Machine Learning

CS 539

Worcester Polytechnic Institute
Department of Computer Science
Instructor: Prof. Kyumin Lee

Midterm Review

Midterm Exam

- The exam will be held at 4pm in the next class, Feb 27.
- Bring your laptop/phone so that you can submit your answers via Canvas.
- Remote exam will not be allowed.
- The exam is closed book.
- You may prepare and use one standard 8.5" by 11" piece of paper with any notes you think appropriate or significant (use only *one-side*).
- You may use a calculator if it make you feel comfortable. But no other electronic devices are allowed (e.g., cell phone, tablet and computer). You will use your laptop/phone for answer submission only!

Previous Class…

Supervised Learning:
→ Regression and Classification

Decision Tree
→ How to choose the best attribute?
→ Information Gain with Entropy

Previous Class…

Overfitting in Decision Tree? How to avoid it?
→ Reduced error pruning,

kNN
→ 1-NN vs k-NN
→ How to reduce the prediction time?

Evaluation
→ Accuracy, Precision and Recall

Previous Class…

Cross-validation

Linear Regression
and
Regularization

Perceptron

Previous Class…

Logistic Regression

VC dimension
Bias vs Variance

Neural Network
→ Feedforward propagation
→ Back propagation

Upcoming Schedule

- Midterm
  - Feb 27 in class
- HW4 will be out on March 1st (postponed)
- Project Workday on March 12 and 15
- Project Proposal
  - Due date is March 18

Project Workday

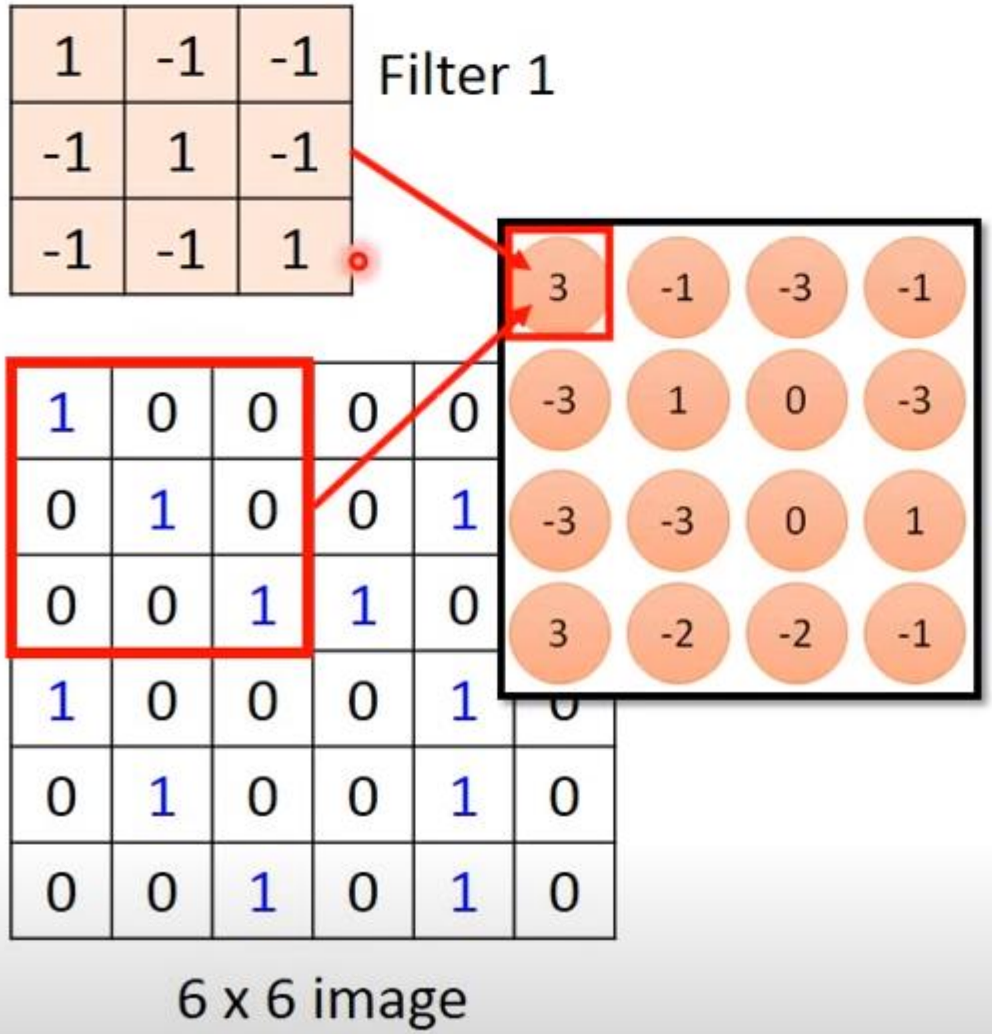
Date	Presenters
3/12	• Yu-Chi Liang, William Ryan, Riley Blair, and Stephen Fanning • Chris Lee, Andrew Kerekon, Amulya Mohan, Alex Siracusa, and Sulaiman Moukheiber • Vagmi Bhagavathula, Deepti Gosukonda, Adina Palayoor, Bishoy Soliman Hanna, and Jared Chan • Sreeram Marimuthu, Oruganty Nitya Phani Santosh, Sarah Olson, and Thomas Pianka • Shubham Dashrath Wagh, Atharva Pradip Kulkarni, Amit Virchandbhai Prajapati, Niveditha Narasimha Murthy • Aria Yan, Alisha Peeriz, Nupur Kalkumbe, Pavan Antala, Rutuja Madhumilind Dongre • Noushin Khosravi Largani, Jinqin Xiong, Kexin Li, Ronit Kapoor, and Yiqun Duan • Phil Brush, Liam Hall, Jared Morgan, Alex
3/15	• Khang Luu, Austin Aguirre, Brock Dubey, Ivan Klevanski, • Adhiraj, Karl, Shariq Madha, Yue Bao, Vasilli Gorbunov • Edward Smith, Michael Alicea, Cutter Beck, Blake Bruell, Anushka Bangal • Rahul Chhatbar, Sonu Tejwani, Deep Suchak, Shoan Bhatambare • Daniel Fox, Bijesh Shrestha, Chad Hucey, Aayush Sangani, Ivan Lim • Devesh Bhangale, Shipra Poojary, Jagruti Chitte, Parth Shroff, Saurabh Pande • Alessandra Serpes, Khushita Joshi, Sanjeeeth Nagappa Chakrasali, Shaun Noronha, Sankalp Vyas • Rohan Rana, Theo Coppola, Olivia Raisbeck

