potential. However this seems implausible in the presence of many tachyonic directions. A similar approach would be to try and set W=0 and then rely entirely on D-terms to stabilise the moduli. Another possibility (discussed recently in [140]) is that an anomalous U(1) might remove the tachyonic directions. While the massive gauge boson will eat the axionic degree of freedom, an axionic direction may survive in the phase of a scalar charged under the U(1).

A third loophole is that stability might not rely on the existence of an actual minimum for the potential. Equation (7.44) involves the Kähler metric  $\mathcal{K}_{u\bar{u}}$ . The kinetic term for  $\tau_u$  is  $\mathcal{K}_{u\bar{u}}\partial_{\mu}\tau_u\partial^{\mu}\tau_u$ . If we just consider the  $\tau_u$  direction, the physical mass is therefore

$$m_{\tau_u}^2 = -2e^{\mathcal{K}}W\bar{W} = -\frac{8}{9}|m_{BF}|^2,$$
 (7.47)

where  $m_{BF}$  is the relevant Breitenlohner-Freedman bound. As tachyonic modes can be stable in AdS, one could argue that it is sufficient simply to solve the F-term equations, without worrying about whether the resulting locus is an actual minimum of the potential.

While this point is more substantial, it does not resolve the problem. The real world is not AdS, and for stability requires a positive definite mass matrix. For any realistic model, the vacuum energy must be uplifted such that it vanishes. After this uplift, the extra geometric advantages of AdS go away and the tachyons can no longer be supported. As there may be many tachyons present - one for each massless axion - the entire problem of moduli stabilisation must necessarily be solved over again in the uplifting. As the uplift is generally the least controlled part of the procedure, this is a hard problem. While the uplift may remove the tachyons, at the present level of understanding this is pure hypothesis. It is then very unclear how useful the original supersymmetric AdS saddle point is, and whether it is a suitable locus to uplift.

The argument above suggests that supersymmetric solutions are unpromising starting points from which to address the strong CP problem. Either all moduli appear in the superpotential, in which case there is no light axion, or a modulus is absent from the superpotential, in which case the potential is tachyonic. The fourth and most obvious loophole is to give up on the requirement of supersymmetric minima, and search for nonsupersymmetric minima of the potential with massless axions.

We shall consider this point in the next section. Before we do so we first note that this result has consequences for several approaches to moduli stabilisation considered in the literature. For example, in the weakly coupled heterotic string, the one-loop corrections to the gauge kinetic function (7.8) imply that gaugino condensation generates a superpotential

$$W_{n.p.} = Ae^{-\alpha S + \beta_i T^i}. (7.48)$$

It has been proposed [141] to use the superpotential (7.48), together with a constant term  $W_0$ , to stabilise both dilaton and Kähler moduli by solving  $D_SW = D_{T^i}W = 0$ . However, there is only one phase - and hence only one axion - explicitly present in (7.48). The above argument shows that the resulting scalar potential will actually be tachyonic at the supersymmetric locus, with signature  $(+, -, \ldots, -)$ .

A similar problem exists in several of the IIA flux compactifications recently studied with both 2-form and 3-form fluxes turned on [142, 143]. In this context it is observed that it is possible to solve all the F-term equations  $D_iW = 0$ , and it has been proposed to use this to stabilise the moduli. However, the solution of these equations is independent of many of the axions present (the imaginary parts of the complex structure moduli, namely those associated with the  $C_3$  field). Consequently as long as  $h^{2,1} \neq 0$  the supersymmetric locus is always tachyonic, with one tachyon present for every unfixed axion. Tachyons have been recognised in particular models [140, 142, 144], but from the above they would seem to be a very generic problem for such approaches.

## 7.2.3 Non-supersymmetric Minima with Massless Axions

The above no-go theorem shows that supersymmetric moduli stabilisation is not a good starting point from which to solve the strong CP problem. This is an argument in favour of non-supersymmetric moduli stabilisation. That there is no no-go theorem for non-supersymmetric minima with massless axions can be shown by construction: for example, the large volume compactifications of chapter 5 all contain a massless axion associated with the large cycle controlling the overall volume. As indicated previously, this cannot be a QCD axion, as any brane on this cycle is very weakly coupled. However, it does serve to show that the no-go theorem does not apply to non-supersymmetric stabilisation. If we want to try and force this cycle into being a QCD axion, we can tune the parameters to force the minimum of this potential to relatively small volumes. This corresponds to lowering  $W_0$  to bring the minimum in to smaller volumes - that this occurs can be confirmed numerically. Another possibility would be a purely perturbative stabilisation of the volume modulus, solely using Kähler corrections (in which axions do not appear). This has been discussed in [145, 146], although without an explicit example.

I shall not dwell on these possibilities. First, because the resulting axion decay constant would be, as we shall see shortly, close to the Planck scale and outside the allowed window and secondly, because at such small volumes there is

no good control parameter. I shall instead focus on a version of the large-volume compactifications of chapter 5, suitably modified to include a QCD axion. In this section I only discuss moduli stabilisation but in section 7.3 I shall show that these can also realise phenomenological values for  $f_a$ .

I will illustrate the discussion with a three-modulus toy model, in which I assume the volume may be expressed in terms of 4-cycles as

$$\mathcal{V} = (T_1 + \bar{T}_1)^{\frac{3}{2}} - (T_2 + \bar{T}_2)^{\frac{3}{2}} - (T_3 + \bar{T}_3)^{\frac{3}{2}}. \tag{7.49}$$

The use of three moduli is because this turns out to be the minimal number required for our purposes: clearly, this is no significant restriction on model-building. Expressed in terms of 2-cycles, (7.49) corresponds to

$$\mathcal{V} = \lambda (t_1^3 - t_2^3 - t_3^3). \tag{7.50}$$

We note (7.50) satisfies the consistency requirement that  $\frac{\partial^2 \mathcal{V}}{\partial t_i \partial t_j}$  have signature (+, -, -). We may perhaps think of this toy model as a  $\mathbb{P}^3$  with two points blown up. Denoting the cycles by 1, 2 and 3, the Kähler potential including perturbative corrections is

$$\mathcal{K} = -2\ln\left((T_1 + \bar{T}_1)^{\frac{3}{2}} - (T_2 + \bar{T}_2)^{\frac{3}{2}} - (T_3 + \bar{T}_3)^{\frac{3}{2}}\right) - \frac{\xi}{g_s^{3/2}\mathcal{V}}.$$
 (7.51)

 $g_s$  is fixed by the fluxes and in (7.51) should be regarded as a tunable parameter. For superpotential, we shall take

$$W = W_0 + e^{-\frac{2\pi}{n}(T_2 + T_3)}. (7.52)$$

This could arise from gaugino condensation on a stack of n branes wrapping the combined cycle  $2+3.^5$  QCD will be realised as a stack of branes wrapping cycle 3.

In the limit  $\mathcal{V} \gg 1$  with  $\tau_2$  and  $\tau_3$  small, the leading functional form of the scalar potential is (omitting numerical factors)

$$V = \frac{(\sqrt{\tau_2} + \sqrt{\tau_3})e^{-\frac{2\pi}{n}2(\tau_2 + \tau_3)}}{\mathcal{V}} - \frac{(\tau_2 + \tau_3)e^{-\frac{2\pi}{n}(\tau_2 + \tau_3)}}{\mathcal{V}^2} + \frac{\xi}{g_s^{3/2}\mathcal{V}^3}.$$
 (7.53)

The minus sign in (7.53) arises from minimising the potential for the axion  $\text{Im}(T_2 + T_3)$ . The axions  $\text{Im}(T_1)$  and  $\text{Im}(T_2 - T_3)$  do not appear in (7.53) and are unfixed. By considering the limit  $\mathcal{V} \to \infty$ ,  $\frac{2\pi(\tau_2 + \tau_3)}{n} \sim \ln \mathcal{V}$ , it follows that as  $\mathcal{V} \to \infty$  the potential (7.53) goes to zero from below. As by adjusting  $g_s$  we

 $<sup>\</sup>overline{\phantom{a}}^5$ The use of gaugino condensation rather than instanton effects is necessary to ensure that the cycle 2+3 is large enough to contain QCD.

can make the third term of (7.53) arbitrarily large, we can ensure the potential remains positive until arbitrarily large volumes, and thus any minimum will be at exponentially large volumes.

Is there a minimum? The potential is clearly symmetric under  $\tau_2 \leftrightarrow \tau_3$ , and the potential restricted to the locus  $\tau_2 = \tau_3$  indeed has a minimum at exponentially large volumes. Because of the symmetry  $\tau_2 \leftrightarrow \tau_3$ , this 'minimum' is also a critical point of the full potential. However, it is not a minimum of the full potential. At fixed  $\tau_2 + \tau_3$  and fixed  $\mathcal{V}$ , (7.53) depends only on  $\sqrt{\tau_2} + \sqrt{\tau_3}$ . For fixed  $\tau_2 + \tau_3$ , this is maximised at  $\tau_2 = \tau_3$ , and so the mode  $\tau_2 - \tau_3$  is tachyonic at this locus. We have not investigated whether this tachyon satisfies the Breitenlohner-Freedman bound for AdS stability. This is for the same reasons as above: once we uplift, the geometric protection of AdS ceases to be relevant. Consequently the fields in (7.53) run away either to  $\tau_2 = 0$  or  $\tau_3 = 0$ , where one of the blow-up cycles collapses.

This result shows that the above toy model does not, by itself, have a minimum of the potential with a massless QCD axion. We may ask whether this is a feature of the geometric details of the model - for example, whether a different choice of triple intersection form in (7.50) would alter this result. We have investigated several other toy models without finding a minimum, and while we have no proof we suspect none exists so long as the Kähler potential is given by (7.51).

This is bad news, but it is controllable bad news. The instability above is very particular: there is no instability either for the overall volume or for the sum of the blow-up volumes  $\tau_1 + \tau_2$ , but only for the difference  $\tau_1 - \tau_2$ . The effect of the instability is to drive one of the blow-up cycles to collapse. Consequently, the instability can be cured by *any* effect that becomes important at small cycle volume and prevents collapse.

For example, the presence of a term

$$\frac{1}{\sqrt{\tau_2}\mathcal{V}^3} + \frac{1}{\sqrt{\tau_3}\mathcal{V}^3} \tag{7.54}$$

in (7.53) would obviously stabilise the cycles  $\tau_2$  and  $\tau_3$  against collapse and generate a minimum of the potential. As this term does not affect the argument that in the  $\mathcal{V} \to \infty$  limit the potential approaches zero from below, the resulting minimum would still be at exponentially large volume.

At this point it is useful to recall the discussion of section 5.2 on the general form of Kähler corrections that are and are not allowed. Here we just note the terms of (7.54) may be generated from a correction to the Kähler potential,

$$\mathcal{K} + \delta \mathcal{K} = -2\ln(\mathcal{V}) + \frac{\epsilon \sqrt{\tau_2}}{\mathcal{V}} + \frac{\epsilon \sqrt{\tau_3}}{\mathcal{V}}.$$
 (7.55)

For simplicity we have kept the  $\tau_2 \leftrightarrow \tau_3$  symmetry. Such a correction is motivated by the fact that it gives corrections to the Kähler metrics  $\mathcal{K}_{2\bar{2}}$  and  $\mathcal{K}_{3\bar{3}}$  suppressed

by factors of  $g^2$  for the field theory on the relevant cycle. More specifically,

$$\mathcal{K}_{2\bar{2}} + \delta \mathcal{K}_{2\bar{2}} = \frac{3}{2\sqrt{2\tau_2}\mathcal{V}} \left( 1 - \frac{\epsilon}{12\sqrt{2}\tau_2} \right). \tag{7.56}$$

As  $\tau_2 = \frac{1}{g^2}$  for a brane wrapping the cycle 2, the correction is suppressed by  $g^2$ .

The inverse metric involves an infinite series of terms diverging in the  $\tau_2 \to 0$  and  $\tau_3 \to 0$  limit. For example,

$$\mathcal{K}^{2\bar{2}} = \frac{2\sqrt{2\tau_2}\mathcal{V}}{3} \left( 1 + \frac{\epsilon}{12\sqrt{2}\tau_2} + \frac{\epsilon^2}{288\tau_2^2} + \dots \right). \tag{7.57}$$

This is easy to understand: at  $\tau_2 = \frac{\epsilon}{12\sqrt{2}}$ , the Kähler metric  $\mathcal{K}_{2\bar{2}}$  goes to zero and the inverse metric diverges. This divergence can be seen by resumming (7.57). In the physical potential, this divergence will create a positive wall at finite values of  $\tau_2$  and  $\tau_3$ . The positivity can be seen from the fact that the divergence in  $\mathcal{K}^{-1}$  will only appear through the term

$$e^{\mathcal{K}} \mathcal{K}^{i\bar{j}} D_i W D_{\bar{i}} \bar{W},$$
 (7.58)

which is manifestly positive definite. Consequently the potential will diverge positively at finite values of  $\tau_2$  and  $\tau_3$ , and so a stable minimum must exist for both  $\tau_2$  and  $\tau_3$ .

