Ma2201/CS2022 Foundations of C.S. Spring, 2021
Quiz 1000
PRINT NAME: ____________________
$\mathcal{SIGN}$: ____________________

1. (**4 pts**) Consider the grammar

$$\begin{aligned} G: S &\rightarrow aBAb \mid \lambda \\ A &\rightarrow a^2B \mid b^2 \\ B &\rightarrow b^2A \mid a^2 \end{aligned} .$$

Create a Chomsky Normal Form Grammar G' with $L(G) = L(G')$.

You can use the conversion method we discussed, or you can you another method, but in that case, justify what you do.

♣ *I split the first S rule in half, so that I could reuse the first part in the first A rule.*

$$\begin{aligned} G: S &\rightarrow XY \mid \lambda \\ A &\rightarrow U_aX \mid U_bU_b \\ B &\rightarrow U_bZ \mid U_aU_a \\ X &\rightarrow U_aB \\ Y &\rightarrow AU_b \\ Z &\rightarrow U_bA \\ U_a &\rightarrow a \\ U_b &\rightarrow b \end{aligned} .$$

Did not simplify it much, though. ♣

2. (**3 pts**) Design a Chomsky Normal Form grammar whose language is the palindromes on $\{a, b\}$ which are of odd length and start with a. Your Chomsky Grammar should have no rule which not allowed in the definition in the Text.

♣ *Our options are to write a grammar, and then convert, or to design it as a Chomsky grammar in the first place, which is what I will do.*

C will derive the center of the string, so it either derives a single letter, or it has to go through a process of adding an initial letter, and a matching terminal letter.

$$\begin{aligned} G: S &\rightarrow a \mid U_aD \\ C &\rightarrow a \mid b \mid U_aD \mid U_bE \\ D &\rightarrow CU_a \\ E &\rightarrow CU_b \\ U_a &\rightarrow a \\ U_b &\rightarrow b \end{aligned} .$$

D derives the center with a extra terminal a, and E derives the center with a extra terminal b. The start also acts as a center, but without the b option. ♣

3. (**3 pts**) Suppose we have a grammar which contains the two variables A and B and all the A and B rules are

$$\begin{array}{lcl} A & \to & AB \mid ab \mid BA \mid ba \mid ABABA \\ B & \to & BB \mid bb \mid AB \mid aa \mid BBABB \end{array}$$

Introduce new variables X and Y to remove the direct left recursion.

♣ *Easiet to just follow the procedure. Actually, B and B are handled exactly the same way.*

$$\begin{array}{lcl} A & \to & ab \mid BA \mid ba \mid abX \mid BAX \mid baX \\ B & \to & bb \mid AB \mid aa \mid bbY \mid ABY \mid aaY \\ X & \to & B \mid BABA \mid BX \mid BABAX \\ Y & \to & B \mid BABB \mid BY \mid BABBY \end{array}$$

♣