# Superfield algorithms for topological field theories

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#### Abstract

A superfield algorithm for master actions of a class of gauge field theories including topological ones in arbitrary dimensions is presented generalizing a previous treatment in two dimensions. General forms for master actions in superspace are given, and possible theories are determined by means of a ghost number prescription and the master equations. The resulting master actions determine the original actions together with their gauge invariances. Generalized Poisson sigma models in arbitrary dimensions are constructed by means of this algorithm, and other simple applications in low dimensions are given including a derivation of the non-abelian Chern-Simon model.

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# 1 The basic formulation

In a previous paper [1] we gave a superfield algorithm for a class of master actions in two dimensions by means of which we derived generalized Poisson sigma models. (A superfield form of the master action for the ordinary Poisson sigma models [2, 3] was given by Cattaneo and Felder [4].) In this communication we generalize this algorithm to arbitrary dimensions. This provides then for a general framework for topological field theories and generalizations. These models all share the following properties: i) The equations of motion are of first order in the derivatives. ii) They are directly defined in terms of their corresponding master actions. iii) There are general simple rules for how the original actions are obtained from these master actions. iv) The quantum theory of the models are obtained by a gauge fixing of the master actions. For other related works see also refs. [5, 6, 7, 8, 9, 10, 11].

The master actions corresponding to an n-dimensional field theory will be entirely expressed in terms of fields on a supermanifold of dimension (n, n), i.e. half of the coordinates are bosonic, even ones, and half are fermionic, odd ones. To begin with we consider a class of master actions expressed in terms of pairs of superfields. They are

$$\Sigma[\Phi, \Phi^*] = \int d^n u d^n \tau \mathcal{L}_n(u, \tau), \tag{1}$$

where the Lagrangian densities  $\mathcal{L}_n(u,\tau)$  are given by

$$\mathcal{L}_n(u,\tau) = \Phi_A^*(u,\tau)D\Phi^A(u,\tau)(-1)^{\varepsilon_A+n} - S(\Phi(u,\tau),\Phi^*(u,\tau)), \tag{2}$$

where  $u^a$ , a = 1, 2, ..., n are bosonic coordinates on the base space and  $\tau^a$ , a = 1, 2, ..., n, the corresponding fermionic ones. D is the odd de Rham differential

$$D \equiv \tau^a \partial_a, \quad \partial_a \equiv \frac{\partial}{\partial u^a} \quad \Rightarrow \quad D^2 = 0.$$
 (3)

Since  $\Sigma$  is required to be even and since the measure  $d^n\tau$  is even or odd depending on whether n is even or odd, it follows that the Grassmann parity of the Lagrangian density is n, i.e.  $\varepsilon(\mathcal{L}_n(u,\tau)) = n$ . This implies that  $\varepsilon(S) = n$  and  $\varepsilon(\Phi_A^*) = \varepsilon_A + 1 + n$ , where  $\varepsilon_A \equiv \varepsilon(\Phi^A)$ . (For even n the form of  $\mathcal{L}_n$  in (2) is a generalization of the two dimensional treatment in [1], and for odd n it is a generalization of the onedimensional expression given in [6, 11]. Such superfield forms for master actions for topological field theories were also proposed in [5] (see e.g. [6]).)

The master actions (1) are required to satisfy the classical master equation

$$(\Sigma, \Sigma) = 0, \tag{4}$$

where the antibracket is defined by

$$(F,G) \equiv \int F \frac{\overleftarrow{\delta}}{\delta \Phi^{A}(u,\tau)} (-1)^{n\varepsilon_{A}} d^{n}u d^{n}\tau \frac{\overrightarrow{\delta}}{\delta \Phi^{*}_{A}(u,\tau)} G - (F \leftrightarrow G)(-1)^{(\varepsilon(F)+1)(\varepsilon(G)+1)},$$
(5)

which in turn may be defined by [14]

$$(F,G) = \Delta(FG)(-1)^{\varepsilon_F} - (\Delta F)G(-1)^{\varepsilon_F} - F(\Delta G) \tag{6}$$

in terms of the basic odd, nilpotent  $\Delta$ -operator

$$\Delta \equiv \int d^n u d^n \tau (-1)^{(n+1)\varepsilon_A} \frac{\delta}{\delta \Phi^A(u,\tau)} \frac{\delta}{\delta \Phi^*_A(u,\tau)}.$$
 (7)

This  $\Delta$ -operator also differentiates the antibracket (5) according to Leibniz' rule, *i.e.* 

$$\Delta(F,G) = (\Delta F,G) + (F,\Delta G)(-1)^{\varepsilon_F+1}.$$
 (8)

The functional derivatives in (5) and (7) are defined by the properties

$$\frac{\delta}{\delta \Phi^{B}(u,\tau)} \Phi^{A}(u',\tau') = \delta_{B}^{A} \delta^{n}(u-u') \delta^{n}(\tau-\tau') = \Phi^{A}(u,\tau) \frac{\overleftarrow{\delta}}{\delta \Phi^{B}(u',\tau')},$$

$$\varepsilon \left(\frac{\delta}{\delta \Phi^{A}}\right) = \varepsilon \left(\frac{\overleftarrow{\delta}}{\delta \Phi^{A}}\right) = \varepsilon_{A} + n,$$

$$F \frac{\overleftarrow{\delta}}{\delta \Phi^{A}(u,\tau)} d^{n}\tau = (-1)^{\varepsilon_{A}(\varepsilon(F)+1)} d^{n}\tau \frac{\delta}{\delta \Phi^{A}(u,\tau)} F,$$
(9)

where the delta-function in the odd coordinates  $\tau^a$  satisfies the properties

$$\int f(\tau')\delta^n(\tau - \tau')d^n\tau' = f(\tau) = \int d^n\tau'\delta^n(\tau' - \tau)f(\tau'). \tag{10}$$

