Ma2201/CS2022 Quiz 0001

Foundations of C.S.

Spring, 2021

PRINT NAME: ____________________

$\mathcal{SIGN}$: ____________________

1. Let $A = \{1, 2, 3\}$, $B = \{1, 3, 5\}$ and $C = \mathcal{P}(A \cap B)$.

a) (1 **pt**) List all the elements of $A \cup (B \cap C)$.

♣ *First $B \cap C$ is empty, since the elements of C are sets, and the elements of B are numbers. So $A \cup (B \cap C) = A = \{1, 2, 3\}$* ♣

b) (1 **pt**) List all the elements of $(\mathcal{P}(A) \cap C) \times (A \cap B)$.

♣ *The elements of $(\mathcal{P}(A) \cap C) = (\mathcal{P}(A) \cap \mathcal{P}(A \cap B)) = \mathcal{P}(A \cap B) = \mathcal{P}(\{1, 3\})$ so the desired elements are pairs, the first coordinate is a subet of $\{1, 3\}$, and the second coordinate is either 1 or 3:*

$$(\emptyset, 1), (\{1\}, 1), (\{3\}, 1), (\{1, 3\}, 1), (\emptyset, 3), (\{1\}, 3), (\{3\}, 3), (\{1, 3\}, 3)$$

♣

c) (1 **pt**) List all the elements of $\mathcal{P}(A) \cap \mathcal{P}(B) \cap \mathcal{P}(A \cup B)$.

♣ *The elements are subsets of A, B and $A \cup B$. The last requirement is automatically satisfied by the first two, and that just gives subsets of $A \cap B = \{1, 3\}$, so:*
$\{\emptyset, \{1\}, \{3\}, \{1, 3\}\}$. ♣

d) (2 **pt**) Define an onto function, f, with domain $A \times B$ and target C.

♣ *There are many onto functions from the 9 elements of $A \times B$ to the 4 elements of C, such as $f((1, 1)) = \emptyset$ $f((1, 3)) = \{1\}$ $f((1, 5)) = \{3\}$ $f((2, 1)) = \{1, 3\}$ $f((2, 3)) = \{1, 3\}$ $f((2, 5)) = \{1, 3\}$ $f((3, 1)) = \{1, 3\}$ $f((3, 3)) = \{1, 3\}$ $f((3, 5)) = \{1, 3\}$*
♣

e) (2 **pt**) Define an **onto** function with domain A and target $A \times A$ which is not one-to-one.

♣ *Oops – this was a copy paste error. The word onto was not supposed to be there. So the problem was impossible. Sorry.* ♣

2. Define a relation X on $\mathbb{N}$, $X \subseteq \mathbb{N} \times \mathbb{N}$, by setting $(n, m) \in X$ if there is a number $k \in \mathbb{N}$ with

$$\min(n, m) \leq k^2 \leq \max(n, m)$$

If you prefer algebra, this is just the same:

$$\frac{n+m}{2} - \frac{|n-m|}{2} \leq k^2 \leq \frac{n+m}{2} + \frac{|n-m|}{2}$$

a) Explain briefly why, or why not, X is reflexive.

b) Explain briefly why, or why not, X is symmetric.

c) Explain briefly why, or why not, X is transitive.

♣ *In non-formal speech, two numbers are related if there there is a perfect square between them:*

Reflexive: The relation is **not** *reflexive because* 5 *has no perfect square between it and itself. It is not a perfect square.*

(The property of reflexivity is not satisfied if there is a single violator. I don't have determine the set of violators. I just have to exhibit a single one of my choice.)

Symmetric: The relation **is** *symmetric since the condition depends only on* $\min(n, m)$ *and* $\max(n, m)$*, and* $\min(n, m) = \min(m, n)$ *and* $\max(n, m) = \max(m, n)$.

Transitive: The relation is **not** *transitive since* 5 *is related to* 6 *and* 6 *is related to* 5*, but, as above,* 5 *is not related to* 5.

Of course there are many ways to explain these correctly. ♣