CS5003 **Foundations of C.S.** Spring, 2020

Quiz 0001

PRINT NAME: ____________________

SIGN: ____________________

1. (6 **pts**) Prove for sets X, Y and Z that

$$X \cap (Y \cup Z^c) = (X \cap Y) \cup (X \cap Z^c)$$

using the double implication method. Your proof should be clear and complete, and use the method indicated.

♣ Proof: We first show that $X \cap (Y \cup Z^c) \subseteq (X \cap Y) \cup (X \cap Z^c)$. Let $u \in X \cap (Y \cup Z^c)$. So $u \in X$ and $u \in Y \cup Z^c$.

Case 1: $u \in Y$. So we have $u \in X$ and $u \in Y$, so $u \in X \cap Y$, hence $u \in (X \cap Y) \cup (X \cap Z^c)$.

Case 2: $u \in Z^c$. So we have $u \in X$ and $u \in Z^c$, so $u \in X \cap Z^c$, hence $u \in (X \cap Y) \cup (X \cap Z^c)$ in this case as well.

So regardless of case, $X \cap (Y \cup Z^c) \subseteq (X \cap Y) \cup (X \cap Z^c)$.

We next show that $(X \cap Y) \cup (X \cap Z^c) \subseteq X \cap (Y^c \cup Z)$. Let $v \in (X \cap Y) \cup (X \cap Z^c)$, so there are two cases.

Case 1: $v \in (X \cap Y)$, so $v \in X$ and $v \in Y$. Since $v \in Y$, $v \in Y \cup Z^c$, so $v \in X \cap (Y \cup Z^c)$.

Case 1: $v \in (X \cap Z^c)$, so $v \in X$ and $v \in Z^c$. Since $v \in Z^c$, $v \in Y \cup Z^c$, so $v \in X \cap (Y \cup Z^c)$ in this case as well.

Thus $(X \cap Y) \cup (X \cap Z^c) \subseteq X \cap (Y \cup Z^c)$.

Thus we have shown $X \cap (Y \cup Z^c) = (X \cap Y) \cup (X \cap Z^c)$.

[As you should have expected, the proof did not deviate from the form of the one we did in class, where the third set written as a single letter, and not an expression like Z^c.]

♣

2. (4 **pts**) Let A and B be sets. For each of the following label it T it it must be true, F if it must be false, and X if it cannot be determined from the given information.

___ $A \cap B \subseteq \mathcal{P}(A \cup B)$.

___ $A \cap (A \cup B)^c = \emptyset$.

___ $\mathcal{P}(A \cap B) \in \mathcal{P}(A \cup B)$

___ $\emptyset \subseteq A^c \cup B^c$.

♣ __X__ *If the symbol was $\in$ and not $\subseteq$, the statement would be always true. For $\in$ it is usually not true, but it might be, say if $A \cap B = \emptyset$.*

__T__ *An element of $A \cap (A \cup B)^c$ would have to be in A, but not in $A \cup B$, which is impossible. You can also use the distributive law and Demorgan's to analyze it.*

__X__ *If the symbol was $\subseteq$ and not $\in$, then the statement would be always true. But as it is, the statement is usually false. However, if $A \cap B = \emptyset$, then $\mathcal{P}(A \cap B) = \{\emptyset\}$, so if $\emptyset \in A \cup B$, the statement is true, and this can happen, say, if $A = \{1, 2, 3\}$ and $B = \{\emptyset\}$. Now their intersection is empty, but their union contains the empty set as an element, so the set containing just the empty set is a subset of $A \cup B$, and so $\{\emptyset\}$ is an element of it's power set. (The was the most difficult point to get on the quiz)*

__T__ *The empty set is a subset of every set.*

♣