Master-Slave Scheme and Controlling Chaos in the Braiman and Goldhirsch Method

Han-Tzong Su[1], Yaw-Hwang Chen[2], and Rue-Ron Hsu[3]

Nonlinear Science Group, Department of Physics

National Cheng Kung University

Tainan,Taiwan 70101, Republic of China

ABSTRACT

This brief report presents a master-slave scheme to demonstrate how control chaos works in the Braiman and Goldhirsch method for the one-dimensional map system. The scheme also naturally explain why the anomalous response arises in a periodically perturbed, unimodal map system.
PACS number(s): 05.45.+b.

Typeset using LaTeX

[1]E-mail address: htsu@mail.ncku.edu.tw
[2]E-mail address: yhchen@ibm65.phys.ncku.edu.tw
[3]E-mail address: rrhsu@mail.ncku.edu.tw

1

arXiv:chao-dyn/9802001v1 31 Jan 1998

Braiman and Goldhirsch (BG) [1] proposed a simple non-feedback method, in contrast to the Ott-Grebogi-Yorke feedback scheme [2], to create stable periodic orbits from a chaos using a weak periodic perturbation. Though there are some successful numerical and experimental demonstrations of the BG method [3]-[5], the periodicity and the stability condition of the targeted stable state was not identified analytically until recently [6]. If one considers a generic one-dimensional chaotic map under the influence of a periodic perturbation,

$$z_{n+1} = f(z_n) - \alpha y_n, \tag{1}$$

where α is a small number and y_n is the added weak perturbation with periodicity p. The desired stable states in response to the period-p perturbation in a chaotic map system can only have the periodicity $q = kp$, where k is an integer number. Furthermore, using the linear stability analysis, one can deduce that the stability condition, for the output with the periodicity $q = kp$, is

$$|M| = \left| \prod_{j=1}^{k} \left[\prod_{\ell=1}^{p} \left(\frac{\partial f}{\partial z} \bigg|_{\bar{z}_{j\ell}} \right) \right] \right| < 1. \tag{2}$$

Here, $\bar{z}_{j\ell}$ is the $j\ell$ times mapping of $\bar{z}_1$, and $\bar{z}_1$ are the roots of the periodicity condition

$$z = f(f(...(f(f(z) - \alpha y_1) - \alpha y_2) - ...) - \alpha y_{kp-1}) - \alpha y_{kp}. \tag{3}$$

Even though, the analysis is presented in an elegant mathematical form in the reference [6]. Providing a more intuitive picture to illustrate how a chaotic

2

system can be controlled by weak periodic perturbation is still a worthwhile effort. In this brief report, we will introduce a conceptual picture, called the master-salve scheme, to explain how control chaos works in the BG method. This picture will also gives us a new handle to understand the anomalous responses in a dynamical system under the influence of periodic perturbation. For example, when a period-2 perturbation with elements $\{y_1 = a,\ y_2 = 0.2\}$ is added to a chaotic logistic map, then the system becomes $z_{n+1} = 4z_n(1 - z_n) - y_n$. The antimonotonicity - concurrent creation and destruction of periodic orbits [7, 8], appears in the bifurcation diagram for the variation of a, see Fig.1. It seems to be in the contrary to the well known numerical fact [9]: *the antimonotonicity could never appear in a unperturbed one-dimensional unimodal map system.* In the next paragraph, we will see that this anomalous response can be interpreted naturally in the master-slave scheme.

To begin with, let us consider a generic map under the influence of a period-p orbit $\{y_1, y_2, ..., y_p\}$, see Eq.(1) . For convenience, we will label the initial data and initial perturbation as z_1 and y_1, respectively. The key idea of the master-slave scheme is as following. We divide the original dynamical variables z_n into p new variables, called $\{x_m^{(1)}, x_m^{(2)}, ..., x_m^{(p)}\}$. The relation between z_n and the new variables $x_m^{(i)}$ is defined as

$$x_m^{(i)} = z_{pm+i}, \quad 1 \leq i \leq p. \tag{4}$$

3

Hence, the original dynamical equation can be separated into p maps:

$$
\begin{aligned}
x_m^{(2)} &= f(x_m^{(1)}) - \alpha y_1, \\
x_m^{(3)} &= f(x_m^{(2)}) - \alpha y_2, \\
&\quad ... \\
x_m^{(p)} &= f(x_m^{(p-1)}) - \alpha y_{p-1}, \\
x_{m+1}^{(1)} &= f(x_m^{(p)}) - \alpha y_p.
\end{aligned}
\tag{5}
$$