We have now outlined, in the context of the large-volume scenario of chapter 5, a way to stabilise all the size moduli while retaining a massless QCD axion. We would like cycle 3 to support QCD: by adjusting  $\epsilon$ , we can always make the correction (7.55) sufficiently large to ensure that  $\tau_3$  is stabilised with the correct size for QCD. For intermediate string scales, this requires  $\tau_3 \sim 10$ . As the actual correction would be very hard to calculate, at this level we just adjust  $\epsilon$  phenomenologically. Of course, the complexity of a real model is much greater than that of (7.55). However we note again that, even though the corrections cannot easily be calculated, our proposal for moduli stabilisation only requires that they exist and come with the right sign to prevent collapse.

While the above has been a toy example, the above approach may be applied to any model in which the moduli are stabilised along the lines of [3, 4]. Keeping an axion massless introduces an instability causing a blow-up cycle to want to collapse. Kähler corrections that become important at small volume can stabilise this cycle but will not affect the overall structure of the potential, and in particular will not affect the stabilisation of the volume at  $\mathcal{V} \gg 1$ .

## 7.2.4 Higher Instanton Effects in the Axion Potential

We have given above a Kähler potential and superpotential that will stabilise the moduli while containing a candidate QCD axion. The nonperturbative terms in

the superpotential are in general just the leading terms in an instanton expansion. Even though the higher order terms may be highly suppressed and irrelevant to moduli stabilisation, they could still lift the flat direction associated with the massless axion. Given that  $\Lambda_{QCD} \ll M_P$ , even highly subleading terms could dominate over QCD instantons.

Let us estimate the general magnitude of such instanton effects. The magnitude of brane instantons depends on the volumes of the cycles they can wrap. Generally there will be many such cycles, whose sizes depend on the stabilised moduli, but minimally there must always exist the cycle which support the QCD stack. It follows from the DBI action that the gauge coupling for a D7-brane stack is

$$\frac{1}{g^2} = \frac{\text{Re}(T)}{2\pi} \Rightarrow \alpha^{-1} = \frac{4\pi}{g^2} = 2\text{Re}(T) = 2\tau.$$

This defines the gauge coupling at the high scale where the effective field theory becomes valid. When  $m_s \sim M_P$ , this is in essence the string scale, but if  $m_s \ll M_P$ , the difference between  $m_{KK}$  and  $m_s$  becomes significant. It is a subtle issue whether  $m_s$  or  $m_{KK}$  is the appropriate high scale. If QCD is supported on a small cycle within a large internal space, the KK modes associated with the bulk will be uncharged under QCD and will not contribute to the running coupling. KK modes of the QCD cycle will contribute, but these will be at masses comparable to the string scale. In considering the running coupling, we therefore use  $m_s$  as the high scale rather than  $m_{KK}$ . We consider a wide range of string scales and take a sampling of high scale values from  $10^8 \to 10^{16}$  GeV. Given a string scale, we determine the appropriate internal volume by  $m_s \sim \frac{M_P}{\sqrt{\mathcal{V}}}$ .

The QCD coupling runs logarithmically with energy scale, with

$$\alpha_{QCD}^{-1}(10^2 {\rm GeV}) \sim 9$$
 and  $\alpha_{QCD}^{-1}(10^{16} {\rm GeV}) \sim 25$ .

The required high-scale couplings and cycle sizes are given in table 7.1, together with the action for a D3-brane instanton wrapping the same cycle as the QCD stack. Its magnitude is set by  $\sim e^{-2\pi T}$  and we show in table 7.1 the approximate magnitude of single- and double-instanton effects. In addition to the QCD cycle, there may be other cycles which instantons may wrap. We do not include these for two reasons: first, whether such instantons would generate a potential for the QCD axion is model-dependent<sup>6</sup>, and secondly, we can always arrange the model such that the QCD cycle is smallest and hence dominates the instanton expansion. We observe that the magnitude of the required cycle volume, and thus the magnitude of potential instanton effects, varies significantly with the string scale. If present, such D-instantons would generate a potential for the

<sup>&</sup>lt;sup>6</sup>It seems odd that such instantons could affect the QCD axion at all. However, if QCD is on cycle 3, and the axion  $b_2 + b_3$  is fixed by effects on cycle 2+3, an instanton solely on cycle 2 effectively generates a potential for the QCD axion.

$\mathrm{E}_{UV}$	$10^8 \; \mathrm{GeV}$	$10^{10}~{\rm GeV}$	$10^{12}~{\rm GeV}$	$10^{14}~{\rm GeV}$	$10^{16}~{\rm GeV}$
$\alpha_{QCD}^{-1}(\mathbf{E}_{UV})$	15.8	18.1	20.4	22.7	25
$Re(T) = \frac{\alpha^{-1}}{2}$	7.9	9.1	10.2	11.4	12.5
$e^{-2\pi T}$	$2.8 \times 10^{-22}$	$1.5 \times 10^{-25}$	$1.5 \times 10^{-28}$	$7.8 \times 10^{-32}$	$7.8 \times 10^{-35}$
$e^{-4\pi T}$	$7.7 \times 10^{-44}$	$2 \times 10^{-50}$	$2 \times 10^{-56}$	$6 \times 10^{-63}$	$6 \times 10^{-69}$

Table 7.1: Cycle sizes and instanton amplitudes for various UV scales

Table 7.2: Magnitude of Axion Potentials from Superpotential Instanton Effects

$\mathrm{E}_{UV}$	$10^8 \text{ GeV}$	$10^{10}~{\rm GeV}$	$10^{12}~{\rm GeV}$	$10^{14}~{\rm GeV}$	$10^{16}~{\rm GeV}$
$\mathcal{V}$	$10^{20}$	$10^{16}$	$10^{12}$	$10^{8}$	$10^{4}$
$V_{1-instanton}$	$10^{-57}M_P^4$	$10^{-56}M_P^4$	$10^{-55}M_P^4$	$10^{-54} M_P^4$	$10^{-53}M_P^4$
$V_{2\text{-instanton}}$	$10^{-79}M_P^4$	$10^{-81}M_P^4$	$10^{-83}M_P^4$	$10^{-85} M_P^4$	$10^{-87} M_P^4$
$V_{3-instanton}$	$10^{-101}M_P^4$	$10^{-106} M_P^4$	$10^{-111}M_P^4$	$10^{-116}M_P^4$	$10^{-121}M_P^4$

QCD axion. To compare their magnitude to that of QCD effects, we need an estimate of their contribution to the scalar potential. In this context we only care about terms containing a phase and so contributing to the axion potential. To this end, the relevant term from the scalar potential (7.19) is

$$V_{axion} = e^{\mathcal{K}} \left( \mathcal{K}^{i\bar{j}} \left( \partial_i W(\partial_{\bar{j}} \mathcal{K}) \bar{W} + c.c. \right) \right). \tag{7.59}$$

A superpotential instanton contribution  $e^{-2\pi nT_i}$  generates a term

$$V_{axion} = \frac{-2a_i \tau_i W_0}{\mathcal{V}^2} e^{-2\pi n \tau_i} \cos(\theta_i). \tag{7.60}$$

The absolute magnitude of (7.60) depends on the value of n, the internal volume  $\mathcal{V}$  and the tree-level superpotential  $W_0$ . As we are looking towards phenomenology we also take  $m_{3/2} = \frac{W_0}{\mathcal{V}} \sim 1 \text{TeV} \sim 10^{-15} M_P$ , as appropriate for gravity-mediated TeV-scale soft terms. Note that for models built around the KKLT scenario, we always have  $m_s \gtrsim M_{GUT}$  and only the largest value of  $E_{UV}$  is achievable. In table 7.2 we give the internal volumes required for each UV scale, as well as the resulting absolute magnitude of 1-, 2- and 3-instanton superpotential corrections to the scalar potential. For the same reasons as above, we only consider instantons wrapping the QCD cycle.

As well as superpotential effects, there are also nonperturbative corrections to the Kähler potential (the perturbative corrections to  $\mathcal{K}$  do not have an axionic dependence). While smaller, these are easier to generate - the instantons can

$\mathrm{E}_{UV}$	$10^8 \text{ GeV}$	$10^{10}~{\rm GeV}$	$10^{12}~{\rm GeV}$	$10^{14}~{\rm GeV}$	$10^{16}~{\rm GeV}$
V	$10^{20}$	$10^{16}$	$10^{12}$	$10^{8}$	$10^{4}$
$V_{1-instanton}$	$10^{-77}M_P^4$	$10^{-72}M_P^4$	$10^{-67}M_P^4$	$10^{-62}M_P^4$	$10^{-57}M_P^4$
$V_{2-instanton}$	$10^{-99}M_P^4$	$10^{-97}M_P^4$	$10^{-95}M_P^4$	$10^{-93}M_P^4$	$10^{-91}M_P^4$
$V_{3-instanton}$	$10^{-121}M_P^4$	$10^{-122}M_P^4$	$10^{-123}M_P^4$	$10^{-124}M_P^4$	$10^{-125}M_P^4$

Table 7.3: Magnitude of Axion Potentials from Kähler Potential Instanton Effects

have four fermionic zero modes rather than only two. A correction

$$\mathcal{K} = -2\ln(\mathcal{V}) \to \mathcal{K} = -2\ln(\mathcal{V} + e^{-2\pi nT}) \tag{7.61}$$

will generate effects in the scalar potential at order

$$V_{\delta\mathcal{K}} \sim \frac{W_0^2}{\mathcal{V}^3} e^{-2\pi nT}.$$

and thus generate a potential for the QCD axion  $\theta$  of the form

$$V_{\delta K}\cos(\theta+\alpha)$$
.

Again assuming TeV-scale (visible) SUSY breaking,  $\frac{W_0}{V} \sim 10^{-15}$ , the magnitudes of such effects are shown in table 7.3.

The axion potential originating from QCD effects and relevant to the strong CP problem is

$$V_{QCD} \sim \Lambda_{QCD}^4 (1 - \cos(\theta)),$$

with  $\Lambda_{QCD} \sim 2 \times 10^{-19} M_P$  and  $\Lambda_{QCD}^4 \sim 10^{-75} M_P^4$ . We require QCD effects to be sufficiently dominant to be consistent with the failure to observe CP violation in strong interactions. Suppose we have a potential

$$V = A(1 - \cos(\theta)) + \epsilon \cos(\theta + \gamma). \tag{7.62}$$

If  $A \gg \epsilon$ , the minimum is displaced from  $\theta = 0$  by  $\delta\theta \sim \frac{\epsilon}{A}$ . Observationally,  $|\theta| < 10^{-10}$ , and thus non-QCD contributions must have absolute magnitude smaller than  $10^{-85} M_P^4$ . By comparison with tables 7.2 and 7.3 it follows that in order for QCD instantons to dominate the axion potential, the single instanton corrections to both the superpotential and the Kähler potential must be absent, while the 2-instanton superpotential correction may or may not be present depending on factors of  $2\pi$  and the precise value of the string scale.

To contribute to a superpotential (Kähler potential) an instanton must have at most 2 (4) fermionic zero modes, to generate  $\int d^4x d^2\theta$  and  $\int d^4x d^2\theta d^2\bar{\theta}$  terms respectively. In the absence of flux, there is a necessary condition on a divisor to

generate a superpotential [77]: it must have holomorphic Euler characteristic one,  $\chi_g(D)=1$ . In the presence of flux, this condition may be relaxed. The number of zero modes on an instanton, and thus its ability to appear in either the Kähler or superpotential, may also be affected by the presence of the stack of QCD branes wrapping the would-be instanton cycle. We shall not attempt to analyse this question for specific 'real' models, but by fiat simply assume the necessary instantons to be absent from the potential. In this regard it is encouraging that the number of instantons required to be suppressed is quite limited.

## 7.3 The Axion Decay Constant

The last section was devoted to solving the strong CP problem: keeping an axion light while stabilising the moduli. However, even achieving this does not resolve all phenomenological problems. Given that a QCD axion exists, as indicated above there are strong bounds on the axion decay constant  $f_a$  of equation (7.3):  $10^9 \text{GeV} \lesssim f_a \lesssim 10^{12} \text{GeV}$ . The lower bound, from supernova coolings is hard. While the upper bound may be relaxed by considering non-standard cosmologies, here we shall also treat this as hard. We want to estimate  $f_a$  in some moduli stabilisation scenarios (the value of  $f_a$  in string compactifications is also discussed in [147, 148]). In IIB compactifications, the axionic coupling to QCD arises from the clean and model-independent Chern-Simons coupling. However, to obtain the physical value of  $f_a$  the axion must be canonically normalised. This depends on the Kähler metric, and in particular on where the moduli are stabilised.

We assume we can write the Kähler potential as

$$\mathcal{K} = \mathcal{K}(T_i + \bar{T}_i), \tag{7.63}$$

with  $\mathcal{K}$  real. This is true in perturbation theory, owing to the axionic shift symmetry, and any nonperturbative violations are small enough to be irrelevant for this purpose. In this case the kinetic terms for the axionic and size moduli do not mix. Noting that  $\mathcal{K}_{i\bar{j}} = \mathcal{K}_{j\bar{i}}$ , we have for any i and j

$$\mathcal{K}_{i\bar{j}}(\partial_{\mu}T^{i}\partial^{\mu}\bar{T}^{j}) + \mathcal{K}_{j\bar{i}}(\partial_{\mu}T^{j}\partial^{\mu}\bar{T}^{i})$$

$$= \mathcal{K}_{i\bar{j}}\left((\partial_{\mu}\tau_{i} + i\partial_{\mu}c_{i})(\partial^{\mu}\tau_{j} - i\partial^{\mu}c_{j}) + (\partial_{\mu}\tau_{j} + i\partial_{\mu}c_{j})(\partial^{\mu}\tau_{i} - i\partial^{\mu}c_{i})\right)$$

$$= \mathcal{K}_{i\bar{j}}(2\partial_{\mu}\tau_{i}\partial^{\mu}\tau_{j} + 2\partial_{\mu}c_{i}\partial^{\mu}c_{j}), \tag{7.64}$$

and the two sets of terms decouple.