An explicit calculation of the antibracket of the master actions (1) yields

$$\frac{1}{2}(\Sigma, \Sigma) = \int d^n u d^n \tau \left( D \mathcal{L}_n(u, \tau) + \frac{1}{2} (S, S)_n(u, \tau) \right)$$
(11)

where we have introduced the local *n*-bracket

$$(f,g)_n \equiv f \frac{\overleftarrow{\partial}}{\partial \Phi^A} \frac{\overrightarrow{\partial}}{\partial \Phi_A^*} g - (f \leftrightarrow g)(-1)^{(\varepsilon(f)+1+n)(\varepsilon(g)+1+n)}, \tag{12}$$

where f and g are local functions of  $\Phi^A(u,\tau)$  and  $\Phi^*_A(u,\tau)$ . For even n this is an antibracket and for odd n it is a Poisson bracket. In other words  $\Phi^A$  and  $\Phi^*_A$  are canonical conjugate field variables on an antisymplectic manifold for even n and on a symplectic manifold for odd n. The condition (4) combined with (11) yields the conditions

$$\int d^n u d^n \tau D \mathcal{L}_n(u, \tau) = 0, \tag{13}$$

which determines the allowed boundary conditions, and

$$(S,S)_n = 0. (14)$$

Furthermore, we have from (1) and (7)

$$\Delta \Sigma = 0 \tag{15}$$

since the  $\tau$ -part yields a factor zero. (As usual we believe that the bosonic part may be regularized appropriately.)  $\Sigma$  satisfies therefore also the quantum master equation

$$\frac{1}{2}(\Sigma, \Sigma) = i\hbar \Delta \Sigma, \tag{16}$$

when (13) and (14) are satisfied, which means that no quantum corrections of the measure in the path integral are required for these models.

If one treats the master action  $\Sigma$  in (1) as an ordinary action, then the equations of motion are

$$D\Phi^{A} = (S, \Phi^{A})_{n}, \quad D\Phi_{A}^{*} = (S, \Phi_{A}^{*})_{n},$$
 (17)

the consistency of which again requires (14). Notice that the equation of motion for S then is DS = 0. Notice also the relations

$$(\Sigma, \Phi^{A}) = (-1)^{n} \left( D\Phi^{A} - (S, \Phi^{A})_{n} \right),$$
  

$$(\Sigma, \Phi_{A}^{*}) = (-1)^{n} \left( D\Phi_{A}^{*} - (S, \Phi_{A}^{*})_{n} \right).$$
(18)

The superfields above may be Taylor expanded in the odd  $\tau$ -coordinates in such a fashion that the coefficient fields are fields and antifields in the ordinary sense on the base manifold. Or in other words such that the antibracket (5) and the  $\Delta$ -operator (7) have their conventional forms:

$$(F,G) = \sum_{r} \int F \frac{\overleftarrow{\delta}}{\delta \Phi^{rA}(u)} d^{n} u \frac{\overrightarrow{\delta}}{\delta \Phi^{*}_{rA}(u)} G - (F \leftrightarrow G)(-1)^{(\varepsilon(F)+1)(\varepsilon(G)+1)},$$

$$\Delta \equiv \sum_{r} \int d^{n} u (-1)^{\varepsilon_{rA}} \frac{\delta}{\delta \Phi^{rA}(u)} \frac{\delta}{\delta \Phi^{*}_{rA}(u)}, \quad \varepsilon_{rA} \equiv \varepsilon(\Phi^{rA}), \tag{19}$$

where  $\Phi^{rA}(u)$  and  $\Phi^*_{rA}(u)$  are the coefficient fields (r denotes antisymmetric u-indices). The antibracket in (19) must yield

$$(\Phi^A(u,\tau), \Phi_B^*(u',\tau')) = (-1)^{n\varepsilon_A} \delta_B^A \delta^n(u-u') \delta^n(\tau-\tau'), \tag{20}$$

which trivially follows from (5). An explicit construction is given in appendix B.

So far we have given superfield actions in arbitrary dimensions which under certain conditions satisfy the master equations. In the following we will always require  $\Sigma$  to actually satisfy the master equation, *i.e.* we require the boundary conditions to be consistent with (13) and the local equation (14) to be satisfied. In order to determine the class of gauge field theories which have such master actions we need also to prescribe ghost numbers to the field. We generalize then the prescription

given in [1] (such ghost numbers were also considered in [4, 6, 8, 11]): We choose the odd coordinates  $\tau^a$  to have ghost number plus one, which implies that D in (3) has ghost number plus one. Since we require the master action  $\Sigma$  to have ghost number zero and since the measure  $d^n\tau$  has ghost number -n the Lagrangian density  $\mathcal{L}_n$  in (2) must have ghost number n, *i.e.* 

$$gh(\Sigma) = 0, \quad gh(d^n\tau) = -n, \quad \Rightarrow \quad gh(\mathcal{L}_n) = n.$$
 (21)

The form (2) of  $\mathcal{L}_n$  leads us to the following general rule for the superfields and the local function S:

$$gh(\Phi^A) + gh(\Phi_A^*) = n - 1, \quad gh(S) = n.$$
 (22)

We assume that S is given by a power expansion in the superfields  $\Phi^A$  and  $\Phi_A^*$  in which case this ghost number prescription will restrict the possible terms in S. In such an analysis it is convenient to use the following convention:

$$gh(\Phi_A^*) \ge gh(\Phi^A),$$
 (23)

which in itself does not impose any restriction. Notice that  $\Phi^A$  and  $\Phi_A^*$  can only have equal ghost number in odd dimensions. (More precisely for n=2m+1 we may have the ghost numbers  $gh(\Phi^A)=gh(\Phi_A^*)=m$ .) Since the local bracket (12) satisfies

$$(\Phi^A(u,\tau), \Phi_B^*(u,\tau))_n = \delta_B^A, \tag{24}$$

it follows from the ghost number prescription (22) that the local bracket  $(,)_n$  in itself carries ghost number 1-n. From (20) and the fact that  $\delta^n(\tau-\tau')$  carries ghost number n, it follows that the antibracket (5) or equivalently (19) carries ghost number plus one which is the standard ghost number prescription for the antibracket in the field-antifield formalism.

