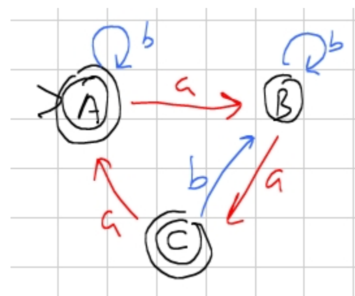
Ma2201/CS2022 Quiz 1100

Foundations of C.S.

Spring, 2021 Online Quiz

1. (2 pts) Consider M, the Deterministic Finite Automaton,



For the word $ababa \in L(M)$, identify three substrings which are pumpable, and say briefly why, and one string which is not pumpable, and say briefly why.

♣ *The easiest way to do the first is to trace out the string in the machine, and use a loop, and it explains the pumpability at the same time. The loops are in brackets,* $ababa = a[b]aba = ab[ab]a = aba[ba]$ *has the first two are loops at* B*, the third a loop at* C*. It doesn't matter for the question, or the lemma, whether the state is accepting or not.)*

For a non-pumpable string, we only have to have one string for which replacing it with a power makes a new string which is not accepted. Not all non-loops are non-pumpable, but the third a *in* $ab[a]ba$ *is non-pumpable since* aba^2ba *is not in the language. (Notice I don't have to show that all powers cause non-acceptance, but only that some do not.* ♣

2.(2 pts) Draw, or give a table you don't like drawing, the diagram of a deterministic finite automaton whose language has a string of length 4 with no pumpable substring.

♣ *The diagram should have only long cycles, but a string of length 4 being accepted. There are lots of correct answers. You can do it with one letter.*

$$> \boxed{A} \xrightarrow{a} \boxed{B} \xrightarrow{a} \boxed{C} \xrightarrow{a} \boxed{D} \xrightarrow{a} \boxed{\boxed{E}} \xrightarrow{a} \boxed{F} \xrightarrow{a} \ldots \textit{back to } A$$

has language $\{a^{4+6k}\}$*, which by length are spaced* 6 *apart. Pumping a substring of* a^4 *gives strings which are too close together in length, so some of them will not be accepted.* ♣

3. (6 pts) Consider the language L consisting of all elements of $w \in \{a, b, c\}^*$ with odd length and with $n_b(w) < n_a(w) + n_c(w)$. So $(a^7b^5c^9 \in L)$.

Use the pumping lemma to show that L is not regular.

Your response will be graded both on correctness and clarity of exposition.

♣ *Let* N *be given. Consider the string* $w = b^N a^N c^1$ *which has odd length and more a's and c's than b's, so* $b^N a^N c^1 \in L$*. So any non-trivial factorization of* $w = uxv$ *with the length of* ux *less than* N *has* $x = a^k$ *with* $k > 0$*, and* x *is not pumpable since* $ux^3v = b^{N+2k}a^N c^1$ *and* $N + 2k > N + 1$*, so* $ux^3v \notin L$*.*

Thus the language L *is not regular.*

♣