Foundations of C.S.

Spring, 2023

PRINT NAME: _____

$\mathcal{SIGN}$: _____

1. (3 pts) Define a grammar G whose language $L(G)$ is the set of strings on $\{a, b, c\}$ which contain a^2 as a substring.

Assign roles for your variables in a few words.

♣ Many correct answers.

Roles: A: Derives anystring. F: derives the forced substring (a^2)

$$G : \begin{aligned} S &\rightarrow AFA \\ A &\rightarrow aA \mid bA \mid cA \mid \lambda \\ F &\rightarrow aa \end{aligned}$$

♣

2. (4 pts) Define a grammar G' whose language $L(G')$ is the set of strings on $\{a, b, c\}$ which do not contain a^2 as a substring.

Assign roles for your variables in a few words.

♣ Design: Any a's must be separated by non-trivial strings of b's or c's. The final a needs no separator. Strings without a's will be done separately.

Roles: A: and a followed by a separator.

A': Final a followed by optional separator.

D: Separator, or b, c string.

$$G : \begin{aligned} S &\rightarrow \lambda \mid D \mid A \mid DA \\ A &\rightarrow aDA \mid A' \\ A' &\rightarrow a \mid aD \\ D &\rightarrow bD \mid cD \mid c \mid d \end{aligned}$$

♣

3. (3 pts) Define a grammar G whose language $L(G)$ is the set of strings on $\{a, b, c\}$ which do not contain either a^2 or b^2 as substrings.

Assign roles for your variables in a few words.

♣ Design: The a's and b's must occur together in alternating strings. Different types are separated by strings of c's.

Roles: S: In terms of the others; nothing, only c's, one ab clump with perhaps C's in front or behind, multiple ab clumps with alternate beginnings and endings.

A: any alternating string. repetitions of ab with an optional b in front and a at the end.

B: repetitions of ab

C: Separator, non-trivial c string.

D: Repetitions of AC.

$$G : S \;\rightarrow\; \lambda \mid C \mid A \mid CA \mid AC \mid D \mid CD \mid DA \mid CDA$$
$$A \;\rightarrow\; a \mid b \mid ab \mid ba \mid B \mid Ba \mid bB \mid bBa$$
$$B \;\rightarrow\; ab \mid abB$$
$$C \;\rightarrow\; c \mid cC$$
$$D \;\rightarrow\; AC \mid ACD$$

♣