Robustness of Decoherence-Free States for Charge Qubits under Local Non-uniformity

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We analyze the robustness of decoherence-free (DF) subspace and subsystem in charge qubits, when difference from the collective decoherence measurement condition is large (∼5%) in the long time period, which is applicable for solid-state qubits using as a quantum memory. We solve master equations of up to four charge qubits and a detector as a quantum point contact (QPC). We show that robustness of DF states is strongly affected by local non-uniformities. We also discuss the possible two-qubit logical states by exactly solving the master equations.

Although decoherence is the largest obstacle for quantum information processing, a lot of powerful active methods for correcting effects of decoherence have been discovered[1]. As to the passive anti-decoherence protection, *decoherence-free* (DF) *subspace*[2,3] and *subsystem*[4] are shown to be very useful for *collective decoherence environment*, in which all qubits suffer the same disturbance from the environment. Singlet state is the only DF state for two qubits and there are two independent DF subspace bases for four qubits, *e.g.*

$$
\begin{aligned}
|\Psi_1^{[4]}\rangle_{(1234)} &= 2^{-1}(|01\rangle - |10\rangle)_{(12)} \otimes (|01\rangle - |10\rangle)_{(34)}, \\
|\Psi_2^{[4]}\rangle_{(1234)} &= 1/(2\sqrt{3})(2|0011\rangle - |0101\rangle - |0110\rangle - |1001\rangle \\
&- |1010\rangle + 2|1100\rangle)_{(1234)} \qquad (1)
\end{aligned}
$$

where $|1001\rangle_{(1234)} = |1\rangle_1|0\rangle_2|0\rangle_3|1\rangle_4$ and so on. The DF subsystem starts from three qubits. Experiments have succeeded until four qubits in photon system[5,6] and three qubits in nuclear magnetic resonance (NMR)[7]. Bacon*et al.*[3] also showed that, even if there is a symmetry breaking perturbation from the collective environment, which is parameterized by a coupling strength η, the DF subspace is robust in the order of $O(\eta)$ when $\eta \ll 1$.

However, in the case of solid-state qubits, even a single qubit is hard to fabricate and the redundancy regarding the number of qubits would be a critical issue in constructing a large qubit system. First of all, we could not prepare plenty of qubits with mathematically exact size. The sizes of Cooper-pair box of Ref.[8] and GaAs quantum dot (QD)[9] where coherent oscillation can be observed are less than hundreds of nm. The requirement of a few % fluctuation between qubits would result in controllability of a few nm in fabrication. This would be unrealistic until future when fabrication process is greatly advanced. The fluctuation of sizes would lead to that of interaction amplitude between qubits and a measurement apparatus, and that of the applied gate bias, in addition to the effect of randomly distributed background traps[10,11]. Note that even a roughness of the order of 1 Å at interfaces affects current characteristics in advanced LSI technologies as shown in Ref.[12] Thus, the non-uniformity of solid-state qubits to collective decoherence environment is much larger and rather *local* compared with that of optical or NMR qubits, and it will be necessary to consider the effects of second order $O(\eta^2)$ or higher symmetry-breaking perturbation.

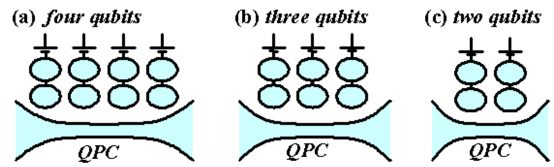

FIG. 1: Qubits that use double dot charged states are capacitively coupled to a QPC detector.

In this paper, we theoretically describe the effect of large non-uniformity (∼5%) of charge qubit parameters on DF states based on time-dependent density matrix (DM) equations considering the measurement process by detector current. Detection of qubit states induces a *backaction* on the qubit state resulting in a corrupting of qubit states. Thus, the measurement is an important interaction with the environment for qubits. The charge qubit is a two-level system[13] controlled by gate electrodes[10,14], and constituted from coupled QDs where one excess electron is inserted, assuming there is one energy level in each QD. The detector discussed here is a quantum point contact (QPC) in the tunneling region depicted in Fig.1. The position of the excess charge affects the QPC current electrically, resulting in detection of charged state. Experiments[15,16] have successfully proved the high sensitivity of QPC detector current $I = G_0 T v_d$ ($G_0 = 2e^2/h \sim 77\mu$S, T is a transmission coefficient v_d is a bias between electrodes) in the tunneling region($T < 1$). The QPC current induces *shot noise* as the basic and unavoidable noise[16], and the cause of decoherence treated here. The purpose of this study is to investigate the robustness of many-qubit DF states for *local* large non-uniformity and show the possibility of using non-DF states with two-qubit singlet states during the QPC measurement. Two-qubit DM is analytically solved and perfectly analyzed.

