CS5003 Quiz 0011

Foundations of C.S.

Spring, 2023

PRINT NAME: ____________________

$\mathcal{SIGN}$: ____________________

1. (7 pts) Let $L \subseteq \{a, b, c, d\}^*$ be defined recursively via

BASIS: $d, abab, bb, \lambda \in L$.

RECURSIVE STEP: If $w \in L$ then awb, bwc, cwa, and w^2 are all in L.

CLOSURE: All elements of the set can be obtained from the closure after a finite number of applications of the recursive step.

Prove carefully by induction that $n_a(w)+n_b(w)+n_c(w)$ is even for every element $w \in L$. Use the back of the page if you need more space.

♣ The problem calls for a careful proof. The proof is more important that the actual result, which is pretty simple. The statement to be proved is about all elements $w \in L$, and there is no specified variable to perform induction on – we have to supply that ourselves.

Proof: By induction on the number of recursive steps applied.

Base Case: No rules applied. Then the element w is in the basis, and $w \in \{d, abab, bb, \lambda\}$, so $n_a(w) + n_b(w) + n_c(w) \in \{0, 2, 4\}$, all of which are even.

Inductive step. Suppose $n_a + n_b + n_c$ is even for all elements in L_n and let $w' \in L_{n+1}$. Either $w' \in L_n$, in which case $n_a(w') + n_b(w') + n_c(w')$ is even by the inductive hypothesis, or w' is obtained from some $w \in L_n$ after one rule application. Since $w \in L_n$, we have $n_a(w) + n_b(w) + n_c(w)$ is even by the inductive hypothesis. Using the rules, $w' \in \{awb, bwc, cwa, w^2\}$ and $n_a(w')+n_b(w')+n_c(w')$ is either $n_a(w)+n_b(w)+n_c(w)+2$ or $2[n_a(w)+n_b(w)+n_c(w)]$, both of which are even. So, in either case $n_a(w')+n_b(w')+n_c(w')$ is even, as required. So $n_a + n_b + n_c$ is even for all elements in L_{n+1}, and so all elements of L by induction. ♣

2. (3 pts) Let $K = \{w \in \{a, b, c\}^* \mid w = a^i b^j c^k; i, j, k \in \mathbb{N}\}$. For each of the following, label it T if the statement is true, F if the statement is false, and X if it cannot be determined from the given information.

Label each of the following as TRUE or FALSE or X, if there is not enough information given.

For each, just give a word or two of explanation.

__ K^* is a regular language.

__ KKK is a regular language.

__ $cba \in KKK$.

♣ All True.

Notice that $K = a^*b^*c^*$ is regular.

The first, the Kleene-$*$ of a regular set is regular.

The Second, Concatenating regular languages yields a regular language.

The Third: $c \in K$, $b \in K$, and $a \in K$, so $cba \in KKK$.

Notice that K^3, which is the same as KKK, is not describing those elements which belong to K cubed, $K^3 \neq \{w^3 \mid w \in K\}$. They are matched separately: $K^3 \neq \{w_1w_2w_3 \mid w_i \in K\}$. ♣