Let us first show that if both the overall volume and the individual cycle volumes are comparable to the string scale, then as expected  $f_a \gtrsim 10^{16} \text{GeV}$ . Suppose an axion  $c_i$  is to be the QCD axion. The Lagrangian for this axion is

$$\mathcal{K}_{i\bar{i}}\partial_{\mu}c_{i}\partial^{\mu}c_{i} + \frac{c_{i}}{4\pi}\int F^{a}\wedge F^{a}.$$
 (7.65)

For simplicity we have not included mixing terms: these will not greatly affect the discussion.

The simplest toy model is that of a factorisable toroidal orientifold, with Kähler potential

$$\mathcal{K} = -\ln\left((T_1 + \bar{T}_1)(T_2 + \bar{T}_2)(T_3 + \bar{T}_3)\right) 
= -\ln(T_1 + \bar{T}_1) - \ln(T_2 + \bar{T}_2) - \ln(T_3 + \bar{T}_3).$$
(7.66)

and Kähler metric

$$\mathcal{K}_{i\bar{j}} = \begin{pmatrix}
(T_1 + \bar{T}_1)^{-2} & 0 & 0 \\
0 & (T_2 + \bar{T}_2)^{-2} & 0 \\
0 & 0 & (T_3 + \bar{T}_3)^{-2}
\end{pmatrix}.$$
(7.67)

If we denote the axions by  $c_1$ ,  $c_2$  and  $c_3$ , the axion kinetic terms are

$$\frac{1}{4\tau_1^2}\partial_{\mu}c_1\partial^{\mu}c_1 + \frac{1}{4\tau_2^2}\partial_{\mu}c_2\partial^{\mu}c_2 + \frac{1}{4\tau_3^2}\partial_{\mu}c_3\partial^{\mu}c_3. \tag{7.68}$$

For definiteness, let us assume QCD is realised on cycle 1. There is no inter-axion mixing and the relevant axion Lagrangian is

$$\frac{1}{4\tau_1^2}\partial_\mu c_1\partial^\mu c_1 + \frac{c_1}{4\pi}\int F^a \wedge F^a. \tag{7.69}$$

If we canonically normalise  $c'_1 = \frac{c_1}{\sqrt{2}\tau_1}$ , this becomes

$$\frac{1}{2}\partial_{\mu}c_1'\partial^{\mu}c_1' + \frac{\sqrt{2}\tau_1}{4\pi}c_1' \int F^a \wedge F^a. \tag{7.70}$$

In units where  $M_P = 1$ , the axion decay constant is

$$f_a = \frac{1}{4\pi\tau_1\sqrt{2}}.$$

If QCD is to be realised on this cycle, we need  $\tau_1 \sim 12$ , and thus  $f_a \sim 10^{16} \text{GeV}$ . Going beyond this toy example, we recall that in general the Kähler metric was given by (7.24),

$$\mathcal{K}_{i\bar{j}} = \frac{G_{i\bar{j}}^{-1}}{\mathcal{V}^2}, \qquad G_{i\bar{j}} = -\frac{3}{2} \left( \frac{k_{ijk}t^k}{\mathcal{V}} - \frac{3}{2} \frac{k_{imn}t^m t^n k_{jpq}t^p t^q}{\mathcal{V}^2} \right). \tag{7.71}$$

If all cycles are string scale in magnitude, then  $\mathcal{K}_{i\bar{j}} \sim \mathcal{O}(1)$  and it is impossible to lower the axion decay constant substantially through canonical normalisation. The same conclusion applies:  $f_a \gtrsim 10^{16} \text{GeV}$ . This conclusion is unsuprising: the axionic coupling to matter is a stringy coupling, and so we expect  $f_a$  to be

comparable to the string scale. If the string and Planck scales are identical,  $f_a$  cannot lie within the allowed window.

If we lower the string scale, phenomenological values for  $f_a$  can be achieved. To analyse this, let us return to the toy model of (7.49), which had Kähler potential

$$\mathcal{K} = -2\ln\left((T_1 + \bar{T}_1)^{\frac{3}{2}} - (T_2 + \bar{T}_2)^{\frac{3}{2}} - (T_3 + \bar{T}_3)^{\frac{3}{2}}\right). \tag{7.72}$$

The Kähler metric for this model is

$$\mathcal{K}_{i\bar{j}} = \begin{pmatrix}
\frac{-3}{2\sqrt{2\tau_{1}}\mathcal{V}} + \frac{9\tau_{1}}{\mathcal{V}^{2}} & -\frac{9\sqrt{\tau_{2}}}{2\mathcal{V}^{5/3}} & -\frac{9\sqrt{\tau_{3}}}{2\mathcal{V}^{5/3}} \\
-\frac{9\sqrt{\tau_{2}}}{2\mathcal{V}^{5/3}} & \frac{3}{2\sqrt{2\tau_{2}}\mathcal{V}} + \frac{9\tau_{2}}{\mathcal{V}^{2}} & \frac{9\sqrt{\tau_{2}\tau_{3}}}{\mathcal{V}^{2}} \\
-\frac{9\sqrt{\tau_{3}}}{2\mathcal{V}^{5/3}} & \frac{9\sqrt{\tau_{2}\tau_{3}}}{\mathcal{V}^{2}} & \frac{3}{2\sqrt{2\tau_{3}}\mathcal{V}} + \frac{9\tau_{3}}{\mathcal{V}^{2}}
\end{pmatrix}.$$
(7.73)

The axion kinetic terms are  $\mathcal{K}_{i\bar{j}}\partial_{\mu}c_{i}\partial^{\mu}c_{j}$ . At small volumes there is substantial mixing between the axions  $c_{1}$ ,  $c_{2}$  and  $c_{3}$ . However, in the limit  $\mathcal{V} \to \infty$  with  $\tau_{1} \gg \tau_{2}, \tau_{3}$ , the Kähler metric has the scaling behaviour

$$\mathcal{K}_{i\bar{j}} \sim \begin{pmatrix} \mathcal{V}^{-4/3} & \mathcal{V}^{-5/3} & \mathcal{V}^{-5/3} \\ \mathcal{V}^{-5/3} & \mathcal{V}^{-1} & \mathcal{V}^{-2} \\ \mathcal{V}^{-5/3} & \mathcal{V}^{-2} & \mathcal{V}^{-1} \end{pmatrix},$$
(7.74)

and is to a good approximation diagonal. The requirement  $\tau_1 \gg 1$  implies that QCD cannot be realised on branes wrapping cycle 1, as the resulting field theory is far too weakly coupled. However, if  $\tau_2 \sim \tau_3 \sim 10$  we may realise QCD by wrapping branes on one of these cycles (for concreteness cycle 3). The resulting axion decay constant is

$$f_a \sim \frac{\sqrt{\mathcal{K}_{3\bar{3}}}}{4\pi} M_P \sim \frac{\mathcal{O}(1)}{4\pi\sqrt{\mathcal{V}}} M_P.$$
 (7.75)

Thus if  $V \sim 10^{14}$  and  $\tau_3 \sim 10$ , the QCD gauge coupling is correct and the axion decay constant  $f_a \sim 10^{10} \text{GeV}$  lies within the narrow phenomenological window. Up to  $\mathcal{O}(1)$  factors, the string and Planck scales are related by

$$m_s = \frac{g_s M_P}{\sqrt{\overline{\mathcal{V}}}}. (7.76)$$

Such a large volume corresponds to lowering the string scale to  $m_s \sim 10^{11} \text{GeV}$ . The lowered axion decay constant is easy to understand physically.  $f_a$  measures the axion-matter coupling, which is an effect localised around the small QCD cycle. Thus the only scale it is sensitive to is the string scale, and so up to numerical factors  $f_a \sim m_s$ . This is illustrated in figure 7.1.

The above is a very particular limit of moduli space, with one cycle taken extremely large while all others are only marginally larger than the string scale.

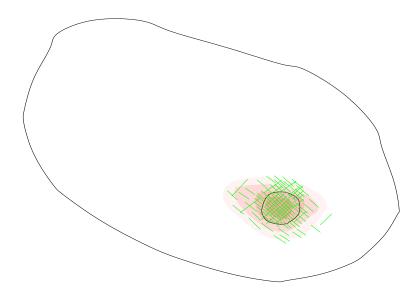


Figure 7.1: The axion and matter wavefunctions are both localised on small string-scale cycles. The axion decay constant measures the (string scale) overlap.

It would thus be essentially a curiosity if it were not also the exact regime in which the moduli are stabilised in the large-volume compactifications described in chapter 5. As the stabilised volume is exponentially sensitive to the stabilised dilaton, a priori the string scale can lie anywhere between the Planck and TeV scales. There is no difficulty, and no fine-tuning, in stabilising the volume so as to achieve an intermediate string scale.

The above result on  $f_a$  is independent of whether an axion remains massless or not. As described in section 7.2.1, the simplest version of the scenarios of [3, 4] makes all axions far too heavy to solve the strong CP problem. In sections 7.2.3 and 7.2.4 we have described the necessary modifications to this scenario such that a massless QCD axion will survive to solve the strong CP problem. Combining this with the above, we have for the first time given a procedure to stabilise all moduli while ensuring a QCD axion exists with a phenomenologically allowed value for  $f_a$ .

In itself this is interesting, as axions within the phenomenological window have always been hard to achieve in string compactifications. However, this scenario compels a further very interesting relationship between the axion decay constant and the (visible) supersymmetry breaking scale.

We argued earlier that if a QCD axion is to be present in IIB flux compactifications, QCD must be realised on a stack of D7-branes. We have also described how to stabilise the moduli such that a QCD axion can exist with a phenomenologically allowed decay constant. As discussed in detail in chapter 6, given a procedure of moduli stabilisation it is possible to calculate the magnitude of soft supersymmetry breaking terms. We recall that for a gauge theory

on D7 branes the scalar and gaugino masses have scaling behaviour (we neglect subleading factors of  $\ln \mathcal{V}$ )

$$m_{D7} \sim M_{D7} \sim m_{3/2} = e^{\mathcal{K}/2} W \sim \frac{M_P}{\mathcal{V}}.$$
 (7.77)

It is an attractive feature of the large-volume compactifications that these scales are all set by the volume V. As we have

$$f_a \sim \frac{M_P}{\sqrt{\mathcal{V}}}$$
 and  $m_{soft} \sim \frac{M_P}{\mathcal{V}}$ .

it follows that up to numerical factors

$$f_a = \sqrt{M_P m_{soft}}. (7.78)$$

Thus in such models the axion decay constant is compelled to be the geometric mean of the Planck scale and the (visible) supersymmetry breaking scale. This is a striking result, as *a priori* these two pieces of physics are entirely unrelated. The relation (7.78) is another example of the phenomenological virtues of an intermediate string scale [149].

## 7.4 Consequences

The purpose of this chapter has been to investigate the conditions under which a QCD axion, ideally with a phenomenological value for  $f_a$ , may coexist with stabilised moduli in string compactifications. This divides into two questions: first, how to stabilise the moduli such that a massless axion survives, and secondly, how to obtain allowed values of  $f_a$ ,  $10^9 \text{GeV} < f_a < 10^{12} \text{GeV}$ .

In the context of the first question, we have shown that the simplest versions of the moduli stabilisation scenarios considered in chapters 4 and 5 do not contain any light axions. If every relevant modulus is stabilised by nonperturbative effects, then their axionic components receive large masses and cannot be a QCD axion. We also proved a negative result, in that supersymmetric moduli stabilisation is disfavoured: there exist no supersymmetric minima of the F-term potential with flat axionic directions. Even if AdS stability is present due to the Breitenlohner-Freedman bound, the tachyons must be removed by the time we are in Minkowski space. Performing this step requires a much greater technical understanding of uplifting AdS vacua to Minkowski space than is currently available, and so it is unclear how relevant the original supersymmetric AdS solutions are.

This result is pure  $\mathcal{N}=1$  supergravity and so makes no assumptions about the particular string model considered. This result is parenthetical to the main

thrust of this thesis, which concerns the large volume non-supersymmetric compactifications of chapter 5. However we include it because it applies to all string compactifications, and in particular shows that in many of the supersymmetric IIA flux compactifications considered recently in the literature the complex structure moduli sector is heavily tachyonic.

We do however view this negative result as a positive feature of the large volume compactifications of chapter 5: as the minimum found there is non-susy it does not suffer from the general problem identified in this result. In the context of these compactifications, we outlined how to stabilise moduli while keeping axions massless. Here we had to rely on Kähler corrections that will become important as a cycle collapses to zero size. While unfortunately not much is known about these, our main requirement was simply that they exist. Clearly progress in determining the form of such corrections would be very interesting. We also specified the extent to which subleading higher-order instantons must be absent in order for a leading-order axion to solve the strong CP problem.