The Hamiltonian for the combined qubits and the QPC is written as $H = H_{\rm qb} + H_{\rm qpc} + H_{\rm int}$. $H_{\rm qb}$ describes the interacting N qubits (Fig.) : $H_{\rm qb} = \sum_{i=1}^{N}(\Omega_i\sigma_{ix} + \epsilon_i\sigma_{iz})$ $+\sum_{i=1}^{N-1} J_{i,i+1}\sigma_{iz}\sigma_{i+1z}$, where Ω_i and ϵ_i are the inter-QD tunnel coupling and energy difference (gate bias) within each qubit. Here the spin operators are expressed by annihilation operators of an electron in the upper and lower QDs of each qubit. $J_{i,i+1}$ is a coupling constant between two nearest qubits, originating from capacitive couplings in the QD system[14]. $|\uparrow\rangle$ and $|\downarrow\rangle$ refer to the two single-qubit states in which the excess charge is localized in the upper and lower dot, respectively. $H_{\rm qpc} = \sum_{\alpha=L,R}\sum_{i_\alpha s} E_{i_\alpha} c^\dagger_{i_\alpha s} c_{i_\alpha s} + \sum_{i_L,i_R s} V_s (c^\dagger_{i_L s} c_{i_R s} + c^\dagger_{i_R s} c_{i_L s})$ describes the QPC. Here $c_{i_L s}(c_{i_R s})$ $s=\uparrow,\downarrow$ is the annihilation operator of an electron in the i_Lth (i_Rth) level ($i_L(i_R) = 1, ..., n$) of the left(right) electrode. $H_{\rm int}$ is the (capacitive) interaction between the qubits and the QPC, that induces *dephasing* between different eigenstates of σ_{iz}[2]. Most importantly, it contains the fact that localized charge near the QPC increases the energy of the system electrostatically, thus affecting the tunnel coupling between the left and right electrodes:

$$H_{\rm int} = \sum_{i_L,i_R,s} \left[\sum_{i=1}^{N} \delta V_{is} \sigma_{iz}\right] (c^\dagger_{i_L s} c_{i_R s} + c^\dagger_{i_R s} c_{i_L s}). \quad (2)$$

Note that the case where δV_i is independent of the qubit corresponds to *collective environment*, thus the DF states are realized. Hereafter we neglect the spin dependence of V and δV_i. We assume that the tunneling rate through the N qubits, Γ, is composed of direct series of each N tunneling rate near i-th qubit, Γ_i, such as $\Gamma^{-1} = \sum_i \Gamma_i^{-1}$, where Γ_i is defined as $\Gamma_i^{(\pm)} \equiv 2\pi \wp_L \wp_R (v_d/N)|V \pm \delta V_i|^2$ ($\wp_L$ and $\wp_R$ are the density of states of the electrodes at the Fermi surface) depending on the qubit state $\sigma_{iz} = \pm 1$. The strength of measurement is parameterized by $\Delta\Gamma_i$ as $\Gamma_i^{(\pm)} = \Gamma_{i0} \pm \Delta\Gamma_i$. We call $|\downarrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$, $|\uparrow\downarrow\rangle$, and $|\uparrow\uparrow\rangle$ as $|A\rangle \sim |D\rangle$ respectively, and four-qubit states are written by $|AA\rangle$, $|AB\rangle$ ...$|DD\rangle$. For uniform two qubits, $\Gamma_A = \Gamma_0(1-\zeta)/2$, $\Gamma_B = \Gamma_C = \Gamma_0(1-\zeta^2)/2$ and $\Gamma_D = \Gamma_0(1+\zeta)/2$ with $\zeta \equiv \Delta\Gamma/\Gamma_0$. The DM equations of N qubits and detector at zero temperature are derived similar to Ref.[17] as

