A Master Formula Approach to Chiral Symmetry Breaking

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We find that various results of current algebra at tree level and beyond can be directly obtained from a master formula, without use of chiral perturbation theory or effective Lagrangians. Application is made to $\pi\pi$ scattering, where it is shown that the bulk of the ρ contribution can be determined in a model independent way.

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Consider an action whose kinetic part is invariant under chiral $SU_L(2)\times SU_R(2)$ with a scalar-isoscalar mass term in the $(2,2)$ representation. Examples are two flavor QCD or sigma models. The symmetry properties of the theory may be expressed by gauging the kinetic part with c-number external fields v^a_μ and a^a_μ, and extending the mass term to include couplings with scalar and pseudoscalar fields s and p^a. For two-flavor QCD, the relevant part of the action reads

$$\mathbf{I} = +\int d^4x\overline{q}\gamma^\mu\left(i\partial_\mu + G_\mu + v^a_\mu\frac{\tau^a}{2} + a^a_\mu\frac{\tau^a}{2}\gamma_5\right)q$$
$$-\frac{\hat{m}}{m_\pi^2}\int d^4x\overline{q}\left(m_\pi^2 + s - i\gamma_5\tau^a p^a\right)q \qquad (1)$$

where m_π is the pion mass. We will assume that $\phi = (v^a_\mu, a^a_\mu, s, p^a)$ are smooth functions that fall off rapidly at infinity.

Currents and densities $\mathcal{O} = (\mathbf{V}, \mathbf{A}, f_\pi\sigma, f_\pi\pi)$ may be introduced as

$$\mathcal{O}(x) = \frac{\delta\mathbf{I}}{\delta\phi(x)} \qquad (2)$$

which obey the Veltman-Bell equations [1]

$$\nabla^\mu\mathbf{V}_\mu + \underline{a}^\mu\mathbf{A}_\mu + f_\pi\underline{p}\,\pi = 0 \qquad (3)$$

$$\nabla^\mu\mathbf{A}_\mu + \underline{a}^\mu\mathbf{V}_\mu - f_\pi(m_\pi^2+s)\,\pi + f_\pi p\,\sigma = 0 \qquad (4)$$

where $\nabla_\mu = \partial_\mu\mathbf{1} + \underline{v}_\mu$ is the vector covariant derivative, $\underline{a}^{ac}_\mu = \epsilon^{abc}a^b_\mu$, $\underline{p}^{ac} = \epsilon^{abc}p^b$, and f_π is the pion decay constant. In the above, we have used the fact that the Bardeen anomaly [2] and the Wess-Zumino term [3] vanish for $SU_L(2)\times SU_R(2)$. Introducing the extended S-matrix $\mathcal{S}$, holding the incoming fields fixed, and using the Schwinger action principle [4] imply

$$<\beta\ \text{in}|\delta\mathcal{S}|\alpha\ \text{in}> = i<\beta\ \text{in}|\mathcal{S}\delta\mathbf{I}|\alpha\ \text{in}> \ . \qquad (5)$$

This result together with asymptotic completeness, yield the Peierls-Dyson formula [5]

$$\mathcal{O}(x) = -i\mathcal{S}^\dagger\frac{\delta\mathcal{S}}{\delta\phi(x)} \ . \qquad (6)$$

It follows from the Veltman-Bell equations (3-4) that

$$\left(\nabla^{ac}_\mu\frac{\delta}{\delta v^c_\mu(x)} + \underline{a}^{ac}_\mu(x)\frac{\delta}{\delta a^c_\mu(x)} + \underline{p}^{ac}(x)\frac{\delta}{\delta p^c(x)}\right)\mathcal{S} =$$
$$\left(\mathbf{X}^a_V(x) + \underline{p}^{ac}(x)\frac{\delta}{\delta p^c(x)}\right)\mathcal{S} = 0 \qquad (7)$$

