Solutions of the Master Virasoro Equation as Conformal Points of Perturbed WZNW Model

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Abstract

It is shown that the master equation of the affine-Virasoro construction on the unitary affine algebra naturally emerges in the fusion algebra of the nonunitary level k WZNW model. Operators corresponding to solutions of the master equation are suitable for performing one-parametrical renormalizable perturbation around the given conformal nonunitary WZNW model. In the large $|k|$ limit, the infrared fixed point of the renormalization group beta function is found. There are as many infrared conformal points as $\frac{1}{2}N(N+1)$, where N is the total number of solutions of the master equation.

August 1994

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1   Introduction

There is a large class of two dimensional CFT's which are described by the affine-Virasoro construction [1],[2]. The latter is the most general bilinear of the affine currents. A study of embeddings of the affine-Virasoro construction into the affine algebra has resulted in a master equation [1],[2], particular solutions of which correspond to the affine-Sugawara [3],[4],[5], coset [3],[4],[6], and spin-orbit [1],[3],[7] constructions. Being exactly solvable the aforementioned conformal theories admit proper Lagrangian formulations. There has been growing interest, recently, in the question of whether or not all other solutions (emeddings) of the master equation (affine-Virasoro construction) can have natural Lagrangian descriptions [8]-[11].

A classical action for generic affine-Virasoro construction has been obtained in [8] by utilizing the Hamiltonian approach to the Wess-Zumino-Novikov-Witten (WZNW) model [12]. Whereas in [9] an affine-Virasoro action is built up in terms of the conformal non-Abelian Thirring model. As yet it is not clear whether there is a link between these two actions. We have shown explicitly the conformal symmetry of the non-Abelian Thirring model at the isoscalar Dashen-Frishman conformal point [13] which turns out to be one of the solutions of the master Virasoro equation. However, it is very difficult to arrive at all other conformal points starting from the nonconformal Thirring model. Such an attempt has been done in ref. [11]. Finally ref. [10] was an effort to get the master equation from the vanishing of renormalization group beta functions of a certain nonlinear sigma model. Unfortunately, the last approach does not lead beyond the classical solutions of the master equation. This result was somewhat programmed by the use of the $1/k$-expansion method, where k is level of the underlying affine algebra. Indeed, in the large k limit, the master equation collapses to the classical approximation [14] which cannot give more than classical solutions.

The aim of the present paper is to suggest another application of the master equation in the quest for new conformal lagrangian field theories. We will describe a situation in which the master equation emerges not as a condition of the conformal symmetry but as a condition of renormalizability of a certain quantum field theory. Additional requirement of

conformal invariance will result in an equation for the *one*-parametrical coupling constant.

Our approach to a certain extent is a generalization of the idea of perturbing the nonunitary WZNW model by its kinetic term [16]. The latter appears to be an appropriate relevant unitary Virasoro primary operator in the spectrum of the nonunitary model [16]. Certainly, the nounitary WZNW model is a highly nontrivial theory. Because it has states with negative norm, its proper definition is quite complicated and has not been yet done completely. Therefore, part of our discussion may seem to be somewhat formal. However, we will argue that all operators and manipulations on them which we will use in this paper are unitary and well defined. We believe that the nonunitary WZNW model itself will be finally properly understood.

The paper is organized as follows. In section 2 the master Virasoro equation associated with the affine-Virasoro construction is derived from the fusion algebra of the nonunitary WZNW model. In section 3 the operators, whose fusion algebra gives rise to the master equation, are utilized to perturb the nonunitary WZNW model. We show that the perturbed theory has as many infrared conformal points as $\frac{1}{2}N(N+1)$, where N is the total number of solutions to the master equations.

