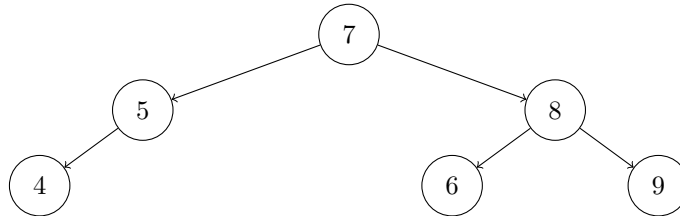
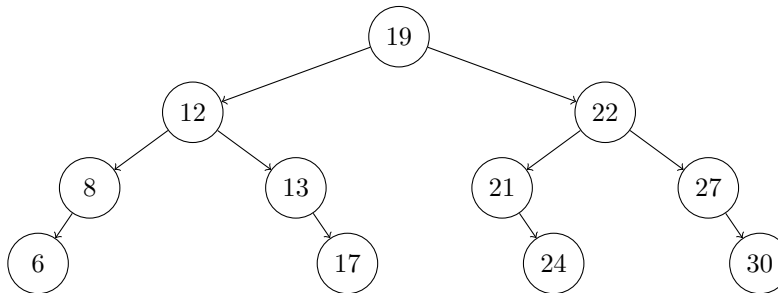


# 1 AVL Trees

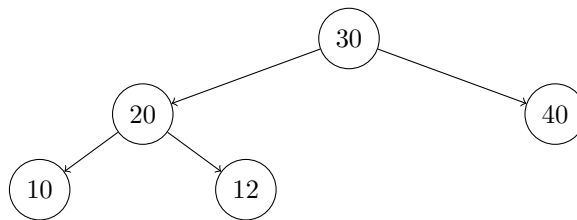
**Problem 1.** Perform a left rotation on the root of the following tree. Be sure to specify the X, Y, and Z subtrees used in the rotation.



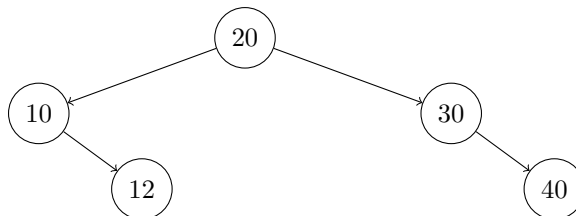
**Problem 2.** Show the right rotation of the subtree rooted at 27. Be sure to specify the X, Y, and Z subtrees used in the rotation.



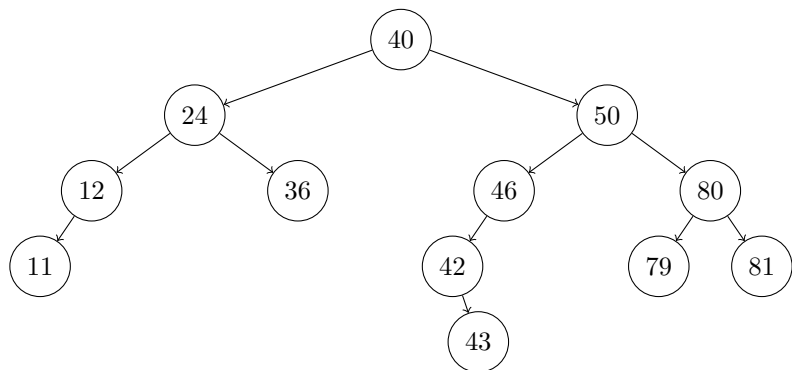
**Problem 3.** Using the appropriate AVL tree algorithm, insert the value 12 into the following tree. Show the tree before and after rebalancing.



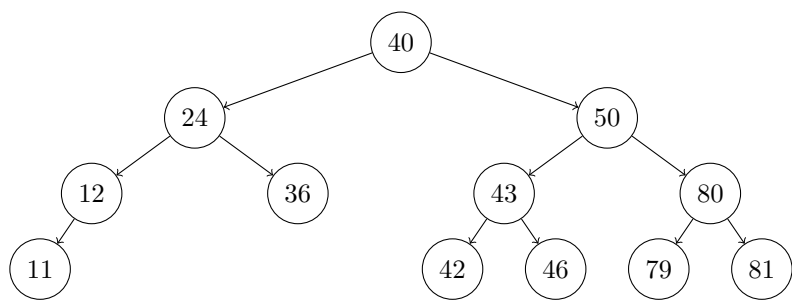
after rebalancing



**Problem 4.** Using the appropriate AVL tree algorithm, remove the value 54 from the following tree. Show the tree before and after rebalancing.

