STUDY OF EARLY UNIVERSE IN AN M THEORETIC MODEL

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THESIS

for

DOCTOR OF PHILOSOPHY

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January 25, 2011

DECLARATION

I, hereby declare that the investigation presented in the thesis has been carried out by me. The work is original and the work has not been submitted earlier as a whole or in part for a degree/diploma at this or any other Institution or University.

Samrat Bhowmick

. . . . . . to my parents and to the nature . . . . . .

ACKNOWLEDGEMENTS

Abstract

In this thesis we study early universe in the frame work of M theory. We assume that the early universe is homogeneous, anisotropic, and is dominated by the mutually BPS intersecting branes of M theory. Certain class of black holes can be described by string/M theory have similar structure of intersecting BPS branes configurations. We are motivated by such black holes to make a similar model for early universe.

But due to the lack of knowledge of the exact brane dynamics, we use U duality symmetry of the M theory to get an equation of states. We also verify the equations of states obtained by duality also hold for known case like black holes. Then we solve Einstein equations to get evolution of early universe.

In particular We assume that the early universe is homogeneous, anisotropic, and is dominated by the mutually BPS $22'55'$ intersecting branes of M theory. The spatial directions are all taken to be toroidal. Using analytical and numerical methods, we study the evolution of such an universe. We find that, asymptotically, three spatial directions expand to infinity and the remaining spatial directions reach stabilised values. From string theory perspective, the dilaton is hence stabilised also. We give a physical description of the stabilisation mechanism.

Any stabilised values can be obtained by a fine tuning of initial brane densities. The constant sizes depend on certain imbalance among initial values. One naturally obtains $M_{11} \simeq M_s \simeq M_4$ and $g_s \simeq 1$ within a few orders of magnitude. Smaller numbers, for example $M_s \simeq 10^{-16} M_4$, are also possible but require fine tuning.

In some sense our $22'55'$ configuration is special. We give some example of other configurations for which stabilisation can not be achieved. We give their asymptotic time evolution. We find only $22'55'$ and its U dual configuration can achieved stabilisation of 7 spacelike dimensions.

Also, from the perspective of four dimensional spacetime, the effective four dimensional Newton's constant G_4 is now time varying. Its time dependence will follow from explicit solutions. We find in the present case that, asymptotically, G_4 exhibits characteristic log periodic oscillations.

List of Papers

- S. Bhowmick, S. Digal, and S. Kalyana Rama,
  "Stabilisation of Seven (Toroidal) Directions and Expansion of the remaining Three in an M theoretic Early Universe Model", *Phys. Rev.* **D79**, 101901 (2009) ,
  `arXiv:0810.4049 [hep-th]`.

- S. Bhowmick and S. Kalyana Rama,
  "From 10+1 to 3+1 dimensions in an early universe with mutually BPS intersecting branes",
  Phys. Rev. **D82**, 083526 (2010) ,
  `arXiv:1007.0205 [hep-th]`.

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Introduction

1.1 Background

In physics one of the problems, mankind has been wondering for all time is "how universe evolves?" In last century advancement of physics gave us enormous amount of knowledge to address such question in a scientific framework. So the study of evolution of universe began, which is known as *cosmology*.

It is understood that gravity plays main role in evolution of universe, because it is long ranged and unlike electromagnetic force, it is only attractive. After the discovery of general relativity people understand that, spacetime has a rich geometric structure and is dynamic in nature. This dynamics is governed by Einstein equation.

It is assumed in cosmology that spacetime is homogeneous and isotropic. On the basis of this fact a well known model has been made after the work of Friedmann, Robertson and Walker. This is known as FRW model. In this model matter is taken as homogeneous and isotropic fluids distributed over the whole space like surface. FRW model is very successful to describe universe for large interval of *time*, except very early universe, and possibly present day universe (late time acceleration).

Another striking discovery in physics of last century is quantum mechanics. It describes motion of atom and subatomic particle with extraordinary precision. Nevertheless GR, the theory of gravity is classical. So a challenge posed to physicist is to make a quantum theory of gravity. Toward the end of last century string/M theory [1, 2, 3] emerges as the most promising candidate for quantum gravity.

String/M -theory unifies all the forces of the nature. In string theory fundamental building blocks of nature are one or higher dimensional objects, (open and closed string – 1 dimensional, Dp-branes – p dimensional, $NS5$-branes – 5 dimensional) instead of point like particle. In M-theory there are two types of objects $M2$ brane and $M5$ branes which are 2 and 5 dimensional object respectively. But a new problem appears in this new theory – spacetime now become higher dimensional. In string theory spacetime is $9+1$ dimensional and in M theory spacetime is $10+1$ dimensional. On the other hand the large spacetime we observe is $3+1$. So other 7 dimensions of M theory must be curled up to a very tiny scale, beyond the reach of present day experiment. So now cosmology has to explain evolution of all 10 space like dimension and how dynamically 7 of them stabilised whereas other 3 continues to expand. We studied this problem in this thesis.

1.2 Preliminaries

M theory is a 11 dimensional theory which consists of $M2$ branes and $M5$ branes. Various string theories are various limits of M theory. Until very recently almost nothing was known about full quantum mechanics of M-branes. Even now we do not have any clear idea. But 11-dimensional supergravity which is supposed to be low energy limit of M theory is well understood.

11 dim supergravity is basically a theory of gravity with super symmetry. It is governed by the action

$$S = \frac{1}{16\pi G_{11}} \int d^{11}x \sqrt{-g} \left(R - \frac{1}{2 \times 4!} F_4^2 \right) \; + \; S_{CS} \; + \; S_f \; . \qquad (1.1)$$

Where F_4 is a 4 form field strength such that, $F_4 = dC_3$. S_f is fermionic part of the action and S_{CS} is Chern-Simon term. $M2$ brane and $M5$ brane are respectively, electrically and magnetically charged under F_4. This theory contains black hole solutions. Black holes are made of stacks of $M2$ or $M5$ branes or intersecting combination of them. These black hole solutions of single $M2$ branes or $M5$ branes preserve all dynamical supersymmetries, that is half of the supersymmetries of the action. For intersection, if they follow certain rules then they also preserve same amount of supersymmetries. These configurations are called BPS configuration.

This black hole solutions are well known and will be discussed in some details in later chapter. We are motivated by this black hole set up to make a model for early universe.

Another novel feature of M theory is duality symmetries. For details of this duality see for example [4, 2, 5]. M theory, compactified on one of the ten space like dimension gives type IIA string theory. In string theory there are S and T duality which connect various string theories. For example T duality on type IIA theory transforms it to a type IIB theory and vice versa.

Unlike point partials closed string can wind aground compact direction. So beside momentum it has another quantum number called winding number. Let p and w be the momentum and winding of a closed string. It wraps a compact direction of radius R. Then T duality is a symmetry where

$$
\begin{aligned}
p &\longleftrightarrow w \\
R &\longleftrightarrow \frac{\alpha'}{R} .
\end{aligned}
$$

(1.2)

Where α' is string length. In case of open string T duality transforms a Dp brane $D(p-1)$ or $D(p+1)$ depending on whether T duality is applied along the brane direction or normal to it respectively. In this case also $R \longleftrightarrow \frac{\alpha'}{R}$.

In type IIB string theory there is another symmetry called S duality. This is a symmetry between strong coupling and weak coupling. That is under S duality coupling constant $\longleftrightarrow$ (coupling constant)$^{-1}$.

Those symmetries also hold in supergravity level, in some cases with a slightly larger symmetry group. When 11 dimensional supergravity compactified on an S^1, it gives type IIA supergravity. Which is governed by

$$
\begin{aligned}
S_{IIA} = \frac{1}{16\pi G_{10}} \int d^{10}x \quad &\sqrt{-g} \left[e^{-2\phi} \left(R + 4\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}H_3^2 \right) - \frac{1}{2}(F_2^2 + \tilde{F}_4^2) \right] \\
&+ \quad S_{CS} + S_f .
\end{aligned}
$$

(1.3)

Where ϕ is dilaton, a scalar field which comes from compactified dimension of M theory. $H_3(= dB_2)$ is NS-NS 3-form field strength and $F_p(= dA_{p-1})$'s are R-R field strength. $\tilde{F}_4$ is

$$
\tilde{F}_4 = dA_3 - A_1 \wedge F_3 .
$$

(1.4)

3

S_{CS} is Chern-Simons term, given by

$$S_{CS} = -\frac{1}{2 \times 16\pi G_{10}} \int B_2 \wedge F_4 \wedge F_4 \ . \tag{1.5}$$

S_f is the action for fermionic fields.

This theory contains Dp branes with p even. They are charged under RR fields, F_{p+2}. Under T duality IIA SuGra becomes IIB SuGra theory. IIB theory also contains Dp branes with p odd. Again S duality is a symmetry of IIB theory with symmetry group $SL(2, \mathbb{R})$. Under these duality operations metric, NS-NS and R-R fields changes. In Appendix **A** we listed the rules for this changes.

1.3   Present Work

In early universe, temperatures and densities reach Planckian scales. Its description then requires a quantum theory of gravity. A promising candidate for such a theory is string/M theory. When the temperatures and densities reach string/M theory scales, the appropriate description is expected to be given in terms of highly energetic and highly interacting string/M theory excitations [6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. [1] In this context, it has been proposed in an earlier work an entropic principle according to which the final spacetime configuration that emerges from such high temperature string/M theory phase is the one that has maximum entropy for a given energy. This principle implies, under certain assumptions, that the number of large spacetime dimensions is $3 + 1$ [15].

High densities and high temperatures also arise near black hole singularities. In string theory black holes are made of branes. These branes have their excitations. So in terms of this excitations black holes have temperature. Since branes stays near singularity, high temperature of black holes near singularity has meaning. Therefore, it is reasonable to expect that the string/M theory configurations which describe such regions of black holes will describe the early universe also.

Consider the case of black holes. Various properties of a class of black holes have been successfully described using mutually BPS intersecting configurations

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[1] Only string theory is considered in these references. But their arguments can be extended for M theory also leading to similar conclusions.

of string/M theory branes. [2] Black hole entropies are calculated from counting excitations of such configurations, and Hawking radiation is calculated from interactions between them.

In the extremal limit, such brane configurations consist of only branes and no antibranes. In the near extremal limit, they consist of a small number of antibranes also. It is the interaction between branes and antibranes which give rise to Hawking radiation. String theory calculations are tractable and match those of Bekenstein and Hawking in the extremal and near extremal limits. But they are not tractable in the far extremal limit where the numbers of branes and antibranes are comparable. However, even in the far extremal limit, black hole dynamics is expected to be described by mutually BPS intersecting brane configurations where they now consist of branes, antibranes, and other excitations living on them, all at non zero temperature and in dynamical equilibrium with each other [17, 18, 19, 20, 21, 22, 23, 24, 25, 26]. For the sake of brevity, we will refer to such far extremal configurations also as brane configurations even though they may now consist of branes and antibranes, left moving and right moving waves, and other excitations.

The entropy S of N stacks of mutually BPS intersecting brane configurations, in the limit where $S \gg 1$, is expected to be given by

$$S \sim \prod_I \sqrt{n_I + \bar{n}_I} \sim \mathcal{E}^{\frac{N}{2}} \tag{1.6}$$

where n_I and $\bar{n}_I$, $I = 1, \cdots, N$, denote the numbers of branes and antibranes of Ith type, $\mathcal{E}$ is the total energy, and the second expression applies for the charge neutral case where $n_I = \bar{n}_I$ for all I . The proof for this expression is given by comparing it in various limits with the entropy of the corresponding black holes [17, 18], see also [17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28]. For $N \leq 4$ and when other calculable factors omitted here are restored, this expression matches that for the corresponding black holes in the extremal and near extremal limit and, in the models based on that of Danielsson et al [19], matches upto a numerical factor in

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[2] Mutually BPS intersecting configurations means, for example, that in M theory two stacks of 2 branes intersect at a point; two stacks of 5 branes intersect along three common spatial directions; a stack of 2 branes intersect a stack of 5 branes along one common spatial direction; waves, if present, will be along a common intersection direction; and each stack of branes is smeared uniformly along the other brane directions. See [16] for more details and for other such string/M theory configurations.

the far extremal limit [17] – [28] also. However, no such proof exists for $N > 4$ since no analogous object, black hole or otherwise, is known whose entropy is $\propto \mathcal{E}^*$ with $* > 2$.

Note that, in the limit of large $\mathcal{E}$, the entropy $S(\mathcal{E})$ is $\ll \mathcal{E}$ for radiation in a finite volume and is $\sim \mathcal{E}$ for strings in the Hagedorn regime. In comparison, the entropy given in (1.6) is much larger when $N > 2$. This is because the branes in the mutually BPS intersecting brane configurations form bound states, become fractional, [3] and support very low energy excitations which lead to a large entropy. Thus, for a given energy, such brane configurations are highly entropic.

Another novel consequence of fractional branes is the following. According to the 'fuzz ball' picture for black holes [30], the fractional branes arising from the bound states formed by intersecting brane configurations have non trivial transverse spatial extensions due to quantum dynamics. The size of their transverse extent is of the order of Schwarzschild radius of the black holes. Therefore, essentially, the region inside the 'horizon' of the black hole is not empty but is filled with fuzz ball whose fuzz arise from the quantum dynamics of fractional strings/branes.

Chowdhury and Mathur have recently extended the fuzz ball picture to the early universe [27, 28]. They have argued that the early universe is filled with fractional branes arising from the bound states of the intersecting brane configurations, and that the brane configurations with highest N are entropically favourable, see equation (1.6).

However, as mentioned below equation (1.6) and noted also in [27, 28], the entropy expression in (1.6) is proved in various limits for $N \leq 4$ only and no proof exists for $N > 4$. Also, we are not certain of the existence of any system whose entropy $S(\mathcal{E})$ is parametrically larger than $\mathcal{E}^2$ for large $\mathcal{E}$. See related discussions in [31, 32]. Therefore, in the following we will assume that $N \leq 4$. Then, a homogeneous early universe in string/M theory may be taken to be dominated by the maximum entropic $N = 4$ brane configurations distributed uniformly in the common transverse space.

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[3]Fractionation of branes states is familiar phenomena in string theory [29], see also [30]. Consider for example a string of length L_T with an wave on it. it wraps a compact direction n times. Each cycle has a length say L, so $L_T = nL$. Total momentum of the wave say $\frac{n_p}{L}$. This momentum can be distributed all of these cycles. So strings along each of these cycles behaves like a fractional string and total entropy, $S \propto \sqrt{n\, n_p}$. This is called fractionation. Similar thing happens in branes also.

Such $N = 4$ mutually BPS intersecting brane configurations in the early universe may then provide a concrete realisation of the entropic principle proposed earlier by one of us to determine the number $(3 + 1)$ of large spacetime dimensions [15]. Indeed, in further works [31, 32, 33, 34], using M theory symmetries and certain natural assumptions, it has been shown that these configurations lead to three spatial directions expanding and the remaining seven spatial directions stabilising to constant sizes.

In this thesis, we assume that the early universe in M theory is homogeneous and anisotropic and that it is dominated by $N = 4$ mutually BPS intersecting brane configurations. [4] In this context, it is natural to assume that all spatial directions are on equal footing to begin with. Therefore we assume that the ten dimensional space is toroidal. We then present a thorough analysis of the evolution of such an universe.

The corresponding energy momentum tensor T_{AB} has been calculated in [27] under certain assumptions. However, general relations among the components of T_{AB} may be obtained [32] using U duality symmetries of M theory which are, therefore, valid more generally. We show in this thesis that these U duality relations alone imply, under a technical assumption, that the $N = 4$ mutually BPS intersecting brane configurations with identical numbers of branes and antibranes will asymptotically lead to an effective $(3 + 1)$ dimensional expanding universe.

In order to proceed further, and to obtain the details of the evolution, we make further assumptions about T_{AB}. We then analyse the evolution equations in D dimensions in general, and then specialise to the eleven dimensional case of interest here.

We are unable to solve explicitly the relevant equations. However, applying the general analysis mentioned above, we describe the qualitative features of the evolution of the $N = 4$ brane configuration. In the asymptotic limit, three spatial directions expand as in the standard FRW universe and the remaining seven spatial directions reach constant, stabilised values. These values depend on the initial conditions and can be obtained numerically. Also, we find that any stabilised values may be obtained, but requires a fine tuning of the initial brane densities.

Using the analysis given here, we present a physical description of the mech-

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[4] There is an enormous amount of work on the study of early universe in string/M theory. For a small, non exhaustive, sample of such works, see [35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47].

anism of stabilisation of the seven brane directions. The stabilisation is due, in essence, to the relations among the components of T_{AB} which follow from U duality symmetries, and to each of the brane directions in the $N = 4$ configuration being wrapped by, and being transverse to, just the right number and kind of branes. This mechanism is very different from the ones proposed in string theory or in brane gas models [37] – [40] to obtain large $3+1$ dimensional spacetime. (See section **I A** below also.)

In the asymptotic limit, the eleven dimensional universe being studied here can also be considered from the perspective of four dimensional spacetime. One then obtains an effective four dimensional Newton's constant G_4 which is now time varying. Its precise time dependence will follow from explicit solutions of the eleven dimensional evolution equations.

We find that, in the case of $N = 4$ brane configuration, G_4 has a characteristic asymptotic time dependence : the fractional deviation δG_4 of G_4 from its asymptotic value exhibits log periodic oscillations given by

$$\delta G_4 \ \propto \ \frac{1}{t^\alpha} \ Sin(\omega \ ln \ t + \phi) \ . \tag{1.7}$$

The proportionality constant and the phase angle ϕ depend on initial conditions and matching details of the asymptotics, but the exponents α and ω depend only on the configuration parameters. Such log periodic oscillations seem to be ubiquitous and occur in a variety of physical systems [48, 49, 50]. But, to our knowledge, this is the first time it appears in a cosmological context. One expects such a behaviour to leave some novel imprint in the late time universe, but its implications are not clear to us.

Since we are unable to solve the evolution equations explicitly, we analyse them using numerical methods. We present the results of the numerical analysis of the evolution. We illustrate the typical evolution of the scale factors showing stabilisation and the log periodic oscillations mentioned above. By way of illustration, we choose a few sets of initial values and present the resulting values for the sizes of the stabilised directions and ratios of the string/M theory scales to the effective four dimensional scale.

We also discuss critically the implications of our assumptions. As we will explain, many important dynamical questions must be answered before one un-

derstands how our known $3 + 1$ dimensional universe may emerge from M theory. Until these questions are answered and our assumptions justified, our assumptions are to be regarded conservatively as amounting to a choice of initial conditions which are fine tuned and may not arise naturally.

1.4   Organisation of the Thesis

The organisation of this thesis is as follows:

In chapter **2** we discuss various black hole solution and their energy momentum tensor. Then we discuss U duality relations in M theory and string theory.

In first part of chapter **2** (sections **2.1** to **2.4**) we discuss some known result of black holes in M theory. In the second part (sections **2.5** to **2.7**) we discuss U duality in M and string theory, and their applications in black hole solution.

In chapter **3**, evolution of early universe has been discussed. In section **3.1**, we present equations of motion fro our cosmological model. In section **3.2**, we discuss consequences of U duality in our cosmological model, and consequently in section **3.3** and **3.4**, we give a general result and make our ansatz for T_{AB}.

In section **3.5** to section **3.7**, we present first a general analysis of D dimensional evolution equations and then we specialise to eleven dimensional case of $N = 4$ intersecting brane configurations and described various result mentioned above.

In section **3.8**, we present mechanism of stabilisation in details and then in section **3.9**, we discuss stabilised values of the brane directions, their ranges, and the necessity of fine tuning.

In section **3.10**, we discuss consequences of some example of intersecting configurations which are not $22'55'$, and we showed in these cases stabilisation can not be achieved.

In section **3.11**, we present the four dimensional perspective and the time variations of G_4 . In section **3.12**, we present the results of numerical analysis. In section **3.13**, we conclude by presenting a brief summary, a few comments on the assumptions made, and by mentioning a few issues which may be studied further.

2

Black Holes and Duality in M-Theory and String Theory

U-duality is a symmetry of M-theory which consists of T-duality, S-duality of string theory and dimensional reduction and dimension upliftment. In certain cases of supergravity solutions this symmetries can be used to get relations among various metric component. Theses relations can be used to get relations among various components of energy-momentum tensors. We will describe the procedure in detail in this chapter. In our model for early universe we use these relations to find equations of states.

We here discuss intersecting M branes system. In this chapter we study black holes made by mutually BPS intersecting M-branes. In chapter **3** we discuss mutually BPS intersecting M-branes dominated universe. To be specific, we consider $22'55'$ configurations.[5]

Such system is dictated by a $(10 + 1)$ dimensional effective action given, in the standard notation, by

$$S_{11} = \frac{1}{16\pi G_{11}} \int d^{11}x \, \sqrt{-g} \, R + S_{br} \tag{2.1}$$

where S_{br} is the action for the fields corresponding to the branes. The corresponding equations of motion are given, in the standard notation and in units where

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[5] In our notation, $22'55'$ denotes two stacks each of 2 branes and 5 branes, all intersecting each other in a mutually BPS configuration. Similarly for other configurations, *e.g.* $55'5''W$ denotes three stacks of 5 branes intersecting in a mutually BPS configuration with a wave along the common intersection direction.

$8\pi G_{11} = 1$, by [6]

$$R_{AB} - \frac{1}{2}\, g_{AB} R = T_{AB} \quad , \quad \sum_A \nabla_A T^A{}_B = 0 \tag{2.2}$$

where $A = (0, i)$ with $i = 1, 2, \cdots, 10$ and T_{AB} is the energy momentum tensor corresponding to the action S_{br} , defined by

$$\delta_g S_{br} = \frac{1}{2} \int d^{11}x \, \sqrt{-g} \, T_{AB} \, \delta g^{AB} \, , \tag{2.3}$$

where δ_g is the variation with respect to gravitational fields g_{AB}. Equations of motion for the brane fields can be found varying the action with respect to brane fields.

$$\frac{\partial S}{\partial\,(\text{Brane fields})} = 0. \tag{2.4}$$

If we know brane fields explicitly we can find equation of motion. For example in black hole cases we show them in next section.

2.1   Black Brane Energy Momentum Tensor and Solutions

For black hole case, T_{AB} is obtained from the action for higher form gauge fields. That is S_{br} is known. With a suitable ansatz for the metric, equations of motion (2.2) can be solved to obtain black hole solutions.

To explain the process consider 11-dimensional supergravity action. The bosonic part of the action is

$$S = \frac{1}{16\pi G_{11}} \int d^{11}x \, \sqrt{-g} \left(R - \frac{1}{2 \times 4!} F_4^2 \right) \, , \tag{2.5}$$

---

[6] In the following, the convention of summing over repeated indices is not always applicable. Hence, we will always write the summation indices explicitly. Unless indicated otherwise, the indices $A, B, \cdots$ run from 0 to 10, the indices $i, j, \cdots$ from 1 to 10, and the indices $I, J, \cdots$ from 1 to N.

where F_4 is a 4 form field, $F_4 = dC_3$. So S_{br} is given by

$$S_{br} = \frac{1}{16\pi G_{11}} \int d^{11}x \sqrt{-g} \left(-\frac{1}{2 \times 4!}F_4^2\right) . \tag{2.6}$$

The existence of 3-form gauge potential suggests that the theory contains 2-branes which couples to 3-form via

$$Q_2 \int d^3\xi \, C_{MNL} \frac{\partial x^M}{\partial \xi^a} \frac{\partial x^N}{\partial \xi^b} \frac{\partial x^L}{\partial \xi^c} \tag{2.7}$$

where ξ^a are world volume coordinates of 2-brane. Equations of motion for F_4 are

$$\nabla_M F^{MNPQ} = 0 , \tag{2.8}$$

where right hand side is set to zero because fermionic currents corresponding to branes are set to zero. This 4-form field also satisfy Bianchi identity,

$$\nabla_M F_{NPQL} + \nabla_L F_{MNPQ} + \nabla_Q F_{LMNP} + \nabla_P F_{QLMN} + \nabla_N F_{PQLM} = 0 . \tag{2.9}$$

The theory given by (2.5) contains a 2 dimensional and a 5 dimensional objects $M2$ and $M5$ brane. $M2$ branes are electrically charged and $M5$ branes are magnetically charged under F_4. For this matter field, F_4 energy-momentum tensor, T_{AB} is given by

$$T_{AB} = \frac{1}{48} \left[4F_{AMNP} F_B{}^{MNP} - \frac{1}{2}g_{AB} F_4^2\right] . \tag{2.10}$$

Second equation of equation (2.2),

$$\sum_A \nabla_A T^A{}_B = 0 \tag{2.11}$$

now follows from equations (2.8), (2.9) and (2.10). to show this consider

$$\begin{aligned}
\sum_A \nabla_A T^A{}_B &= 4(\nabla_A F^{AMNP})F_{BMNP} \\
&+ 4F^{AMNP}(\nabla_A F_{BMNP}) - g^A{}_B (\nabla_A F_{MNPL})F^{MNPL} .
\end{aligned}$$

First term is 0 by equation (2.8). Last two terms can be written as

$$-F^{MNPL} \left(\nabla_L F_{BMNP} + \nabla_P F_{LBMN} + \nabla_N F_{PLBM} + \nabla_M F_{NPLB} + \nabla_B F_{MNPL} \right) ,$$

which is zero by equation (2.9).

This theory contains black brane solution, which are solutions with solitonic objects. These black holes are actually made of stack of $M2$ or $M5$ branes or BPS intersecting combinations of them. With a suitable ansatz for metric and fields Einstein equation can be solved to obtain black hole solutions. First we calculate energy momentum tensor for various intersecting brane configurations. To do that we make suitable ansatz for metric and justify it.

2.2   General Black Brane solutions

Now we are in a position to get black hole solution for BPS configuration. *[reference] To get the solutions, let the spacetime coordinates be $x^A = (r, x^\alpha)$ where $x^\alpha = (x^0, x^i, \theta^a)$ with $x^0 = t$, $i = 1, \cdots, q$, $a = 1, \cdots, m$, and $q + m = 9$. The x^i directions may be taken to be toroidal, some or all of which are wrapped by branes, and θ^a are coordinates for an m dimensional space of constant curvature given by $\epsilon = \pm 1$ or 0 . The metric and brane fields depend only on r coordinate, and defined by $r^2 = \sum_{\alpha=q+1}^{q+m} (x^\alpha)^2$. We write the line element ds, in an obvious notation, as

$$ds^2 = -e^{2\lambda_0} dt^2 + \sum_i e^{2\lambda^i} (dx^i)^2 + e^{2\lambda} dr^2 + e^{2\sigma} d\Omega^2_{m,\epsilon} . \qquad (2.12)$$

Black hole solutions are given by $\epsilon = +1$. But the analysis is true for any maximally symmetric non-compact space.

