



CS5003
Quiz 1100

Foundations of C.S.

Spring, 2022

PRINT NAME: _____

SIGN: _____

1. (6 pts) Consider the language $L = \{w \in \{a, b\}^* \mid n_a(w), n_b(w) \text{ both even}\}$. We know for several reasons that L is a regular language.

Find a number N so that every string $w \in L$ of length at least N factors as $w = upv$, with the length of up no more than N , and $up^k v \in L$ for all $k \geq 0$.

Explain why your value of N works.

♣ You can take the key number to be 4. Let w be a string of length at least 4 in L . If among the first four characters there is an a^2 or a b^2 , then take that square to be the pumpable string. Otherwise, the first four characters are either $abab$ or $baba$, in which case we take the first four characters to be the pumpable string. ♣

2. (4 pts) Use the pumping lemma to show that the language $X = \{w \in \{a, b, c\}^* \mid w = a^i b^j c^{4k} b^j a^5; i, j, k \geq 0\}$ is not regular.

♣ Let N be given. Consider the string $b^N c^4 b^N a^5 \in X$. The pumping lemma would guarantee a pumpable substring among the first N characters. The pumpable substring would have to be b^j for $1 \leq j \leq N$. But that is impossible since $b^{N+j} c^4 b^N a^5 \notin X$. So X violates the conclusion of the pumping lemma, and is not regular.

Note that, even though I gave you an extra 10 minutes, and most people were still working near the end, the solutions are very short. That is normal, since sometimes short solutions take time and care to formulate and assemble – and, without study, practice and familiarity, may even be impossible to formulate within half an hour. ♣