2 Master Virasoro equation from nonunitary WZNW model

Let us consider the level k WZNW model whose action is given as follows [15]

$$S(k) = -\frac{k}{4\pi}\{\int \text{Tr}|g^{-1}\text{d}g|^2 \ + \ \frac{i}{3}\int \text{d}^{-1}\text{Tr}(g^{-1}\text{d}g)^3\}, \tag{1}$$

where g is the matrix field taking its values on the Lie group G. At this point we do not specify whether k is positive or negative. The theory possesses the affine symmetry $\hat{G}\times\hat{G}$ which entails an infinite number of conserved currents [5],[15]. The latter can be derived from the basic currents J and $\bar{J}$,

$$J = J^a t^a = -\frac{k}{2}g^{-1}\partial g, \qquad \bar{J} = \bar{J}^a t^a = -\frac{k}{2}\bar{\partial}gg^{-1}, \tag{2}$$

satisfying the equations of motion

$$\bar{\partial}J = 0, \qquad \partial\bar{J} = 0. \tag{3}$$

In eqs. (2) t^a are the generators of the Lie algebra $\mathcal{G}$ associated with the Lie group G,

$$[t^a, t^b] = f^{abc} t^c, \tag{4}$$

with f^{abc} the structure constants.

The spectrum of the WZNW model contains states which correspond to the primary fields of the underlying affine symmetry [5]. By definition, ϕ_i is an affine (chiral) primary field, if it has the following operator product expansion (OPE) with the affine current J [5]

$$J^a(w)\phi_i(z) = \frac{t_i^a}{w - z}\phi_i(z) \ + \ reg., \tag{5}$$

where the matrices t_i^a correspond to the (left) representation of $\phi_i(z)$. In the WZNW model, any affine primary field is Virasoro primary and its conformal dimension is given by [5]

$$\Delta_i = \frac{c_i}{c_V + k}, \tag{6}$$

where $c_i = t_i^a t_i^a$ and c_V is defined according to

$$f^{acd} f^{bcd} = c_V \ \delta^{ab}. \tag{7}$$

There are Virasoro primary states in the spectrum of the WZNW model which are the descendants of the affine primary vectors. One of such states was considered in [9],[16]. In what follows we are going to describe a class of such fields.

Let us take the following composite field

$$O^{L,\bar{L}} = L_{ab} \ \bar{L}_{\bar{a}\bar{b}} \ : J^a \ \bar{J}^{\bar{a}} \phi^{b\bar{b}} : \tag{8}$$

which is defined as a normal ordered product of the affine currents J^a, $\bar{J}^{\bar{a}}$ with the isospin (1,1) affine-Virasoro primary field $\phi^{b\bar{b}}$ in the adjoint representation of $G \times G$. The product of the three operators in eq. (8) can be properly defined according to

$$O^{L,\bar{L}}(z, \bar{z}) = L_{ab}\bar{L}_{\bar{a}\bar{b}} \ \oint \frac{dw}{2\pi i} \ \oint \frac{d\bar{w}}{2\pi i} \frac{J^a(w)\bar{J}^{\bar{a}}(\bar{w}) \ \phi^{b\bar{b}}(z, \bar{z})}{|z - w|^2}, \tag{9}$$

where the product in the numerator of the integrand is understood as an OPE. It is easy to see that the given product does not contain singular terms provided the matrices L_{ab} and $\bar{L}_{\bar{a}\bar{b}}$ are *symmetrical*.

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From the definition it follows that the operator $O^{L,\bar{L}}$ is an affine descendant of the affine-Virasoro primary field ϕ. Indeed, $O^{L,\bar{L}}$ can be presented in the form

$$O^{L,\bar{L}}(0) = L_{ab}\bar{L}_{\bar{a}\bar{b}}\ J^a_{-1}\bar{J}^{\bar{a}}_{-1}\phi^{b\bar{b}}(0), \tag{10}$$

where

$$J^a_m = \oint \frac{dw}{2\pi i}w^m J^a(w), \qquad \bar{J}^{\bar{a}}_m = \oint \frac{d\bar{w}}{2\pi i}\bar{w}^m \bar{J}^{\bar{a}}(\bar{w}). \tag{11}$$

