SUPERFIELD BRST CHARGE AND THE MASTER ACTION

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ABSTRACT. Using a superfield formulation of extended phase space, we propose a new form of the Hamiltonian action functional. A remarkable feature of this construction is that it directly leads to the BV master action on phase space. Conversely, superspace can be used to construct nilpotent BRST charges directly from solutions to the classical Lagrangian Master Equation. We comment on the relation between these constructions and the specific master action proposal of Alexandrov, Kontsevich, Schwarz and Zaboronsky.

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1. Introduction

In two recent papers [1, 2], it has been shown that quantization, in both the Hamiltonian operator language and a phase space path integral, has an equivalent superfield formulation. The superspace consists of ordinary time t and a new Grassmann-odd direction θ . All original phase space coordinates $z_0^A(t)$ are just the zero-components of super phase-space coordinates

(1)
$$z^{A}(t,\theta) = z_{0}^{A}(t) + \theta z_{1}^{A}(t) .$$

It follows that $z^A(t,\theta)$ has the same statistics as $z_0^A(t)$, denoted by $\epsilon(z^A)$. The superspace derivative

$$D \equiv \frac{d}{d\theta} + \theta \frac{d}{dt} ,$$

replaces the ordinary time derivative. It satisfies

$$(3) D^2 = \frac{d}{dt} ,$$

and one can indeed show that the usual Heisenberg equations of motion on the original phase space variables are obtained by applying D twice on the extended variables.

We shall here consider another extension of the usual time derivative,

$$\mathcal{D} \equiv \theta \frac{d}{dt} ,$$

which obviously satisfies $\mathcal{D}^2 = 0$. This derivative will turn out to play a central role in understanding BV-quantization [3] in phase space [4]. We note that \mathcal{D} can be assigned a definite ghost number according to the ghost number we assign to θ , and we choose $gh(\theta) = 1$.

We now make some general observations concerning a relation between the even Poisson bracket on the original phase-space manifold and an associated (Grassmann-odd) antibracket on the super path space that can be induced by it [1, 7]:*

(5)
$$\begin{aligned} \left(z_0^A(t), z_0^B(t') \right) &= 0 \\ \left(z_0^A(t), z_1^B(t') \right) &= \omega^{AB}(z_0) \delta(t - t') \\ \left(z_1^A(t), z_1^B(t') \right) &= z_1^C \partial_C \omega^{AB}(z_0) \delta(t - t') \,. \end{aligned}$$

where $\omega^{AB}(z) = \{z^A, z^B\}$. This antibracket obviously carries one unit of Grassmann parity and one unit of ghost number. Another important property of (5) is the following: let f(z) be a function on phase space and let the functional F be defined by

(6)
$$F[z] = \int dt d\theta \ f(z(t,\theta)) \ ,$$

so that the Grassmann parity of F is opposite that of f. Then for any functions f, g on the phase space we have

(7)
$$(F,G) \equiv \int dt d\theta \{f,g\} .$$

where F, G are the corresponding functionals obtained by (6). In particular, if a Grassmann-odd function f satisfies the "Hamiltonian Master Equation" $\{f, f\} = 0$ then the corresponding Grassmann-even functional $F = \int dt d\theta f$ satisfies the BV Master Equation with respect to the antibracket:

$$(8) (F,F) = 0.$$

This has an obvious generalization, because even if $f = f(z, t, \theta)$ is a function on the original phase space that also explicitly depends on time t and its super-partner θ , and f still satisfies the Hamiltonian Master Equation $\{f, f\} = 0$ for any t and θ , then the corresponding functional $F = \int dt d\theta f(z(t, \theta), t, \theta)$ also satisfies the Master Equation with respect to the antibracket (7).

^{*}See section 4 for details.