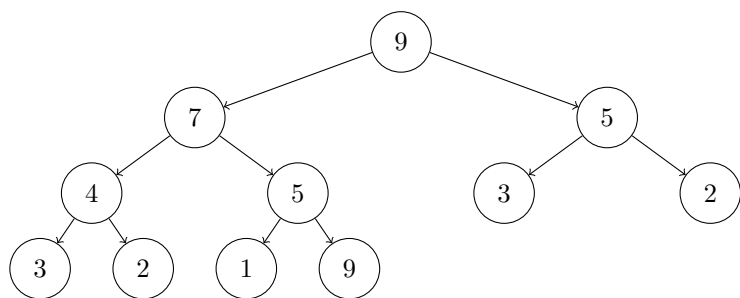


after rebalancing

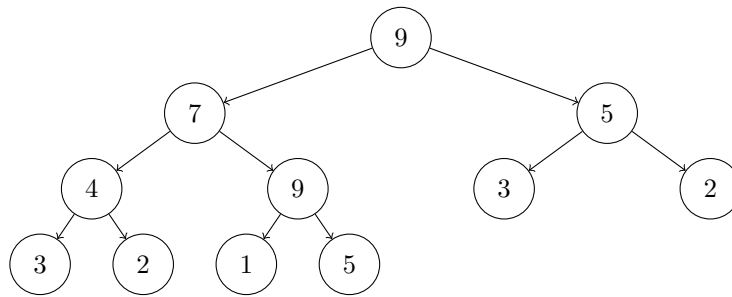


## 2 Heaps

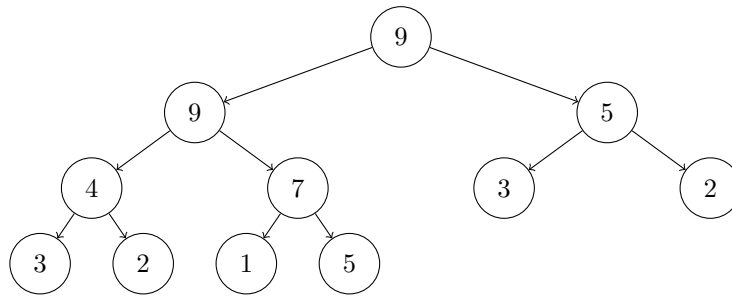
**Problem 1.** Show the addition of the element 9 to the max-heap below. First, show the addition of 9 to the tree; then, show each bubbling step.



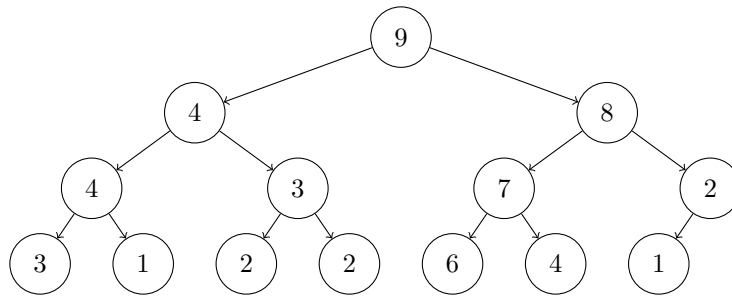
next step



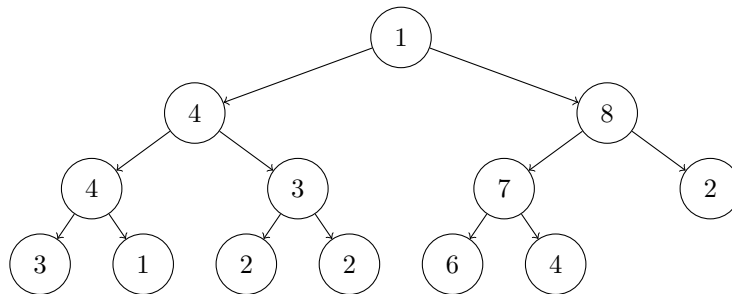
next step



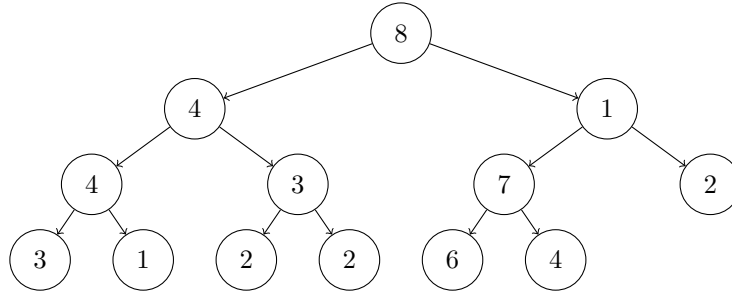
**Problem 2.** Show the removal of the top element of this max-heap. First, show the swap of the root node; then, show each bubbling step.



