Kraus representation for maps and master equation in spin star model with layered environment

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Abstract

Quantum operations are usually defined as completely positive (CP), trace preserving (TP) maps on quantum states, and can be represented by operator-sum or Kraus representations. In this paper, we calculate operator-sum representation and master equation of an exactly solvable dynamic of one-qubit open system in layered environment . On the other hand, we obtain exact Nakajima-Zwanzig (NZ) and time-convolutionless (TCL) master equation from the maps. Finally, we study a simple example to consider the relation between CP maps and initial quantum correlation and show that vanishing initial quantum correlation is not necessary for CP maps.

Keywords : Kraus representations, Quantum Correlation, Nakajima-Zwanzing and Time-convolutionless Master Equation

1 Introduction

In quantum information theory, it is widely accepted that any physical process can be described by a quantum operation or quantum channel which is CP (this means that not only map $\Phi : B_1 \rightarrow B_2$ on $c^\star$-algebra is positive, but also the combined operation $\Phi \otimes I_N : B_1 \otimes N \rightarrow B_2 \otimes N$ for all dimensions N is positivity preserving) and TP maps (this means that $\text{Tr}\Phi(\rho) = \text{Tr}\rho$ for all trace class operator ρ) between spaces of operators[1]-[5]. These maps play an important role in the description of nonunitary time evolutions of open quantum systems that interact with an environment. On the other hand, the environment is classified as Markovian with no memory effect and non-Markovian with memory effect [6]-[19].

It is well-known that for closed quantum system, its time evolution can be described by a unitary operator. However, for an open system, the time evolution is not necessarily unitary. The evolution of an open system is usually described by the Kraus representation [20]. The Kraus representation of an open system is usually constructed by considering a large closed system. Let's assume an interaction between the open system denoted as (s) and the envi-

ronment (b). This environment is a quantum system with the Hilbert space of an arbitrary dimension. The whole $(s)+(b)$ system evolves unitarily. In most of the studies on dynamics of open systems, it is assumed that the open system and its environment are at the initial moment of their joint evolution factorized[21, 22], that is they are described by the density operator of the form

$$\rho_{sb} = \rho_s \otimes \rho_b,$$

where ρ_s is the initial state of system and ρ_b is the initial state of the environment. As the combined system is a closed one, its evolution is unitary,

$$\rho_{sb}(t) = U_{sb}\rho_{sb}(0)U_{sb}^\dagger, \tag{1.1}$$

where U_{sb} is the unitary operator.The interested system, as an open one, than evolves in the following way

$$\rho_s(t) = tr_b\{U_{sb}\rho_{sb}U_{sb}^\dagger\} = tr_b\{\rho_{sb}(t)\}. \tag{1.2}$$

The above equation is obtained by doing partial-trace on bath. Kraus[20, 23] and Choi[24] showed that if the above equation can be equivalently expressed in the form

$$\rho_s(t) = \Phi(\rho_s(0)) = \sum_{\mu\nu} M_{\mu\nu}(t)\rho_s(0)M_{\mu\nu}^\dagger(t), \tag{1.3}$$

where the map Φ is described by $\Phi : \rho_s(0) \to \rho_s(t)$ and $M_{\mu\nu}(t)$ satisfy the following equation

$$\sum_{\mu\nu} M_{\mu\nu}^\dagger(t)M_{\mu\nu}(t) = I, \tag{1.4}$$

it is said that evolution of $\rho_s(t)$ has the form of the Kraus representation and the above equation is condition of CP map[25].

In this paper, we consider a model of a spin star configuration which consists of N+1 localized spin-$\frac{1}{2}$ particles. One of the spins is located at the center of the star, while the other spins on concentric circles with different radii surrounding the central spin layer by layer[26]. The difference in radius is because the coupling coefficients between layers spins and

the central spin is taken differently. At first, we obtain Kraus representations and then arrive master equation with the use of Kraus representations and show that they satisfy Eq.(4).