2. Superfield Realization of the Phase Space Antifield Formalism

Let us now consider a system with first class constraints. We thus have a Grassmann-odd BRST generator $\Omega = \Omega(z)$ and an Hamiltonian H = H(z), which are taken to satisfy

(9)
$$\{\Omega, \Omega\} = 0 \quad \text{and} \quad \{H, \Omega\} = 0.$$

They combine nicely into one Grassmann-odd object Q [1]

(10)
$$Q(z,\theta) \equiv \Omega(z) + \theta H(z) ,$$

which is nilpotent due to eq. (9):

$$\{Q, Q\} = 0.$$

Let us in addition consider the following action functional:

(12)
$$S[z] = \int dt d\theta \left[V_A(z(t,\theta)) \mathcal{D} z^A(t,\theta) - Q \right],$$

where the symplectic potential V_A is related to the symplectic metric ω_{AB} ($\omega_{AC}\omega^{CB} = \delta_A^B$) via

(13)
$$\omega_{AB} = \left(\partial_A V_B - (-1)^{\epsilon(A)\epsilon(B)} \partial_B V_A\right) (-1)^{\epsilon_B}.$$

As usual, we assume the symplectic form to be exact. We emphasize that this construction is quite different from that of refs. [1, 2]. In particular, in the present formulation the superpartners $z_1(t)$ cannot be viewed as Pfaffian ghosts. To construct the path integral one must thus in addition specify the formally invariant super Liouville measure, as usual. If in addition to eq. (11) we also assume that there are no boundary terms from Q

(14)
$$\int dt d\theta \, \mathcal{D}Q = \int dt \, \dot{\Omega}(z_0) = 0 \,,$$

then it follows from the previous considerations that this action satisfies the Master Equation

$$(S,S) = 0.$$

It is simple to integrate out the additional θ -variable from eq. (12), and one finds:

(16)
$$S[z] = \int dt \left[V_A(z_0(t)) \dot{z}_0^A(t) - H(z_0(t)) - z_1^A(t) \partial_A \Omega(z_0(t)) \right].$$

Except for the last term, this is simply the conventional phase-space action if there were no constraints. Let us rewrite this last term:

(17)
$$z_1^A \partial_A \Omega(z_0(t)) = z_{0A}^* \{ z_0^A, \Omega \} ,$$

where we have defined $z_{0A}^* \equiv z_1^B \omega_{BA}$. This is the general phase space action extended with antifields to satisfy the classical BV Master Equation (S,S)=0. Note that the variables z_0^A and z_{0A}^* are canonically conjugate within the antibracket, but the precise identification of which plays the role of "field" or "antifield" becomes apparent upon identification of ghost number [4]. All these assignments follow automatically from the superfield approach.

To make these considerations more concrete, let us consider the case of first class constraints $T_{\alpha}(z)$ with the usual algebra

(18)
$$\{T_{\alpha}, T_{\beta}\} = C_{\alpha\beta}^{\gamma} T_{\gamma}, \qquad \{T_{\alpha}, H_{0}\} = V_{\alpha}^{\beta} T_{\beta}.$$

where for simplicity we assume all the original phase space variables to be bosonic. According to the BFV prescription [5] one introduces ghosts c^{α} together with their conjugate momenta \mathcal{P}_{α} , and the BRST charge and the extended Hamiltonian are then given by

$$\Omega = T_{\alpha}c^{\alpha} - \frac{1}{2}\mathcal{P}_{\gamma}C^{\gamma}_{\alpha\beta}c^{\alpha}c^{\beta} + \dots$$

$$(19) H = H_0 + \mathcal{P}_{\alpha} V_{\beta}^{\alpha} c^{\beta} + \dots$$

where the dots denote the higher order terms in the expansion of Ω and H with respect to the ghost momenta \mathcal{P} . The extended Poisson bracket is given in coordinates by

(20)
$$\left\{q^{i}, p_{j}\right\} = \delta_{j}^{i}, \qquad \left\{c^{\alpha}, \mathcal{P}_{\beta}\right\} = \delta_{\beta}^{\alpha}.$$

We now allow all the phase space coordinates to depend on t and θ . Let us write explicitly their expansion with respect to θ :