Being an affine descendant, the operator $O^{L,\bar{L}}$ continues to be a Virasoro primary operator. Indeed, one can check that the state $O^{L,\bar{L}}(0)|0\rangle$ is a highest weight vector of the Virasoro algebra, with $|0\rangle$ the $SL(2,C)$ invariant vacuum. That is,

$$L_0 O^{L,\bar{L}}(0)|0\rangle = \Delta_O\ O^{L,\bar{L}}(0)|0\rangle,$$

$$\tag{12}$$

$$L_{m>0}O^{L,\bar{L}}(0)|0\rangle = 0.$$

Here the generators L_n are given by the contour integrals

$$L_n = \oint \frac{dw}{2\pi i}\ w^{n+1}T(w), \tag{13}$$

where $T(w)$ is holomorphic component of the Sugawara stress tensor of the conformal WZNW model,

$$T(z) = \frac{:J^a(z)J^a(z):}{k+c_V}. \tag{14}$$

In eqs. (12), Δ_O is the conformal dimension of the operator $O^{L,\bar{L}}$. We find

$$\Delta_O = \bar{\Delta}_O = 1 + \frac{c_V}{k+c_V}. \tag{15}$$

Here $\bar{\Delta}_O$ is the conformal dimension of $O^{L,\bar{L}}$ associated with antiholomorphic conformal transformations. In what follows, we will discuss the WZNW model with negative level. This theory has states with negative norm. Remarkably the given operator $O^{L,\bar{L}}$ corresponds to a unitary highest weight vector of the Virasoro algebra when $|k| > c_V$ [17]. Therefore, all correlation functions of this operator make sense even when k is negative.

Clearly, operators $O^{L,\bar{L}}$ with arbitrary symmetrical matrices L_{ab}, $\bar{L}_{\bar{a}\bar{b}}$ are Virasoro primary vectors with the same conformal dimensions. However, their fusion algebras may

be different. We would like to focus on a particular subclass of operators $O^{L,\bar{L}}$ which obey the following fusion

$$O^{L,\bar{L}} \cdot O^{L,\bar{L}} = [O^{L,\bar{L}}] \ + \ [I] \ + ... \tag{16}$$

where the square brackets denote the contributions of $O^{L,\bar{L}}$ and identity operator I and the corresponding descendants of $O^{L,\bar{L}}$ and I, whereas dots stand for all other admitted operators with different conformal dimensions. It is important to emphasize that the operator $O^{L,\bar{L}}$ on the left and right hand sides of eq. (16) has one and the same pairs of matrices L_{ab} and $\bar{L}_{\bar{a}\bar{b}}$. We will show that eq. (16) leads to algebraic equations for the given matrices. These equations will be the main subject of this section.

Due to the theorem of holomorphic factorization, we can forget for a while about antiholomorphic part of the operator $O^{L,\bar{L}}$. First of all, we have to compute the following OPE

$$\phi^a(w) \ \phi^b(z) = \sum_I (w-z)^{\Delta_I - 2\Delta_\phi} \ C_I^{ab}[\Phi^I(z)], \tag{17}$$

where $[\Phi^I]$ are conformal classes of all Virasoro primaries Φ^I arising in the fusion of two ϕ's. For our purposes, it is sufficient to calculate $C_c^{ab} \ [\phi^c]$.

Let us set z to zero in eq. (17). Then after acting on the $SL(2,C)$ vacuum $|0\rangle$, eq. (17) gives

$$\phi^a(w)|\phi^b\rangle = w^{-\Delta_\phi}C_c^{ab} \ [|\phi^c\rangle] \ + \ ... \tag{18}$$

The structure constants C_c^{ab} and the terms of $[|\phi^c\rangle]$ can be deduced from the invariance of eq. (18) under the affine symmetry. Indeed, by acting with J_0^a and J_1^a on both sides of eq. (18), we find

$$C_c^{ab} \ [|\phi^c\rangle] = A\{f^{abc}|\phi^c\rangle \ + \ w \ J_{-1}^a|\phi^b\rangle \ + \ ...\}, \tag{19}$$

where A is an overall constant whose value is not essential for our consideration. It is instructive to verify that for $G = SU(2)$ and positive k the same expression (19) can be derived by using the parafermion representation [18] of the affine primary ϕ^a which corresponds to the field Φ^J with $J = 1$ in notations of ref. [18]. Besides, in the limit $c_V \to \infty$, the field ϕ^a acquires dimension (1,0) of the affine current. Note that

$$\Delta_\phi = \frac{c_V}{k + c_V}. \tag{20}$$

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Apparently, eq. (19) is consistent with this limit.

