Foundations of C.S.

PRINT NAME: _____

$\mathcal{SIGN}$: _____

1. (5 pts) Label each of the following as True or False. Read them carefully.

___ If $L(G)$ is context free, then G is in Chomsky Normal Form.

___ If G is a grammar which both in Chomsky Normal Form and in Greibach normal form, then G is a regular grammar.

___ If G is in Chomsky Normal Form, then G is essentially non-contracting.

___ If G is in Greibach Normal Form with variable set V, then every sentential form w of G can be factored as $w = uv$ where $u \in \Sigma^*$ and $v \in V^*$.

___ If G is a regular grammar, then every derivation is Leftmost.

♣ False. But the language of any grammar can be represented by a Chomsky Grammar.

True. The only rules allowed are to send variables to letters, and only Start may go to λ. So the form is a regular grammar. Moreover, all variables other than start are unreachable, hence useless, and the language is finite.

True. This is just technical. Only S can have a λ rule.

False. This might seem to be so you are only thinking of *leftmost* derivations.

True. This is obviously true since there is never more than one variable in any sentential form.   ♣

2. (3 pts) Let G be the grammar

$$\begin{aligned} G : S &\to ABABC \mid \lambda \\ A &\to aaa \mid aB \mid a \\ B &\to aab \mid bC \mid b \\ C &\to aac \mid cA \mid \lambda \end{aligned}$$

Convert to an equivalent grammar H in Chomsky Normal Form.

♣ Null = {S,C} and that gives us the equivalent:

$$\begin{aligned} G' : S &\to ABABC \mid ABAB \mid \lambda \\ A &\to aaa \mid aB \mid a \\ B &\to aab \mid bC \mid b \\ C &\to aac \mid cA \end{aligned}$$

Then taking the usual U_a, U_b and U_c we can define $D \to U_a U_a$ and $E = AB$ and $F = EE$. That gives us the equivalent

$$\begin{aligned} G'' : S &\to FC \mid EE \mid \lambda \\ A &\to DU_a \mid U_a B \mid U_a \\ B &\to DU_b \mid U_b C \mid U_b \\ C &\to DU_c \mid U_c A \end{aligned}$$

which is in Chomsky normal form   ♣

3. (2 pts) Professor Startiwitz defines Context-free grammar to be Startiwitz if every variable has at least one rule for which $A \Rightarrow w$ with $n_S(w) > 0$. Is a Startiwitz Grammar a normal form? Justify why or why not. (Put your answer on the back.)

♣ We have to show that we can convert any context free grammar to an equivalent Startiwitz grammar. Every variable will require at least one rule using start. So add useless new variable X and add the new rule $\to SX$ to every variable, including X. That makes X unterminable, hence useless, and these addition rules recursing back to start do change the language.   ♣