Maps are one of the main bases to obtain Kraus representations. In addition, they are effective in obtaining exact NZ and TCL master equation[27]. NZ and TCL are used when the exact solution of the master equation is a difficult task, therefore we can approximate the master equation to solve[28, 8]. These techniques were introduced by Nakajima (1958) and Zwanzig (1960) and independently by the Brussels school (Prigogine, 1962)[8]. They are widely used in non-equilibrium statistical mechanics and have played an important role in various fields of physics (e.g. in the theory of transport phenomena and relaxation processes, in quantum theory damping, in super-radiance and laser theory)[29]. Details of the application of this method in the various fields and numerous references are given in books and reviews articles of $Haken$[30, 31], $Agarwal$[32] and $Haake$ [33]. By considering the above model and exploiting the knowledge of the exact unitary evolution, and the reduced dynamics, as well as a suitable matrix representation of the dynamical maps, we can exhibit the exact TCL and NZ equation of motion.

The paper is organized as follows. In Sec.(II), we will explain about how to obtain map and Kraus representation and master equation. Exact NZ and TCL master equation are discussed in Sec.(III). Finally, in Sec.(IV), we consider a simple example that shows vanishing quantum discord is not necessary for CP maps.

2 Map, Kraus representation and Master equation

2.1 Background

The first step in relating a general, non-Markovian master equation $\dot{\rho}(t) = \Lambda[\rho(t)]$ to a corresponding CP map $\rho(t) = \Phi_t[\rho(0)]$ relies on expressing this relationship in matrix form. We need to write the matrix form to the basis of $\{W_a\}$, that the basis denotes every convenient

orthonormal basis set for the Hermitian operators on the Hilbert space, i.e.

$$W_a^\dagger = W_a,$$

$$tr[W_a W_b] = \delta_{ab}.$$

Now, according to the introduced basis, for any maps Φ (though we will be concerned mainly with CP maps below) we can express both ρ and $\Phi(\rho)$ in matrix form as follows[34],

$$\Phi(\rho) = (Fr)^T W, \tag{2.5}$$

where the evolution matrix F is the following

$$F_{kl} := tr[W_k \Phi(W_l)], \tag{2.6}$$

and also vector r is defined as

$$r_l := tr[W_l \rho].$$

Indicating time-dependence of Φ and F via Φ_t and $F(t)$, and defining

$$\rho(t) := \Phi_t[\rho(0)]. \tag{2.7}$$

Time evolution of ρ can be expressed as

$$\dot{\rho} = [\dot{F}(t)r(0)]^T W.$$

Now, suppose that $\rho(t)$ satisfies a master equation of the form

$$\dot{\rho} = \Lambda(\rho). \tag{2.8}$$

Note that the linear map Λ may be time-dependent. Now, this master equation can be rewritten as

$$\dot{\rho} = [\dot{L}(t)r(t)]^T W.$$

Note that L (and F) are necessarily real, but not symmetric in general. Comparing the results of the last two paragraphs gives

$$\dot{F}r(0) = Lr(t) = LFr(0). \tag{2.9}$$

Hence, since by linearity this equation must hold for all vectors $r(0)$ (whether or not they correspond to a density operator).

Choi has shown that complete positivity of Φ is equivalent to the positivity of the particular matrix S [35, 36], and that the Kraus decompositions of Φ are related to the outer product decomposition of S. The master equation gives us matrix L, which in turn gives us a matrix F, characterising the linear evolution map Φ_t. In order to proceed from F to a Kraus decomposition, all we need to do is finding S from F, and then diagonalise it. In fact, our construction of S from L allows us to *determine* whether or not, for a given proposed master equation, the corresponding map is completely positive. Choi further demonstrated that Φ is completely positive if and only if S is positive, i.e., $S \geq 0$ [35, 36]. The Hermitian property implies that one can always decompose S as a sum of outer products, i.e.,

$$S = \sum_i V_i(t)V_i^\dagger(t) \tag{2.10}$$

for some set vectors$\{V(i)\}$ that we take $\{V(i)\}$ as follows

$$V(i) = \sqrt{|\lambda_i|}\Pi(i), \tag{2.11}$$

where Π coefficients are eigenvectors of matrix S and also the Kraus operators are given by

$$M_i := \sum_a V(i)_a \beta_a. \tag{2.12}$$

Noting that the β_a form an orthonormal (non-Hermitian) basis for the operators on the Hilbert space[34].

