Ma2201/CS2022

Foundations of C.S.

Spring, 2021

Quiz 0100

PRINT NAME: ____________________

$\mathcal{SIGN}$: ____________________

Consider the following languages on $\Sigma = \{a, b\}$ defined by the regular expressions.

$$
\begin{aligned}
L_1 : &\quad (\lambda^* \cup a \cup b \cup a^3 \cup b^3)^* \\
L_2 : &\quad \lambda \cup a \cup b \cup ((a \cup b)^3)^* \\
L_3 : &\quad \lambda \cup (a \cup b) \cup (a^* \cup b^*)^3 \\
L_4 : &\quad \lambda \cup (a \cup b) \cup (a^3 \cup b^3)^* \\
L_5 : &\quad \lambda \cup (a \cup b) \cup (a^3)^* \cup (b^3)^*
\end{aligned}
$$

1. (**2 pts**) Find two numbers i and j with $L_i \subseteq L_j$ and $L_j \nsubseteq L_i$.

2. (**2 pts**) Find two numbers n and m with $L_n = L_m$.

3. (**2 pts**) List all L_i, if any, which contain only strings of finite length.

4. (**2 pts**) List all L_i, if any, which contain the string $a^3b^3a^3b^3$.

5. (**2 pts**) Pick any two distinct languages above and describe them set theoretically.

♣ *Here are all the languages decoded*

$$
\begin{aligned}
L_1 &= \{w \in \{a, b\}^*\} \\
L_2 &= \{w \in \{a, b\}^* \mid w \in \{a, b\} \text{ or } \mathit{length}(w) = 3k, k \in \mathbb{N}\} \\
L_3 &= \{w \in \{a, b\}^* \mid w \in \{a, b\} \text{ or } w = u_1u_2u_3, n_a(u_i)n_b(u_i) = 0\} \\
L_4 &= \{w \in \{a, b\}^* \mid w \in \{a, b\} \text{ or } w = u_1 \cdots u_k, k \geq 0, u_i \in \{a^3, b^3\}\} \\
L_5 &= \{w \in \{a, b\}^* \mid w \in \{a, b\} \text{ or } w = a^{3k}, \text{ or } w = b^{3k}, k \in \mathbb{N}\}
\end{aligned}
$$

For L_1 the first, fourth and fifth terms are redundant. For L_2, $(a \cup b)^3$ matches any string of length three, and with a $*$ matches any strings a non-trivial multiple of 3. For L_3, $(a^* \cup b^*)$ matches any string consisting of only a's or only b's, so one of n_a and n_b must be zero, and cubing gives a product of three such strings, not *necessarily the same one or type. For L_4, $(a^3 \cup b^3)$ matches either a^3 or b^3, not much choice, and with the $*$ gives a string of such elements. For L_5, $(a^3)^*$ matches matches any string of a's of length divisibly by 3, similarly for $(b^3)^*$. Note that λ would have been in each language even if it wasn't specifically mentioned.*

For 1, taking $j = 1$ is a good move.

For 2, there is no answer. That was for a different draft. Sorry. If you said any two of these were the same, then I should take off, but I won't.

For 3, NOBODY should get this wrong. All strings are are of finite length. This is from the recursive definition of string. So you should list all 5.

For 4, L_1, L_2 and L_4.

For 5, examples above. ♣