Plugging the first $(p-1)$ maps, $x_m^{(2)}, x_m^{(3)}, ... x_m^{(p)}$, into $x_{m+1}^{(1)}$, one find the map between $x_{m+1}^{(1)}$ and $x_m^{(1)}$, which characterise the dynamical properties of original system, and let us call it *the master equation*:

$$
\begin{aligned}
x_{m+1}^{(1)} &= F(x_m^{(1)}; \alpha y_1, \alpha y_2, ..., \alpha y_p) \\
&= f(f(...(f(f(x_m^{(1)}) - \alpha y_1) - \alpha y_2) - ...) - \alpha y_{p-1}) - \alpha y_p,
\end{aligned}
\tag{6}
$$

The remained $p-1$ maps, which just are mappings of $x_m^{(1)}$, and are designated as *the slave equations*:

$$
\begin{aligned}
x_m^{(2)} &= f(x_m^{(1)}) - \alpha y_1, \\
x_m^{(3)} &= f(x_m^{(2)}) - \alpha y_2, \\
&\quad ... \\
x_m^{(p)} &= f(x_m^{(p-1)}) - \alpha y_{p-1}.
\end{aligned}
\tag{7}
$$

Since, the dynamics of the slave equations are completely controlled by the master equation, hence the name-*the master-slave scheme*. As long as the

4

master equation, Eq.(6), is in a stable period-k orbit, then the slave equations indicate that $(p-1)$ images will appear simultaneously. It means that there exists a period-kp orbit in the original system.

From linear stability analysis, one can deduce that the stability condition for the period-k orbit in the master equation is $|M| < 1$. The stability quantity M now simply is

$$M = \left[\frac{\partial}{\partial x} F^k(x_m^{(1)}; \alpha y_1, \alpha y_2, ..., \alpha y_p) \right] \Big|_{\bar{x}^{(1)}}, \tag{8}$$

where $\bar{x}^{(1)}$ is one of the roots of the periodicity condition

$$\bar{x}^{(1)} = F^k(\bar{x}_m^{(1)}; \alpha y_1, \alpha y_2, ..., \alpha y_p). \tag{9}$$

Obviously, in terms of the original dynamical variable z_n and the map f, Eq.(8) and Eq.(9) will reduce to Eq.(2) and Eq.(3), respectively.

Now, to be more specific, let us take $\alpha = 1$ and the perturbation y_n be of period-2 with elements $\{y_1 = a, \ y_2 = 0.2\}$. We will further assume that the system is a chaotic logistic map, $f(z) = 4z(1 - z)$, before we turn on the perturbation. In this special case, the master equation becomes

$$x_{m+1}^{(1)} = 4(4x_m^{(1)}(1 - x_m^{(1)}) - a)(1 - 4x_m^{(1)}(1 - x_m^{(1)}) + a) - 0.2, \tag{10}$$

and the slave map is

$$x_m^{(2)} = 4x_m^{(1)}(1 - x_m^{(1)}) - a. \tag{11}$$

Here, $x_m^{(1)}$ ($x_m^{(2)}$) denotes the odd (even) part of z_m, i.e. $x_m^{(1)} = z_{2m+1}$ ($x_m^{(2)} = z_{2m+2}$), and the initial value is labelled as $x_0^{(1)} = z_1$. The bifurcation diagram

5

of the master equation, Eq.(10), for the perturbation a with values between 0.0 and 0.55, is shown in Fig.2. The bifurcation diagram indicates that the desired stable period-k orbit will appear if a suitable perturbation is applied on this chaotic logistic map. For example, period-1 orbit occurs when a is between $(0.194, 0.240)$; and period-2 orbits can be generated when a is at $(0.170, 0.194)$, $(0.240, 0.290)$, or $(0.425, 0.464)$; etc. From Eq.(11), the bifurcation diagram the slave map is plotted in Fig.3. One can see that the same periodic orbits also appear exactly at the same regions of the perturbation a. Obviously, the combination of Fig.2 and Fig.3 leads to Fig.1 exactly. Also, from the numerical simulation presented in the Fig.1, one can clearly see that the periodicity of the stable response in the chaotic map under the influence of a period-2 perturbation is $2k$ - as one would have expected.

For a careful reader, she/he may have noticed that there are some anomalous responses in the bifurcation diagrams of this perturbed logistic map, which occurs at the region $a \in (0.25, 0.45)$, see Fig. 1-3. These anomalous responses are called the antimonotonicity that does not exist in a unperturbed logistic map. Since the logistic map $f(z) = rx(1 - x)$ only has one critical point at $x = 0.5$, it seems to be a dilemma for those who are familiar with the work of Dawson, Grebogi and Koçak [9] : *if a one-dimensional map $x_{n+1} = F(x_n, \alpha)$ has at least two critical points that lie in a chaotic attractor for a parameter $\alpha = \alpha^*$, then generically, F is antimonotone at α^*.* However, as it has been mentioned in the last paragraph, the dynamical of a periodi-

cally perturbed system is governed by the master equation. The right hand side of Eq.(10) is a fourth order polynomial of $x_m^{(1)}$, and this implies that the system may have three critical points. Therefore, the antimonotonicity could arise naturally in a periodically perturbed logistic map.