$$\left(\nabla^{ac}_\mu\frac{\delta}{\delta a^c_\mu(x)} + \underline{a}^{ac}_\mu(x)\frac{\delta}{\delta v^c_\mu(x)}\right.$$
$$\left.-(m_\pi^2+s(x))\frac{\delta}{\delta p^a(x)} + p^a(x)\frac{\delta}{\delta s(x)}\right)\mathcal{S} =$$
$$\left(\mathbf{X}^a_A(x) - (m_\pi^2+s(x))\frac{\delta}{\delta p^a(x)} + p^a(x)\frac{\delta}{\delta s(x)}\right)\mathcal{S} = 0 \qquad (8)$$

where $\mathbf{X}_V$ and $\mathbf{X}_A$ are the generators of local $SU_L(2)\times SU_R(2)$.

We further require

$$<0|\mathbf{A}^a_\mu(x)|\pi^b(p)> = if_\pi\delta^{ab}p_\mu\ e^{-ip\cdot x} \ . \qquad (9)$$

In the absence of stable axial vector or other pseudoscalar mesons, this is equivalent to the asymptotic conditions ($x^0\to\mp\infty$)

$$\mathbf{A}^a_\mu(x) \to -f_\pi\partial_\mu\pi^a_{\text{in,out}}(x)$$

and

$$\partial^\mu\mathbf{A}^a_\mu(x) \to +f_\pi m_\pi^2\pi^a_{\text{in,out}}(x) \qquad (10)$$

where π_{in} and π_{out} are free incoming and outgoing pion fields. Comparison of (10) with (4) shows that π is a normalized interpolating field.

To incorporate (10) into (7-8) we introduce a modified action

$$\hat{\mathbf{I}} = \mathbf{I} - f_\pi^2\int d^4x\left(s(x) + \frac{1}{2}a^\mu(x)\cdot a_\mu(x)\right) \ , \qquad (11)$$

the corresponding extended S-matrix

$$\hat{\mathcal{S}} = \mathcal{S}\ \exp\left(-if_\pi^2\int d^4x\left(s(x) + \frac{1}{2}a^\mu(x)\cdot a_\mu(x)\right)\right) \ , \qquad (12)$$

and a change of variable $p = J/f_\pi - \nabla^\mu a_\mu$. Taking $\hat{\phi} = (v^a_\mu, a^a_\mu, s, J^a)$ as independent variables, modified currents and densities $\hat{\mathcal{O}} = (\mathbf{j}_V, \mathbf{j}_A, f_\pi\hat{\sigma}, \hat{\pi})$ may be defined as

$$\hat{\mathcal{O}}(x) = \frac{\delta\hat{\mathbf{I}}}{\delta\hat{\phi}} = -i\hat{\mathcal{S}}^\dagger\frac{\delta\hat{\mathcal{S}}}{\delta\hat{\phi}} \ . \tag{13}$$

The chain rule yields

$$\begin{aligned}
&\mathbf{V}^a_\mu(x) = \mathbf{j}^a_{V\mu}(x) + f_\pi \underline{a}^{ac}_\mu(x)\hat{\pi}^c(x)\\
&\mathbf{A}^a_\mu(x) = \mathbf{j}^a_{A\mu}(x) + f^2_\pi a^a_\mu(x) - f_\pi(\nabla_\mu\hat{\pi})^a(x)\\
&\sigma(x) = \hat{\sigma}(x) + f_\pi\\
&\pi^a(x) = \hat{\pi}^a(x)
\end{aligned} \tag{14}$$

Substitution into (3) gives

$$\nabla^\mu\mathbf{j}_{V\mu} + \underline{a}^\mu\mathbf{j}_{A\mu} + \underline{J}\pi = 0 \tag{15}$$

and therefore

$$\left(\mathbf{X}_V + \underline{J}\frac{\delta}{\delta J}\right)\hat{\mathcal{S}} = 0 \ . \tag{16}$$