$$\begin{aligned}\frac{d\rho_{z_1 z_2}}{dt} &= i[J_{z_2} - J_{z_1}]\rho_{z_1 z_2} - i\sum_{j=1}^{N} \Omega_j (\rho_{g_j(z_1), z_2} - \rho_{z_1, g_j(z_2)}) \\ &- \left[\sqrt{\Gamma^{z_1}} - \sqrt{\Gamma^{z_2}}\right]^2 \rho_{z_1 z_2} \end{aligned} \quad (3)$$

where $z_1, z_2 = AA, AB, ..., DD$ for four qubits (256 equations) and $z_1, z_2 = A, B, C, D$ for two qubits (16 equations). $J_{AA} = \sum_i^4 \epsilon_i + J_{12} + J_{23}$, $J_{AB} = \sum_i^3 \epsilon_i - \epsilon_4 + J_{12} - J_{23}$, ..., $J_{DD} = -\sum_i^4 \epsilon_i + J_{12} + J_{23}$. $g_l(z_i)$ and $g_r(z_i)$ are introduced for the sake of notational convenience and determined by the relative positions between qubit states. For two qubits, $g_1(A) = B$, $g_2(A) = C$, $g_1(B) = A$, $g_2(B) = D$, $g_1(C) = D$, $g_2(C) = A$, $g_1(D) = C$, $g_2(D) = B$. Because the detector current is $I = e\sum_{z=A,..,D} \Gamma^z \rho_{zz}$, we have $\Delta T/T \sim \Delta\Gamma/\Gamma_0 (= \zeta)$[18]. In the two-qubit case, we use entangled Bell basis: $|a\rangle \equiv (|A\rangle + |D\rangle)/\sqrt{2}$, $|b\rangle \equiv (|A\rangle - |D\rangle)/\sqrt{2}$, $|c\rangle \equiv (|B\rangle + |C\rangle)/\sqrt{2}$, $|d\rangle \equiv (|B\rangle - |C\rangle)/\sqrt{2}$.

Decoherence rates– From Eq. (3), the dephasing is expected to be relevant to the *dephasing rate* $\mathit{\Gamma}_{\rm d}(z_1, z_2) \equiv [\sqrt{\Gamma_{z_1}} - \sqrt{\Gamma_{z_2}}]^2$. To see the decoherence effect explicitly, we study time-dependent *fidelity*, $F(t) \equiv {\rm Tr}[\rho(0)\rho'(t)]$ on the rotating coordinate as $\hat{\rho}'(t) = e^{i\sum \Omega'_i \sigma_{ix} t} \hat{\rho}(t) e^{-i\sum \Omega'_i \sigma_{ix} t}$ ($\Omega'_i \equiv \sqrt{\Omega_i^2 + \epsilon_i^2/4}$) to eliminate the bonding-antibonding coherent oscillations of free qubits (trace is carried out over qubit states). $F(t)$ can be expanded in time as $F(t) = 1 - \sum_{n=1}(1/n!)(t/\tau^{(n)})^n$ where $1/\tau^{(n)} = \{-{\rm Tr}[\rho(0) d^n \rho(0)/dt^n]\}^{1/n}$ (*decoherence rates*). Using Eq.(3) for two qubits, $1/\tau_c^{(1)} = 1/\tau_d^{(1)} = (1/2)\mathit{\Gamma}_{\rm d}(B, C) \sim (\Gamma_0/8)(1-\zeta^2)\zeta^2\eta^2$ when $|B\rangle$ and $|C\rangle$ are not symmetric such as the left qubit has a local fluctuation $\Gamma^{(\mp)}(1-\eta)$. Moreover, $(1/\tau_c^{(2)})^2 = (\Omega_1 + \Omega_2)^2 + (\epsilon_1 - \epsilon_2)^2 + \mathit{\Gamma}_{\rm d}^2(B, C)/4$ and $(1/\tau_d^{(2)})^2 = (\Omega_1 - \Omega_2)^2 + (\epsilon_1 - \epsilon_2)^2 + \mathit{\Gamma}_{\rm d}^2(B, C)/4$. Thus, the symmetry-breaking terms start from $O(\eta^2)$[3]. We have similar expressions for three-qubit DF *subsystem* and four-qubit DF *subspace*. For $|\Psi_1^{[3]}\rangle \equiv (|010\rangle - |100\rangle)/\sqrt{2}$[4], $1/\tau_{1[3]}^{(1)} = (1/2)\mathit{\Gamma}_{\rm d}(010, 100)$ and for $|\Psi_1^{[4]}\rangle_{(1234)}$,

