Linear Algebra

Prof. Gerhard Jäger, winter term 2023/2024

Assignment 06

1. (4 points) Find the largest possible number of independent vectors of

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \quad \mathbf{v}_4 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} \quad \mathbf{v}_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \quad \mathbf{v}_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

   Justify your answer.

2. (2 points) Decide the dependence or independence of

   (a)

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \quad \mathbf{u}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \quad \mathbf{u}_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

   (b)

$$\mathbf{w}_1 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} \quad \mathbf{w}_2 = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix} \quad \mathbf{w}_3 = \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$$

3. (4 points) Choose three independent columns of U. Then make two other choices. Do the same for A.

$$U = \begin{bmatrix} 2 & 3 & 4 & 1 \\ 0 & 6 & 7 & 0 \\ 0 & 0 & 0 & 9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 & 4 & 1 \\ 0 & 6 & 7 & 0 \\ 0 & 0 & 0 & 9 \\ 4 & 6 & 8 & 2 \end{bmatrix}$$