When we for a given set of superfields which satisfy (22) and boundary conditions consistent with (13) also find an S satisfying gh(S) = n and  $(S, S)_n = 0$ , then we have a consistent master action  $\Sigma$  given by (1). From such a master action we may then extract the original model according to the following rules: First remove the integration measure. Then perform the following replacements:

$$D \longrightarrow \text{ exterior derivative } d$$

$$\Phi^{A}, gh(\Phi^{A}) = k \ge 0 \longrightarrow k\text{-form field } \phi^{A}, \ \varepsilon(\phi^{A}) = \varepsilon_{A} + k,$$

$$\Phi^{*}_{A}, gh(\Phi^{*}_{A}) = (n - 1 - k) \ge 0 \longrightarrow (n - 1 - k)\text{-form field } \phi^{*}_{A},$$

$$\varepsilon(\phi^{*}_{A}) = \varepsilon_{A} + k,$$

$$\Phi^{A}, gh(\Phi^{A}) < 0, \longrightarrow 0, \text{ or } \Phi^{*}_{A}, gh(\Phi^{*}_{A}) < 0, \longrightarrow 0,$$
ordinary multiplication  $\longrightarrow \text{ wedge products,}$ 

$$(25)$$

where the form fields,  $\phi^A$  and  $\phi_A^*$ , are fields on the *n*-dimensional *u*-space. These rules are easily extracted from the fact that the original fields,  $\phi^A(u)$  and  $\phi_A^*(u)$ , are

component fields with ghost number zero of the superfields,  $\Phi^A(u,\tau)$  and  $\Phi^*_A(u,\tau)$ . (Signs depend on how the coefficient fields are defined precisely.)

The superfield master actions determine also the gauge invariance of the original action. The appropriate gauge transformations may be obtained as follows: First identify the components of the superfields that are antifields to the original fields. These antifields have ghost number minus one and if  $\Phi^A$  contains the field then  $\Phi_A^*$  contains the corresponding antifield or vice versa. Identify then the terms in the master Lagrangian  $\int d^n \tau \mathcal{L}_n$  which are linear in these antifields. The terms in the coefficients of these antifields which contain no antifields represent then the gauge transformations of the field. The ghost fields with ghost number plus one are then to be interpreted as gauge parameters.

# 2 Master actions in terms of general superfields

In [1] we generalized the two-dimensional treatment along the lines of the generalized antisymplectic formulation in [12]. Here we give the corresponding treatment in arbitrary dimensions. Let the superfields  $Z^I(u,\tau)$ ,  $\varepsilon(Z^I) \equiv \varepsilon_I$ , represent arbitrary coordinates on a (anti)symplectic manifold. The local brackets (12) are then defined by

$$(f,g)_n \equiv f(Z(u,\tau)) \frac{\overleftarrow{\partial}}{\partial Z^I} E^{IK}(Z(u,\tau)) \frac{\overrightarrow{\partial}}{\partial Z^K} g(Z(u,\tau)), \tag{26}$$

which implies

$$(Z^{I}(u,\tau), Z^{K}(u,\tau))_{n} = E^{IK}(Z(u,\tau)).$$
 (27)

The functions  $E^{IK}$  have the properties

$$\varepsilon(E^{IK}) = \varepsilon_I + \varepsilon_K + 1 + n, \quad E^{KI} = -E^{IK}(-1)^{(\varepsilon_I + 1 + n)(\varepsilon_K + 1 + n)}.$$
 (28)

These properties follow from the requirements

$$\varepsilon((f,g)_n) = \varepsilon_f + \varepsilon_g + 1 + n, \quad (f,g)_n = -(g,f)_n(-1)^{(\varepsilon_f + 1 + n)(\varepsilon_g + 1 + n)}. \quad (29)$$

The Jacobi identities of the bracket (26), i.e.

$$((f,g)_n,h)_n(-1)^{(\varepsilon_f+1+n)(\varepsilon_h+1+n)} + cycle(f,g,h) = 0,$$
(30)

require in turn

$$E^{IL}\partial_L E^{JK}(-1)^{(\varepsilon_I+1+n)(\varepsilon_K+1+n)} + cycle(I,J,K) = 0.$$
(31)

We assume that  $Z^I$  span the bracket (26) which implies that  $E^{IK}$  is invertible. The inverse,  $E_{IK}$ , is defined by

$$E_{IL}E^{LK} = \delta_I^K = E^{KL}E_{LI}. (32)$$

It satisfies the properties

$$\varepsilon(E_{IK}) = \varepsilon_I + \varepsilon_K + 1 + n, \quad E_{KI} = -E_{IK}(-1)^{(\varepsilon_I + n)(\varepsilon_K + n) + n},$$
  
$$\partial_I E_{JK}(Z)(-1)^{(\varepsilon_I + n)\varepsilon_K} + cycle(I, J, K) = 0.$$
 (33)

The last relation follows from (32) and the Jacobi identities (31).

We define the generalized antibracket by

$$(F,G) \equiv F \int \frac{\overleftarrow{\delta}}{\delta Z^{I}(u,\tau)} d^{n}u d^{n}\tau (Z^{I}(u,\tau), Z^{K}(u',\tau')) d^{n}u' d^{n}\tau' \frac{\overrightarrow{\delta}}{\delta Z^{K}(u',\tau')} G,$$
(34)

where

$$(Z^{I}(u,\tau), Z^{K}(u',\tau')) = E^{IK}(Z(u,\tau))(-1)^{n\varepsilon_K} \delta^n(u-u')\delta^n(\tau-\tau') =$$

$$= \delta^n(u-u')\delta^n(\tau-\tau')(-1)^{n\varepsilon_I} E^{IK}(Z(u,\tau)). \tag{35}$$

The Jacobi identities are here

$$(Z^{I}(u,\tau), (Z^{J}(u',\tau'), Z^{K}(u'',\tau'')))(-1)^{(\varepsilon_{I}+1)(\varepsilon_{K}+1)} + cycle(I, u, \tau; J, u', \tau'; K, u'', \tau'') = 0,$$
(36)

which are satisfied due to (31). In fact, the properties of  $E^{IK}$  imply that the antibracket (34), (35) satisfies all required properties. Inserting (35) into (34) yields the expression

$$(F,G) \equiv F \int \frac{\overleftarrow{\delta}}{\delta Z^{I}(u,\tau)} d^{n}u d^{n}\tau E^{IK} (-1)^{n\varepsilon_{K}} \frac{\overrightarrow{\delta}}{\delta Z^{K}(u,\tau)} G. \tag{37}$$

This antibracket follows also from the relation (6) and the  $\Delta$ -operator

$$\Delta \equiv \frac{1}{2} \int d^n u d^n \tau (-1)^{\varepsilon_I} \rho^{-1} \frac{\delta}{\delta Z^I(u,\tau)} \rho E^{IK}(Z(u,\tau)) (-1)^{n\varepsilon_K} \frac{\delta}{\delta Z^K(u,\tau)},$$
(38)

where  $\rho(Z(u,\tau))$  is a measure density on the field-antifield space. This  $\Delta$ -operator differentiates the antibracket (37) according to the rules (8).