$$\frac{1}{\tau_{1[4]}^{(1)}} = \frac{1}{8} \sum_{z_1,z_2=BB,BC,CB,CC} \mathit{\Gamma}_{\rm d}(z_1, z_2), \quad (4)$$

both of which also start from $O(\eta^2)$ (decoherence rate of $|\Psi_2^{[4]}\rangle_{(1234)}$ has a similar but more complicated form). The robustness of $N \geq 3$ qubits changes depends on which qubit includes local fluctuation. For example, if the leftmost qubit of $|\Psi_1^{[3]}\rangle$ fluctuates, $1/\tau_{1[3]}^{(1)} \sim (\Gamma_0/(3+\zeta)^3)(1-\zeta^2)\zeta^2\eta^2$, but $1/\tau_{1[3]}^{(1)} = 0$ when the rightmost qubit fluctuates. In the latter case, there is a symmetry between the leftmost and middle qubit and we can say that if some symmetry remains, DF states are robust. To see the dependence of spatial arrangement of qubits, we also consider $|\Psi_3^{[4]}\rangle_{(1234)} \equiv |\Psi_1^{[4]}\rangle_{(1423)}$ (hereafter we omit the subscript). $|\Psi_3^{[4]}\rangle$ is expected to be more susceptible than $|\Psi_1^{[4]}\rangle$, because the former is not a product of two singlet states as $|\Psi_1^{[4]}\rangle$ and less symmetric exchanges of qubit states are possible when qubit parameters fluctuate. Thus, the DF states are strongly affected by the distribution of local non-uniformities.

Numerical calculations support these analyses. Here we add fluctuations *locally* to Ω_i, ϵ_i and Γ_i respectively to various DF states of $N \leqq 4$ qubits. In case (i), only 3rd qubit fluctuates as $\Omega_3 \to \Omega_3(1-\eta)$, $\epsilon_3 \to \epsilon_3(1-\eta)$ and $\Gamma_3 \to \Gamma_3(1-\eta)$. In case (ii), the 2nd and 3rd qubits fluctuate. In case (iii) only 4th qubit fluctuates. Fig. 2 (a) shows a time-dependent $F(t)$ in a strong measurement case of $\zeta = 0.6$ of 1% fluctuations ($\eta = 0.01$) and Fig.2 (b) shows that of a weak measurement case of $\zeta = 0.2$ of 5% fluctuations ($\eta = 0.05$), both in degeneracy point $\epsilon = 0$ (*relaxation* decoherence region[10,11]). Although Fig. 2 (a) indicates that the four-qubit DF states are fairly robust, Fig. 2 (b) shows that the large fluctuation (5%) greatly degrades $|\Psi_3^{[4]}\rangle$ in case (iii). If we take Γ_0=100MHz, its lifetime ($F(t) > 0.75$) is about $50\Gamma_0^{-1} \sim 500ns$, which is much shorter than lowest order estimation of $\eta^{-2}\Gamma_0^{-1} \sim 40\mu s$. This shows that higher symmetry-breaking perturbation terms cannot be neglected and there is a case where a product state is more preferable than four-qubit DF states. $F(t)$ for $|\Psi_2^{[4]}\rangle$ is in the same order of $|\Psi_1^{[4]}\rangle$, which means that DF states composed of many entangled states seem fairly robust at the degeneracy point. The three-qubit DF states are fairly robust for larger η, and show susceptibilities to measurement strength as the singlet state and other Bell states (Fig.2(a)). In the case of finite bias ϵ (*pure dephasing* region), the effect of which appears from $1/\tau^{(2)}$, $F(t)$ of $|\Psi_2^{[4]}\rangle$ and $|\Psi_3^{[4]}\rangle$ seem more susceptible than the singlet state and the singlet type $|\Psi_1^{[4]}\rangle$ (Fig.3). The other Bell states and three-qubit DF states are less susceptible than $|\Psi_2^{[4]}\rangle$ and $|\Psi_3^{[4]}\rangle$, which would again be due to the higher order symmetry-breaking terms. To summarize these results, DF states of many qubit ($N=4$) are robust for most cases but there is a case where even product states might be better under large symmetry-breaking fluctuations ($\eta \gtrsim 5\%$) in the long term.

