Ma2201/CS2022
Quiz 0101

Foundations of C.S.

Spring, 2022

PRINT NAME: _____

$\mathcal{SIGN}$: _____

1. (5 pts) Consider the grammar

$$\begin{array}{rcl} G: S & \to & AB \\ A & \to & aAb^2 \mid ab \\ B & \to & b^2Bc \mid bc \end{array}$$

Write $L(G)$ in set notation. You do not have to prove that your set is correct.

♣ *Let* $L' = \{w \in \{a,b,c\}^* \mid a^{k+1}b^{2k+2j+2}c^{j+1}; j,k \geq 0\}$. *Then* $L(G) = L'$
First $L' \subseteq L(G)$, *which we show with a derivation:*

$$S \Rightarrow AB \stackrel{k}{\Rightarrow} a^kAb^{2k}B \Rightarrow a^{k+1}b^{2k+1}B \stackrel{j}{\Rightarrow} a^{k+1}b^{2k+1}b^{2j}Bc^j \Rightarrow a^{k+1}b^{2k+1}b^{2j+1}c^{j+1} = a^{k+1}b^{2k+2j+2}c^{j+1}$$

Second, to show $L(G) \subseteq L'$ *we show that all sentential forms are of one of five types in the following list.*

$$S, \quad a^kAb^{2k+2j}Bc^j, \quad a^{k+1}b^{2k+2j+1}Bc^j, \quad a^kAb^{2k+2j+1}c^{j+1}, \quad a^{k+1}b^{2k+2j+2}c^{j+1}$$

We show that by induction on the number of rules applied. For zero rules we have S, *which is on the list. Suppose after* n *rules have been applied that the resulting sentential form is on the list. So we only need to observe that applying one rule to any element of the list, gives another element of the list. Specifically, the start rule takes the first type to the second, with* $k = j = 0$*; the first* A *or* B *rules take sentential forms of the second, third and fourth type to a sentential form of the very same type with* k *or* j *incremented by* 1*; the second* A *and* B *rules take words of the second type to the third or fourth, and sentential forms of the third and fourth type to the fifth. That covers all five rules. So, by induction, all sentential forms are included on the list, so* $L(G) \subseteq L'$.

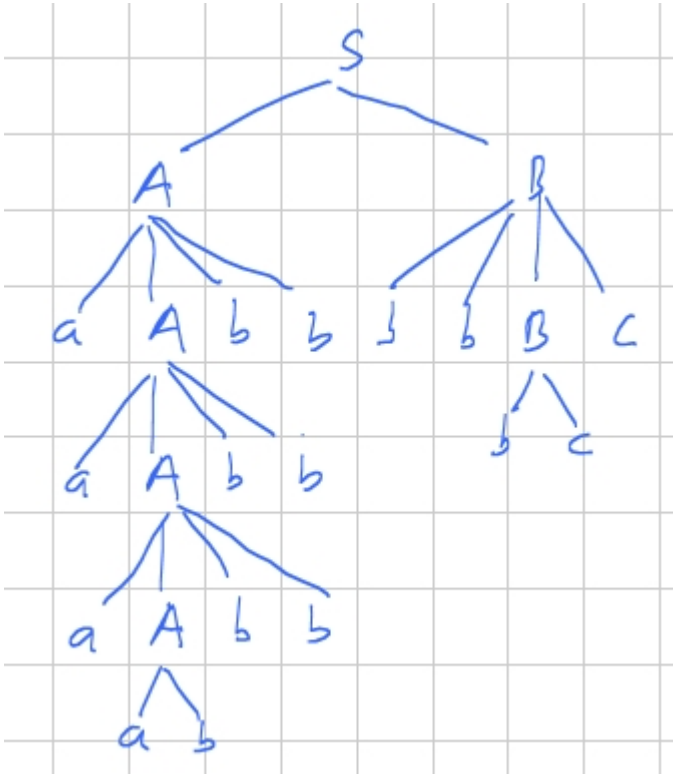
You were not asked for a proof, but by next time you should be ready for one! ♣

2. (3 pts) In the grammar

$$\begin{aligned} G: S &\rightarrow AB \\ A &\rightarrow aAb^2 \mid ab \\ B &\rightarrow b^2Bc \mid bc \end{aligned}$$

Give a derivation tree for the string $aaaabbbbbbbbbbcc$.

♣



♣

3. (2 pts) Write a regular expression for the language of the following grammar

$$\begin{aligned} G: S &\rightarrow ABC \\ A &\rightarrow a^2A \mid Aa \mid \lambda \\ B &\rightarrow b^4B \mid Bb \mid \lambda \\ C &\rightarrow aC \mid Cb \mid \lambda \end{aligned}$$

♣

$$a^*b^*a^*b^*$$

You should notice that two of the rules can be eliminated with out affecting the language, but they DO effect the sentential forms. ♣

Bonus: (2pts) Find any string of length ten not in this language, and state why not.

♣ *You can take abababbabab which is not in the language A can only derive strings of a's, B can only derive strings of b's and C can only derive a string of a followed by a string of b, so at most three switches from between a and b are possible.* ♣