Vector Semantics & Embeddings

Jurafsky and Martin, chapter 6

Word Meaning

Word embedding（词嵌入）是自然语言处理（NLP）中一种常用的技术，
用于将单词或短语映射到实数向量空间中。
这种技术的核心思想是将文本中的单词或短语表示为连续的实值向量，
以便计算机可以更好地理解和处理文本信息。

Word embedding 的目标是将单词的语义信息捕捉到向量中，
使得在向量空间中语义相似的单词在向量空间中的距离也相近。
这种表示能够保留词语之间的语义关系，比如近义词在向量空间中的距离会比较近，
反义词则会在向量空间中有一定的方向关系。

常见的 word embedding 方法包括 Word2Vec、GloVe、FastText
等。这些方法通常在大型语料库上进行训练，学习出单词的向量表示。
这些向量可以用于各种 NLP 任务，例如文本分类、情感分析、语义相似度计算等。

What do words mean?

In the context of N-gram or text classification methods we've seen so far:

- Words are just strings (or indices w_i in a vocabulary list)
- Strings can be sorted alphabetically (like lexical entries in a dictionary).
- That's not very satisfactory! Why?

Homonymy and Polysemy

Word meanings are structured in different ways:

- By different senses:
  - Homonymy: words that are identical in form (e.g. *bear*), but have unrelated meanings. Homonyms are listed under different lexical entries in a dictionary and often involve different word classes (e.g. nouns and verbs)

Meaning

  - Polysemy: a word with related senses (e.g. *music*). Polysemous words are listed under a single lexical entry in a dictionary and involve a single word class.

Lemmas and senses

lemma

mouse (N)

sense

1. any of numerous small rodents...
2. a hand-operated device that controls a cursor...

Modified from the online thesaurus WordNet; https://wordnet.princeton.edu/

A sense or “concept” is the meaning component of a word

Lemmas can be polysemous (have multiple senses)

Word similarity and relatedness

- By relatedness or association (e.g. *table* and *chair*; *coffee* and *mug*)
- By semantic fields (e.g. physical objects, events, beliefs or desires)
- By semantic and conceptual relations such as synonymy, hyponymy, antonymy, meronymy

Relations between senses: Synonymy

Synonyms have the same meaning in some or all contexts.

- filbert / hazelnut
- couch / sofa
- big / large
- automobile / car
- vomit / throw up
- water / H_20

Relations between senses: Synonymy

Note that there are probably no examples of perfect synonymy.

- Even if many aspects of meaning are identical
- Still may differ based on politeness, slang, register, genre, etc.

Relation: Synonymy?

water/H_20

"H_20" in a surfing guide?

big/large

my big sister != my large sister

The Linguistic Principle of Contrast

Difference in form → difference in meaning

Abbé Gabriel Girard 1718

Re: "exact" synonyms

"je ne crois pas qu'il y ait de-
mot ſynonime dans aucune
Langue."

[I do not believe that there is a synonymous word in any language]

LA JUSTESSE
DE LA
LANGUE FRANÇOISE,
OU
LES DIFFERENTES SIGNIFICATIONS
DES MOTS QUI PASSENT
POUR
SYNONIMES.

Par M. l'Abbé GIRARD C. D. M. D. D. B.

A PARIS,
Chez LAURENT D'HOURY, Imprimeur-
Libraire, au bas de la rue de la Harpe, vis-
à vis la rue S. Severin, au Saint Eſprit.

M. DCC. XVIII.
Avec Approbation & Privilege du Roy.

Thanks to Mark Aronoff!

Relation: Similarity

Words with similar meanings. Not synonyms, but sharing some element of meaning

`car, bicycle`

`cow, horse`

Ask humans how similar 2 words are

word1	word2	similarity
vanish	disappear	9.8
behave	obey	7.3
belief	impression	5.95
muscle	bone	3.65
modest	flexible	0.98
hole	agreement	0.3

SimLex-999 dataset (Hill et al., 2015)

Relation: Word relatedness

Also called "word association"

Words can be related in any way, perhaps via a semantic frame or field

- `coffee, tea:` **similar**
- `coffee, cup:` **related**, not similar

Semantic field

Words that
- cover a particular semantic domain
- bear structured relations with each other.

hospitals

surgeon, scalpel, nurse, anaesthetic, hospital

restaurants

waiter, menu, plate, food, menu, chef

houses

door, roof, kitchen, family, bed

Relation: Antonymy

Senses that are opposites with respect to only one feature of meaning

Otherwise, they are very similar!

`dark/light    short/long fast/slow    rise/fall`

`hot/cold      up/down        in/out`

More formally: antonyms can

- define a binary opposition or be at opposite ends of a scale
  - `long/short, fast/slow`
- *reversives*:
  - `rise/fall, up/down`

Connotation (sentiment)

- Words have **affective** meanings
  - Positive connotations (*happy*)
  - Negative connotations (*sad*)
- Connotations can be subtle:
  - Positive connotation: *copy, replica, reproduction*
  - Negative connotation: *fake, knockoff, forgery*
- Evaluation (sentiment!)
  - Positive evaluation (*great, love*)
  - Negative evaluation (*terrible, hate*)

Connotation

Osgood et al. (1957)

Words seem to vary along 3 affective dimensions:

- **valence**: the pleasantness of the stimulus
- **arousal**: the intensity of emotion provoked by the stimulus
- **dominance**: the degree of control exerted by the stimulus

Closer to 1, More relevant

	Word	Score		Word	Score
Valence	love	1.000		toxic	0.008
	happy	1.000		nightmare	0.005
Arousal	elated	0.960		mellow	0.069
	frenzy	0.965		napping	0.046
Dominance	powerful	0.991		weak	0.045
	leadership	0.983		empty	0.081

Values from NRC VAD Lexicon (Mohammad 2018)

So far

Concepts or word senses

- Have a complex many-to-many association with **words** (homonymy, multiple senses)

Have relations with each other

- Synonymy
- Antonymy
- Similarity
- Relatedness
- Connotation

Vector Semantics & Embeddings

Word Meaning

Vector Semantics & Embeddings

Vector Semantics

Computational models of word meaning

Can we build a theory of how to represent word meaning, that accounts for at least some of the desiderata?