(21)
$$q^{i} = q_{0}^{i} - \theta p_{*}^{i} \qquad p_{i} = p_{i}^{0} + \theta q_{i}^{*}$$

$$c^{\alpha} = c_{0}^{\alpha} - \theta u^{\alpha} \qquad \mathcal{P}_{\alpha} = u_{\alpha}^{*} + \theta c_{\alpha}^{*}.$$

where we have chosen some defining signs in order to facilitate comparison with the existing literature. According to our choice $gh(\theta) = 1$ the ghost numbers of the new variables read

(22)
$$\begin{aligned} \operatorname{gh}(q_0^i) &= 0 \,, \quad \operatorname{gh}(p_i^0) &= 0 \,, \quad \operatorname{gh}(q_i^*) &= -1 \,, \quad \operatorname{gh}(p_*^i) &= -1 \,, \\ \operatorname{gh}(c_0^\alpha) &= 1 \,, \quad \operatorname{gh}(u^\alpha) &= 0 \,, \quad \operatorname{gh}(c_\alpha^*) &= -2 \,, \quad \operatorname{gh}(u_\alpha^*) &= -1 \,. \end{aligned}$$

The action (16) then takes the form

(23)
$$S = \int dt \left[p_i^0 \dot{q}_0^i - H_0 + T_\alpha u^\alpha - p_*^i \{ p_i^0, T_\alpha \} c_0^\alpha - q_i^* \{ q_0^i, T_\alpha \} c_0^\alpha + u^* \dot{c}_0^\alpha - u_\alpha^* V_\beta^\alpha c_0^\beta + \frac{1}{2} c_\gamma^* C_{\alpha\beta}^\gamma c_0^\alpha c_0^\beta - u_\gamma^* C_{\alpha\beta}^\gamma u^\alpha c_0^\beta + \dots \right],$$

where the dots denote higher order terms in powers of u_{α}^* and c_{α}^* . It is easy to see that the first three terms in (23) are nothing but the extended Hamiltonian action

(24)
$$S = \int dt \, (p_i^0 \dot{q}_0^i - H_0 + T_\alpha u^\alpha),$$

corresponding to the system under consideration. Indeed, these terms enter only with ghost number zero variables, and should thus be understood in the BV formalism as the initial action. Making use of the ghost number assignments (22) it is also easy to infer the gauge generators from eq. (23): they are precisely the gauge generators of the extended Hamiltonian action (24). It follows that u^{α} are simply the Lagrangian multipliers corresponding to the constraints T_{α} . All other assignments coincide exactly with those of the extended phase space action first identified by Fisch and Henneaux [4]. We have thus explicitly confirmed the remarkable fact that the whole extended phase-space BV formalism is precisely encoded in this superspace path integral approach.

3. An Inverse Construction

There are two equivalent ways two perform path integral quantization of Hamiltonian systems with first-class constraints:

- (i) via the BFV formalism based on an extended Poisson bracket and BRST charge Ω .
- (ii) via the BV formalism based on the antibracket and the master action corresponding to the extended Hamiltonian action.

As we have shown above, the superfield approach allows one to explicitly derive the BV formulation from the BFV prescription on phase space. The space of field histories (which is the BV configuration space) thus appears as the space of super-paths of the initial BFV phase space. Remarkably, this space comes with a BV antibracket which originates directly from the BFV Poisson bracket. Similarly, the master action derives directly from the BRST charge Ω and the BFV extended Hamiltonian.

It is natural to ask if there, conversely, exists a "phase space" description of any Lagrangian gauge theory which is dual to the standard BV description. As we shall now show, the answer to this is affirmative. Moreover, we will again directly arrive at the correct dual description by means of the superfield approach. A quite different superfield formulation of BV Lagrangian quantization was first proposed in ref. [6].

