Assignment 08.

1. Given $^{5\times5}$ Matrix A, if $|A| = 2$

- $|2A| = 2^n \cdot |A| = 2^5 \cdot 2 = 64.$ ✓
- $|-A| = -|A| = -2$ ✓
- $|A^2| = |A \cdot A| = (|A|)^2 = 4$ ✓
- $|A^{-1}| = \frac{1}{|A|} = \frac{1}{2}$ $(\because |A^{-1}| \cdot |A| = 1).$ ✓

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2. $|A| = aei + bfg + cdh - ceg - bdi - afh$

$= a(ei - fh) \cancel{+} - b(di - fg) + c(dh - eg)$

$= 0 - 1 \times (1 \times 3 - 2 \times 1) + 1 \times (1 \times 2 - 2 \times 1)$

$= \cancel{1}\ -1$

1

$|B| = 1 \times (0 - 3 \times 3) - 2 \times (2 \times 3 - 3 \times 3) + 3 \times (2 \times 3 - 0) = 15.$ ✓

3. ~~$|A| = \sum_i a_{ij} C_{ij}$~~ Using LU decomposition, the.

~~$\therefore |U| = u_{11}C_{11} + u_{12}C_{12} + u_{13}C_{13} + u_{14}C_{14}$.~~

$$U = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 2 & 0 & \cancel{0}1 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

$\therefore |U| = U_{11} \times U_{22} \times U_{33} \times U_{44}$

$= 1 \times 2 \times 3 \times 5 = 30.$ ✓

$\therefore |U^{-1}| = \frac{1}{30}.$ ✓

2

$|U^2| = |U|^2 = 900.$ ✓

4. Given ~~the~~ rules, $|A^T A| = |A^T| \cdot |A| = (|A|)^2.$

$\because$ A has non-independent columns, thus A has row(s) with all 0, resulting in $|A| = 0.$

✓ 2

$\therefore |A^T A| = 0.$

5. Yes. If $|A| \neq 0$, then the elements on the diagonal are not 0, meaning each column has a rank, thus independent.

✓ 2