We note the result of the no-go theorem also favours gravity-mediated supersymmetry breaking. If the moduli potential must break supersymmetry in order to solve the strong CP problem, then this suggests that supersymmetry should be broken at the string scale. Gravity mediation therefore always contributes to the visible soft terms and, unless the string scale is drastically lowered, will tend to dominate over gauge mediated effects. An intermediate string scale may then be preferred in order to obtain TeV-scale soft terms.

In the context of the second question, the fact that  $f_a$  is hierarchically lower than the Planck scale implies that compactifications with  $m_s \sim M_P$  are unlikely to give allowed values for  $f_a$ . This problem can be avoided in the models of chapter 5 in which the string scale is hierarchically lower than the Planck scale. In these models,  $f_a \sim M_s$  and  $M_{SUSY} \sim \frac{M_s^2}{M_P}$ . An intermediate string scale therefore gives both  $10^9 \text{GeV} < f_a < 10^{12} \text{GeV}$  and visible susy breaking at  $\mathcal{O}(1\text{TeV})$ . It is hard to find models with phenomenological values for  $f_a$ , and so it is very interesting that in the above model this also implies TeV-scale supersymmetry breaking.

A more general point argued in this chapter is that in the context of the landscape the strong CP problem may serve as an experimentum crucis. Assuming that the solution to the strong CP problem is a Peccei-Quinn axion and that string theory is a correct description of nature, this is a solution that is extremely sensitive to the physics of moduli stabilisation. Requiring an axion to remain (essentially) massless while all other moduli are stabilised is a technically clean problem directly addressing the issue of vacuum selection. Indeed, as seen above imposing this requirement directly rules out many scenarios of moduli stabilisation. The further condition  $10^9 \text{GeV} < f_a < 10^{12} \text{GeV}$  is even more constraining: the large-volume models of chapter 5 are, as far as we know, the only models capable of producing axions with the required decay constants.

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We now turn our attention from particle physics problems to cosmological problems, and consider a way to realise inflation in the large-volume compactifications of chapter 5.

# Chapter 8

# An Inflationary Model

This chapter is based on the paper [5].

The previous two chapters have described particle physics applications of the moduli stabilisation scenario developed in chapter 5. As discussed there, one of the main applications of moduli potentials is the determination of the magnitude and pattern of supersymmetry breaking. However, the moduli potential may also have important cosmological applications in the theory of inflation.

Inflation is the dominant theory for the origin of structure in the universe. The universe is observed to be homogeneous on scales larger than that of galactic superclusters ( $\sim 100 \mathrm{Mpc}$ ) and inhomogeneous below it. This behaviour can be seen primordially in the Cosmic Microwave Background (CMB), which has a uniform temperature T=2.73 K with perturbations at the level of one part in  $10^5$ . Simulations show that these perturbations are of the right magnitude to generate the large-scale inhomogeneities seen today.

The term 'inflation' simply refers to any time in the history of the universe where the scale factor a(t) is accelerating. If the metric of the universe can be written

$$ds^{2} = -dt^{2} + a(t)^{2}(dx^{2} + dy^{2} + dz^{2}), (8.1)$$

then inflation corresponds to any period in the history of the universe during which

$$\ddot{a}(t) > 0.$$

During a long period of inflation, the size of the universe expands exponentially. The original advantage of inflation was that it would dilute unwanted relics (such as gravitinos or magnetic monopoles) produced in the very early universe and would solve the horizon and flatness problems. It was later realised that inflation can also generate density perturbations through the vacuum fluctuations of a scalar field. We shall not review all of the theory behind inflation - useful references are the books [150, 151].

The standard inflationary paradigm is that of slow-roll inflation. It is well known that a universe dominated by a vacuum energy  $\Lambda$  expands exponentially, with

$$a(t) = e^{\Lambda(t-t_0)}a(t_0).$$

Slow-roll inflation occurs as a scalar field slowly rolls down a very flat potential. During this time, the energy density of the universe is dominated by the vacuum energy of the scalar field. As long as the scalar potential is sufficiently flat, the universe undergoes  $N_e \gg 1$  efolds of inflation during this period,

$$\ln \frac{a(t_{end})}{a(t_{start})} = N_e.$$
(8.2)

The flatness of the potential can be quantified through the slow-roll parameters,  $\epsilon$ ,  $\eta$  and  $\xi$ . For single-field inflation, these are defined by

$$\epsilon = \frac{M_P^2}{2} \left(\frac{V'}{V}\right)^2, \tag{8.3}$$

$$\eta = M_P^2 \frac{V''}{V}, \tag{8.4}$$

$$\eta = M_P^2 \frac{V''}{V},$$

$$\xi = M_P^4 \frac{V'V'''}{V^2},$$
(8.4)

taking derivatives with respect to the canonically normalised inflaton field. Slowroll inflation occur if  $\epsilon, \eta \ll 1$ . Inflation continues until  $\eta \sim 1$  (note that typical models have  $\epsilon \ll \eta$ ), whereupon it ends rapidly. If these conditions hold, many e-folds of inflation can be expected:  $N \gtrsim 60$  is required. The primordial density perturbations are generated by the quantum fluctations of the inflaton field. The task of inflationary model-building is to find (well-motivated) potentials realising these conditions. Such questions are timely as observations can now provide precision tests for inflationary models [152, 153].

As slow-roll inflation is driven by the potential of a scalar field, the moduli potential may in principle give rise to inflation. There are many scalar fields in string theory, and so many candidates for the inflaton field. These can be classified by their origin in either the open or closed string sector [154, 155, 156. The most common open string inflaton is a brane/antibrane separation [157, 158, 159], whereas closed string inflatons typically correspond to geometric moduli [160]. As inflation is driven by the moduli potential, the rapid recent advances in moduli stabilisation have been accompanied by much effort devoted to inflationary model building in string theory [161, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176].

In phenomenological applications, the low energy limit of string theory is generally  $\mathcal{N}=1$  supergravity. In this case there is a standard problem - the  $\eta$ problem - attendant on building inflationary models, whether involving brane or

modular fields as the inflaton. This states that for the  $\mathcal{N}=1$  F-term potential the slow-roll  $\eta$  parameter is  $\mathcal{O}(1)$  unless a finely tuned cancellation occurs. The F-term potential is

$$V_F = e^{\mathcal{K}} \left( \mathcal{K}^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2 \right). \tag{8.6}$$

Examining the potential along the  $\varphi$  direction, we then have

$$\frac{\partial^2 V}{\partial \varphi^2} = \mathcal{K}'' V_F + \dots \tag{8.7}$$

By considering  $\varphi$  to be canonically normalised (i.e.  $\mathcal{K} \sim \phi \bar{\phi} + \ldots$ ), we see that generically  $\eta \sim 1$ . The  $\eta$  problem is manifest for F-term modular inflation. In brane inflation it is not manifest, but reappears once this is embedded into a moduli stabilisation scenario [161].

In this chapter I present a simple inflationary scenario within the framework of the moduli stabilisation mechanism developed in chapter 5. The inflaton is one of the Kähler moduli and inflation proceeds by reducing the F-term energy. The  $\eta$  problem can be evaded by use of the no-scale properties of the Kähler potential. This mechanism in principle applies to a large class of Calabi-Yau compactifications that will be further specified below.

# 8.1 Keeping $\eta$ Small

#### 8.1.1 General Idea

Slow-roll inflation requires almost flat directions in the scalar potential. A natural source of flat directions would be a field only appearing exponentially in the potential. Denoting this field by  $\tau$ , an appropriate (and textbook [151]) potential would be

$$V(\tau) = V_0 \left( 1 - Ae^{-\tau} + \dots \right), \tag{8.8}$$

where the dots represent higher exponents.

As discussed above, nonperturbative effects are relevant for stabilising many string moduli. Examples are the Kähler moduli of IIB flux compactifications and the dilaton and Kähler moduli in heterotic Calabi-Yau compactifications. In principle this discussion applies to all such fields, but we focus here on the IIB Kähler moduli  $(T_i = \tau_i + ic_i)$  whose potential was studied at length in chapter 5. We recall these appear nonperturbatively in the superpotential,

$$W = \int G_3 \wedge \Omega + \sum_i A_i e^{-a_i T_i}, \tag{8.9}$$

with the threshold corrections  $A_i$  independent of the Kähler moduli.

The  $\eta$  problem states that the potential from generic  $\mathcal{K}$  has  $\eta \sim \mathcal{O}(1)$ . However, the key word is 'generic', and Kähler potentials arising from string theory are (by definition) not generic. A common way these potentials fail to be generic is by being no-scale, corresponding to

$$\mathcal{K}^{i\bar{j}}\partial_i \mathcal{K}\partial_{\bar{j}} \mathcal{K} = 3. \tag{8.10}$$

For a constant superpotential  $W = W_0$ , the no-scale scalar potential vanishes:

$$V_F = e^{\mathcal{K}} \left( \mathcal{K}^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2 \right) = 0. \tag{8.11}$$

In contrast to the 'generic' behaviour predicted by the  $\eta$  problem, all directions are exactly flat. Indeed, the tree-level Kähler potential for the IIB size moduli is no-scale,

$$\mathcal{K} = -2\ln(\mathcal{V}),\tag{8.12}$$

with  $\mathcal{V}$  the internal volume. Suppose we lift the no-scale behaviour through nonperturbative terms in the superpotential as in (8.9). The scalar potential becomes

$$V_F = e^{\mathcal{K}} \mathcal{K}^{i\bar{j}} \left[ a_i A_i a_j \bar{A}_j e^{-a_i T_i - a_j \bar{T}_j} - ((\partial_i \mathcal{K}) W a_j \bar{A}_j e^{-a_j \bar{T}_j} + c.c) \right]. \tag{8.13}$$

The  $T_i$  directions are no longer exactly but exponentially flat as in (8.8). It is then natural to ask whether this flatness can drive inflation.

While the potential (8.13) is exponentially flat, it also appears exponentially small. However, this is only true so long as all  $T_i$  fields are large. In the presence of several Kähler moduli the variation of V along the  $T_i$  direction is in general uncorrelated with the magnitude of V. We can then hope to build an inflationary model in a multi-modulus scenario.

## 8.1.2 Embedding in IIB Flux Compactifications

The above gives the motivation. We now embed the above idea in the moduli stabilisation mechanism developed in chapter 5.

The  $\mathbb{P}^4_{[1,1,1,6,9]}$  model used in chapter 5 has two moduli. It will turn out that to realise inflation we need a three-modulus model. As the determination of nonperturbative superpotential corrections for a given Calabi-Yau is very difficult, we use a toy model in which the existence of appropriate superpotential corrections is assumed. We denote the 4-cycle volumes by  $\tau_i = \text{Re}(T_i)$  and take the following simplified form for the Calabi-Yau volume,

$$\mathcal{V} = \alpha(\tau_1^{3/2} - \sum_{i=2}^n \lambda_i \tau_i^{3/2})$$

$$= \frac{\alpha}{2\sqrt{2}} \left[ (T_1 + \bar{T}_1)^{3/2} - \sum_{i=1}^n \lambda_i (T_i + \bar{T}_i)^{3/2} \right]. \tag{8.14}$$

 $\tau_1$  controls the overall volume and  $\tau_2, \ldots, \tau_n$  are blow-ups whose only non-vanishing triple intersections are with themselves.  $\alpha$  and  $\lambda_i$  are positive model-dependent constants. The minus signs are necessary to ensure  $\frac{\partial^2 \mathcal{V}}{\partial \tau_i \partial \tau_j}$  has signature  $(1, h^{1,1} - 1)$  [29]. The  $\mathbb{P}^4_{[1,1,1,6,9]}$  case corresponds to (8.14) with n = 1.

The dilaton and complex structure moduli are as usual flux-stabilised. We take the Kähler moduli superpotential to be<sup>1</sup>

$$W = W_0 + \sum_{i=2}^{n} A_i e^{-a_i T_i}, \tag{8.15}$$

where  $a_i = \frac{2\pi}{q_s N}$ . The  $\alpha'$ -corrected Kähler potential is

$$\mathcal{K} = \mathcal{K}_{cs} - 2\ln\left[\alpha\left(\tau_1^{3/2} - \sum_{i=2}^n \lambda_i \tau_i^{3/2}\right) + \frac{\xi}{2}\right],\tag{8.16}$$

where  $\xi = -\frac{\zeta(3)\chi(M)}{2(2\pi)^3}$ . As the dilaton is fixed and regarded as a constant, we can define the moduli using either string or Einstein-frame volumes; we use the former. The latter would correspond to replacing  $a_i \to a_i g_s$  and  $\xi \to \xi g_s^{-3/2}$  in (8.15) and (8.16) - the physics is of course the same.