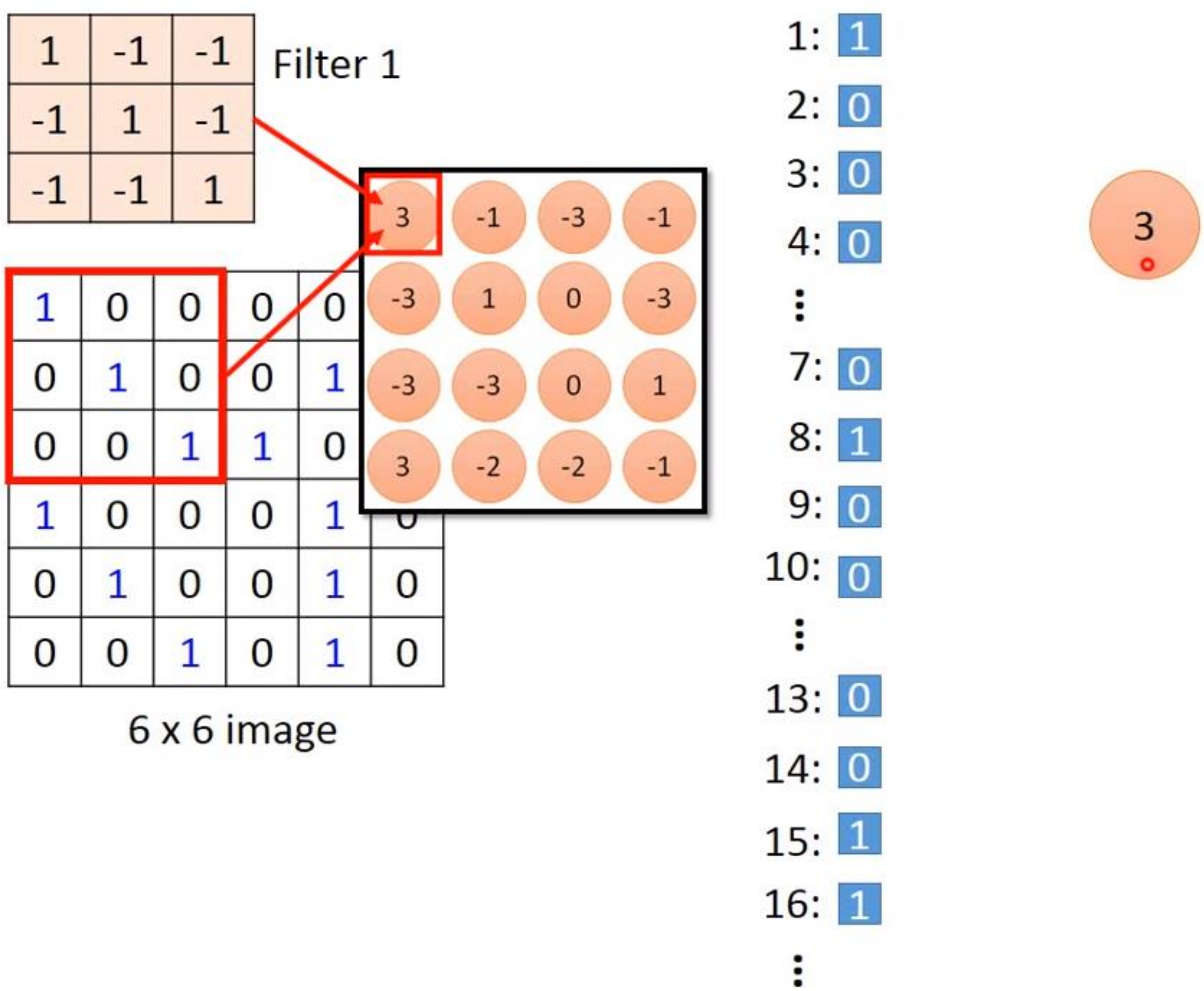
More detailed logistics will be announced on March 1st

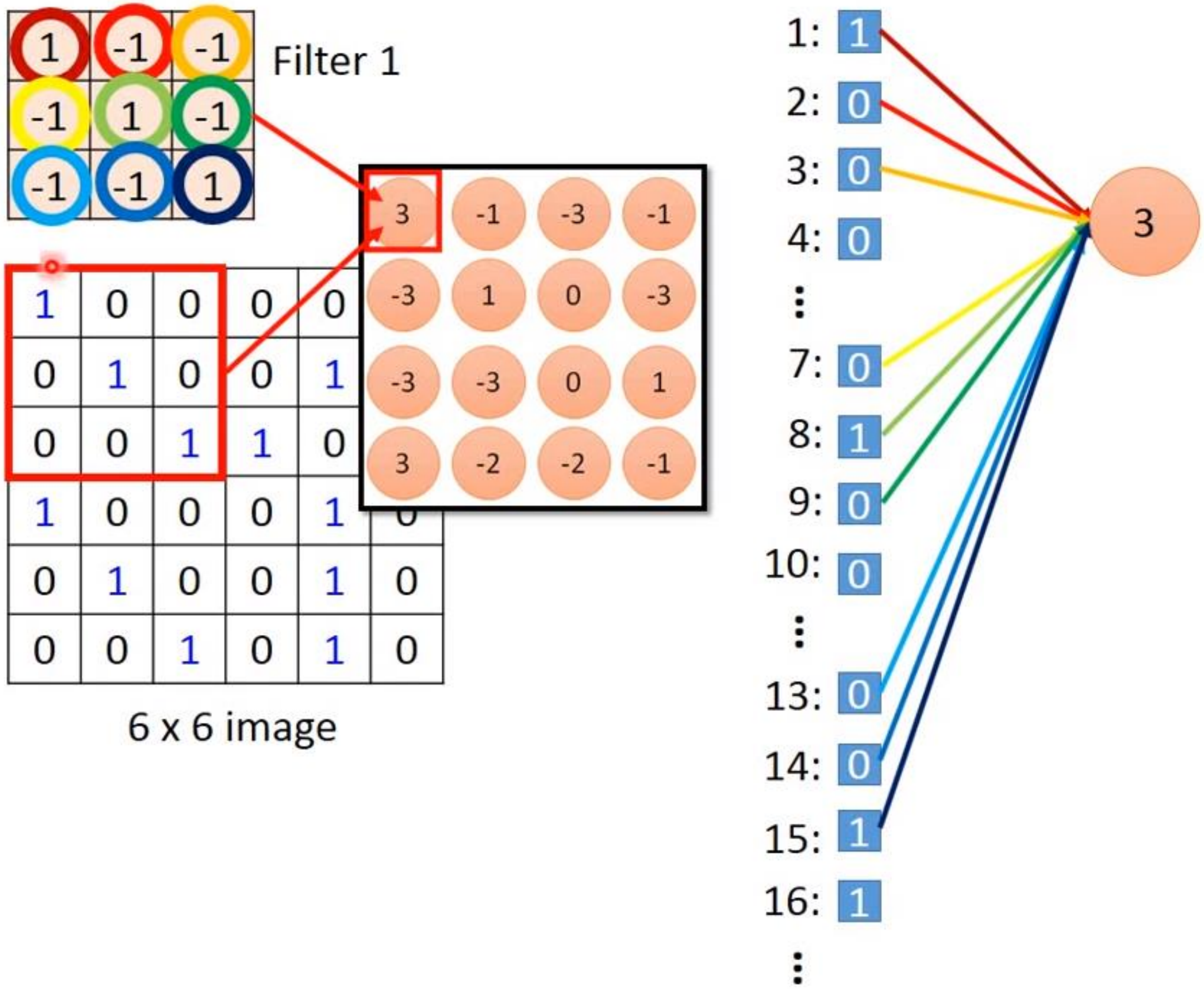
Convolutional Neural Network (CNN)

Convolution:

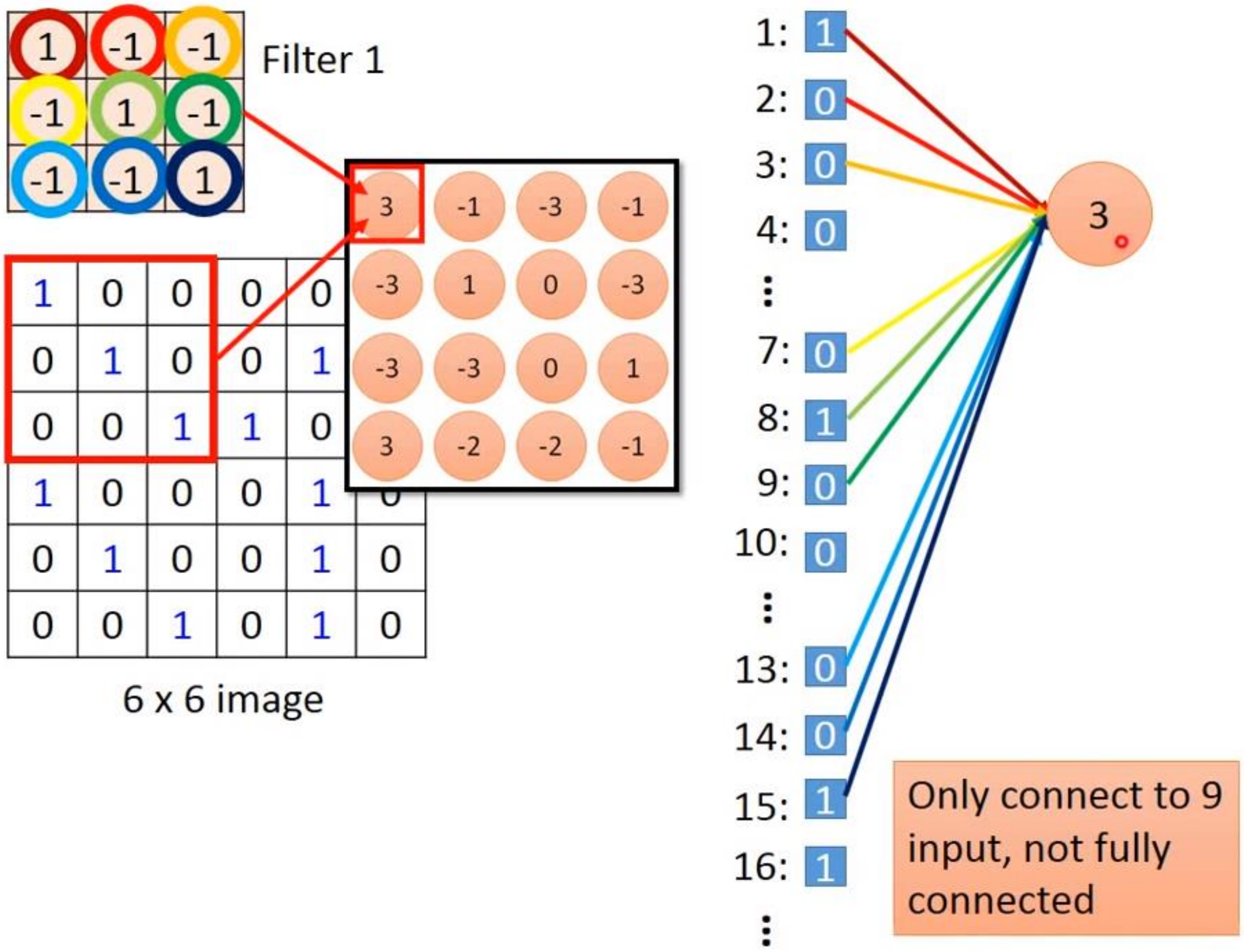
Convolution is a mathematical operation on two functions to produce a third function that expresses how the shape of one is modified by the other. It is a term that the neural network community (Hinton, specifically) adopted from the signal and image processing communities to architect the “feature detection” layers of “deep” neural networks.


Filter 1
6 x 6 image

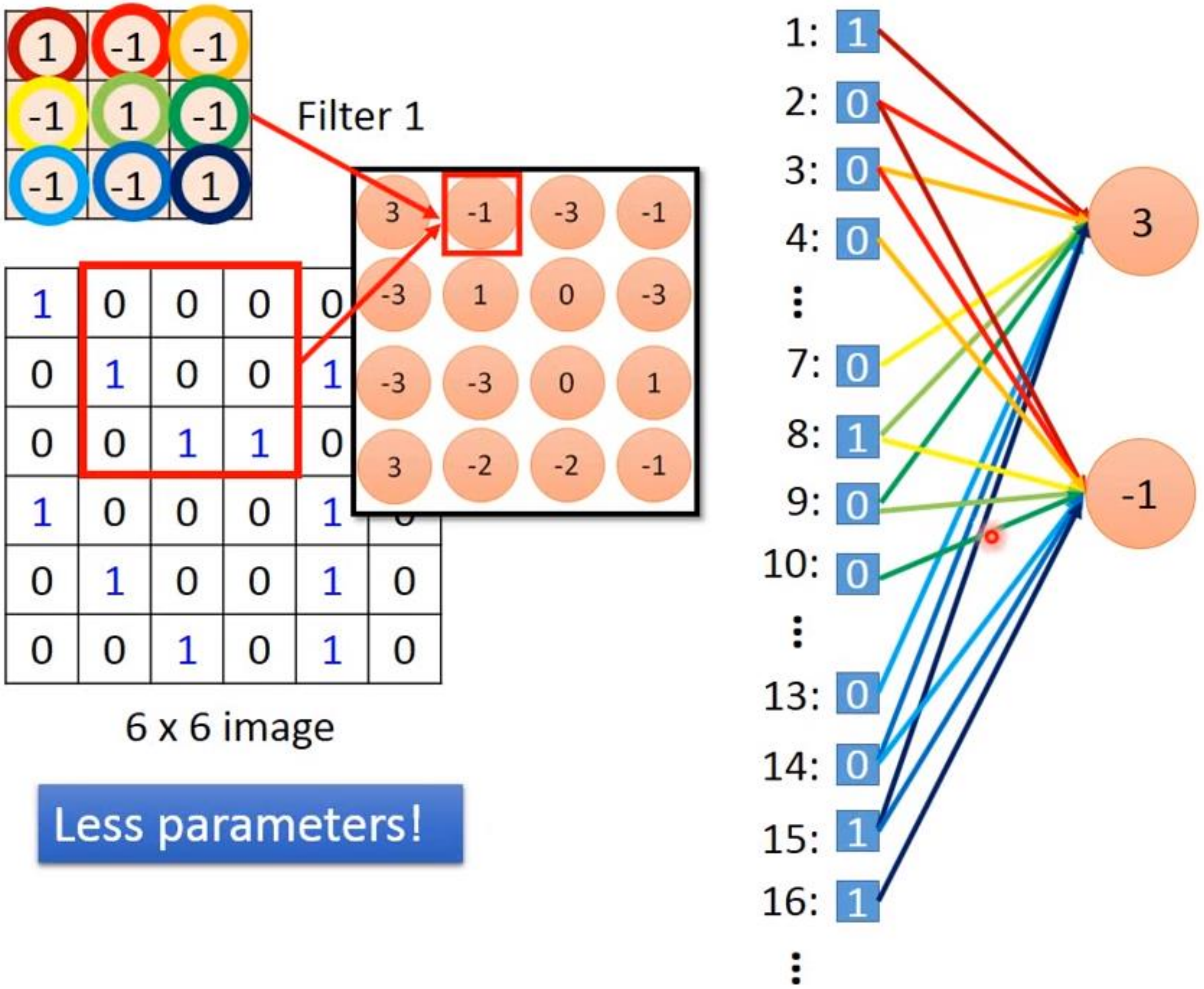

Filter 1
1 -1 -1
-1 1 -1
-1 -1 1
6 x 6 image
1 0 0 0 0 1
0 1 0 0 1 0
0 0 1 1 0 0
1 0 0 0 1 0
0 1 0 0 1 0
0 0 1 0 1 0
3 -1 -3 -1
-3 1 0 -3
-3 -3 0 1
3 -2 -2 -1
1: 1
2: 0
3: 0
4: 0
7: 0
8: 1
9: 0
10: 0
13: 0
14: 0
15: 1
16: 1
3



