Ma2201/CS2022

Quiz 0110

Foundations of C.S.

Spring, 2022

PRINT NAME: ____________________

$\mathcal{SIGN}$: ____________________

1. (5 pts) Let $L(G)$ be the language of the grammar

$$\begin{aligned} G : S &\to a^2Ab^2 \mid a^2Bb^2 \mid \lambda \\ A &\to a^2Ab^2 \mid a^2Bb^2 \mid \lambda \\ B &\to b^2Ba^2 \mid a^2Ab^2 \end{aligned}$$

And set $M = \{w \in \{a,b\}^* \mid n_a(w) = n_b(w)\}$. Prove that $L(G) \subseteq M$.

♣ *We will prove by induction that, after* n *moves, every sentential form* W *satisfies* $n_a(W) = n_b(W)$.

Base Case: $n = 0$, *so the sentential form is* S. $n_a(S) = n_b(S) = 0$.

Inductive step. Let $n \geq 0$ *be given. Suppose that* $S \overset{n}{\Rightarrow} W$, *and* $n_a(W) = n_b(W)$. *We will show that for all sentential forms with* $W \to W'$, *we must have* $n_a(W) = n_b(W)$, *There are only two types of rules.* λ *rules, do not change* n_a *or* n_b, *in which case* $n_a(W') = n_a(W) = n_b(W) = n_b(W')$, *as required. The other type or rule adds exactly two a's and exactly two b' to the sentential form, in which case* $n_a(W') = n_a(W) + 2 = n_b(W) + 2 = n_b(W')$. *In either case* $n_a(W') = n_b(W')$ *for all* W' *with* $S \overset{n+1}{\Rightarrow} W$.

So the requirement is satisfied for every sentential form W *derivable from* S *by induction, and hence for all elements of the language.* ♣

2. (5 pts) Let $L(G)$ be the language of the grammar

$$\begin{aligned} G : S &\to a^2Ab^2 \mid a^2Bb^2 \mid \lambda \\ A &\to a^2Ab^2 \mid a^2Bb^2 \mid \lambda \\ B &\to b^2Ba^2 \mid a^2Ab^2 \end{aligned}$$

And set $N = \{w \in \{a,b\}^* \mid w = a^{2k}b^{2k}a^{2k}b^{2k}a^{2k}b^{2k}\}$. On the back of this page, prove that $N \subseteq L(G)$.

♣ *We give a derivation sequence. For* $k = 0$, *we have* $w = \lambda$, *and* $S \Rightarrow \lambda$ *using the* S *rule. For* $k = 1$, *we use* $S \Rightarrow a^2Bb^2 \Rightarrow a^2b^2Ba^2b^2 \Rightarrow a^2b^2a^2Ab^2a^2b^2 \Rightarrow a^2b^2a^2\lambda b^2a^2b^2$.

For $k \geq 2$, *we use* $S \Rightarrow a^2Ab^2 \overset{k-2}{\Rightarrow} a^{2k-2}Ab^{2k-2} \Rightarrow a^{2k}Bb^{2k} \overset{k}{\Rightarrow} a^{2k}b^{2k}Ba^{2k}b^{2k} \Rightarrow a^{2k}b^{2k}a^2Ab^2a^{2k}b^{2k} \overset{k-1}{\Rightarrow} a^{2k}b^{2k}a^{2k}Ab^{2k}a^{2k}b^{2k} \Rightarrow a^{2k}b^{2k}a^{2k}b^{2k}a^{2k}b^{2k}$. ♣