The independent non vanishing components of $T^A{}_B$ are given by $T^r{}_r = f$ and $T^\alpha{}_\alpha = p_\alpha$ where $\alpha = (0, i, a)$. These components can be calculated explicitly using the action S_{br} . For example, for an electric p-brane along $(x^1, \cdots, x^p)$ directions, they are given by (see equation (2.10))

$$p_0 = p_\parallel = -p_\perp = -p_a = f = \frac{1}{4} F_{01\cdots pr} F^{01\cdots pr} \qquad (2.13)$$

where $p_{\parallel} = p_i$ for $i = 1, \cdots, p$, $p_{\perp} = p_i$ for $i = p+1, \cdots, q$, and note that f is negative. For mutually BPS N intersecting brane configurations, it turns out [51, 52, 53, 54, 55, 56, 57, 58, 59, 60] that the respective energy momentum tensors $T^A_{\ B}$ and $T^A_{\ B(I)}$ obey conservation equations separately. We explained this point in detail in previous section.

$$T^A_{\ B} = \sum_I T^A_{\ B(I)} \ , \quad \sum_A \nabla_A T^A_{\ B(I)} = 0 \ . \tag{2.14}$$

Equations of motion may now be obtained from equations (2.2) and (2.14). After some manipulations, they may be written as follows:

$$\Lambda_r^2 - \sum_\alpha (\lambda_r^\alpha)^2 \ = \ 2\,f + \epsilon\,m(m-1)e^{-2\sigma} \tag{2.15}$$

$$\lambda_{rr}^\alpha + \Lambda_r \lambda_r^\alpha \ = \ -p_\alpha + \frac{1}{9}\,(f + \sum_\beta p_\beta)$$
$$+\,\epsilon\,(m-1)e^{-2\sigma}\,\delta^{\alpha a} \tag{2.16}$$
$$f_r + f\Lambda_r - \sum_\alpha p_\alpha \lambda_r^\alpha \ = \ 0 \tag{2.17}$$

where $\Lambda = \sum_\alpha \lambda^\alpha = \lambda^0 + \sum_i \lambda^i + m\sigma$ and the subscripts r denote r-derivatives. One can see equation (2.17), which is same as (2.11) in this case is consequence of equations of motion for 4-form gauge field and definition of energy momentum tensor. In case of intersecting branes $p_\alpha = \sum_I p_{\alpha(I)}$, $f = \sum_I f_I$ and equation (2.17) may be written as

$$f_{Ir} + f_I \Lambda_r - \sum_\alpha p_{\alpha(I)} \lambda_r^\alpha = 0 \ . \tag{2.18}$$

This is because of, as we already claimed, $T^A_{\ B(I)}$ obey conservation equation separately. See [59] particularly, whose set up and the equations of motions are closest to the present ones.

The other equations of motion are the equations for 4-form field., which is same as equations (2.8) and (2.9). Equation (2.18) can be derived from equations (2.8) and (2.9).

If we assume $p_{\alpha(I)} = -(1 - u_\alpha^I)\,f_I$, then equation (2.18) can be solved. The

solution is found to be

$$f_I = - \, e^{l^I - 2\Lambda} \ , \quad l^I = \sum_\alpha u_\alpha^I \lambda^\alpha + l_0^I \ . \tag{2.19}$$

Now we define the matrices $G_{\alpha\beta}$ and $\mathcal{G}^{IJ}$ as

$$G_{\alpha\beta} = 1 - \delta_{\alpha\beta} \ , \quad \mathcal{G}^{IJ} = \sum_{\alpha,\beta} G^{\alpha\beta} \, u_\alpha^I \, u_\beta^J \ , \tag{2.20}$$

where $G^{\alpha\beta}$ is the inverse of $G_{\alpha\beta}$ and is given by

$$G^{\alpha\beta} = \frac{1}{9} - \delta^{\alpha\beta} \ . \tag{2.21}$$

If we define a new coordinate τ by $d\tau = e^{-\Lambda} \, dr$, equation (2.16) becomes

$$e^{-2\Lambda} \, \lambda_{\tau\tau}^\alpha = - \, p_\alpha + \frac{1}{9} \, (f + \sum_\beta p_\beta) + \ \epsilon \, (m-1) e^{-2\sigma} \, \delta^{\alpha a} \ . \tag{2.22}$$

If one multiplies both side by u_α and take a sum over α and then use equation (2.19) in (2.22) one finds

$$l_{\tau\tau}^I = - \sum_J \mathcal{G}^{IJ} \, e^{l^J} + \sum_{a \in \Omega} u_a \, \epsilon \, (m-1) \, e^{2(\Lambda - \sigma)} \ . \tag{2.23}$$

Specific values of $\mathcal{G}^{IJ}$ depend on intersecting configuration. In case of black holes we can calculate them from explicit calculation of energy momentum tensor or by using duality. In section **2.6** we discuss this point in detail.

If the components of energy momentum tensor follow a relation like

$$\sum_\alpha c_\alpha \left(-p_\alpha + \frac{1}{9} \, (f + \sum_\beta p_\beta) \right) = 0 \tag{2.24}$$

then this immediately implies a relation among metric components λ^α and σ. We will see, for various brane solutions from explicit calculation and also using duality same type of relations come. For example, when $\alpha \neq a$, then equations (2.22) and

(2.24) implies

$$\sum_\alpha c'_\alpha \lambda^\alpha = 0 \ . \tag{2.25}$$

Explicit intersecting configurations give values of c_α and c'_α. We will see some examples in the next section.

2.3 Energy-Momentum Tensor of Black Brane Solution

In this section we calculate Energy-Momentum Tensor for various black brane configurations. First we calculate for $M2$ and $M5$ brane case. Then we also give energy-momentum tensor of various intersecting combinations of them, both BPS and non-BPS.

We show here that, for BPS intersecting brane configurations, total energy momentum tensor is just sum of energy momentum tensors created by individual set of branes. On the other hand we give an example of 2 sets of intersecting $M2$ branes configuration, which does not follow BPS rule, total energy momentum tensor is not just sum over that of individual brane solutions.

In BPS cases we also notice, certain relations hold among various components of $T^A{}_B$. In a later section we will show that, these relations can be obtained by using U duality without computing explicit form of $T^A{}_B$.

2.3.1 $M2$ Branes

Consider a stack of $M2$ branes along (x^1, x^2) directions. x^1 and x^2 are taken to be compact. x^3 and x^4 are also taken to be compact. In this case the solution is known, it is given by in extremal case

$$ds^2 = H_2^{-\frac{2}{3}}\left(-dt^2 + \sum_{i=1}^{2}(dx^i)^2\right) + H_2^{\frac{1}{3}}\left(\sum_{i=3}^{4}(dx^i)^2 + dr^2 + r^2 d\Omega_7^2\right) \ , \tag{2.26}$$

where $H_2(r) = 1 - \frac{1}{r^4}$ and $r^2 = \sum_{i=5}^{10} (x^i)^2$. In general line element of this solution can be taken to be

$$ds^2 = -e^{2\lambda^0(r)} dt^2 + \sum_{i=1}^{4} e^{2\lambda^i(r)} (dx^i)^2 + e^{2\lambda(r)} \left(dr^2 + r^2 d\Omega_5^2 \right) . \tag{2.27}$$

Ansatz for field, C_{MNP} is

$$C_{012} = f(r) , \tag{2.28}$$

which gives

$$F_{012r} = f'(r) = E(r) , \tag{2.29}$$

where $'$ indicate derivative with respect to r. This actually means $M2$ branes are electrically charged under the field F_4. Energy momentum tensor is given by

$$T^0{}_0 = T^{\parallel}{}_{\parallel} = -T^{\perp}{}_{\perp} = T^r{}_r = -T^a{}_a = -\frac{1}{4} e^{-2(\lambda^0 + \lambda^1 + \lambda^2 + \lambda)} (E(r))^2 , \tag{2.30}$$

where indices $\parallel$ and $\perp$ indicate parallel and perpendicular to the brane directions and a indicates directions in Ω_5 respectively.

As we have shown in last section that, conservation equation follows from equations (2.8), (2.9) and (2.10) here also conservation equation $\sum_A \nabla_A T^A{}_B = 0$ is satisfied.

One can see from equations (2.30) and (2.16) that the constraining relation among scale factor, mentioned before turns out to be

$$\lambda^0 = \lambda^{\parallel} = -2\lambda^{\perp} . \tag{2.31}$$

2.3.2   BPS Intersection of 2 Sets of $M2$ Branes

Now consider 2 sets of intersecting $M2$ branes along (x^1, x^2) and (x^3, x^4). we denote first set by 2 and second set by $2'$. Black brane solution of intersecting branes was first identified in [61], then many solutions were quickly constructed and governing rules of their existence were studied. For a review of intersecting branes solution see book [5] and reference there in. This intersecting configuration follows BPS rules. BPS intersecting solution the intersection rules are discussed in

Appendix **C**. For this configuration our metric ansatz remains same as above.

$$ds^2 = -e^{2\lambda^0(r)}\,dt^2 + \sum_{i=1}^{4} e^{2\lambda^i(r)}\,(dx^i)^2 + e^{2\lambda(r)}\left(dr^2 + r^2 d\Omega_5^2\right)\ .$$

Nevertheless, we have now two sets of electrically charged branes, so our ansatz for C_{MNP} gets changed. Now we have

$$
\begin{aligned}
C_{012} &= f_2(r) \\
C_{034} &= f_{2'}(r)\ .
\end{aligned}
\tag{2.32}
$$

So now non vanishing components are C_{012} and C_{034} and their cyclic permutations. Above potential gives field strength of the form

$$
\begin{aligned}
F_{012r} &= f_2'(r) = E_2(r) \\
F_{034r} &= f_{2'}'(r) = E_{2'}(r)\ .
\end{aligned}
\tag{2.33}
$$

Energy momentum tensors in this case found to be

$$
\begin{aligned}
T^0{}_0 &= -\frac{1}{4}\,e^{-2(\lambda^0+\lambda)}\left\{e^{-2(\lambda^1+\lambda^2)}(E_2(r))^2 + e^{-2(\lambda^3+\lambda^4)}(E_{2'}(r))^2\right\} \\
T^1{}_1 = T^2{}_2 &= -\frac{1}{4}\,e^{-2(\lambda^0+\lambda)}\left\{e^{-2(\lambda^1+\lambda^2)}(E_2(r))^2 - e^{-2(\lambda^3+\lambda^4)}(E_{2'}(r))^2\right\} \\
T^3{}_3 = T^4{}_4 &= \frac{1}{4}\,e^{-2(\lambda^0+\lambda)}\left\{e^{-2(\lambda^1+\lambda^2)}(E_2(r))^2 - e^{-2(\lambda^3+\lambda^4)}(E_{2'}(r))^2\right\} \\
T^r{}_r &= -\frac{1}{4}\,e^{-2(\lambda^0+\lambda)}\left\{e^{-2(\lambda^1+\lambda^2)}(E_2(r))^2 + e^{-2(\lambda^3+\lambda^4)}(E_{2'}(r))^2\right\} \\
T^a{}_a &= \frac{1}{4}\,e^{-2(\lambda^0+\lambda)}\left\{e^{-2(\lambda^1+\lambda^2)}(E_2(r))^2 + e^{-2(\lambda^3+\lambda^4)}(E_{2'}(r))^2\right\}\ .
\end{aligned}
\tag{2.34}
$$

One can see easily from above equations, that total energy momentum tensors are just the sum of individual brane configurations,

$$T^A{}_B = \sum_I T^A{}_{B\,(I)}\ .$$

From general analysis of section **2.1** we can say that conservation equation $\sum_A \nabla_A T^A{}_B = 0$ is satisfied for total energy momentum tensor as well as individual energy mo-

mentum tensors. Last conclusion is confirmed by the fact that, conservation of $T^A{}_{B(I)}$ is verified in section **2.3.1**.

Like in previous subsection here also a relation among scale factors comes out using equations (2.34) and equation of motion.

$$
\begin{aligned}
\lambda^1 &= \lambda^2 \\
\lambda^3 &= \lambda^4 \\
2\lambda^1 + 2\lambda^3 + \lambda^0 &= 0 \ .
\end{aligned}
\tag{2.35}
$$

2.3.3 $M5$ Branes

In case of $M5$ brane, just like $M2$ brane case metric ansatz is taken of the same form, except now 5 of the 10 spacelike dimensions $(x^1, x^2, x^3, x^4, x^5)$ are compact and $M5$ branes wrap them. In general we may take some of the other directions are also compact. In that case they will be treated as directions perpendicular to branes and will be in same footing as directions of Ω_4. Metric ansatz is taken to be

$$
ds^2 = -e^{2\lambda^0(r)}dt^2 + \sum_{i=1}^{5} e^{\lambda^i}(dx^i)^2 + e^{2\lambda(r)}\left(dr^2 + r^2 d\Omega_4^2\right) \ ,
\tag{2.36}
$$

where now $r^2 = \sum_{i=6}^{10}(x^i)^2$. $M5$ branes are magnetically charged under the gauge field F_4. Its dual field is a 7-form field F_7. It is electrically charged under this 7-form F_7. F_4 and F_7 are related by

$$
(F_4)_{MNPQ} = \sqrt{-g} \ \epsilon_{012345rMNPQ}(F_7)^{01\cdots r} \ .
\tag{2.37}
$$

So one may take ansatz for for 3-form potential is of the form

$$
C_{NPQ} = \frac{1}{4}\epsilon_{012345rMNPQ}f(r)x^M \ .
\tag{2.38}
$$

So F_4 takes non zero value only when $(MNPQ) \in \{7, 8, 9, 10\}$. For computational simplicity and to match with standard notations, we take $f(r)$ as

$$
f(r) = \sqrt{-g} \ g^{00}g^{11}g^{22}g^{33}g^{44}g^{55}g^{rr}E(r) \ ,
\tag{2.39}
$$

where g_{AB} is the metric, g^{AB} is its inverse and g is its determinant. With this ansatz energy momentum tensor turns out to be

$$T^0{}_0 = T^{\parallel}{}_{\parallel} = T^r{}_r = -T^a{}_a = -\frac{1}{4}\, e^{-2(\lambda^0 + \lambda^1 + \lambda^2 + \lambda^3 + \lambda^4 + \lambda^5 + \lambda)} (E(r))^2 \; , \qquad (2.40)$$

where index $\parallel$ indicates parallel to brane directions and a indicates directions in Ω_4 respectively.

Here we have only one type of branes. Again general argument of section **2.1** is applicable. So

$$\sum_A \nabla_A T^A{}_B = 0$$

is true.

Equations (2.40) and equations of motion imply relations among scale factors. These equations are same as (2.31).

$$2\lambda^0 = 2\lambda^{\parallel} = -\lambda^{\perp} \; . \qquad (2.41)$$

2.3.4 BPS Intersection of $M2$ Branes and $M5$ Branes

In this subsection we give an example of BPS intersecting configuration of a stack of $M2$ branes, stretched along (x^1, x^2) and that of $M5$ branes are stretched along $(x^1, x^3, x^4, x^5, x^6)$. As before all these $x^1 \cdots x^6$ are compact, and the system is localised in common transverse space $x^7 \cdots x^{10}$. Again ansatz for black brane metric is similar to previous cases. It is taken of the form

$$ds^2 = -e^{2\lambda^0(r)}dt^2 + \sum_{i=1}^{6} e^{\lambda^i}(dx^i)^2 + e^{2\lambda(r)}\left(dr^2 + r^2 d\Omega_3^2\right) \; . \qquad (2.42)$$

Here $r^2 = \sum_{i=7}^{10}(x^i)^2$. Under 4-form gauge field $M2$ branes are electrically charged and $M5$ branes are charged magnetically. So we take an ansatz for our gauge potential as

$$\begin{aligned} C_{012} &= f_2(r) \\ C_{NPQ} &= \frac{1}{4}\epsilon_{013456rMNPQ}f_5(r)\,x^M \; , \end{aligned} \qquad (2.43)$$

where again $f_5(r)$ is taken of the form

$$f_5(r) = \sqrt{-g}\, g^{00} g^{11} g^{33} g^{44} g^{55} g^{66} g^{rr} E_5(r) \; . \tag{2.44}$$

Note that, here $\{M, N, P, Q\}$ can take value in x^3 and Ω_3, otherwise C_{MPQ} will be zero. Now the non-zero components of energy momentum tensor for this set of fields are

$$
\begin{aligned}
T^0{}_0 &= -\frac{1}{4}\, e^{-2(\lambda^0+\lambda^1+\lambda)} \left\{ e^{-2\lambda^2}(E_2(r))^2 + e^{-2(\lambda^3+\lambda^4+\lambda^5+\lambda^6)}(E_5(r))^2 \right\} \\
T^1{}_1 &= -\frac{1}{4}\, e^{-2(\lambda^0+\lambda^1+\lambda)} \left\{ e^{-2\lambda^2}(E_2(r))^2 + e^{-2(\lambda^3+\lambda^4+\lambda^5+\lambda^6)}(E_5(r))^2 \right\} \\
T^2{}_2 &= -\frac{1}{4}\, e^{-2(\lambda^0+\lambda^1+\lambda)} \left\{ e^{-2\lambda^2}(E_2(r))^2 - e^{-2(\lambda^3+\lambda^4+\lambda^5+\lambda^6)}(E_5(r))^2 \right\} \\
T^3{}_3 &= T^4{}_4 = T^5{}_5 = T^6{}_6 \\
&= \frac{1}{4}\, e^{-2(\lambda^0+\lambda^1+\lambda)} \left\{ e^{-2\lambda^2}(E_2(r))^2 - e^{-2(\lambda^3+\lambda^4+\lambda^5+\lambda^6)}(E_5(r))^2 \right\} \\
T^r{}_r &= -\frac{1}{4}\, e^{-2(\lambda^0+\lambda^1+\lambda)} \left\{ e^{-2\lambda^2}(E_2(r))^2 + e^{-2(\lambda^3+\lambda^4+\lambda^5+\lambda^6)}(E_5(r))^2 \right\} \\
T^a{}_a &= \frac{1}{4}\, e^{-2(\lambda^0+\lambda^1+\lambda)} \left\{ e^{-2\lambda^2}(E_2(r))^2 + e^{-2(\lambda^3+\lambda^4+\lambda^5+\lambda^6)}(E_5(r))^2 \right\} \; . \tag{2.45}
\end{aligned}
$$

So it is again just the sum of individual $T^A{}_B$ created by $M2$ branes and $M5$ branes separately. Same argument of section **2.1** goes through about conservation equation. Also and $T^A{}_{B\,(I)}$'s are conserved separately as they are, for single $M2$ brane and single $M5$ brane case.

Equations (2.45) and equation of motion imply constraining relations, like before, among scale factors.

$$
\begin{aligned}
\lambda^0 &= \lambda^1 \\
\lambda^3 &= \lambda^4 = \lambda^5 = \lambda^6 \\
\lambda^2 + 2\lambda^3 &= 0 \; . \tag{2.46}
\end{aligned}
$$

2.3.5 Non-BPS Intersection of Branes

Now we will consider an almost similar configuration of subsection **2.3.2**, except now our configuration is non-BPS. In this case lets take two sets of 2 branes along (x^1, x^2) and (x^2, x^3). It is a non-BPS intersection of branes. We take x^1, x^2 and x^3

as compact. Both set are electrically charged. So the 3-form potential components in this case are

$$
\begin{aligned}
C_{012} &= f_2(r) \\
C_{023} &= f_{2'}(r) \ .
\end{aligned}
\tag{2.47}
$$

So corresponding fields are

$$
\begin{aligned}
F_{012r} &= f_2'(r) = E_2(r) \\
F_{023r} &= f_{2'}'(r) = E_{2'}(r) \ .
\end{aligned}
\tag{2.48}
$$

For metric we may start with an ansatz like (2.42), but it turns out that this ansatz is inconsistent. The reason is explained below. Because of the first term in the expression of energy momentum tensor (equations (2.10)), T_{13} is non-zero,

$$
T_{13} = \frac{1}{12} g^{00} g^{22} g^{rr} F_{102r} F_{302r} \times 3! \ .
\tag{2.49}
$$

But since the metric is diagonal, $(R_{13} - \frac{1}{2} g_{13} R)$ is zero. So obviously Einstein equations are not satisfied. Therefore one has to take a different ansatz, simplest one is diagonal metric with only g_{13} non-zero. That is,

$$
ds^2 = -e^{2\lambda^0(r)} dt^2 + \sum_{i=1}^{3} e^{2\lambda^i(r)} (dx^i)^2 + 2 e^{2\lambda^c} dx^1 dx^3 + + e^{2\lambda(r)} \left(dr^2 + r^2 d\Omega_6^2 \right). \tag{2.50}
$$

With this ansatz it turns out non vanishing components of Einstein tensor are all diagonal components and G_{13}. So now we can equate G_{MN} and T_{MN}. So we will take the above line element as our ansatz.

We calculate here $T^M{}_N$. The non-zero components of them turn out to be

$$
\begin{aligned}
T^0{}_0 &= \frac{1}{4}\left(g^{00}g^{11}g^{22}g^{rr}F_{012r}F_{012r} + g^{00}g^{22}g^{33}g^{rr}F_{023r}F_{023r} + g^{00}g^{13}g^{22}g^{rr}F_{012r}F_{032r}\right) \\
T^1{}_1 &= \frac{1}{4}\left(g^{00}g^{11}g^{22}g^{rr}F_{012r}F^{012r} - g^{00}g^{22}g^{33}g^{rr}F_{023r}F_{023r} + g^{00}g^{13}g^{22}g^{rr}F_{012r}F_{032r}\right) \\
T^2{}_2 &= \frac{1}{4}\left(g^{00}g^{11}g^{22}g^{rr}F_{012r}F^{012r} + g^{00}g^{22}g^{33}g^{rr}F_{023r}F_{023r} + g^{00}g^{13}g^{22}g^{rr}F_{012r}F_{032r}\right) \\
T^3{}_3 &= \frac{1}{4}\left(-g^{00}g^{11}g^{22}g^{rr}F_{012r}F_{012r} + g^{00}g^{22}g^{33}g^{rr}F_{023r}F_{023r} + g^{00}g^{13}g^{22}g^{rr}F_{012r}F_{032r}\right) \\
T^r{}_r &= \frac{1}{4}\left(g^{00}g^{11}g^{22}g^{rr}F_{012r}F_{012r} + g^{00}g^{22}g^{33}g^{rr}F_{023r}F_{023r} + g^{00}g^{13}g^{22}g^{rr}F_{012r}F_{032r}\right) \\
T^a{}_a &= -\frac{1}{4}\left(g^{00}g^{11}g^{22}g^{rr}F_{012r}F_{012r} + g^{00}g^{22}g^{33}g^{rr}F_{023r}F_{023r} + g^{00}g^{13}g^{22}g^{rr}F_{012r}F_{032r}\right) \ ,
\end{aligned}
$$

$$(2.51)$$

where index a denotes coordinates in Ω_6. There is another component T_{13}. Now because of $T^1{}_3 = g^{11}T_{13} + g^{13}T_{33}$ and $T^3{}_1 = g^{33}T_{13} + g^{13}T_{11}$, $T^1{}_3$ and $T^3{}_1$ are not symmetric. They turned out to be various combination of F_{012r} and F_{023r}, and are non-zero. From above equations one can see clearly that, total energy momentum tensor is not just the sum of individual energy momentum tensors created by each sets of branes separately. For example in equations (2.51) first two terms in each equation give energy momentum tensor for individual brane configuration, but the third term is extra. Also T_{13} is a new component, which was not in case of single $M2$ brane system. So we may conclude that, for non-BPS intersection equation (2.14) is not satisfied.

2.4 $22'55'$ **Black Holes**

BPS intersection of multiple branes describes multicharge extremal black holes. We will now discuss of 2 $M2$ branes and 2 $M5$ branes intersecting configuration. This $22'55'$ configuration is the matter of our cosmological model. Here we take x^1 to x^7 are compact. First set of $M2$ branes wrap (x^1, x^2) directions, second set of $M2$ branes wrap (x^3, x^4), first $M5$ branes wrap $(x^1, x^3, x^5, x^6, x^7)$ and second $M5$ branes wrap $(x^2, x^4, x^5, x^6, x^7)$. $M2$ branes are electrically charged and $M5$ branes are magnetically charged under F_4. Let Q_2, $Q_{2'}$, Q_5, $Q_{5'}$ are the 4 parameter which determine mass and charges of the extremal black hole solution we get. So

the components of the gauge field potential are

$$C_{012} = H_2^{-1}(r) - 1 \tag{2.52}$$

$$C_{034} = H_{2'}^{-1}(r) - 1 \tag{2.53}$$

$$C_{LMN} = \frac{1}{4}\epsilon_{013567rPLMN}f_5(r)\, x^P \tag{2.54}$$

$$C_{ABC} = \frac{1}{4}\epsilon_{024567rPABC}f_{5'}(r)\, x^P\ ; \tag{2.55}$$

where

$$f_5(r) = \sqrt{-g}\, g^{00}g^{11}g^{33}g^{55}g^{66}g^{77}g^{rr}H_5^{-2}(r) \tag{2.56}$$

$$f_{5'}(r) = \sqrt{-g}\, g^{00}g^{22}g^{44}g^{55}g^{66}g^{77}g^{rr}H_{5'}^{-2}(r)\ . \tag{2.57}$$

H_I are the harmonic function, and given by $H_I = 1 + \frac{Q_I}{r}$. For this configuration geometry turns out to be

$$
\begin{aligned}
ds^2 =\ & (H_2 H_{2'})^{-2/3}(H_5 H_{5'})^{-1/3}[-dt^2 + H_{2'}H_{5'}(dx^1)^2 + H_{2'}H_5(dx^2)^2 + \\
& H_2 H_{5'}(dx^3)^2 + H_2 H_5(dx^4)^2 + H_2 H_{2'}\left\{(dx^5)^2 + (dx^6)^2 + (dx^7)^2\right\} \\
& + H_2 H_{2'} H_5 H_{5'}\left(dr^2 + r^2 d\Omega_2^2\right)]
\end{aligned}
\tag{2.58}
$$

Here one can calculate energy momentum tensor for each type of branes separately using equation (2.10). They are

$$
\begin{aligned}
T^0{}_{0\,(2)} = T^1{}_{1\,(2)} = T^2{}_{2\,(2)} = -T^3{}_{3\,(2)} = \cdots = -T^8{}_{8\,(2)} \\
= T^r{}_{r\,(2)} = -T^a{}_{a\,(2)} = -\frac{1}{4}\frac{(H_2'/H_2)^2}{(H_2 H_{2'})^{1/3}(H_5 H_{5'})^{2/3}}
\end{aligned}
\tag{2.59}
$$

$$
\begin{aligned}
T^0{}_{0\,(2')} = T^3{}_{3\,(2')} = T^4{}_{4\,(2')} = -T^1{}_{1\,(2')} = \cdots = -T^8{}_{8\,(2')} \\
= T^r{}_{r\,(2')} = -T^a{}_{a\,(2')} = -\frac{1}{4}\frac{(H_{2'}'/H_{2'})^2}{(H_2 H_{2'})^{1/3}(H_5 H_{5'})^{2/3}}
\end{aligned}
\tag{2.60}
$$

$$
\begin{aligned}
T^0{}_{0\,(5)} = T^1{}_{1\,(5)} = T^3{}_{3\,(5)} = T^5{}_{5\,(5)} = \cdots = T^7{}_{7\,(5)} = \\
-T^2{}_{2\,(5)} = -T^4{}_{4\,(5)} = T^r{}_{r\,(5)} = -T^a{}_{a\,(5)} = -\frac{1}{4}\frac{(H_5'/H_5)^2}{(H_2 H_{2'})^{1/3}(H_5 H_{5'})^{2/3}}
\end{aligned}
\tag{2.61}
$$

$$T^0{}_{0\,(5')} = T^2{}_{2\,(5')} = T^4{}_{4\,(5')} = T^5{}_{5\,(5')} = \cdots = T^7{}_{7\,(5')} =$$

$$-T^1{}_{1\,(5')} = -T^3{}_{3\,(5')} = T^r{}_{r\,(5')} = -T^a{}_{a\,(5')} = -\frac{1}{4}\frac{(H'_{5'}/H_{5'})^2}{(H_2 H_{2'})^{1/3}(H_5 H_{5'})^{2/3}} \quad (2.62)$$

Then one can see they satisfy conservation equation separately. Total $T^A{}_B$ is just sum of individual contributions as we have seen in previous examples.

$$T^A{}_B = \sum_I T^A{}_{B\,(I)} \ . \tag{2.63}$$

2.5   U Duality Relations In M Theory

We now describe the relations which follow from U duality symmetries, involving chains of dimensional reduction and uplifting and T and S dualities of string theory. See [32]. To explain the concept let us consider a solution of the form

$$ds^2_{11} = -e^{2\lambda^0}dt^2 + \sum_{\mu=1}^{10} e^{2\lambda^\mu}(dx^\mu)^2 \ , \tag{2.64}$$

where we assume for $\mu = i,j,k$ x^i are compact and metric does not depends on them. That is $\lambda^\mu = \lambda^\mu(t,X)$, where X includes space like coordinates except x^i, x^j and x^k. Let $\downarrow_k$ and $\uparrow_k$ denote dimensional reduction and uplifting along k^{th} direction between M theory and type IIA string theory. We are applying operation $\downarrow_k$. To do that we write ds_{11} as

$$ds^2_{11} = e^{-\frac{2}{3}\phi}\,ds^2_{10} + e^{\frac{4}{3}\phi}\,(dx^k)^2 \ , \tag{2.65}$$

where ds_{10} is 10 dimensional line element of type IIA theory. Type IIA string theory metric is given by

$$ds^2_{10} = -e^{2\lambda'^0}dt^2 + \sum_{\mu\neq k} e^{2\lambda'^\mu}(dx^\mu)^2 \ . \tag{2.66}$$

ϕ is dilaton and is independent of x^i, x^j and x^k. It is function of (t,X) only. If we integrate over x^k with above metric we will get type IIA super gravity action. Here comparing equations (2.64), (2.65) and (2.66) one can see ϕ and λ'^μ are given

by

$$\phi = \frac{3}{2}\lambda^k$$
$$\lambda'^{\mu} = \lambda^{\mu} + \frac{1}{3}\phi = \lambda^{\mu} + \frac{1}{2}\lambda^k \tag{2.67}$$

Now in string theory one can perform T duality along a compact direction. It converts type IIA theory to type IIB theory and back. It also converts a Dp brane to $D(p-1)$ or $D(p+1)$ branes depending on whether T duality is applied along brane or perpendicular to brane respectively. This is a symmetry of the IIA/IIB string theory. A solution, for example, (2.66) breaks it. So applying this transformation generates new solution. We denote T duality operation along i^{th} direction by T_i. S duality is a symmetry of type IIB theory. This is asymmetry between coupling constant $\leftrightarrow$ (coupling constant)$^{-1}$. Rules for the these duality transformations are listed in Appendix **A**.

