Assignment 07.

1. (a) $p = \dfrac{b^T a}{a^T a} \cdot a = \dfrac{[\cos\theta \cdot \sin\theta]\begin{bmatrix}1\\0\end{bmatrix}}{[1\ 0]\begin{bmatrix}1\\0\end{bmatrix}} \cdot \begin{bmatrix}1\\0\end{bmatrix}$

$= \dfrac{\cos\theta \times 1 + \sin\theta \times 0}{1\times 1 + 0\times 0} \cdot \begin{bmatrix}1\\0\end{bmatrix}$

$= \cos\theta \begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}\cos\theta\\0\end{bmatrix}$ ✓

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(b) $p = \dfrac{[1\ 1]\begin{bmatrix}1\\-1\end{bmatrix}}{[1\ -1]\begin{bmatrix}1\\-1\end{bmatrix}} \cdot \begin{bmatrix}1\\-1\end{bmatrix} = \dfrac{1\times 1 + 1\times(-1)}{1\times 1 + (-1)\times(-1)} \cdot \begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$. ✓

2. (a) $P = A(A^TA)^{-1}A^T$. $A^T = [-1\ 2\ 2]$. $B^T = [2\ 2\ -1]$

$(A^TA)^{-1} = \left([-1\ 2\ 2]\begin{bmatrix}-1\\2\\2\end{bmatrix}\right)^{-1}$

$= [(-1)\times(-1) + 2\times 2 + 2\times 2]^{-1} = 1/9$

$\therefore P_A = \begin{bmatrix}-1\\2\\2\end{bmatrix} \times \frac{1}{9} \times [-1\ 2\ 2] = \begin{bmatrix}1/9 & -2/9 & -2/9\\ -2/9 & 4/9 & 4/9\\ -2/9 & 4/9 & 4/9\end{bmatrix}$ ✓

$(B^TB)^{-1} = [2\ 2\ -1]\begin{bmatrix}2\\2\\-1\end{bmatrix} = 1/9$

$\therefore P_B = \begin{bmatrix}2\\2\\-1\end{bmatrix} \times 1/9 \times [2\ 2\ -1] = \begin{bmatrix}4/9 & 4/9 & -2/9\\ 4/9 & 4/9 & -2/9\\ -2/9 & -2/9 & -1/9\end{bmatrix}$ ✓

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(b) $P_A \cdot P_B = 1/9 \cdot 1/9 \cdot \begin{bmatrix}1 & -2 & -2\\ -2 & 4 & 4\\ -2 & 4 & 4\end{bmatrix}\begin{bmatrix}4 & 4 & -2\\ 4 & 4 & -2\\ -2 & -2 & 1\end{bmatrix} = \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$. ✓

Because they're orthogonal.

3. (a) (b). $A_0 = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$ $\therefore A(\text{after removed}) = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\\0&0&0\end{bmatrix}$.

✓

$P = A(A^TA)^{-1}A^T = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&0&0\end{bmatrix}$ $P*b = [1\ 2\ 3\ 0]$

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4. Yes, p is symmetric.

Symmetric means $P = P^T$.

Let's see:

$$P^T = (A(A^TA)^{-1}A^T)^T$$

$$= (A^T) \cdot [(A^TA)^{-1}]^T A^T$$

$$= A \cdot [A^T(A^T)^T]^{-1} \cdot A^T$$

$$= A \cdot (A^TA)^{-1} \cdot A^T$$

which $= P$.

$\therefore P^T = P$

$\therefore$ symmetric.

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