2.2 Map, Master Equation and Kraus representation for spin star model for one-qubit system with layered environment in a channel

We obtain all the issues described in the previous section for the model which at first was mentioned. The Hamiltonian of the XX central spin model that is composed by a localized spin, hereafter called central spin, coupled to N spins of bath layers with the different coupling constant α, takes the form[26]

$$H = 2(\sigma_+ \Xi_- + \sigma_- \Xi_+), \tag{2.13}$$

where $\Xi_\pm$ is denoted as

$$\Xi_+ = \sum_{\mu=1}^{n} \alpha_\mu J_+^\mu, \tag{2.14}$$

$$\Xi_- = \sum_{\mu=1}^{n} \alpha_\mu J_-^\mu, \tag{2.15}$$

where $\sigma_\pm$ and $J_\pm^\mu$ are the Pauli operators referring to the central spin and the surrounding spins respectively , and $J_\pm^\mu$ is denoted as

$$J_\pm^\mu \equiv \sum_{j=1}^{N_\mu} \sigma_\pm^j. \tag{2.16}$$

So, evolution density matrix of system is as follows

$$\rho_S(t) = \frac{1}{2} \begin{pmatrix} 1 + v_3(0)f_3 & v_-(0)f_{12} \\ v_+(0)f_{12} & 1 - v_3(0)f_3 \end{pmatrix} \tag{2.17}$$

where we have introduced the functions[26]

$$f_{12}(t) \equiv tr_B\{\cos[2th_1(\alpha_1, \ldots, \alpha_n)] \cos[2th_2(\alpha_1, \ldots, \alpha_n)] \otimes 2^{-N} I_B\}, \tag{2.18}$$

and

$$f_3(t) \equiv tr_B\{\cos[2th_1(\alpha_1, \ldots, \alpha_n)] \otimes 2^{-N} I_B\}, \tag{2.19}$$

where $h_1(\alpha_1, \alpha_2, \ldots, \alpha_n)$ and $h_2(\alpha_1, \alpha_2, \ldots, \alpha_n)$ are

$$h_1 = h_1(\alpha_1, \alpha_2, \ldots, \alpha_n) = \sqrt{\sum_{\mu=1}^{n} \alpha_\mu^2 J_+^\mu J_-^\mu}, \tag{2.20}$$

$$h_2 = h_2(\alpha_1, \alpha_2, \ldots, \alpha_n) = \sqrt{\sum_{\mu}^{n} \alpha_\mu^2 J_-^\mu J_+^\mu}. \tag{2.21}$$

To obtain the master equation, the matrix F firs must be determined, and checked as whether it is invertible. The basis $\{W_a\}$ is chosen as $W_a = \frac{1}{\sqrt{2}}\sigma_a$, where σ_a are the Pauli operators. One finds

$$\Phi(W_0) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{2.22}$$

$$\Phi(W_1) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & f_{12} \\ f_{12} & 0 \end{pmatrix}, \tag{2.23}$$

$$\Phi(W_2) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -if_{12} \\ if_{12} & 0 \end{pmatrix}, \tag{2.24}$$

$$\Phi(W_3) = \frac{1}{\sqrt{2}} \begin{pmatrix} f_3 & 0 \\ 0 & f_3 \end{pmatrix}. \tag{2.25}$$

The matrix F follows via Eqs.(5) and (6) as

$$F = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & f_{12} & 0 & 0 \\ 0 & 0 & f_{12} & 0 \\ 0 & 0 & 0 & f_3 \end{pmatrix}, \tag{2.26}$$

The solution for L follows via Eq.(9)as

$$L = \dot{F}F^{-1} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{f'_{12}}{f_{12}} & 0 & 0 \\ 0 & 0 & \frac{f'_{12}}{f_{12}} & 0 \\ 0 & 0 & 0 & \frac{f'_3}{f_3} \end{pmatrix}. \tag{2.27}$$

