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2. WAVE MOTION.

Wave:

Wave is a disturbance that transfer of energy from one place to another place.

Energy:

→ ability to do work.

Eg:- of energy in terms of wave.
Sound, sunlight, water, etc..

Medium:

→ A medium is a material that a mechanical wave travels through.

Eg: water, air, rope / string.

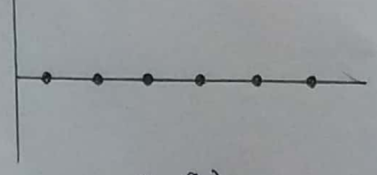
→ Light does not require a medium but above said examples require a medium. Also light is not a mechanical wave.

Mechanical Wave:

Mechanical waves are created when a source of energy causes a medium to vibrate

Vibration:

→ is a repeated back and forth and up & down motion.



→ when no energy is applied all the points are at rest.

Fig (i)

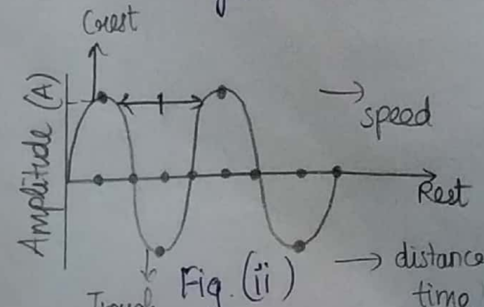


Fig. (ii)

→ when Energy is applied energy is transfered with the particle vibrations.

classification of waves:

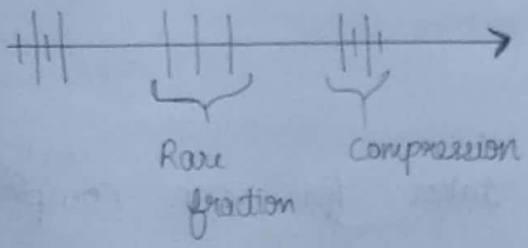
* Transverse waves
* Longitudinal / compressional waves.

Transverse waves:

They move the medium at right angle to the direction that the wave is travelling. The higher point (maximum point) of the wave is called crest. The low point (minimum point) of the wave is called trough.

Longitudinal waves:

They move the medium parallel to the direction the wave is travelling. The part of the longitudinal wave that are close together are called compression. The part of the longitudinal wave which is spread out is called rare fraction.



Medium:

*) Only solids can support tranverse waves.
*) All three ⇒ solids, liquids & gases can support longitudinal waves.

Types of waves:

1) Mechanical waves
2) Electro Magnetic waves.

In Mechanical wave:

*) it is in the pulse form.

*) It requires a medium for propagation Eg:- sound, water, etc.

In electromagnetic wave:

*) It is in continuous pulse form. 

*) Does not require a medium. Eg: visible light, X-rays, Gamma rays, UV rays, etc.

Properties of Wave:

Amplitude:

The height of the wave, measured in meters.

Wavelength:

The distance between adjacent crests, measured in meters.

Frequency:

The number of complete waves that pass a point in one second, measured in inverse seconds (or) Hertz (Hz).

Time Period:

The time it takes for one complete wave to pass a given point, measured in seconds.

$$T = \frac{1}{f}.$$

Speed:

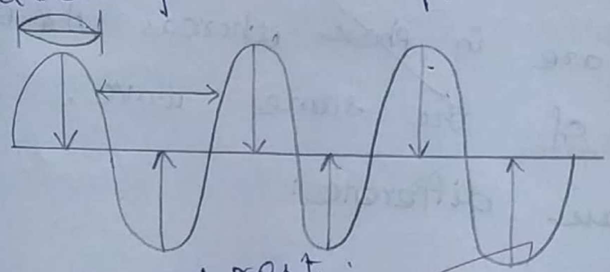
The horizontal speed of a point on a wave as it propagates, measured in meters/second.

$$\text{Speed} = \frac{\text{Wavelength}}{\text{Period}} = \text{Wavelength} \times \text{frequency}.$$

Standing wave & Progressive wave

Standing wave:

A wave remains in a constant position as a result of interference between two waves travelling in opposite direction.

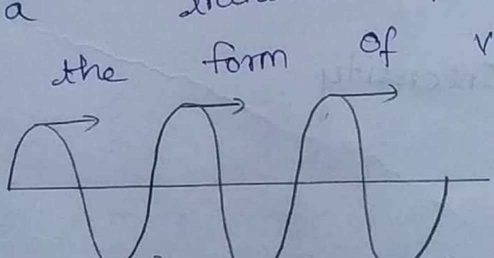


Node will be at rest.

Antinode will be at high energy.

Progressive wave:

The profile of the wave is moving which indicates a transfer of energy. Energy is moved in the form of vibrating particle.