Let us start with the standard BV formulation of any Lagrangian gauge theory. Let \mathcal{M} be the antisymplectic manifold of the BV configuration space, the antisymplectic structure of which determines the BV antibracket (\cdot, \cdot) . We let S denote the master action defined on this BV configuration space \mathcal{M} . This master action S is required to be of ghost number zero: gh(S) = 0, and will of course classically satisfy the Master Equation (S, S) = 0. We also let Γ^A denote local coordinates on \mathcal{M} (in Darboux coordinates Γ^A are just the fields ϕ and antifields ϕ^*). In terms of local coordinates the antibracket is determined by the odd Poisson bivector $E^{AB} = (\Gamma^A, \Gamma^B)$.

Let us now turn to the superfield formulation. In this case we simply consider one odd coordinate θ , which we here take to be of opposite ghost number as compared with the previous section: $gh(\theta) = -1$. We will consider θ as a Grassmann-odd version of ordinary time in exactly the same manner as above, and we thus allow Γ^A to depend on θ , *i.e.*,

(25)
$$\Gamma^A = \Gamma_0^A + \theta \Gamma_1^A,$$

Obviously

(26)
$$\epsilon(\Gamma_0^A) = \epsilon(\Gamma_1^A) + 1 = \epsilon(\Gamma^A), \quad \operatorname{gh}(\Gamma_0^A) = \operatorname{gh}(\Gamma_1^A) - 1 = \operatorname{gh}(\Gamma^A),$$

and the path space will thus have an *even* symplectic structure (see Sec. 4). The corresponding Poisson bracket is given in coordinates by

(27)
$$\begin{cases}
\Gamma_0^A, \Gamma_0^B \} &= 0 \\
\{\Gamma_0^A, \Gamma_1^B \} &= E^{AB}(\Gamma_0) \\
\{\Gamma_1^A, \Gamma_1^B \} &= \Gamma_1^C \partial_C E^{AB}(\Gamma_0) .
\end{cases}$$

This Poisson bracket obviously carries zero ghost number. Now we define the quantity

(28)
$$\Omega(\Gamma_0, \Gamma_1) \equiv \int d\theta \ S(\Gamma(\theta)),$$

By construction $gh(\Omega) = 1$, and we note that Ω will be nilpotent:

$$\{\Omega, \Omega\} = 0.$$

In fact, this nilpotency condition is here to be viewed as a Poisson-bracket Master Equation. But it immediately raises the question: Can this Ω also be formally considered as the BRST charge corresponding to a Hamiltonian system with constraints? Although we do not allow the fields Γ^A to depend in addition on a *new* bosonic coordinate "time", this is in fact the case.

It is not difficult to understand the nature of the associated Hamiltonian system of constraints. Let S be the extended master action of a gauge theory described by an initial action $S_0(q^i)$ and gauge generators R^i_{α} which we for simplicity take to be linearly independent (the discussion can be easily generalized to the reducible case). They form a possibly open algebra

$$[R_{\alpha}, R_{\beta}] = C_{\alpha\beta}^{\gamma} R_{\gamma} + \dots$$

where dots means the terms vanishing on the stationary surface of the action S_0 . Thus in the BV formulation we need, for the minimal sector, the fields of the initial theory q^i , $gh(q^i) = 0$ (which we for simplicity take to be bosonic), ghosts c^{α} , $gh(c^{\alpha}) = 1$, and all their antifields. As usual, we combine fields into ϕ^A , and antifields into ϕ^A . The BV antibracket and ghost number assignments are

(31)
$$(\phi^A, \phi_B^*) = \delta_B^A, \quad gh(\phi_A^*) = -gh(\phi^A) - 1.$$

The master action constructed according to the BV prescription to satisfy the classical Master Equation (S, S) = 0 is then

(32)
$$S = S_0 + q_i^* R_\alpha^i c^\alpha - \frac{1}{2} c_\gamma^* C_{\alpha\beta}^\gamma c^\alpha c^\beta + \dots$$

where the dots denotes possible terms of higher order in antifields.

