PRN No. Total No. of Questions: 12 OP Code:

JSPM's

Rajarshi Shahu College of Engineering, Tathawade, Pune- 411033 (An autonomous institute affiliated to Savitribai Phule Pune University

Examinations: End Semester (ESE) Nov/Dec24-25

Semester: I

Academic Year: 2024-25

Class: F. Y. B. Tech. (CS)

Department: Engineering Sciences and Humanities

Course Code: ES1204T

Course Name and pattern: Discrete Mathematics (2023 Pattern)

Duration: 2 Hours.

Max. Marks: 50 Marks

Instructions to the Candidates

1. Attempt Q.1, Q.2 and Q.3 from section-A.

2. Attempt Q.4, Q.5, Q.6, Q.7 or Q.8, Q.9 or Q.10, Q.11 or Q.12 from section-B.

3. Figures to the right indicate full marks.

4. Assume suitable data if necessary.

5. Use of non-programmable electronics pocket calculator is allowed.

Section A

- a) In how many ways can the letters in the word 'MISSISSIPPI' be arranged [01] BL1 CO₄ 0.1 so that 'S' are always together?
 - b) Find the generating functions of the sequence 1, -1,1,-1,1, ...

CO4 [01] BL1

a) Determine, if the following graph is having the Eulerian circuit. Justify the [01] BL1 CO₅ 0.2 answer.



- b) Determine the number of edges in a graph with 5 vertices out of which two [02] CO₅ vertices of degree 3 and three vertices with degree 2.
- a) How many edges will be there in a tree with n nodes? Q.3

CO₅ [01] BL1

- b) How many total numbers of vertices are there in a full 4-ary tree having 19 [02] BL2 **CO5** pendant vertices?
- Solve (any one): Q.4
 - a) Obtain the principal conjunctive normal form (PCNF) and principal [04] BL3 disjunctive normal form (PDNF) of the statement $(\sim (p \lor q)) \leftrightarrow (p \land q)$ by using Truth table method.

b) Translate each of the following statement into symbols, using quantifiers, [04] BL3 CO1 variables and predicate: For the universe of all integers.

There exists a positive integer that is even.

If x is even then x is not divisible by 7. ii)

Solve (any one): Q.5

a) Let M_R be the matrix representation of a partial ordered relation R defined [04] BL3 CO2 on set $A = \{a,b,c,d,e\}$

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Draw its Hasse diagram and also find its maximal elements.

b) Let f and g be the functions from the set of all real numbers to real numbers [04] BL3 defined by f(x) = x - 1 and $g(x) = x^2$. Find

(i) $(f \circ g)(x)$

(ii)(gof)(x) (iii)(gof)(2)

(iv) $(f \circ g)$ (2)

Q.6 Solve (any one):

- a) Show that the set Q^+ of all positive rational numbers forms an abelian [04] BL3 CO3 group under the operation * defined by $a * b = \frac{ab}{3}$, $a, b \in Q^+$.
- b) Given the parity check matrix $H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$ corresponding to [04] BL3 CO3 the encoding function $e: B^3 \to B^6$, use it to decode the following received

words and hence, to find the original message.

- i) 111001
- ii) 100101.
- a) In a box, there are 40 pen drives of which 4 are defective. Determine Q.7 [05] BL3 CO4

i) In how many ways we can select five pen drives?

- ii) In how many ways we can select five non defective pen drives?
- (iii In how many ways we can select five pen drives containing exactly three defective pen drives?
- b) Solve the recurrence relation $a_n - a_{n-1} - 2a_{n-2} = 4$, where $a_0 = -1$ and $a_1 = 3$.

[05] BL3 CO4

[05] BL3 **CO4**

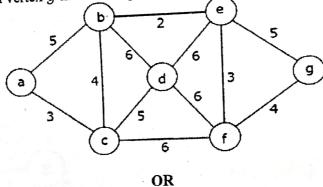
a) Use mathematical induction to prove that **Q.8**

Use mathematical induction to prove and
$$\sum_{i=1}^{n} i = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2} \text{ for all integers } n \ge 1$$

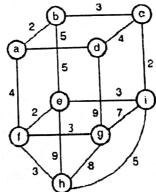
CO₄ BL3 [05]

b) Solve the recurrence relation
$$a_n = -10a_{n-1} - 25a_{n-2} + 72$$
, where $a_0 = 3$ and $a_1 = 17$.

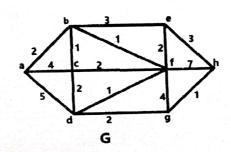
- CO₅ a) Determine whether the graphs k_7 and $k_{3,4}$ are isomorphic or not. Justify the BL₃ [05] Q.9 answer. BL3 CO5
 - b) Use Dijkstra's algorithm, to find the shortest path and its length between [05] vertex a and vertex g in following weighted graph.



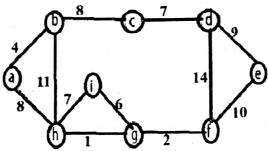
a) Find the solution to the travelling salesman problem for the following figure [05] BL3 CO5 Q.10 using nearest neighbor method, starting vertex h.



b) Use Dijkstra's algorithm, to find the shortest path and its length between [05] BL3 CO5 vertex a and vertex h in following weighted graph G.



Q.11 a) Find the minimal spanning tree and its minimum weight using Kruskal's [05] BL₃ Cos

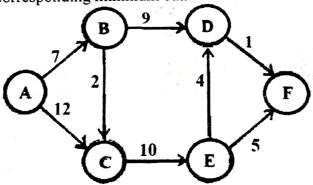


- b) A file contains the string "ENGINEERING". If Huffman Coding is used for [05] BL3 CO5 data compression, then determine
 - i) Huffman Code for each character
 - ii) Average code length
 - iii) Length of Huffman encoded message (in bits).

OR

Q.12 a) Find maximum flow in the transport network using labeling procedure. [05] BL3 C05

Determine the corresponding minimum cut.



- b) A file contains the string "DATAANALYST". If Huffman coding is used [05] BL3 CO5 for data compression, then determine
 - i) Huffman Code for each character.
 - ii) Average code length.
 - iii) Length of Huffman encoded message (in bits).