Ma2201/CS2022 Quiz 0110

Foundations of C.S.

Spring, 2023

PRINT NAME: ____________________

$\mathcal{SIGN}$: ____________________

1. (5 pts) Let $L(G)$ be the language of the grammar

$$\begin{aligned} G: S &\rightarrow AB \mid \lambda \\ A &\rightarrow aAa \mid bAb \mid cAc \mid \lambda \\ B &\rightarrow acacB \mid bcbcB \mid \lambda \end{aligned}$$

Set $M = \{w \in \{a,b,c\}^* \mid n_a(w), n_b(w), n_c(w) \text{ are all even}\}$. Prove that $L(G) \subseteq M$.
Hint: You will need induction.

♣ *We will prove by induction on the number of rules applied that every sentential w form has $n_a(w)$, $n_b(w)$, $n_c(w)$ all even.*

Base Case: With no rules applied the sentential form is S, and $n_a(S) = n_b(S) = n_c(S) = 0$, which is even.

For the inductive step, let $n \geq 0$ be given and assume that all sentential forms generated after n moves have an even number of a's, b's and c's. Since the $n+1$ rule application adds 0 or 2 to the number of a's, b's and c's, every sentential form w' obtained after $n+1$ rules also has $n_a(w')$, $n_b(w')$, and $n_c(w')$ even.

So the result is true for all sentential forms by induction, and for all elements of $L(G)$.
♣

2. (2 pts) Let $k \in \mathbb{N}$. Prove $a^{2k}(ac)^{2k} \in L(G)$. Hint: You will not need induction.

♣ $S \Rightarrow AB \overset{k}{\Rightarrow} a^k A a^k B \overset{k}{\Rightarrow} a^k a^k ((ac)^2)^k B \Rightarrow a^k a^k ((ac)^2)^k = a^{2k}(ac)^{2k}$. ♣

3. (3 pts) Let $L(G)$ be the language of the grammar

$$\begin{aligned} G: S &\rightarrow AB \mid \lambda \\ A &\rightarrow aAa \mid bAb \mid cAc \mid \lambda \\ B &\rightarrow acacB \mid bcbcB \mid \lambda \end{aligned}$$

$N = \{w \in \{a,b,c\}^* \mid w = w^R, \text{length}(w) \text{ is even.}\}$. On the back of this page, prove that $N \subseteq L(G)$.

Hint: You might need an induction.

♣ *Every even length palindrome is of the form $w = uu^R$. We will use induction on the length of u to show that uAu^R is derivable from A.*

Base Case: Length 0, $\lambda A \lambda^R = A$ is derivable from A.

Inductive Step: Let $n \geq 0$ be given and suppose that $A \overset{n}{\Rightarrow} uAu^R$ for any string u of length n. Every string of length $n+1$ is of the form ux, with $x \in \{a,b,c\}$ and u of length n. Then $A \overset{n}{\Rightarrow} uAu^R \Rightarrow u(xAx)u^R = uxAx^Ru^R = uxA(ux)^R$, as required.

So $A \overset{*}{\Rightarrow} uu^R$ for all strings u by induction.

So for any $w = uu^R$, we have $S \Rightarrow AB \overset{*}{\Rightarrow} uAu^RB \Rightarrow uu^RB \Rightarrow uu^R = w$ and $w \in L(G)$.
♣