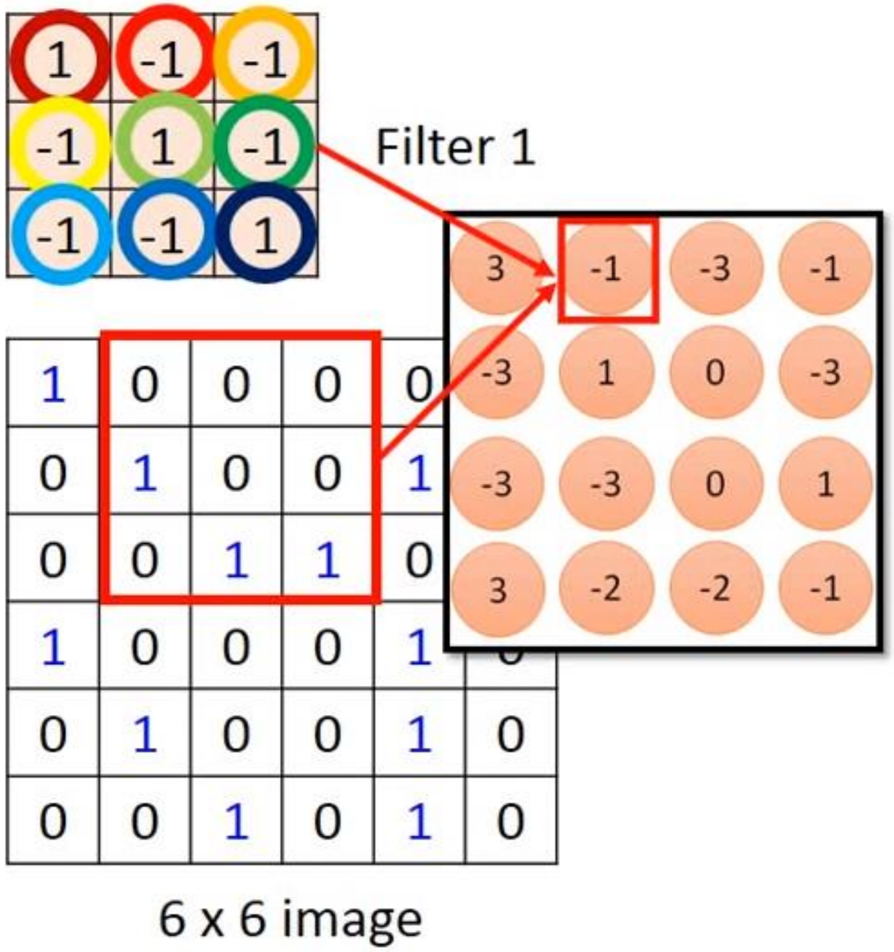
Filter 1
1 -1 -1
-1 1 -1
-1 -1 1
6 x 6 image
1 0 0 0 0 1
0 1 0 0 1 0
0 0 1 1 0 0
1 0 0 0 1 0
0 1 0 0 1 0
0 0 1 0 1 0
3 -1 -3 -1
-3 1 0 -3
-3 -3 0 1
3 -2 -2 -1
1: 1
2: 0
3: 0
4: 0
7: 0
8: 1
9: 0
10: 0
13: 0
14: 0
15: 1
16: 1
3

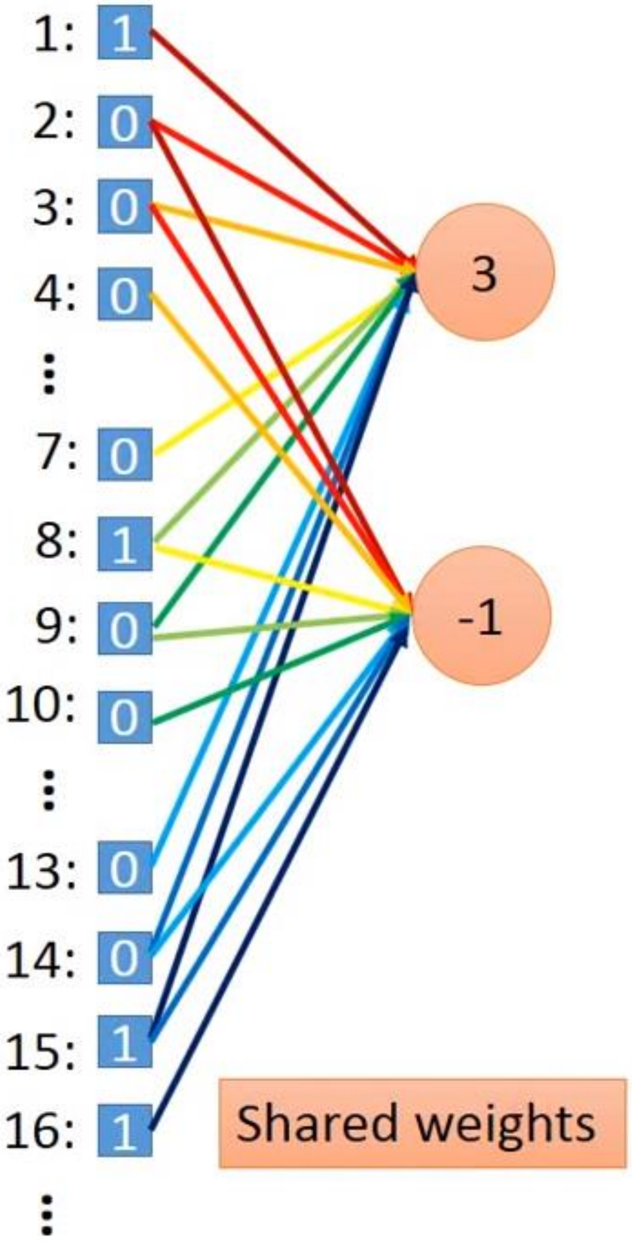


Filter 1
6 x 6 image
1: 1
2: 0
3: 0
4: 0
7: 0
8: 1
9: 0
10: 0
13: 0
14: 0
15: 1
16: 1
3
Only connect to 9 input, not fully connected



Less parameters!