Applying a T duality along say, x^j which we denote by T_j, generates a new solution, given by (see equation (A.2) and (A.7))

$$ds'^2_{10} = -e^{2\lambda'^0}dt^2 + \sum_{\mu \neq \{j,k\}} e^{2\lambda'^{\mu}}(dx^{\mu})^2 + e^{-2\lambda'^j}(dx^j)^2 \,,$$
$$\phi' = \phi - \lambda'^j = \lambda^k - \lambda^j \,, \tag{2.68}$$

where equation (2.67) has been used. Note that metric along x^j, g_{jj} goes to $(g_{jj})^{-1}$ according to equation (A.2). This solution is of type IIB theory. Again application of T_i generate a new solution of IIA theory.

$$ds''^2_{10} = -e^{2\lambda'^0}dt^2 + \sum_{\mu \neq \{i,j,k\}} e^{2\lambda'^{\mu}}(dx^{\mu})^2 + e^{-2\lambda'^j}(dx^j)^2 + e^{-2\lambda'^i}(dx^i)^2$$
$$\phi'' = \phi - \lambda'^i - \lambda'^j = \frac{1}{2}\lambda^k - \lambda^i - \lambda^j \,. \tag{2.69}$$

Again g_{ii} goes to $(g_{ii})^{-1}$ according to equation (A.2).

Now dimensional upliftment to 11 dimensional theory can be done via

$$ds'^2_{11} = e^{-\frac{2}{3}\phi''}\,ds''^2_{10} + e^{\frac{4}{3}\phi''}\,(dx^k)^2 \,. \tag{2.70}$$

Using equation (2.69) in (2.70) one gets

$$ds'^2_{11} = -e^{2\lambda''^0} dt^2 + \sum_{\mu=1}^{10} e^{2\lambda''^\mu} (dx^\mu)^2 \ , \tag{2.71}$$

where these λ''^i's are given in terms of λ^i's by (using equation (2.67)),

$$
\begin{aligned}
\lambda''^i &= \lambda^j - \frac{2}{3}(\lambda^i + \lambda^j + \lambda^k) \\
\lambda''^j &= \lambda^i - \frac{2}{3}(\lambda^i + \lambda^j + \lambda^k) \\
\lambda''^k &= \lambda^k - \frac{2}{3}(\lambda^i + \lambda^j + \lambda^k) \\
\lambda''^l &= \lambda^l + \frac{1}{3}(\lambda^i + \lambda^j + \lambda^k) \qquad \forall\, l \neq \{i,j,k\}
\end{aligned}
\tag{2.72}
$$

In general, simplifying notation, we can write, application of U duality $\uparrow_k T_i T_j \downarrow_k$ in (2.64),transforms the λ^is in the scale factors to λ'^is given by

$$
\lambda'^i = \lambda^j - 2\lambda \ , \quad \lambda'^j = \lambda^i - 2\lambda \ , \quad \lambda'^k = \lambda^k - 2\lambda
$$
$$
\lambda'^l = \lambda^l + \lambda \ , \quad l \neq \{i,j,k\} \ , \quad \lambda = \frac{\lambda^i + \lambda^j + \lambda^k}{3} \ . \tag{2.73}
$$

2.6 Application of U duality relations in Black Holes

Note that, as can be seen from the above steps, the U duality relations follow as long as the directions involved in the U duality operations are isometry directions. Since none of the common transverse directions are involved in obtaining the relations above, it follows that they are valid even if the common transverse directions are not compact. Thus the U duality relations are applicable in such cases also.

Similarly, the time dependence of λ^is played no role in obtaining the U duality relations here. Hence, these relations may be expected to arise for the black hole case also. We will describe this case in detail in this section.

Consider black holes in $m+2$ dimensional spacetime described by mutually BPS intersecting brane configurations in M theory. The brane action S_{br} in equation (2.1) is the standard one for higher form gauge fields and given by equation (2.6). Corresponding black hole solutions are given in section **2.3**. So here we only

highlight the points related to U duality symmetries. Also, for illustration, we consider only 2 branes and 5 branes.

As mentioned in section **2.5**, the method of U duality symmetries applies here also and leads to the same relations between λ^i . They are best seen in the extremal case. (The non extremal case requires further analysis and is more involved.)

The eleven dimensional line element ds for 2 brane is

$$ds^2 = H_2^{-\frac{2}{3}} \left(-dt^2 + \sum_{i=1}^{2}(dx^i)^2 \right) + H_2^{\frac{1}{3}} \left(dr^2 + r^2 d\Omega_7^2 \right) \ . \tag{2.74}$$

For $M5$ brane ds is

$$ds^2 = H_5^{-1/3} \left(-dt^2 + \sum_{i=1}^{5}(dx^i)^2 \right) + H_5^{2/3} \left(dr^2 + r^2 d\Omega_4^2 \right) \ , \tag{2.75}$$

where H_2 and H_5 are harmonic function and function of r.

In general line element for 2 or 5 brane can be written in the form

$$ds^2 = -e^{2\lambda^0} dt^2 + \sum_i e^{2\lambda^i} (dx^i)^2 \tag{2.76}$$

where (λ^0, λ^i) depend on r, the radial coordinate of the $m+1$ dimensional transverse space. For 2 branes and 5 branes, as can be seen from, equation (2.74) and (2.75), the λ^is may be written as

$$\lambda^1 = \lambda^2 = -\frac{2\tilde{h}}{6} \quad , \qquad \lambda^3 = \cdots = \lambda^{10} = \frac{\tilde{h}}{6} \tag{2.77}$$

$$\lambda^1 = \cdots = \lambda^5 = -\frac{\tilde{h}}{6} \quad , \qquad \lambda^6 = \cdots = \lambda^{10} = \frac{2\tilde{h}}{6} \tag{2.78}$$

where $e^{\tilde{h}} = H = 1 + \frac{Q}{r^{m-1}}$ is the corresponding harmonic function and Q is the charge. See, for example, [62] for more detail.

We see now relations among λ's [equations (2.77) or (2.78)] follow from U duality relations. Consider a solution of $M2$ brane along (x^1, x^2). Also take (x^3, x^4, x^5) as compact and are isometry direction. An obvious symmetry implies

$$\lambda^1 = \lambda^2 \ . \tag{2.79}$$

It also implies

$$\lambda^3 = \lambda^4 = \lambda^5 \ . \tag{2.80}$$

Directions x^a ($\in \{\Omega_4$ and $r\}$) are also transverse to brane directions. So we may assume

$$\lambda^3 = \lambda^4 = \lambda^5 = \lambda^6 = \lambda^7 = \lambda^8 = \lambda^9 = \lambda^{10} \ . \tag{2.81}$$

Now apply U duality operations $\downarrow_5 T_3 T_4 \uparrow_5$. It transforms $M2$ brane to $M5$ brane.

$$M2(12) \xrightarrow{\downarrow_5} D2(12) \xrightarrow{T_4} D3(124) \xrightarrow{T_3} D4(1234) \xrightarrow{\uparrow_5} M5(12345) \ .$$

This new solution is $M5$ branes may be given by

$$ds_{11}'^2 = -d\hat{t}^2 + \sum_{i=1}^{10} e^{2\lambda'^i(\hat{t})}(dx^i)^2 \ . \tag{2.82}$$

Where using equations (2.73) we find these λ'^i's are given in terms of λ^i's by

$$\begin{aligned}
\lambda'^3 &= \lambda^4 - \frac{2}{3}(\lambda^3 + \lambda^4 + \lambda^5) \\
\lambda'^4 &= \lambda^3 - \frac{2}{3}(\lambda^3 + \lambda^4 + \lambda^5) \\
\lambda'^5 &= \lambda^5 - \frac{2}{3}(\lambda^3 + \lambda^4 + \lambda^5) \\
\lambda'^i &= \lambda^i + \frac{1}{3}(\lambda^3 + \lambda^4 + \lambda^5) \qquad \forall\, i \neq \{3,4,5\} \ .
\end{aligned} \tag{2.83}$$

There are also obvious symmetry relation for $M5$ brane.

$$\lambda'^1 = \lambda'^2 = \lambda'^3 = \lambda'^4 = \lambda'^5 \ , \quad \lambda'^6 = \lambda'^7 = \lambda'^8 = \lambda'^9 = \lambda'^{10} \ . \tag{2.84}$$

So now one can see equations (2.77) and (2.78) are satisfied by the metric components of black hole solutions. in short we can write the relations among λ's as

$$\lambda^{\parallel} + 2\lambda^{\perp} = 0, \quad 2\lambda'^{\parallel} + \lambda'^{\perp} = 0 \ . \tag{2.85}$$

where the superscripts $\parallel$ and $\perp$ denote spatial directions along and transverse to the branes respectively. Note that, to find these relation we have use duality relations only. Explicit form of $\tilde{h}$ can only be known by solving equations of motion

and putting proper boundary conditions, like asymptotic flatness.

For the extremal $22'55'$ configuration $(12, 34, 13567, 24567)$, the transverse space is three dimensional and the λ^is may be written as [62] (see equation 2.58),

$$
\begin{aligned}
\lambda^1 &= \frac{1}{6} \left(-2\tilde{h}_1 + \tilde{h}_2 - \tilde{h}_3 + 2\tilde{h}_4 \right) \\
\lambda^2 &= \frac{1}{6} \left(-2\tilde{h}_1 + \tilde{h}_2 + 2\tilde{h}_3 - \tilde{h}_4 \right) \\
\lambda^3 &= \frac{1}{6} \left(\tilde{h}_1 - 2\tilde{h}_2 - \tilde{h}_3 + 2\tilde{h}_4 \right) \\
\lambda^4 &= \frac{1}{6} \left(\tilde{h}_1 - 2\tilde{h}_2 + 2\tilde{h}_3 - \tilde{h}_4 \right) \\
\lambda^5 = \lambda^6 = \lambda^7 &= \frac{1}{6} \left(\tilde{h}_1 + \tilde{h}_2 - \tilde{h}_3 - \tilde{h}_4 \right) \\
\lambda^8 = \lambda^9 = \lambda^{10} &= \frac{1}{6} \left(\tilde{h}_1 + \tilde{h}_2 + 2\tilde{h}_3 + 2\tilde{h}_4 \right)
\end{aligned}
\tag{2.86}
$$

where $e^{\tilde{h}_I} = H^I = 1 + \frac{Q_I}{r}$ are the corresponding harmonic functions and Q_Is are the charges. Furthermore, if 2 and $2'$ branes are identical then $\tilde{h}_1 = \tilde{h}_2$ and we get $\lambda^1 = \lambda^3$, and similarly other relations when different sets of branes are identical.

Note that obvious symmetry relations for $22'55'$ black hole are

$$
\lambda^5 = \lambda^6 = \lambda^7 \ , \quad \lambda^8 = \lambda^9 = \lambda^{10} \ ,
\tag{2.87}
$$

and the U duality relation comes out following above steps are

$$
\lambda^1 + \lambda^4 + \lambda^5 = \lambda^2 + \lambda^3 + \lambda^5 = 0 \ .
\tag{2.88}
$$

These equations (2.87) and (2.88) lead to equation (2.86).

We further illustrate the U duality methods by interpreting a U duality relation $\sum_i c_i \lambda^i = 0$ as implying a relation among the components of the energy momentum tensor T_{AB} . The relations thus obtained are indeed obeyed by the components of T_{AB} calculated explicitly using the corresponding higher form gauge field action S_{br} in section **2.3**.

Consider now the case of 2 branes or 5 branes. We assume that $p_a = p_\perp$ which is natural since θ^a directions are transverse to the branes. Applying the U duality relations in equation (2.85) then implies, for both 2 branes and 5 branes,

the relation

$$p_{\parallel} = p_0 + p_{\perp} + f \qquad (2.89)$$

among the components of their energy momentum tensors. See equations (2.30) and (2.40). Note that it is also natural to take $p_0 = p_{\parallel}$ since $x^0 = t$ is one of the worldvolume coordinates and may naturally be taken to be on the same footing as the other ones $(x^1, \cdots, x^p)$. Equation (2.89) then implies that $p_{\perp} = -f$. The relation between $p_{\parallel}$ and f is to be specified by an equation of state which, in the black hole case, is that given in equations (2.30) and (2.40).

For now, however, we take p_0 and $p_{\parallel}$ to be different. Keeping in mind that f is negative, we assume the equations of state to be of the form $p_{\alpha I} = -(1 - u_\alpha^I)\, f_I$ where $\alpha = (0, i, a)$, $I = 1, \cdots, N$, and u_α^I are constants, mentioned in section **2.2**. Here we give explicitly u_α's for 2 and 5 black brane solution.

$$2 \;:\; u_\alpha = (\; u_0, \; u_{\parallel}, \; u_{\parallel}, \; u_{\perp}, \; u_{\perp}, \; u_{\perp}, \; u_{\perp}, \; u_{\perp}, \; u_{\perp}, \; u_{\perp})$$

$$5 \;:\; u_\alpha = (\; u_0, \; u_{\parallel}, \; u_{\parallel}, \; u_{\parallel}, \; u_{\parallel}, \; u_{\parallel}, \; u_{\perp}, \; u_{\perp}, \; u_{\perp}, \; u_{\perp}) \qquad (2.90)$$

where the I superscripts have been omitted here and $u_{\parallel} = u_0 + u_{\perp}$ which follows from equation (2.89). Note that $u_{\perp} = 0$ and $u_0 = u_{\parallel} = 2$ in the black hole case given in equation (2.13) or specifically for 2 brane and 5 brane in equations (2.30) and (2.40).

Using definition of $\mathcal{G}^{IJ}$, l^I and τ equation (2.23) now becomes

$$l^I_{\tau\tau} = -\sum_J \mathcal{G}^{IJ}\, e^{l^J} + u_{\perp}\, \epsilon\, m(m-1)\, e^{2(\Lambda - \sigma)} \quad . \qquad (2.91)$$

We will see that same type of equation will appear in cosmological case. Since now we know u_α's, using equations (2.90) and (2.20), it is now straightforward to calculate $\mathcal{G}^{IJ}$ for N intersecting brane configurations. les first calculate $\mathcal{G}^{IJ}$ for 2 brane using (2.20).

$$\mathcal{G}^{IJ} = 2u_0\, (u_{\perp} - u_0 \delta^{IJ}) \quad . \qquad (2.92)$$

Same expression comes for 5 brane. It turns out, for any BPS intersecting configuration that, $\mathcal{G}^{IJ}$ is of the same form, namely

$$\mathcal{G}^{IJ} = 2u_0\, (u_{\perp} - u_0 \delta^{IJ}) \quad . \qquad (2.93)$$

The corresponding $\mathcal{G}_{IJ}$ is given by

$$\mathcal{G}_{IJ} = \frac{1}{2u_0^2} \left(\frac{u_\perp}{Nu_\perp - u_0} - \delta_{IJ} \right) \ .$$

(2.94)

Now take $p_0 = p_\parallel$. Then equation (2.89) gives $p_\perp + f = 0$. In terms of u_α , we now have $u_0 = u_\parallel$ and $u_\perp = 0$. Clearly, then $\mathcal{G}^{IJ} \propto \delta^{IJ}$ and equations (2.91) can be solved for $l^I(\tau)$. See [59] for such solutions, with $u_0 = 2$ as follows from equation (2.13), and their analysis.

2.7 S and T Dualities in String Theory

String theory has S and T duality symmetries. T duality transforms type IIA theory to type IIB and back. So it is a symmetry of IIA and IIB theory combined together. S duality is a symmetry of type IIB theory which transform coupling constant to (coupling constant)$^{-1}$. Just like M-theory case, for certain supergravity solutions these symmetries can be used to get relations among various metric components and dilaton, and hence relations among energy-momentum tensors. To illustrate this, we consider a general solution of string theory (type IIA or IIB supergravity). Line element ds_{Dp} in Einstein frame is

$$ds_{Dp}^2 = -e^{2\lambda_p^0} Z dt^2 + \sum_{i=1}^{p} e^{2\lambda_p^\parallel}(dx^i)^2 + \sum_{i=p+1}^{q} e^{2\lambda_p^\perp}(dx^i)^2 + e^{2\sigma}\left(\frac{dr^2}{Z} + r^2 d\Omega_{n+1,\epsilon}^2\right)$$

(2.95)

and dilaton, $\phi_p = \phi_p(t, r)$. here we assume $x^1, \cdots, x^q$ are compact and are isometry direction, and $i = 1, \cdots, p$ are directions parallel to Dp-branes. Obvious symmetries ensure us to take all λ^i parallel to branes are equal and we denote them as before by $\lambda_p^\parallel$, similarly for $\lambda_p^\perp$. Here all λs, Z and σ are function of r. $d\Omega_{n+1,\epsilon}$ is metric of constant curvature $n + 1$-dimensional space. In fact we don't even need non-compact directions in above form. It can be any curved geometry.

Here to illustrate duality relation we consider black p-brane solution and this is of the form given in equation (2.95). The subscript Dp indicate metric is for Dp-branes. This metric is a of general black p-brane solution. q is the total number of compact directions, and $q + n = 7$. This system physically describes geometry created by D-brane localised in space. Technically relations we will get

in this section can be used when λ's are function of t only. So if we consider no noncompact direction and $Z = 1$ we can as well describe cosmological case.

This solution is in Einstein frame. To apply duality rules we first convert it in string frame. String frame metric is

$$
\begin{aligned}
ds^2_{s,Dp} &= -e^{2\lambda^0_p + \frac{\phi_p}{2}} Z dt^2 + \sum_{i=1}^{p} e^{2\lambda^{\|}_p + \frac{\phi_p}{2}} (dx^i)^2 + \sum_{i=p+1}^{q} e^{2\lambda^{\perp}_p + \frac{\phi_p}{2}} (dx^i)^2 \\
&+ e^{2\sigma + \frac{\phi_p}{2}} \left(\frac{dr^2}{Z} + r^2 d\Omega^2_{n+1,\epsilon} \right)
\end{aligned}
\tag{2.96}
$$

Now if we perform a T-duality along p^{th} direction we get $D(p-1)$ branes solution.

$$
\begin{aligned}
ds^2_{s,Dp-1} &= -e^{2\lambda^0_p + \frac{\phi_p}{2}} Z dt^2 + \sum_{i=1}^{p-1} e^{2\lambda^{\|}_p + \frac{\phi_p}{2}} (dx^i)^2 + e^{-(2\lambda^{\|}_p + \frac{\phi_p}{2})} (dx^p)^2 \\
&+ \sum_{i=p+1}^{q} e^{2\lambda^{\perp}_p + \frac{\phi_p}{2}} (dx^i)^2 + e^{2\sigma + \frac{\phi_p}{2}} \left(\frac{dr^2}{Z} + r^2 d\Omega^2_{n+1,\epsilon} \right)
\end{aligned}
\tag{2.97}
$$

$$
\phi_{p-1} = \phi_p - \frac{1}{2} ln(e^{2\lambda^{\|}_p + \frac{\phi_p}{2}}) = \frac{3\phi_p}{4} - \lambda^{\|}_p .
\tag{2.98}
$$

Note that in p^{th} direction metric component changes sign according to equation (A.2). Equation (2.98) can be found using equation (A.7).

The Einstein frame metric for $D(p-1)$ solution can be found by multiplying $e^{-\phi_{p-1}/2}$ to $ds^2_{s,Dp-1}$.

$$
\begin{aligned}
ds^2_{Dp-1} &= e^{-\phi_{p-1}/2} ds^2_{s,Dp-1} \\
&= -e^{2\lambda^0_{p-1}} Z dt^2 + \sum_{i=1}^{p-1} e^{\frac{5\lambda^{\|}_p}{2} + \frac{\phi_p}{8}} (dx^i)^2 + e^{-\frac{3\lambda^{\|}_p}{2} - \frac{7\phi_p}{8}} (dx^p)^2 \\
&+ \sum_{i=p+1}^{q} e^{2\lambda^{\perp}_p + \frac{\lambda^{\|}_p}{2} \frac{\phi_p}{8}} (dx^i)^2 + ds^2_{\perp} ,
\end{aligned}
\tag{2.99}
$$

where $ds^2_{\perp}$ is metric for transverse space. Equations (2.97) and (2.98 have been used to get above expression. This line element can be written as

$$
ds^2_{Dp-1} = -e^{2\lambda_0} Z dt^2 + \sum_{i=1}^{p-1} e^{2\lambda^{\|}_{p-1}} (dx^i)^2 + \sum_{i=p}^{q} e^{2\lambda^{\perp}_{p-1}} (dx^i)^2 + ds^2_{\perp} ,
\tag{2.100}
$$

where λ_p can be given in terms of λ_{p-1} by

$$2\lambda_{p-1}^{\parallel} \;=\; \frac{5\lambda_p^{\parallel}}{2} + \frac{\phi_p}{8} \tag{2.101}$$

$$2\lambda_{p-1}^{\perp} \;=\; -\frac{3}{2}\lambda_p^{\parallel} - \frac{7}{8}\phi_p = 2\lambda_p^{\perp} + \frac{\lambda_p^{\parallel}}{2} + \frac{\phi_p}{8} \tag{2.102}$$

and

$$\phi_{p-1} = \frac{3\phi_p}{2} - \lambda_p^{\parallel}. \tag{2.103}$$

Simplification of equation (2.102) shows that,

$$\lambda_p^{\perp} + \lambda_p^{\parallel} + \frac{\phi_p}{2} \;=\; 0. \tag{2.104}$$

Now consider a $D3$ brane, S-duality of $D3$ brane gives $D3$ brane. So S duality rule (A.14) gives $\phi_3 = -\phi_3$, which implies $\phi_3 = 0$. Consider $D2$ brane solution now, $\phi_2 = -\lambda_3^{\parallel}$, (using equation (2.103)). Equation (2.104) gives $\lambda_3^{\parallel} = -\lambda_3^{\perp}$. Denoting $\lambda_3^{\perp}$ by λ, one find

$$\lambda_p^{\perp} = \lambda, \quad \lambda_p^{\parallel} = -\lambda, \quad \phi_p = 0 \times \lambda. \tag{2.105}$$

In general using induction it is easy to show that,

$$\lambda_p^{\perp} = \frac{p+1}{4}\lambda, \quad \lambda_p^{\parallel} = -\frac{7-p}{4}\lambda, \quad \phi_p = (3-p)\lambda, \tag{2.106}$$

where λ is now the only parameter determining full line element. If one uses different set of duality operations one can also find similar relations for $F1$-string or $NS5$-brane. In general

$$\lambda_p^{\perp} = \frac{p+1}{4}\lambda, \quad \lambda_p^{\parallel} = -\frac{7-p}{4}\lambda, \quad \phi_p = z(3-p)\lambda, \tag{2.107}$$

where $z = 1$ for Dp-brane and $z = -1$ for $F1$-string or $NS5$-brane.

Just like before we take energy-momentum tensors has only diagonal component.

$$T^{\mu}{}_{\nu} \;=\; diag(P_0, P_{\parallel}, P_{\perp}, P_r, P_a), \tag{2.108}$$

where $P_{\parallel}$ for $i = 1, 2, \cdots, p$; $P_{\perp}$ for $i = p+1, \cdots, q$; $T^r{}_r = P_r$ and $T^{\theta_a}{}_{\theta_a} = P_a$. It

is natural to assume $P_\perp = P_a$ because all of them are transverse to brane direction but for the time being we keep this notation. We also have another component, T_ϕ coming from ϕ-variation of action

$$\delta_\phi S = \int d^{10}x \; \sqrt{-g} \; T_\phi \; \delta\phi \;, \tag{2.109}$$

where δ_ϕ denote variation with respect to ϕ. Equation of motions are now Einstein equations (2.2) together with

$$\nabla^2 \phi = -T_\phi \;. \tag{2.110}$$

The equations of motion for the black brane case (metric given by (2.95)) turns out to be

$$\lambda''^i + L'\lambda'^i = \frac{1}{Z} e^{2\sigma} (P - P_i) \tag{2.111}$$

$$\phi'' + L'\phi' = -\frac{1}{Z} e^{2\sigma} T_\phi \tag{2.112}$$

where $L = \frac{Z'}{Z} + \frac{n+1}{r} + n\sigma + \lambda^0 + \Lambda$ and $P = \frac{1}{8}(P_0 + \sum P_i + P_r + (n+1)P_a)$.

Using equation (2.107) in equation (2.111) one finds

$$-\frac{p+1}{7-p} = \frac{P - P_\perp}{P - P_\parallel} \;, \tag{2.113}$$

which on simplification gives

$$P_\parallel + (7-q)P_\perp = P_0 + P_r + (n+1)P_a \;. \tag{2.114}$$

Similarly use of equation (2.107) in equation (2.112) gives

$$T_\phi = -\frac{3-p}{2} z \left(P_0 + P_r + (n+1)P_a \right) - (8-q)P_\perp) \tag{2.115}$$

Just like black brane in M-theory case taking $P_\perp = P_a$, one can do the same calculation to solve for black D-brane or black string solution. Also note that in cosmological case there is no transverse metric and $q = 9$. Then above relation

will translate to

$$\rho + P_{\parallel} - 2P_{\perp} = 0 \tag{2.116}$$

$$T_{\phi} = -\frac{3-p}{2} z \left(-\rho + P_{\perp}\right), \tag{2.117}$$

where P_0 is taken as $(-\rho)$. Then again same analysis can be done as in both cosmological case and black hole case of M-theory to get same conclusions.