We now allow ϕ^A, ϕ_A^* to depend on θ . The expansion of ϕ, ϕ^* in θ thus reads

(33)
$$q^i = q_0^i - \theta \gamma^i, \qquad q_i^* = \pi_i + \theta p_i, \qquad c^\alpha = c_0^\alpha + \theta \eta^\alpha, \qquad c_\alpha^* = \rho_\alpha + \theta \mathcal{P}_\alpha.$$

Moreover, it follows from eq. (26) that ghost number assignments are:

(34)
$$\operatorname{gh}(q_0^i) = \operatorname{gh}(p_i) = 0, \quad \operatorname{gh}(c^{\alpha}) = \operatorname{gh}(\eta^{\alpha}) = 1, \quad \operatorname{gh}(\mathcal{P}_{\alpha}) = \operatorname{gh}(\pi_i) = -1, \\ \operatorname{gh}(\eta^{\alpha}) = 2, \quad \operatorname{gh}(\rho_{\alpha}) = -2.$$

The Poisson bracket (27) becomes explicitly

(35)
$$\{q_0^i, p_j\} = \delta_j^i, \qquad \{\gamma^i, \pi_j\} = \delta_j^i, \\ \{c_0^\alpha, \mathcal{P}_\beta\} = \delta_\beta^\alpha, \qquad \{\eta^\alpha, \rho_\beta\} = \delta_\beta^\alpha,$$

Substituting (32) in (28) and integrating over θ we arrive at

$$(36) \qquad \Omega = -\gamma^{i}\partial_{i}S_{0} + p_{i}R_{\alpha}^{i}c_{0}^{\alpha} - \pi_{i}R_{\alpha}^{i}\eta^{\alpha} + \pi_{i}(\gamma^{j}\partial_{j}R_{\alpha}^{i})c_{0}^{\alpha} - \frac{1}{2}\mathcal{P}_{\gamma}C_{\alpha\beta}^{\gamma}c_{0}^{\alpha}c_{0}^{\beta} - \rho_{\alpha}C_{\alpha\beta}^{\gamma}\eta^{\alpha}c_{0}^{\beta} + \dots,$$

where dots denote higher order terms in the variables \mathcal{P}, π and ρ . We will see that they are to be identified with ghost momenta. Eq. (36) can formally be identified with the BRST charge of a system with constraints. Using the ghost number assignments it is easy to see that γ^i and c_0^{α} are the ghosts associated with first class constraints $\partial_i S$ and $p_i R_{\alpha}^i$, and π_i , \mathcal{P}_{α} are their conjugate momenta. The variables η^{α} and ρ_{α} are simply the ghosts for ghosts and their momenta associated with the reducible constraints $\partial_i S$. To be precise, the Lagrangian gauge generators R_{α}^i are the reducibility functions for the constraints $\partial_i S_0$ due to the Noether identity $R_{\alpha}^i \partial_i S_0 = 0$. The corresponding term $\pi_i R_{\alpha}^i \eta^{\alpha}$ indeed enters (36). It should be emphasized that all these identifications are in an algebraic sense only: There is no analogue of the ordinary time coordinate of the Hamiltonian system.

An interesting open question concerns the role of quantum corrections to the master action S. Suppose we expand the solution to the full quantum Master Equation

$$\frac{1}{2}(S,S) = i\hbar\Delta S$$

in powers of \hbar , and insert this full solution into the definition (28). The nilpotency condition (29) will then be broken by \hbar -corrections on the right hand side. What is the interpretation of the \hbar -corrections to the BRST charge Ω ? Perhaps this is related to canonical quantization of the Poisson bracket and the corresponding Hamiltonian quantum Master Equation $[\hat{\Omega}(\hbar), \hat{\Omega}(\hbar)] = 0$. In any case, the question deserves a more detailed study.

