

## Foundations of C.S.

Spring, 2023

1. (3 pts) Find a regular expression for the set of strings on  $\{a, b, c\}$  which contain  $a^2$  as a substring.

Justify your design in a few words.

 $\clubsuit$  Design: This one everyone should get. Every such string decomposes as any initial string, followed by  $a^2$  and then some terminal string. The fact that a string has many have several such decompositions is irrelevant. That is the nature of regular expressions.

$$(a \cup b \cup c)^*a^2(a \cup b \cup c)^*$$

works fine. (The string  $a^2ba^3$  matches it in three ways, and ababc doesn't match it at all.) There are other correct solutions.

2. (4 pts) Find a regular expression for the set of strings on  $\{a, b, c\}$  which do not contain  $a^2$  as a substring.

Justify your design in a few words.

♣ Design: Every non-final a must be followed by a non-empty string of b's and c's. So  $(a(b \cup c)^+)^*a$  will match all such strings which start and end with a. To allow for other beginnings and endings, and those strings which have no a's at all, we have:

$$(b \cup c)^* (a(b \cup c)^+)^* a(b \cup c)^* \cup (b \cup c)^*$$

There are other correct solutions.  $\blacksquare$ 

3. (3 pts) Find a regular expression for the set of strings on  $\{a, b, c\}$  which do not contain either  $a^2$  or  $b^2$  as substrings.

Justify your design in a few words.

 $\clubsuit$  Design: All substrings consisting of just a's and b's have to alternate back and forth. So  $a(ba)^*(b \cup \lambda)$  if it starts with a, and  $b(ab)^*(a \cup \lambda)$  if it starts with b. Such segments must be separated by a nonempty string of c's, so

 $((a(ba)^*(b \cup \lambda) \cup b(ab)^*(a \cup \lambda))c^+)^*(a(ba)^*(b \cup \lambda) \cup b(ab)^*(a \cup \lambda))$ , and it may start or end with c's. So the final expression is

$$c^{*}((a(ba)^{*}(b \cup \lambda) \cup b(ab)^{*}(a \cup \lambda))c^{+})^{*}(a(ba)^{*}(b \cup \lambda) \cup b(ab)^{*}(a \cup \lambda))c^{*},$$

There are other correct solutions.

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