Now, to obtain the master equation in the form $\dot{\rho} = \Lambda_t(\rho)$, we calculate the Choi matrix R in Ref.[34],

$$R = \begin{pmatrix} \frac{f_3'}{2f_3} & \frac{f_{12}'}{f_{12}} & 0 & 0 \\ \frac{f_{12}'}{f_{12}} & \frac{f_3'}{2f_3} & 0 & 0 \\ 0 & 0 & -\frac{f_3'}{2f_3} & 0 \\ 0 & 0 & 0 & -\frac{f_3'}{2f_3} \end{pmatrix}, \tag{2.28}$$

The master equation immediately follows via equation as

$$\Lambda_t(\rho) := \sum_{ab} R_{ab}(t)\beta_a \rho \beta_b^\dagger,$$

$$\dot{\rho}(t) = \Lambda_t[\rho(t)] = \left(\frac{f_3'}{2f_3} + \frac{f_{12}'}{f_{12}}\right)\{\sigma_+\sigma_-, \rho\sigma_-\sigma_+\} - \frac{f_3'}{2f_3}(\sigma_+\rho\sigma_+ + \sigma_-\rho\sigma_-). \tag{2.29}$$

Finally, we obtain Kraus representations via using of Eqs.(11) and (12). At first, we calculate matrix S as follows

$$S = \frac{1}{2}\begin{pmatrix} 1 + 2f_{12} + f_3 & 0 & 0 & 0 \\ 0 & 1 - f_3 & 0 & 0 \\ 0 & 0 & 1 - f_3 & 0 \\ 0 & 0 & 0 & 1 - 2f_{12} + f_3 \end{pmatrix}. \tag{2.30}$$

Now, we obtain Kraus representations

$$M_1 = \sqrt{\frac{1}{2}(1 + 2f_{12} + f_3)}W_0, \tag{2.31}$$

$$M_2 = \sqrt{\frac{1}{2}(1 - f_3)}W_1, \tag{2.32}$$

$$M_3 = \sqrt{\frac{1}{2}(1 - f_3)}W_2, \tag{2.33}$$

$$M_4 = \sqrt{\frac{1}{2}(1 - 2f_{12} + f_3)}W_3, \tag{2.34}$$

that they satisfy Eq.(4).

3 exact Nakajima-Zwanzig and time-convolutionless master equation

3.1 Background

With the aid of the knowledge of the exact time evolution, and using the representation of maps in terms of matrices, we now explicitly obtain two kinds of exact equations of motion for the reduced system's dynamics. We first consider a master equation in differential form with a generator local in time, that is, the TCL master equation. Assuming the existence of such a generator $K_{TCL}(t)$, it should obey the equation[27]

$$\dot{\rho}(t) = K_{TCL}(t)\rho(t), \tag{3.35}$$

which, due to $\rho(t) = \Phi(t)\rho(0)$, is satisfied upon identifying

$$K_{TCL} = \dot{\Phi}(t)\Phi^{-1}(t) \tag{3.36}$$

or, in terms of matrices,

$$P_{TCL} = \dot{F}(t)F^{-1}(t), \tag{3.37}$$

this expression holds when F is invertible.

Now, we can obtain the NZ master equation according to the TCL master equation when memory kernel exists. In this case the memory kernel $K_{NZ}(t)$ should obey the convolution equation

$$\dot{\rho}(t) = (K_{NZ} \circ \rho)(t), \tag{3.38}$$

so that in view of Eq.(7), one has the relation

$$\widehat{K}_{NZ}(u) = u\mathbb{I} - \widehat{\Phi}^{-1}(u), \tag{3.39}$$

where the hat denotes the Laplace transform, and therefore in matrix representation,

$$\widehat{P}_{NZ}(u) = u\mathbb{I} - \widehat{F}^{-1}(u). \tag{3.40}$$

3.2 Nakajima-Zwanzig(NZ) and Time-Convolutionless(TCL) for spin star model for one-qubit system with layered environment in a channel

The technique used in the previous subsection to obtain the TCL and NZ equation of motion for a model whose evolution is known, by exploiting the representation of maps in terms of matrices, is applicable for a detailed study of Model in Sec.(IIB).

