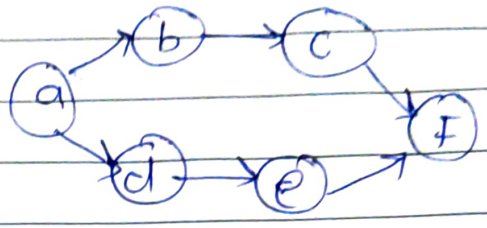
CS5084
Assignment 3

1. given fig. 3.10



from given graph G
node
- 'a' should comes first & 'f' last
- 'b' preceeds 'c' And 'd' preceeds 'e'
(for the middle topology)

Following cases to be considered.

i) if 'b' comes first then 'c' or 'd'
   i) ordering would be : b, c, d, e
OR ii) ordering would be b, d, c, e
OR iii) b, d, e, c

ii) If 'd' comes first then 'e' or 'b'
   i) ordering would be d, e, b, c
OR ii) d, b, e, c
OR iii) d, b, c, e

Thus total '6'

5] We can prove by indution on number of nodes in 'T' (tree) (2)

Let $k_0(T)$ denotes ⇒ no. of binary tree leaves ('T')

Let $k_2(T)$ denotes ⇒ no. of nodes with 2 children ...(given condition)

Basis of Induction: Tree with single node, & this node is the only leaf of tree & has no nodes with 2 children.

let 'T' be an general binary tree with more than one node, & Let 'L' be the leaf.

Since 'T' has more than one node & 'L' is leaf (because no child), we can delete / remove From tree. Let [T'] be that new tree.

Since, 'L' was leaf no then it must have had parent. Lets 'P' be that parent

If 'p' had no other child in 'T' then it can be leaf in [T']

$\therefore k_0(T') = k_0(T)$

& $k_2(T') = k_2(T')$

lets apply Induction hypothesis to T' (induct^n step done)

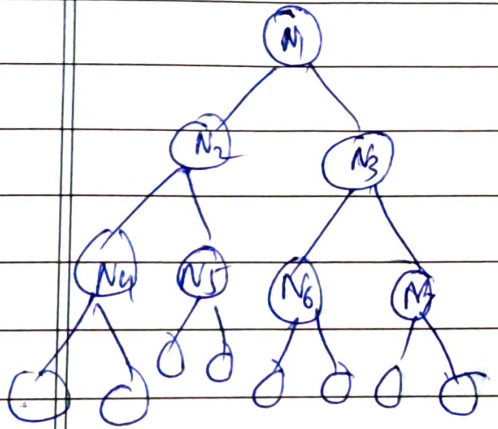
if P had another child in T, then it does not become a leaf after deletion, but it used to have 2 children & now it doesn't.

Thus, we can conclude

$$k_0(T') = k_0(T) - 1$$

$$k_2(T') = k_2(T') - 1$$

Again after applying induction hypothesis on T' we can prove that in any ~~B.O.~~ Binary tree the no. of nodes with 2 children is exactly one less than ~~the~~ no. of leaves.



no. of nodes = 7
no. of levels = 8.

6] lets assume that graph 'G' has an edge $e = \{p, q\}$ that $\notin T$ — (I)

∵ T is dfs tree, one of the 2 ends must be ancestor of the other.

Suppose 'p' is that ancestor of 'q'
if T is bfs tree then the distance of 2 nodes from given 'u' should have difference of 1 unit maximum.

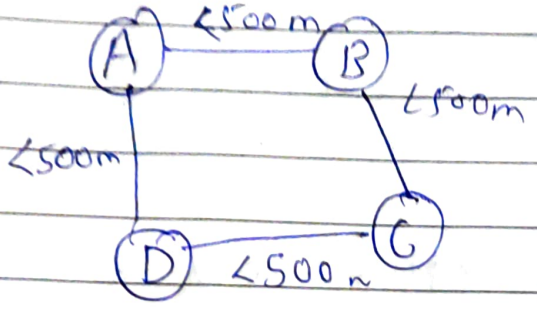
But, if 'p' is ancestor of 'q' & distance from 'u' to 'q' in T is at most greater than the distance from 'u' to 'p'
the 'p' must be direct parent of 'q' in T

from this we can say $\{p, q\}$ is an edge of T

which contradicts eqn (I).

7] Claim is true; to support the claim cosider following scenario.

4 devices i.e 4 nodes say A, B, C, D in the graph are connected within no more than 500 m range.



here, every device is in the range of at least 2 devices i.e.

A is in range with B, D
B ——— A, C
C ——— B, D
D ——— A, C

i.e. $\frac{n}{2} = \frac{4}{2} = 2$ ⇒ every node has degree '2' at least.

Hence, network remains connected hence claim is true.

2] We can apply DFS to detect undirected graph.

We can strat from arbitrary node in the graph.
- mark current node as visited.
- Repeat above steps for each non-visited neighbor node.
- if (neighbor node → already visited ✓ && ≠ parent node)
  ⇒ there is a cycle in the graph.
- if none of the neighbors form cyle the continue with next node.

The running time of algorithm is $O(m+n)$
m - edges., n - nodes
because edges to be checked twice one for each direction & each node visited exactly once in that direction.