A master action  $\Sigma$  in terms of the superfields  $Z^{I}(u,\tau)$  may e.g. have the form (1) with a Lagrangian density given by

$$\mathcal{L}_n(u,\tau) = V_I(Z(u,\tau))DZ^I(u,\tau)(-1)^{\varepsilon_I+n} - S(Z(u,\tau)), \tag{39}$$

where the (anti)symplectic potential  $V_I(Z(u,\tau))$  has the Grassmann parity  $\varepsilon_I+n+1$ . Here we obtain the left functional derivative of  $\Sigma$  in the form:

$$\frac{\delta \Sigma}{\delta Z^I} = (-1)^{n\varepsilon_I} \left( E_{IK} D Z^K (-1)^{\varepsilon_K + n} - \partial_I S \right), \tag{40}$$

where  $E_{IK}$  is defined by

$$E_{IK}(Z) = \partial_I V_K(Z) - \partial_K V_I(Z) (-1)^{(\varepsilon_I + n)(\varepsilon_K + n) + n}.$$
 (41)

The equations of motion are therefore

$$DZ^I = (S, Z^I)_n. (42)$$

Consistency requires then again  $(S, S)_n = 0$ . A still more general form for the master action  $\Sigma$  is obtained from the Lagrangian density

$$\mathcal{L}_n(u,\tau) = Z^K(u,\tau)\bar{E}_{KI}(Z(u,\tau))DZ^I(u,\tau)(-1)^{\varepsilon_I+n} - S(Z(u,\tau)), \quad (43)$$

where

$$\bar{E}_{KI}(Z) \equiv (Z^J \partial_J + 2)^{-1} E_{KI}(Z) = \int_0^1 d\alpha \alpha E_{KI}(\alpha Z). \tag{44}$$

(Such an expression was first given in the symplectic case in [13].) Notice also the general form of the expression (18):

$$(\Sigma, Z^{I}) = (-1)^{n} \left( DZ^{I} - (S, Z^{I})_{n} \right). \tag{45}$$

When calculating  $(\Sigma, \Sigma)$  in terms of the antibracket (34) or (37), where  $\Sigma$  is expressed in terms of (39) or (43), we again find the relation (11). This means that the master equation  $(\Sigma, \Sigma) = 0$  also here requires the conditions (13), *i.e.*  $\int d^n u d^n \tau D \mathcal{L}_n = 0$ , which determines the allowed boundary conditions, and  $(S, S)_n = 0$  in terms of the bracket (26) which together with the ghost number prescription determine the allowed form of S.

The ghost number prescriptions are here

$$gh(S) = n$$
,  $gh(V_I) + gh(Z^I) = n - 1$ , each  $I$ , (46)

for the Lagrangian (39), and

$$gh(S) = n$$
,  $gh(Z^K) + gh(Z^I) + gh(\bar{E}_{IK}) = n - 1$ , each  $I$  and  $K$ , (47)

for the Lagrangian (43). The last relation together with (44), or (46) and (41) imply

$$gh(E_{IK}(Z)) = n - 1 - gh(Z^{I}) - gh(Z^{K}),$$
  

$$gh(E^{IK}(Z)) = 1 - n + gh(Z^{I}) + gh(Z^{K}),$$
(48)

where the last relation follows from (32). The last equality implies then that the local bracket  $(, )_n$  carries the extra ghost number 1 - n (see (27)) exactly what we had in the previous section. Only for n = 1 do we have the standard Poisson bracket.

# 3 Further generalizations

1) As in [1] we may choose arbitrary coordinates  $u^a$  on the surface. For the measure

$$d^n u \left( \det h_a^b(u) \right)^{-1} \tag{49}$$

we have the nilpotent D-operator

$$D = \tau^a T_a + \frac{1}{2} \tau^b \tau^a U_{ab}^c(u) \frac{\partial}{\partial \tau^c}, \tag{50}$$

where

$$T_a \equiv h_a^b(u) \frac{\partial}{\partial u^b}, \quad U_{ab}^c(u) \equiv -h_a^f(u) h_b^d(u) \frac{\partial}{\partial u^f} \left(h^{-1}\right)_d^c(u) - (a \leftrightarrow b).$$
 (51)

Notice that

$$D^2 = 0 \implies [T_a, T_b] = U_{ab}^c(u)T_c.$$
 (52)

- 2) If the original field theory is a superfield theory on a space with coordinates  $u^a$  having Grassmann parities  $\varepsilon_a$ , then the  $\tau^a$ -coordinates have Grassmann parities  $\varepsilon_a + 1$ . In this case we must assume that the superfields  $\Phi^A(u,\tau)$  still are possible to expand as a power series in  $\tau^a$  which then is an infinite expansion for the bosonic  $\tau^a$ -coordinates. (The fact that this implies an infinite number of component fields with arbitrarily low ghost numbers suggests that infinite reducibility is a generic feature in the superfield case.) If the original base manifold has dimension (n,m) then the supermanifold has dimension (n+m,n+m) and we get a subdivision into the two cases n+m odd or even. This means that the dimension of the base space still determines the odd and even cases.
- 3) The local functions S may also have explicit  $\tau$ -dependence, since the master actions above still satisfy the master equations provided S satisfies (14). However, such master actions are not of the same geometric nature as the previous ones since they do not lead to topological field theories due to the terms with explicit  $\tau$ -dependence. (This generalization was first considered in the one-dimensional treatments in [6, 11].)
- 4) A natural extension of the formalism in section 2 is to let  $E^{IK}(Z)$  be singular. It may e.g. be a Dirac bracket in which case we have

$$E_{(\mathcal{D})}^{IK} \equiv (Z^I, Z^K)_{n(\mathcal{D})} \equiv (Z^I, Z^K)_n - (Z^I, \Theta^{\mu}(Z))_n C_{\mu\nu}(\Theta^{\nu}(Z), Z^K)_n, \quad (53)$$

where  $\Theta^{\mu}(Z)$  are constraint variables such that  $C^{\mu\nu} \equiv (\Theta^{\mu}, \Theta^{\nu})_n$  is invertible with the inverse  $C_{\mu\nu}$ . The basic Dirac antibracket is then according to (35) given by

$$(Z^{I}(u,\tau), Z^{K}(u',\tau'))_{(\mathcal{D})} = E^{IK}_{(\mathcal{D})}(Z(u,\tau))(-1)^{n\varepsilon_{K}}\delta^{n}(u-u')\delta^{n}(\tau-\tau').$$
(54)

The local action S should be replaced by

$$S(Z(u,\tau)) \longrightarrow S(Z(u,\tau)) + \Pi_{\mu}(u,\tau)\Theta^{\mu}(Z(u,\tau)),$$
 (55)

where  $\Pi_{\mu}$  is a Lagrange multiplier superfield.