All in all, we arrive at the following formula for the OPE of two ϕ's

$$\phi^a(w)\,\phi^b(z) = \frac{[I]}{(w-z)^{2\Delta_\phi}} \;+\; A\left\{\frac{f^{abc}}{(w-z)^{\Delta_\phi}}\phi^c(z) \;+\; \frac{1}{(w-z)^{\Delta_\phi-1}}J^a_{-1}\phi^b(z) \;+\; ...\right\} \;+\; ... \tag{21}$$

The point to be made is that in the case $G = SL(n)$, the coefficient A in eq. (21) can be fixed explicitly using the free field representation method [19],[20].

Taking into account eq. (21), we obtain for the holomorphic part of the operator $O^{L,\bar{L}}$ the following fusion

$$O(w)\,O(z) \;=\; \frac{[I]}{(w-z)^{2\Delta_O}} \;+\; A\frac{L_{ab}\,L_{mn}}{(w-z)^{\Delta_O}}(\frac{k}{2}\delta^{am}\,J^b_{-1}\phi^n(z) \;+\; f^{amk}f^{bnc}\,J^k_{-1}\phi^c(z) \tag{22}$$

$$+\;\; f^{amk}f^{kbc}\,J^c_{-1}\phi^n(z) \;+\; f^{amk}f^{knc}\,J^b_{-1}\phi^c(z)) \;+\; ...$$

Here dots stand for conformal operators with different conformal dimensions.

From formula (22) one can see that in general two operators $O^{L,\bar{L}}$ fuse into another operator $O^{\tilde{L},\bar{\tilde{L}}}$. Also, it becomes clear that in order to have $O^{L,\bar{L}}$ on the right hand side of eq. (22) with the same matrices L, $\bar{L}$, we have to impose the following condition

$$L_{ab} = -\frac{k}{2}\,L_{ac}L_{cb} \;-\; L_{cd}L_{ef}f^{cea}f^{dfb} \;-\; L_{cd}f^{cef}f^{dfa}L_{be} \;-\; L_{cd}f^{cef}f^{dfb}L_{ae}, \tag{23}$$

with a similar equation for $\bar{L}_{\bar{a}\bar{b}}$. It is rather amazing that the given equation is almost identical to the celebrated master Virasoro equation [1],[2]. The only difference is in the sign of the first term on the right hand side of (23). So, if we have dealt with the operator $O^{L,\bar{L}}$ associated with the unitary affine algebra, then the equation we would get is the master equation of the nonunitary affine-Virasoro construction. Whereas *the master Virasoro equation for the unitary affine-Virasoro construction originates from the fusion algebra of the operator $O^{L,\bar{L}}$ corresponding to the nonunitary affine algebra.* This will be a crucial observation for the further discussion. In particular, in the case of the unitary affine algebra, the existence of Virasoro primary operators with the fusion given by eq. (16) depends on the existence of solutions to the nonunitary master Virasoro equation. While in the case of the nonunitary affine algebra, there are definitely Virasoro primary operators $O^{L,\bar{L}}$ in the spectrum of the nonunitary WZNW model, obeying the fusion in

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eq. (16). In other words, we have shown that the unitary master equation resides in the fusion algebra of the nonunitary WZNW model.

3 $O^{L,\bar{L}}$-perturbation

It is obvious that in the limit $k \to -\infty$, $O^{L,\bar{L}}$ becomes a relevant quasimarginal operator. Indeed, in this limit, the conformal dimension Δ_O is just slightly less than one. This operator will satisfy the fusion algebra given by eq. (16) provided the algebraic equation (23) is fulfilled. In order to use this operator as a perturbation around the nonunitary WZNW model, we should be aware of the fact that there are no other relevant operators on the right hand side of eq. (16) but $[I]$ and $[O^{L,\bar{L}}]$. Otherwise, the perturbed theory will require the inclusion of additional relevant operators with corresponding coupling constants.