From the results of chapter 5 we anticipate that at the minimum  $\tau_1 \gg \tau_i$  and  $\mathcal{V} \gg 1$ . In this limit the scalar potential is

$$V = e^{\mathcal{K}} \left[ \mathcal{K}^{i\bar{j}} \partial_i W \partial_{\bar{j}} \bar{W} + \mathcal{K}^{i\bar{j}} \left( (\partial_i K) W \right) \partial_{\bar{j}} \bar{W} + c.c. \right] + \frac{3\xi W_0^2}{4\mathcal{V}^3}. \tag{8.17}$$

As we need  $\xi > 0$ , we require  $h^{2,1} > h^{1,1}$ . For the Kähler potential (8.12), we have

$$\mathcal{K}^{i\bar{j}} \sim \frac{8\mathcal{V}\sqrt{\tau_i}}{3\alpha\lambda_i}\delta_{ij} + \mathcal{O}(\tau_i\tau_j). \tag{8.18}$$

 $\mathcal{K}^{i\bar{j}}$  is real and satisfies  $\mathcal{K}^{i\bar{j}}\partial_{\bar{j}}\mathcal{K} = 2\tau_i(1 + \mathcal{O}(\mathcal{V}^{-1}))$ . At large volume only the leading part of  $\mathcal{K}^{i\bar{j}}$  is relevant and the scalar potential becomes

$$V = \sum_{i} \frac{8(a_i A_i)^2 \sqrt{\tau_i}}{3\mathcal{V}\lambda_i \alpha} e^{-2a_i \tau_i} - \sum_{i} 4 \frac{a_i A_i}{\mathcal{V}^2} W_0 \tau_i e^{-a_i \tau_i} + \frac{3\xi W_0^2}{4\mathcal{V}^3}.$$
 (8.19)

More generally we could take  $W = W_0 + \sum_{i=2}^n A_i e^{-a_{ij}T_j}$ . The effect of this is to alter the condition (8.25) in a model-dependent fashion. As long as the modified form of (8.25) can be satisfied, inflation occurs and the results for the inflationary parameters are unaffected. In general we expect this to be possible, although we note that there do exist models, such as the  $\mathcal{F}_{11}$  model of [91], for which this cannot be achieved.

The minus sign in the second term arises from setting the  $b_i$  axion to its minimum. There are terms subleading in  $\mathcal{V}$ , but importantly these only depend on  $\tau_i$  through the overall volume. This is crucial, as it ensures that at large  $\tau_i$  the variation of the potential with  $\tau_i$  is exponentially suppressed. We can find the global minimum by extremising (8.19) with respect to  $\tau_i$ . Doing this at fixed  $\mathcal{V}$ , we obtain

$$(a_i A_i) e^{-a_i \tau_i} = \frac{3\alpha \lambda_i W_0}{2\mathcal{V}} \frac{(1 - a_i \tau_i)}{(\frac{1}{2} - 2a_i \tau_i)} \sqrt{\tau_i}.$$
 (8.20)

If we approximate  $a_i \tau_i \gg 1$  (which is valid at large volume as  $a_i \tau_i \sim \ln(\mathcal{V})$ ), then substituting this into the potential (8.19) generates a contribution

$$\frac{-3\lambda_i W_0^2}{2\mathcal{V}^3} \tau_{i,min}^{3/2} \alpha = \frac{-3\lambda_i W_0^2 \alpha}{2\mathcal{V}^3 a_i^{3/2}} (\ln \mathcal{V} - c_i)^{3/2}$$
(8.21)

where  $c_i = \ln(\frac{3\alpha\lambda_i W_0}{2a_i A_i})$ . At large values of  $\ln \mathcal{V}$ , the resulting effective potential for the volume  $\mathcal{V}$  once all  $\tau_i$  fields are minimised is

$$V = \frac{-3W_0^2}{2\mathcal{V}^3} \left( \sum_{i=2}^n \left[ \frac{\lambda_i \alpha}{a_i^{3/2}} \right] (\ln \mathcal{V})^{3/2} - \frac{\xi}{2} \right).$$
 (8.22)

This effective potential is another way of understanding why the potential of chapter 5 has a minimum at exponentially large volumes. To ensure the global minimum is at V = 0, we must include an uplift term:

$$V = \frac{-3W_0^2}{2\mathcal{V}^3} \left( \sum_{i=2}^n \left[ \frac{\lambda_i \alpha}{a_i^{3/2}} \right] (\ln \mathcal{V})^{3/2} - \frac{\xi}{2} \right) + \frac{\gamma W_0^2}{\mathcal{V}^2}, \tag{8.23}$$

where  $\gamma \sim \mathcal{O}(\frac{1}{\mathcal{V}})$  parametrises the uplift. By tuning  $\gamma$ , the potential (8.23) (and by extension its full form (8.19)) has a Minkowski or small de Sitter minimum.

To obtain inflation we consider the potential away from the minimum. We take one of the 'small' moduli, say  $\tau_n$ , and displace it far from its minimum. At constant volume the potential is exponentially flat along this direction, and the modulus rolls back in an inflationary fashion. We do not need to worry about initial conditions. While we do not know how the moduli evolution starts, we do know how it must end, namely with all moduli at their minima. Given this - we have nothing new to say here on the overshoot problem [177] - inflation occurs as the last Kähler modulus rolls down to its minimum.

It is necessary that all other moduli, and in particular the volume, are stable during inflation. Displacing  $\tau_n$  from its minimum nullifies the effective contribution (8.21) made by the stabilised  $\tau_n$  to the volume potential. During inflation the effective volume potential is

$$V = \frac{-3W_0^2}{2\mathcal{V}^3} \left( \sum_{i=2}^{n-1} \left[ \frac{\lambda_i \alpha}{a_i^{3/2}} \right] (\ln \mathcal{V})^{3/2} - \frac{\xi}{2} \right) + \frac{\gamma W_0^2}{\mathcal{V}^2}.$$
 (8.24)

Provided that the ratio

$$\rho \equiv \frac{\lambda_n}{a_n^{3/2}} : \sum_{i=2}^n \frac{\lambda_i}{a_i^{3/2}} \tag{8.25}$$

is sufficiently small<sup>2</sup>, there is little difference between (8.23) and (8.24) and the volume modulus will be stable during inflation. As we obviously require  $\rho < 1$ , it follows that at least three Kähler moduli are necessary. While (8.25) can always be satisfied by an appropriate choice of  $a_i$ , the presence of the summation implies that this becomes easier the more Kähler moduli are present.

We illustrate the form of the resulting inflationary potential in figure 8.1, showing the inflaton and volume directions.

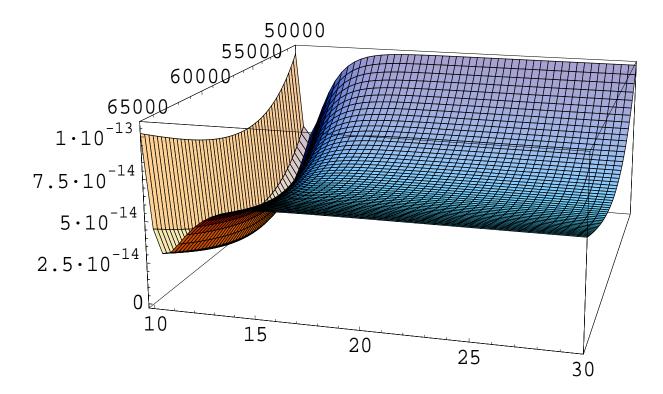


Figure 8.1: Inflationary potential: the inflaton lies along the x-direction and the volume along the y-direction.

<sup>&</sup>lt;sup>2</sup>This can be quantified in explicit models. For a 3-modulus model, the condition on the ratio  $\rho$  is  $9.5(\ln V)\rho < 1$ . To obtain inflation with correct density perturbations, the appropriate volumes are  $\mathcal{O}(10^5 - 10^7)$ , which can be satisfied using sensible values for  $\lambda_i$  and  $a_i$ .

# 8.2 Inflationary Potential and Parameters

Let us quantify the resulting potential and compute the inflationary parameters. The inflationary potential is read off from (8.19) to be

$$V_{inf} = V_0 - \frac{4\tau_n W_0 a_n A_n e^{-a_n \tau_n}}{\mathcal{V}^2},\tag{8.26}$$

as the double exponential in (8.19) is irrelevant during inflation. During inflation  $V_0$  is constant and can be parametrised as

$$V_0 = \frac{\beta W_0^2}{\mathcal{V}^3}. (8.27)$$

 $(\frac{1}{\mathcal{V}^3})$  sets the scale of the potential during inflation). However,  $\tau_n$  is not canonically normalised, as to leading order in volume

$$\mathcal{K}_{n\bar{n}} = \frac{3\lambda}{8\sqrt{\tau_n}\mathcal{V}}.\tag{8.28}$$

The canonically normalised field is

$$\tau_n^c = \sqrt{\frac{4\lambda}{3\mathcal{V}}} \tau_n^{\frac{3}{4}}.\tag{8.29}$$

In terms of  $\tau_n^c$ , the inflationary potential is

$$V = V_0 - \frac{4W_0 a_n A_n}{\mathcal{V}^2} \left(\frac{3\mathcal{V}}{4\lambda}\right)^{2/3} (\tau_n^c)^{4/3} \exp\left[-a_n \left(\frac{3\mathcal{V}}{4\lambda}\right)^{2/3} (\tau_n^c)^{4/3}\right]. \tag{8.30}$$

This is similar, but not identical, to the textbook potential  $V = V_0(1 - e^{-\tau})$ . Note however the enormous ( $\gtrsim 10^3$ ) factor of  $\mathcal{V}^{2/3}$  in the exponent. Although  $\tau_n^c$  is canonically normalised, it has no natural geometric interpretation and for clarity we shall express the inflationary parameters in terms of  $\tau_n$ , the 4-cycle volume.

The slow-roll parameters are defined by

$$\epsilon = \frac{M_P^2}{2} \left(\frac{V'}{V}\right)^2, \tag{8.31}$$

$$\eta = M_P^2 \frac{V''}{V}, \tag{8.32}$$

$$\xi = M_P^4 \frac{V'V'''}{V^2}, (8.33)$$

with the derivatives being with respect to  $\tau_n^c$ . These can be evaluated to give

$$\epsilon = \frac{32\mathcal{V}^3}{3\beta^2 W_0^2} a_n^2 A_n^2 \sqrt{\tau_n} (1 - a_n \tau_n)^2 e^{-2a_n \tau_n},$$

$$\eta = -\frac{4a_n A_n \mathcal{V}^2}{3\lambda \sqrt{\tau_n} \beta W_0} \left[ (1 - 9a_n \tau_n + 4(a_n \tau_n)^2) e^{-a_n \tau_n} \right],$$

$$\xi = \frac{-32(a_n A_n)^2 \mathcal{V}^4}{9\beta^2 \lambda^2 W_0^2 \tau_n} (1 - a_n \tau_n) \left( 1 + 11a_n \tau_n - 30(a_n \tau_n)^2 + 8(a_n \tau_n)^2 \right) e^{-2a_n \tau_n}.$$
(8.34)

Then  $\xi \ll \epsilon, \eta \ll 1$  provided that  $e^{-a_n \tau_n} \ll \frac{1}{V^2}$ .

Within the slow-roll approximation, the spectral index and its running are given by

$$n-1 = 2\eta - 6\epsilon + \mathcal{O}(\xi), \tag{8.35}$$

$$\frac{dn}{d\ln k} = 16\epsilon\eta - 24\epsilon^2 - 2\xi. \tag{8.36}$$

The number of efoldings is given by

$$N_e = \int_{\phi_{end}}^{\phi} \frac{V}{V'} d\phi, \tag{8.37}$$

which may be expressed as

$$N_e = \frac{-3\beta W_0 \lambda_n}{16 \mathcal{V}^2 a_n A_n} \int_{\tau_n^{end}}^{\tau_n} \frac{e^{a_n \tau_n}}{\sqrt{\tau_n} (1 - a_n \tau_n)} d\tau_n.$$
 (8.38)

Matching the COBE normalisation for the density fluctuations  $\delta_H = 1.92 \times 10^{-5}$  requires

$$\frac{V^{3/2}}{M_D^3 V'} = 5.2 \times 10^{-4},\tag{8.39}$$

where the LHS is evaluated at horizon exit,  $N_e = 50 - 60$  efoldings before the end of inflation. This condition can be expressed as

$$\left(\frac{g_s^4}{8\pi}\right) \frac{3\lambda\beta^3 W_0^2}{64\sqrt{\tau_n} (1 - a_n \tau_n)^2} \left(\frac{W_0}{a_n A_n}\right)^2 \frac{e^{2a_n \tau_n}}{\mathcal{V}^6} = 2.7 \times 10^{-7}.$$
(8.40)

We have here included a factor of  $\frac{g_s^4}{8\pi}$  that should properly be included as an overall normalisation in V. This arises from the prefactor in W and is discussed in the Appendix. The condition (8.39) determines the normalisation of the potential and in practice we use it as a constraint on the stabilised volume.

Finally, the tensor-to-scalar ratio is

$$r \sim 8\epsilon.$$
 (8.41)

#### 8.2.1Model Footprints

What are the predictions of the above model? There are various undetermined parameters arising from the detailed microphysics, such as the threshold correction A or tree-level superpotential  $W_0$ . In principle, these are determined by the specific Calabi-Yau with its brane and flux configurations, but they are prohibitively difficult to calculate in realistic examples. However, it turns out that the most important results are independent of these parameters. In particular, solving equations (8.35) to (8.41) numerically, we find the robust results

$$\eta \approx -\frac{1}{N_c},\tag{8.42}$$

$$\epsilon < 10^{-12},$$
 (8.43)

$$\xi \approx -\frac{2}{N_e^2}. (8.44)$$

These results are not so surprising, given the similarity of the potential to the textbook form  $V_0(1-e^{-\tau})$ . Taking a range  $N_e=50\to60$ , we obtain in the slow-roll approximation

$$0.960 < n < 0.967,$$
 (8.45)

$$-0.0006 < \frac{dn}{d \ln k} < -0.0008,$$

$$0 < |r| < 10^{-10},$$
(8.46)

$$0 < |r| < 10^{-10}, (8.47)$$

where the above uncertainties arise principally from the number of e-foldings. If we go beyond the slow-roll approximation, the expression for n will receive  $\mathcal{O}(\xi)$ corrections - these are minimal and can be neglected.