Analytical solution for two-qubit case– The probability that the unexpected non-uniformities induce symmetry-breaking effects becomes higher as the number of qubits increases as shown above. Because preparing many solid-state qubits is not easy, the redundancy of coding qubits is a trade-off against the fabrication difficulty. Thus, using the singlet state and one of the non-DF states, $\{\rho_{aa}, \rho_{bb}, \rho_{cc}\}$ in two qubit space would sometimes be a realistic solution to construct two logical states $|0\rangle_L$ and $|1\rangle_L$. Here we investigate which of $\{\rho_{aa}, \rho_{bb}, \rho_{cc}\}$ is appropriate for the second basis in two-qubit space. When we move to four Bell bases under the conditions that the two qubits are identical with no interaction between them ($J_{ij}=0$) and no bias $\epsilon_i=0$ (collective environment), Eqs.(3) are divided into the following five groups:

$$\dot{\rho}_{dd} = 0 \tag{5}$$

$$\dot{\rho}_{bd} = -\gamma^B \rho_{bd} \tag{6}$$

$$\begin{cases} \dot{\rho}_{ad} &= -2i\Omega\rho_{cd} - \gamma^B \rho_{ad} \\ \dot{\rho}_{cd} &= -2i\Omega\rho_{ad} \end{cases} \tag{7}$$

$$\begin{cases} \dot{\rho}_{ab} &= -2i\Omega\rho_{cb} + \frac{1}{2}\gamma^D(\rho_{ba} - \rho_{ab}) \\ \dot{\rho}_{bc} &= 2i\Omega\rho_{ba} - \gamma^B \rho_{bc} \end{cases} \tag{8}$$

$$\begin{cases} \dot{\rho}_{aa} &= 2i\Omega(\rho_{ac} - \rho_{ca}) - \frac{1}{2}\gamma^D(\rho_{aa} - \rho_{bb}) \\ \dot{\rho}_{bb} &= \frac{1}{2}\gamma^D(\rho_{aa} - \rho_{bb}) \\ \dot{\rho}_{cc} &= -2i\Omega(\rho_{ac} - \rho_{ca}) \\ \dot{\rho}_{ac} &= 2i\Omega(\rho_{aa} - \rho_{cc}) - \gamma^B \rho_{ac} \end{cases} \tag{9}$$

where $\gamma^D = \Gamma_{\rm d}(A, D)$ and $\gamma^B = \Gamma_{\rm d}(A, B)$. These equations can be solved analytically. First, singlet state ρ_{dd} is time-independent (DF state) and $\rho_{bd}(t) = \rho_{bd}(0)e^{-\gamma^B t}$. $\rho_{ad}(t)$ and $\rho_{cd}(t)$ behave like $e^{-(\gamma^B \pm \sqrt{(\gamma^B)^2 - 16\Omega^2})t}$, and $\rho_{ab}(t)$ and $\rho_{bc}(t)$ behave like $e^{-(\gamma^B + \gamma^D \pm \sqrt{(\gamma^B + \gamma^D)^2 - 16\Omega^2})t/2}$, thus decay rates of $\rho_{ad}(t)$, $\rho_{cd}(t)$, $\rho_{ab}(t)$,and $\rho_{bc}(t)$ depend on whether $4\Omega > \gamma^B$, $\gamma^B + \gamma^D$ or $4\Omega < \gamma^B$, $\gamma^B + \gamma^D$, respectively. From Eq.(9), $\rho_{aa}(t) + \rho_{bb}(t) + \rho_{cc}(t)$ is conserved and $\rho_{ac}(t) + \rho_{ca}(t) = (\rho_{ac}(0) + \rho_{ca}(0))e^{-\gamma_B t}$. Thus, $\rho_{aa}(t)$, $\rho_{bb}(t)$, $\rho_{cc}(t)$ and