4. Geometry of the Super Path Space

It is useful to clarify the geometrical meaning of the structures entering the above superspace formulations, and view them in greater generality. In particular, it is instructive to see how the antibracket and the usual Poisson bracket enter on equal footing. In this section, which will be a bit more abstract, we find it convenient to even use the same symbol for the two, namely $[\cdot, \cdot]_{\mathcal{M}}$. One must of course keep in mind that the odd and even brackets have odd and even Grassmann parities, respectively. Now let \mathcal{M} be a symplectic manifold (which can be even or odd), and let $[\cdot, \cdot]_{\mathcal{M}}$ be the corresponding Poisson bracket or antibracket, depending on the Grassmann parity. We denote by n the dimension of \mathcal{M} and κ the Grassmann parity of the bracket $[\cdot, \cdot]_{\mathcal{M}}$, i.e.,

(38)
$$\epsilon([f,g]_{\mathcal{M}}) = \epsilon(f) + \epsilon(g) + \kappa,$$

The exchange relation, the Leibniz rule and the Jacobi identity are then neatly summarized, for both brackets, by

$$(39) \qquad \begin{aligned} [f,g]_{\mathcal{M}} &= -(-1)^{(\epsilon(f)+\kappa)(\epsilon(g)+\kappa)} [g,f]_{\mathcal{M}} \\ [f,gh]_{\mathcal{M}} &= [f,g]_{\mathcal{M}} h + (-1)^{(\epsilon(f)+\kappa)(\epsilon(g))} g[f,h]_{\mathcal{M}} \\ [f,[g,h]_{\mathcal{M}}]_{\mathcal{M}} &= [[f,g]_{\mathcal{M}},h]_{\mathcal{M}} + [g,[f,h]_{\mathcal{M}}]_{\mathcal{M}} (-1)^{(\epsilon(f)+\kappa)(\epsilon(g)+\kappa)} .\end{aligned}$$

for any functions f, g and h on \mathcal{M} . In local coordinates Γ^A on \mathcal{M} we write generically $E^{AB} = [\Gamma^A, \Gamma^B]_{\mathcal{M}}$ for both kinds of brackets.

Let in addition Σ be a supermanifold of dimension k and of coordinates x^i . We assume for simplicity that it is compact. Let there in addition be a volume form $d\mu(x) = \rho(x)dx = \rho(x)dx^1 \dots dx^k$ on Σ . We denote by \mathcal{E} the super-path space, i.e. the space of smooth maps from Σ to \mathcal{M} . In local coordinates each map is described by the functions $\Gamma^A(x)$. As \mathcal{M} is symplectic, and Σ has the above volume form, then [7] the super path space \mathcal{E} is also symplectic (see also section 4.3 of ref. [1] and ref. [8]). Indeed, for any functionals F, G we define

$$(40) [F,G]_{\mathcal{E}} = (-1)^{(\epsilon(F) + \epsilon(d\mu))\epsilon(d\mu)} \int d\mu(x) (F \frac{\overleftarrow{\delta}}{\delta \Gamma^{A}(x)} E^{AB}(\Gamma(x)) \frac{\delta}{\delta \Gamma^{B}(x)} G).$$

Here we have made use of the following conventions for the functional derivatives: for infinitesimal variation $\delta\Gamma^A(x)$ we write

(41)
$$\delta F[\Gamma] = \int d\mu(x) \delta \Gamma^{A}(x) (\frac{\delta}{\delta \Gamma^{A}(x)} F[\Gamma]) = \int (F[\Gamma] \frac{\overleftarrow{\delta}}{\delta \Gamma^{A}(x)}) \delta \Gamma^{A}(x) d\mu(x).$$

In particular, left and right derivatives are then related by

(42)
$$\frac{\delta}{\delta\Gamma^{A}(x)}F = (-1)^{\epsilon(d\mu)+(\epsilon(F)+\epsilon(d\mu)+1)(\epsilon(\Gamma^{A})+\epsilon(d\mu))}F\frac{\overleftarrow{\delta}}{\delta\Gamma^{A}(x)},$$

where $\epsilon(d\mu)$ is the Grassmann parity of the measure. Note that the Grassmann parity of the functional derivative $\frac{\delta}{\delta\Gamma^A(x)}$ is $\epsilon(\Gamma^A) + \epsilon(d\mu)$.