We'll introduce **vector semantics**

The standard model in language processing!

Handles many of our goals!

Ludwig Wittgenstein

PI #43:

"The meaning of a word is its use in the language"

Let's define words by their usages

One way to define "usage":

words are defined by their environments (the words around them)

Zellig Harris (1954):

If A and B have almost identical environments we say that they are synonyms.

The Distributional Approach to Word Meaning

The distribution of an element will be understood as the sum of all of its environments. An environment of an element A is an existing array of its co-occurrents. (Harris 1954, p. 146)

The fact that, for example, not every adjective occurs with every noun can be used as a measure of meaning difference. (Harris 1954, p. 156)

You shall know a word by the company it keeps. (J.R. Firth 1957, p. 2)

The Distributional Hypothesis (Sahlgren 2008)

- *Words which are similar in meaning occur in similar contexts.*

  (Rubenstein & Goodenough 1965)

- *Words with similar meanings will occur with similar neighbors if enough text material is available.* (Schütze & Pedersen 1995)

Sketch-Engine: Collocates of *Sweden* and *Norway*

Verify distribution hypothesis.
Interpret: appear tgt

WORD SKETCH DIFFERENCE

English Web 2020 (enTenTen20)

Account expires in April 2022 »
Get more space

Sweden 691,676× Norway 557,184×

"Sweden/Norway" and/or ...		
Uppsala	3,037	0
Gothenburg	6,277	13
Stockholm	18,481	60
Norway	27,277	372
Finland	15,853	8,526
Switzerland	8,988	5,785
Denmark	18,415	15,332
Netherlands	6,046	7,118
Iceland	1,350	7,960
Sweden	394	27,277
Oslo	64	12,659
Bergen	15	5,046

verbs with "Sweden/Norway" as object		
non-align	21	0
captain	32	0
defeat	590	199
beat	709	326
neighbour	145	110
tour	238	186
neighbor	64	53
rule	151	206
invade	152	830
Christianise	0	9
Christianize	0	21
cede	0	120

verbs with "Sweden/Norway" as subject		
average	128	0
defeat	287	80
invade	101	29
cede	132	42
ban	147	82
rank	188	164
pledge	45	103
rename	24	75
export	46	187
pine	0	40
rat	0	45
spruce	0	136

adjective predicates of "Sweden/Norway"		
Swedish	13	0
U17	10	0
s	18	0
spectacular	84	11
neutral	102	35
international	108	77
socialist	14	13
national	36	52
mountainous	0	17
Norwegian	0	10
+47	0	14
spruce	0	42

What does recent English borrowing *ongchoi* mean?

Suppose you see these sentences:

- Ong choi is delicious **sautéed with garlic**.
- Ong choi is superb **over rice**
- Ong choi **leaves** with salty sauces

And you've also seen these:

- …spinach **sautéed with garlic over rice**
- Chard stems and **leaves** are **delicious**
- Collard greens and other **salty** leafy greens

Conclusion:

- Ongchoi is a leafy green like spinach, chard, or collard greens
  - We could conclude this based on words like "leaves" and "delicious" and "sauteed"

Ongchoi: *Ipomoea aquatica* *"Water Spinach"*

空心菜
kangkong
rau muống
…

Yamaguchi, Wikimedia Commons, public domain

Idea 1: Defining meaning by linguistic distribution

Let's define the meaning of a word by its distribution in language use, meaning its neighboring words or grammatical environments.

Idea 2: Meaning as a point in space (Osgood et al. 1957)

3 affective dimensions for a word

- **valence**: pleasantness
- **arousal**: intensity of emotion
- **dominance**: the degree of control exerted

	Word	Score		Word	Score
Valence	love	1.000		toxic	0.008
	happy	1.000		nightmare	0.005
Arousal	elated	0.960		mellow	0.069
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Dominance	powerful	0.991		weak	0.045
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NRC VAD Lexicon
(Mohammad 2018)

Hence the connotation of a word is a vector in 3-space

Reminder: Vectors in linear/logistic regression

- Encoding data points by a set of features in a high-dimensional vector space
- We will now generalize this type of encoding to the representation of words
- Consult JM, Chapter 5 and the slides on linear/logistic regression and on vectors and vector operators (see readings on moodle for the current chapter

Idea 1: Defining meaning by linguistic distribution

Idea 2: Meaning as a point in multidimensional space

Defining meaning as a point in space based on distribution

Each word = a vector   (not just "good" or "w_{45}")

Similar words are "**nearby in semantic space**"

We build this space automatically by seeing which words are **nearby in text**

We define meaning of a word as a vector

Called an "embedding" because it's embedded into a space (see textbook)

The standard way to represent meaning in NLP

Every modern NLP algorithm uses embeddings as the representation of word meaning

Fine-grained model of meaning for similarity

Intuition: why vectors?

Consider sentiment analysis:

- With **words**, a feature is a word identity
  - Feature 5: 'The previous word was "terrible"'
  - requires **exact same word** to be in training and test
- With **embeddings**:
  - Feature is a word vector
  - 'The previous word was vector [35,22,17…]
  - Now in the test set we might see a similar vector [34,21,14]
  - We can generalize to **similar but unseen** words!!!