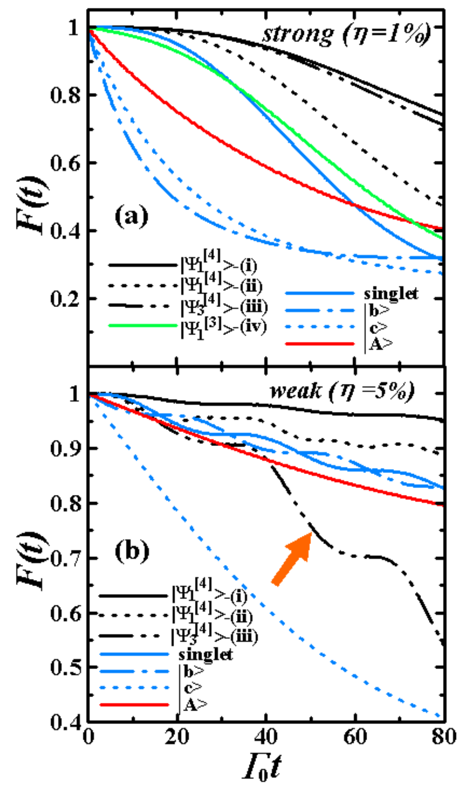

FIG. 2: Time-dependent fidelity of four-qubit DF states ($|\Psi_1^{[4]}\rangle$ and $|\Psi_3^{[4]}\rangle$) three-qubit DF state of $|\Psi_1^{[3]}\rangle$, and two-qubit states (singlet, $|b\rangle$, $|c\rangle$, and product state $|A\rangle$) under various fluctuations : (i)$\Omega_3 = (1-\eta)\Omega$, $\epsilon_3 = \eta\Gamma_0$ and $\Gamma_3^{(\pm)} = (1-\eta)\Gamma^{(\pm)}$. (ii)$\Omega_2 = \Omega_3 = (1-\eta)\Omega$, $\epsilon_2 = \epsilon_3 = \eta\Gamma_0$ and $\Gamma_2^{(\pm)} = \Gamma_3^{(\pm)} = (1-\eta)\Gamma^{(\pm)}$. (iii)$\Omega_4 = (1-\eta)\Omega$, $\epsilon_4 = \eta\Gamma_0$ and $\Gamma_4^{(\pm)} = (1-\eta)\Gamma^{(\pm)}$. (iv)$\Omega_2 = (1-\eta)\Omega$, $\epsilon_2 = \eta\Gamma_0$ and $\Gamma_2^{(\pm)} = (1-\eta)\Gamma^{(\pm)}$. (iv) is also applied to the two qubits. (a) η=0.01 and ζ=0.6 (strong measurement), (b) η=0.05 and ζ=0.2 (weak measurement). $\Omega = 2\Gamma_0$, $J_{ij} = 0$ $\epsilon_i = 0$.

$\rho_{ac}(t)$ can be analytically obtained by solving three-order polynomial equations. In the case of high qubit oscillation $\Omega \gg \gamma^B, \gamma^D$ and weak measurement such as $\zeta \ll 1$, we obtain $\gamma^D \sim 4\gamma^B$ in order of ζ^2. Then we have the eigenvalues of time-dependent matrix equations for $\{\rho_{aa}, \rho_{cc}, \rho_{ac} - \rho_{ca}\}$ in order of $\delta \equiv \gamma^B/(4\Omega)$ as $-3\gamma^B$, $-\gamma^B \pm 4\Omega i\left(1 - 5/3\delta^2\right)$, which shows that the period of qubit oscillation is delayed by the measurement through γ^B and γ^D. Thus we found that the qubit behavior strongly depends on the relative magnitude of Ω to γ and this is also an important factor in the selection of the possible candidate of the logical basis. In Fig. 2(b) of $\Omega = 2\Gamma > \gamma^D \sim 0.2\Gamma$, $F(t)$s of $|b\rangle$ and $|d\rangle$ are larger than those of $|c\rangle$ (and $|a\rangle$). In Ref.[19], we argued the coherent oscillation of the DM without the detector and showed that the DM for $|b\rangle$ and $|d\rangle$ are time-independent at $\epsilon=0$. This indicates that $|b\rangle$ is the candidate in the $\Omega \gg \gamma_B, \gamma_D$ region, reflecting that $|b\rangle$ becomes an unpolarized triplet suffering less degradation from the repulsive Coulomb in-

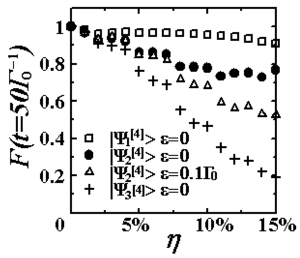

FIG. 3: Fidelity of four-qubit DF states at $t = 50\Gamma_0^{-1}$in the case of (iii) as a function of non-uniformity. $\Omega = 2\Gamma_0$, $J_{ij} = 0$ and ζ=0.2.

