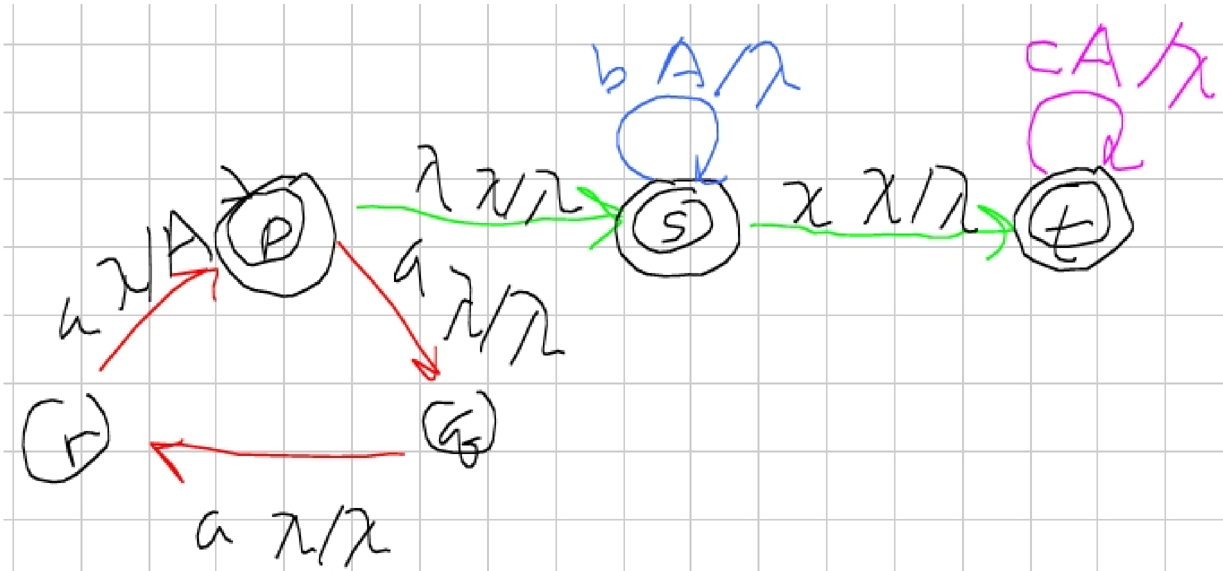
Ma2201/CS2022 | Foundations of C.S. | Spring, 2021

Quiz 1101 | | Online Quiz

1. (5 pts) Let $L = \{a^{3i+3j}b^ic^j \in \{a,b,c\}^* \mid i,j \in \mathbb{N}\}$. Design a push down automaton, standard and not extended, for this language. Describe briefly how it works.

♣ *Of course, there are many ways to do this.*



The machine is basically non-deterministic, the red cycle producing 3 a's on each circuit, and recording one A per circuit, so that the number of b's and c's will properly balance if producing each b and c requires popping one of those recorded A's off the stack. Each state is an accepting state, since once the stack is empty, the a's, b's, and c's are in the form $a^{3i+3j}b^ic^j$ with $i+j$ the number of circuits of the red triangle. Strings are accepted in state p if $i = j = 0$, state s if $j = 0$ and $i > 0$, and state t if $j > 0$. ♣

2. (5 pts) Let $M = \{a^{3i+3j}b^ic^i \in \{a,b,c\}^* \mid i,j \in \mathbb{N}\}$. Us the pumping lemma for context free languages to show that L is not context free.

♣ *Unfortunately, this problem has two copy/paste errors. First, the language was M, and the question asked about L, which was definitely not the point. You were supposed to show M was non context free. More importantly, there were not supposed be any j's in the the definition of M. It was supposed to be $M = \{a^{3i}b^ic^i \in \{a,b,c\}^* \mid i \in \mathbb{N}\}$. The one above is not context free, but you can't show it directly from the context free pumping lemma. You need a little trick first. Here is the proof I wrote for the problem I intended. (When I grade this, I just won't take points on problem 2.)*

Let K be given. If M were context free then any string of length at least K would be factorable so as to have two simultaneously pumpable substrings. Specifically, for $a^{3K}b^Kc^K$, we would have $a^{3K}b^Kc^K = xuyvz$ with uyv of length no more than K, u and v not both empty, and $xu^nyv^nz \in M$ for all $n \geq 1$. First of all, neither u nor v could contain two different letters, since otherwise, pumping would result in a b for an a, or a c before a b, which is not possible. Thus one of the letters does not occur either u or v. But then it is also impossible to pump u and v, since the number of occurrences of an any letter in a string in M, determines the whole string. ♣