On the other hand, substitution into (4) gives

$$\begin{aligned}
&\nabla^\mu\mathbf{j}_{A\mu} + \underline{a}^\mu\mathbf{j}_{V\mu} =\\
&-f^2_\pi\nabla^\mu a_\mu + f_\pi\nabla^\mu\nabla_\mu\pi\\
&-f_\pi\underline{a}^\mu\underline{a}_\mu\pi + f_\pi(m^2_\pi + s)\pi\\
&-(J - f_\pi\nabla^\mu a_\mu)(\hat{\sigma} + f_\pi) \ .
\end{aligned} \tag{17}$$

This equation may be integrated by introducing the retarded and advanced Green's functions

$$\left(-\Box - m^2_\pi - \mathbf{K}\right)G_{R,A} = \mathbf{1} \tag{18}$$

$$\mathbf{K} = 2\underline{v}^\mu\partial_\mu + (\partial^\mu\underline{v}_\mu) + \underline{v}^\mu\underline{v}_\mu - \underline{a}^\mu\underline{a}_\mu + s \tag{19}$$

where we have adopted a condensed matrix notation. We have the Yang-Feldman-Kallen type-equations [6]

$$\begin{aligned}
\pi &= \left(1 + G_R\mathbf{K}\right)\pi_{\rm in} - G_RJ + G_R\left(\nabla^\mu a_\mu - J/f_\pi\right)\hat{\sigma}\\
&\quad -\frac{1}{f_\pi}G_R\left(\nabla^\mu\mathbf{j}_{A\mu} + \underline{a}^\mu\mathbf{j}_{V\mu}\right)\\
&= \left(1 + G_A\mathbf{K}\right)\pi_{\rm out} - G_AJ + G_A\left(\nabla^\mu a_\mu - J/f_\pi\right)\hat{\sigma}\\
&\quad -\frac{1}{f_\pi}G_A\left(\nabla^\mu\mathbf{j}_{A\mu} + \underline{a}^\mu\mathbf{j}_{V\mu}\right) \ .
\end{aligned} \tag{20}$$

Noting that $\pi_{\rm out} = \hat{\mathcal{S}}^\dagger\pi_{\rm in}\hat{\mathcal{S}}$, and using (13) we arrive at

$$\begin{aligned}
\frac{\delta}{\delta J}\hat{\mathcal{S}} &= -iG_RJ\hat{\mathcal{S}} + i\hat{\mathcal{S}}\left(1 + G_R\mathbf{K}\right)\pi_{\rm in}\\
&\quad +\frac{1}{f_\pi}G_R\left(\nabla^\mu a_\mu - J/f_\pi\right)\frac{\delta\hat{\mathcal{S}}}{\delta s} - \frac{1}{f_\pi}G_R\mathbf{X}_A\hat{\mathcal{S}}\\
&= -iG_AJ\hat{\mathcal{S}} + i\left(1 + G_A\mathbf{K}\right)\pi_{\rm in}\hat{\mathcal{S}}\\
&\quad +\frac{1}{f_\pi}G_A\left(\nabla^\mu a_\mu - J/f_\pi\right)\frac{\delta\hat{\mathcal{S}}}{\delta s} - \frac{1}{f_\pi}G_A\mathbf{X}_A\hat{\mathcal{S}} \ .
\end{aligned} \tag{21}$$

Evidently, any result which is a consequence of (10) and symmetry (7-8) must be contained in (16,21). Since (16) simply represents local isospin invariance, the nontrivial results of current algebra must be basically contained in (21).