(→) indicates transformation of energy with respective to particles in a medium.

Angular frequency (ω)

Angular displacement per unit time is called angular frequency.

$$\omega = \frac{2\pi}{T}$$

Unit → rad/s.

Wave number (k):

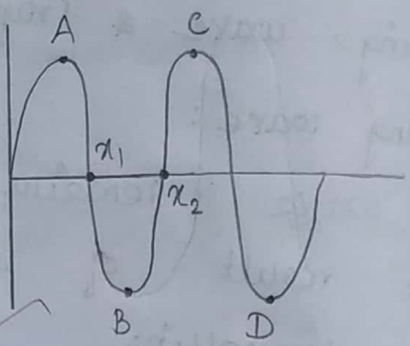
No. of waves in a unit distance.

$$k = \frac{2\pi}{\lambda}$$

Unit ⇒ rad/m.

Phase

It is represented by ϕ.



$$\phi = \frac{2\pi}{\lambda}(x_1 - x_2)$$

$$\phi = \frac{2\pi}{\lambda}(x).$$

→ Point A & C are in phase whereas A & B are out of phase of the same wave.

Phase and Phase difference:

In phase:

When the waves are in phase, some of the displacement is large. Thus the amplitude is increased. So, the intensity of the wave is also increased.

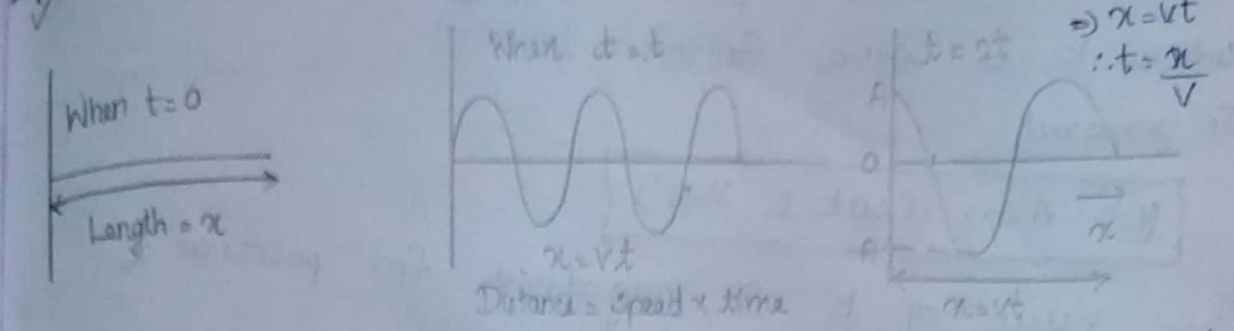
ie, Amplitude ∝ Intensity

$A/a \propto I$.

Out of Phase:

The max displacement of one wave can coincide with minimum displacement of the other wave. These waves are out of phase. If the waves are 180° out of phase. The sum of displacements of the two waves is 0.

Equation of Plane Progressive Wave :



$\Rightarrow x = vt$
$\therefore t = \frac{x}{V}$

$y = A\cos\omega t$ [Equation of Plane progressive wave w.r.t t] consider a string of length x, when time t=0, then x=0 where x is the distance travelled by the string. ~~A small oscillation~~

A small oscillation or a force is applied to a string so that it oscillates and when time t=t, the distance travelled will be vt

Then, $y = A\cos\omega t \longrightarrow (1)$ the wave equation w.r.t to which represents only time.

where A ⇒ amplitude of the wave.

when the string oscillates, the time interval b/w t=0 & t=t is given as, $(t - x/v)$, the wave equation w.r.t to t and x is given below,

$$y = A\cos\omega\left(t - \frac{x}{v}\right) \rightarrow (2).$$

$$y = A\cos 2\pi\nu\left(t - \frac{x}{v}\right)$$

$$y = A\cos 2\pi\left(\nu t - \frac{\nu x}{v}\right)$$

$$y = A\cos 2\pi\left(\nu t - \frac{x}{\lambda}\right)$$

$$y = A\cos\left(2\pi\nu t - \frac{2\pi x}{\lambda}\right).$$

$$\left[\begin{array}{l} c = \nu\lambda \\ v = \nu\lambda \\ \lambda = \frac{v}{\nu} \\ \frac{1}{\lambda} = \frac{\nu}{v} \end{array}\right]$$

$y = A\cos(\omega t - kx)$ $\left[\because k = \frac{2\pi}{\lambda}\right]$

where k is wave factor.

In general

$$\boxed{y = A\cos(\omega t \pm kx)}$$

The value of k is negative for positive x direction of wave motion

k value is positive for negative x direction of wave motion.

