Ma2201/CS2022 **Foundations of C.S.** Spring, 2022

Quiz 0100

PRINT NAME: ____________________

$\mathcal{SIGN}$: ____________________

1. (**7 pts**) Find a regular expression for all strings on a, b, c in which the substring abc does not occur.

Explain your principles of design so that your expression makes sense.

♣ *How can a string starting in a, consisting of a and b alone, continue so as to be unable to continue to participate in an abc? By ending it with b^2. The first b^2 renders all the a's and b's harmless. In the absence of b^2, the string has to end in ac, or the word must terminate. We have to make sure that the very short segments such as ac and abb are matched:* $(a^+b)^+b \cup a^+(ba^+)^*c$

So that leads me to $(b \cup c \cup (a^+b)^+b \cup a^+(ba^+)^*c)^*(a \cup b)^*$

I did not ask for a proof that the expression is true, but let's sketch one out anyway.

By the argument in the design, any string which matches the above is partitioned into subwords which not only do not contain an abc, but which cannot combine to form one. So no string matching the expression contains an abc, and all that is left to show is that every string with no abc does in fact match it, and we show that by induction on the length of the string.

The empty string matches the expression, so, inductively, we consider of a string w of length n and assume every shorter string with no abc matches the expression. Suppose w begins with a b or a c, so $w = bw'$ or cw'. Then w' contains no abc and hence matches the expression, and the initial b or c matches $(b \cup c \cup (a^+b)^+b \cup a^+(ba^+)^*c)$, so w matches the expression.

So we may assume w begins with an initial string of a's, a^kw'. If w' has no c's, then $w = \lambda w$ matches $(a \cup b)^*$. So we can assume w' has at least one c. Now, either the first c in w' has no b^2 in front of it, or it does have a b^2 in front of it.

If $w = a^kw'$ has no b^2 before the first c, then the character before the first c is a, and the initial segment containing first c matches $a^+(ba^+)^*c$, and the rest of the string has no abc and so matches $(b \cup c \cup (a^+b)^+b \cup a^+(ba^+)^*c)^*(a \cup b)^*$ by the induction hypothesis, so w does as well.

On the other hand, if a b^2 does occur before the first c, then the initial string matches $(a^+b)^+b$, and what follows has no abc, so again w matches $(b \cup c \cup (a^+b)^+b \cup a^+(ba^+)^*c)^*(a \cup b)^*$.

So in all cases, the string matches the regular expression, and the result is true by induction. ♣

2. (**2 pts**) Find a regular expression which matches an infinite set of strings, but does not match any one of the three strings a^2, b^2, or c^2.

Explain your principles of design so that your expression makes sense.

♣ *There are so many ways to do this. To match an infinite set of string, we just need any Kleene star. To avoid all those squares we can use* $(abc)^*$

That solution does not contain any squared letter even as a subword, which is not a requirement. You can also take a^3a^+, which also doesn't match any string of length 2. ♣

3. (**1 pts**) Given the regular expression $(a \cup b)^*(((c \cup d)(a \cup b)^*)^2)^*$, describe the language of the expression set theoretically.

♣ *If L is the language,* $L = \{w \in \{a, b, c, d\}^* \mid n_c + n_d \text{ is even}\}$. ♣