

Linear Algebra 03.

1. Augmented matrix

$$\left[\begin{array}{cc|cc} a & b & 1 & 0 \\ c & d & 0 & 1 \end{array} \right]$$

elimination step

$$\left[\begin{array}{cc|cc} a & b & 1 & 0 \\ 0 & d - \frac{bc}{a} & -\frac{c}{a} & 1 \end{array} \right]$$

dividing by pivots

$$\left[\begin{array}{cc|cc} 1 & \frac{b}{a} & \frac{1}{a} & 0 \\ 0 & 1 & -\frac{c}{a(d - \frac{bc}{a})} & \frac{1}{d - \frac{bc}{a}} \end{array} \right]$$

inversed matrix

$$\left[\begin{array}{cc|cc} \frac{1}{a} & 0 & \frac{1}{a(d - \frac{bc}{a})} & \frac{1}{d - \frac{bc}{a}} \\ \frac{c}{a(d - \frac{bc}{a})} & \frac{1}{d - \frac{bc}{a}} & \frac{1}{a} & 0 \end{array} \right] \text{ or } \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

2. (a) $9x + 12y = 9$ ✓

(b) $9x + 12y = 30$ ✓

3. (a) $E_{13} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ✓

(b) $E_{13} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ ✓

(c) $E_{13} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ ✓

4. (a) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$ ✓

(b) $\begin{bmatrix} 1 \times 1 + 0 \times 1 + 0 \times 1 & 1 \times 2 + 0 \times 3 + 0 \times 4 & 1 \times 3 + 0 \times 7 + 0 \times 0 \\ (-1) \times 1 + 1 \times 1 + 0 \times 1 & (-1) \times 2 + 1 \times 3 + 0 \times 4 & (-1) \times 3 + 1 \times 7 + 0 \times 0 \\ (-1) \times 1 + 0 \times 1 + 1 \times 1 & (-1) \times 2 + 0 \times 3 + 1 \times 4 & (-1) \times 3 + 0 \times 7 + 1 \times 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & 2 & -3 \end{bmatrix}$ ✓

5. $A^T = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

It's reversed matrix is its transpose matrix ✓