Linear Algebra

Prof. Gerhard Jäger, winter term 2023/2024

Assignment 05

1. (4 points) Reduce A and B to their *row echelon form* and their *reduced row echelon form*. Which columns are free? Find all special solutions for $A\mathbf{x} = \mathbf{0}$ and $B\mathbf{y} = \mathbf{0}$.

$$A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 4 & 4 \\ 0 & 8 & 8 \end{bmatrix}$$

2. (4 points) True or false (with reason if true or example to show it is false):

   (a) A the reduced row echelon form of a square matrix has no free columns.

   (b) The reduced row echelon form of an invertible matrix has no free columns.

   (c) An $m \times n$ matrix has no more than n pivot variables.

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3. (2 points) Construct a square matrix A where the null space of A is different from the null space of A^T.