Let us first state some obvious properties of the bracket structure (40). First, the Grassmann parity k' of the bracket (40) is related to the Grassmann parity of the bracket $[\cdot, \cdot]_{\mathcal{M}}$ by $\kappa' = \kappa + \epsilon(d\mu)$. The bracket (40) obviously satisfies (39) with κ being the κ' and is thus a Poisson bracket or an antibracket, depending on its Grassmann parity. Taking F and G in (40) to be

(43)
$$F[\Gamma] = \int d\mu(x) f(\Gamma(x)), \qquad G[\Gamma] = \int d\mu(x) g(\Gamma(x)),$$

for some functions f, g on \mathcal{M} we arrive at

$$[F,G]_{\mathcal{E}} = \int d\mu \, [f,g]_{\mathcal{M}} .$$

Let there in addition be given a vector field $q = q^i \frac{\partial}{\partial x^i}$ on Σ . We assume that $div_{d\mu}(q) = 0$, which implies that $\int d\mu \ qf = 0$ for any function f on Σ . The vector field q can be lifted to a vector field Q on the super path space \mathcal{E} [7]. In coordinates we have for any functional $F[\Gamma]$

(45)
$$QF[\Gamma] = \int d\mu(x) ((q^{i}(x)\frac{\partial}{\partial x^{i}}\Gamma^{A}(x))\frac{\delta}{\delta\Gamma^{A}(x)}F[\Gamma]).$$

An important observation is that Q is an Hamiltonian vector field with respect to the bracket (40). Indeed, let $V_A(\Gamma)$ be the symplectic potential on \mathcal{M} ; for the symplectic 2-form we have

(46)
$$E_{AB} = (\partial_A V_B - (-1)^{\epsilon(A)\epsilon(B)} \partial_B V_A) (-1)^{\epsilon(B)(\kappa+1)},$$

where κ is the parity of the symplectic form. Then for an arbitrary functional $F[\Gamma]$ we have

(47)
$$QF = -[C, F]_{\mathcal{E}}, \qquad C = \int d\mu(x) \left(q \Gamma^{A}(x)\right) V_{A}(\Gamma(x)),$$

with $\epsilon(C) = \epsilon(d\mu) + \epsilon(q) + \kappa$. Note that if q is an odd nilpotent vector field on Σ , then the corresponding Hamiltonian C automatically satisfies the classical Master Equation $[C, C]_{\mathcal{E}} = 0$. Another important property of Q is that for any functional F of the form (43) we have $QF = -[C, F]_{\mathcal{E}} = 0$.

These properties of the super path space bracket allows one to directly construct a BV master action

$$(48) W = \alpha C + \beta F,$$

for some functional F from (43) and any f satisfying $[f, f]_{\mathcal{M}} = 0$. This holds for arbitrary coefficients α and β . When the Grassmann parity of this W is odd, it simply has the interpretation as a BRST-like charge Ω . It was shown in ref [7] that the BV master actions corresponding to Chern-Simons theory and 2D topological sigma models have precisely the same structure. In all these cases one chooses Σ to be an odd tangent bundle over the even manifold Σ_0 , with the odd nilpotent vector field q being the De Rham differential on Σ_0 . Remarkably, the BV master action of the 2D Poisson sigma model used in [9] for the construction of the Kontsevich star product [10] also has just the form (48).

Surprising relations between the Poisson brackets of Hamiltonian BFV quantization and antibrackets of Lagrangian quantization have recently been discovered for topological gauge theories in a quite different manner [11] (see also refs. [12, 13, 14]). It would also be interesting to consider the isomorphism between the Poisson bracket and the antibracket [15] in the light of this superfield construction.