# 4 Generalized Poisson sigma models in any dimension.

Consider dimension n and consider first superfield pairs  $X^i$  and  $X_i^*$  with the properties

$$\varepsilon(X_i^*) = \varepsilon_i + 1 + n, \quad \varepsilon_i \equiv \varepsilon(X^i),$$
  

$$gh(X_i^*) = n - 1 - gh(X^i), \tag{56}$$

which are required by our general rules in section 1. Consider then the local function

$$S = \frac{1}{2} X_j^* X_i^* \omega^{ij}(X) (-1)^{\varepsilon_j + n}. \tag{57}$$

From the general prescription

$$\varepsilon(S) = n, \quad gh(S) = n,$$
 (58)

it follows that the functions  $\omega^{ij}(X)$  must satisfy the properties

$$\varepsilon(\omega^{ij}) = n + \varepsilon_i + \varepsilon_j,$$
  

$$gh(\omega^{ij}) = 2 - n + gh(X^i) + gh(X^j).$$
 (59)

The expression (57) implies furthermore that

$$\omega^{ij} = -\omega^{ji}(-1)^{(\varepsilon_i + n)(\varepsilon_j + n)}. (60)$$

The master equation  $(S, S)_n = 0$  requires then

$$\omega^{il}\partial_l \omega^{jk} (-1)^{(\varepsilon_i + n)(\varepsilon_k + n)} + cycle(i, j, k) = 0.$$
(61)

From these results it follows that  $\omega^{ij}(X)$  has exactly the same properties as the local bracket (26) for n-1. Therefore, we may make the identification

$$\omega^{ij}(X) = (X^i, X^j)_{n-1}, \tag{62}$$

where  $( , )_0$  may be identified with the conventional antibracket in the field-antifield formalism. For even n  $\omega^{ij}$  is a Poisson bracket and for odd n  $\omega^{ij}$  is an antibracket. But only for n = 1, 2 do these brackets have the conventional ghost number prescriptions.  $(( , )_1$  is the conventional Poisson bracket.) The boundary conditions must be consistent with (13).

When  $\omega^{ij}(X)$  satisfies the above properties and when the boundary conditions are consistent with (13) then the action

$$\Sigma[X, X^*] = \int d^n u d^n \tau \left( X_i^* D X^i (-1)^{\varepsilon_i + n} - S \right)$$
(63)

satisfies the master equation  $(\Sigma, \Sigma) = 0$ . However, for this case the original models may only be written down after we have given a more explicit form for  $\omega^{ij}(X)$ . The

only exception is for n = 2 and  $gh(X^i) = 0$  in which case, according to the rules (25), we directly obtain

$$A = \int \left( x_i^* \wedge dx^i - \frac{1}{2} x_j^* \wedge x_i^* \omega^{ij}(x) \right), \tag{64}$$

where  $x_i^*$  and  $x^i$  are one-form and zero-form fields respectively. (The Grassmann parities are  $\varepsilon(x_i^*) = \varepsilon(x^i) \equiv \varepsilon_i$ .) This is just the well-known Poisson sigma model [2, 3] for which Cattaneo and Felder also gave the superfield master action given here for  $\varepsilon(x^i) = 0$  [4]. One may easily check that the boundary conditions given in [4] are consistent with (13).

In [1] generalized Poisson sigma models for n = 2 were constructed by means of the algorithm presented here. This construction may also be generalized to arbitrary n. We consider then the following expression for S:

$$S = \frac{1}{2} X_j^* X_i^* \omega^{ij}(X) (-1)^{\varepsilon_j + n} + \Lambda_\alpha^* \theta^\alpha(X), \tag{65}$$

where we have introduced new superfield pairs  $\Lambda_{\alpha}^*$ ,  $\Lambda^{\alpha}$  with the properties

$$\varepsilon(\Lambda_{\alpha}^{*}) = \varepsilon_{\alpha} + n, \quad \varepsilon(\Lambda^{\alpha}) = \varepsilon_{\alpha} + 1, \quad \varepsilon_{\alpha} \equiv \varepsilon(\theta^{\alpha}), 
gh(\Lambda^{\alpha}) = gh(\theta^{\alpha}) - 1, \quad gh(\Lambda_{\alpha}^{*}) = n - gh(\theta^{\alpha}).$$
(66)

The local master equation  $(S, S)_n = 0$  requires now apart from (61) also

$$\theta^{\alpha}(X) \overleftarrow{\partial}_{i} \omega^{ji}(X) = 0, \tag{67}$$

which means that  $\omega^{ji}(X)$  may be interpreted as a Dirac bracket of the n-1 type, i.e. a bracket  $(\cdot,\cdot)_{n-1}$  satisfying  $(\cdot,\theta^{\alpha}(X))_{n-1}=0$ .