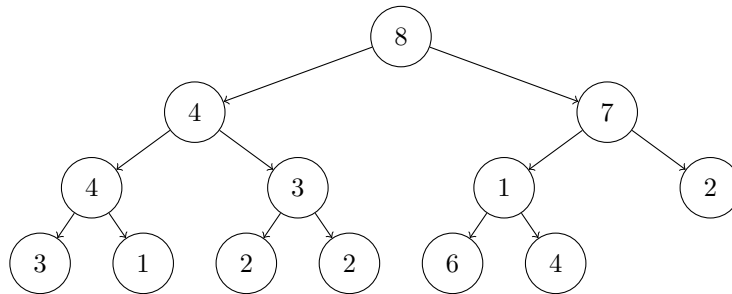
next step



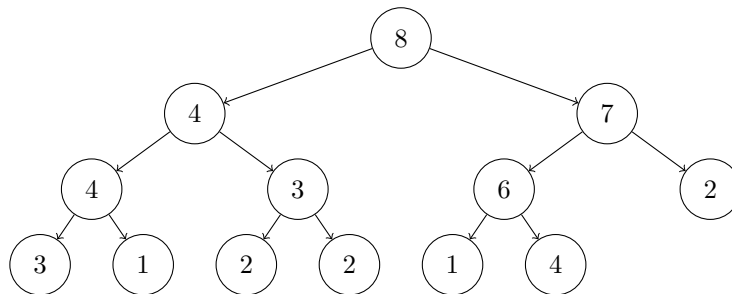
next step



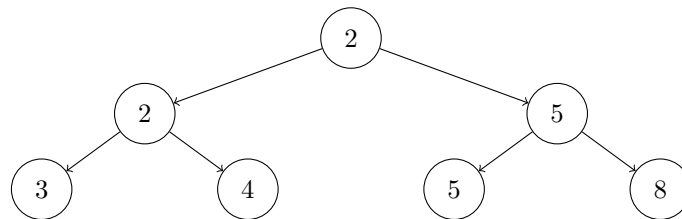
next step

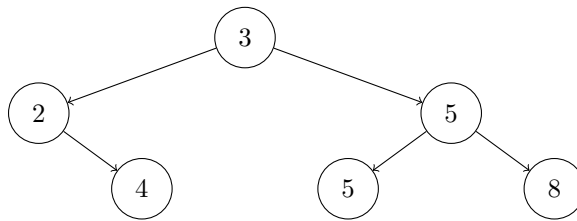


next step

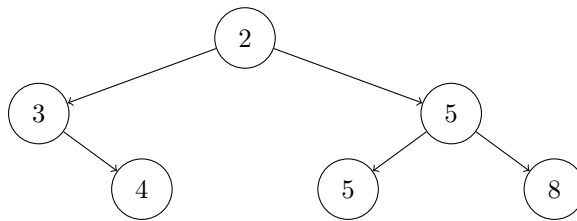


**Problem 3.** Consider the sequence of elements  $[5, 4, 2, 3, 2, 8, 5]$ . Using the representation discussed in class, show the tree to which this sequence corresponds. Then, show the *heapification* of this tree; that is, show how this tree is transformed into a heap. Demonstrate each bubbling step.

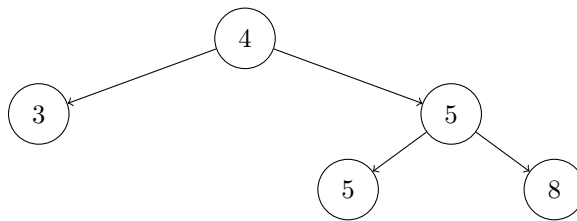




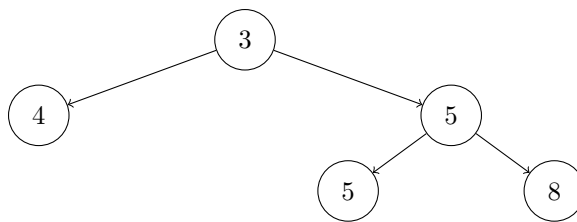
next step



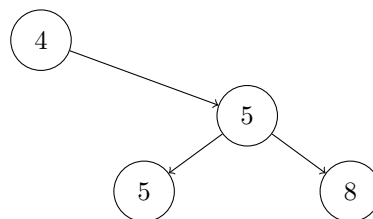
next step



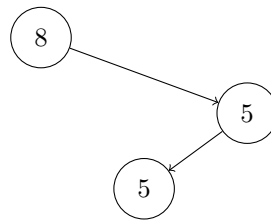
next step



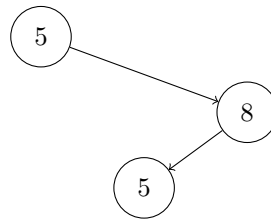
next step



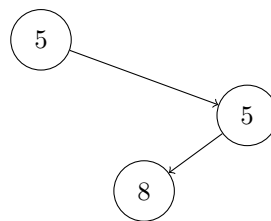
next step



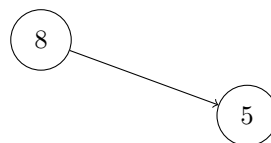
next step



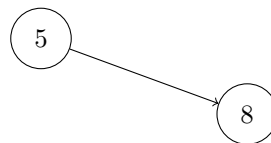
next step



next step



next step



next step



which gives us: 2234558