To show that this is the case and that (21) is the desired master formula, we note that

$$\begin{aligned}
G_{R,A} &= \Delta_{R,A} + \Delta_{R,A}\mathbf{K}G_{R,A}\\
&= \Delta_{R,A} + G_{R,A}\mathbf{K}\Delta_{R,A}
\end{aligned} \tag{22}$$

where $\Delta_{R,A}$ are the Green's functions for free fields. Multiplying (21) by $(1 + G_A\mathbf{K})^{-1} = 1 - \Delta_A\mathbf{K}$ and Fourier decomposing yield

$$\begin{aligned}
\left[a^a_{\rm in}(k), \hat{\mathcal{S}}\right] &= \int d^4yd^4ze^{ik\cdot y}\left(1 + \mathbf{K}G_R\right)^{ac}(y,z)\\
&\quad\times\left(-i\hat{\mathcal{S}}(\mathbf{K}\pi_{\rm in})^c(z) + i\hat{\mathcal{S}}J^c(z)\right.\\
&\quad -\frac{1}{f_\pi}\left(\nabla^\mu a_\mu - J/f_\pi\right)^c(z)\frac{\delta\hat{\mathcal{S}}}{\delta s(z)}\\
&\quad \left.+\frac{1}{f_\pi}\mathbf{X}^c_A(z)\hat{\mathcal{S}}\right)
\end{aligned} \tag{23}$$

$$\begin{aligned}
\left[\hat{\mathcal{S}}, a^{a\,\dagger}_{\rm in}(k)\right] &= \int d^4yd^4ze^{-ik\cdot y}\left(1 + \mathbf{K}G_R\right)^{ac}(y,z)\\
&\quad\times\left(-i\hat{\mathcal{S}}(\mathbf{K}\pi_{\rm in})^c(z) + i\hat{\mathcal{S}}J^c(z)\hat{\mathcal{S}}\right.\\
&\quad -\frac{1}{f_\pi}\left(\nabla^\mu a_\mu - J/f_\pi\right)^c(z)\frac{\delta\hat{\mathcal{S}}}{\delta s(z)}\\
&\quad \left.+\frac{1}{f_\pi}\mathbf{X}^c_A(z)\hat{\mathcal{S}}\right)
\end{aligned} \tag{24}$$

where $a^a_{\rm in}(k)$ and $a^{a\dagger}_{\rm in}(k)$ are the annihilation and creation operators of incoming pions with momentum k and isospin a. Iterations give the two and higher pion reduction formulas, *e.g.* to order $\mathcal{O}(\phi)$

$$\begin{aligned}
&\left[a^b_{\rm in}(k_2), \left[\hat{\mathcal{S}}, a^{a\dagger}_{\rm in}(k_1)\right]\right] =\\
&\int d^4ye^{-ik_1\cdot y}\frac{1}{f_\pi}\mathbf{X}^a_A(y)\left[a^b_{\rm in}(k_2), \hat{\mathcal{S}}\right] .
\end{aligned} \tag{25}$$

The Bogoliubov causality condition [7] implies that

$$T^*\left(\hat{\mathcal{O}}(x_1)....\hat{\mathcal{O}}(x_n)\right) = (-i)^n\hat{\mathcal{S}}^\dagger\frac{\delta^n}{\delta\hat{\phi}(x_1)...\delta\hat{\phi}(x_n)}\hat{\mathcal{S}} \ . \tag{26}$$

With this in mind, using (23-25), sandwiching between nucleon states and switching off the external fields, give the familiar πN scattering formula

$$< N(p_2)|\Big[a^b_{\rm in}(k_2),\Big[\mathbf{S},a^{a\dagger}_{\rm in}(k_1)\Big]\Big]|N(p_1)>=$$
$$-\frac{i}{f_\pi}m_\pi^2\delta^{ab}\int d^4y e^{-i(k_1-k_2)\cdot y} < N(p_2)|\hat\sigma(y)|N(p_1)>$$
$$-\frac{1}{f_\pi^2}k_1^\alpha k_2^\beta\int d^4y_1 d^4y_2 e^{-ik_1\cdot y_1+ik_2\cdot y_1}$$
$$\times < N(p_2)|T^*\Big(\mathbf{j}^a_{A\alpha}(y_1)\mathbf{j}^b_{A\beta}(y_2)\Big)|N(p_1)>$$
$$+\frac{1}{f_\pi^2}k_1^\alpha\int d^4y e^{-i(k_1-k_2)\cdot y}\epsilon^{abe} < N(p_2)|\mathbf{V}^e_\alpha(y)|N(p_1)> \tag{27}$$

