

# Linear Algebra

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Prof. Gerhard Jäger, winter term 2023

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## Assignment 07

1. (2 points) Compute the projections of  $\mathbf{b}$  onto  $\mathbf{a}$ .

(a)

$$\mathbf{b} = \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix} \quad \mathbf{a} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

(b)

$$\mathbf{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \mathbf{a} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

2. (4 points) Consider the following matrices:

$$A = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$$

(a) Construct the projection matrices  $P_A$  and  $P_B$  for  $A$  and  $B$ .

(b) Compute the product  $P_A P_B$ . Explain in words why it has this value.

3. (2 points) (a) Suppose  $A$  is the  $4 \times 4$  identity matrix with its last column removed.  $A$  is  $4 \times 3$ .

Project  $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$  onto the column space of  $A$ .

(b) What is the projection matrix  $P$  that maps each vector to its projection onto the column space of  $A$ ?

4. (2 points) Is  $P = A(A^T A)^{-1} A^T$  symmetric? Prove your answer!