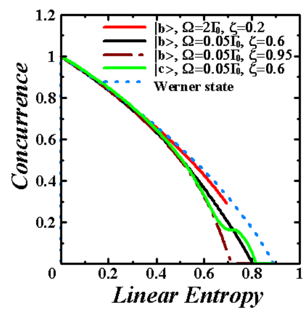

FIG. 4: Linear entropy S_L and concurrence C plane under the measurement during $0 \leqq t \leqq 100\Gamma_0$ for two qubit states $|b\rangle$ and $|c\rangle$ ($\epsilon = 0$). In weak measurement case of $\zeta \leqq 0.2$, C does not degrade to 0. All data start closely to Werner state.

teraction of the detector. On the other hand, in the $\Omega = 0$ case which can be in the limit of $\Omega \ll \gamma^D, \gamma^B$, Eq. (3) is easily solved even with a finite bias ϵ and ρ_{cc} is found to be a time-independent unpolarized triplet, being the candidate of logical state.

Next, we compare ρ_{bb} and ρ_{cc} in the *purity plane*, that is a relation between *linear entropy* and *concurrence*[20]. Linear entropy $S_L = 4/3(1 - \mathrm{Tr}(\hat{\rho}^2))$ expresses purity of qubits, ranging from 0 (pure state) and 1 (maximally-mixed state). From Eq.(9), $S_L = 4/3[1 - \rho_{aa}^2 - \rho_{bb}^2 - \rho_{cc}^2 - 2|\rho_{ac}|^2]$, and we have $dS_L/dt = (2/3)\gamma^D(\rho_{bb} - \rho_{aa})^2 + (8/3)\gamma^B|\rho_{ac}|^2$. Concurrence C, which is a measure of entanglement, is also given from Eq.(9). Starting from $\rho_{bb}(0) = 1$, at $t \sim 0$ in the case of $\Omega \gg \gamma_D = 4\gamma_B$, we have $C \sim (-1 + 4e^{-3\gamma^B t})/3$ ($dC/dt \sim -\gamma^D\rho_{bb}/2$) and $S_L \sim 8/9(1 - e^{-6\gamma^B t})$. Then dC/dS_L ∼-3/4 near $t \sim 0$. If we start $\rho_{cc}(0) = 1$ in the case of $\Omega \ll \gamma_D = 4\gamma_B$, we have $C \sim (-1 + 4e^{-(2/3)\Omega t})$ and $S_L \sim 8/9(1 - e^{-(4/3)\Omega t})$, and thus dC/dS_L ∼-3/4. If we check the Werner state $\gamma_w|b\rangle\langle b| + (1 - \gamma_w)/4\hat{I}_2 \otimes \hat{I}_2$ where $\hat{I}_2$ is 2x2 unit matrix $(1 > \gamma_w > 0)$[20,21], we obtain $dC^{(w)}/dS_L^{(w)} = -3/4$ at $\gamma_w = 1$. Thus both ρ_{bb} and ρ_{cc} coincide with the Werner state at $t \sim 0$, which shows that the two states are good entangled states, because the Werner state is a mixture of the maximally entangled state[20]. Figure 4 shows that both ρ_{bb} and ρ_{cc} evolve close to the Werner state. Thus, the two states behave similarly in the purity plane, and are equal candidates.

The noise spectrum $S(\omega)$ of the QPC without qubits is given by $S(\omega) = e^2\Gamma/\pi$ (white noise) in the present model, thus, the shot noise affects qubit states in full frequency domain. Astafiev *et al.*[22] experimentally showed that the main causes of the noise in the Josephson qubits are f noise and the background charge noises or $1/f$ noise, which we do not include. These noises would locally affect qubits and degrade the robustness of the DF states more than discussed here.

In conclusion, we have solved master equations of many qubits and QPC detector, and discuss the robustness of DF states under the large non-uniformities ($\sim 5\%$) during the long time period. Two-qubit non-DF states are shown to be one solution in constructing logical qubits.

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[18] To differentiate the DF states, local measurements are required[5]. In Fig.1, this means that other independent detectors are required or Γ_i should be different for each qubit as in Ref.[17]. However, because the aim of this paper is to see the effect of local non-uniformity, we take the simple setup of Fig.1.
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