Ma2201/CS2022 # Foundations of C.S. Spring, 2023

Quiz 1100

PRINT NAME: ____________________

$\mathcal{SIGN}$: ____________________

1. (5 pts) For each of the following, mark it as regular or non-regular.

$A = \{w \in \{a,b,c\}^* \mid w = a^n x; n > 2; x \in \{a,b,c\}^*\}$

♣ Regular $a^3(a \cup b \cup c)^*$. ♣

$B = \{w \in \{a,b,c\}^* \mid w = a^i b^j c^k a^l b^m c^n; i+j+k+l+m+n \text{ even}\}$

♣ Regular. You make a DFA with 13 states, whose roles describe which of the six types a^i, $a^i b^j$, $a^i b^j c^k$, $a^i b^j c^k a^l$, $a^i b^j c^k a^l b^m$, and $a^i b^j c^k a^l b^m c^n$ of string you have, and, in each case, whether the length is odd or even, and a failure state. ♣

$C = \{w \in \{a,b,c\}^* \mid w = a^n u b^n; u \in \{c\}^*; n \leq 2^{10}\}$

♣ Regular. It is a finite union of regular sets, $a^k c^* a^k$, with k varying from $k = 0$ to $k = 2^{10} - 1$. ♣

$D = \{w \in \{a,b,c\}^* \mid w = a^n u b^n; u \in \{c\}^*; n \geq 2^{10}\}$

♣ Irregular. proved below. ♣

$E = \{w \in \{a,b,c\}^* \mid w = xy; x \in \{a,b\}^*; n_a(x) + n_b(x) > 2^{10}\}$

♣ Regular: $(a \cup b)^{2^{10}}(a \cup b \cup c)^*$. ♣

2. (2 pts) For one of the languages above, you choose, prove that it is regular.

♣ All done above except B, which is merely sketched. ♣

3. (3 pts) For one of the languages above, you choose, prove that it is not regular.

♣ It has to be D. Let N be given. Consider the string $w = a^{N+2^{10}} b^{N+2^{10}}$ which is in the language. According to the pumping lemma w must factor as $w = upv$, with up of length at most N with p non-empty and pumpable. But then p consists only of a's, and $w' = up^2v$ has $n_a(w') > n_b(w')$ and is not in D.

You can also do the finite state test. Consider the infinite set of strings of the form $a^{i+2^{10}}$. For any pair of them $a^{i+2^{10}}$ and $a^{i+2^{10}}$ with $j \neq i$, set $w_{ij} = b^{i+2^{10}}$. Then $a^{i+2^{10}} w_{ij}$ is the language but $a^{j+2^{10}} w_{ij}$ is not. This violates the Finite State Test. ♣