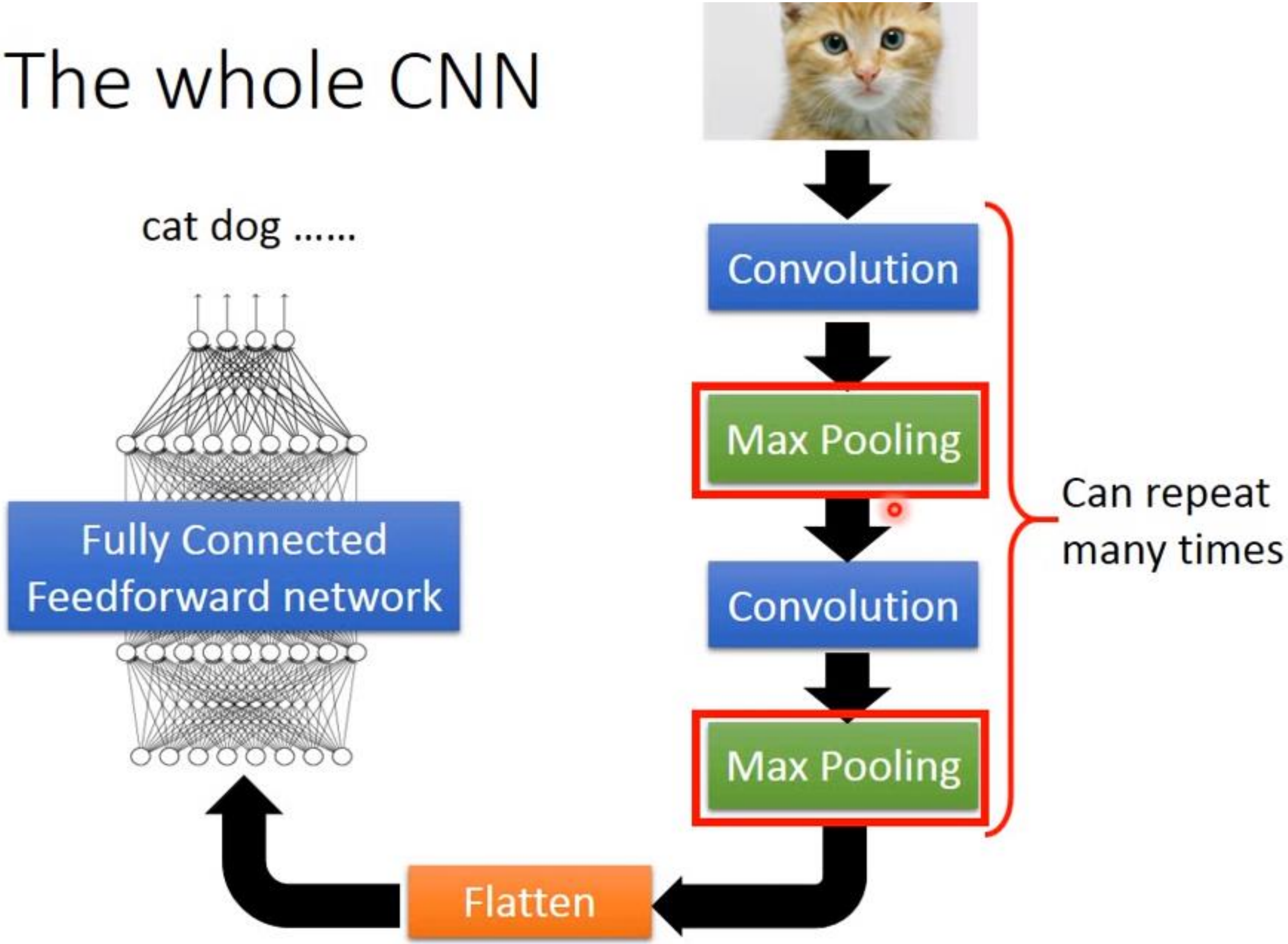




Less parameters!

Even less parameters!

The whole CNN



VGGNet

Small filters, Deeper networks

16 - 19 layers (VGG16Net)

Only 3x3 CONV stride 1, pad 1
and 2x2 MAX POOL stride 2

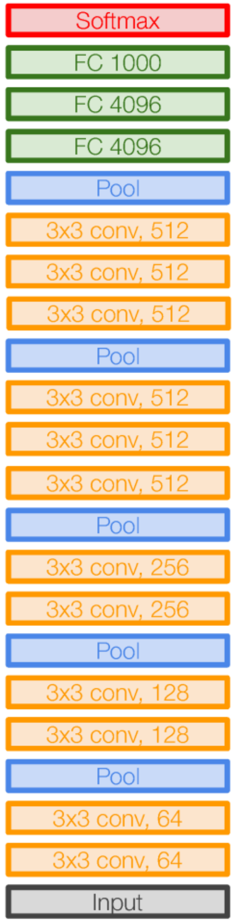
Reference: Very Deep Convolutional Networks for Large-Scale Image Recognition
https://arxiv.org/abs/1409.1556

architecture from [Simonyan and Zisserman 2014].

VGGNet

(not counting biases)

INPUT: [224x224x3] memory: 224*224*3=150K params: 0
CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*3)*64 = 1,728
CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*64)*64 = 36,864
POOL2: [112x112x64] memory: 112*112*64=800K params: 0
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*64)*128 = 73,728
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*128)*128 = 147,456
POOL2: [56x56x128] memory: 56*56*128=400K params: 0
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*128)*256 = 294,912
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824
POOL2: [28x28x256] memory: 28*28*256=200K params: 0
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*256)*512 = 1,179,648
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296
POOL2: [14x14x512] memory: 14*14*512=100K params: 0
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
POOL2: [7x7x512] memory: 7*7*512=25K params: 0
FC: [1x1x4096] memory: 4096 params: 7*7*512*4096 = 102,760,448
FC: [1x1x4096] memory: 4096 params: 4096*4096 = 16,777,216
FC: [1x1x1000] memory: 1000 params: 4096*1000 = 4,096,000



VGG16

VGGNet

(not counting biases)

INPUT: [224x224x3] memory: 224*224*3=150K params: 0

CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*3)*64 = 1,728

CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*64)*64 = 36,864

POOL2: [112x112x64] memory: 112*112*64=800K params: 0

CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*64)*128 = 73,728

CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*128)*128 = 147,456

POOL2: [56x56x128] memory: 56*56*128=400K params: 0

CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*128)*256 = 294,912

CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824

CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824

POOL2: [28x28x256] memory: 28*28*256=200K params: 0

CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*256)*512 = 1,179,648

CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296

CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296

POOL2: [14x14x512] memory: 14*14*512=100K params: 0

CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296

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CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296

POOL2: [7x7x512] memory: 7*7*512=25K params: 0

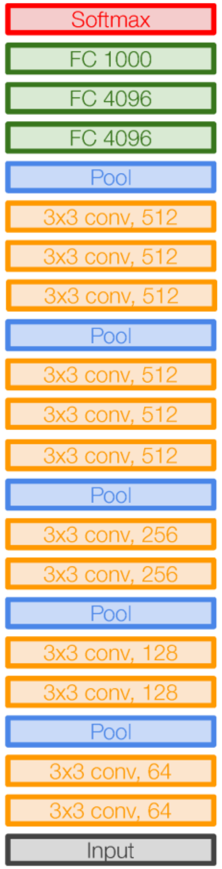
FC: [1x1x4096] memory: 4096 params: 7*7*512*4096 = 102,760,448

FC: [1x1x4096] memory: 4096 params: 4096*4096 = 16,777,216

FC: [1x1x1000] memory: 1000 params: 4096*1000 = 4,096,000

TOTAL memory: 24M * 4 bytes ~= 96MB / image (for a forward pass)

TOTAL params: 138M parameters



VGG16

VGGNet

(not counting biases)

INPUT: [224x224x3] memory: 224*224*3=150K params: 0
CONV3-64: [224x224x64] memory: **224*224*64=3.2M** params: (3*3*3)*64 = 1,728
CONV3-64: [224x224x64] memory: **224*224*64=3.2M** params: (3*3*64)*64 = 36,864
POOL2: [112x112x64] memory: 112*112*64=800K params: 0
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*64)*128 = 73,728
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*128)*128 = 147,456
POOL2: [56x56x128] memory: 56*56*128=400K params: 0
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*128)*256 = 294,912
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824
POOL2: [28x28x256] memory: 28*28*256=200K params: 0
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*256)*512 = 1,179,648
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296
POOL2: [14x14x512] memory: 14*14*512=100K params: 0
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
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CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
POOL2: [7x7x512] memory: 7*7*512=25K params: 0
FC: [1x1x4096] memory: 4096 params: 7*7*512*4096 = **102,760,448**
FC: [1x1x4096] memory: 4096 params: 4096*4096 = 16,777,216
FC: [1x1x1000] memory: 1000 params: 4096*1000 = 4,096,000

TOTAL memory: 24M * 4 bytes ~= 96MB / image (only forward! ~*2 for bwd)
TOTAL params: 138M parameters

Note:

Most memory is in early CONV

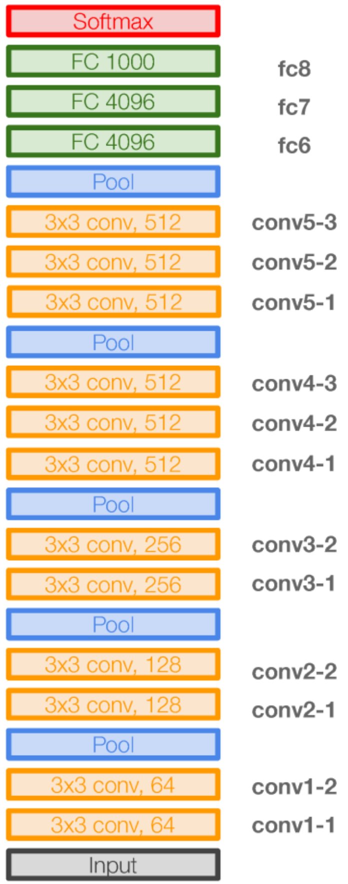
Most params are in late FC

VGGNet

(not counting biases)