Finally, a brief note is given as a concluding remark. The master-slave scheme gives us a conceptual picture that the periodic perturbation indeed makes controlling chaos feasible. The scheme also helps us to understand how the anomalous responses arise in a periodically perturbed one-dimensional map.

Acknowledgements

References

[1] Y. Braiman and I. Goldhirsch, **Phys. Rev. Lett. 66**, 2545 (1991).

[2] E. Ott, C. Grebogi, and J.A. Yorke, **Phys. Rev. Lett. 64**, 1196 (1990). T. Shinbrot, C. Grebogi, E. Ott and J.A. Yorke, **Nature 363**, 411 (1993). V. In, S.E. Mahan, W.L. Ditto and M.L. Spano, **Phys. Rev. Lett. 74**, 4420 (1995). Also see *Coping with chaos*, ed. E. Ott., T. Sauer and J.A. Yorke (Wiley, New York, 1995) for more references.

[3] S.T. Vohra, L. Fabing and F. Bucholtz, **Phys. Rev. Lett. 75**, 65 (1995).

[4] P. Colet and Y. Braiman, **Phys. Rev. E 53**, 200 (1996).

[5] H.-J. Li and J.-L. Chern, **Phys. Rev. E 54**, 2118 (1996).

[6] R.-R. Hsu, H.-T. Su, J.-L. Chern, and C.-C. Chen, **Phys. Rev. Lett. 78**, 2936 (1997).

[7] I. Kan, H. Koçak, and J. A. Yorke, **Ann. Math. 136**, 219 (1992).

[8] T. C. Newell, V. Kovanis, and A. Gavrielides, **Phys. Rev. Lett. 77**, 1747 (1996).

[9] S. P. Dawson, C. Grebogi, and H. Koçak, **Phys. Rev. E 48**, 1676 (1993).

Figure Captions :

Figure 1. Bifurcation diagram for a periodically perturbed chaotic logistic map, $z_{n+1} = 4z_n(1 - z_n) - y_n$, where y_n is of period-2 and with elements $\{y_1 = a, \ y_2 = 0.2\}$. The initial point for z_1 is 0.54, and 100 data points are plotted after 4000 transient iterations.

Figure 2. Bifurcation diagram for the master equation Eq.(10), versus the perturbation a. The initial point for $x_0^{(1)}$ is 0.54, and 50 data points are plotted after 2000 transient iterations.

Figure 3. The image $x_m^{(2)}$, which is determined by the slave equation Eq.(11), of the master equation Eq.(10) verus a.
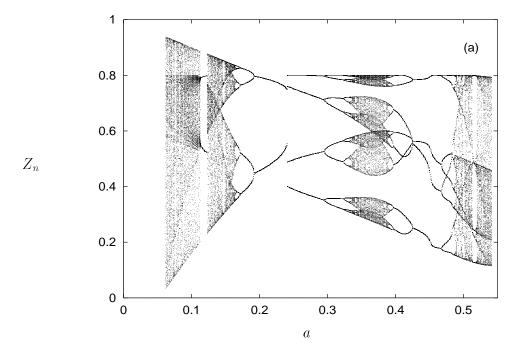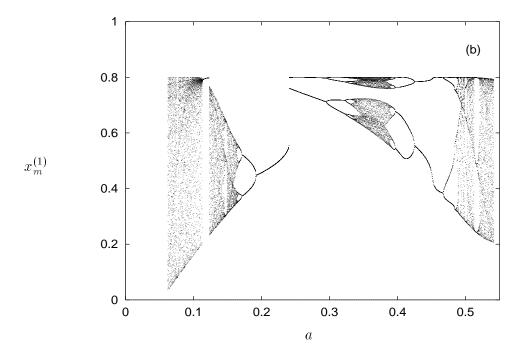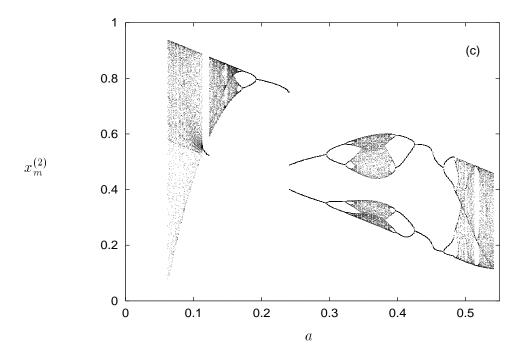
Fig.1

Fig.2

Fig.3