We'll discuss 2 kinds of embeddings

tf-idf

- Information Retrieval workhorse!
- A common baseline model
- **Sparse** vectors
- Words are represented by (a simple function of) the **counts** of nearby words

Word2vec

- **Dense** vectors
- Representation is created by training a classifier to **predict** whether a word is likely to appear nearby
- Later we'll discuss extensions called **contextual embeddings**

From now on:
Computing with meaning representations instead of string representations

荃者所以在鱼，得鱼而忘荃 Nets are for fish;
Once you get the fish, you can forget the net.
言者所以在意，得意而忘言 Words are for meaning;
Once you get the meaning, you can forget the words

庄子(Zhuangzi), Chapter 26

Vector Semantics & Embeddings

Vector Semantics

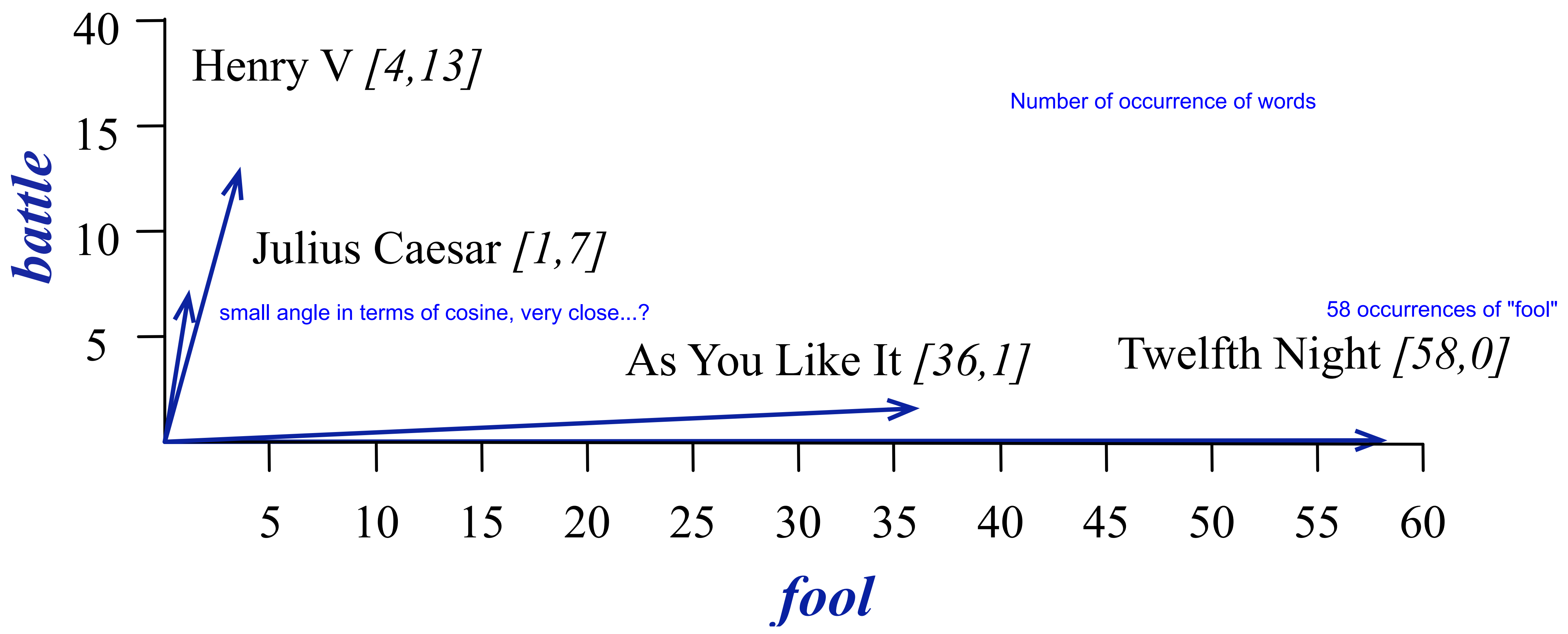
Words and Vectors

Term-document matrix

Each document is represented by a vector of words

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	0	7	13
good	114	80	62	89
fool	36	58	1	4
wit	20	15	2	3

Visualizing document vectors



Vectors are the basis of information retrieval

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	0	7	13
good	114	80	62	89
fool	36	58	1	4
wit	20	15	2	3

Vectors are similar for the two comedies

But comedies are different than the other two

Comedies have more *fools* and *wit* and fewer *battles*.

Idea for word meaning: Words can be vectors too!!!

columns for document vectors, rows for word vectors (?Correct?)

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	0	7	13
good	114	80	62	89
fool	36	58	1	4
wit	20	15	2	3

battle is "the kind of word that occurs in Julius Caesar and Henry V"

fool is "the kind of word that occurs in comedies, especially Twelfth Night" not word-doc matrix anymore

More common: word-word matrix (or "term-context matrix")