The expression (65) is not the general expression of S in terms of these fields. We may also add the terms

$$(-1)^{\varepsilon_j+n+(\varepsilon_i+n)(\varepsilon_k+n)} \frac{1}{6} X_i^* X_j^* X_k^* \omega_\alpha^{kji}(X) \Lambda^\alpha + (-1)^{\varepsilon_i+n} X_i^* \Lambda_\alpha^* \omega_\beta^{\alpha i}(X) \Lambda^\beta + \dots,$$

$$(68)$$

where

$$\varepsilon(\omega_{\alpha}^{kji}(X)) = \varepsilon_i + \varepsilon_j + \varepsilon_k + \varepsilon_{\alpha}, \quad \varepsilon(\omega_{\beta}^{\alpha i}(X)) = \varepsilon_i + \varepsilon_{\alpha} + \varepsilon_{\beta} + n, 
gh(\omega_{\alpha}^{kji}(X)) = gh(X^i) + gh(X^j) + gh(X^k) - gh(\theta^{\alpha}) + 4 - 2n, 
gh(\omega_{\beta}^{\alpha i}(X)) = gh(X^i) + gh(\theta^{\alpha}) - gh(\theta^{\beta}) + 2 - n,$$
(69)

and where  $\omega_{\alpha}^{kji}$  is totally antisymmetric in i, j, k with Grassmann parity  $\varepsilon_{ijk} = (\varepsilon_i + n)(\varepsilon_j + n) + (\varepsilon_j + n)(\varepsilon_k + n) + (\varepsilon_k + n)(\varepsilon_i + n)$ . In this case the master equations yield a weak form of the Jacobi identities (61) and the degeneracy condition (67), which means that  $\omega^{ij}(X)$  now is a weak Dirac bracket of the n-1 type. For all these forms of S the master action is given by

$$\Sigma[X, X^*, \Lambda, \Lambda^*] = \int d^n u d^n \tau \left( X_i^* D X^i (-1)^{\varepsilon_i + n} + \Lambda_\alpha^* D \Lambda^\alpha (-1)^{\varepsilon_\alpha + n + 1} - S \right).$$

$$(70)$$

We may also add further superfield pairs  $\Xi^{\alpha_k}$ ,  $\Xi^*_{\alpha_k}$ , k = 1, ..., L, which enter S to leading order in the form  $\sum_{k=0}^{L-1} \Xi^*_{\alpha_{k+1}} Z^{\alpha_{k+1}}_{\alpha_k}(X) \Xi^{\alpha_k}$   $(k \geq 0, \alpha_0 \equiv \alpha, \Xi^{\alpha_0} \equiv \Lambda^{\alpha})$  where

$$\varepsilon(Z_{\alpha_k}^{\alpha_{k+1}}) = \varepsilon_{\alpha_{k+1}} + \varepsilon_{\alpha_k}, \quad \varepsilon(\Xi^{\alpha_k}) \equiv \varepsilon_{\alpha_k} + k + 1,$$
  

$$gh(Z_{\alpha_k}^{\alpha_{k+1}}) = gh(\Xi^{\alpha_{k+1}}) - gh(\Xi^{\alpha_k}) + 1.$$
(71)

The master action is here

$$\Sigma[X, X^*, \Xi, \Xi^*] =$$

$$= \int d^n u d^n \tau \left( X_i^* D X^i (-1)^{\varepsilon_i + n} + \sum_{k=0}^L \Xi_{\alpha_k}^* D \Xi^{\alpha_k} (-1)^{\varepsilon_{\alpha_k} + k + 1 + n} - S \right), \quad (72)$$

and the master equations tell us that  $\theta^{\alpha}$  is reducible to stage L, which means  $Z_{\alpha}^{\alpha_1}\theta^{\alpha}=0, Z_{\alpha_2}^{\alpha_3}Z_{\alpha_1}^{\alpha_2}\approx 0, \ldots, Z_{\alpha_{L-1}}^{\alpha_L}Z_{\alpha_{L-2}}^{\alpha_{L-1}}\approx 0.$ 

The original model and its properties of all these modified forms of S can only be written down when we have given a more explicit form for the ghost dependent functions. However, for n = 2 and  $gh(X^i) = 0$  we obtain according to the rules (25) for all the above cases the original action

$$A = \int \left( x_i^* \wedge dx^i - \frac{1}{2} x_j^* \wedge x_i^* \omega^{ij}(x) - \lambda_\alpha \theta^\alpha(x) \right), \tag{73}$$

where  $\lambda_{\alpha}$  is a Lagrange multiplier 2-form field.

Still more generalized forms of Poisson sigma models might be possible to derive if we allow S to have explicit  $\tau$ -dependence which is possible according to the generalization 3 in section 3.

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# A Further applications

#### A.1 Models in n=1

Consider first superfield pairs  $X^i$  and  $X_i^*$  with ghost number zero and arbitrary Grassmann parities  $\varepsilon(X_i^*) = \varepsilon(X^i) \equiv \varepsilon_i$ . If these are the only superfields then

S=0, since S must have ghost number one. The original model is then trivial and is of the form  $A=\int x_i^*dx^i$ . If we also have the superfield pairs  $\Lambda_\alpha^*$  and  $\Lambda^\alpha$  with ghost numbers one and minus one respectively, then S has the general form  $(\varepsilon(\Lambda_\alpha^*)=\varepsilon(\Lambda^\alpha)=\varepsilon_\alpha+1,\,\varepsilon_\alpha\equiv\varepsilon(\theta^\alpha))$ 

$$S = \Lambda_{\alpha}^* \theta^{\alpha}(X, X^*) + \frac{1}{2} \Lambda_{\alpha}^* \Lambda_{\beta}^* U_{\gamma}^{\beta \alpha}(X, X^*) \Lambda^{\gamma} (-1)^{\epsilon_{\alpha}} + \dots, \tag{74}$$

where the dotted terms are determined by the condition  $(S, S)_1 = 0$ . Notice that S is odd and that  $(, )_1$  is a conventional Poisson bracket. In fact,

$$(S,S)_1 = 0 \quad \Rightarrow \quad (\theta^{\alpha}, \theta^{\beta})_1 = U_{\gamma}^{\alpha\beta} \theta^{\gamma}.$$
 (75)

Thus, the master action

$$\Sigma[X, X^*, \Lambda, \Lambda^*] = \int du d\tau \left( X_i^* D X^i (-1)^{\varepsilon_i + 1} + \Lambda_\alpha^* D \Lambda^\alpha (-1)^{\varepsilon_\alpha} - S \right), \quad (76)$$

where S is given in (74) satisfies the master equation provided the constraint variables  $\theta^{\alpha}$  are in involutions. We may also add further pairs  $\Xi_{\alpha_k}^*$ ,  $\Xi^{\alpha_k}$  with ghost number k > 1 and -k. The resulting S satisfying  $(S, S)_1 = 0$  will then imply that the constraint variables  $\theta^{\alpha}$  are reducible (linearly dependent) up to a certain stage. Thus, S may attain the general form of a BFV-BRST charge for a constraint theory where the constraints are in arbitrary involutions on the phase space spanned by the canonical coordinates  $X^i$  and  $X_i^*$ .  $\Lambda_{\alpha}^*$  are ghosts and  $\Xi_{\alpha_k}^*$  ghost for ghosts. The corresponding BRST charge for the original model is the  $\tau = 0$  component of the  $S(u,\tau)$  in (74). The boundary condition (13) requires the conservation of this charge, i.e.  $S(u_2,0) - S(u_1,0) = 0$  where  $u_1$  and  $u_2$  are the limits of the u-integration in  $\Sigma$ . The original action is obtained from the corresponding master action using the rules (25). We find for all the above cases the original action

