Thapar Institute of Engineering and Technology, Patiala
School of Mathematics
Probability and Statistics (UCS410)
Practice Sheet: 5

1. Define the following: (a) two-dimensional random variable (b) marginal and conditional probability distributions.
2. If X denotes the number of kings and Y denotes number of aces when two cards are drawn at random without replacement from a deck of well shuffled pack of 52 cards,
   find (i) The joint probability distribution of (X, Y).
   (ii) The marginal distribution
   $(iii) P(X = 2/Y = 1)$ $(iv)\ P(X < 2/0 < Y < 2)$ $(v)\ P(1 \le X \le 2/Y = 0,2)$
   (Ans- (iii) 0, (iv) 1, (v) 13/81)
3. Let the joint pdf of a random variable (X, Y) is defined as
   $$f(x,y) = k(xy + y^2), \qquad 0 \le x \le 2, 0 \le y \le 1.$$
   Find $(i) the\ value\ of\ k$ $(ii) P(X > 1) (iii) P(X + Y < 1)$ $(iv) P\left(X < 1, Y > \frac{1}{2}\right)$ (v) $f_X(x), f_Y(y)$. Also test whether X and Y are independent?
   (Ans- k=3/5, 13/20, 3/40, 23/80, not independent)

4. The pdf of (X, Y) be defined as
   $$f(x,y) = \left(\frac{1}{4}\right) e^{-|x|+|y|}, \qquad -\infty \le x \le \infty, -\infty \le y \le \infty$$
   Are X and Y independent? Find the probability that $X \le 1\ and\ Y \le 0$.
   (Ans- independent, ½-1/4e)

5. Random variable (X, Y) has a joint probability density function $f(x,y) = (2x + y)/27$, where x and y can assume only integer values 0, 1, 2. Find the conditional distribution of $Y\ for\ X = x$.

6. Two ideal dice are thrown. Let X1 be the score on the first die and X2 the score on the other die. Let Y denote the maximum of $X_1\ and\ X_2\ i.e. max(X_1, X_2)$.
   (a) Write down the joint distribution of Y and X_1,
   (b) (b) Find E(Y) and Var(Y).

   (Ans- E(Y)=161/36, Var(Y)=2555/1296).

7. Let $f(x,y) = \begin{cases} 21x^2y^2, & 0 \le x < y \le 1. \\ 0 & elsewhere \end{cases}$ be the joint pdf X, Y.
   Find the conditional mean and variance of X given Y=y and 0<y<1.
   (Ans- E(X/Y) =3y/4, Var(X/Y) =3/80$y^2, 0 \le y < 1$).

8. A pair of fair dice is thrown and let X be the number of 6's turned up. Find the moment generating function (MGF), mean and variance of X.

   (Ans. MGF= $\frac{25}{36} + \frac{10}{36}(1+\frac{t}{1!}+\frac{t^2}{2!}+\ldots) + \frac{1}{36}(1+\frac{2t}{1!}+\frac{4t^2}{2!}+\ldots)$; Mean=1/3, Var=5/18)

9. Find MGF of X whose probability density function is given by $f(x) = k\,\frac{e^{-|x|}}{5}$, $-\infty < x < \infty$. Find first three moments of X about the origin. What is the variance of X?

   (Ans. Three moments are 0, 2,0; Var(X)= 2)

10. The joint pdf of a two-dimensional random variables (X, Y) is given as:

$f(x,y) = \begin{cases} \frac{3}{2}(x^2+y^2), if\ 0 \le x,y \le 1 \\ 0, otherwise. \end{cases}$

Find C_{XY} , E (XY) andρ_{XY}. (Ans. $C_{XY} = -\frac{1}{64}$, E (XY) $= \frac{3}{8}$, $\rho_{XY} = -\frac{15}{73}$)

11. The joint pdf of a two-dimensional random variables (X, Y) is $P_X(k) = \frac{1}{k!}e^{-2}2^k$ and $P_Y(k) = \frac{1}{k!}e^{-3}3^k$. Compute the MGF of Z = 2X + 3Y.

    (Ans. $e^{(2e^{2t}+3e^{2t}-5)}$)