LA 04.

1. Simplify expression: $(((A^T)^{-1}(B^T)^{-1})^T)^{-1}$

$((A^T)^{-1}(B^T)^{-1})^T = ((A^T)^{-1})^T((B^T)^{-1})^T = (A^{-1})^T(B^{-1})^T = A^{-T}B^{T}$

$\therefore$ Final result $\Rightarrow (A^{-T}B^{T})^{-1}$

2.(a). $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 6 & 3 & 4 \end{bmatrix}$ $U = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 0 & 0 & 4 \end{bmatrix}$

$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$ eliminate 6

$E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{1}{2} & 1 \end{bmatrix}$ eliminate 0

$E = E_1 E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & -\frac{1}{2} & 0 \end{bmatrix}$

(b). $L = (E_1)^{-1} \cdot (E_2)^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & -\frac{3}{2} & 1 \end{bmatrix}$

$U = E_2 \cdot E_1 \cdot A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 0 & 0 & 3 \end{bmatrix}$

3. $U = \begin{bmatrix} a & b \\ 0 & d-\frac{bc}{a} \end{bmatrix}$ $L = \begin{bmatrix} 1 & 0 \\ \frac{c}{a} & 1 \end{bmatrix}$

4. $ad - bc \neq 0$ or $ad \neq bc$

5. If pivots $= 0$, namely if $a = 0$ or $c = 0$, then a row permutation is needed prior to LU decomposition.

6. $B = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}$ (a) $L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ $D = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b-a & 0 & 0 \\ 0 & 0 & c-b & 0 \\ 0 & 0 & 0 & d-c \end{bmatrix}$ $U = \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & 0 & d-c \end{bmatrix}$