3

Evolution of Early Universe

In this chapter we discuss evolution of early universe made of mutually BPS intersecting $M2$ and $M5$ branes. To be specific, we consider $22'55'$ configurations. To do this we use duality relations discuss in previous chapter. Before going into details analysis we give a general consequences U duality in our cosmological context.

3.1   Equations of Motion for Our Cosmological Model

For cosmological case, T_{AB} is often determined using equations of state of the dominant constituent of the universe. Such equations of state may be obtained if the underlying physics is known; or, one may assume a general ansatz for them and proceed. [7]

T_{AB} for intersecting branes in the early universe has been calculated in [27] assuming that the branes and antibranes in the intersecting brane configurations are non interacting and that their numbers are all equal, *i.e.* $n_I = \bar{n}_I$ for $I = 1, 2, \cdots, N$ and $n_1 = \cdots = n_N$. However, general relations among the components of T_{AB} may be obtained [32] using U duality symmetries of M theory, involving chains of dimensional reduction and uplifting and T and S dualities of string theory, using which 2 branes and 5 branes or $22'55'$ and $55'5''W$ configurations can be interchanged. Such relations are valid more generally, for example even when n_I and $\bar{n}_I$ are all different.

---

[7] This is similar to the FRW case. Equation of state $p = \frac{\rho}{3}$ for radiation, or $p = 0$ for pressureless dust, may be obtained from the physics of radiation or of massive particles; or, one may assume a general ansatz $p = w\rho$ and proceed.

These general relations on the equations of state are sufficient to show, under a technical assumption, that the $N = 4$ mutually BPS intersecting brane configurations with identical numbers of branes and antibranes, *i.e.* with $n_1 = \cdots = n_4$ and $\bar{n}_1 = \cdots = \bar{n}_4$, will asymptotically lead to an effective $(3+1)$ dimensional expanding universe. To obtain the details of the evolution, however, we need further assumptions and an ansatz of the type $p = w\rho$ [32, 33, 34].

We now present the details. Let the line element ds be given by

$$ds^2 = -dt^2 + \sum_i e^{2\lambda^i}(dx^i)^2 \tag{3.1}$$

where e^{λ^i} are scale factors and, due to homogeneity, λ^i are functions of the physical time t only. (Parametrising the scale factors as e^{λ^i} turns out to be convenient for our purposes.) It then follows that T_{AB} depends on t only and that it is of the form

$$T^A{}_B = diag(-\rho,\ p_i)\ . \tag{3.2}$$

We assume that $\rho > 0$. From equations (2.2) one now obtains

$$\Lambda_t^2 - \sum_i (\lambda_t^i)^2 \;=\; 2\,\rho \tag{3.3}$$

$$\lambda_{tt}^i + \Lambda_t \lambda_t^i \;=\; p_i + \frac{1}{9}\,(\rho - \sum_j p_j) \tag{3.4}$$

$$\rho_t + \rho\Lambda_t + \sum_i p_i \lambda_t^i \;=\; 0 \tag{3.5}$$

where $\Lambda = \sum_i \lambda^i$ and the subscripts t denote time derivatives. Note, from equation (3.3), that Λ_t cannot vanish. Hence, if $\Lambda_t > 0$ at an initial time t_0 then it follows that e^Λ increases monotonically for $t > t_0$. We assume the evolution to be such that $e^\Lambda \to \infty$ eventually.

In the context of early universe in M theory, it is natural to assume that all spatial directions are on equal footing to begin with. Therefore we assume that the ten dimensional space is toroidal. Further, we assume that the early universe is homogeneous and is dominated by the $22'55'$ configuration where, with no loss of generality, we take two stacks each of 2 branes and 5 branes to be along (x^1, x^2) , (x^3, x^4) , $(x^1, x^3, x^5, x^6, x^7)$, and $(x^2, x^4, x^5, x^6, x^7)$ directions respectively, and take

these intersecting branes to be distributed uniformly along the common transverse space directions (x^8, x^9, x^{10}) . Note that the total brane charges must vanish, *i.e.* $n_I = \bar{n}_I$ for all I , since the common transverse space is compact. We denote this $22'55'$ configuration as $(12, 34, 13567, 24567)$. The meaning of this notation is clear and, below, such a notation will be used to denote other configurations also.

3.2 Equations of States as Consequence of U Duality

Application of such U duality operations generate new solution from old one. We discuss this technique in section **2.6**, but for we repeat the steps since here they are our main tools to find equations of states. Lets take an example of $M2$ brane along (x^1, x^2). We consider cosmological solution of the form,

$$ds_{11}^2 = -dt^2 + \sum_{i=1}^{10} e^{2\lambda^i(t)} (dx^i)^2 \ , \tag{3.6}$$

where we assume all the special directions are compact and toroidal. Here obvious symmetry implies

$$\lambda^1 = \lambda^2, \quad \lambda^3 = \lambda^4 = \lambda^5 = \lambda^6 = \lambda^7 = \lambda^8 = \lambda^9 \ . \tag{3.7}$$

If we apply a chain of operations $\downarrow_5 T_3 T_4 \uparrow_5$ in the way mentioned in previous section, **2.5**, we can generate a $M5$ brane solution.

$$M2(12) \xrightarrow{\downarrow_5} D2(12) \xrightarrow{T_4} D3(124) \xrightarrow{T_3} D4(1234) \xrightarrow{\uparrow_5} M5(12345) \ .$$

This new solution is $M5$ branes may be given by

$$ds_{11}'^2 = -d\hat{t}^2 + \sum_{i=1}^{10} e^{2\lambda'^i(\hat{t})} (dx^i)^2 \ . \tag{3.8}$$

Where using equations (2.73) we find these λ'^i's are given in terms of λ^i's by

$$
\begin{aligned}
\lambda'^3 &= \lambda^4 - \frac{2}{3}(\lambda^3 + \lambda^4 + \lambda^5) \\
\lambda'^4 &= \lambda^3 - \frac{2}{3}(\lambda^3 + \lambda^4 + \lambda^5) \\
\lambda'^5 &= \lambda^5 - \frac{2}{3}(\lambda^3 + \lambda^4 + \lambda^5) \\
\lambda'^i &= \lambda^i + \frac{1}{3}(\lambda^3 + \lambda^4 + \lambda^5) \qquad \forall\, i \neq \{3, 4, 5\} .
\end{aligned}
\tag{3.9}
$$

Note that in case of our cosmological solution we can always redefine our time coordinate to go to comoving frame, as we have redefine time as $\hat{t}$ here.

Obvious symmetry of $M5$ brane solution demands

$$
\lambda'^1 = \lambda'^2 = \lambda'^3 = \lambda'^4 = \lambda'^5 , \quad \lambda'^6 = \lambda'^7 = \lambda'^8 = \lambda'^9 = \lambda'^{10} .
\tag{3.10}
$$

So using equations (3.7), (3.10) and the relations among λ'^i and λ^i one can show that

$$
\lambda^\| + 2\lambda^\perp = 0, \quad 2\lambda'^\| + \lambda'^\perp = 0 .
\tag{3.11}
$$

where the superscripts $\|$ and $\perp$ denote spatial directions along and transverse to the branes respectively.

Putting back these relations in the equation of motion (3.4) one can show

$$
p_\| = 2p_\perp - \rho .
\tag{3.12}
$$

In general $M2$ brane or $M5$ brane or wave above relation turns out to be

$$
p_\| = z(\rho - p_\perp) + p_\perp ,
\tag{3.13}
$$

where $z = -1$ for 2 and 5 branes and $= +1$ for waves.

Similarly one can go through all such operations in intersecting brane configuration to generate new solutions. In case of our $22'55'$ configuration $(12, 34, 13567, 24567)$ the obvious symmetry relations are

$$
22'55' \; : \quad \lambda^5 = \lambda^6 = \lambda^7 , \quad \lambda^8 = \lambda^9 = \lambda^{10} .
\tag{3.14}
$$

Proceeding as in the case of 2 and 5 branes above, and using the U duality $\uparrow_5$ $T_1 T_2 \downarrow_5$ which relates the $22'55'$ and $W55'5''$ configurations, one obtains two more relations given by [32]

$$\lambda^1 + \lambda^4 + \lambda^5 = \lambda^2 + \lambda^3 + \lambda^5 = 0 \ .\tag{3.15}$$

Clearly, the U duality relations in equations (2.87) and (2.88) are valid here also. Then again like before, use of equation (3.4) yields relations among various components of energy momentum tensor.

$$p_5 = p_6 = p_7 \tag{3.16}$$

$$p_8 = p_9 = p_{10} \tag{3.17}$$

$$p_1 + p_4 + p_5 = p_2 + p_3 + p_5 = 0 \tag{3.18}$$

In general, for an N intersecting brane configuration, the U duality symmetries will lead to $10 - N$ relations among the λ^is, leaving only N of them independent. These relations are of the form $\sum_i c_i \lambda^i = 0$ where c_i are constants. Clearly, such a relation can be violated by constant scaling of x^i coordinates. Hence, we interpret it as implying a relation among the components of T_{AB} . In view of equation (3.4), we interpret a U duality relation $\sum_i c_i \lambda^i = 0$ as implying that

$$\sum_i c_i f^i = 0 \quad , \quad f^i = p_i + \frac{1}{9} \left(\rho - \sum_j p_j \right) \ . \tag{3.19}$$

Substituting equation (3.19) in equation (3.4), it follows upon an integration that

$$\sum_i c_i \lambda_t^i = c \, e^{-\Lambda} \tag{3.20}$$

where c is an integration constant. If $\sum_i c_i \lambda_t^i = 0$ initially at $t = t_0$ then $c = 0$. In such cases then $\sum_i c_i \lambda_t^i = 0$ for all t and, hence, $\sum_i c_i \lambda^i = v$ where v is another integration constant.

In general $\sum_i c_i \lambda_t^i \neq 0$ initially at $t = t_0$ and, hence, $c \neq 0$. Let the evolution be such that $e^\Lambda \sim t^\beta \to \infty$ in the limit $t \to \infty$. Then it follows from equation (3.20) that $\sum_i c_i \lambda_t^i \to 0$ in this limit. If, furthermore, $\beta > 1$ then equation (3.20)

also gives

$$\sum_i c_i \lambda^i = v + \mathcal{O}(t^{1-\beta}) \to v \tag{3.21}$$

where v is an integration constant. If $\beta \leq 1$ then $\sum_i c_i \lambda^i$ is a function of t. We will see later that, in the solutions we obtain with further assumptions, β turns out to be > 1 for $N > 1$.

Note that, in previous chapter, for black hole case we showed that U duality relations hold by calculating T_{AB} explicitly. On the other hand here we use U duality to get equations of states.

3.3   A General Result

We now consider a general result for the $22'55'$ configuration that follows from the U duality relations alone [32]. The λ^is for this configuration obey the relations given in equations (3.14) and (3.15). Note that a suitable U duality, for example $\uparrow_6 T_4 T_5 \downarrow_6$, can transform 2 branes and 5 branes into each other. Hence, we will refer to two types of branes as being identical if they have identical numbers of branes and antibranes, *i.e.* I^{th} type is identical to J^{th} type if $n_I = n_J$ and $\bar{n}_I = \bar{n}_J$.

Consider the case when 2 and $2'$ branes in the $22'55'$ configurations are identical. This will enhance the obvious symmetry relations. It is easy to see that we now have one more independent relation $\lambda^1 = \lambda^3$. If, instead, 5 and $5'$ branes are identical, then the extra independent relation is $\lambda^1 = \lambda^2$. Similarly, if 2 and $5'$ branes are identical then, after a few steps involving U duality $\uparrow_6 T_4 T_5 \downarrow_6$ which interchanges 2 and $5'$ branes, it follows that the extra independent relation is $\lambda^2 = \lambda^5$.

Now if all the four types of branes in the $22'55'$ configuration are identical, *i.e.* if $n_1 = \cdots = n_4$ and $\bar{n}_1 = \cdots = \bar{n}_4$, then, we have three extra independent relations

$$\lambda^1 = \lambda^2 = \lambda^3 = \lambda^5 \quad . \tag{3.22}$$

Combined with equations (3.14) and (3.15), we get $\lambda^1 = \cdots = \lambda^7 = 0$ which is to be interpreted as $f^1 = \cdots = f^7 = 0$, see equation (3.19). Hence, as described earlier, it follows for $i = 1, \cdots, 7$ that if $\lambda^i_t = 0$ initially at $t = t_0$ then $\lambda^i_t = 0$ and $\lambda^i = v^i$ for all t where v^i are constants. Or, if $e^\Lambda \sim t^\beta \to \infty$ in the limit

$t \to \infty$ with $\beta > 1$, it then follows for $i = 1, \cdots, 7$ that $\lambda_t^i \to 0$ and $\lambda^i \to v^i$ in this limit. Obtaining the values of the asymptotic constants v^i , however, requires knowing the details of evolution. It also follows similarly that $e^{\lambda^i} \sim e^{\frac{\Lambda}{3}} \to \infty$ for $i = 8, 9, 10$. It is straightforward to show that same results are obtained for the equivalent $55'5''W$ configuration also.

Thus, assuming either that $\lambda_t^1 = \cdots = \lambda_t^7 = 0$ initially at $t = t_0$ or that $e^{\Lambda} \sim t^{\beta} \to \infty$ in the limit $t \to \infty$ with $\beta > 1$, we obtain that the $N = 4$ mutually BPS intersecting brane configurations with identical numbers of branes and antibranes, i.e. with $n_1 = \cdots = n_4$ and $\bar{n}_1 = \cdots = \bar{n}_4$, will asymptotically lead to an effective (3+1) dimensional expanding universe with the remaining seven spatial directions reaching constant sizes. This result follows as a consequence of U duality symmetries alone, which imply relations of the type given in equation (3.19) among the components of the energy momentum tensor T_{AB} . This result is otherwise independent of the details of the equations of state, and also of the ansatzes for T_{AB} we make in the following in order to proceed further.

3.4   Ansatz for T_{AB}

The dynamics underlying the general result given above may be understood in more detail, and the asymptotic constants v^i can be obtained, if an explicit solution for the evolution is available. In the following, we will make a few assumptions which enable us to obtain such details.

Consider now the case of 2 branes or 5 branes only. From the U duality relations, it follows from equation (3.12), that $p_{\parallel} = -\rho + 2p_{\perp}$. For the case of waves, the relation is given by equation (3.13). A similar relation for black hole is derived in equation (2.89). In general, ρ , $p_{\parallel}$, and $p_{\perp}$ are functions of the numbers n and $\bar{n}$ of branes and antibranes, satisfying the U duality relations (3.13). If $n = \bar{n}$ then $p_{\parallel}$ and $p_{\perp}$ may be thought of as functions of ρ satisfying equation (3.13) [32].

Consider now mutually BPS N intersecting brane configuration. In the black hole case, following the discussion of section **2.1** and **2.6**, it turns out that the energy momentum tensors $T^A_{B(I)}$ of the Ith type of branes are mutually non

interacting and separately conserved [51] – [60]. That is,

$$T^A_B = \sum_I T^A_{B(I)} \ , \quad \sum_A \nabla_A T^A_{B(I)} = 0 \ . \tag{3.23}$$

We assume that this is the case in the context of early universe also where $T^A_B = diag(-\rho, \ p_i)$, $T^A_{B(I)} = diag(-\rho_I, \ p_{iI})$, $\rho_I > 0$, and $(\rho_I, \ p_{iI})$ satisfy the U duality relations in (3.13) for all I . Equations (2.14) now give

$$\rho = \sum_I \rho_I \ , \quad p_i = \sum_I p_{iI} \tag{3.24}$$

$$(\rho_I)_t + \rho_I \Lambda_t + \sum_i p_{iI} \lambda^i_t = 0 \ . \tag{3.25}$$

We have verified explicitly for a variety of mutually BPS N intersecting brane configurations that equations (3.13) and (3.24) are sufficient to satisfy all the relations of the type $\sum_i c_i f^i = 0$ implied by U duality symmetries. See [32] for more details.

To solve the evolution equations (3.3), (3.4), (3.24), and (3.25), we need the functions ρ_I , $p_{\|I}$, and $p_{\perp I}$. To proceed further, we assume that $n_I = \bar{n}_I$ for all I . This is necessary if, as is the case here, the common transverse directions are compact and hence the net charges must vanish. Then $p_{\|I}$ and $p_{\perp I}$ may be thought of as functions of ρ_I satisfying equation (3.13).

It is natural to expect that $p_{\perp I}(\rho_I)$ is the same function for waves, 2 branes, and 5 branes since they can all be transformed into each other by U duality operations which do not involve the transverse directions. We assume that this is the case. We further assume that this function $p_\perp(\rho)$ is given by

$$p_\perp = (1 - u) \, \rho \tag{3.26}$$

where u is a constant. Such a parametrisation of the equation of state, instead of the usual one $p = w\rho$, leads to elegant expressions as will become clear in the following, see [41, 42] also. The results of [27] correspond to the case where $u = 1$. Here, we assume only that $0 < u < 2$. The constant u is arbitrary otherwise.

It now follows that p_{iI} in equation (3.24) are of the form $p_{iI} = (1 - u^I_i) \, \rho_I$ and that the constants u^I_i can be obtained in terms of u using equations (3.13) and (3.26). Thus, for 2 branes, 5 branes, and waves, we have $u_\perp = u$, $u_\| = (1 - z) \, u$,

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and hence

$$
\begin{aligned}
2 \;\; &: \;\; u_i = (\;\; 2,\;\; 2,\;\; 1,\;\; 1,\;\; 1,\;\; 1,\;\; 1,\;\; 1,\;\; 1,\;\; 1)\, u \\
5 \;\; &: \;\; u_i = (\;\; 2,\;\; 2,\;\; 2,\;\; 2,\;\; 2,\;\; 1,\;\; 1,\;\; 1,\;\; 1,\;\; 1)\, u \\
W \;\; &: \;\; u_i = (\;\; 0,\;\; 1,\;\; 1,\;\; 1,\;\; 1,\;\; 1,\;\; 1,\;\; 1,\;\; 1,\;\; 1)\, u
\end{aligned}
\tag{3.27}
$$

where the I superscripts have been omitted since $N = 1$. Similarly, u_i^I for the $22'55'$ configuration are given by

$$
\begin{aligned}
2 \;\; &: \;\; u_i^1 = (\;\; 2,\;\; 2,\;\; 1,\;\; 1,\;\; 1,\;\; 1,\;\; 1,\;\; 1,\;\; 1,\;\; 1)\, u \\
2' \;\; &: \;\; u_i^2 = (\;\; 1,\;\; 1,\;\; 2,\;\; 2,\;\; 1,\;\; 1,\;\; 1,\;\; 1,\;\; 1,\;\; 1)\, u \\
5 \;\; &: \;\; u_i^3 = (\;\; 2,\;\; 1,\;\; 2,\;\; 1,\;\; 2,\;\; 2,\;\; 2,\;\; 1,\;\; 1,\;\; 1)\, u \\
5' \;\; &: \;\; u_i^4 = (\;\; 1,\;\; 2,\;\; 1,\;\; 2,\;\; 2,\;\; 2,\;\; 2,\;\; 1,\;\; 1,\;\; 1)\, u \;\; .
\end{aligned}
\tag{3.28}
$$

This completes our ansatz for the energy momentum tensor T_{AB} for the intersecting brane configurations in the early universe.

3.5 General Analysis of Evolution Equations

The evolution of the universe can now be analysed. In this section, we first present the analysis in a general form which is applicable to a D dimensional homogeneous, anisotropic universe. We specialise to the intersecting brane configurations in the next section.

The D dimensional line element ds is given by equation (3.1), now with $i = 1, 2, \cdots, D-1$. The total energy momentum tensor T_{AB} of the dominant constituents of the universe is given by equation (3.2). The equations of motion for the evolution of the universe is given, in units where $8\pi G_D = 1$, by equations (3.3) – (3.5) with 9 in equation (3.4) now replaced by $D-2$. Defining

$$
G_{ij} = 1 - \delta_{ij} \;\; , \quad G^{ij} = \frac{1}{D-2} - \delta_{ij} \;\; ,
\tag{3.29}
$$

the equations (3.3) and (3.4), with 9 replaced by $D-2$, may be written compactly

as

$$\sum_{i,j} G_{ij}\lambda_t^i \lambda_t^j \;=\; 2\,\rho \tag{3.30}$$

$$\lambda_{tt}^i + \Lambda_t\lambda_t^i \;=\; \sum_j G^{ij}\,(\rho - p_j) \tag{3.31}$$

where $i, j, \cdots$ run from 1 to $D-1$.

Let the universe be dominated by N types of mutually non interacting and separately conserved matter labelled by $I = 1, \cdots, N$. Then the corresponding energy momentum tensors $T_{AB(I)}$ and their components ρ_I and p_{iI} satisfy equations (3.23) – (3.25).

Further, let the equations of state be given by $p_{iI} = (1 - u_i^I)\,\rho_I$ where u_i^I are constants. Equations (3.25), (3.30), and (3.31) may now be simplified and cast in various useful forms as follow.

Using $p_{iI} = (1 - u_i^I)\,\rho_I$, equation (3.25) can be integrated to give

$$\rho_I = e^{l^I - 2\Lambda} \;\;,\;\;\; l^I = \sum_i u_i^I\lambda^i + l_0^I \tag{3.32}$$

where l_0^I are integration constants. Further using equations (3.24) and (3.32), equations (3.30) and (3.31) become

$$\sum_{i,j} G_{ij}\lambda_t^i \lambda_t^j \;=\; 2\sum_J e^{l^J - 2\Lambda} \tag{3.33}$$

$$\lambda_{tt}^i + \Lambda_t\lambda_t^i \;=\; \sum_J u^{iJ}\,e^{l^J - 2\Lambda} \tag{3.34}$$

where $u^{iJ} = \sum_j G^{ij}\,u_j^J$. Let the initial conditions at an initial time t_0 be given, with no loss of generality, by

$$\left(\lambda^i,\; \lambda_t^i,\; l^I,\; l_t^I,\; \rho_I\right)_{t=t_0} = \left(0,\; k^i,\; l_0^I,\; K^I,\; \rho_{I0}\right) \tag{3.35}$$

where

$$\rho_{I0} = e^{l_0^I} \;\;,\;\;\; K^I = \sum_i u_i^I k^i \;\;,\;\;\; \sum_{i,j} G_{ij}k^i k^j = 2\sum_J e^{l_0^J} \;\;. \tag{3.36}$$

Equations (3.33) and (3.34) may now be solved for the $D-1$ variables λ^i with

the above initial conditions. Or, instead, these equations may be manipulated so that one needs to solve for N variables l^I only, see equations (3.37), (3.41), (3.44), and (3.46) below. We now perform these manipulations.

First define a variable $\tau(t)$ as follows:

$$d\tau = e^{-\Lambda} dt \ , \quad \tau(t_0) = 0 \ . \tag{3.37}$$

Then, for $\lambda^i(t)$ or equivalently $\lambda^i(\tau(t))$, we have

$$\lambda_t^i = e^{-\Lambda} \lambda_\tau^i \ , \quad \lambda_{tt}^i + \Lambda_t \lambda_t^i = e^{-2\Lambda} \lambda_{\tau\tau}^i \tag{3.38}$$

where the subscripts τ denote τ–derivatives. Note that the initial values at $\tau(t_0) = 0$ remain unchanged since $\Lambda = 0$, and hence $\lambda_t^i = \lambda_\tau^i$ at $t = t_0$. Equations (3.33) and (3.34) now become

$$\sum_{i,j} G_{ij}\lambda_\tau^i \, \lambda_\tau^j \ = \ 2 \sum_J e^{l^J} \tag{3.39}$$

$$\lambda_{\tau\tau}^i \ = \ \sum_J u^{iJ} \, e^{l^J} \ . \tag{3.40}$$

Also, from $l^I = \sum_i u_i^I \lambda^i + l_0^I$, it follows that

$$l_{\tau\tau}^I = \sum_J \mathcal{G}^{IJ} \, e^{l^J} \tag{3.41}$$

where

$$\mathcal{G}^{IJ} = \sum_i u_i^I \, u^{iJ} = \sum_{i,j} G^{ij} \, u_i^I \, u_j^J \ . \tag{3.42}$$

We assume that $\mathcal{G}_{IJ}$ exists such that $\sum_J \mathcal{G}_{IJ} \, \mathcal{G}^{JK} = \delta_I{}^K$, *i.e.* that the matrix $\mathcal{G}$ formed by $\mathcal{G}^{IJ}$ is invertible. [8] Then, from equation (3.41), we have

$$\sum_J \mathcal{G}_{IJ} \, l_{\tau\tau}^J = e^{l^I} \ . \tag{3.43}$$

Note that equation (3.41) is of the form equation (2.91). But here we can not

---

[8]This is not always the case. For example, $u_i^I = u^I$ for all i in the isotropic case. Then $\mathcal{G}^{IJ} \propto u^I u^J$ and $\mathcal{G}$ is not invertible. This is not a problem, it just means that the set of variables l^I can be reduced to a smaller independent set; one then proceeds with the smaller set.

calculate $\mathcal{G}^{IJ}$ explicitly because we do not know explicit form of $T^A{}_{B\,(I)}$ now. But using equation of states and duality relations we will be able to calculate it, it turns out $\mathcal{G}^{IJ}$ is not diagonal in this case. Substituting this expression for e^{l^I} into equation (3.40), then integrating it twice and incorporating the initial conditions given in equation (3.35), we get

$$\lambda^i = \sum_I u^i_I \, (l^I - l^I_0) + \; L^i \, \tau \quad , \quad u^i_I = \sum_{j,J} \mathcal{G}_{IJ} \, G^{ij} \, u^J_j \qquad (3.44)$$

where L^i are integration constants. It follows from $\lambda^i_\tau(0)$ that L^i are related to initial values k^i and K^I by $k^i = \sum_I u^i_I \, K^I + \; L^i$. Using this expression for k^i in the relation $K^I = \sum_i u^I_i k^i$, or substituting the expression for λ^i given in equation (3.44) into the equation (3.32) for l^I , leads to the following N constraints on L^i :

$$\sum_i u^I_i L^i = 0 \quad , \quad I = 1, 2, \cdots , N \quad . \qquad (3.45)$$

Now, using equations (3.44) and (3.45), equation (3.39) may be written in terms of l^I as follows:

$$\sum_{I,J} \mathcal{G}_{IJ} \, l^I_\tau \, l^J_\tau = 2 \, (E + \sum_I e^{l^I}) \quad , \quad 2E = - \sum_{i,j} G_{ij} L^i L^j \quad . \qquad (3.46)$$

One may now solve equations (3.41) and (3.46) for N variables $l^I(\tau)$. Then $\lambda^i(\tau)$ are obtained from equation (3.44) and $t(\tau)$ from equation (3.37). Inverting $t(\tau)$ then gives $\tau(t)$, and thereby $\lambda^i(t)$.

3.5.1 $N = 1$ Case

Consider the $N = 1$ case. Note that we are still considering the general D dimensional universe, not the eleven dimensional one. We assume here that $\mathcal{G}^{11} = \mathcal{G} > 0$. Now, as shown in Appendix **C**, it follows in general that if $\sum_i u_i L^i = 0$ and $\sum_{i,j} G^{ij} u_i u_j > 0$ then $E \geq 0$ and E vanishes if and only if L^i all vanish. Since $\sum_i u^1_i L^i = 0$, see equation (3.45), and we assume that $\mathcal{G}^{11} = \sum_{i,j} G^{ij} u^1_i u^1_j > 0$, we have $E \geq 0$. We further assume that $E > 0$, equivalently that L^is do not all vanish.