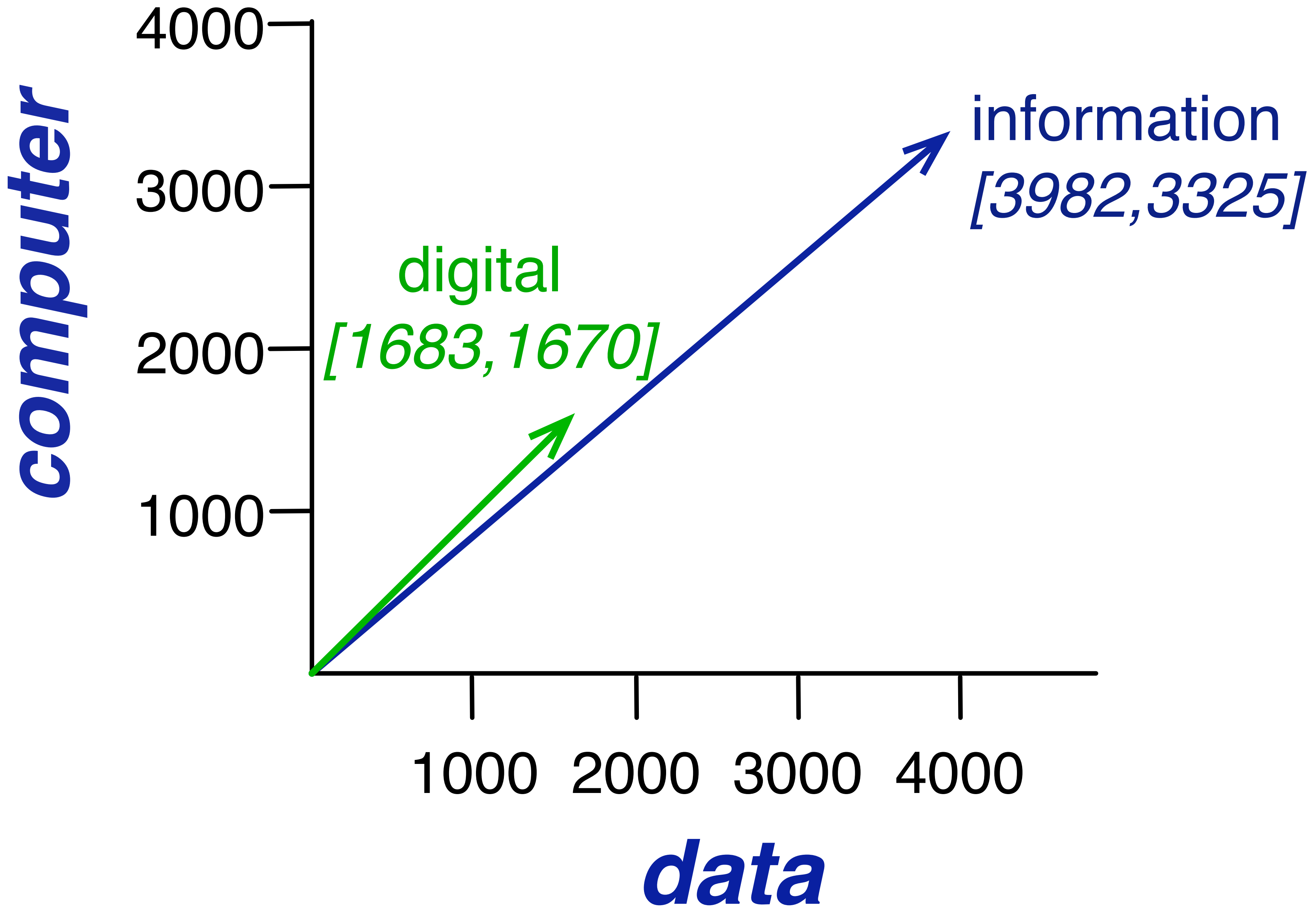
Two **words** are similar in meaning if their context vectors are similar

is traditionally followed by	**cherry**	pie, a traditional dessert
often mixed, such as	**strawberry**	rhubarb pie. Apple pie
computer peripherals and personal	**digital**	assistants. These devices usually
a computer. This includes	**information**	available on the internet

Also from vocab

Target words in vocab

	aardvark	...	computer	data	result	pie	sugar	...
cherry	0	...	2	8	9	442	25	...
strawberry	0	...	0	0	1	60	19	...
digital	0	...	1670	1683	85	5	4	...
information	0	...	3325	3982	378	5	13	...


4000
3000
2000
1000
computer
information
[3982,3325]
digital
[1683,1670]
1000
2000
3000
4000
data

Vector Semantics & Embeddings

Words and Vectors

Vector Semantics & Embeddings

Cosine for computing word similarity

Computing word similarity: Dot product and cosine

The dot product between two vectors is a scalar:

$$\text{dot product}(\mathbf{v}, \mathbf{w}) = \mathbf{v} \cdot \mathbf{w} = \sum_{i=1}^{N} v_i w_i = v_1 w_1 + v_2 w_2 + \ldots + v_N w_N$$

The dot product tends to be high when the two vectors have large values in the same dimensions

Dot product can thus be a useful similarity metric between vectors

Problem with raw dot-product

Dot product favors long vectors

Dot product is higher if a vector is longer (has higher values in many dimension)

Vector length:

$$|\mathbf{v}| = \sqrt{\sum_{i=1}^{N} v_i^2}$$

Frequent words (*of, the, you*) have long vectors (since they occur many times with other words).

So dot product overly favors frequent words

Alternative: cosine for computing word similarity

$$\text{cosine}(\vec{v},\vec{w}) = \frac{\vec{v}\cdot\vec{w}}{|\vec{v}||\vec{w}|} = \frac{\sum_{i=1}^{N} v_i w_i}{\sqrt{\sum_{i=1}^{N} v_i^2}\sqrt{\sum_{i=1}^{N} w_i^2}}$$