When $|k|$ is very large, the operator $O^{L,\bar{L}}$ takes the form

$$O^{L,\bar{L}} = G_{\mu\nu} \ \partial x^\mu \bar{\partial} x^\nu, \tag{24}$$

where x^μ are coordinates on the group manifold G, whereas

$$G_{\mu\nu} = -\frac{k^2}{8} L_{ab}\bar{L}_{\bar{a}\bar{b}} \ \phi^{b\bar{b}} e^a_{(\mu} \bar{e}^{\bar{a}}_{\nu)}. \tag{25}$$

Here e^a_μ and $\bar{e}^{\bar{a}}_\nu$ define left- and right-invariant Killing vectors respectively. Thus, in the classical limit ($|k| \to \infty$), the operator $O^{L,\bar{L}}$ becomes the nonlinear sigma model term with metric given by eq. (25). The renormalizability of the sigma model will allow only kinetic sigma model term and identity operator to appear on the right hand side of the fusion in eq. (16). In turn, the equation (23) will guarantee that this kinetic term will have the same structure as $O^{L,\bar{L}}$. Thus, in the large $|k|$ limit, dots on the right hand side of eq. (16) can be dropped out. Also, it is useful to point out that in the given limit, the operator $\phi^{a\bar{a}}$ goes to identity. Therefore, the operator $O^{L,\bar{L}}$ can be presented as a Thirring like current-current interaction

$$O^{L,\bar{L}} \to S_{a\bar{a}} \ J^a \bar{J}^{\bar{a}}, \tag{26}$$

with

$$S_{a\bar{a}} = L_{ab}\bar{L}_{b\bar{a}}. \tag{27}$$

All in all, in the large $|k|$ limit, the operator $O^{L,\bar{L}}$ is suitable for performing a renormalizable one-parametrical perturbation around the nonunitary WZNW model. A study of such a perturbation will be the aim of this section.

It is convenient to rescale the matrices L, $\bar{L}$ as follows

$$L = -\frac{2}{k}\hat{L}, \qquad \bar{L} = -\frac{2}{k}\hat{\bar{L}}. \tag{28}$$

Correspondingly, eq. (23) takes the form

$$\hat{L}_{ab} = \hat{L}_{ac}\hat{L}_{cb} + \frac{2}{k}\left(\hat{L}_{cd}\hat{L}_{ef}f^{cea}f^{dfb} + \hat{L}_{cd}f^{cef}f^{dfa}\hat{L}_{be} + \hat{L}_{cd}f^{cef}f^{dfb}\hat{L}_{ae}\right), \tag{29}$$

with a similar equation for $\hat{\bar{L}}$.

Let us consider the following theory

$$S(\epsilon) = S_{WZNW}(k) - \epsilon \int d^2z \, O^{\hat{L},\hat{\bar{L}}}(z,\bar{z}), \tag{30}$$

where ϵ is thought of being a small parameter. Note that k is chosen to be negative and $|k| \to \infty$. The theory is renormalizable if and only if the matrices $\hat{L}$, $\hat{\bar{L}}$ fulfill the master equation (29). Suppose it is the case. Then to leading orders in ϵ, one can write down the renormalization group equation for the coupling ϵ. To second order, one finds (see e.g. [21])

$$\frac{d\epsilon}{dt} \equiv \beta = (2 - 2\Delta_O)\epsilon - \pi \, C \, \epsilon^2 + \mathcal{O}(\epsilon^3), \tag{31}$$

with C being computed from the formula

$$\langle O^{\hat{L},\hat{\bar{L}}}(z_1,\bar{z}_1)O^{\hat{L},\hat{\bar{L}}}(z_2,\bar{z}_2)O^{\hat{L},\hat{\bar{L}}}(z_3,\bar{z}_3)\rangle = C||O^{\hat{L},\hat{\bar{L}}}||^2\Pi^3_{i<j}\frac{1}{|z_{ij}|^{2\Delta_O}}, \tag{32}$$

where

$$||O^{\hat{L},\hat{\bar{L}}}||^2 = \langle O^{\hat{L},\hat{\bar{L}}}(1)O^{\hat{L},\hat{\bar{L}}}(0)\rangle. \tag{33}$$