Omitting the I labels, equations (3.41) and (3.46) for $l(\tau)$ become

$$l_{\tau\tau} = \mathcal{G}\, e^l \quad , \quad (l_\tau)^2 = 2\, \mathcal{G}\, (E + e^l) \ . \tag{3.47}$$

The initial values are $l_0 = l(0)$ and $K = l_\tau(0)$ obeying $K^2 = 2\, \mathcal{G}\, (E + e^{l_0})$. We take $K > 0$ with no loss of generality. Then the solution for $l(\tau)$ is given by

$$e^l = \frac{E}{Sinh^2\, \alpha(\tau_\infty - \tau)} \tag{3.48}$$

where

$$2\alpha^2 = \mathcal{G}E \quad , \quad Sinh^2\, \alpha\tau_\infty = E\, e^{-l_0} \quad , \quad K = 2\alpha\, Coth\, \alpha\tau_\infty \ . \tag{3.49}$$

The sign of α is immaterial but, just to be definite, we take it to be positive. The sign of τ_∞ is same as that of K , hence $\tau_\infty > 0$. Also, $\lambda^i(\tau)$ and $t(\tau)$ may now be obtained but are not needed here for our purposes.

Note that $e^l \to 4E\, e^{2\alpha(\tau-\tau_\infty)}$ and vanishes in the limit $\tau \to -\infty$, whereas $e^l \to \frac{2}{\mathcal{G}}\, (\tau_\infty - \tau)^{-2}$ and diverges in the limit $\tau \to \tau_\infty$. The value of τ_∞ depends on the initial values l_0 and K, or equivalently E , as given in equations (3.49). It is finite and can be evaluated exactly. However, if $e^{l_0} \ll E$ then τ_∞ may be approximated in a way that will be useful later on.

From the exact solution given above, we have $Sinh^2\, \alpha\tau_\infty = E\, e^{-l_0}$ and $K = 2\alpha\, Coth\, \alpha\tau_\infty$. In the limit $e^{l_0} \ll E$, we then have $e^{2\alpha\tau_\infty} \simeq 4\, E\, e^{-l_0}$ and $K \simeq 2\alpha$. It, therefore, follows that

$$\tau_\infty \simeq \frac{1}{K}\, (ln\, E - l_0 + ln\, 4) \ . \tag{3.50}$$

In the limit $e^{l_0} \ll E$, the evolution of $l(\tau)$ may also be thought of as follows. Consider E to be fixed and e^{l_0} to be very small so that $e^{l_0} \ll E$. It then follows from equations (3.47) that, at initial times, $l_{\tau\tau}$ is very small and that $l_\tau \simeq \sqrt{2\mathcal{G}E} = 2\alpha$ is independent of e^l . Hence, $l(\tau)$ evolves as if there is no 'force', i.e. $l(\tau) \simeq l_0 + K\tau$ where $K = l_\tau(0) > 0$ is the initial 'velocity'. Once e^l becomes of $\mathcal{O}(E)$ then it affects l_τ . But, from then on, e^l evolves quickly and diverges soon after.

This suggests that one may well approximate τ_∞ by the time τ_a required for

l , which starts from l_0 with a velocity K and evolves freely with no force, to reach $ln\ E$ – namely, to reach a value where $e^l = e^{l_0 + K\tau_a} = E$. In other words, if $e^{l_0} \ll E$ then

$$\tau_\infty \simeq \tau_a = \frac{1}{K}\ (ln\ E - l_0)\ . \tag{3.51}$$

A comparison with equation (3.50) shows that the exact τ_∞ which follows from solving the evolution equations is indeed well approximated by τ_a in equation (3.51) in the limit $e^{l_0} \ll E$. Note that τ_a is calculated using only the initial values, requiring no knowledge of the exact solution.

3.5.2 $N > 1$ **Case**

When $N > 1$, the equations of motion can be solved if $\mathcal{G}^{IJ}$ are of certain form [41] – [47]. For example, if $\mathcal{G}^{IJ} \propto \delta^{IJ}$ then the solutions are similar to those in the $N = 1$ case described above. For general forms of $\mathcal{G}^{IJ}$, we are unable to obtain explicit solutions. Nevertheless, the general evolution can still be analysed if one assumes suitable asymptotic forms for the scale factors e^{λ^i} .

It follows from equations (3.29) and (3.30) that Λ_t cannot vanish. With no loss of generality, let $\Lambda_t > 0$ initially at $t = t_0$. Then e^Λ decreases monotonically for $t < t_0$, equivalently $\tau < 0$, and increases monotonically for $t > t_0$, equivalently $\tau > 0$. Further features of the evolution depend on the structure of $\mathcal{G}^{IJ}$ and u_i^I . In the cases of interest here, it turns out that e^Λ and also all e^{l^I} vanish in the limit $\tau \to -\infty$, and diverge in the limit $\tau \to \tau_\infty$ where τ_∞ is finite. We assume such a behaviour and analyse the asymptotic solutions.

3.5.2.1 **Asymptotic evolution:** $e^\Lambda \to 0$

We assume that $(e^\Lambda, e^{l^I}) \to 0$ in the limit $\tau \to -\infty$. Then, equations (3.40) and (3.41) can be solved since their right hand sides depend only on e^{l^I}s which now vanish. Hence, in the limit $\tau \to -\infty$, we write

$$e^{l^I}\ =\ e^{\tilde{c}^I \tau}\ =\ t^{\tilde{b}^I}\ \ ,\ \ \ e^{\lambda^i}\ =\ e^{\tilde{c}^i \tau}\ =\ t^{\tilde{b}^i} \tag{3.52}$$

which are valid up to multiplicative constants and where $(\tilde{c}^I,\ \tilde{c}^i,\ \tilde{b}^I,\ \tilde{b}^i)$ are constants. Also, the equalities in the asymptotic expressions here and in the following are valid only up to the leading order. Equation (3.44) now implies that

$\tilde{c}^i = \sum_I u_I^i \tilde{c}^I + L^i$. Also, $e^\Lambda = e^{\tilde{c}\tau}$ where $\tilde{c} = \sum_i \tilde{c}^i$. Then it follows from equation (3.37) that $t \sim e^{\tilde{c}\tau}$. Hence,

$$\tilde{b}^I = \frac{\tilde{c}^I}{\tilde{c}} \quad , \quad \tilde{b}^i = \frac{\tilde{c}^i}{\tilde{c}} \quad , \quad \sum_i \tilde{b}^i = 1 \ . \tag{3.53}$$

Furthermore, equation (3.39) implies that $(\sum_i \tilde{b}^i)^2 - \sum_i (\tilde{b}^i)^2 = 0$. Thus the evolution is of Kasner type in the limit $\tau \to -\infty$. The constants $\tilde{c}^I$s in equations (3.52) must be such that the resulting $\sum_i \tilde{b}^i = \sum_i (\tilde{b}^i)^2 = 1$, but are otherwise arbitrary. In an actual evolution, however, $\tilde{c}^I$s can be determined in terms of the initial values l_0^I and K^I with no arbitrariness, but this requires complete solution for $l^I(\tau)$.

3.5.2.2 Asymptotic evolution: $e^\Lambda \to \infty$

We assume that $e^\Lambda \to \infty$ in the limit $\tau \to \tau_\infty$ where τ_∞ is finite. Whether this limit is reached at a finite or infinite physical time t depends on the values of u_i^I , see below. $\Lambda(\tau)$ may be obtained in terms of $l^I(\tau)$ using equation (3.44). Hence, in the limit $e^\Lambda \to \infty$, some or all of the e^{l^I}s diverge. Consider the following ansatz in the limit $\tau \to \tau_\infty$:

$$e^{l^I} = e^{c^I} (\tau_\infty - \tau)^{-2\gamma^I} \quad , \quad e^{\lambda^i} = e^{c^i} (\tau_\infty - \tau)^{-2\gamma^i} \quad , \tag{3.54}$$

where c^I and γ^I are constants, and some or all of the γ^Is must be > 0 so that some or all of the e^{l^I}s diverge. Equation (3.44) now implies that

$$\gamma^i = \sum_I u_I^i \gamma^I \quad , \quad c^i = \sum_I u_I^i (c^I - l_0^I) + L^i \tau_\infty \ . \tag{3.55}$$

Also, $e^\Lambda = e^c (\tau_\infty - \tau)^{-2\gamma}$ where $c = \sum_i c^i$ and $\gamma = \sum_i \gamma^i$. For the ansatz in equations (3.54) to be consistent, it is necessary that $\gamma > 0$ so that $e^\Lambda \to \infty$ in the limit $\tau \to \tau_\infty$. Now $t(\tau)$ follows from equation (3.37) and is given by

$$t - t_s = \frac{1}{2\gamma - 1} e^c (\tau_\infty - \tau)^{-(2\gamma - 1)} \quad , \quad \gamma = \sum_{i,I} u_I^i \gamma^I \tag{3.56}$$

where t_s is a finite constant. If $2\gamma < 1$ then $t \to t_s$ which means that $e^\Lambda \to \infty$ at a finite physical time t_s. If $2\gamma > 1$ then $t \to \infty$ in the limit $e^\Lambda \to \infty$. Which case is realised, *i.e.* whether $2\gamma < 1$ or > 1, depends on the values of u_i^I.

Using equation (3.56), the asymptotic behaviour of e^{l^I} and e^{λ^i} can be obtained in terms of t. For example, let $2\gamma > 1$ and

$$e^{l^I} = e^{b^I + 2b} \, t^{\beta^I} \quad , \quad e^{\lambda^i} = e^{b^i} \, t^{\beta^i} \quad , \quad e^\Lambda = e^b \, t^\beta \tag{3.57}$$

in the limit $t \to \infty$. It then follows that

$$\beta^I = \frac{2\,\gamma^I}{2\gamma - 1} \quad , \quad \beta^i = \frac{2\,\gamma^i}{2\gamma - 1} \quad , \quad \beta = \frac{2\,\gamma}{2\gamma - 1} \ . \tag{3.58}$$

Note that, in this case, we have $e^\Lambda \sim t^\beta$ in the limit $t \to \infty$ with $\beta > 1$. See the discussion below equation (3.19) for the relevance of this feature.

To obtain the values of γ^I, and thereby γ^i, in equation (3.54), consider equation (3.43) from which it follows that

$$2 \sum_J \mathcal{G}_{IJ} \, \gamma^J = e^{c^I} \, (\tau_\infty - \tau)^{2(1 - \gamma^I)} \ . \tag{3.59}$$

The left hand side in the above equation is a constant but the right hand side depends on τ. This is consistent if $\gamma^I = 1$ in which case the right hand side becomes a positive constant, or if $\gamma^I < 1$ in which case the right hand side vanishes in the limit $\tau \to \tau_\infty$. Thus, there are two possibilities:

$$\text{(i)} \quad \gamma^I = 1 \implies 2 \sum_J \mathcal{G}_{IJ} \, \gamma^J = e^{c^I} > 0 \tag{3.60}$$

$$\text{(ii)} \quad \gamma^I \neq 1 \implies \sum_J \mathcal{G}_{IJ} \, \gamma^J = 0 \quad , \quad \gamma^I < 1 \ . \tag{3.61}$$

For a given $\mathcal{G}_{IJ}$, the possible consistent solutions for $(\gamma^I, \, e^{c^I})$ are to be obtained as follows. Assume that some I's are of type **(i)** and the remaining ones are of type **(ii)**. Then solve equations (3.60) and (3.61) for e^{c^I} in type **(i)** and for γ^I in type **(ii)**. Such a solution is consistent if the resulting $(e^{c^I}, \, \gamma^I)$ satisfy $e^{c^I} > 0$ for Is in type **(i)** and $\gamma^I < 1$ for Is in type **(ii)**. Also, some or all of the γ^Is must be > 0 as required in equation (3.54). (It is further necessary that the resulting

$\gamma > 0$ so that $e^\Lambda \to \infty$, but calculating γ requires u_i^I.)

Consider an example, which will be useful later, where $\mathcal{G}^{IJ}$ and $\mathcal{G}_{IJ}$ are given by

$$\mathcal{G}^{IJ} = a \, (b - \delta^{IJ}) \quad , \qquad \mathcal{G}_{IJ} = \frac{1}{a} \, (\frac{b}{Nb-1} - \delta_{IJ}) \tag{3.62}$$

with $a > 0$ and $Nb > 1$. It is then easy to show that the only possibility is the one given in **(i)**. Also $\sum_J \mathcal{G}_{IJ} = \frac{1}{a(Nb-1)} > 0$, and thus $\gamma^I = 1$ for all I is a consistent solution as required by equation (3.60). In the $N = 1$ case, we get $\mathcal{G}^{11} = \mathcal{G} = a \, (b-1) > 0$, and e^l in the limit $\tau \to \tau_\infty$ obtained as described above agrees with that obtained from the explicit solution, see below equation (3.49).

Thus e^{c^I} and γ^I, and thereby $\gamma^i = \sum_I u_I^i \, \gamma^I$ and $\gamma = \sum_{i,I} u_I^i \, \gamma^I$, are all determined ultimately by u_i^I. The constants c^i are given by equation (3.55) and they depend on u_i^I, on the initial values l_0^I and L^i, and also on τ_∞. But determining τ_∞, and hence determining c^i when L^i do not all vanish, requires complete solution for $l^I(\tau)$.

3.5.3   Deviations from $e^{l^I} \to \infty$ Asymptotics

We consider the deviations of $l^I(\tau)$ from its asymptotic behaviour given in equation (3.54), which will turn out to be of interest. Let the deviations $s^I(\tau)$ for $I = 1, 2, \cdots, N$ be defined, in the limit $\tau \to \tau_\infty$, by

$$e^{l^I} = e^{c^I} \, (\tau_\infty - \tau)^{-2\gamma^I} \, e^{s^I(\tau)} \tag{3.63}$$

where c^I and γ^I are determined as described earlier. For the purpose of illustration, and also for later use, we now assume that all the Is are of type **(i)**, namely that $\gamma^I = 1$ and $e^{c^I} = 2 \sum_J \mathcal{G}_{IJ} > 0$ for all I. It then follows straightforwardly from equation (3.41) that

$$(\tau_\infty - \tau)^2 \, s_{\tau\tau}^I = 2 \sum_{K,L} \mathcal{G}^{IK} \, \mathcal{G}_{KL} \, (e^{s^K} - 1) \, . \tag{3.64}$$

Consider the example of $\mathcal{G}^{IJ}$ given in equation (3.62). Then $\sum_J \mathcal{G}_{IJ} = \frac{1}{a(Nb-1)}$

and, for any σ^K , one has

$$\sum_{K,L} \mathcal{G}^{IK}\, \mathcal{G}_{KL}\, \sigma^K = -\, \frac{1}{Nb-1}\, (\sigma^I - b \sum_K \sigma^K) \ . \tag{3.65}$$

In equation (3.64), $\sigma^K = 2\,(e^{s^K} - 1)$. It now follows easily that, up to the leading order in $\{s^K\}$, the difference $s^I - s^J$ obeys the equation

$$(\tau_\infty - \tau)^2\, (s^I - s^J)_{\tau\tau} + \frac{2}{Nb-1}\, (s^I - s^J) = 0 \ . \tag{3.66}$$

The solutions to these equations are of the form

$$(s^I - s^J) \ \sim \ (\tau_\infty - \tau)^{\frac{1}{2}\,(1\pm\sqrt{\Delta})} \ , \quad \Delta = 1 - \frac{8}{Nb-1} \ . \tag{3.67}$$

Note that $s^I - s^J = l^I - l^J$ since γ^I and c^I are same for all I , see equation (3.63). Hence, equations (3.66) and (3.67) can be used to understand in more detail the behaviour of l^Is as they all diverge in the limit $\tau \to \tau_\infty$ as given in equation (3.54). We will discuss these features in next two sections.

3.6 Intersecting Branes

We now analyse the evolution of the universe dominated by mutually BPS N intersecting brane configurations of M theory. The number of spacetime dimensions $D = 11$. We describe general case first, then we specialise on $22'55'$ configuration. The equations of state are assumed to be given by $p_{iI} = (1 - u_i^I)\, \rho_I$ where, as a consequence of U duality symmetries, u_i^I are parametrised in terms of one constant u . The indices $i, j, \cdots$ run from 1 to 10 and the indices $I, J, \cdots$ from 1 to N . For 2 branes, 5 branes, and waves, $N = 1$ and the corresponding u_i^I are given in equations (3.27). For $22'55'$ configuration, $N = 4$ and the corresponding u_i^I are given in equations (3.7).

3.6.1   Evolution Equations

The evolution of λ^i describing the scale factors is given by the equations described earlier which, for ease of reference, we summarise below:

$$\lambda^i_{\tau\tau} = \sum_J u^{iJ} \, e^{l^J} \tag{3.68}$$

$$l^I_{\tau\tau} = \sum_J \mathcal{G}^{IJ} \, e^{l^J} \tag{3.69}$$

$$\lambda^i = \sum_J u^i_J \, (l^J - l^J_0) + L^i \, \tau \tag{3.70}$$

where

$$u^{iI} = \sum_j G^{ij} \, u^I_j \quad , \quad \mathcal{G}^{IJ} = \sum_{i,j} G^{ij} \, u^I_i \, u^J_j \quad , \quad u^i_I = \sum_{j,J} \mathcal{G}_{IJ} \, G^{ij} \, u^J_j \tag{3.71}$$

with G^{ij} and $\mathcal{G}_{IJ}$ as defined earlier, and L^i are arbitrary constants satisfying the constraints $\sum_i u^I_i L^i = 0$ for all I. Also, l^I_τ obey the constraint

$$\sum_{I,J} \mathcal{G}_{IJ} \, l^I_\tau \, l^J_\tau = 2 \, (E + \sum_J e^{l^J}) \tag{3.72}$$

where $2E = - \sum_{i,j} G_{ij} L^i L^j$. Equations (3.69) and (3.72) are to be solved for $l^I(\tau)$ with initial conditions $l^I(0) = l^I_0 = ln \, \rho_{I0}$ and $l^I_\tau(0) = K^I$ where ρ_{I0} are initial densities and

$$\sum_{I,J} \mathcal{G}_{IJ} \, K^I \, K^J = 2 \, (E + \sum_J e^{l^J_0}) \; . \tag{3.73}$$

Then $\lambda^i(\tau)$ follow from equation (3.70) and the physical time $t(\tau)$ from $dt = e^\Lambda \, d\tau$. Inverting $t(\tau)$ then gives $\tau(t)$, and thereby $\lambda^i(t)$.

We can now calculate $\mathcal{G}^{IJ}$ for the mutually BPS intersecting brane configurations. As explained in footnote **2**, in the BPS configurations two stacks of 2 branes intersect at a point; two stacks of 5 branes intersect along three common spatial directions; a stack of 2 branes intersects a stack of 5 branes along one common spatial direction; and, waves, if present, will be along a common intersection direction. With these rules given, it is now straightforward to calculate $\mathcal{G}^{IJ}$ using

equations (3.27) and (3.71). It turns out because of the BPS intersection rules that the resulting $\mathcal{G}^{IJ}$ are given by

$$\mathcal{G}^{IJ} = 2u^2 \left(1 - \delta^{IJ}\right) \ . \tag{3.74}$$

The corresponding $\mathcal{G}_{IJ}$ exists for $N > 1$, and is given by

$$\mathcal{G}_{IJ} = \frac{1}{2u^2} \left(\frac{1}{N-1} - \delta_{IJ}\right) \ . \tag{3.75}$$

Note that, for $N > 1$, the above $\mathcal{G}^{IJ}$ is a special case of the example considered earlier in equation (3.62), now with $a = 2u^2$ and $b = 1$,

3.6.2 $22'55'$ **Case**

It is also straightforward to calculate u^{iI} and u_I^i for the $22'55'$ configuration using the definitions in equation (3.71) and the u_i^I in equation (3.7). They are given by

$$
\begin{array}{rrl}
2 & : & u^{i1} \propto (-2, -2, \ 1, \ 1, \ 1, \ \ 1, \ \ 1, \ 1, \ 1, \ 1) \\
2' & : & u^{i2} \propto (\ 1, \ \ 1, -2, -2, \ 1, \ \ 1, \ \ 1, \ 1, \ 1, \ 1) \\
5 & : & u^{i3} \propto (-1, \ \ 2, -1, \ \ 2, -1, -1, -1, \ 2, \ 2, \ 2) \\
5' & : & u^{i4} \propto (\ 2, -1, \ \ 2, -1, -1, -1, -1, \ 2, \ 2, \ 2)
\end{array}
\tag{3.76}
$$

where the proportionality constant is $\frac{u}{3}$, and by

$$
\begin{array}{rrl}
2 & : & u_1^i \propto (\ 2, \ \ 2, -1, -1, -1, -1, -1, \ 1, \ 1, \ 1) \\
2' & : & u_2^i \propto (-1, -1, \ \ 2, \ \ 2, -1, -1, -1, \ 1, \ 1, \ 1) \\
5 & : & u_3^i \propto (\ 1, -2, \ \ 1, -2, \ \ 1, \ \ 1, \ \ 1, \ 0, \ 0, \ 0) \\
5' & : & u_4^i \propto (-2, \ \ 1, -2, \ \ 1, \ \ 1, \ \ 1, \ \ 1, \ 0, \ 0, \ 0)
\end{array}
\tag{3.77}
$$

where the proportionality constant is $\frac{1}{6u}$.

We are unable to solve equations (3.69), (3.72), and (3.74) for $N > 1$.[9] How-

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[9] In the case of black holes, the equations of motion for the corresponding harmonic functions $H^I = 1 + \frac{Q^I}{r} \equiv e^{\tilde{h}_I}$ can also be written in a form similar to that of equation (3.69). The main steps are indicated in Appendix **2.6**. The analogous $\mathcal{G}^{IJ}$ in the black hole case turns out to be $\propto \delta^{IJ}$, and the equations can then be solved.

ever, applying the general analysis described in section **3.5** and making further use of the explicit forms of u_i^I and $\mathcal{G}^{IJ}$ given in equations (3.7) and (3.74), one can understand the qualitative features of the evolution of the $22'55'$ configuration.

We first make several remarks which will lead to an immediate understanding of the evolution of this configuration.

(1) Let $u_i = \sum_I u_i^I$. It can then be checked that $\sum_{i,j} G^{ij} u_i u_j > 0$. Also, $\sum_i u_i L^i = 0$ since $\sum_i u_i^I L^i = 0$ for all I . Hence, as shown in Appendix **C**, it follows that E given in equation (3.72) is ≥ 0 and that it vanishes if and only if L^i all vanish.

(2) The constraints $\sum_i u_i^I L^i = 0$ imply that

$$L^1 - L^4 = L^2 - L^3 = L^5 + L^6 + L^7 = 0$$
$$L^8 + L^9 + L^{10} = -3 \left(L^1 + L^2 \right) \ . \tag{3.78}$$

Thus, for example, we may take $(L^1, L^2, L^6, L^7, L^8, L^9)$ to be independent. The remaining L^is are then determined by the above equations. Also, we have

$$L \equiv \sum_i L^i = -(L^1 + L^2) \ . \tag{3.79}$$

Using equations (3.78), (3.79), and the Schwarz inequality (C.1) in Appendix **C**, we write E as

$$
\begin{aligned}
2E &= \sum_i (L^i)^2 - \left(\sum_i L^i \right)^2 \\
&= 3(L)^2 + \sum_{i=5}^7 (L^i)^2 + 2\sigma_2^2 + \sigma_3^2 \\
&= 3(L^1)^2 + (L^1 + 2L^2)^2 + \sum_{i=5}^7 (L^i)^2 + \sigma_3^2 \tag{3.80}
\end{aligned}
$$

where the first line is the definition of E , $\sigma_2 = 0$ if and only if $L^1 = L^2$, and

---

Also, note that if $L^i = 0$ for all i then λ^i in equation (3.70) here may be written as in equation (2.86) in Appendix **2.6**. The role of $\tilde{h}_I$ there is played by the functions $2uh_I = 2u \sum_J \mathcal{G}_{IJ} (l^J - l_0^J)$ here. Such a similarity is present for other intersecting brane configurations also.

$\sigma_3 = 0$ if and only if $L^8 = L^9 = L^{10}$. See the Schwarz inequality given in equation (C.1). It is easy to show that the above expressions for E imply that $(L^i)^2$ for all i are bounded above by E as follows: $E \geq c_i (L^i)^2 \geq 0$ where c_i are constants of $\mathcal{O}(1)$. In particular, note the inequality $2E \geq 3(L)^2$ which is required in Appendix **D**.

(3) It follows from equations (3.70), (3.77), and (3.79) that

$$\Lambda_\tau = \sum_i \lambda_\tau^i = \frac{1}{6u} (2l_\tau^1 + 2l_\tau^2 + l_\tau^3 + l_\tau^4) + L \quad. \tag{3.81}$$

Using the explicit form of $\mathcal{G}_{IJ}$ given in equation (3.75) with $N = 4$, equation (3.72) becomes

$$\left(\sum_I l_\tau^I\right)^2 - 3 \sum_I (l_\tau^I)^2 = 12u^2 \left(E + \sum_I e^{l^I}\right) > 0 \tag{3.82}$$

where the inequality follows since $E \geq 0$ and $e^{l^I} > 0$. We show in Appendix **D** that this inequality implies that none of (Λ_τ, l_τ^I) may vanish, and that they must all have same sign. Hence, for all τ throughout the evolution, (Λ_τ, l_τ^I) must all be non vanishing, and be all positive or all negative.

3.7 Asymptotic Evolution

With no loss of generality, let $\Lambda_t > 0$ initially at $t = t_0$. Then it follows from the above result that (Λ_τ, l_τ^I) must all be positive and non vanishing for all τ. Hence, (Λ, l^I) are all monotonically increasing functions for all τ throughout the evolution.

Equation (3.69) may be written, using equation (3.74), as

$$l_{\tau\tau}^I = 2u^2 \sum_{J \neq I} e^{l^J} \quad. \tag{3.83}$$

In the past, τ and all l^I decrease continuously. Hence, the right hand side in equation (3.83) becomes more and more negligible. The asymptotic solution in the limit $\tau \to -\infty$ is then given by $l^I = \tilde{c}^I \tau + \tilde{d}^I$ where $\tilde{c}^I > 0$. Thus $e^{l^I} \to 0$ in this

limit.

Similarly, in the future, τ and all l^I increase continuously. However, the right hand side in equation (3.83) increases exponentially now. It is then obvious that all $e^{l^I} \to \infty$ within a finite interval of τ, *i.e.* at a finite value τ_∞ of τ. In this context, see equations (3.47) and (3.48), and the general analysis given in asymptotic evolution in section **3.5.2.2**.

We now analyse the corresponding asymptotic solutions.

3.7.1 Asymptotic Evolution: $e^\Lambda \to 0$

It follows from the above discussion that $e^\Lambda \to 0$ in the limit $\tau \to -\infty$. Also, in this limit, we have

$$e^{l^I} \;=\; e^{\tilde{c}^I \tau} \;=\; t^{\tilde{b}^I} \quad , \quad e^{\lambda^i} \;=\; e^{\tilde{c}^i \tau} \;=\; t^{\tilde{b}^i} \tag{3.84}$$

up to multiplicative constants where $(\tilde{c}^I,\ \tilde{c}^i,\ \tilde{b}^I,\ \tilde{b}^i)$ are constants. The evolution is then of Kasner type and is similar to that described in section **3.5.2.1**. The constants $\tilde{c}^I$s are determined by the initial values l_0^I and K^I, but obtaining the exact dependence in the general case requires complete solution for $l^I(\tau)$. However, if the initial values l_0^I are large and negative then we have $e^{l^I} \ll 1$ for all $\tau < 0$ and, hence, $\tilde{c}^I = K^I$ to a good approximation.