Based on the definition of the dot product between two vectors a and b

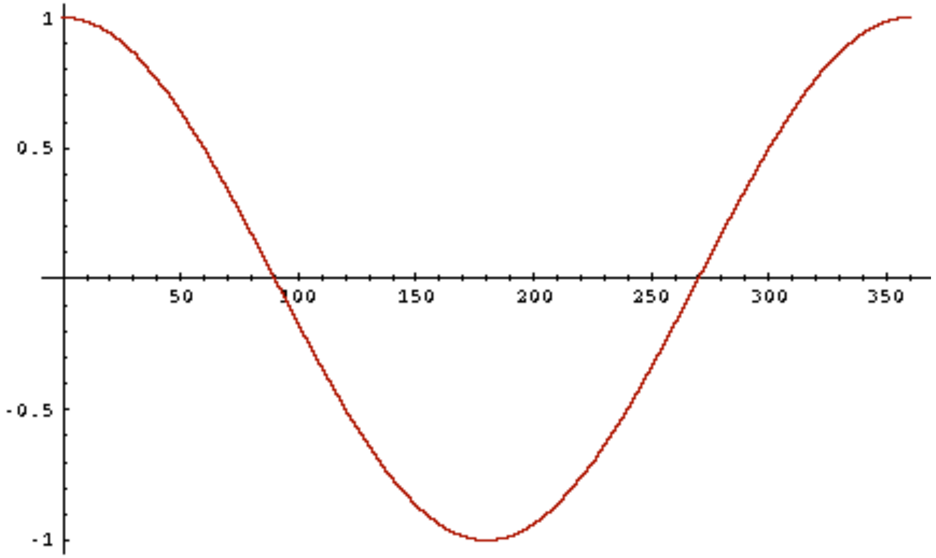
$$\begin{aligned}\mathbf{a}\cdot\mathbf{b} &= |\mathbf{a}||\mathbf{b}|\cos\theta \\ \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|} &= \cos\theta\end{aligned}$$

Cosine as a similarity metric

-1: vectors point in opposite directions

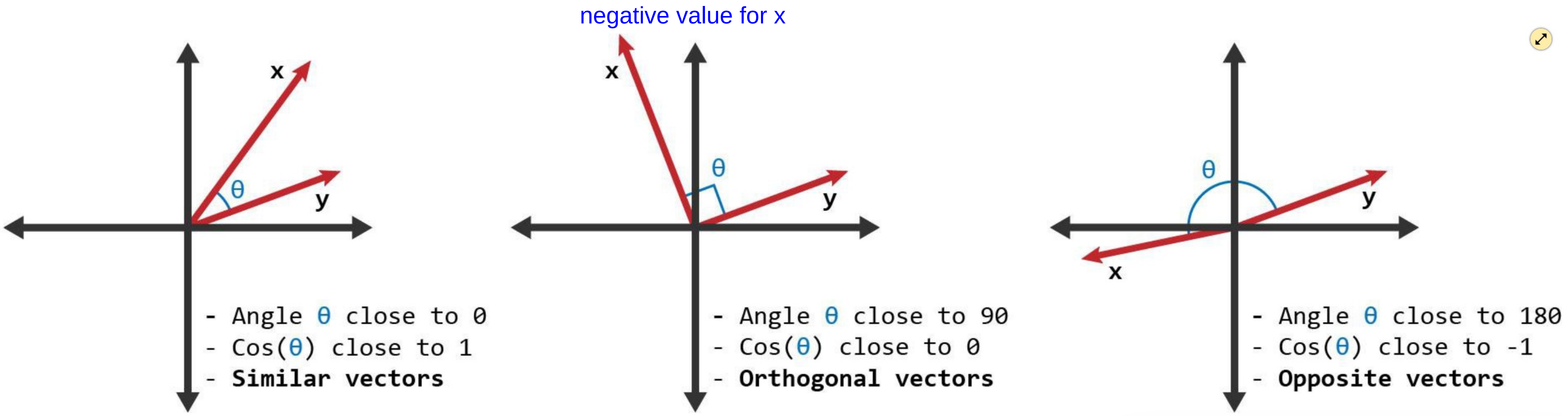
+1: vectors point in same directions

0: vectors are orthogonal



But since raw frequency values are non-negative, the cosine for term-term matrix vectors ranges from 0–1

Angles and Cos



Credit to: https://www.learndatasci.com/glossary/cosine-similarity/

Recall: the following cosine values: +1: vectors point in the same direction
0: vectors are orthogonal
-1: vectors point in opposite directions