By using results of [17], we find the coefficient C to leading order in $1/k$ is

$$C = \frac{1}{c_V}\frac{\hat{L}_{al}\hat{L}_{bm}\hat{L}_{cn} \, f^{abc}f^{lmn} \, \hat{\bar{L}}_{\bar{a}\bar{l}}\hat{\bar{L}}_{\bar{b}\bar{m}}\hat{\bar{L}}_{\bar{c}\bar{n}} \, f^{\bar{a}\bar{b}\bar{c}}f^{\bar{l}\bar{m}\bar{n}}}{\hat{L}_{dd} \, \hat{\bar{L}}_{\bar{d}\bar{d}}} + \mathcal{O}(1/k). \tag{34}$$

In this formula, the matrices $\hat{L}$ and $\hat{\bar{L}}$ are computed perturbatively in $1/k$ [14]. For example,

$$\hat{L}_{ab} = \hat{L}_{ab}^{(0)} + \mathcal{O}(1/k),$$

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$$(35)$$

$$L_{ab}^{(0)} = \frac{1}{2}\sum_c \Omega_{ac}\Omega_{bc}\ \theta^c,$$

where the quantities Ω_{ac} and θ^c are defined by [14]

$$\Omega\ \Omega^{\mathrm{T}} = 1, \qquad \theta^a = 0 \ \ \mathrm{or} \ \ 1, \qquad a = 1,...,\dim G. \tag{36}$$

Besides, there is the quantization condition [14]

$$0 = \sum_{cd}\theta^c(\theta^a + \theta^b - \theta^d)\ \hat{f}_{cda}\hat{f}_{cdb}, \qquad a < b, \tag{37}$$

where

$$\hat{f}_{abc} = f^{lmn}\ \hat{L}_{al}^{(0)}\hat{L}_{bm}^{(0)}\hat{L}_{cn}^{(0)}. \tag{38}$$

Similarly one can find expression for $\hat{\bar{L}}$.

Given a solution of the master equation, we can find the constant C and, correspondingly, a fixed point ϵ^* of the beta function in eq. (31). This fixed point is an infrared conformal point of the theory described by action (30). We find

$$\epsilon^* = -\frac{2c_V}{\pi C k}. \tag{39}$$

Since different L's and $\bar{L}$'s give rise to different C's, there will be as many infrared conformal points as $\frac{1}{2}N(N+1)$ with N the total number of solutions to the master equation.

The perturbative expression for the Virasoro central charge at the point ϵ^* is given by the Cardy-Ludwig formula [21]

$$c(\epsilon^*) = c_{WZNW}(k) - \frac{(2 - 2\Delta_O)^3}{C^3}||O^{\hat{L},\hat{\bar{L}}}||^2, \tag{40}$$

where

$$||O^{\hat{L},\hat{\bar{L}}}||^2 = \frac{k^2\hat{L}_{aa}\hat{\bar{L}}_{\bar{a}\bar{a}}}{4\dim G}. \tag{41}$$

Unfortunately, it is very difficult to identify the given perturbative conformal points with some exact conformal theories. In particular, we could not clarify whether or not our action given by eq. (30) provides a Lagrangian to the affine-Virasoro construction. But the remarkable fact that conformal points of our theory are related to the solutions of the master Virasoro equation makes such a hypothesis quite plausible. We believe that there

is a certain combination of the matrices L and $\bar{L}$ which again is a solution of the master equation. For some particular pairs of L and $\bar{L}$ this combination is likely to appear in the affine-Virasoro stress tensor at the corresponding conformal point ϵ^*.

There are two more interesting issues which we left for further investigation. It might be interesting to use the operator $O^{L,\bar{L}}$ to perturb gauged WZNW models. Another interesting question is the fusion algebra of operators $O^{L,\bar{L}}$ with different pairs of matrices L, $\bar{L}$ obeying the master equation. We hope to return to these problems in future publications.

Acknowledgement: I would like to thank S. J. Gates Jr., C. M. Hull, E. Kiritsis and E. Verlinde for fruitful conversations. I thank also W. Sabra for reading the manuscript.

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