To evaluate the inflationary energy scale, it is convenient to reformulate the COBE normalisation of density perturbations  $\delta_H = 1.92 \times 10^{-5}$  as

$$\frac{V^{1/4}}{\epsilon^{1/4}} = 6.6 \times 10^{16} \text{GeV}. \tag{8.48}$$

Unlike the predictions for the spectral index, the required internal volume is dependent on the microscopic parameters. For typical values of these this is found numerically to take a range of values

$$10^5 l_s^6 \le \mathcal{V} \le 10^7 l_s^6, \tag{8.49}$$

where  $l_s = (2\pi)\sqrt{\alpha'}$ . As the moduli stabilisation mechanism of chapter 5 naturally generates exponentially large volumes, these can be achieved without difficulty. The range of  $\epsilon$  at horizon exit is  $10^{-13} \ge \epsilon \ge 10^{-15}$ , and thus the inflationary energy scale is rather low,

$$V_{inf} \sim 10^{13} \text{GeV}.$$
 (8.50)

This implies in particular that tensor perturbations would be unobservable in this model.

There is no practical upper limit on the number of efoldings attainable. The potential is exponentially flat as the inflaton 4-cycle increases in volume. A very large number of efoldings is achieved by a very small variation in the original inflaton value and barring cancellations we therefore expect  $N_{e,total} \gg 60$  in these models.

In these compactifications, the lightest non-axionic modulus has a mass [4]

$$M \sim \frac{M_P}{\mathcal{V}^{3/2}}.\tag{8.51}$$

Thus even at the larger end of volumes  $M \gg \mathcal{O}(10)$ TeV and there is no cosmological moduli problem. Of course, this is somewhat trivial: the cosmological moduli problem concerns a tension between TeV-scale supersymmetry and long-lived moduli. Focusing on the inflation scale removes the long-lived moduli, but it also removes TeV-scale supersymmetry. It remains an open problem to build well-motivated models containing both inflationary and weak scales.

As inflation takes place within supergravity, reheating in the above model can be described within field theory. As the cycle volume is the gauge coupling for a brane wrapped on the cycle, there exists a coupling

$$\tau_n \int F_{\mu\nu} F^{\mu\nu}.$$

If  $\tau_n$  is the inflaton, it can decay to radiation, rehating the universe and recovering the Hot Big Bang.

As indicated earlier, we do not need to concern ourselves with initial conditions for inflation. Given that the moduli attain their minimum, the inflaton is simply the last Kähler modulus to roll down to the minimum. We do not need to worry about interference from the evolution of the other moduli, as once they roll down to their minimum they become heavy and rapidly decouple from inflationary dynamics.

#### 8.2.2 Additional Corrections and Extensions

The inflationary mechanism presented here relies on the exponential flatness of the  $\tau_n$  direction at constant volume. This is unbroken by the tree-level Kähler potential, the included  $\alpha'^3$  correction of [90] and the uplift term. The uplift terms have several possible sources [84, 85, 86], but all scale inversely with the volume

$$V_{uplift} \sim \frac{1}{\mathcal{V}^{\alpha}},$$
 (8.52)

where  $\frac{4}{3} \leq \alpha \leq 2$ . As the modular dependence is encoded through the overall volume, rather than depending explicitly on all the moduli, at constant volume the  $T_n$  direction is extremely flat for large values of  $T_n$ .

Let us discuss possible effects that may spoil the exponential flatness, first considering superpotential effects. The nonrenormalisation theorems guarantee that the Kähler moduli cannot appear perturbatively in W. However, the flatness could be spoiled if the functions  $A_i$  of (8.9) depended polynomially on the Kähler moduli. From (8.19), a term  $A(T_j)e^{-T_i}$  in the superpotential would source an effective polynomial term for  $T_j$  once  $T_i$  was stabilised. However, as the  $A_i$  must both be holomorphic in  $T_i$  and respect the axion shift symmetries, this polynomial dependence on  $T_i$  cannot occur. Indeed, in the models for which these threshold corrections have been computed explicitly, the functions  $A_i$  do not depend on the Kähler moduli [54]. Combined with non-renormalisation results, this means that the exponential flatness cannot be lifted by superpotential effects.

The other possibility is that the exponential flatness may be lifted by corrections to the Kähler potential depending on  $\tau_n$  such as were discussed in chapter 7. Both the tree-level Kähler potential and the  $\mathcal{O}(\alpha'^3)$  corrections of [90] have the property that their contribution to the scalar potential depends only on the overall volume. This does not lift the exponential flatness of the  $\tau_n$  direction at constant  $\mathcal{V}$ . The possible existence of such corrections is difficult to analyse explicitly and may depend on whether or not branes are wrapped on the relevant cycle. An example of such corrections would be the open string corrections computed in [99, 145]. However this calculation does not directly apply, as it is on an orbifold and does not involve blow-up modes such as we have used for the inflaton.

The upshot is that the exponential flatness of the  $\tau_n$  direction is not broken by any of the known corrections. In general, any correction that can be expressed in terms of the overall volume will not alter the exponential flatness of the  $\tau_n$ direction. If corrections existed which did break this exponential flatness, it would be necessary to examine their form and magnitude - it is not after all necessary that the exponential flatness survive for all values of  $\tau_n$ , but merely for those relevant during the last sixty e-folds.

Finally, we have used an oversimplified form for the Calabi-Yau, picturing it as simply a combination of a volume cycle and blow-up modes. This is not necessary for the inflationary mechanism described here. Whilst in (8.14) we assumed  $h^{1,1}-1$  moduli to be blow-ups whose only nonvanishing triple intersection was with themselves, a single such modulus would be perfectly adequate as an inflaton. Indeed, even this is not necessary - the minimal requirement is simply a flat direction, which originates from the no-scale behaviour and is broken by nonperturbative effects. The condition necessary to ensure the volume is stable during inflaton will then be a generalisation of (8.25).

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### 8.3 Discussion

This chapter has described a general but simple scenario of inflation in string theory. Its advantages are that it does not require fine tuning of parameters, applies to a very large class of compactifications and is predictive at the level that can be ruled out within a few years. This scenario realises large field inflation in a natural way. The main properties of these models are the existence of flat directions broken by non-perturbative effects. The flat directions have their origins in the no-scale property of the Kähler potential and are generic for IIB Kähler moduli, as is the appearance of instanton-generated nonperturbative superpotentials. The scenario is embedded in the exponentially large volume compactifications of [3, 4] and requires  $h^{2,1} > h^{1,1}$  and  $h^{1,1} > 2$ . This last requirement is necessary to ensure that the volume is stabilised during inflation.

The main, robust numerical prediction is for a spectral index

$$n_s = 1 - \frac{2}{N_e} = 0.96 \to 0.967.$$
 (8.53)

This is in good agreement with the three-year WMAP results [153]

$$n_s = 0.951 \pm 0.016. \tag{8.54}$$

Tensor perturbations are unobservable in this model, consistent with current observations which see no evidence for them.

Notice that the volumes required to obtain inflation, while large, are not extremely large as the string scale is only a few orders of magnitude below the Planck scale. The necessary volumes of  $\mathcal{O}(10^5 - 10^7)$  in string units can be obtained by natural choices of the exponential parameters  $a_i$ .

Although there are many moduli, the inflationary period reduces to a single-field case. This is because the inflaton is simply the last modulus to roll down to its minimum, and once other moduli attain their minimum they rapidly become heavy and decouple from inflationary dynamics. In principle there are at least two other fields that may have a nontrivial role during the cosmological evolution. One is the axion partner of the inflaton field. We have chosen this to sit at the minimum of its oscillatory potential, at least for the last sixty efolds. This is not a strong assumption - because the inflaton direction is so flat, there is a lot of time for the axion to relax from a possibly non-zero inital value to its minimum before the last sixty efolds start.

There is one substantial problem with the above model (which also applies for almost all models of string inflation). The energy scale of inflation cannot be too far removed from the GUT scale. If string moduli are used for inflation, the same potential that gives inflation will also generally give GUT-scale supersymmetry breaking. We saw in chapter 6 that a realistic phenomenology required a

volume  $V \sim 10^{14}$ , much larger than the values  $V \sim 10^5 \rightarrow 10^7$  encountered here. Thus while this inflationary model may be appealing by itself, it is difficult to reconcile with supersymmetry broken at the TeV scale, and this is a problem.

This problem may be less serious in the scenario of chapter 5 than in other stringy models. The minimum of the potential is exponentially sensitive to certain parameters (such as  $g_s$ ). It is then possible that such parameters change during inflation in such a way that the location of the minimum changes from the volumes  $\mathcal{V} \sim 10^5 \to 10^7$  suitable for inflation to the values  $\mathcal{V} \sim 10^{14}$  suitable for low-energy supersymmetry. It would be interesting to build an explicit model realising this scenario.

Let us finally discuss the generality of this scenario. The main technical assumption we have used is the direct expression for the volume in terms of the Kähler moduli (8.14). This was overkill - the only part of the assumption we actually used was that the inflaton modulus appears alone in the volume as  $\mathcal{V} = \ldots - (T_n + \bar{T}_n)^{\frac{3}{2}}$ . As indicated above, we can relax even this: the absolute minimal requirement is simply the existence of a flat direction broken by nonperturbative effects. There may be several possible inflationary directions - in the above model,  $\tau_2, \ldots, \tau_n$  are all good candidates - with the particular one chosen determined by which Kähler modulus is last to attain its minimum. In each case we expect similar physics to emerge with a robust prediction for the spectral index of density perturbations.

# Part IV Conclusions and Outlook

# Chapter 9

# Conclusions

Let us conclude by summarising the results of this thesis and by outlining the prospects for future work.

The thesis has been concerned with moduli stabilisation in IIB string theory and its phenomenological applications. The two chapters of Part I were introductory. They motivated the use of string theory as a framework for physics beyond the Standard Model and reviewed the use of fluxes to stabilise moduli, from both a four-dimensional and ten-dimensional perspective. I also described the flux compactifications of [31] which serve as background to most of this thesis.

Part II was concerned with developing a detailed understanding of moduli stabilisation. In a IIB context 3-form fluxes stabilise the complex structure moduli. The large degeneracy of flux choices suggests the use of statistical methods to understand the loci of the stabilised moduli. In chapter 3 we used Monte-Carlo techniques to study this explicitly on a particular Calabi-Yau, finding good agreement with the statistical predictions of Douglas and collaborators.

The stabilisation of Kähler moduli requires the use of nonperturbative effects. In chapter 4 we reviewed the KKLT scenario in which all moduli are stabilised by nonperturbative effects, coming from either D3-brane instantons or gaugino condensation. Such effects are nonperturbative in both the  $g_s$  and  $\alpha'$  expansions. These can only give the dominant contribution to the scalar potential under very particular circumstances. We gave a careful analysis of this and concluded that in general  $\alpha'$  corrections must be included to study the scalar potential.

In chapter 5 we gave a detailed analysis of the scalar potential incorporating  $\alpha'$  corrections. We showed that, for arbitrary values of  $W_0$ , there in general exists a non-supersymmetric minimum of the scalar potential at exponentially large volumes. This minimum requires at least two moduli, one of which is a blow-up mode. The exponentially large volumes come from a competition between correc-

tions perturbative in the overall volume and corrections nonperturbative in the volume of the small blow-up cycle. These compete in a logarithmic fashion leading to the exponentially large volume. We studied this potential quantitatively on  $\mathbb{P}^4_{[1,1,1,6,9]}$ , explicitly finding the minimum and the spectrum of moduli masses. The overall volume is exponentially sensitive to the stabilised dilaton, and so in effect different flux choices allow the string scale to be dialled arbitrarily. In particular, the choice of an intermediate string scale naturally produces a TeV scale gravitino mass, giving a dynamic solution of the hierarchy problem. The small cycle volumes also have the right order of magnitude to support Standard Model gauge couplings.

The use of some  $\alpha'$  corrections invites the concern that the full (unknown)  $\alpha'$  expansion will also be needed. However at very large volumes this is not true as the  $\alpha'$  expansion is controlled. Because of the no-scale structure, the first correction is required (as it corrects zero) but higher corrections are not. We analysed this in chapter 5 by studying the dimensional reduction of the local terms in the ten-dimensional IIB action, showing that the neglected terms give subleading contributions compared to those included. We also discussed loop corrections not coming from local ten-dimensional terms, showing how the known examples are subleading to the  $\alpha'^3$  correction we included. We also described how one can use the existence of a classical geometric limit to constrain the form of corrections to the Kähler potential.

This large-volume scenario does not require fine-tuning, breaks supersymmetry and stabilises all moduli in such a way as to address the hierarchy problem. It therefore deserves a more detailed analysis of the phenomenology, which was carried out in Part III. The phenomenology is primarily determined by the value of the string scale. There is here a tension between the values of the string scale appropriate to TeV scale supersymmetry breaking and the values giving inflationary potentials with the correct normalisation of density perturbations.