Cosine examples

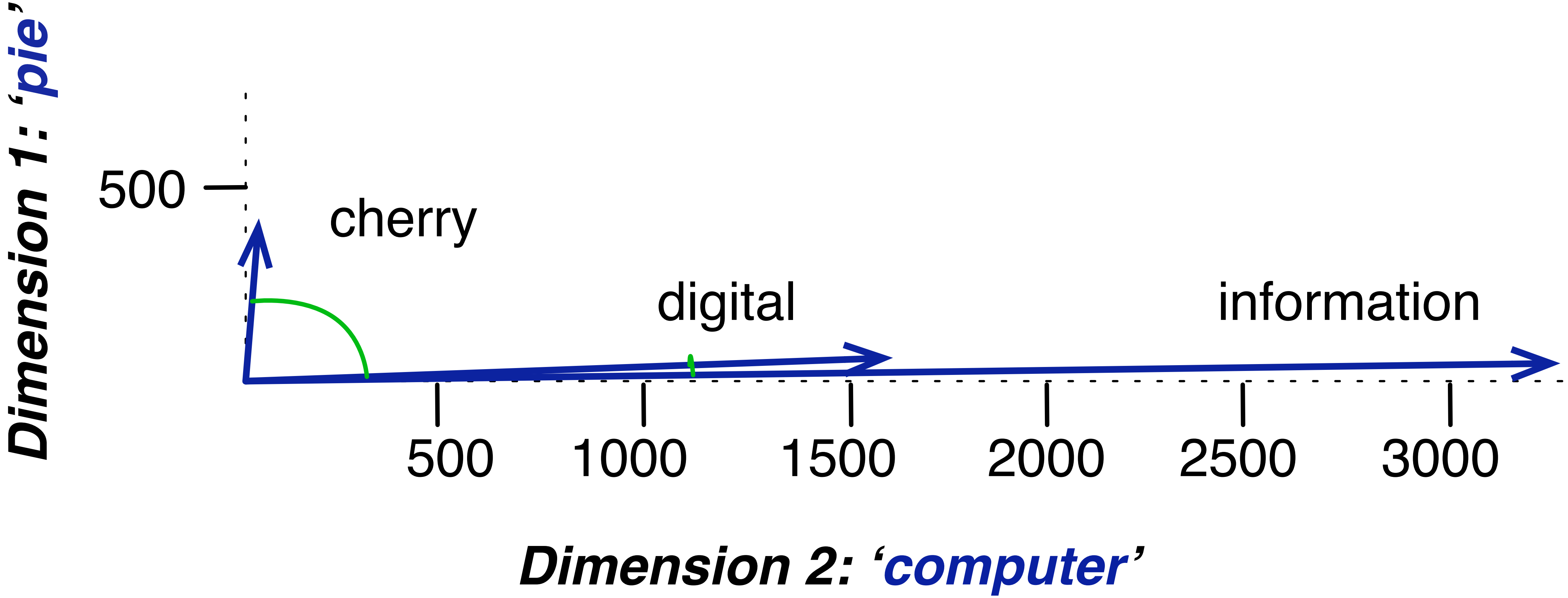
$$\cos(\vec{v},\vec{w}) = \frac{\vec{v}\bullet\vec{w}}{|\vec{v}||\vec{w}|} = \frac{\vec{v}}{|\vec{v}|}\bullet\frac{\vec{w}}{|\vec{w}|} = \frac{\sum_{i=1}^{N} v_i w_i}{\sqrt{\sum_{i=1}^{N} v_i^2}\sqrt{\sum_{i=1}^{N} w_i^2}}$$

	pie	data	computer
cherry	442	8	2
digital	5	1683	1670
information	5	3982	3325

$$\cos(\text{cherry}, \text{information}) = \frac{442 * 5 + 8 * 3982 + 2 * 3325}{\sqrt{442^2 + 8^2 + 2^2}\sqrt{5^2 + 3982^2 + 3325^2}} = .017$$

$$\cos(\text{digital}, \text{information}) = \frac{5 * 5 + 1683 * 3982 + 1670 * 3325}{\sqrt{5^2 + 1683^2 + 1670^2}\sqrt{5^2 + 3982^2 + 3325^2}} = .996$$

Visualizing cosines (well, angles)



More common: word-word matrix (or "term-context matrix")

Two **words** are similar in meaning if their context vectors are similar

is traditionally followed by	**cherry**	pie, a traditional dessert
often mixed, such as	**strawberry**	rhubarb pie. Apple pie
computer peripherals and personal	**digital**	assistants. These devices usually
a computer. This includes	**information**	available on the internet

	aardvark	...	**computer**	**data**	**result**	**pie**	**sugar**	...
cherry	0	...	2	8	9	442	25	...
strawberry	0	...	0	0	1	60	19	...
digital	0	...	1670	1683	85	5	4	...
information	0	...	3325	3982	378	5	13	...

Vector Semantics & Embeddings

Cosine for computing word similarity

Vector Semantics & Embeddings

TF-IDF

But raw frequency is a bad representation

- The co-occurrence matrices we have seen represent each cell by word frequencies.
- Frequency is clearly useful; if *sugar* appears a lot near *apricot*, that's useful information.
- But overly frequent words like *the*, *it,* or *they* are not very informative about the context
- It's a paradox! How can we balance these two conflicting constraints?

Two common solutions for word weighting

tf-idf: tf-idf value for word t in document d:

$$w_{t,d} = \text{tf}_{t,d} \times \text{idf}_t$$

Words like "the" or "it" have very low idf

PMI: (Pointwise mutual information)

- $\textbf{PMI}(w_1, w_2) = log \frac{p(w_1,w_2)}{p(w_1)p(w_2)}$

See if words like "good" appear more often with "great" than we would expect by chance

Term frequency (tf)

$\text{tf}_{t,d} = \text{count}(t,d)$

Instead of using raw count, we squash a bit:

$\text{tf}_{t,d} = \log_{10}(\text{count}(t,d)+1)$

Document frequency (df)

df_t *is* the number of documents *t* occurs in.

(note this is not collection frequency: total count across all documents)

"*Romeo*" is very distinctive for one Shakespeare play:

	Collection Frequency	**Document Frequency**
Romeo	113	1
action	113	31

Inverse document frequency (idf)

$$\text{idf}_t = \log_{10}\left(\frac{N}{\text{df}_t}\right)$$

N is the total number of documents in the collection

Word	**df**	**idf**
Romeo	1	1.57
salad	2	1.27
Falstaff	4	0.967
forest	12	0.489
battle	21	0.246
wit	34	0.037
fool	36	0.012
good	37	0
sweet	37	0

What is a document?

Could be a play or a Wikipedia article

But for the purposes of tf-idf, documents can be **anything**; we often call each paragraph a document!

Final tf-idf weighted value for a word

$$w_{t,d} = \text{tf}_{t,d} \times \text{idf}_t$$

Raw counts:

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	0	7	13
good	114	80	62	89
fool	36	58	1	4
wit	20	15	2	3

tf-idf:

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	0.074	0	0.22	0.28
good	0	0	0	0
fool	0.019	0.021	0.0036	0.0083
wit	0.049	0.044	0.018	0.022

Summary

$\text{tf-idf}_{t,d} = \text{tf}_{t,d} \times \text{idf}_f$ assigns to term *t* a weight in document *d* that is

1. highest when *t* occurs many times within a small set of documents (thus lending high discriminating power to those documents
2. lower when the term occurs fewer times in a document, or occurs in many documents (thus offering a less pronounced relevance signal)
3. Lowest when the term occurs in virtually all documents.

