

# Linear Algebra

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Prof. Gerhard Jäger, winter term 2023

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## Assignment 04

1. (1 point) Simplify the expression

$$(((A^T)^{-1}(B^T)^{-1})^T)^{-1}$$

2. Consider the matrix  $A$ :

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 6 & 3 & 4 \end{bmatrix}$$

- (a) (1 point) Find the elimination matrix  $E$  which performs Gauss elimination, i.e.

$$EA = U$$

- (b) (1 point) Perform LU decomposition of  $A$ .

3. (2 points) Perform LU decomposition of

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

4. (1 point) Under what conditions is the matrix from exercise 3 invertible?
5. (1 point) Under what conditions do we have to perform a row permutation prior to LU decomposition with the matrix in exercise 3?
6. Consider the matrix

$$B = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}$$

- (a) (2 points) Perform LDU decomposition, i.e. find

- a lower triangular matrix  $L$  with 1s on the diagonal,
- an upper triangular matrix  $U$  with 1s on the diagonal, and
- a diagonal matrix  $D$  such that

$$B = LDU$$

- (b) (1 point) Is there a special relationship between  $L$  and  $U$ ? If so, which one?