Now, starting from Eq.(17) and exploiting the same strategy used in Sec.(IIIA), one immediately obtains, for the matrix representation of the TCL generator, the expression

$$
P_{TCL}(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \gamma_1 & 0 & 0 \\ 0 & 0 & \gamma_1 & 0 \\ 0 & 0 & 0 & \gamma_2 \end{pmatrix},
\tag{3.41}
$$

where γ_1 and γ_2 are

$$
\gamma_1 = -2(h_1 \tan[2h_1 t] + h_2 \tan[2h_2 t]), \gamma_2 = -4h_1 \tan[4h_1 t].
$$

And one can also determine the expression of the NZ memory kernel, whose Laplace transform is given by

$$
\widehat{P}_{NZ}(u) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \eta_1 & 0 & 0 \\ 0 & 0 & \eta_1 & 0 \\ 0 & 0 & 0 & \eta_2 \end{pmatrix},
\tag{3.42}
$$

where η_1 and η_2 are

$$
\eta_1 = u - \frac{2^N[u^4 + 8(h_1^2 + h_2^2)u^2 + 16(h_1^2 - h_2^2)^2]}{u^3 + 4(h_1^2 + h_2^2)u}, \eta_2 = u - \frac{2^N[u^2 + 16h_1^2]}{u}.
$$

So the master equation in operator form reads[27],

$$
k_{TCL}(t)\rho = -\frac{1}{2}\eta_2(\sigma_+\rho\sigma_- - \frac{1}{2}\{\sigma_-\sigma_+, \rho\}) - \frac{1}{2}\eta_2(\sigma_-\rho\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-, \rho\}) + \frac{1}{4}(\eta_2 - 2\eta_1)(\sigma_z\rho\sigma_z - \rho),
\tag{3.43}
$$

and

$$\widehat{K}_{NZ}(u)\rho = -\frac{1}{2}\gamma_2(\sigma_+\rho\sigma_- - \frac{1}{2}\{\sigma_-\sigma_+, \rho\}) - \frac{1}{2}\gamma_2(\sigma_-\rho\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-, \rho\}) + \frac{1}{4}(\gamma_2 - 2\gamma_1)(\sigma_z\rho\sigma_z - \rho).$$

(3.44)

It immediately appears that, given the time evolution map from the relation Eqs.(41) and (42), one can directly obtain the generator of the master equation in TCL form or the memory kernel for the NZ form, respectively, without resorting to the evolution of the whole perturbative series.Obviously, given the exact time evolution, one dose not need the equations of motion.

4 Initial quantum correlation and CP maps

4.1 Background

Recently, a relation between CP maps and quantum discord has been put forth [37, 38]. quantum discord, first introduced by Ollivier and Zurek [39], that captures the difference of two natural quantum extensions of the classical mutual information, can be used as a measure of the quantum correlations. Although quantum discord is equal to the entanglement for pure states, it includes the quantum correlation which are contained in mixed states that are not entangled. In [37], it is shown that if the initial system-bath state has vanishing quantum discord, then the dynamics of system can be described by a CP map. In [38] Shabani and Lidar made a strong claim: *The reduced dynamics of a system is completely-positive, for any coupling with the bath, only if the initial system-bath state has vanishing quantum discord as measured by the system.*

Here we show that underlying assumption in Ref.[38] limit the generality of the constructed reduced dynamics. To do this we consider two different paradigms to ask the question: given any unitary evolution for the system-bath, can we define a CP map taking a family of initial states $\{\rho^S(0)\}$ to final states $\{\rho^S(t)\}$?

4.2 Vanishing quantum discord is not necessary for CP maps

Now, we prove our claim that vanishing quantum discord is not necessary for CP with notice to below example. In this example, first we obtain Kraus representation with map and Eq.(3) and then we prove that map is CP with notice to Eq.(4). On the other hand, we compute quantum discord and it shows that we can choose conditions for nonzero quantum discord.