$$A = \int \left( x_i^* dx^i - \lambda_\alpha \theta^\alpha(x, x^*) \right), \tag{77}$$

where  $\lambda_{\alpha}$  is a Lagrange multiplier one-form field. This action is defined on the phase space where  $x_i^*$  and  $x^i$  are canonical conjugate variables ( $x_i^*$  are conjugate momenta to  $x^i$ ). It is gauge invariant under the gauge transformations

$$\delta x^{i} = (x^{i}, \theta^{\alpha})_{1} \beta_{\alpha}, \quad \delta x_{i}^{*} = (x_{i}^{*}, \theta^{\alpha})_{1} \beta_{\alpha},$$

$$\delta \lambda_{\alpha} = d\beta_{\alpha} (-1)^{\varepsilon_{\alpha}} - \beta_{\mu} \lambda_{\nu} U_{\alpha}^{\nu\mu} (-1)^{\varepsilon_{\mu}},$$
(78)

where  $\beta^{\alpha}$  are the gauge parameters. The theory (77) is an arbitrary constraint theory with zero Hamiltonian, which implies that the theory is reparametrization invariant. In fact, any one-dimensional Hamiltonian model may be cast into this form, since any such theory may be cast into a reparametrization invariant form. However, a nonzero Hamiltonian  $H(X, X^*)$  is possible to introduce if we allow S to have an explicit  $\tau$ -dependence (generalization 3 in section 3, see also [6]). Condition (14) requires then  $H(X, X^*)$  to have zero local Poisson bracket with the S in (74). We may also choose fields which are arbitrary symplectic coordinates if we make use of the general master actions following from (39) and (43). These results agree with those obtained in [6].

### A.2 The Chern-Simon model and generalizations

In n=3 the natural choice of superfield pairs are  $X^i$  and  $X_j^*$  both with ghost number one. Since S must have ghost number three, S must be trilinear in  $X^i$  and  $X_j^*$ . This case is simpler to analyze if we let the superfield  $Z^I$  represent both  $X^i$  and  $X_j^*$ .  $Z^I$  are then Darboux coordinates on a symplectic manifold. The local Poisson bracket is then

$$(Z^I, Z^K)_3 = E^{IK},$$
 (79)

where  $E^{IK}$  is constant. (Notice that the bracket ( , )<sub>3</sub> carries an extra ghost number -2 which implies  $gh(E^{IK})=0$  for  $gh(Z^I)=1$ .) The general form of S is in this case

$$S = \frac{1}{6}C_{IJK}Z^IZ^JZ^K. (80)$$

If we let the fields  $Z^I$  be odd fields corresponding to original fields which are even one-form fields, then  $C_{IJK}$  in (80) are even real constants. Notice also that  $E^{IK}$  in (79) is symmetric for odd fields. The master action with the Lagrangian density (39) becomes in this case

$$\Sigma[Z] = \frac{1}{2} \int d^3u d^3\tau \left( E_{IK} Z^K D Z^I - \frac{1}{3} C_{IJK} Z^I Z^J Z^K \right). \tag{81}$$

The master equation  $(\Sigma, \Sigma) = 0$  allows us then to interpret  $C_{IJK}$  as structure coefficients of a Lie group, since  $C_{IJK}$  is totally antisymmetric from (80) and since  $(S, S)_3 = 0$  requires them to satisfy the Jacobi identities.  $E^{IK}$  acts as a group metric. According to the rules (25) the original model is

$$A = \frac{1}{2} \int E_{IK} z^I \wedge \left( dz^K - \frac{1}{3} E^{KL} C_{LMN} z^M \wedge z^N \right), \tag{82}$$

which is a non-abelian Chern-Simon model.  $z^I$  are even, one-form fields. (The Chern-Simon model was also treated in [5].)

We may also introduce superfield pairs with ghost numbers two and zero respectively, and pairs with ghost numbers three and minus one. These superfields allow for many more terms in S which are consistent with the ghost number prescription. It seems likely that the local master equation (14) then will allow for new nontrivial solutions, in which case one will find generalized Chern-Simon models. All fields must satisfy boundary conditions which are consistent with (13).

In n=4 we may obtain something similar to the Chern-Simon model if we choose superfield pairs  $X^i$  and  $X_i^*$  with ghost number one and two respectively. It is then natural to let  $X^i$  be odd and  $X_i^*$  even since the corresponding original one-and two-form fields,  $x^i$  and  $x_i^*$ , then are even according to the rule (25). The general form of S is then

$$S = \frac{1}{2}C^{ij}X_i^*X_j^* + \frac{1}{2}C^i{}_{jk}X_i^*X^jX^k + \frac{1}{24}C_{ijkl}X^iX^jX^kX^l,$$
(83)

where the coefficients are even real constants. In this case the condition  $(S, S)_4 = 0$  yields the following three conditions:

$$C^{i}_{jk}C^{kl}X_{i}^{*}X_{l}^{*}X^{j} = 0, (84)$$

$$\left(C^{i}_{jm}C^{m}_{kl} + \frac{1}{3}C_{jklm}C^{mi}\right)X_{i}^{*}X^{j}X^{k}X^{l} = 0,$$
(85)

$$C_{ijkn}C^n_{lm}X^iX^jX^kX^lX^m = 0. (86)$$

We notice the following special solutions:

- i) If  $C^{ij} = C_{ijkl} = 0$  then  $C^{i}_{jk}$  may be interpreted as structure coefficients of a Lie group since  $(S, S)_4 = 0$  requires the Jacobi identities due to (85).
- ii) If only  $C_{ijkl} = 0$  then the same interpretation of  $C^i_{jk}$  is possible. However, in this case  $(S, S)_4 = 0$  requires not only the Jacobi identities for  $C^i_{jk}$  but also the condition

$$C^{i}_{jl}C^{lk} + C^{k}_{jl}C^{li} = 0, (87)$$

from (84). If  $C^{ij}$  is an invertible matrix, then  $C_{ij}$  exists satisfying  $C^{ij}C_{jk} = \delta_k^i$  and (87) may be rewritten as

$$C_{ijk} + C_{kji} = 0, \quad C_{ijk} \equiv C_{il}C^l_{\ ik}. \tag{88}$$

Thus, the symmetric matrix  $C^{ij}$  may be interpreted as a group metric and  $C_{ijk}$  is totally antisymmetric as required by a semi-simple Lie group. This corresponds to what we had in the Chern-Simon model in three dimensions. The original model is according to the rule (25):

$$A = \int \left( x_i^* \wedge dx^i - \frac{1}{2} C^{ij} x_i^* \wedge x_j^* - \frac{1}{2} C^i_{jk} x_i^* \wedge x^j \wedge x^k \right), \tag{89}$$

where  $x^i$  and  $x_i^*$  are even one- and two-form fields respectively. (They are the ghost number zero coefficients of  $X^i$  and  $X_i^*$ .) This action is gauge invariant under the transformations

$$\delta x^{i} = d\beta^{i} + C^{ij}\gamma_{j} + C^{i}_{jk}\beta^{j}x^{k},$$
  

$$\delta x^{*}_{i} = d\gamma_{i} + C^{k}_{ij}x^{*}_{k}\beta^{j} + C^{k}_{ij}\gamma_{k} \wedge x^{j},$$
(90)

where the gauge parameters  $\beta^i$  and  $\gamma_i$  are zero- and one-forms respectively.

Also here we may introduce further superfield pairs with ghost numbers three and zero respectively and pairs with ghost numbers four and minus one etc which will allow for new terms in S. The local master equation (14) should then allow for new nontrivial solutions in which case one obtains generalized models. Again all fields must satisfy boundary conditions which are consistent with (13).

From the kinetic terms in the superfield Lagrangians (2) it is clear that we get a BF-theory in any dimension n, and in any dimension we may also have a Lagrangian multiplier which is an n-form field in the original actions. However, in higher and higher dimensions there are more and more different k-form fields allowed. The most general structure will therefore be more and more complex in higher and higher dimensions.

# B Superfields in terms of component fields

A decomposition of the superfields in component fields satisfying (19) may be obtained by means of the following recursive formula. We start with zero components  $\Phi_0^A$  and  $\Phi_{0B}^*$  satisfying

$$(\Phi_0^A(u), \Phi_{0B}^*(u')) = \delta_B^A \delta^n(u - u'). \tag{91}$$

We write then the superfields as follows

$$\Phi^{A}(u,\tau^{1},\ldots,\tau^{n-1},\tau^{n}) = \Phi_{0}^{A}(u,\tau^{1},\ldots,\tau^{n-1}) + \tau^{n}\Phi_{1}^{A}(u,\tau^{1},\ldots,\tau^{n-1}), (92)$$

$$\Phi_A^*(u,\tau^1,\ldots,\tau^{n-1},\tau^n) = \\
= \left( (-1)^{n-1} \Phi_{1A}^*(u,\tau^1,\ldots,\tau^{n-1}) - \Phi_{0A}^*(u,\tau^1,\ldots,\tau^{n-1})\tau^n \right) (-1)^{\varepsilon_A}, \quad (93)$$

where

$$(\Phi^{A}(u,\tau^{1},\ldots,\tau^{n-1}),\Phi_{B}^{*}(u',\tau'^{1},\ldots,\tau'^{n-1})) =$$

$$= \delta^{n}(u-u')\delta^{n-1}(\tau-\tau')\delta_{B}^{A}(-1)^{(n-1)\varepsilon_{A}}.$$
(94)

Then by construction the *n*-parametric superfields (92) and (93) satisfy the relations (94) with the formal replacement  $n \to n+1$ . One may easily chech that the following expressions satisfy these recursion formulas:

$$\Phi^{A}(u,\tau) = \Phi_{0}^{A}(u) + \tau^{a}\Phi_{a}^{A}(u) + \frac{1}{2}\tau^{a_{1}}\tau^{a_{2}}\Phi_{a_{1}a_{2}}^{A}(u) + \cdots 
\cdots + \frac{1}{k!}\tau^{a_{1}}\cdots\tau^{a_{k}}\Phi_{a_{1}\cdots a_{k}}^{A}(u) + \cdots + \frac{1}{n!}\tau^{a_{1}}\cdots\tau^{a_{n}}\Phi_{a_{1}a_{2}\cdots a_{n}}^{A}(u), 
\Phi^{*}_{A}(u,\tau) = \left(\frac{1}{n!}\Phi_{A}^{*a_{1}a_{2}\cdots a_{n}}(u) - \frac{1}{(n-1)!}\Phi_{A}^{*a_{1}a_{2}\cdots a_{n-1}}(u)\tau^{a_{n}} + \right) 
+ \frac{1}{2}\frac{1}{(n-2)!}\Phi_{A}^{*a_{1}a_{2}\cdots a_{n-2}}(u)\tau^{a_{n-1}}\tau^{a_{n}} + \cdots 
\cdots + (-1)^{k}\frac{1}{k!(n-k)!}\Phi_{A}^{*a_{1}\cdots a_{n-k}}(u)\tau^{a_{n-k+1}}\cdots\tau^{a_{n}} + \cdots 
\cdots + (-1)^{n}\frac{1}{n!}\Phi_{A}^{*0}(u)\tau^{a_{1}}\cdots\tau^{a_{n}}\right)(-1)^{n\varepsilon_{A}}\varepsilon_{a_{1}a_{2}\cdots a_{n}}, \tag{95}$$

where  $\varepsilon_{a_1 a_2 \cdots a_n}$  is totally antisymmetric such that  $\varepsilon_{12 \cdots n} = 1$ . We also define  $\varepsilon_1 \equiv 1$ ,  $\varepsilon_0 \equiv 1$ . Notice that  $\Phi^A_{a_1 \cdots a_k}$  and  $\Phi^{*a_1 \cdots a_k}_A$  are totally antisymmetric, and that we have

$$(\Phi_{a_1 \cdots a_k}^A(u), \Phi_B^{*a_1 \cdots a_k}(u')) = \delta_B^A \delta^n(u - u'). \tag{96}$$

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