Summary of definitions

- **collection frequency** (cf_t): the total number of occurrences of the term *t* in a collection.
- **document frequency** (df_t): the number of documents that contain the term *t* at least once.
- **term frequency** ($tf_{t,d}$): the number of occurrences of a term *t* in a document *d*.

Vector Semantics & Embeddings

TF-IDF

Vector Semantics & Embeddings

PPMI

Pointwise Mutual Information

Pointwise mutual information:
Do events x and y co-occur more than if they were independent?

$$\text{PMI}(X,Y) = \log_2 \frac{P(x,y)}{P(x)P(y)}$$

PMI between two words: (Church & Hanks 1989)
Do words x and y co-occur more than if they were independent?

$$\text{PMI}(word_1, word_2) = \log_2 \frac{P(word_1, word_2)}{P(word_1)P(word_2)}$$

Positive Pointwise Mutual Information

- PMI ranges from $-\infty$ to $+\infty$
- But the negative values are problematic
  - Things are co-occurring **less than** we expect by chance
  - Unreliable without enormous corpora
    - Imagine w1 and w2 whose probability is each 10^{-6}
    - Hard to be sure p(w1,w2) is significantly different than 10^{-12}
  - Plus it's not clear people are good at "unrelatedness"
- So we just replace negative PMI values by 0
- Positive PMI (**PPMI**) between word1 and word2:

$$\mathrm{PPMI}(word_1, word_2) = \max\left(\log_2 \frac{P(word_1, word_2)}{P(word_1)P(word_2)}, 0\right)$$

Computing PPMI on a term-context matrix

Matrix F with W rows (words) and C columns (contexts)

f_{ij} is # of times w_{i} occurs in context c_{j}

$$p_{ij} = \frac{f_{ij}}{\sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}} \qquad p_{i*} = \frac{\sum_{j=1}^{C} f_{ij}}{\sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}} \qquad p_{*j} = \frac{\sum_{i=1}^{W} f_{ij}}{\sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}}$$

	computer	data	result	pie	sugar	count(w)
cherry	2	8	9	442	25	486
strawberry	0	0	1	60	19	80
digital	1670	1683	85	5	4	3447
information	3325	3982	378	5	13	7703
count(context)	4997	5673	473	512	61	11716

$$pmi_{ij} = \log_2 \frac{p_{ij}}{p_{i*}p_{*j}} \qquad ppmi_{ij} = \begin{cases} pmi_{ij} & \text{if } pmi_{ij} > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$p_{ij} = \frac{f_{ij}}{\sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}}$$

	computer	data	result	pie	sugar	count(w)
cherry	2	8	9	442	25	486
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information	3325	3982	378	5	13	7703
count(context)	4997	5673	473	512	61	11716

p(w=information,c=data) = 3982/111716 = .3399

p(w=information) = 7703/11716 = .6575

p(c=data) = 5673/11716 = .4842

$$p(w_i) = \frac{\sum_{j=1}^{C} f_{ij}}{N} \qquad p(c_j) = \frac{\sum_{i=1}^{W} f_{ij}}{N}$$

	p(w,context)					p(w)
	computer	data	result	pie	sugar	p(w)
cherry	0.0002	0.0007	0.0008	0.0377	0.0021	0.0415
strawberry	0.0000	0.0000	0.0001	0.0051	0.0016	0.0068
digital	0.1425	0.1436	0.0073	0.0004	0.0003	0.2942
information	0.2838	0.3399	0.0323	0.0004	0.0011	0.6575
p(context)	0.4265	0.4842	0.0404	0.0437	0.0052	

$$pmi_{ij} = \log_2 \frac{p_{ij}}{p_{i*}p_{*j}}$$

	p(w,context)					p(w)
	computer	data	result	pie	sugar	p(w)
cherry	0.0002	0.0007	0.0008	0.0377	0.0021	0.0415
strawberry	0.0000	0.0000	0.0001	0.0051	0.0016	0.0068
digital	0.1425	0.1436	0.0073	0.0004	0.0003	0.2942
information	0.2838	0.3399	0.0323	0.0004	0.0011	0.6575
p(context)	0.4265	0.4842	0.0404	0.0437	0.0052	

pmi(information,data) = $\log_2$ (.3399 / (.6575*.4842)) = .0944

Resulting PPMI matrix (negatives replaced by 0)

	computer	data	result	pie	sugar
cherry	0	0	0	4.38	3.30
strawberry	0	0	0	4.10	5.51
digital	0.18	0.01	0	0	0
information	0.02	0.09	0.28	0	0

Weighting PMI

PMI is biased toward infrequent events

- Very rare words have very high PMI values

Two solutions:

- Give rare words slightly higher probabilities
- Use add-one smoothing (which has a similar effect)

Weighting PMI: Giving rare context words slightly higher probability

Raise the context probabilities to $\alpha = 0.75$:

$$\text{PPMI}_\alpha(w,c) = \max(\log_2 \frac{P(w,c)}{P(w)P_\alpha(c)}, 0)$$

$$P_\alpha(c) = \frac{count(c)^\alpha}{\sum_c count(c)^\alpha}$$

This helps because $P_\alpha(c) > P(c)$ for rare *c*

Consider two events, P(a) = .99 and P(b)=.01

$$P_\alpha(a) = \frac{.99^{.75}}{.99^{.75} + .01^{.75}} = .97 \quad P_\alpha(b) = \frac{.01^{.75}}{.01^{.75} + .01^{.75}} = .03$$

Vector Semantics & Embeddings

Word2vec

Sparse versus dense vectors

tf-idf (or PMI) vectors are

- **long** (length |V|= 20,000 to 50,000)
- **sparse** (most elements are zero)

Alternative: learn vectors which are

- **short** (length 50-1000)
- **dense** (most elements are non-zero)

Sparse versus dense vectors

Why dense vectors?