We choose the same model as that Ref.[15] has used. That is to consider a combined system composed of two spin-1/2 subsystems with the interaction Hamiltonian

$$H_{sb} = \sigma_x \otimes \frac{1}{2}(I - \sigma_z) + I \otimes \frac{1}{2}(I + \sigma_z), \tag{4.45}$$

where σ_x and σ_z are Pauli spin operators. In this model, the first qubit plays the role of the open system while the second qubit plays the role of the environment. The interaction described by the Hamiltonian corresponds to the well-known controlled- NOT gate [22, 13]. The unitary evolution operator is given by $U_{sb}(t) = e^{-iH_{sb}t}$, explicitly

$$U_{sb}(t) = \begin{pmatrix} e^{-it} & 0 & 0 & 0 \\ 0 & \cos t & 0 & -i\sin t \\ 0 & 0 & e^{-it} & 0 \\ 0 & -i\sin t & 0 & \cos t \end{pmatrix}, \tag{4.46}$$

So, first we obtain $\rho_{sb}(0)$ as follows

$$\rho_{sb}(0) = \frac{1}{4}(I^s \otimes I^b + \sum_{i=1}^{3} c_i \sigma_i^s \otimes \sigma_i^b), \tag{4.47}$$

and also we can obtain $\rho_s(0)$ with trace of $\rho_{sb}(0)$ as follows

$$\rho_s(0) = \frac{1}{2}I, \tag{4.48}$$

where $\sigma_i s$ are Pauli matrices. From Eqs. (2), (10) and (11), we get the density matrix of the system

$$\rho_s(t) = \frac{1}{4} \begin{pmatrix} 2 + c_3(1 - \cos(2t)) & -ic_3\sin(2t) \\ ic_3\sin(2t) & 2 - c_3(1 - \cos(2t)) \end{pmatrix}. \tag{4.49}$$

So, maps can be written, with notice to Eqs (7) and (49), as

$$\Phi(W_0) = \frac{1}{2\sqrt{2}} \begin{pmatrix} 2 + c_3(1 - \cos(2t)) & -ic_3 \sin(2t) \\ ic_3 \sin(2t) & 2 - c_3(1 - \cos(2t)) \end{pmatrix},$$

$$\Phi(W_1) = \Phi(W_2) = \Phi(W_3) = 0. \tag{4.50}$$

Now, we can obtain matrix of F and then matrix of S by having maps. On the other hand, we can arrive Kraus representations with have matrix of S as follows

$$M_1 = \frac{1}{2}\Gamma(t)\sin(t)W_0 + \frac{i}{2}\Gamma(t)\cos(t)W_1 - \frac{\sqrt{2}}{4}W_3, \tag{4.51}$$

$$M_2 = -\frac{1}{2}\Gamma(t)\cos(t)W_0 + \frac{i}{2}\Gamma(t)\sin(t)W_1 + \frac{\sqrt{2}}{4}W_2, \tag{4.52}$$

$$M_3 = \frac{1}{2}\Gamma(t)\sin(t)W_0 + \frac{i}{2}\Gamma(t)\cos(t)W_1 + \frac{\sqrt{2}}{4}W_3, \tag{4.53}$$

$$M_4 = \frac{1}{2}\Gamma(t)\cos(t)W_0 - \frac{i}{2}\Gamma(t)\sin(t)W_1 + \frac{\sqrt{2}}{4}W_2, \tag{4.54}$$

where $\Gamma(t) = \sqrt{1 - c_3 \sin(t)}$ and Kraus operators $\{M_i\}$ satisfy Eq.(4) which expresses that our choose map is CP.

However, given the quantum discord calculated in Ref.[40] and Eq.(4), we can conclude that according to Kraus representations obtained, Eq.(4) will always be satisfied regardless of the values of c_3 and the values of quantum discord and it is clearly violation of claim of Ref.[38]. So, we understand that vanishing quantum discordC is not necessary for CP map.

5   conclusion

In this paper, we have started from obtained maps of spin star model for one-qubit system with layered environment and then by use of these maps, computed the matrix F. With the matrix F, first we have obtained the Choi matrix S and then by use of matrix S, have arrived Kraus representation and master equation that clearly, we understood that Kraus representation satisfy Eq.(4). It expresses that maps are CP. On the other hand, by use of the matrix F we

have arrived matrix representations of the TCL and NZ generator and then with use of matrix representations we have obtained operator representations of the TCL and NZ generator. In the end, with the two subjects expressed (satisfies the equation (4) and the calculated quantum discord), we have proven that vanishing quantum discord is not necessary for CP.

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