where $\mathbf{S}=\hat{\mathcal{S}}|_{\phi=0}$ is the on-shell S-matrix. The disconnected part in (27) can be checked to cancel. At threshold, (27) yields the Tomozawa-Weinberg relation [8].

The extension to $\pi\pi$ scattering is straightforward in principle, although lengthy in practice. We find that the transition amplitude $i\mathcal{T}(p_2d,k_2b\leftarrow k_1a,p_1c)$ is a sum of four contributions

$$i\mathcal{T}_{\rm tree}=\frac{i}{f_\pi^2}\Big(s-m_\pi^2\Big)\ \delta^{ac}\delta^{bd}+\ 2\ {\rm perm.} \tag{28}$$

$$i\mathcal{T}_{\rm rho}=\frac{i}{f_\pi^2}\epsilon^{abe}\epsilon^{cde}\Big(\mathbf{F}_V(t)-1-\frac{t}{4f_\pi^2}\mathbf{\Pi}_V(t)\Big)$$
$$+\ 2\ {\rm perm.} \tag{29}$$

$$i\mathcal{T}_{\rm sigma}=-\frac{2im_\pi^2}{f_\pi}\delta^{ab}\delta^{cd}\Big(\mathbf{F}_S(t)+\frac{1}{f_\pi}-\frac{1}{2f_\pi^2} < 0|\hat\sigma|0>\Big)$$
$$+\frac{m_\pi^4}{f_\pi^2}\delta^{ab}\delta^{cd}\int d^4y e^{-i(k_1-k_2)\cdot y}$$
$$\times < 0|T^*\Big(\hat\sigma(y)\hat\sigma(0)\Big)|0>_{\rm conn.}+\ 2\ {\rm perm.} \tag{30}$$

$$i\mathcal{T}_{\rm rest}=+\frac{1}{f_\pi^4}k_1^\alpha k_2^\beta p_1^\gamma p_2^\delta$$
$$\times\int d^4y_1d^4y_2d^4y_3\, e^{-ik_1\cdot y_1+ik_2\cdot y_2-ip_1\cdot y_3}$$
$$< 0|T^*\Big(\mathbf{j}^a_{A\alpha}(y_1)\mathbf{j}^b_{A\beta}(y_2)\mathbf{j}^c_{A\gamma}(y_3)\mathbf{j}^d_{A\delta}(0)\Big)|0>_{\rm conn.} \tag{31}$$

where s,t,u are the Mandelstam variables,

$$< 0|a^d_{\rm in}(p_2)\mathbf{V}^e_\alpha(y)a^{c\dagger}_{\rm in}(p_1)|0>_{\rm conn.}=$$
$$i\epsilon^{dec}(p_1+p_2)_\alpha\ \mathbf{F}_V(t)e^{-i(p_1-p_2)\cdot y} \tag{32}$$

is the pion electromagnetic form factor,

$$i\int d^4x e^{iq\cdot x} < 0|T^*\Big(\mathbf{V}^a_\alpha(x)\mathbf{V}^b_\beta(0)\Big)|0>=$$
$$\delta^{ab}\Big(-g_{\alpha\beta}q^2+q_\alpha q_\beta\Big)\mathbf{\Pi}_V(q^2) \tag{33}$$

is the isovector correlation function, and

$$< 0|a^d_{\rm in}(p_2)\sigma(y)a^{c\dagger}_{\rm in}(p_1)|0>_{\rm conn.}=$$
$$\delta^{cd}\ \mathbf{F}_S(t)e^{-i(p_1-p_2)\cdot y} \tag{34}$$

is the scalar form factor. Experimentally, (32-33) are well described by ρ dominance.

