Ma2201/CS2022 Quiz 0010

Foundations of C.S.

Spring, 2021

PRINT NAME: ____________________

$\mathcal{SIGN}$: ____________________

1. (6 pts) Let P, Q and R be sets. Prove carefully, **using the double inclusion method** that

$$(R \cup P) \cap Q = (P \cap Q) \cup (Q \cap R).$$

♣ *Let $x \in (R \cup P) \cap Q$. Then $x \in R \cup P$ and $x \in Q$. Since $x \in R \cup P$ there are two cases.*

Case 1: If $x \in R$, then we have $x \in R$ and $x \in Q$, so $x \in Q \cap R$. Since $x \in Q \cap R$ we have $x \in (P \cap Q) \cup (Q \cap R)$, as desired.

Case 1: If $x \in P$, then we have $x \in P$ and $x \in Q$, so $x \in P \cap Q$. Since $x \in P \cap Q$ we have $x \in (P \cap Q) \cup (Q \cap R)$, in this case as well.

Hence, in either case, $x \in (P \cap Q) \cup (Q \cap R)$, so $(R \cup P) \cap Q \subseteq (P \cap Q) \cup (Q \cap R)$.

Now let $y \in (P \cap Q) \cup (Q \cap R)$, so $y \in P \cap Q$ or $y \in Q \cap R$.

Case 1: If $y \in P \cap Q$ then $y \in P$ and $y \in Q$. Since $y \in P$, we have $y \in R \cup P$. Thus $y \in (R \cup P) \cap Q$.

Case 2: If $y \in Q \cap R$ then $y \in Q$ and $y \in R$. Since $y \in R$, we have $y \in R \cup P$. Thus $y \in (R \cup P) \cap Q$ in this case as well.

Hence, in either case, $y \in (R \cup P) \cap Q$, so $(P \cap Q) \cup (Q \cap R) \subseteq (R \cup P) \cap Q$.

Therefore $(R \cup P) \cap Q = (P \cap Q) \cup (Q \cap R)$.

♣

2. (4 pts) Suppose $f : \mathbb{Z} \to \mathcal{P}(\mathbb{Z})$ and $g : \mathcal{P}(\mathbb{Z}) \to \mathbb{Z}$ are functions.

Check each of the following statements which *must* be true, and write a brief explanation why each box is, or is not, checked.

☐ f is not one-to-one.

☐ f is not onto.

☐ g is one-to-one.

☐ g is onto.

♣ *$\mathbb{Z}$ is countable and $\mathcal{P}(\mathbb{Z})$ is uncountable, so $|\mathbb{Z}| < |\mathcal{P}(\mathbb{Z})|$, so there is no onto function from $\mathbb{Z}$ to $\mathcal{P}(\mathbb{Z})$ and no one-to-one function from $\mathcal{P}(\mathbb{Z})$ to $\mathbb{Z}$.*

f might or might not be one-to-one, so the first should not be checked.

f cannot be onto, so the second one must be checked.

g cannot be one-to-one. So of course, the third one should not be checked.

g might or might not be onto, so the forth one should not be checked. ♣