Differential Equation of a Plane Progressive Wave.

$$y = A\cos(\omega t - kx) \quad \text{—— (1)}$$

*) $(-) \Rightarrow$ wave is moving in positive direction

Diff on B.S: w.r.t to t.

$$\frac{dy}{dt} = -A\sin(\omega t - kx)\,\omega$$

$$\frac{dy}{dt} = -A\omega\sin(\omega t - kx)$$

$$\frac{d^2y}{dt^2} = -A\omega^2\cos(\omega t - kx)$$

(or)

$$\frac{d^2y}{dt^2} = -y\omega^2 \rightarrow (2) \quad \left[\because y = \frac{-1}{\omega^2}\cdot\frac{d^2y}{dt^2}\right] \rightarrow (3)$$

Now, w.r.t x:

$$\frac{dy}{dx} = -Ak\sin(\omega t - kx)$$

$$\frac{d^2y}{dx^2} = -Ak^2\cos(\omega t - kx)$$

$$\frac{d^2y}{dx^2} = -yk^2 \rightarrow (4) \qquad \left[\because y = \frac{-1}{k^2} \cdot \frac{d^2y}{dx^2}\right] \rightarrow (5).$$

By Equating (3) & (5)

$$\frac{-1}{\omega^2} \cdot \frac{d^2y}{dt^2} = \frac{-1}{k^2} \frac{d^2y}{dx^2}.$$

(or)

$$\frac{d^2y}{dt^2} = \frac{\omega^2}{k^2} \frac{d^2y}{dx^2}.$$

$$= \frac{(2\pi \nu)^2}{(2\pi/\lambda)^2} \cdot \frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dt^2} = \frac{\cancel{4\pi^2}\nu^2\lambda^2}{\cancel{4\pi^2}} \cdot \frac{d^2y}{dx^2} \qquad \left[\begin{matrix} v = \nu\lambda \\ v^2 = \nu^2\lambda^2 \end{matrix}\right].$$

$$\boxed{\frac{d^2y}{dt^2} = v^2 \times \frac{d^2y}{dx^2}}$$

For 1 dimension.

For 3 dimension:

$$\frac{d^2y}{dt^2} = v^2 \times \left[\frac{d^2y}{dx^2} + \frac{d^2y}{dy^2} + \frac{d^2y}{dz^2}\right]$$

Put $y = u$

$$\frac{d^2u}{dt^2} = v^2 \times \left[\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2}\right]$$

For 2 dimension:

$$\frac{d^2y}{dt^2} = v^2 \times \left[\frac{d^2y}{dx^2} + \frac{d^2y}{dy^2}\right]$$

Speed of the wave:

(3)

$$v = \frac{\text{distance}}{\text{time}} = \frac{x}{t}$$

t = time period.

For wave $\Rightarrow x = \lambda$, $t = T$

$$v = \frac{\lambda}{T} = \lambda\left(\frac{1}{T}\right)$$

$$= \frac{2\pi}{k} \times \frac{\omega}{2\pi} \qquad \left[k = \frac{2\pi}{\lambda} \text{ \& } T = \frac{2\pi}{\omega}\right]$$

$$\omega = \frac{2\pi}{T}$$

$$\boxed{v = \frac{\omega}{k}}$$

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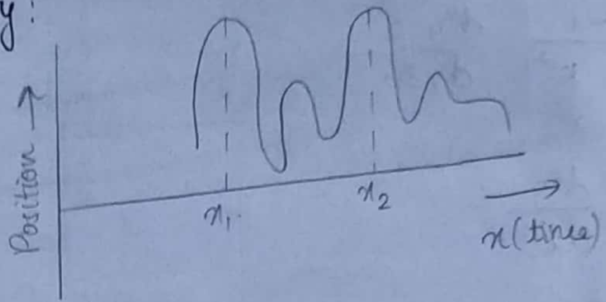
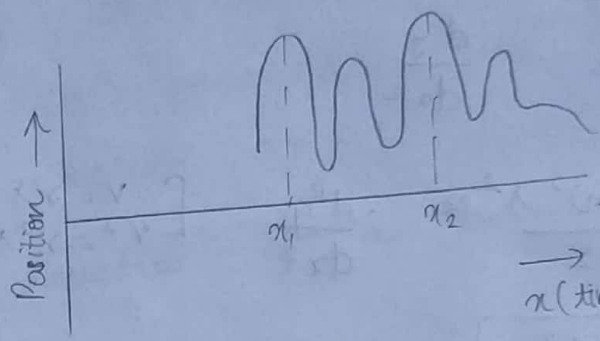
$\rightarrow$ amplitude decreases

Phase velocity:





When the wave propagates, the particles does not move but the disturbance is transfered from one particle to another. When a wave propagates the size, shape of the waves are same. In fig. 1, when time $t = t_1$ sec, then the waves are at the position x_1 and x_2. When $t = t_2$ sec, x_1 is moved to the right and similarly ~~so it~~ x_2. When time increases, the particle moves in the positive x direction and the general equation is given as,

$$y = f(x - vt)$$

When $t = t_1$

$$y_1 = f(x_1 - vt_1)$$

When $t = t_2$

$$y_2 = f(x_2 - vt_2)$$

If $y_1 = y_2$

$$f(x_1 - vt_1) = f(x_2 - vt_2)$$

$$x_2 - x_1 = -vt_1 + vt_2$$

$$= -v(t_1 - t_2)$$

$$x_2 - x_1 = -v(t_1 - t_2)$$

$$\vartheta = \frac{x_2 - x_1}{t_2 - t_1} \Rightarrow \therefore V = \frac{x}{t} \Rightarrow \frac{V(t_2 - t_1)}{t_2 - t_1}$$

So, $\boxed{\vartheta = V}$

Velocity of the wave (group velocity) is equals to phase velocity.

Attennation of waves:

$$a(x) = a_0 e^{-\alpha x}$$

x = displacement

a_0 = amplitude when $x = 0$

α = ~~attent~~ attennation constant.

Attennation means loss of ~~chara~~ energy when any wave propagates through the media it is stopped in between due to loss of energy. In any wave motion, the wave slows down due to the loss of energy and the attennation is given by the eqn.

$$\boxed{a(x) = a_0 e^{-\alpha x}}$$

The loss is exponential and the α is an attennation constant.

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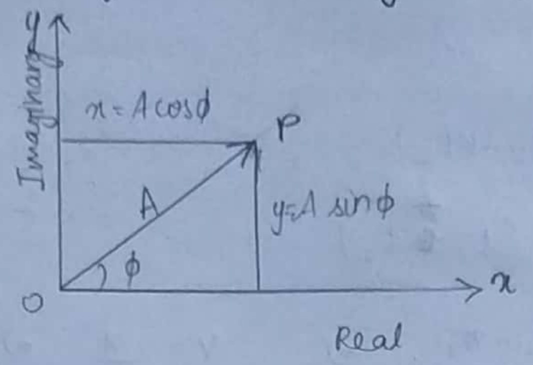
Representation of wave using complex number:



$$P = x + iy$$

$$P = A(\cos\phi + i\sin\phi)$$

Euler's eqn:

$$\exp(i\phi) = \cos\phi + i\sin\phi$$

$$P = A\ \exp(i\phi)$$

$$= Ae^{(i\phi)}$$

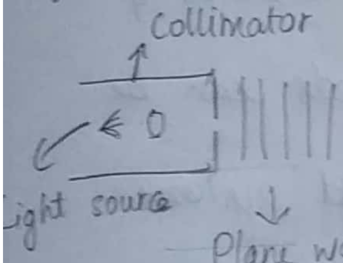
→ Consider a point $P = (x, y)$ on a two dimensional plane. Let x coordinate be the real part & y coordinate be an imaginary part of the complex number, $x = A\cos\phi$ & $y = A\sin\phi$

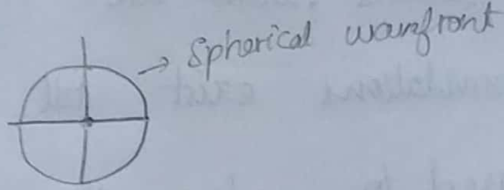
Importance of Spherical & Plane Wave front:

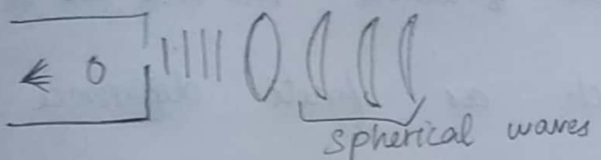
A wave front is the locus of the points characterised by propagation of position of the same phase.

A propagation of line in one dimension, a curve in two dimension or the wave is 3 dimensions. A lens can be used to change the shape of the wavefronts.

Plane wavefronts becomes spherical after going through the lens. The simplest form of a wavefront is plane wave, where the ~~waves~~ rays are parallel to one another. The light from this type of wave is refered to as collimated light.







When source is near, we get spherical wavefront & when it is far, we get plane wavefront.

Note:

Source → monochromatic light — sodium lamp

Polychromatic light - White light

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Introduction to numerical methods for the solution of Wave equation:

It is known that the differential equation that can be solved by exact analytical formulae are only few in numbers. Therefore, the development of accurate numerical methods is essential for extracting ~~quantitive~~ quantitative information as well as achieving qualitative understanding of various behaviour of that solution.

Even in cases, such as heat & wave equations, there are explicit or clear solutions & formulations exist till numerical methods can be used to solve the above mentioned systems. Many other basic numerical solution methods can be fit into broad types such as finite difference method is one among them. The second type is numerical solution technique which is called as finite element method.