The unknown terms (30-31) may be estimated at low energies by expanding in $1/f_\pi$. The master equation (21) then truncates to

$$\frac{\delta\hat{\mathcal{S}}_0}{\delta J}=-i\hat{\mathcal{S}}_0G_RJ+i\hat{\mathcal{S}}_0\Big(1+G_R\mathbf{K}\Big)\pi_{\rm in}$$
$$=-i\hat{\mathcal{S}}_0G_AJ+i\Big(1+G_A\mathbf{K}\Big)\pi_{\rm in}\hat{\mathcal{S}}_0 \tag{35}$$

corresponding to the quadratic action

$$\mathbf{I}_Q=\frac12\int d^4x\Big((\nabla^\mu\pi)^a(\nabla_\mu\pi)^a-(\underline{a}^\mu\pi)^a(\underline{a}_\mu\pi)^a$$
$$-(m_\pi^2+s)\pi^a\pi^a\Big)+\int d^4x\ J^a\pi^a\ . \tag{36}$$

In (35-36), s and a^a_μ enter only through the combination $\hat s=s\mathbf{1}-\underline{a}_\mu\underline{a}^\mu$. If we take this to be true for $\hat{\mathcal{S}}_0$, we obtain a two-parameter fit to pionic data at one-loop level, which reproduces the KSFR relation [9]. Also, since $\hat s$ is isospin symmetric, the bulk of the ρ contribution to $\pi\pi$ scattering at low energies is given by (29) in a model independent manner.

With (35-36) and the assumption above, the sum (30-31) is given by

$$+\frac{i}{f_\pi^4}m_\pi^2\delta^{ab}\delta^{cd}\Big(2t-\frac52m_\pi^2\Big)\Big(\hat c_1+\mathcal{J}(t)\Big)$$
$$+\frac{i}{4f_\pi^4}\Big(2\delta^{ab}\delta^{cd}+\delta^{ac}\delta^{bd}+\delta^{ad}\delta^{bc}\Big)$$
$$\times\Big(t-2m_\pi^2\Big)^2\Big(\hat c_1+\mathcal{J}(t)\Big)$$
$$+\ 2\ {\rm perm.} \tag{37}$$

whereas (32-33) become

$$\mathbf{F}_V(t)=1+\frac{1}{2f_\pi^2}\Big(c_1t+\frac{t}{72\pi^2}+\frac13(t-4m_\pi^2)\mathcal{J}(t)\Big) \tag{38}$$

$$\Pi_V(t)=c_1+\frac{1}{72\pi^2}+\frac13\Big(1-\frac{4m_\pi^2}{t}\Big)\mathcal{J}(t) \tag{39}$$

where c_1 and $\hat c_1$ are the two constants and

$$\mathcal{J}(q^2)=-i\int\frac{d^4k}{(2\pi)^4}\Big(\frac{1}{k^2-m_\pi^2+i0}\frac{1}{(k-q)^2-m_\pi^2+i0}$$
$$-\Big(\frac{1}{k^2-m_\pi^2+i0}\Big)^2\Big)$$
$$=\frac{1}{16\pi^2}\int_0^1dx\frac{x(1-2x)q^2}{x(1-x)q^2-m_\pi^2+i0}\ . \tag{40}$$

