Linear Algebra

Prof. Gerhard Jäger, winter term 2023

Assignment 08

1. (2 points) If a 5×5 matrix has $|A| = 2$, find
   - $|2A|$
   - $|-A|$
   - $|A^2|$
   - $|A^{-1}|$

(Note that A^2 is defined as AA.)

2. (2 points) Find the determinants of the following matrices:

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 3 \\ 3 & 3 & 3 \end{bmatrix}$$

3. (2 points) Find the determinants of U and U^{-1} and U^2:

$$U = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 6 & 6 & 1 \\ -1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 6 \end{bmatrix}$$

4. (2 points) Let A be a 3×2 matrix with non-independent columns. What is the determinant of $A^T A$?

5. (2 points) Let A be a 4×4 matrix with $|A| \neq 0$. Are the columns of A independent? Justify your answer.