Finally, let us explicitly make contact to the examples we gave in the previous sections. For the first case we choose Σ to be a (1|1) supermanifold with coordinates t and θ . We also choose \mathcal{M} to be an even symplectic manifold, and simply take as measure $d\mu = dt d\theta$, as well as an odd operation $\mathcal{D} = \theta \frac{d}{dt}$. A smooth map $\Sigma \to \mathcal{M}$ is given by the set of functions $\Gamma^A(t,\theta) = \Gamma_0^A + \theta \Gamma_1^A$. Using the representation

(49)
$$\Gamma_0^A(t) = \int d\theta \, \theta \, \Gamma^A(t,\theta) \,, \qquad \Gamma_1^A(t) = \int d\theta \, \Gamma^A(t,\theta) \,,$$

and explicitly integrating over θ in the definition (40) we arrive at

(50)
$$\begin{aligned}
\left(\Gamma_0^A(t), \Gamma_0^B(t')\right) &= 0 \\
\left(\Gamma_0^A(t), \Gamma_1^B(t')\right) &= \omega^{AB}(\Gamma_0)\delta(t - t') \\
\left(\Gamma_1^A(t), \Gamma_1^B(t')\right) &= \Gamma_1^C \partial_C \omega^{AB}(\Gamma_0)\delta(t - t').
\end{aligned}$$

In its turn the odd nilpotent vector field D, considered as acting on functionals, is a Hamiltonian vector field with Hamiltonian

(51)
$$C = \int dt \, d\theta \, V_A \mathcal{D}\Gamma^A.$$

Choosing Darboux coordinates p, q on \mathcal{M} one arrives at the standard form $\int dt p_i^0 \dot{q}_0^i$. Thus we see that the Hamiltonian action (12) has precisely the "geometrical" form (48) with f being the super BRST charge $\Omega + \theta H$. The only difference is that f in this case explicitly depends on θ .

In the case of the inverse construction of Sec. 3 one chooses Σ to be a one dimensional space with Grassmann-odd coordinate θ and \mathcal{M} as an antisymplectic manifold. Using the general formula (40) one arrives directly at the explicit form of the odd path space Poisson bracket (27). In this case we simply take $\alpha = 0$ and F to be the master action S in eq. (48). We have thus shown how both of these cases follow directly from the above general framework.

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References

- [1] I.A. Batalin, K. Bering and P.H. Damgaard, Nucl. Phys. **B515** (1998) 455.
- [2] I.A. Batalin, K. Bering and P.H. Damgaard, Phys. Lett. **B446** (1999) 175.
- [3] I.A. Batalin and G.A. Vilkovisky, Phys. Lett. **B102** (102) (1981) 27; Phys. Rev. **D28** (1983) 2527.

- [4] J.M.L. Fisch and M. Henneaux, Phys. Lett. **B226** (1989) 80.
 A. Dresse, J.M.L. Fisch, P. Gregoire and M. Henneaux, Nucl. Phys. **B354** (1991) 191.
- [5] E.S. Fradkin and G.A. Vilkovisky, Phys. Lett. **B55** (1975) 224.
 - I.A. Batalin and G.A. Vilkovisky, Phys. Lett. B69 (1977) 309.
 - E.S. Fradkin and E.E. Fradkina, Phys. Lett. **B72** (1978) 343.
 - I.A. Batalin and E.S. Fradkin, Phys. Lett. **B122** (1983) 157.
- P.M. Lavrov, P.Yu. Moshin and A.A. Reshetnyak, Mod. Phys. Lett. A10 (1995) 2687.
 P.M. Lavrov, Phys. Lett. B366 (1996) 160.
- [7] M. Alexandrov, M. Kontsevich, A. Schwarz and O. Zaboronsky, Int. J. Mod. Phys. 12 (1997) 1405.
- [8] K. Bering, J. Math. Phys. **39** (1998) 2507.
- [9] A. S. Cattaneo and G. Felder, q-alg/9902090.
- [10] M. Kontsevich, q-alg/9709040.
- [11] M.A. Grigoriev, hep-th/9906209.
- [12] A. Nersessian and P.H. Damgaard, Phys. Lett. **B355** (1995) 150.
- [13] M.A. Grigoriev, A.M. Semikhatov and I.Yu. Tipunin, J. Math. Phys. 40 (1999) 1792.
- [14] I.A. Batalin and R. Marnelius, Phys. Lett. **B434** (1998) 312; hep-th/9809210.
- [15] G. Barnich and M. Henneaux, J. Math. Phys. 37 (1996) 5273.