3.7.2 Asymptotic Evolution: $e^\Lambda \to \infty$

It follows from the above discussion that $e^\Lambda \to \infty$ in the limit $\tau \to \tau_\infty$ where τ_∞ is finite. Also, $e^{l^I} \to \infty$ in this limit and τ_∞ depends on the initial values l_0^I and K^I.

Although solutions for $l^I(\tau)$ are not known, their asymptotic forms in the limit $\tau \to \tau_\infty$, and hence those of $\lambda^i(\tau)$, may be obtained following the analysis given in section **3.5.2.2**. $\mathcal{G}^{IJ}$ in equation (3.74) is a special case of the example (3.62) where, now, $N = 4$, $a = 2u^2$, and $b = 1$. Hence, it can be shown to correspond to the possibility **(i)** given in equation (3.60). Therefore, we have $\gamma^I = 1$ and $e^{c^I} = 2 \sum_J \mathcal{G}_{IJ} = \frac{1}{3u^2}$.

It then follows from equation (3.54) that e^{l^I} and e^{λ^i} are given in the limit

$\tau \to \tau_\infty$ by

$$e^{l^I} = \frac{1}{3u^2} \frac{1}{(\tau_\infty - \tau)^2} \tag{3.85}$$

$$e^{\lambda^i} = e^{v^i} \left(\frac{1}{3u^2} \frac{1}{(\tau_\infty - \tau)^2} \right)^{\sum_I u^i_I} \tag{3.86}$$

where, since $\rho_{I0} = e^{l^I_0}$, we have

$$v^i = -\sum_J u^i_J \, l^J_0 + L^i \, \tau_\infty \ , \qquad e^{v^i} = e^{L^i \, \tau_\infty} \prod_J (\rho_{J0})^{-u^i_J} \ . \tag{3.87}$$

Also, since $\gamma = \sum_{i,I} u^i_I = \frac{1}{u}$, we have from equation (3.56) that the physical time t is given in this limit by

$$t - t_s = A \, (\tau_\infty - \tau)^{-\frac{2-u}{u}} \tag{3.88}$$

where t_s and A are finite constants. Clearly, $t \to \infty$ in the limit $\tau \to \tau_\infty$ since it is assumed that $0 < u < 2$. In this limit, the scale factors e^{λ^i} may be written in terms of t as

$$e^{\lambda^i} = e^{v^i} \, (B \, t)^{\beta^i} \tag{3.89}$$

where B is a constant and $\beta^i = \frac{2u}{2-u} \sum_J u^i_J$. Using equation (3.77) for u^i_I , the exponents β^i are given by

$$\beta^i \propto (0, \ 0, \ 0, \ 0, \ 0, \ 0, \ 0, \ 1, \ 1, \ 1) \tag{3.90}$$

where the proportionality constant is $\frac{2}{3(2-u)}$. Note that $\beta = \sum_i \beta^i = \frac{2}{2-u} > 1$. Hence, we have $e^\Lambda \sim t^\beta$ in the limit $t \to \infty$ with $\beta > 1$. See the discussion below equation (3.19) for the relevance of this feature.

Thus, asymptotically in the limit $t \to \infty$, we obtain that $e^{\lambda^i} \to t^{\frac{2}{3(2-u)}}$ for the common transverse directions $i = 8, 9, 10$. Hence, these directions continue to expand, their expansion being precisely that of a $(3+1)$ – dimensional homogeneous, isotropic universe containing a perfect fluid whose equation of state is $p = (1-u) \, \rho$. Also, $e^{\lambda^i} \to e^{v^i}$ for the brane directions $i = 1, \cdots, 7$. Hence, these directions cease to expand or contract. Their sizes are thus stabilised and are given by e^{v^i} . Note that this result is in accord with the general result described

in section **3.3** since, in the limit $\tau \to \tau_\infty$, the brane densities $\rho_I \propto e^{l^I}$ all become equal and hence the four types of branes all become identical; and, $t \to \infty$ and $e^\Lambda \sim t^\beta \to \infty$ with $\beta > 1$.

3.8   Mechanism of Stabilisation

Using the asymptotic solutions, we can now give a physical interpretation of the dynamical mechanism underlying the stabilisation of the brane directions seen above for the $22'55'$ configuration.

We first study the stabilisation process. Consider equation (3.68) for $\lambda^1_{\tau\tau}$, for example. Using the values of u^{iI} given in equation (3.76), we have

$$\lambda^1_{\tau\tau} \propto (-2e^{l^1} + e^{l^2} - e^{l^3} + 2e^{l^4}) \ . \tag{3.91}$$

In the $22'55'$ configuration, x^1 direction is wrapped by 2 branes and 5 branes and is transverse to $2'$ branes and $5'$ branes. Thus, from the above equation for λ^1 and from similar equations for $\lambda^2, \cdots, \lambda^7$, we see that 2 brane and 5 brane directions 'contract with a force' proportional to $2\rho_{(2)}$ and $\rho_{(5)}$ respectively, whereas the directions transverse to them 'expand with a force' proportional to $\rho_{(2)}$ and $2\rho_{(5)}$ respectively, where $\rho_{(*)} \propto e^{l^{(*)}}$ are the time dependent densities of the corresponding branes.

When $\rho_I \propto e^{l^I}$ all become equal, the forces of expansion cancel the forces of contraction resulting in vanishing net force for the x^1 direction. Then, using equation (3.38), one has

$$\lambda^1_{\tau\tau} = e^{2\Lambda} (\lambda^1_{tt} + \Lambda_t \lambda^1_t) = 0 \ . \tag{3.92}$$

Now, as described earlier in the context of equations (3.20) and (3.21), the transient 'velocity' λ^1_t is damped and λ^1 reaches a constant value in the expanding universe here since we have $e^\Lambda \sim t^\beta$ in the limit $t \to \infty$ with $\beta > 1$. The result is the stabilisation of the x^1 direction.

The stabilised size e^{v^1} of x^1 direction is given by

$$e^{v^1} = e^{L^1 \tau_\infty} \left(\frac{\rho_{20} \, \rho_{40}^2}{\rho_{30} \, \rho_{10}^2} \right)^{\frac{1}{6u}} \ , \tag{3.93}$$

see equation (3.87). Note that e^{v^1} can be interpreted as arising from the imbalance among the initial brane densities ρ_{I0}, and from the parts L^1 of $\lambda_t^1(0)$ which indicate the transients. The above analysis can be similarly applied to the stabilisation of other brane directions $(x^2, \cdots, x^7)$ in the $22'55'$ configuration.

Thus, three conditions need to be satisfied for stabilisation: **(1)** the time dependent brane densities $\rho_I \propto e^{l^I}$ all become equal; **(2)** the forces of expansion and contraction for each of the brane directions be just right so that the net force vanishes; **(3)** the universe be expanding as $e^\Lambda \sim t^\beta$ in the limit $t \to \infty$ with $\beta > 1$ so that the transient velocities are damped and the corresponding scale factors reach constant values.

For any mutually BPS $N > 1$ intersecting brane configurations with the equations of state as assumed here, it is straightforward to show using the earlier analysis that the evolution equations ensure that e^{l^I} all become equal asymptotically even if they were unequal initially, and that $e^\Lambda \sim t^\beta$ in the limit $t \to \infty$ with $\beta > 1$. Thus conditions **(1)** and **(3)** are satisfied. Condition **(2)** requires the brane configuration to be such that each of the brane directions is wrapped by, and is transverse to, just the right number and kind of branes. This condition is satisfied for the $N = 4$ configurations $22'55'$ and $55'5''W$, both of which result in the stabilisation of seven brane directions and the expansion of the remaining three spatial directions. To our knowledge, the only other configurations which satisfy the condition **(2)** are the $N = 3$ configurations $22'2''$ and $25W$, both of which result in the stabilisation of six brane directions and the expansion of the remaining four spatial directions [32]. However it is the $N = 4$ configurations that are entropically favourable, see equation (1.6).

Note that, as described in section **3.3** and up to certain technical assumptions regarding the equality of brane densities and the asymptotic behaviour of e^Λ, the stabilisation here follows essentially as a consequence of U duality symmetries. In particular, it is independent of the ansatz for energy momentum tensors, or of the assumptions about equations of state, as long as the components of the energy momentum tensors obey the U duality constraints of the type given in equation (3.19). Obtaining the details of the stabilisation, however, requires further assumptions *e.g.* of the type made here.

Note also that the present mechanism of stabilisation of seven brane directions, and the consequent emergence of three large spatial directions, is very different

from the ones proposed in string theory or in brane gas models [37, 38, 39, 40].

3.9   Stabilised Sizes of Brane Directions

We thus see for the $22'55'$ configuration that, asymptotically in the limit $e^\Lambda \to \infty$, the initial $(10+1)$ – dimensional universe effectively becomes $(3+1)$ – dimensional. Also, if $v^s = min\{v^1, \cdots, v^7\}$ then a dimensional reduction of the $(10+1)$ – dimensional M theory along the corresponding x^s direction gives type IIA string theory with its dilaton now stabilised. Using the standard relations, one can obtain the string coupling constant g_s , the string scale M_s , and the four dimensional Planck scale M_4 in terms of the M theory scale M_{11} and the stabilised values e^{v^i} . Defining $v^c = \sum_{i=1}^{7} v^i$ and assuming, with no loss of generality, that the coordinate sizes of all spatial directions are of $\mathcal{O}(M_{11}^{-1})$, we obtain

$$g_s^2 = e^{3v^s} \quad , \qquad M_4^2 = e^{v^c - v^s} M_s^2 = e^{v^c} M_{11}^2 \qquad (3.94)$$

where the equalities are valid up to numerical factors of $\mathcal{O}(1)$ only and

$$e^{v^c} = e^{L^c \tau_\infty} \left(\frac{\rho_{10} \; \rho_{20}}{\rho_{30} \; \rho_{40}} \right)^{\frac{1}{6u}} \quad , \qquad L^c = \sum_{i=1}^{7} L^i \qquad (3.95)$$

as follows from equations (3.77), (3.87), and $\rho_{I0} = e^{l_0^I}$. Also, note that $g_s = \left(\frac{M_s}{M_{11}}\right)^3$.

Since we have an asymptotically $3 + 1$ dimensional universe evolving from a $10 + 1$ dimensional one, it is of interest to study the resulting ratios $\frac{M_{11}}{M_4}$ and $\frac{M_s}{M_4}$, and study their dependence on the initial values (l_0^I, K^I, L^i) . In particular, one may like to know the generic values of these ratios and to know whether arbitrarily small values are possible. Setting $M_4 = 10^{19} \; GeV$, one then knows the generic scales of M_{11} and M_s and, for example, whether $M_{11} = 10^{-15} \, M_4 = 10 \; TeV$ is possible.

In view of the relations between (M_{11}, M_s, M_4) given in equation (3.94), this requires studying the stabilised values e^{v^c} and $e^{v^c - v^s}$, their dependence on (l_0^I, K^I, L^i) , and knowing whether they can be arbitrarily large. Note that if $L^i = 0$ for all i then v^i are all determined in terms of l_0^I only, see equation

(3.87). It is then obvious from equations (3.87) and (3.95) that any values for e^{v^c} and $e^{v^c - v^s}$, no matter how large, may be obtained by fine tuning ρ_{I0} correspondingly. [10]

This statement remains true even when L^is do not all vanish. In this case, however, one may question the necessity of fine tuning since, for example, the relation $e^{v^c} \propto e^{L^c \tau_\infty}$ suggests that large values such as $10^{30} \sim e^{70}$ may be obtained by tuning L^is, or τ_∞ , or both to within a couple of orders of magnitude only. It turns out, as we explain below, that fine tuning is still necessary to obtain such large values.

Consider first the possibility of tuning L^i . Note that equations (3.69) and (3.72) are invariant under the scaling

$$(E, \ e^{l^I}, \ \tau) \quad \longrightarrow \quad (\sigma^2 E, \ \sigma^2 e^{l^I}, \ \frac{\tau}{\sigma}) \tag{3.96}$$

where σ is a positive constant. The initial values scale correspondingly as

$$(e^{l_0^I}, \ K^I, \ L^i) \quad \longrightarrow \quad (\sigma^2 e^{l_0^I}, \ \sigma K^I, \ \sigma L^i) \ . \tag{3.97}$$

It then follows from equation (3.70) that λ^i, and hence e^{v^i} , remain invariant.[11] This scaling property merely reflects the choice of a scale for time. For example, using this scaling, one may set $\sum_J e^{l_0^J} = 1$ or, when $E > 0$ as is the case here, set $E = 1$. The corresponding σ then provides a natural time scale for evolution. We set $E = 1$ using the above scaling.

With $E = 1$, the value of τ_∞ now depends only on l_0^I and K^I . Since $2E = \sum_i (L^i)^2 - (\sum_i L^i)^2$, it is still plausible to have a range of non zero measure where L^i are large and $E = 1$, and thereby obtain large values for e^{v^c} and $e^{v^c - v^s}$. However, L^is are further constrained by $\sum_i u_i^I L^i = 0$, $I = 1, \cdots, 4$, and consequently their magnitudes are all bounded from above. For example, with $E = 1$, we obtain $(L^c)^2 \leq \frac{8}{3}$. See remark **(2)** in section **3.6.1**. Thus, large values of e^{v^i} cannot be obtained by tuning L^i alone.

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[10] It follows from equation (3.73) and the definition of E that the generic ranges of the initial values may be taken to be given by $|L^i| \simeq K^I \simeq \sqrt{E} \simeq \sqrt{\rho_{I0}}$ within a couple of orders of magnitude. If the initial values lie way beyond such a range then we consider it as fine tuning.

[11] This invariance is equivalent to that of equations (3.30) and (3.31) under the scaling $(\lambda^i, \rho, p_i, t) \to (\lambda^i, \sigma^2 \rho, \ \sigma^2 p_i, \ \frac{t}{\sigma})$.

Consider now the possibility of tuning τ_∞ . Obtaining the dependence of τ_∞ on (l_0^I, K^I) requires explicit solutions which are not available. Hence, we obtain τ_∞ numerically. We will present the numerical results in the next section. Here we point out that an approximate expression for τ_∞ can be given in the limit when $e^{l_0^I} \ll E$ for all I . The reasoning involved is analogous to that used in obtaining τ_a in equation (3.51). Using similar reasoning and setting $E = 1$ now, we have that if $e^{l_0^I} \ll 1$ for all I then

$$\tau_\infty \simeq \tau_a = \ min\ \{\tau_I\}\ , \quad \tau_I = - \ \frac{l_0^I}{K^I}\ . \tag{3.98}$$

Note that τ_a can be calculated easily and requires no knowledge of explicit solutions. Our numerical results show that τ_a given above indeed provides a good approximation to τ_∞ when $e^{l_0^I} \ll 1$ for all I .

Note also that K^I must satisfy equation (3.73) with $E = 1$. It then follows from an analysis similar to that given in Appendix **D** that K^I are all positive, cannot be too small, and are of $\mathcal{O}(1)$ generically. Hence, in the limit $e^{l_0^I} \ll 1$ for all I, τ_a in equation (3.98) are of $\mathcal{O}(min\{-l_0^I\})$. This indicates that large values of τ_∞ , and hence of e^{v^i} , cannot be obtained by tuning K^I alone; a tuning of l_0^I , which translates to fine tuning of $\rho_{I0} = e^{l_0^I}$, is required. Our numerical analysis also supports this conclusion.

We thus find that, even when L^is do not all vanish, a fine tuning of $\rho_{I0} = e^{l_0^I}$ is necessary to obtain large values for $e^{v^c - v^s}$ and e^{v^c} . We will see some example in section **3.12**.

3.10 Discussion with Other Intersecting Configurations

In this section we will discuss some intersecting configuration which are not $22'55'$ distributed in said way. We mentioned all our assumption in last section. There we say our $22'55'$ is actually a fine tuned initial condition, and how this configuration comes out naturally form M theory is not understood. To discuss this point we here give some example configuration where stabilisation of 7 directions is not achieved.

Even $22'55'$ branes intersecting in some other way following BPS intersection rule may not produces required stabilisation. As an example consider following configuration:

$$2 : -,-,-,-,-,-,\times,\times,-,-$$
$$2' : \times,\times,-,-,-,-,-,-,-,-$$
$$5 : \times,-,\times,\times,\times,-,-,\times,-,-$$
$$5' : -,\times,\times,\times,-,\times,-,\times,-,-$$

where $\times$ denotes brane direction and $-$ direction are perpendicular to brane. For this configuration general analysis of section **3.5** gives

$$\beta^i = \frac{1}{6 - 3u} \{0,0,0,0,1,1,1,-1,2,2\} \ . \tag{3.99}$$

So one can see here only x^1, x^2, x^3 and x^4 get stabilised. This configuration is different from ours. Here stabilisation is achieved only in the directions where exactly two branes are presents. But in our $22'55'$ case exactly 2 sets of branes are present in all 7 directions. Similarly we can also do the analysis for $N > 4$ case. None of these cases stabilisation of 7 directions occurs. As discussed in section **3.5**, using equations (3.58), (3.60), (3.61) and (3.62) one can find general formula for β^i. In our case all $\gamma^I = 1$. So in this case if we define

$$x = \sum_{i,I} u_I^i \ ,$$

then β^i will be given by

$$\beta^i = \frac{2}{2x - 1} \sum_I u_I^i \ , \tag{3.100}$$

where u_I^i are defined in equation (3.44). u_i^I's depend on particular intersection. For our ansatz of: T_{AB}, mentioned in section **3.4**, u_i^I's are given by equation (3.27).

All possible configurations are given for example in [63]. In Table **3.2**, **3.3**, **3.4**, **3.5** and **3.6** we list a few intersecting M brane configurations and their corresponding late time evolution. We use notation of [63], which is following: in square bracket numbers of each type of branes are indicated. For example $[2^n, 5^m]$ means

n number of 2 branes and m number of 5 branes. In curly bracket how many branes are there in each direction is indicated. Position of the number indicates number of branes and corresponding number indicates number of directions. For example $\{p, q, r, s, \ldots\}$ means there is only 1 brane in p directions, there are 2 branes presents in q directions and so on. For example take $([2^2, 5^2], \{3, 4, 1, 0\})$. This is the configuration we discuss in last page. This means 2 sets of 2 branes and 2 sets of 5 branes. Then $\{3, 4, 1, 0\}$) denotes following: 1)there are 1 brane in 3 directions, in this case x^5, x^6 and x^7. 2) There are 2 branes in 4 directions, namely x^1, x^2, x^3 and x^4. 3) There are 3 branes in 1 direction, namely x^8. 4) And there is no direction which is populated by 4 branes. We also list the configurations explicitly for the shake of reader's convenience.

We first give some example for $N \leq 3$ case in Table **3.1** and **3.2**. Next we give some example for $N = 4$ in Table **3.3**. Finally we give example of $N > 4$ in table **3.4**, **3.5** and **3.6**.

Type of intersection	β^i
$[2^2], \{4, 0\}$	$\frac{1}{8-3u}\{-1, -1, -1, -1, 2, 2, 2, 2, 2, 2\}$
$[2^1, 5^1], \{5, 1\}$	$\frac{1}{3-u}, \{-1, 0, 0, 0, 0, 0, 1, 1, 1, 1\}$
$[5^2], \{4, 3\}$	$\frac{2}{10-3u}\{-2, -2, -2, 1, 1, 1, 1, 4, 4, 4\}$

Table 3.1: *Table for various type of intersection vs β^i, for $N = 2$.*

The explicit configuration mentioned in above table, **3.1** given below:

$$[2^2], \{4, 0\}$$

$$2 : \times, \times, -, -, -, -, -, -, -, -$$

$$2' : -, -, \times, \times, -, -, -, -, -, -$$

$$[2^1, 5^1], \{5, 1\}$$

$$2 : \times, \times, -, -, -, -, -, -, -, -$$

$$5 : \times, -, \times, \times, \times, \times, -, -, -, -$$

$$[5^2], \{4, 3\}$$

$$5 : \times, \times, \times, \times, \times, -, -, -, -, -$$

$$5' : \times, \times, \times, -, -, \times, \times, -, -, -$$

Type of intersection	β^i
$[5^3], \{0, 6, 1\}$	$\frac{1}{5-2u}\{-1, 0, 0, 0, 0, 0, 0, 2, 2, 2\}$
$[5^3], \{6, 0, 3\}$	$\frac{1}{5-2u}\{1, 1, 1, 1, 1, 1, 1, 1, 1, 2\}$
$[2^1, 5^2], \{5, 2, 1\}$	$\frac{5}{14-6u}\{-4, 2, -2, -2, 2, 2, 2, 2, 5, 5\}$
$[2^1, 5^2], \{2, 5, 0\}$	$\frac{1}{14-6u}\{-1, -1, -1, -1, -1, 2, 2, 5, 5, 5\}$
$[2^3], \{6, 0, 0\}$	$\frac{1}{4-2u}\{0, 0, 0, 0, 0, 0, 1, 1, 1, 1\}$

Table 3.2: *Table for various type of intersection vs β^i, for $N = 3$.*

The explicit configuration mention in above table, **3.2** given below:

$$[5^3], \{0, 6, 1\}$$

$$5 : \times, \times, \times, \times, \times, -, -, -, -, -$$

$$5' : \times, \times, \times, -, -, \times, \times, -, -, -$$

$$5'' : \times, -, -, \times, \times, \times, \times, -, -, -$$

$$[5^3], \{6, 0, 3\}$$

$$5 : \times, \times, \times, \times, \times, -, -, -, -, -$$

$$5' : -, -, \times, \times, \times, \times, \times, -, -, -$$

$$5'' : -, -, \times, \times, \times, -, -, \times, \times, -$$

$$[2^1, 5^2], \{5, 2, 1\}$$

$$2 : \times, \times, -, -, -, -, -, -, -, -$$

$$5' : \times, -, \times, \times, \times, \times, -, -, -, -$$

$$5'' : \times, -, \times, \times, -, -, \times, \times, -, -$$

$$[2^1, 5^2], \{2, 5, 0\}$$

$$2 : \times, \times, -, -, -, -, -, -, -, -$$

$$5' : \times, -, \times, \times, \times, \times, -, -, -, -$$

$$5'' : -, \times, \times, \times, \times, -, \times, -, -, -$$

$$[2^3], \{6, 0, 0\}$$

$$2 : \times, \times, -, -, -, -, -, -, -, -$$

$$2' : -, -, \times, \times, -, -, -, -, -, -$$

$$2'' : -, -, -, -, \times, \times, -, -, -, -$$

Type of intersection	β^i
$[5^4], \{3, 3, 1, 2\}$	$\frac{1}{20-9u}\{-1, -4, -4, 2, 5, 2, 5, 2, 5, 8\}$
$[5^4], \{4, 0, 4, 1\}$	$\frac{1}{20-9u}\{-4, -1, -1, -1, 5, -1, 5, 5, 5, 8\}$
$[5^4], \{0, 6, 0, 2\}$	$\frac{2}{20-9u}\{-2, -2, 1, 1, 1, 1, 1, 1, 4, 4\}$
$[2^1, 5^3], \{2, 3, 3, 0\}$	$\frac{1}{19-9u}\{-2, 1, -2, -2, 1, 4, 1, 4, 7, 7\}$
$[2^2, 5^2], \{0, 7, 0, 0\}$	$\frac{2}{6-3u}\{0, 0, 0, 0, 0, 0, 0, 1, 1, 1\}$
$[2^2, 5^2], \{3, 4, 1, 0\}$	$\frac{1}{6-3u}\{\frac{1}{6-3u}\{0, 0, 0, 0, 1, 1, 1, -1, 2, 2\}$
$[2^2, 5^2], \{6, 1, 2, 0\}$	$\frac{1}{6-3u}\{-1, 1, -1, 1, 0, 1, 1, 1, 1, 2\}$

Table 3.3: *Table for various type of intersection vs β^i, for $N = 4$.*

The explicit configuration mentioned in above table, **3.3** given below:

$$[5^4], \{3, 3, 1, 2\}$$

$$5 : \times, \times, \times, \times, \times, -, -, -, -, -$$

$$5' : \times, \times, \times, -, -, \times, \times, -, -, -$$

$$5'' : \times, \times, \times, -, -, -, -, \times, \times, -$$

$$5''' : -, \times, \times, \times, -, \times, -, \times, -, -$$

$$[5^4], \{4, 0, 4, 1\}$$

$5 : \times, \times, \times, \times, \times, -, -, -, -, -$

$5' : \times, \times, \times, -, -, \times, \times, -, -, -$

$5'' : \times, -, \times, \times, -, \times, -, \times, -, -$

$5''' : \times, \times, -, \times, -, \times, -, -, \times, -$

$$[5^4], \{0, 6, 0, 2\}$$

$5 : \times, \times, \times, \times, \times, -, -, -, -, -$

$5' : \times, \times, \times, -, -, \times, \times, -, -, -$

$5'' : \times, \times, -, \times, -, \times, -, \times, -, -$

$5''' : \times, \times, -, -, \times, -, \times, \times, -, -$

$$[2^1, 5^3], \{2, 3, 3, 0\}$$

$2 : \times, \times, -, -, -, -, -, -, -, -$

$5 : \times, -, \times, \times, \times, \times, -, -, -, -$

$5' : -, \times, \times, \times, \times, -, \times, -, -, -$

$5' : \times, -, \times, \times, -, -, \times, \times, -, -$

$$[2^2, 5^2], \{0, 7, 0, 0\} \ ^{12}$$

$2 : \times, \times, -, -, -, -, -, -, -, -$

$2' : -, -, \times, \times, -, -, -, -, -, -$

$5 : \times, -, \times, -, \times, \times, \times, -, -, -$

$5' : -, \times, -, \times, \times, \times, \times, -, -, -$

---

[12]This configuration is our configuration.

$$[2^2, 5^2], \{3, 4, 1, 0\}$$

$$2 : -, -, -, -, -, -, \times, \times, -, -$$

$$2' : \times, \times, -, -, -, -, -, -, -, -$$

$$5 : \times, -, \times, \times, \times, -, -, \times, -, -$$

$$5' : -, \times, \times, \times, -, \times, -, \times, -, -$$

$$[2^2, 5^2], \{6, 1, 2, 0\}$$

$$2 : \times, \times, -, -, -, -, -, -, -, -$$

$$2' : -, -, \times, \times, -, -, -, -, -, -$$

$$5 : \times, -, \times, -, \times, \times, \times, -, -, -$$

$$5' : \times, -, \times, -, \times, -, -, \times, \times, -$$

Type of intersection	β^i
$[5^5], \{0, 2, 4, 1, 1\}$	$\frac{1}{25-12u}\{-5, -2, 1, 4, 1, 1, 4, 1, 10, 10\}$
$[2^1, 5^4], \{0, 4, 2, 2, 0\}$	$\frac{3}{8-4u}\{0, 0, -1, -1, 1, 1, 1, 1, 3, 3\}$
$[2^2, 5^3], \{1, 3, 4, 0, 0\}$	$\frac{1}{23-12u}\{-1, 2, 2, -1, -1, -1, 2, 5, 8, 8\}$
$[2^3, 5^2], \{4, 3, 2, 0, 0\}$	$\frac{1}{22-12u}\{-2, -4, -2, 4, 1, 1, 1, 4, 4, 7\}$