INPUT: [224x224x3] memory: 224*224*3=150K params: 0
CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*3)*64 = 1,728
CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*64)*64 = 36,864
POOL2: [112x112x64] memory: 112*112*64=800K params: 0
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*64)*128 = 73,728
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*128)*128 = 147,456
POOL2: [56x56x128] memory: 56*56*128=400K params: 0
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*128)*256 = 294,912
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824
POOL2: [28x28x256] memory: 28*28*256=200K params: 0
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*256)*512 = 1,179,648
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296
POOL2: [14x14x512] memory: 14*14*512=100K params: 0
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
POOL2: [7x7x512] memory: 7*7*512=25K params: 0
FC: [1x1x4096] memory: 4096 params: 7*7*512*4096 = 102,760,448
FC: [1x1x4096] memory: 4096 params: 4096*4096 = 16,777,216
FC: [1x1x1000] memory: 1000 params: 4096*1000 = 4,096,000

TOTAL memory: 24M * 4 bytes ~= 96MB / image (only forward! ~*2 for bwd)
TOTAL params: 138M parameters

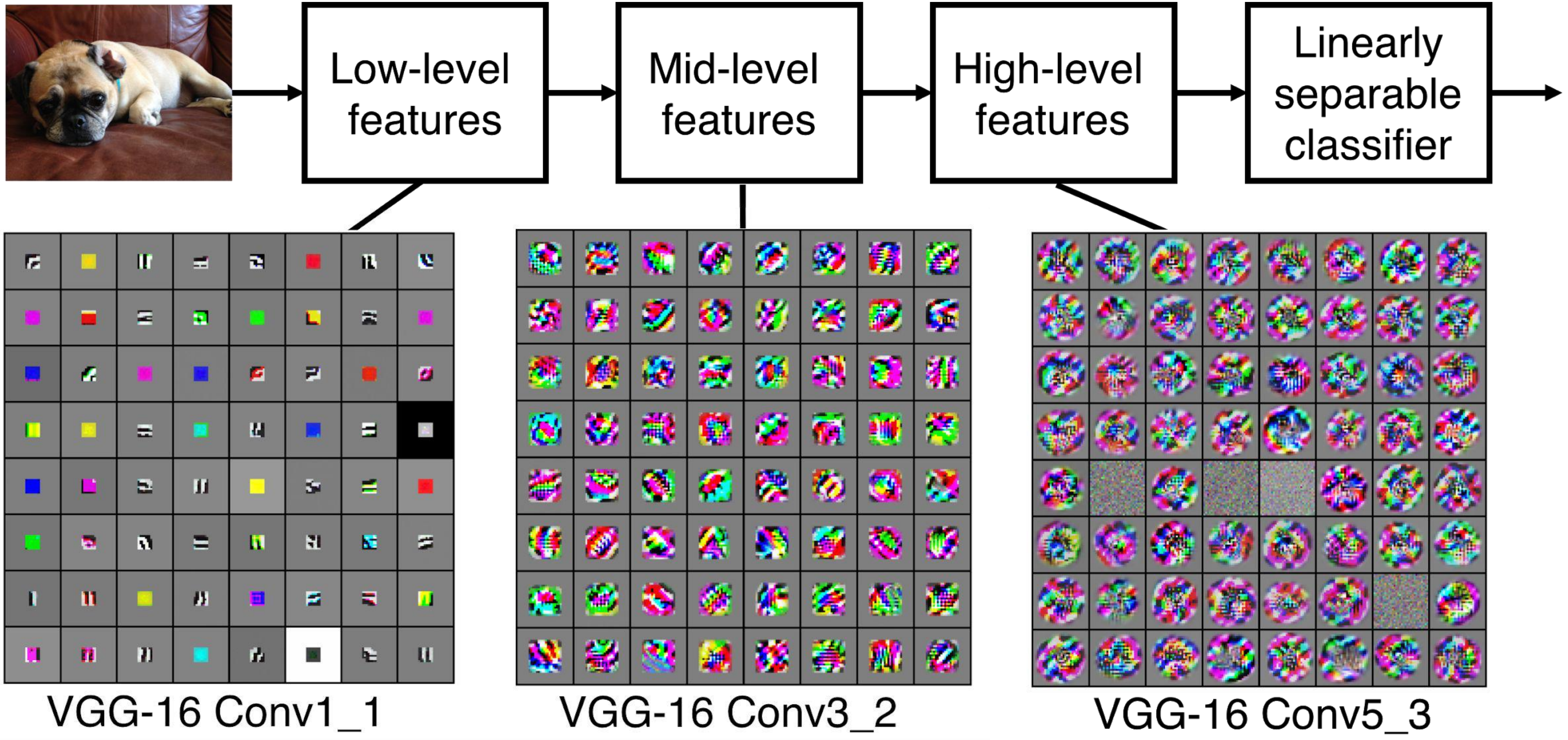


VGG16

Common names

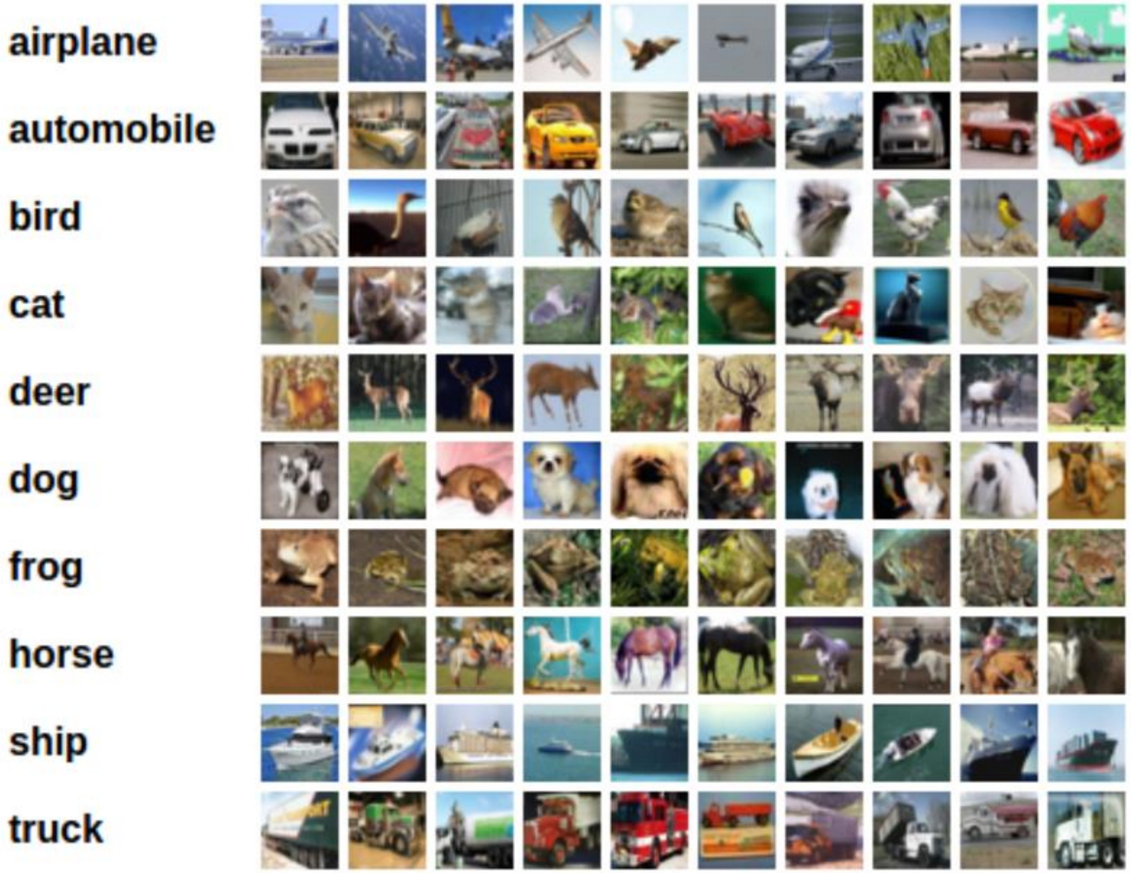
[Zeiler and Fergus 2013]

Visualization of VGG-16 by Lane McIntosh. VGG-16 architecture from [Simonyan and Zisserman 2014].