Chapters 6 and 7 assume an intermediate string scale  $m_s \sim 10^{11} \text{GeV}$ . In this case the gravitino mass is  $\sim 1 \text{TeV}$  and the phenomenology is that of a TeV-scale MSSM - we assume a brane configuration can be found realising the MSSM spectrum. The geometric picture is of a very large bulk space, with approximate volume  $10^{14} l_s^6$ , together with some small blow-up cycles on which branes containing Standard Model matter live. The visible breaking of supersymmetry is gravity-mediated. In chapter 6 we studied the soft breaking terms in this scenario. We first calculated the overall scale of the soft terms and then the fine structure, obtaining a small hierarchy between scalar and gaugino masses.

In chapter 7 we examined the strong CP problem and the axionic solution to it. This requires an axion to remain exactly flat down to the QCD scale, which is in some tension with most approaches to moduli stabilisation. We studied this in some generality for arbitrary string compactifications and proved a no-go theorem on the existence of supersymmetric minima of the F-term potential consistent

with unfixed axions. For a QCD axion to exist, the moduli potential must break supersymmetry, which fits well with the large volume models described in Part II. Lowering the string scale is one of the few ways to ensure the axion decay constant is in a viable regime, and we described a simple modification of the large volume models to ensure a possible QCD axion survives.

Another possible application of moduli potentials is to inflation. A correct normalisation of density perturbations tends to require a relatively high string scale. In chapter 8 we described an inflationary model developed in the context of the large volume models of part II. This requires a string scale close to the GUT scale. The model gives a sharp prediction of  $n_s = 0.96 \rightarrow 0.967$  with negligible tensors. The no-scale properties of the scalar potential are used to avoid the  $\eta$ -problem.

The above clearly does not exhaust the possible phenomenology. The results of chapters 6 and 7 suggest a scenario in which the gluino and scalars are relatively heavy while the other gauginos are relatively light. It is interesting to investigate the phenomenology of such a scenario [128] in detail and in particular the low-energy spectrum.

The quantitative accuracy of phenomenological questions is limited by the lack of an explicit construction of the Standard Model. If such a construction could be found, it would enable a far more detailed study of the phenomenology. The Standard Model gauge groups would have to be supported on the blow-up cycles, and one can envisage the possibility of developing models like that of [178].

There are several cosmological questions of interest. The inflationary scale and the MSSM scale are quite different. This leads to the problem that models addressing one problem generally do not address the other. During inflation, the stable values of moduli fields change. In the scenario described here, the stabilised volume is exponentially sensitive to the stabilised dilaton. If the dilaton were to change its value during the inflationary epoch, the scale of the potential could conceivably be dynamically driven after inflation from the GUT to the TeV scales. It may then be possible to merge the models of chapters 6 and 7 with that of chapter 8. It would be interesting to see if this possibility can be realised explicitly.

Reheating and the thermal history of the universe is another interesting question. The potential described in chapter 5 has a rather distinctive and unusual form. It would be interesting to study the cosmological evolution of moduli in this potential: this may relate to the problems of the paragraph above.

All the above topics I leave for work in the (near) future.

# Appendix A

In the Appendix we collect together some technical results whose derivation would interrupt the main flow of the text. Section A.3 is based on a part of [4] primarily due to Kerim Suruliz.

## A.1 Dimensional Reduction of the IIB Action

In this section we derive the prefactors in the Gukov-Vafa-Witten superpotential.

The bosonic type IIB supergravity action in string frame is [23]

$$S_{IIB} = \frac{1}{(2\pi)^7 \alpha'^4} \int d^{10}x \sqrt{-g} \left\{ e^{-2\phi} \left[ \mathcal{R} + 4(\nabla \phi)^2 \right] - \frac{F_1^2}{2} - \frac{1}{2 \cdot 3!} G_3 \cdot \bar{G}_3 - \frac{\tilde{F}_5^2}{4 \cdot 5!} \right\}. \tag{A.1}$$

It is convenient to work in Einstein frame. We redefine

$$g_{MN} = e^{(\phi - \phi_0)/2} \tilde{g}_{MN},$$
 (A.2)

where  $\phi_0 = \langle \phi \rangle$ . The factor of  $e^{\frac{\phi_0}{2}}$  is to ensure that g and  $\tilde{g}$  are identical in the physical vacuum. The action is then

$$\frac{2\pi e^{-2\phi_0}}{l_s^8} \int d^{10}x \sqrt{-\tilde{g}} \left\{ \tilde{\mathcal{R}} - \frac{\partial_M S \partial^M \bar{S}}{2(\text{Re } S)^2} - \frac{e^{\phi_0} G_3 \cdot \bar{G}_3}{12 \text{ Re } S} - \frac{e^{2\phi_0} \tilde{F}_5^2}{4 \cdot 5!} \right\}, \tag{A.3}$$

where  $l_s = 2\pi\sqrt{\alpha'}$  and  $S = e^{-\phi} + iC_0$ . We neglect warping effects; as discussed in section 2.4.1, these are subleading in the large volume limit. In the orientifold limit and in the absence of warping,  $\tilde{F}_5 = 0$  and  $\partial_M S = 0$ . The dimensional reduction of (A.3) then gives

$$S = \frac{2\pi}{g_s^2 l_s^8} \left( \int d^4 x \sqrt{-\tilde{g}_4} \tilde{\mathcal{R}}_4 \mathcal{V} - \int d^4 x \sqrt{-\tilde{g}_4} \left( \int d^6 x \sqrt{\tilde{g}_6} \frac{e^{\phi_0} G_3 \cdot \bar{G}_3}{12 \operatorname{Re} S} \right) \right). \quad (A.4)$$

We use  $g_s = e^{\phi_0}$  and define

$$\mathcal{V} \equiv \int d^6 x \sqrt{\tilde{g}_6}. \tag{A.5}$$

In 4d Einstein frame, the canonically normalised Einstein-Hilbert action must be

$$S_{EH} = \frac{1}{16\pi G} \int d^4x \sqrt{-g_E} \mathcal{R}_E \equiv \frac{M_P^2}{2} \int d^4x \sqrt{-g_E} \mathcal{R}_E. \tag{A.6}$$

This implies  $\tilde{g}_4$  and  $g_E$  are related by  $\tilde{g}_4 = g_E \frac{\mathcal{V}_s^0}{\mathcal{V}_s}$ , where  $\mathcal{V} \equiv \mathcal{V}_s l_s^6$  and  $\mathcal{V}_s^0 = \langle \mathcal{V}_s \rangle$ . This gives

$$M_P^2 = \frac{4\pi V_s^0}{g_s^2 l_s^2}$$
 and  $m_s = \frac{g_s}{\sqrt{4\pi V_s^0}} M_P$ . (A.7)

Dimensional reduction determines the Kähler potential to be [50]

$$\frac{\mathcal{K}}{M_P^2} = -2\ln(\mathcal{V}_s) - \ln(S + \bar{S}) - \ln\left(-i\int\Omega\wedge\bar{\Omega}\right). \tag{A.8}$$

The superpotential can be found from  $V_{flux}$ . If

$$W = \int G_3 \wedge \Omega, \tag{A.9}$$

we would obtain

$$V_{flux} = \frac{4\pi (\mathcal{V}_s^0)^2}{g_s l_s^8} \int d^4x \sqrt{-g_E} e^{\mathcal{K}/M_P^2} \left[ \mathcal{K}^{a\bar{b}} D_a W D_{\bar{b}} W - 3W \bar{W} \right], \tag{A.10}$$

with a, b running over all moduli. Thus if

$$\hat{W} = \frac{g_s^{\frac{3}{2}} M_P^3}{\sqrt{4\pi} l_s^2} \int G_3 \wedge \Omega, \tag{A.11}$$

the scalar potential takes the standard  $\mathcal{N}=1$  form

$$V = \int d^4x \sqrt{-g_E} e^{\mathcal{K}/M_P^2} \left[ \mathcal{K}^{a\bar{b}} D_a \hat{W} D_{\bar{b}} \bar{\hat{W}} - \frac{3}{M_P^2} \hat{W} \bar{\hat{W}} \right]. \tag{A.12}$$

In verifying this it is necessary to use the relations (A.7).

The Kähler potential will receive perturbative corrections and the superpotential non-perturbative corrections. The  $\alpha'$  corrections to the Kähler potential modify the 4-dimensional kinetic terms and arise from the higher-derivative terms in the ten-dimensional IIB action. In principle the dimensional reduction of these give the perturbative corrections to the Kähler potential, although this is not the best way to determine them. We obtain [90]

$$\frac{\mathcal{K}}{M_P^2} = -2\ln\left(\mathcal{V}_s + \frac{\xi g_s^{3/2}}{2e^{3\phi/2}}\right) - \ln(S + \bar{S}) - \ln\left(-i\int\Omega\wedge\bar{\Omega}\right). \tag{A.13}$$

where  $\xi = -\frac{\chi(M)\zeta(3)}{2(2\pi)^3}$ . The superpotential can receive non-perturbative corrections causing it to depend on the Kähler moduli. These can arise from D3-brane instantons or gaugino condensation. The generic form of the superpotential is then

$$\hat{W} = \frac{g_s^{3/2} M_P^3}{\sqrt{4\pi}} \left( W_0 + \sum A_i e^{-a_i T_i} \right), \tag{A.14}$$

with

$$W_0 = \frac{1}{l_s^2} \left\langle \int G_3 \wedge \Omega \right\rangle. \tag{A.15}$$

#### A.2 Canonical Normalisation of Kähler Moduli

In this section we describe how to canonically normalise the Kähler moduli for the 2-modulus  $\mathbb{P}^4_{[1,1,1,6,9]}$  example discussed in chapter 5. The Kähler potential is given by

$$\mathcal{K} = \mathcal{K}_{cs} - 2\ln\left(\left(T_5 + \bar{T}_5\right)\right)^{\frac{3}{2}} - \left(T_4 + \bar{T}_4\right)^{\frac{3}{2}}\right) + \text{ constant }.$$
 (A.16)

Here  $T_4 = \tau_4 + ib_4$  and  $T_5 = \tau_5 + ib_5$ . All terms depending on dilaton and complex structure moduli have been absorbed into  $\mathcal{K}_{cs}$ . We also recall that as  $\tau_5 \gg 1$ ,  $\frac{1}{\tau_5}$  serves as a good expansion parameter.

It may be verified that

$$\partial_{T_5}\partial_{\bar{T}_5}K = \frac{3\tau_5^{-\frac{1}{2}}}{4(\tau_5^{\frac{3}{2}} - \tau_4^{\frac{3}{2}})} + \frac{9\tau_4^{\frac{3}{2}}\tau_5^{-\frac{1}{2}}}{8(\tau_5^{\frac{3}{2}} - \tau_4^{\frac{3}{2}})^2},$$

$$\partial_{T_4}\partial_{\bar{T}_5}K = \partial_{T_5}\partial_{\bar{T}_4}K = \frac{-9\tau_4^{\frac{1}{2}}\tau_5^{\frac{1}{2}}}{8(\tau_5^{\frac{3}{2}} - \tau_4^{\frac{3}{2}})},$$

$$\partial_{T_4}\partial_{\bar{T}_4}K = \frac{3\tau_4^{-\frac{1}{2}}}{4(\tau_5^{\frac{3}{2}} - \tau_4^{\frac{3}{2}})} + \frac{9\tau_4}{8(\tau_5^{\frac{3}{2}} - \tau_4^{\frac{3}{2}})^2}.$$

These results are summarised by

$$\mathcal{K}_{i\bar{j}} = \begin{pmatrix} \mathcal{K}_{4\bar{4}} & \mathcal{K}_{4\bar{5}} \\ \mathcal{K}_{5\bar{4}} & \mathcal{K}_{5\bar{5}} \end{pmatrix} = \begin{pmatrix} \frac{3\tau_{4}^{-\frac{1}{2}}}{8\tau_{5}^{\frac{3}{2}}} + \mathcal{O}(\frac{1}{\tau_{5}^{3}}) & \frac{-9\tau_{4}^{\frac{1}{2}}}{8\tau_{5}^{\frac{5}{2}}} + \mathcal{O}(\frac{1}{\tau_{5}^{4}}) \\ \frac{-9\tau_{4}^{\frac{1}{2}}}{8\tau_{5}^{\frac{5}{2}}} + \mathcal{O}(\frac{1}{\tau_{5}^{4}}) & \frac{3}{4\tau_{5}^{2}} + \mathcal{O}(\frac{1}{\tau_{5}^{\frac{7}{2}}}) \end{pmatrix}.$$
(A.17)

We denote the values of the fields  $(b_4, b_5, \tau_4, \tau_5)$  at the minimum by  $(b_4^0, b_5^0, \tau_4^0, \tau_5^0)$  and now define

$$\tau_{5}^{'} = \sqrt{\frac{3}{2}} \frac{\tau_{5}}{\tau_{5}^{0}}, \quad \tau_{4}^{'} = \sqrt{\frac{3}{4}} \frac{\tau_{4}}{(\tau_{5}^{0})^{\frac{3}{4}} (\tau_{4}^{0})^{\frac{1}{4}}}, \quad b_{5}^{'} = \sqrt{\frac{3}{2}} \frac{b_{5}}{\tau_{5}^{0}}, \quad b_{4}^{'} = \sqrt{\frac{3}{4}} \frac{b_{4}}{(\tau_{5}^{0})^{\frac{3}{4}} (\tau_{4}^{0})^{\frac{1}{4}}},$$

and 
$$au_4^{'0} = \sqrt{\frac{3}{4}} \frac{ au_4^0}{( au_5^0)^{\frac{3}{4}} ( au_4^0)^{\frac{1}{4}}}, au_5^{'0} = \sqrt{\frac{3}{2}}.$$

