Précis: Quantum Feedback

H. M. Wiseman

The following is the body of page ix of the PhD thesis Quantum Trajectories and Feedback by H.M. Wiseman (Physics Department, University of Queensland, 1994), which is downloadable as a postscript file at http://www.sct.gu.edu.au/~sctwiseh/PhDThesis.ps.z. It is (as it describes itself) a very brief technical summary of the most important results therein.

Consider an optical field in one dimension with canonical commutation relations

$$[b(z,t),b^{\dagger}(z',t)] = v\delta(z-z'), \tag{1}$$

propagating at speed v. Let it be coupled to a system at position z=0 by the Hamiltonian

$$V_{\text{dipole}}(t) = i[b^{\dagger}(0, t)c(t) - c^{\dagger}(t)b(0, t)], \tag{2}$$

where a rotating-wave approximation has been used ¹. Let the output photocurrent be fed back to control the system via

$$V_{\text{feedback}}(t) = Z(t)b^{\dagger}(v\tau, t)b(v\tau, t). \tag{3}$$

For a vacuum input field [zero eigenstate of b(z,t) for z < 0], the explicit increment in an arbitrary system operator is

$$ds = i[H, s]dt - [s, c^{\dagger}] \left(\frac{1}{2}c + b_0\right) dt + \left(\frac{1}{2}c^{\dagger} + b_0^{\dagger}\right) [s, c]dt + [c^{\dagger}(t - \tau) + b_0^{\dagger}(t - \tau)] \left(e^{iZ}se^{-iZ} - s\right) [c(t - \tau) + b_0(t - \tau)]dt,$$
(4)

where $b_0(t) \equiv b(0^-, t)$ commutes with s(t) and obeys

$$b_0(t)b_0^{\dagger}(t)dt = 1, (5)$$

with other such moments vanishing ². For finite delay τ , the output field operator $b_0(t-\tau) + c(t-\tau)$ commutes with s(t), but for $\tau = 0$ it does not. In the latter case, one obtains the corresponding feedback master equation

$$\dot{\rho} = -\mathrm{i}[H,\rho] + e^{-\mathrm{i}Z}c\rho c^{\dagger}e^{\mathrm{i}Z} - \frac{1}{2}\{c^{\dagger}c,\rho\}. \tag{6}$$

This has an obvious interpretation in terms of the quantum jumps associated with photodetection in the theory of quantum trajectories ².

¹C. W. Gardiner, *Quantum Noise* (Springer-Verlag, Berlin, 1991).

²H. J. Carmichael, An Open Systems Approach to Quantum Optics (Springer-Verlag, Berlin, 1993)