Implementation of CNN in PyTorch

For this tutorial, we will use the CIFAR10 dataset. It has the classes: 'airplane', 'automobile', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', 'ship', 'truck'. The images in CIFAR-10 are of size 3x32x32, i.e. 3-channel color images of 32x32 pixels in size.



cifar10

https://pytorch.org/tutorials/beginner/blitz/cifar10_tutorial.html

Implementation of CNN in PyTorch

```
class Net(nn.Module):
    def __init__(self):
        super().__init__()
        self.conv1 = nn.Conv2d(3, 6, 5)
        self.pool = nn.MaxPool2d(2, 2)
        self.conv2 = nn.Conv2d(6, 16, 5)
        self.fc1 = nn.Linear(16 * 5 * 5, 120)
        self.fc2 = nn.Linear(120, 84)
        self.fc3 = nn.Linear(84, 10)

    def forward(self, x):
        x = self.pool(F.relu(self.conv1(x)))
        x = self.pool(F.relu(self.conv2(x)))
        x = torch.flatten(x, 1) # flatten all dimensions except batch
        x = F.relu(self.fc1(x))
        x = F.relu(self.fc2(x))
        x = self.fc3(x)
        return x


net = Net()
print(net)
```

```
Net(
  (conv1): Conv2d(3, 6, kernel_size=(5, 5), stride=(1, 1))
  (pool): MaxPool2d(kernel_size=2, stride=2, padding=0, dilation=1, ceil_mode=False)
  (conv2): Conv2d(6, 16, kernel_size=(5, 5), stride=(1, 1))
  (fc1): Linear(in_features=400, out_features=120, bias=True)
  (fc2): Linear(in_features=120, out_features=84, bias=True)
  (fc3): Linear(in_features=84, out_features=10, bias=True)
)
```

https://pytorch.org/tutorials/beginner/blitz/cifar10_tutorial.html

Implementation of CNN in PyTorch

```
class Net(nn.Module):
    def __init__(self):
        super().__init__()
        self.conv1 = nn.Conv2d(3, 6, 5)
        self.pool = nn.MaxPool2d(2, 2)
        self.conv2 = nn.Conv2d(6, 16, 5)
        self.fc1 = nn.Linear(16 * 5 * 5, 120)
        self.fc2 = nn.Linear(120, 84)
        self.fc3 = nn.Linear(84, 10)

    def forward(self, x):
        x = self.pool(F.relu(self.conv1(x)))
        x = self.pool(F.relu(self.conv2(x)))
        x = torch.flatten(x, 1) # flatten all dimensions except batch
        x = F.relu(self.fc1(x))
        x = F.relu(self.fc2(x))
        x = self.fc3(x)
        return x


net = Net()
print(net)
```

```
Net(
  (conv1): Conv2d(3, 6, kernel_size=(5, 5), stride=(1, 1))
  (pool): MaxPool2d(kernel_size=2, stride=2, padding=0, dilation=1, ceil_mode=False)
  (conv2): Conv2d(6, 16, kernel_size=(5, 5), stride=(1, 1))
  (fc1): Linear(in_features=400, out_features=120, bias=True)
  (fc2): Linear(in_features=120, out_features=84, bias=True)
  (fc3): Linear(in_features=84, out_features=10, bias=True)
)
```

Increase # of filters in deeper layers:

As we move forward in the layers, the patterns get more complex; **hence there are larger combinations of patterns to capture**.

https://pytorch.org/tutorials/beginner/blitz/cifar10_tutorial.html

Implementation of CNN in Keras

```
from keras import layers
from keras import models

model = models.Sequential()
model.add(layers.Conv2D(32, (3, 3), activation='relu',
                        input_shape=(150, 150, 3)))
model.add(layers.MaxPooling2D((2, 2)))
model.add(layers.Conv2D(64, (3, 3), activation='relu'))
model.add(layers.MaxPooling2D((2, 2)))
model.add(layers.Conv2D(128, (3, 3), activation='relu'))
model.add(layers.MaxPooling2D((2, 2)))
model.add(layers.Conv2D(128, (3, 3), activation='relu'))
model.add(layers.MaxPooling2D((2, 2)))
model.add(layers.Flatten())
model.add(layers.Dense(512, activation='relu'))
model.add(layers.Dense(1, activation='sigmoid'))
```

Refer to Section 5.1 and 5.2 in Deep Learning with Python

Number of Parameters

```
from keras import layers
from keras import models

model = models.Sequential()
model.add(layers.Conv2D(32, (3, 3), activation='relu'
                        input_shape=(150, 150, 3)))
model.add(layers.MaxPooling2D((2, 2)))
model.add(layers.Conv2D(64, (3, 3), activation='relu'
model.add(layers.MaxPooling2D((2, 2)))
model.add(layers.Conv2D(128, (3, 3), activation='relu
model.add(layers.MaxPooling2D((2, 2)))
model.add(layers.Conv2D(128, (3, 3), activation='relu
model.add(layers.MaxPooling2D((2, 2)))
model.add(layers.Flatten())
model.add(layers.Dense(512, activation='relu'))
model.add(layers.Dense(1, activation='sigmoid'))
```

Layer (type)	Output Shape	Param #
conv2d_1 (Conv2D)	(None, 148, 148, 32)	896
max_pooling2d_1 (MaxPooling2	(None, 74, 74, 32)	0
conv2d_2 (Conv2D)	(None, 72, 72, 64)	18496
max_pooling2d_2 (MaxPooling2	(None, 36, 36, 64)	0
conv2d_3 (Conv2D)	(None, 34, 34, 128)	73856
max_pooling2d_3 (MaxPooling2	(None, 17, 17, 128)	0
conv2d_4 (Conv2D)	(None, 15, 15, 128)	147584
max_pooling2d_4 (MaxPooling2	(None, 7, 7, 128)	0
flatten_1 (Flatten)	(None, 6272)	0
dense_1 (Dense)	(None, 512)	3211776
dense_2 (Dense)	(None, 1)	513

Total params: 3,453,121
Trainable params: 3,453,121
Non-trainable params: 0

Number of parameters in a CONV layer would be : **((m * n * d)+1)* k)**

m = shape of filter width

n = shape of filter height

d = number of filters/channels in the previous layer

1 = bias term

k = number of filters

CNN

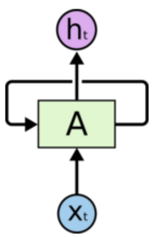
- Video:
  - https://www.youtube.com/watch?v=3JQ3hYko51Y&feature=youtu.be

- Interactive visualization:
  - https://adamharley.com/nn_vis/cnn/3d.html

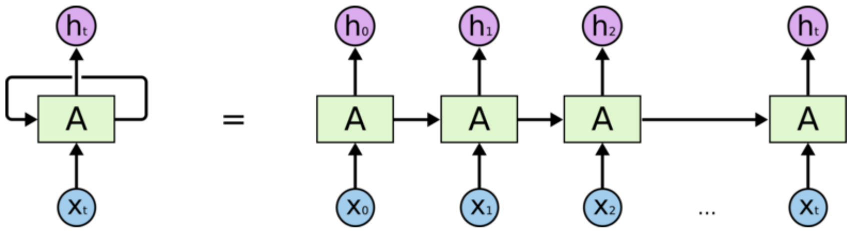
Recurrent Neural Network (RNN)

Recurrent Neural Networks

- Recurrent Neural Networks are networks with loops in them, allowing information to persist.



Recurrent Neural Networks have loops.



