CS5003
Quiz 0110

Foundations of C.S.

Spring, 2023

PRINT NAME: ____________________

$\mathcal{SIGN}$: ____________________

1. (6 pts) Let G be the grammar

$$\begin{array}{rcl} G: S & \to & ABCABC \mid \lambda \\ A & \to & abc \mid a \\ B & \to & bca \mid b \\ C & \to & bcbc \mid aC \mid \lambda \end{array}$$

Convert to an equivalent grammar, H, in Chomsky Normal Form.

♣ This is one way

$$\begin{array}{rcll} G': S & \to & WW \mid \lambda & \\ A & \to & U_aY \mid a & \\ B & \to & TU_a \mid b & \\ C & \to & YY \mid U_aC \mid \lambda & \\ W & \to & AX & \quad (W \text{ produces } ABC) \\ X & \to & BC & \\ Y & \to & U_bU_c & \quad \text{(Occurs several times.)} \\ U_a & \to & a & \\ U_b & \to & b & \\ U_c & \to & c & \end{array}$$

This is still not in Chomsky Normal Form because of the λ-rule in C. $\text{Null}(G') = \{S, C\}$, giving

$$\begin{array}{rcl} G'': S & \to & WW \mid \lambda \\ A & \to & U_aY \mid a \\ B & \to & TU_a \mid b \\ C & \to & YY \mid U_aC \mid a \\ W & \to & AX \\ X & \to & BC \mid C \\ Y & \to & U_bU_c \\ U_a & \to & a \\ U_b & \to & b \\ U_c & \to & c \end{array}$$

Now, removing the one chain rule gives

$$\begin{array}{rcl} H: S & \to & WW \mid \lambda \\ A & \to & U_aY \mid a \\ B & \to & TU_a \mid b \\ C & \to & YY \mid U_aC \mid a \\ W & \to & AX \\ X & \to & BC \mid YY \mid U_aC \mid a \\ Y & \to & U_bU_c \\ U_a & \to & a \\ U_b & \to & b \\ U_c & \to & c \end{array}$$

♣

2. (2 pts) Professor Practicus defines Context-free grammar to be Practical if every variable is reachable and terminable.

Is a Practical Grammar a normal form?

Justify why or why not.

♣ Of course. Just use the procedure for removing useless symbols to get an equivalent Practical Grammar. ♣

3. (2 pts) Define a grammar G which is both in Chomsky Normal Form and in Greibach Normal Form and for which $|L(G)| > 3$, or prove on the back of the sheet why it cannot be done.

♣ No sentential form can both be in the form AB and $aBCD\ldots$, so the only rules allowed are $V \to a$. And all variables except S are unreachable. So we have to take something like

$$G : S \to a \mid b \mid c \mid \lambda.$$

for which $|L(G)| = |\{a, b, c, \lambda\}| = 4 > 3$. ♣