Then after some manipulation, we obtain

$$\begin{split} \sum_{i,j} \mathcal{K}_{i\bar{j}} \partial_{\mu} T^{i} \partial^{\mu} \bar{T}^{j} &= \\ &\frac{1}{2} \bigg[ \left( \frac{\tau_{5}^{'0}}{\tau_{5}^{'}} \right)^{\frac{3}{2}} \left( \frac{\tau_{4}^{'0}}{\tau_{4}^{'}} \right)^{\frac{1}{2}} \partial_{\mu} b_{4}^{'} \partial^{\mu} b_{4}^{'} + \left( \frac{\tau_{5}^{'0}}{\tau_{5}^{'}} \right)^{2} \partial_{\mu} b_{5}^{'} \partial^{\mu} b_{5}^{'} - 4 \sqrt{3} \left( \frac{\tau_{4}^{'}}{\tau_{4}^{'0}} \right)^{\frac{1}{2}} \left( \frac{\tau_{5}^{'0}}{\tau_{5}^{'}} \right)^{\frac{5}{2}} \tau_{4}^{'0} \partial_{\mu} b_{4}^{'} \partial^{\mu} b_{5}^{'} \\ &+ \left( \frac{\tau_{5}^{'0}}{\tau_{5}^{'}} \right)^{\frac{3}{2}} \left( \frac{\tau_{4}^{'0}}{\tau_{4}^{'}} \right)^{\frac{1}{2}} \partial_{\mu} \tau_{4}^{'} \partial^{\mu} \tau_{4}^{'} + \left( \frac{\tau_{5}^{'0}}{\tau_{5}^{'}} \right)^{2} \partial_{\mu} \tau_{5}^{'} \partial^{\mu} \tau_{5}^{'} - 4 \sqrt{3} \left( \frac{\tau_{4}^{'}}{\tau_{4}^{'0}} \right)^{\frac{1}{2}} \left( \frac{\tau_{5}^{'0}}{\tau_{5}^{'}} \right)^{\frac{5}{2}} \tau_{4}^{'0} \partial_{\tau} \tau_{4}^{'} \partial^{\tau} \tau_{5}^{'} \bigg]. \end{split}$$

The moduli are now canonically normalised except for the crossterm, which is suppressed by  $(\tau_5^0)^{\frac{3}{4}}$  and is thus very small. All such crossterms could be eliminated by field redefinitions order by order in  $\frac{1}{\tau_5}$ ; however, negelecting the subleading corrections it is sufficient to use  $\tau_5'$  and  $\tau_4'$  as canonically normalised fields.

# A.3 Soft Term Computations

Let us introduce the notation  $\mathcal{V}' = (T_5 + \bar{T}_5)^{3/2} - (T_4 + \bar{T}_4)^{3/2}$  (which differs by a factor of 36 from the Calabi-Yau volume  $\mathcal{V}$ ). We write

$$\hat{\mathcal{K}}_{T_5} = -3 \frac{(T_5 + \bar{T}_5)^{1/2}}{\mathcal{V}' + \xi'/2},$$

$$\hat{\mathcal{K}}_{T_4} = 3 \frac{(T_4 + \bar{T}_4)^{1/2}}{\mathcal{V}' + \xi'/2}.$$
(A.18)

We denote  $(T_4 + \bar{T}_4)^{1/2} = X$  and  $(T_5 + \bar{T}_5)^{1/2} = Y$ . Then  $\mathcal{V}' = Y^3 - X^3$  and we can calculate the metric  $\mathcal{K}_{T_i\bar{T}_i}$ :

$$\mathcal{K}_{T_i\bar{T}_j} = \begin{pmatrix} \frac{3}{2Xx} + \frac{9X^2}{2x^2} & -\frac{9XY}{2x^2} \\ -\frac{9XY}{2x^2} & \frac{-3}{2Yx} + \frac{9Y^2}{2x^2} \end{pmatrix}, \tag{A.19}$$

and the inverse metric  $\mathcal{K}^{\bar{T}_i T_j}$ 

$$\begin{pmatrix} \frac{-2X(\mathcal{V}'+\xi/2)(2Y^3+X^3-\xi'/2)}{3(\xi'/2-2\mathcal{V}')} & -2X^2Y^2\frac{(\mathcal{V}'+\xi'/2)}{(\xi'/2-2\mathcal{V}')} \\ -2X^2Y^2\frac{(\mathcal{V}'+\xi'/2)}{(\xi'/2-2\mathcal{V}')} & \frac{-2Y(\mathcal{V}'+\xi'/2)(2X^3+Y^3+\xi'/2)}{3(\xi'/2-2\mathcal{V}')} \end{pmatrix}.$$
 (A.20)

In these expressions  $x = Y^3 - X^3 + \xi'/2 = \mathcal{V}' + \xi'/2$ . We can now calculate the F-terms as given by formula (6.6). We assume that  $\partial_5 W \ll \partial_4 W$  (where

<sup>&</sup>lt;sup>1</sup>We do not simply use  $\tau_4$  and  $\tau_5$  as this would not be valid if D3-branes were present.

 $\partial_i \equiv \partial_{T_i}$ ) and only include the nonperturbative contribution corresponding to  $T_4$ . The result is

$$F^{4} = e^{\hat{\mathcal{K}}/2} \frac{2\mathcal{V}' + \xi'}{2\mathcal{V}' - \xi'/2} \left( -X^{2}W + \frac{X}{3} (2Y^{3} + X^{3} - \frac{\xi}{2})(\partial_{4}W) \right)$$

$$F^{5} = e^{\hat{\mathcal{K}}/2} \frac{2\mathcal{V}' + \xi}{2\mathcal{V}' - \xi'/2} \left( -Y^{2}W + X^{2}Y^{2}(\partial_{4}W) \right)$$
(A.21)

After a redefinition of c and d, the prefactor of the kinetic term for the brane modulus  $\phi$ ,  $\tilde{\mathcal{K}}_i$ , can be rewritten as

$$\tilde{\mathcal{K}}_i = \frac{cX + dY}{\mathcal{V}' + \xi'/2}.\tag{A.22}$$

To calculate scalar masses we need

$$\partial_m \partial_{\bar{n}} \log \tilde{\mathcal{K}}_i = \partial_m \partial_{\bar{n}} \left( \log(cX + dY) - \log\left(\mathcal{V}' + \frac{\xi'}{2}\right) \right).$$

As  $\hat{\mathcal{K}} = -2\log(\mathcal{V}' + \xi'/2) + \text{constant},$ 

$$-\partial_m \partial_{\bar{n}} \log \left( \mathcal{V}' + \frac{\xi'}{2} \right) = \frac{1}{2} \hat{\mathcal{K}}_{m\bar{n}}$$

and so

$$\partial_m \partial_{\bar{n}} \log \tilde{\mathcal{K}}_i = \partial_m \partial_{\bar{n}} \left( \log(cX + dY) \right) + \frac{1}{2} \hat{\mathcal{K}}_{m\bar{n}}. \tag{A.23}$$

The necessary derivatives of  $\log(cX+dY)$  can be calculated using  $\partial_4 X = 1/(2X)$ ,  $\partial_5 Y = 1/(2Y)$ , and likewise with respect to  $\bar{4}$  and  $\bar{5}$ . The results are

$$\partial_4 \partial_{\bar{4}} \log(cX + dY) = -\frac{c(2cX + dY)}{4X^3(cX + dY)^2},$$

$$\partial_4 \partial_{\bar{5}} \log(cX + dY) = -\frac{cd}{4XY(cX + dY)^2},$$

$$\partial_5 \partial_{\bar{5}} \log(cX + dY) = -\frac{d(2dY + cX)}{4Y^3(cX + dY)^2}.$$
(A.24)

We now wish to find the soft masses, expressing the result as a small deviation from the no-scale result, which is zero. To do this we make an expansion assuming that  $(\partial_4 W)$  is small (we know it is  $\mathcal{O}(\mathcal{V}^{-1})$  at the minimum of the scalar potential) and  $\xi/\mathcal{V}$  is small. For example, we can note that  $F^m \bar{F}^{\bar{n}}$  always has a prefactor of

$$\frac{(2\mathcal{V}'+\xi')^2}{(2\mathcal{V}'-\xi'/2)^2},$$

which may be expanded as

$$1 + \frac{3\xi'}{2\mathcal{V}'} + \mathcal{O}\left(\frac{1}{\mathcal{V}^2}\right). \tag{A.25}$$

From the above analysis we expect that  $F^m \bar{F}^{\bar{n}} \partial_m \partial_{\bar{n}} \log(cX + dY)$  can be written as  $e^{\hat{\mathcal{K}}} \left( -\frac{1}{2} |W|^2 + \mathcal{O}(\mathcal{V}^{-\alpha}) \right)$  for some  $\alpha > 0$ . Indeed, an explicit computation gives

$$F^{m}\bar{F}^{\bar{n}}\partial_{m}\partial_{\bar{n}}\log(cX+dY) = e^{\hat{\mathcal{K}}}\left(1+\frac{3\xi'}{2\mathcal{V}'}\right) \times \left(-\frac{1}{2}|W|^{2} + \frac{1}{3(cX+dY)}\left(-\frac{\xi c}{2} + 3dX^{2}Y + cX^{3} + 2cY^{3}\right)W\left(\partial_{4}W\right)\right).$$

Similarly we have

$$F^{m}\bar{F}^{\bar{n}}\hat{\mathcal{K}}_{m\bar{n}} = e^{\hat{\mathcal{K}}} \frac{3\mathcal{V}'\left(2\mathcal{V}' - \frac{\xi'}{2}\right)}{2\left(\mathcal{V}' + \frac{\xi'}{2}\right)^{2}} \left(|W|^{2} - \frac{W(\partial_{4}W)}{3\mathcal{V}'}\left(2X^{2}\frac{\xi'}{2} + 2X^{2}\mathcal{V}'\right)\right). \tag{A.26}$$

Expanding

$$\frac{\mathcal{V}'(2\mathcal{V}'-\xi'/2)}{2(\mathcal{V}'+\xi'/2)^2}$$

as  $1 - 3\xi'/4\mathcal{V}'$  we get

$$F^{m}\bar{F}^{\bar{n}}\partial_{m}\partial_{\bar{n}}\log\tilde{\mathcal{K}}_{i} = e^{\hat{\mathcal{K}}}\left(1 + \frac{3\xi'}{2\mathcal{V}'}\right)\left[-|W|^{2} + \frac{1}{3(cX + dY)}\left(-\frac{\xi'c}{2} + 3dX^{2}Y + cX^{3} + 2cY^{3}\right)W(\partial_{4}W) - W(\partial_{4}W)X^{2}\left(1 + \frac{\xi'}{2\mathcal{V}'}\right)\right]$$
(A.27)

Further simplifying and neglecting  $\mathcal{O}(1/\mathcal{V}^2)$  terms we obtain

$$e^{\hat{\mathcal{K}}} \left[ -|W|^2 + \frac{3\xi'}{2\mathcal{V}'}|W|^2 + \frac{W}{3(cX+dY)} \left( \frac{-\xi'c}{2} + 3dX^2Y + cX^3 + 2cY^3 \right) (\partial_4 W) - W(\partial_4 W)X^2 \right].$$

The  $-e^{\hat{\mathcal{K}}}|W|^2$  cancels with  $m_{3/2}^2$  so the mass squared of the brane modulus can be seen to have the form

$$\frac{3\xi'}{2\mathcal{V}'} + f(X,Y)(\partial_4 W),\tag{A.28}$$

which is what we expect naively - before the no-scale structure is broken, the mass is simply zero. There are two sources of no-scale breaking:  $\alpha'$  corrections

corresponding to the first term in (A.28) and nonperturbative corrections corresponding to the second term. To estimate the size of the soft terms, note that all the constants involved in (A.28) are expected to be  $\mathcal{O}(1)$  (including  $c, d, \xi'$ ) and so the first term is  $\mathcal{O}(1/\mathcal{V})$ . As the volume is  $\mathcal{V} \sim Y^3$ , we can estimate

$$f(u,v) \sim \frac{Y^3}{Y} = Y^2 \sim \mathcal{V}^{2/3}.$$
 (A.29)

We also note that, at the minimum,  $\partial_4 W \sim \frac{W_0}{\nu}$ . Then the volume scaling of the second term is  $\mathcal{V}^{2/3}/\mathcal{V} = \mathcal{V}^{-1/3}$ , and as  $e^{\hat{\mathcal{K}}} \sim 1/\mathcal{V}^2$  the moduli masses squared scale as  $\mathcal{O}(1/\mathcal{V}^{7/3})$ . Finally, by considering the form of the superpotential (A.14), we see that  $W(\partial_4 W) \sim W_0^2 g_s^4$ . Putting the factors together, we get

$$m_i^2 = \mathcal{O}(1) \frac{g_s^4 W_0^2}{4\pi (\mathcal{V}_s^0)^{7/3}} M_P^2.$$
 (A.30)

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