Table 3.4: *Table for various type of intersection vs β^i, for $N = 5$.*

The explicit configuration mentioned in above table, **3.4** given below:

$$[5^5], \{0, 2, 4, 1, 1\}$$

$$5 : \times, \times, \times, \times, \times, -, -, -, -, -$$

$$5' : \times, \times, \times, -, -, \times, \times, -, -, -$$

$$5'' : \times, \times, -, \times, -, \times, -, \times, -, -$$

$$5''' : \times, \times, -, -, \times, -, \times, \times, -, -$$

$$5'''' : \times, -, \times, -, \times, \times, -, \times, -, -$$

$$[2^1, 5^4], \{0, 4, 2, 2, 0\}$$

$$2 : \times, \times, -, -, -, -, -, -, -, -$$

$$5 : \times, -, \times, \times, \times, \times, -, -, -, -$$

$$5' : -, \times, \times, \times, \times, -, \times, -, -, -$$

$$5'' : \times, -, \times, \times, -, -, \times, \times, -, -$$

$$5''' : -, \times, \times, \times, -, \times, -, \times, -, -$$

$$[2^2, 5^3], \{1, 3, 4, 0, 0\}$$

$$2 : \times, \times, -, -, -, -, -, -, -, -$$

$$2' : -, -, \times, \times, -, -, -, -, -, -$$

$$5 : \times, -, \times, -, \times, \times, \times, -, -, -$$

$$5' : -, \times, -, \times, \times, \times, \times, -, -, -$$

$$5'' : \times, -, -, \times, \times, \times, -, \times, -, -$$

$$[2^3, 5^2], \{4, 3, 2, 0, 0\}$$

$$2 : \times, \times, -, -, -, -, -, -, -, -$$

$$2' : -, -, \times, \times, -, -, -, -, -, -$$

$$2'' : -, -, -, -, \times, \times, -, -, -, -$$

$$5 : \times, -, \times, -, \times, -, \times, \times, -, -$$

$$5' : \times, -, \times, -, -, \times, \times, -, \times, -$$

Type of intersection	β^i
$[5^6], \{0, 3, 4, 0, 0, 2\}$	$\frac{1}{10-5u}\{-2, -2, 1, 1, 1, 1, 2, 2, 2, 4\}$
$[2^1, 5^5], \{1, 2, 4, 1, 0, 1\}$	$\frac{1}{29-15u}\{-7, -1, 2, 5, 2, 2, 5, 2, 8, 11\}$
$[2^2, 5^4], \{0, 2, 4, 2, 0, 0\}$	$\frac{1}{28-15u}\{1, 1, 1, 1, -2, -2, 4, 4, 10, 10\}$
$[2^3, 5^3], \{2, 3, 3, 1, 0, 0\}$	$\frac{1}{9-5u}\{-1, 2, 1, 0, 0, 1, 0, 1, 2, 3\}$

Table 3.5: *Table for various type of intersection vs β^i, for $N = 6$.*

The explicit configuration mention in above table, **3.5** given below:

$$[5^6], \{0, 0, 4, 3, 0, 1\}$$

$5 : \times, \times, \times, \times, \times, -, -, -, -, -$

$5' : \times, \times, \times, -, -, \times, \times, -, -, -$

$5'' : \times, \times, -, \times, -, \times, -, \times, -, -$

$5''' : \times, \times, -, -, \times, -, \times, \times, -, -$

$5'''' : \times, -, \times, -, \times, \times, -, \times, -, -$

$5''''' : \times, -, -, \times, \times, \times, \times, -, -, -$

$$[2^1, 5^5], \{1, 2, 4, 1, 0, 1\}$$

$2 : \times, -, -, -, -, -, -, -, \times, -$

$5 : \times, \times, \times, \times, \times, -, -, -, -, -$

$5' : \times, \times, \times, -, -, \times, \times, -, -, -$

$5'' : \times, \times, -, \times, -, \times, -, \times, -, -$

$5''' : \times, \times, -, -, \times, -, \times, \times, -, -$

$5'''' : \times, -, \times, -, \times, \times, -, \times, -, -$

$$[2^2, 5^4], \{0, 2, 4, 2, 0, 0\}$$

$2 : \times, \times, -, -, -, -, -, -, -, -$

$2' : -, -, \times, \times, -, -, -, -, -, -$

$5 : \times, -, \times, -, \times, \times, \times, -, -, -$

$5' : -, \times, -, \times, \times, \times, \times, -, -, -$

$5'' : \times, -, -, \times, \times, \times, -, \times, -, -$

$5''' : -, \times, \times, -, \times, \times, -, \times, -, -$

$$[2^3, 5^3], \{2, 3, 3, 1, 0, 0\}$$

$$2 : \times, \times, -, -, -, -, -, -, -, -$$

$$2' : -, -, \times, \times, -, -, -, -, -, -$$

$$2'' : -, -, -, -, \times, \times, -, -, -, -$$

$$5 : \times, -, \times, -, \times, -, \times, \times, -, -$$

$$5' : \times, -, -, \times, -, \times, \times, \times, -, -$$

$$5'' : \times, -, -, \times, \times, -, \times, -, \times, -$$

Type of intersection	β^i
$[5^7], \{0, 0, 0, 7, 0, 0, 1\}$	$\frac{1}{35-18u}\{-7, 2, 2, 2, 2, 2, 2, 2, 14, 14\}$
$[2^3, 5^4], \{0, 0, 6, 2, 0, 0, 0\}$	$\frac{1}{32-18u}\{2, 2, 2, 2, -1, -1, 2, 2, 2, 11, 11\}$

Table 3.6: *Table for various type of intersection vs β^i, for $N = 7$.*

The explicit configuration mentioned in above table, **3.6** given below:

$$[5^7], \{0, 0, 0, 7, 0, 0, 1\}$$

$$5 : \times, \times, \times, \times, \times, -, -, -, -, -$$

$$5' : \times, -, -, \times, \times, \times, \times, -, -, -$$

$$5'' : \times, \times, \times, -, -, \times, \times, -, -, -$$

$$5''' : \times, -, \times, \times, -, \times, -, \times, -, -$$

$$5'''' : \times, \times, -, -, \times, \times, -, \times, -, -$$

$$5''''' : \times, \times, -, \times, -, -, \times, \times, -, -$$

$$5'''''' : \times, -, \times, -, \times, -, \times, \times, -, -$$

$$[2^3, 5^4], \{0, 0, 6, 2, 0, 0, 0\}$$

$$2 : \times, \times, -, -, -, -, -, -, -, -$$

$$2' : -, -, \times, \times, -, -, -, -, -, -$$

$$2'' : -, -, -, -, -, -, \times, \times, -, -$$

$$5 : \times, -, \times, -, \times, \times, \times, -, -, -$$

$$5' : -, \times, -, \times, \times, \times, \times, -, -, -$$

$$5'' : \times, -, -, \times, \times, \times, -, \times, -, -$$

$$5''' : -, \times, \times, -, \times, \times, -, \times, -, -$$

3.11 Time Varying Newton's Constant

The evolution of the eleven dimensional early universe which is dominated by the $22'55'$ configuration described here can also be considered from the perspective of four dimensional spacetime. Indeed, in general, let the eleven dimensional line element ds be given by

$$ds^2 = g_{\mu\nu} \, dx^\mu dx^\nu + \sum_{i=1}^{7} e^{2\lambda^i} (dx^i)^2 \qquad (3.101)$$

where $x^\mu = (x^0, x^8, x^9, x^{10})$, with $x^0 = t$, describes the four dimensional spacetime, and the fields $g_{\mu\nu}$ and λ^i, $i = 1, \cdots, 7$, depend on x^μ only. Also, let $\Lambda^c = \sum_{i=1}^{7} \lambda^i$. It is then straightforward to show that the gravitational part of the eleven dimensional action S_{11} given in equation (2.1) becomes

$$S_4 = \frac{V_7}{16\pi G_{11}} \int d^4x \, \sqrt{-g_{(4)}} \, e^{\Lambda^c} \left\{ R_{(4)} + (\nabla_{(4)} \Lambda^c)^2 - \sum_{i=1}^{7} (\nabla_{(4)} \lambda^i)^2 \right\} \qquad (3.102)$$

where V_7 is the coordinate volume of the seven dimensional space and the subscripts (4) indicate that the corresponding quantities are with respect to the four dimensional metric $g_{\mu\nu}$. The action S_4 describes four dimensional spacetime in which the effective Newton's constant G_4 is spacetime dependent and is given by

$$G_4(x^\mu) = e^{-\Lambda^c(x^\mu)} \, \frac{G_{11}}{V_7} \, . \qquad (3.103)$$

In the case of early universe, the fields $g_{\mu\nu}$ and λ^i depend on t only. Then G_4 is time dependent and we have, for G_4 and its fractional time derivative,

$$G_4(t) = e^{-\Lambda^c(t)} \, \frac{G_{11}}{V_7} \quad , \qquad \frac{(G_4)_t}{G_4} = -\, \Lambda^c_t \ . \tag{3.104}$$

For the four dimensional spacetime arising from the $22'55'$ configuration, the $g_{\mu\nu}$ fields are just the scale factors $(e^{\lambda^8}, e^{\lambda^9}, e^{\lambda^{10}})$ for (x^8, x^9, x^{10}) directions, and all $\lambda^i(\tau)$ are given in equation (3.70) in terms of $l^I(\tau)$, $I = 1, \cdots, 4$. Then, using equation (3.77) and the definitions of Λ^c, L^c, and v^c, we have

$$\Lambda^c = -\,\frac{1}{6u}(l^1 + l^2 - l^3 - l^4) - L^c(\tau_\infty - \tau) + v^c \ . \tag{3.105}$$

In the limit $t \to \infty$, we have from the results given earlier that $\tau \to \tau_\infty$ and the fields l^I all become equal. Then $\Lambda^c \to v^c$ where e^{v^c} is given in equation (3.95), and the three dimensional scale factors evolve as in the standard FRW case, namely $e^{\lambda^8} = e^{\lambda^9} = e^{\lambda^{10}} \sim t^{\frac{2}{3(2-u)}}$ as given in equations (3.89) and (3.90).

It thus follows that the effective Newton's constant G_4 varies with time in the early universe and, in the case of $22'55'$ configuration, approaches a constant value $= e^{-v^c} \frac{G_{11}}{V_7}$ as the four dimensional universe expands to large size. The precise time dependence of G_4 will follow from explicit solutions to equations (3.69) and (3.72). The consequences of a such a time dependent G_4 are clearly interesting, and are likely to be important too. But their study is beyond the scope of the present thesis.

However, we like to point out here a characteristic feature of the time dependence of G_4 which arises in the case of $22'55'$ configuration. Consider the behaviour of the differences $l^I - l^J$ in the limit $\tau \to \tau_\infty$ which, in our case, vanish to the leading order. These quantities have been analysed in section **3.5.3** and, for the example of the $\mathcal{G}^{IJ}$ given in equation (3.62), they are given by equations (3.66) and (3.67) to the non trivial leading order. The case of $22'55'$ configuration corresponds to $N = 4$, $a = 2u^2$, and $b = 1$. Noting that $s^I - s^J = l^I - l^J$ and that $\Delta < 0$ in our case, equation (3.67) now gives

$$(l^I - l^J) \ \sim \ (\tau_\infty - \tau)^{\frac{1}{2}(1 \pm i \sqrt{\frac{5}{3}})} \tag{3.106}$$

to the leading order. Clearly, $\Lambda^c(\tau)$ given in equation (3.105) will also have the same form as above to the non trivial leading order. Thus, taking the real part and writing in terms of t using equation (3.88), we have

$$\Lambda^c = v^c + \frac{b}{t^\alpha} \; Sin(\omega \; ln \; t + \phi) \tag{3.107}$$

to the non trivial leading order in the limit $t \to \infty$ where b and ϕ are constants, $\alpha = \frac{u}{2(2-u)}$, and $\omega = \sqrt{\frac{5}{3}} \; \frac{u}{2(2-u)}$. Correspondingly, the time varying Newton's constant is given by

$$G_4 \; \propto \; e^{-\Lambda^c} \; = \; e^{-v^c} \; (1 - \frac{b}{t^\alpha} \; Sin(\omega \; ln \; t + \phi) \;) \tag{3.108}$$

to the leading order in the limit $t \to \infty$. Note that the constants b and ϕ depend on the details of matching. The constants α and ω arise as real and imaginary parts of an exponent on time variable, see equation (3.106). They do not depend on the initial values (l_0^I, K^I, L^i) and thus are independent of the details of evolution, but depend only on the configuration parameters N and u .

The amplitude of time variation of G_4 is dictated by α , and it vanishes in the limit $t \to \infty$. Hence, the time variation of G_4 in equation (3.108) is unlikely to contradict any late time observations. The time variation of G_4 has log periodic oscillations also: G_4 has an oscillatory behaviour where the nth and $(n+1)^{th}$ nodes occur at times t_n and t_{n+1} which are related by $ln \; t_{n+1} = \frac{\pi}{\omega} + ln \; t_n$, $i.e.$ by $t_{n+1} = e^{\frac{\pi}{\omega}} \; t_n$. The characteristic signatures and observational consequences of such log periodic variations of G_4 are not clear to us.

Log periodic behaviour occurs in many physical systems with 'discrete self similarity' or 'discrete scale symmetry': for example, in quantum mechanical systems with strongly attractive $\frac{1}{r^2}$ potentials near zero energy [48]; in Choptuik scaling and brane – black hole merger transitions [49]; and in a variety of dynamical systems [50]. Algebraically, the log periodicity arises when an exponent on an independent variable becomes complex for certain values of system parameters. The relevant equations and solutions can often be cast in a form given in equations (3.66) and (3.67). But we are not aware of a physical reason which explains the ubiquity of the log periodicity.

To our knowledge, this is the first time a log periodic behaviour appears in a

cosmological context. One expects such a behaviour to leave some novel imprint in the universe. But it is not clear to us which effects to look for, or which observables are sensitive to the log periodic variations of G_4 .

3.12   Numerical Results

We are unable to solve explicitly the equations (3.69) – (3.72) describing the early universe evolution. Hence, we have analysed these equations numerically. In this section, we briefly describe our procedure and present a few illustrative results. We have analysed both the $u = \frac{2}{3}$ and $u = 1$ cases which would correspond to four dimensional universe dominated by radiation and pressureless dust respectively. The results are qualitatively the same and, hence, we take $u = \frac{2}{3}$ in the following. Note that ω in equation (3.108) is then determined and, for $u = \frac{2}{3}$, the nth and $(n+1)^{th}$ nodes in the log periodic oscillations occur at times t_n and t_{n+1} related by $ln(\frac{t_{n+1}}{t_n}) = 4\pi\sqrt{\frac{3}{5}} \simeq 9.734$.

We proceed as follows. We start at an initial time $\tau = 0$ and choose a set of initial values $l_0^I = ln\ \rho_{I0}$. For each set of l_0^I , we further choose numerous arbitrary sets of (K^I, L^i) such that $K^I > 0$, $E = 1$, and equations (3.73) and (3.78) are satisfied. [13] For each set of initial values (l_0^I, K^I, L^i) , we then numerically analyse the evolution for $\tau > 0$ and obtain the value of τ_∞ ; the evolution of l^I, $(\lambda^1, \cdots, \lambda^{10})$, and t ; the stabilised values $(v^1, \cdots, v^7)$; and the resulting values for $(g_s, \frac{M_{11}}{M_4}, \frac{M_s}{M_4})$. For a few sets of initial values, we have analysed the evolution for $\tau < 0$ also.

We find that the numerical results we have obtained confirm the asymptotic features described in this thesis:

(1) e^{λ^i} and l^I all vanish in the limit $\tau \to -\infty$. In this limit, the evolution of the scale factors e^{λ^i} is of Kasner type.

(2) l^I and the physical time t all diverge in the limit $\tau \to \tau_\infty$ where τ_∞ is finite. In this limit, the scale factors $(e^{\lambda^8}, e^{\lambda^9}, e^{\lambda^{10}})$ evolve as in the standard FRW case and $(e^{\lambda^1}, \cdots, e^{\lambda^7})$ reach constant values.

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[13] There are two special choices for the set of K^I . One is where $K^1 = \cdots = K^4$ and another is the one which maximises the approximation τ_a given in equation (3.98). The later set may be determined by the algorithm given in Appendix **E**.

(3) τ_a given in equation (3.98) provides a good approximation to τ_∞ when $e^{l_0^I} \ll 1$ for all I.

(4) Any values for the ratios $\frac{M_{11}}{M_4}$ and $\frac{M_s}{M_4}$ can be obtained, but a corresponding fine tuning of $\rho_{I0} = e^{l_0^I}$ is necessary.

(5) The log periodic oscillations of $l^I - l^J$, equivalently of $(\lambda^1, \cdots, \lambda^7)$, can also be seen in the limit $\tau \to \tau_\infty$. They can be matched to solutions of the type given in equation (3.106).

To illustrate the values of τ_∞ and the ratios $(\frac{M_{11}}{M_4}, \frac{M_s}{M_4})$ one obtains, and to give an idea of their dependence on the initial values l_0^I, we tabulate these quantities in Table **3.7** for a few sets of initial values (l_0^I, K^I, L^i). We have also tabulated the values of τ_a as given by equation (3.98). The value of g_s follows from $g_s = (\frac{M_s}{M_{11}})^3$ and, hence, is not tabulated.

	$-\,(l_0^1, l_0^2, l_0^3, l_0^4)$	τ_a	τ_∞	$\frac{M_{11}}{M_4}$	$\frac{M_s}{M_4}$
(1)	$(2, 5, 8, 8)$	1.88	3.21	$5.77 * 10^{-2}$	$2.86 * 10^{-2}$
(2)	$(5, 4, 6, 9)$	2.96	4.16	$4.56 * 10^{-2}$	$1.93 * 10^{-2}$
(3)	$(15, 12, 10, 16)$	4.88	6.61	$5.95 * 10^{-2}$	$1.96 * 10^{-2}$
(4)	$(25, 26, 27, 28)$	22.00	22.59	$1.99 * 10^{-7}$	$7.30 * 10^{-10}$
(5)	$(41, 30, 50, 43)$	25.80	28.30	$1.87 * 10^{-10}$	$2.92 * 10^{-11}$
(6)	$(44.5, 34, 49, 49.5)$	34.82	36.20	$2.59 * 10^{-14}$	$3.80 * 10^{-15}$

Table 3.7: *The initial values $-\,(l_0^1, l_0^2, l_0^3, l_0^4)$ and the resulting values of τ_a , τ_∞ , $\frac{M_{11}}{M_4}$, and $\frac{M_s}{M_4}$. The values in the last four columns have been rounded off to two decimal places.*

In Table **3.8**, the corresponding initial values (K^I, L^i), $i = 1, 2, 6, 7, 8, 9$, are tabulated up to overall positive constants. The remaining L^is are given by equations (3.78) and the overall positive constants are determined by $E = 1$ and equation (3.73). All the sets of initial values (l_0^I, K^I, L^i) are chosen arbitrarily with no particular pattern and are presented here to give an idea of the typical results.

	$(K^1, K^2, K^3, K^4) \propto$	$(L^1, L^2, L^6, L^7, L^8, L^9) \propto$
(1)	(4.65, 9.14, 4.57, 6.87)	(0.60, 0.62, 0.76, 0.72, −0.94, −0.26)
(2)	(8.86, 8.26, 6.01, 6.62)	−(0.08, −0.93, 0.08, −0.72, 0.54, 0.63)
(3)	(1.61, 2.65, 0.69, 2.1)	−(0.2, −0.68, −0.14, 0.3, 0.08, 0.19)
(4)	(1.03, 1.18, 1.17, 1.27)	(0.08, 0.58, 0.27, 0.27, −0.66, −0.66)
(5)	(5.24, 4.83, 4.30, 4.96)	(0.74, 0.02, 0.24, −0.22, −0.61, −0.75)
(6)	(33.79, 24.23, 35.4, 32.29)	(11.72, 9.31, 4.59, −6.46, −21.02, −21.02)

Table 3.8: *The initial values of (K^I, L^i) for the data shown in Table **3.7**, tabulated here up to overall positive constants. These constants and the remaining L^is are to be fixed as explained in the text.*

We find, by analysing numerous sets of initial values, that changing the values of (K^I, L^i) for a given set of l_0^I changes the values of $\frac{M_{11}}{M_4}$ and $\frac{M_s}{M_4}$ only up to about four orders of magnitude. Any bigger change requires changing $e^{l_0^I}$ to a similar order, confirming that any values for $\frac{M_{11}}{M_4}$ and $\frac{M_s}{M_4}$ can be obtained but only by fine tuning $\rho_{I0} = e^{l_0^I}$.

We illustrate the evolution of the universe for the data set (3) given in Tables **3.7** and **3.8** where many features can be seen clearly. The evolution with respect to τ of l^I is shown in Figures **3.1**, **3.2 a**, and **3.2 b**. For negative values of τ not
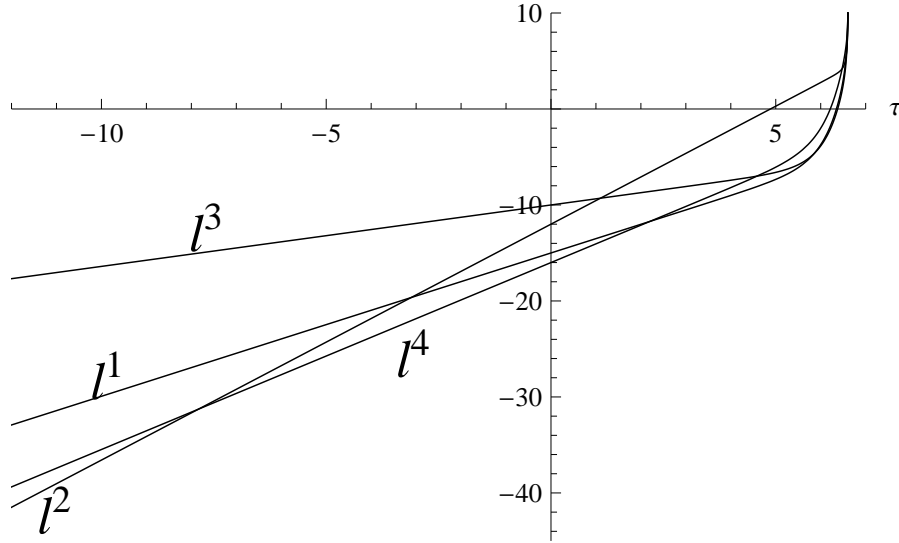
Figure 3.1: *The plots of l^I with respect to τ. The lines continue with no further crossings for negative values of τ not shown in the figure. All l^I diverge at $\tau_\infty \simeq 6.612$. All figures in this thesis are for the data set (3) given in Tables **3.7** and **3.8**.*

shown in Figure **3.1**, all l^I evolve along straight lines with no further crossings and their evolution is of Kasner type. Also, all l^I diverge at a finite value $\tau_\infty \simeq 6.612$ of τ. The magnified plots in Figures **3.2 a** and **3.2 b** for $\tau > 6.40$ and for $\tau > 6.55$ respectively show the continually criss-crossing evolution of l^I which, near τ_∞, represent the log periodic oscillations and are well described by equation (3.106).

The evolution with respect to $\ln t$ of $(\lambda^1, \cdots, \lambda^7)$ is shown in Figure **3.3**. It can be seen that $(\lambda^1, \cdots, \lambda^7)$, and hence the scale factors $(e^{\lambda^1}, \cdots, e^{\lambda^7})$ of the brane directions, all stabilise to constant values as $t \to \infty$.
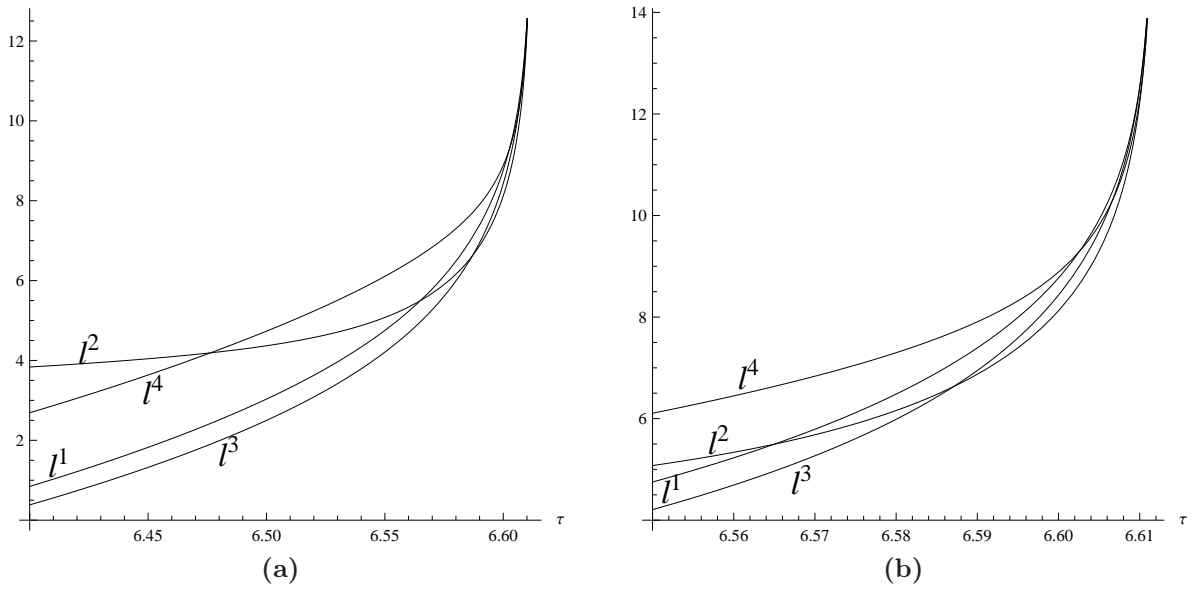
Figure 3.2: (a), (b) *The magnified plots of l^I with respect to τ for $\tau > 6.40$ and for $\tau > 6.55$ showing the continually criss-crossing evolution of l^I . Near $\tau_\infty \simeq 6.612$, these crossings are well described by equation (3.106).*
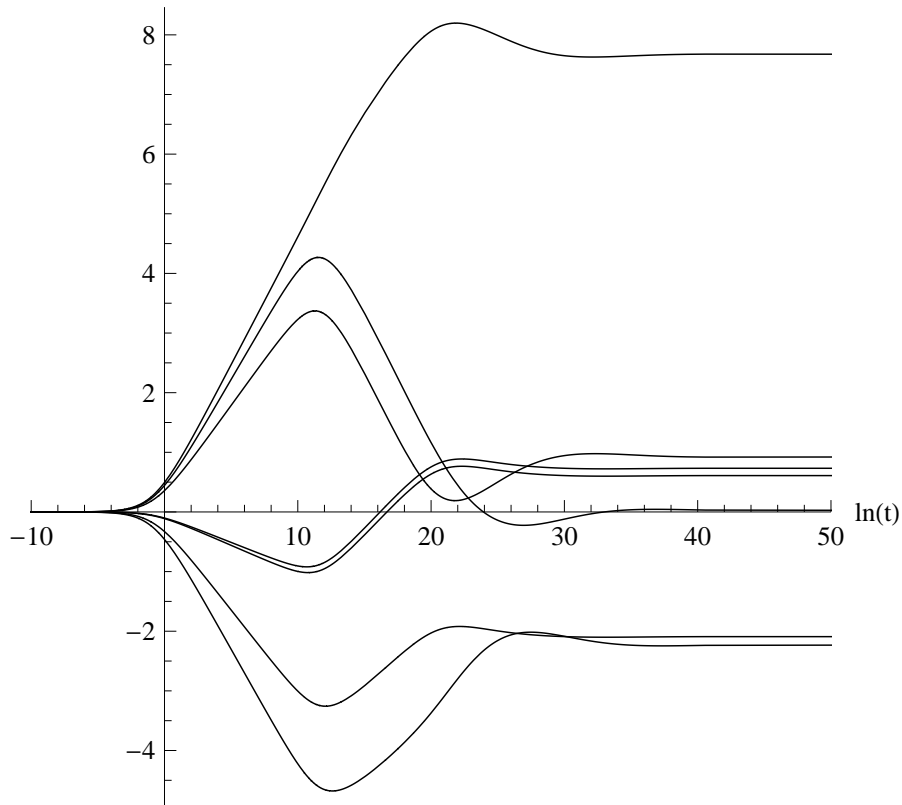