The ρ data gives $c_1 = 0.035$ whereas a fit to the $\pi\pi$ scattering data [10] leads to seven determinations of $\hat{c}_1$

$$\begin{aligned}
16\pi^2\hat{c}_1 &= \frac{1024\pi^3}{63}\frac{f_\pi^4}{m_\pi^4}\left(a_0^0(\mathrm{exp}) - a_0^0(\mathrm{tree})\right) - \frac{14}{9} \\
&= 8 \pm 5 \\
\\
16\pi^2\hat{c}_1 &= \frac{64\pi^3}{9}\frac{f_\pi^4}{m_\pi^2}\left(b_0^0(\mathrm{exp}) - b_0^0(\mathrm{tree}) - b_0^0(\mathrm{rho})\right) \\
&\quad -\frac{91}{108} \\
&= 3 \pm 1 \\
\\
16\pi^2\hat{c}_1 &= 320\pi^3 f_\pi^4\left(a_2^0(\mathrm{exp}) - a_2^0(\mathrm{rho})\right) + \frac{73}{180} \\
&= 2 \pm 1 \\
\\
16\pi^2\hat{c}_1 &= 384\pi^3 \frac{f_\pi^4}{m_\pi^2}\left(a_1^1(\mathrm{exp}) - a_1^1(\mathrm{tree}) - a_1^1(\mathrm{rho})\right) \\
&\quad +\frac{1}{4} \\
&= 2 \pm 5 \\
\\
16\pi^2\hat{c}_1 &= \frac{512\pi^3}{3}\frac{f_\pi^4}{m_\pi^4}\left(a_0^2(\mathrm{exp}) - a_0^2(\mathrm{tree})\right) - \frac{4}{3} \\
&= 26 \pm 21 \\
\\
16\pi^2\hat{c}_1 &= \frac{128\pi^3}{3}\frac{f_\pi^4}{m_\pi^2}\left(b_0^2(\mathrm{exp}) - b_0^2(\mathrm{tree}) - b_0^2(\mathrm{rho})\right) \\
&\quad -\frac{35}{36} \\
&= -1 \pm 2 \\
\\
16\pi^2\hat{c}_1 &= 640\pi^3 f_\pi^4\left(a_2^2(\mathrm{exp}) - a_2^2(\mathrm{rho})\right) + \frac{19}{90} \\
&= 3 \pm 1
\end{aligned} \tag{41}$$

which is seen to be consistent, to the possible exception of $16\pi^2\hat{c}_1 = -1 \pm 2$. Here a_l^I and b_l^I stand respectively for the scattering lengths and range parameters with isospin I and orbital momentum l.

A comprehensive discussion of the present formulation, further applications and detailed comparison with previous work by other authors will be given elsewhere [11]. Extension to $SU_L(3) \times SU_R(3)$ is currently under investigation.

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[1] M. Veltman, Phys. Rev. Lett. **17** (1966) 553; J.S. Bell, Nuovo Cimento (Ser. X) **50 A** (1967) 129.

[2] W.A. Bardeen, Phys. Rev. **184** (1969) 1848.

[3] J. Wess and B. Zumino, Phys. Lett. **B37** (1971) 95.

[4] J. Schwinger, "Quantum Kinematics and Dynamics", W.A. Benjamin, N.Y. (1970).

[5] R.E. Peierls, Proc. Roy. Soc. **A214** (1952) 143.

[6] C.N. Yang and D. Feldman, Phys. Rev. **79** (1950) 972; G. Kallen, Ark. Fys. **2** (1950) 187, 371.

[7] N.N. Bogoliubov and D.V. Shirkov, *Introduction to the Theory of Quantized Fields*, John Wiley and Sons, 1980.

[8] Y. Tomozawa, Nuovo Cim. (ser. X) **46 A** (1966) 707; S. Weinberg, Phys. Rev. Lett. **17** (1966) 616; See also : K. Raman and E.C. Sudarshan, Phys. Lett. **21** (1966) 450.

[9] K. Kawarabayashi and M. Suzuki, Phys. Rev. Lett. **16** (1966) 255; Riazzuddin and Fayyazuddin, Phys. Rev. **147** (1966) 1071.

[10] J.L. Petersen, Phys. Rep. **C2** (1971) 155; M.M. Nagels *et al.*, Nucl. Phys. **B147** (1978) 189.

[11] H. Yamagishi and I. Zahed, SUNY-NTG-94-57.