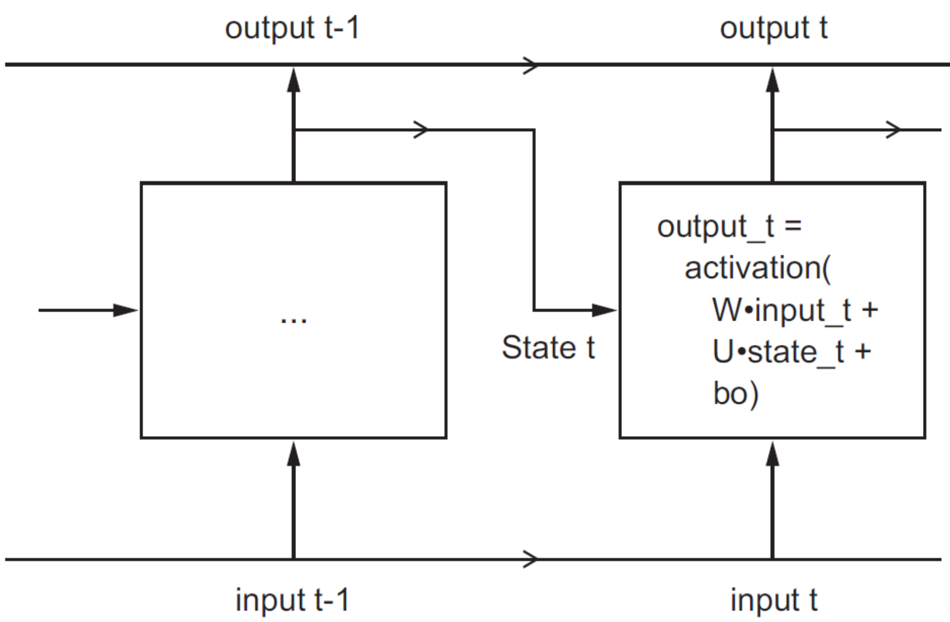
An unrolled recurrent neural network.

In the above diagram, a chunk of neural network, ***A***, looks at some input $\boldsymbol{x_t}$ and outputs a value $\boldsymbol{h_t}$.
A loop allows information to be passed from one step of the network to the next.

A recurrent neural network can be thought of as multiple copies of the same network, each passing a message to a successor.
The diagram above shows what happens if we unroll the loop.

Recurrent Neural Networks

- Intuition of Recurrent Neural Networks
  - Human thoughts have persistence; humans don't start their thinking from scratch every second.
    - As you read this sentence, you understand each word based on your understanding of previous words.
  - One of the appeals of RNNs is the idea that they are able to connect previous information to the present task
    - E.g., using previous video frames to inform the understanding of the present frame.
    - E.g., a language model tries to predict the next word based on the previous ones.



A simple RNN, unrolled over time

Listing 6.21 Numpy implementation of a simple RNN

```
import numpy as np

timesteps = 100
input_features = 32
output_features = 64

inputs = np.random.random((timesteps, input_features))

state_t = np.zeros((output_features,))

W = np.random.random((output_features, input_features))
U = np.random.random((output_features, output_features))
b = np.random.random((output_features,))

successive_outputs = []
for input_t in inputs:
    output_t = np.tanh(np.dot(W, input_t) + np.dot(U, state_t) + b)
    successive_outputs.append(output_t)

    state_t = output_t

final_output_sequence = np.concatenate(successive_outputs, axis=0)
```

- Number of timesteps in the input sequence
- Dimensionality of the input feature space
- Dimensionality of the output feature space
- Input data: random noise for the sake of the example
- Initial state: an all-zero vector
- Creates random weight matrices
- input_t is a vector of shape (input_features,).
- Combines the input with the current state (the previous output) to obtain the current output
- Stores this output in a list
- Updates the state of the network for the next timestep
- The final output is a 2D tensor of shape (timesteps, output_features).

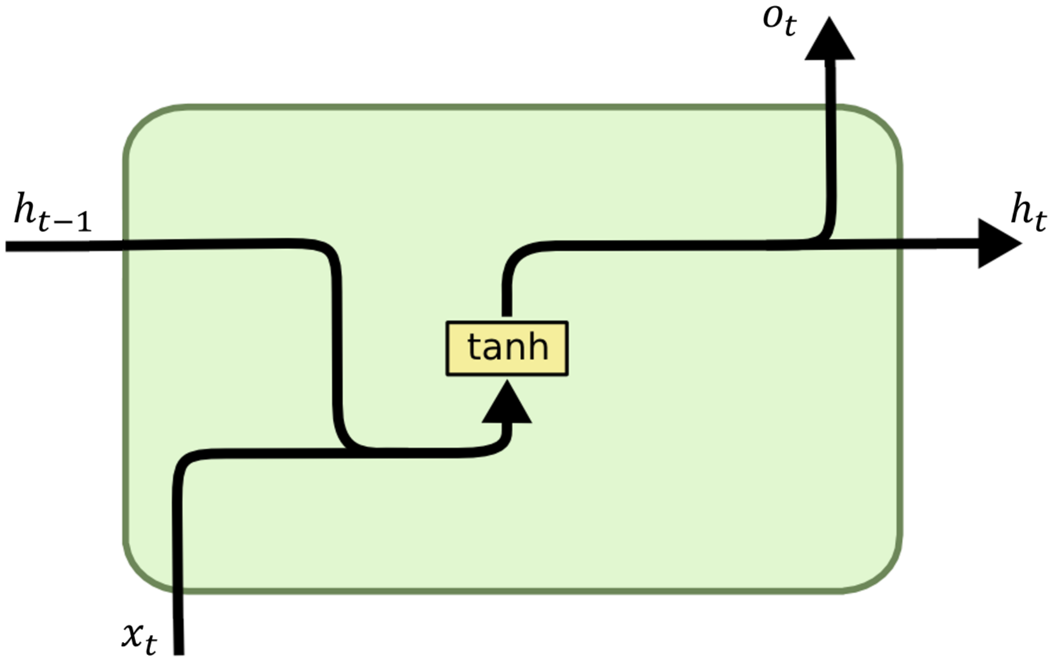
Figures are adapted from “Deep Learning with Python” writtend by F. Chollet
Refer to Section 6.2 of the book

RNN/SimpleRNN in PyTorch

CLASS torch.nn.RNN(*self*, *input_size*, *hidden_size*, *num_layers=1*, *nonlinearity='tanh'*, *bias=True*, *batch_first=False*, *dropout=0.0*, *bidirectional=False*, *device=None*, *dtype=None*) [SOURCE]

Apply a multi-layer Elman RNN with $\tanh$ or ReLU non-linearity to an input sequence. For each element in the input sequence, each layer computes the following function:

$$h_t = \tanh(x_t W_{ih}^T + b_{ih} + h_{t-1} W_{hh}^T + b_{hh})$$




```python
class RNNTagger(torch.nn.Module):

    def __init__(self, embedding_dim, hidden_dim, vocab_size, tagset_size):
        super(RNNTagger, self).__init__()
        self.hidden_dim = hidden_dim

        self.word_embeddings = torch.nn.Embedding(vocab_size, embedding_dim)

        # The RNN takes word embeddings as inputs, and outputs hidden states
        # with dimensionality hidden_dim.
        self.rnn = torch.nn.RNN(embedding_dim, hidden_dim)

        # The linear layer that maps from hidden state space to tag space
        self.hidden2tag = torch.nn.Linear(hidden_dim, tagset_size)

    def forward(self, sentence):
        embeds = self.word_embeddings(sentence)
        rnn_out, _ = self.rnn(embeds.view(len(sentence), 1, -1))
        tag_space = self.hidden2tag(rnn_out.view(len(sentence), -1))
        tag_scores = F.log_softmax(tag_space, dim=1)
        return tag_scores
```

Pytorch reference: https://pytorch.org/docs/stable/generated/torch.nn.RNN.html