Figure 3.3: *The plots of $(\lambda^1, \cdots, \lambda^7)$ with respect to $\ln t$. The lines, from top to bottom at the right most end, correspond to $(\lambda^2, \lambda^3, \lambda^5, \lambda^6, \lambda^4, \lambda^7, \lambda^1)$. $(\lambda^1, \cdots, \lambda^7)$ all stabilise to constant values as $t \to \infty$.*

The evolution with respect to $ln\ t$ of $(\lambda^8, \lambda^9, \lambda^{10})$ and $\Lambda^c = \sum_{i=1}^{7} \lambda^i$ is shown in Figure **3.4**. Note that the seven dimensional volume of the brane directions $\propto e^{\Lambda^c}$ and that it stabilises to a constant value e^{v^c} as $t \to \infty$. We have also verified that the evolution of $(\lambda^8, \lambda^9, \lambda^{10})$ as $t \to \infty$ is same as that of the corresponding ones in a four dimensional radiation dominated FRW universe.
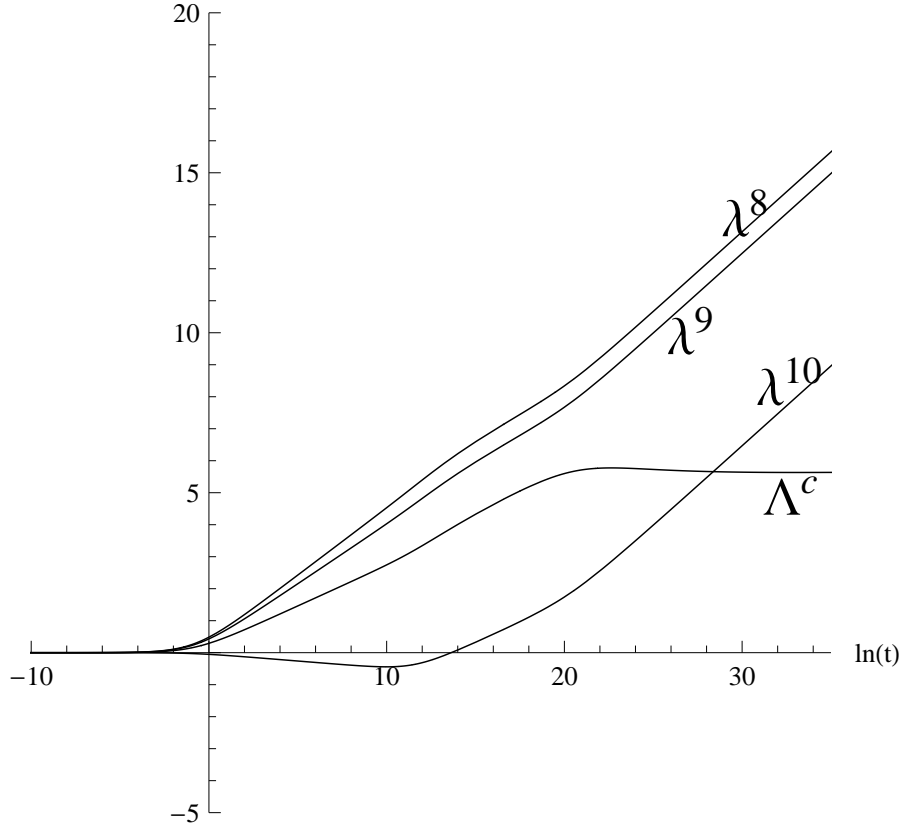

Figure 3.4: *The plots of $(\lambda^8, \lambda^9, \lambda^{10}, \Lambda^c)$ with respect to ln t . The seven dimensional volume of the brane directions $\propto e^{\Lambda^c}$. The evolution of $(\lambda^8, \lambda^9, \lambda^{10})$ as $t \to \infty$ is same as that of the corresponding ones in a four dimensional radiation dominated FRW universe.*

The log periodic oscillations of Λ^c are illustrated in Figures **3.5 a** and **3.5 b** by magnifying the plots of $(\Lambda^c - v^c)$ with respect to $ln\ t$ for $ln\ t > 20$ and for $ln\ t > 30$. The internode separations can be seen in these figures, and they match the value $\simeq 9.734$ obtained in equation (3.107) from the asymptotic analysis.
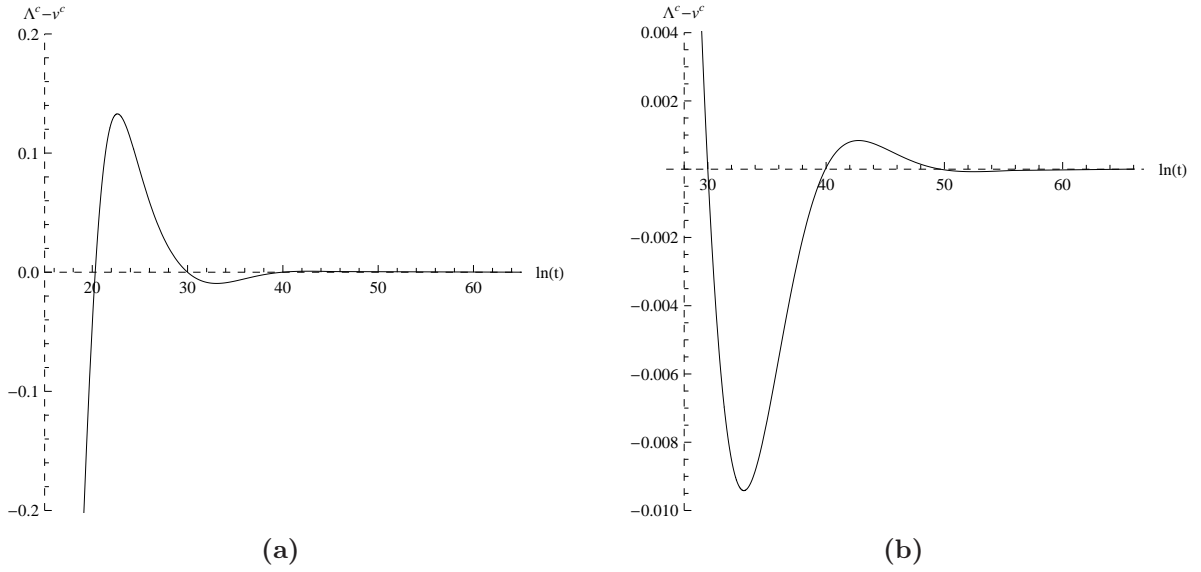
Figure 3.5: (a), (b) *The magnified plots of $(\Lambda^c - v^c)$ with respect to ln t for ln t > 20 and for ln t > 30 showing log periodic oscillations and the internode separation which is $\simeq 9.734$.*

In all the cases we have analysed, the evolutions of (l^I, λ^i) are qualitatively similar to the ones shown in the figures above. The details, such as the rise and fall of λ^i in the initial times or the value of τ_∞ or the stabilised values of the brane directions, depend on the initial values but the asymptotic features described in the beginning of this section are all the same. Hence, we have presented the plots for one illustrative set of initial values only.

3.13 Summary and Conclusions

We summarise the main results of this thesis. We assume that the early universe in M theory is homogeneous, anisotropic, and is dominated by $N = 4$ mutually BPS $22'55'$ intersecting brane configurations which are assumed to be the most entropic ones. Also, the ten dimensional space is assumed to be toroidal. We further assumed that the brane antibrane annihilation effects are negligible during the evolution of the universe at least until the brane directions are stabilised resulting in an effective $3 + 1$ dimensional universe.

We then present a thorough analysis of the evolution of such an universe. We obtain general relations among the components of the energy momentum tensor

T_{AB} using U duality symmetries of M theory and show that these relations alone imply, under a technical assumption, that the $N = 4$ mutually BPS $22'55'$ intersecting brane configurations with identical numbers of branes and antibranes will asymptotically lead to an effective $(3 + 1)$ dimensional expanding universe.

To obtain further details of the evolution, we make further assumptions about T_{AB} . We then analyse the evolution equations in D dimensions in general, and then specialise to the eleven dimensional case of interest here. Since explicit solutions are not available, we apply the general analysis and describe the qualitative features of the evolution of the $N = 4$ brane configuration : In the asymptotic limit, three spatial directions expand as in the standard FRW universe and the remaining seven spatial directions reach constant, stabilised values. These values depend on the initial conditions and can be obtained numerically. Also, any stabilised values may be obtained but it requires a fine tuning of the initial brane densities.

We also present a physical description of the mechanism of stabilisation of the seven brane directions. The stabilisation is due, in essence, to the relations among the components of T_{AB} which follow from U duality symmetries, and to each of the brane directions in the $N = 4$ configuration being wrapped by, and being transverse to, just the right number and kind of branes. This mechanism is very different from the ones proposed in string theory or in brane gas models.

In the asymptotic limit, from the perspective of four dimensional spacetime, we obtain an effective four dimensional Newton's constant G_4 which is now time varying. Its precise time dependence will follow from explicit solutions of the eleven dimensional evolution equations. We find that, in the case of $N = 4$ brane configuration, G_4 has characteristic log periodic oscillations. The oscillation 'period' depends only on the configuration parameters.

Using numerical analysis, we have confirmed the qualitative features mentioned above.

We now make a few comments on the assumptions made in this thesis. Note that the assumptions mentioned above in the first paragraph of this section pull a rug over many important dynamical questions that must be answered in a final analysis. Some of these questions, [14] in the context of M theory, are:

* Starting from the highly energetic and highly interacting M theory excita-

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[14]Many of the questions listed below have been raised by the referee also.

tions, which are expected to describe the high temperature state of the universe, how does a eleven dimensional spacetime emerge?

* What determines the topology of the ten dimensional space? Here, we assumed it to be toroidal. How does the universe evolve if its spatial topology is not toroidal?

* From what stage onwards, is the eleven dimensional 'low energy' effective action a good description of further evolution?

* What are the relevant 'low energy' configurations of M theory? Here, based on the black hole studies, we have assumed that the $N = 4$ mutually BPS $22'55'$ intersecting brane configurations are the most entropic ones and, hence, that they are the dominant configurations in the early universe studied here.

This raises further questions: Are the $22'55'$, and not some other mutually BPS $N \geq 4$ or some other non BPS, configurations really the most entropic and the dominant ones? Even assuming that mutually BPS $N = 4$ is the answer, are there other $N = 4$ configurations beside the $22'55'$ ones and, if so, how do they affect the evolution described here? What are the effects of the sub-dominant configurations? In particular, will the effects of other brane configurations mentioned above undo the stabilisation of seven directions presented here?

Note that unless these questions are answered and, furthermore, it is shown that other brane configurations mentioned above do not undo the stabilisation presented here, our assumption that the evolution of the universe is dictated by the $22'55'$ configuration amounts to a fine tuning: The $22'55'$ configuration assumed here, where the sets of 2 branes and 5 branes wrap the directions $(x^1, \cdots, x^7)$ homogeneously everywhere in the mutually transverse three dimensional space, may not arise generically. Also, the implicitly required absence of other brane configurations is not natural in the context of early universe. Then the problem of the emergence of an effective $3 + 1$ dimensional universe, a solution for which is presented here, gets shifted to answering how the required, finely tuned, initial conditions may arise naturally from M theory.

* What is the time scale of brane antibrane annihilations in the $22'55'$ configuration studied here? Is it long enough for the brane directions to be stabilised as described in this thesis? Here, based on the black hole studies, we have assumed

it to be long enough.

* A related question, but applicable after stabilisation of brane directions, is the following: If all the branes and antibranes will eventually decay, as seems natural, then what are the decay products? How can one obtain the known constituents of our present universe?

Although one of us have presented a principle in [15] that may be of help, the fact is that we do not know even where to begin in answering these questions quantitatively, much less know the answers. Nevertheless we present the above list of questions, unlikely to be complete, in order to emphasise the further work required to understand how our known $3 + 1$ dimensional universe may emerge from M theory.

In the present work, with many attendant assumptions, we considered the $22'55'$ configurations and explained a mechanism by which seven directions stabilise and an effective $3 + 1$ dimensional universe results. Clearly, it is important to answer the questions listed above and thereby determine the relevance of this mechanism.

Within the present framework, there are many other issues that may be studied further. We conclude by mentioning a sample of them. We have shown here that a large stabilised seven dimensional volume can be obtained but it requires a corresponding fine tuning of initial brane densities. This is within the context of our ansatzes for T_{AB} and the equations of state. It will be of interest to prove or disprove the necessity of such a fine tuning in more general contexts.

The $N = 4$ intersecting brane configuration studied here is the entropically favourable one and, as proposed in [15], may be thought of as emerging from the high temperature phase of M theory in the early universe. Such an emergence suggests that there may be novel solutions to the horizon problem and to the primordial density fluctuations, perhaps similar to those explored recently in the Hagedorn phase of string theory by Nayeri et al [64] (see also [65]). Note that this involves answering many of the questions listed above.

It may be of interest to study further the consequences of time varying Newton's constant which appears here, in particular possible imprints of its asymptotic log periodic oscillations.

In the case of a class of black holes, the brane configurations describe well their entropy and Hawking radiation. In the present description of a four dimensional

early universe in terms of $N = 4$ intersecting branes, it is not clear which quantities to calculate which, analogously to entropy or Hawking radiation in the black hole case, may provide further validation. It is important to study this further.
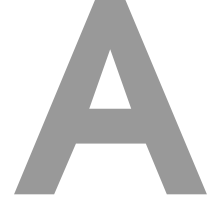
T and S duality rules

A.1   T duality rules for background fields

Lets take a solution of type II supergravity in string frame

$$ds^2 = g_{\mu\nu} \, dx^\mu dx^\nu \tag{A.1}$$

where μ, ν vary from 0 to 9. For simplicity assume all special dimensions are compact. NS sector fields $B_{\mu\nu}$ and dilaton ϕ are switched on. Consider a metric such that it does not depend on x^9. we denote x^9 by z. Now μ, ν run over $(0, \cdots, 8)$. Under T duality background fields transform to a new set of fields. Rule for this transformation is known as Buscher's rules. See for more details [4, 5, 2] for example. According to these set of rules fields goes like following:

$$g_{zz} \quad \rightarrow \quad g'_{zz} = \frac{1}{g_{zz}} \tag{A.2}$$

$$g_{\mu z} \quad \rightarrow \quad g'_{\mu z} = \frac{B_{\mu z}}{g_{zz}} \tag{A.3}$$

$$g_{\mu\nu} \quad \rightarrow \quad g'_{\mu\nu} = g_{\mu\nu} - \frac{g_{\mu z} g_{\nu z} - B_{\mu z} B_{\nu z}}{g_{zz}} \tag{A.4}$$

$$B_{\mu z} \quad \rightarrow \quad B'_{\mu z} = \frac{g_{\mu z}}{g_{zz}} \tag{A.5}$$

$$B_{\mu\nu} \quad \rightarrow \quad B'_{\mu\nu} = B_{\mu\nu} + \frac{2g_{[\mu|z|} B_{\nu]z}}{g_{zz}} \tag{A.6}$$

$$\phi \quad \rightarrow \quad \phi' = \phi - \frac{1}{2} \ln(g_{zz}) \tag{A.7}$$

When R-R fields are switched on they also transform under T-duality. As T duality transform IIA theory to IIB theory and vice versa, it transform odd form fields to even form and even to odd. If we start with a type IIA theory, transformation rules are given by [2]

$$C_z \quad \rightarrow \quad C \tag{A.8}$$

$$C_\mu \,,\, C_{\mu\nu z} \quad \rightarrow \quad C_{\mu z} \,,\, C_{\mu\nu} \tag{A.9}$$

$$C_{\mu\nu\lambda} \quad \rightarrow \quad C_{\mu\nu\lambda z} \tag{A.10}$$

So we see the theory goes to type IIB theory.

A.2 S duality rules for background fields

S duality is a duality of type IIB theory. This duality relates strongly coupled theory to weakly coupled theory. Assuming solution of the form equation (A.1) in string frame rules for transformation of background fields under S duality are following:

$$g_{\mu\nu} \quad \rightarrow \quad g'_{\mu\nu} = e^{-\phi} \, g_{\mu\nu} \tag{A.11}$$

$$B_{\mu\nu} \quad \rightarrow \quad B'_{\mu\nu} = B_{\mu\nu} \,,\, (B^{NS}_{\mu\nu} \leftrightarrow B^{R}_{\mu\nu}) \,^{15} \tag{A.12}$$

$$D_{\mu\nu\lambda\sigma} \quad \rightarrow \quad D'_{\mu\nu\lambda\sigma} = D_{\mu\nu\lambda\sigma} \tag{A.13}$$

$$\phi \quad \rightarrow \quad \phi' = -\phi \tag{A.14}$$

So we see it transform $D1$ brane to $F1$ string and $D5$ brane to $NS5$ brane. $D3$ brane remains unchanged and coupling constant goes to (coupling constant)$^{-1}$.

---

[15]This notation means S duality transform NS-NS 2-form potential to R-R 2-form, and back.

B

BPS Intersection Rules

In general a single p-brane solutions break half of the supersymmetries present in the theory. If we add another set of q-branes intersecting the first set, the new solution breaks another half of the supersymmetries. But it is possible to add second set in such a way that, it breaks the same supersymmetries as the first set does. Then we have a solution which preserves half of the total supersymmetries. We call, this particular way of adding more sets of branes, BPS intersection rules. A compact form of these rules can be found in [5], [See also reference given in [5].]

We list a few of these rules hare. We give for both 10-dimensional and 11-dimensional theory.

$$
\begin{array}{c}
F1 \parallel NS5,\ F1 \perp Dp\,(0), \\
NS5 \perp NS5\,(1),\ NS5 \perp NS5\,(3),\ NS5 \perp Dp\,(p-1)\,(p>1), \\
Dp \perp Dq\,(m)\ :\ p+q=4+2m, \\
W \parallel F1,\ W \parallel NS5,\ W \parallel Dp, \\
KK6 \perp Dp\,(p-2)
\end{array}
$$

Table B.1: *BPS intersections in 10 dimensions.*

$$
\begin{array}{c}
M2 \perp M2\,(0),\ M2 \perp M5\,(1),\ M5 \perp M5\,(1),\ M2 \perp M2\,(3), \\
W \parallel M2,\ W \parallel M5, \\
KK7M \parallel M2,\ KK7M \perp M2\,(0),\ KK7M \parallel M5,\ KK7M \perp M5\,(1),\ KK7M \perp M5\,(3) \\
W \parallel KK,\ W \perp KK7M\,(2),\ W \perp KK7M\,(4)
\end{array}
$$

Table B.2: *BPS intersections in 11 dimensions.*

The notation of the above tables is follows:

$Xp \parallel Yq$ means X and Y type branes are considered. X brane is p dimensional and Y brane is q dimensional. $\parallel$ indicates all of the worldvolume directions of Xp and Yq are parallel. $Xp \perp Yq\,(m)$ means Xp and Yq branes worldvolume intersects only in m directions.
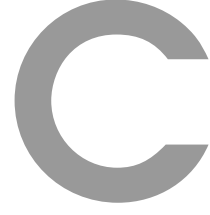
To Show $E \geq 0$

Let $\vec{1} = (1,\ 1,\ \cdots,1)$ and $\vec{v} = (v_1,\ v_2, \cdots,\ v_n)$ be the standard n – component vectors with the standard vector product. Let θ_n be the angle between them. Then $\vec{1} \cdot \vec{1} = n$, $\vec{v} \cdot \vec{v} = \sum_a v_a^2$, $(\vec{1} \cdot \vec{v})^2 = (\sum_a v_a)^2 = n\ cos^2\theta_n \sum_a v_a^2$, and we have the Schwarz inequality in the form

$$n \sum_{a=1}^{n} v_a^2 - (\sum_{a=1}^{n} v_a)^2 = n\ \sigma_n^2\ \geq\ 0 \tag{C.1}$$

where $\sigma_n^2 = sin^2\theta_n \sum_{a=1}^{n} v_a^2$. The equality is valid, *i.e.* $\sigma_n = 0$, if and only if $sin\ \theta_n = 0$, equivalently $v_1 = \cdots = v_n$.

We now show the following:

Let G^{ij} and G_{ij} be given by equation (3.29). If u_i and L^i satisfy the relations $\sum_i u_i L^i = 0$ and $\sum_{ij} G^{ij} u_i u_j > 0$ then $2E = -\sum_{ij} G_{ij} L^i L^j \geq 0$. E vanishes if and only if L^i all vanish.

Proof : It is clear that E vanishes if L^i all vanish. Now, let $\vec{1} = (1,\ 1,\ \cdots,1)$, $\vec{u} = (u_1,\ \cdots, u_{D-1})$, and θ be the angle between them. Then $(\sum_i u_i)^2 = (D-1)\ cos^2\theta \sum_i u_i^2$. Hence, $\sum_{ij} G^{ij} u_i u_j = \frac{1}{D-2} (\sum_i u_i)^2 - \sum_i u_i^2 > 0$ implies that

$$1 - (D-1)\ sin^2\theta > 0\ . \tag{C.2}$$

The vector $\vec{L} = (L^1, \cdots, L^{D-1})$ is perpendicular to $\vec{u}$ since $\sum_i u_i L^i = 0$. Let $\vec{L} = \vec{L}_\perp + \vec{L}_\parallel$ where $\vec{L}_\perp$ is perpendicular to the plane defined by $\vec{1}$ and $\vec{u}$, and $\vec{L}_\parallel$ lies in it. Then $\sum_i (L^i)^2 = L^2_\perp + L^2_\parallel$ where $L^2_\perp = \vec{L}_\perp \cdot \vec{L}_\perp$ and $L^2_\parallel = \vec{L}_\parallel \cdot \vec{L}_\parallel$. Since $\vec{L}$ and $\vec{u}$ are perpendicular and $\vec{L}_\parallel$ lies in the plane defined by $\vec{1}$ and $\vec{u}$, it follows that $\vec{L}_\parallel$ is perpendicular to $\vec{u}$, and that the angle between the vectors $\vec{1}$ and $\vec{L}_\parallel$ is $\frac{\pi}{2} \pm \theta$. We then have

$$
\begin{aligned}
2E &= -\sum_{ij} G_{ij} L^i L^j = \sum_i (L^i)^2 - \left(\sum_i L^i\right)^2 \\
&= L^2_\perp + L^2_\parallel - (D-1) \, L^2_\parallel \, sin^2\theta \;\; \geq \;\; 0
\end{aligned}
$$

where the inequality follows from equation (C.2). The equality holds, and hence E vanishes, only when $L^2_\perp = L^2_\parallel = 0$, $i.e.$ only when L^i all vanish. This completes the proof.

D

Signs and non vanishing of $(\Lambda_\tau, \, l_\tau^I)$

Here, we show that the inequality in equation (3.82) implies that none of $(\Lambda_\tau, \, l_\tau^I)$ may vanish, and that they must all have same sign.

Setting $x_I = l_\tau^I$, equation (3.82) becomes $X = 12u^2 \left(E + \sum_I e^{l^I}\right) > 0$ where the polynomial $X = (x_1 + x_2 + x_3 + x_4)^2 - 3(x_1^2 + x_2^2 + x_3^3 + x_4^2)$. Now, if any of the x_I vanishes then $X \leq 0$, see the Schwarz inequality given in equation (C.1). Hence, none of the x_I may vanish. Rewrite X as

$$
\begin{aligned}
X &= \left\{(x_1 + x_2 + x_3)^2 - 3(x_1^2 + x_2^2 + x_3^3)\right\} - 2x_4^2 + 2x_4\,(x_1 + x_2 + x_3) \\
&= \left\{(x_1 + x_2)^2 - 2(x_1^2 + x_2^2)\right\} + \left\{(x_3 + x_4)^2 - 2(x_3^2 + x_4^2)\right\} \\
&\quad -(x_1^2 + x_2^2 + x_3^3 + x_4^2) + 2(x_1 + x_2)\,(x_3 + x_4)
\end{aligned}
$$

and note that $\{\cdots\} \leq 0$ for each curly bracket, see equation (C.1). Hence, the necessary conditions for $X > 0$ are

$$
x_4\,(x_1 + x_2 + x_3) > 0 \quad , \quad (x_1 + x_2)\,(x_3 + x_4) > 0 \;.
$$

Let one of the x_I , e.g. x_4 , be negative and the other three positive. This violates the first inequality above and, hence, is not possible. Let two of the x_I , e.g. x_3 and x_4 , be negative and the other two positive. This violates the second inequality above and, hence, is not possible. Similarly, three of the x_I being negative and one positive is also not possible. Thus, the only possibility is that all x_I have same sign. Thus we have that none of the l_τ^I may vanish, and that they must all have same sign.

Appendix D. Signs and non vanishing of $(\Lambda_\tau,\; l_\tau^I)$

With l_τ^I denoted as x_I , equation (3.81) for Λ_τ becomes

$$6u\,\Lambda_\tau = 2x_1 + 2x_2 + x_3 + x_4 + 6u\,L \; .$$

Note that $u > 0$. If $L = 0$ then it follows that Λ_τ does not vanish and has the same sign as x_I . Consider now the case where $L \neq 0$. Using equation (C.1) to eliminate $\sum_I x_I^2$ in the polynomial X, we obtain

$$X = \frac{1}{4}\,(x_1 + x_2 + x_3 + x_4)^2 - 3\sigma_4^2 = 12u^2\,(E + \sum_I e^{l^I}) \; .$$

Using the inequality $2E > 3(L)^2$, see equation (3.80), it follows that $(x_1 + x_2 + x_3 + x_4)^2 > 72u^2(L)^2$. Combined with the earlier result on l_τ^I , this inequality implies that $(x_1 + x_2 + x_3 + x_4 + 6uL)$, and hence Λ_τ given above, may not vanish and must have the same sign as $x_I = l_\tau^I$, irrespective of whether L is positive or negative. This completes the proof.

E

Set of K^I which maximises τ_a

With no loss of generality, let $0 < -l_0^1 \le \cdots \le -l_0^4$. The corresponding set of K^I which satisfies equation (3.73), with $E = 1$, and which maximises $\tau_a = min\{\tau_I\}$, where $\tau_I = -\frac{l_0^I}{K^I}$, may be obtained by the following algorithm. The required analysis is straightforward but a little tedious and, hence, is omitted.

- Let $K^1 = -l_0^1 K$. It will turn out that $\tau_a = \tau_1 = \frac{1}{K}$.

- Choose $K^2 = -l_0^2 K$. Then $\tau_2 = \tau_1$.

- If $-l_0^1 - l_0^2 \le -l_0^3$ then choose $K^3 = K^4 = -(l_0^1 + l_0^2) K$. Then $\tau_4 \ge \tau_3 \ge \tau_2 = \tau_1$.

- If $-l_0^1 - l_0^2 > -l_0^3$ then choose $K^3 = -l_0^3 K$. Then $\tau_3 = \tau_2 = \tau_1$.

- If $-l_0^1 - l_0^2 > -l_0^3$ and if $-l_0^1 - l_0^2 - l_0^3 \le -2l_0^4$ then choose $K^4 = -\frac{1}{2} (l_0^1 + l_0^2 + l_0^3) K$. Then $\tau_4 \ge \tau_3 = \tau_2 = \tau_1$.

- If $-l_0^1 - l_0^2 > -l_0^3$ and if $-l_0^1 - l_0^2 - l_0^3 > -2l_0^4$ then choose $K^4 = -l_0^4 K$. Then $\tau_4 = \cdots = \tau_1$.

- K^I are all thus determined in terms of K . Equation (3.73), with $E = 1$, will now determine K .

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