- Short vectors may be easier to use as **features** in machine learning (fewer weights to tune)
- Dense vectors may **generalize** better than explicit counts
- Dense vectors may do better at capturing synonymy:
  - *car* and *automobile* are synonyms; but are distinct dimensions
    - a word with *car* as a neighbor and a word with *automobile* as a neighbor should be similar, but aren't
- **In practice, they work better**

Common methods for getting short dense vectors

“Neural Language Model”-inspired models

- Word2vec (skipgram, CBOW), GloVe

Singular Value Decomposition (SVD)

- A special case of this is called LSA – Latent Semantic Analysis

Alternative to these "static embeddings":

- Contextual Embeddings (ELMo, BERT)
- Compute distinct embeddings for a word in its context
- Separate embeddings for each token of a word

Simple static embeddings you can download!

Word2vec (Mikolov et al)

https://code.google.com/archive/p/word2vec/

GloVe (Pennington, Socher, Manning)

http://nlp.stanford.edu/projects/glove/

Word2vec

Popular embedding method

Very fast to train

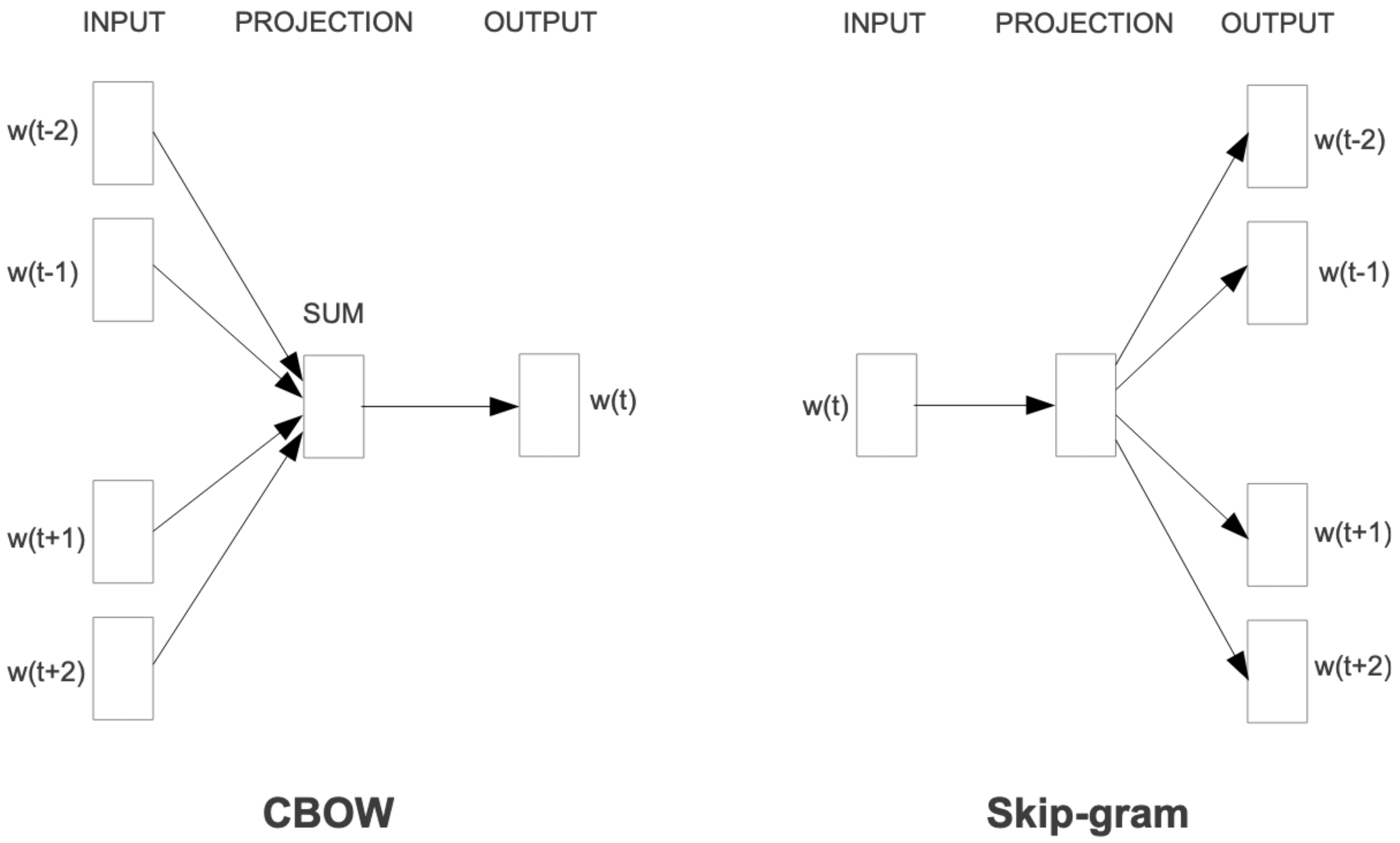
Code available on the web

Idea: **predict** rather than **count**

Word2vec provides various options. We'll do:

skip-gram with negative sampling (SGNS)

Word Embeddings created by Self-Supervision from large data sets (Mikolov et al. 2013)



Word2vec

Instead of **counting** how often each word *w* occurs near "*apricot*"

- Train a classifier on a binary **prediction** task:
  - Is *w* likely to show up near "*apricot*"?

We don’t actually care about this task

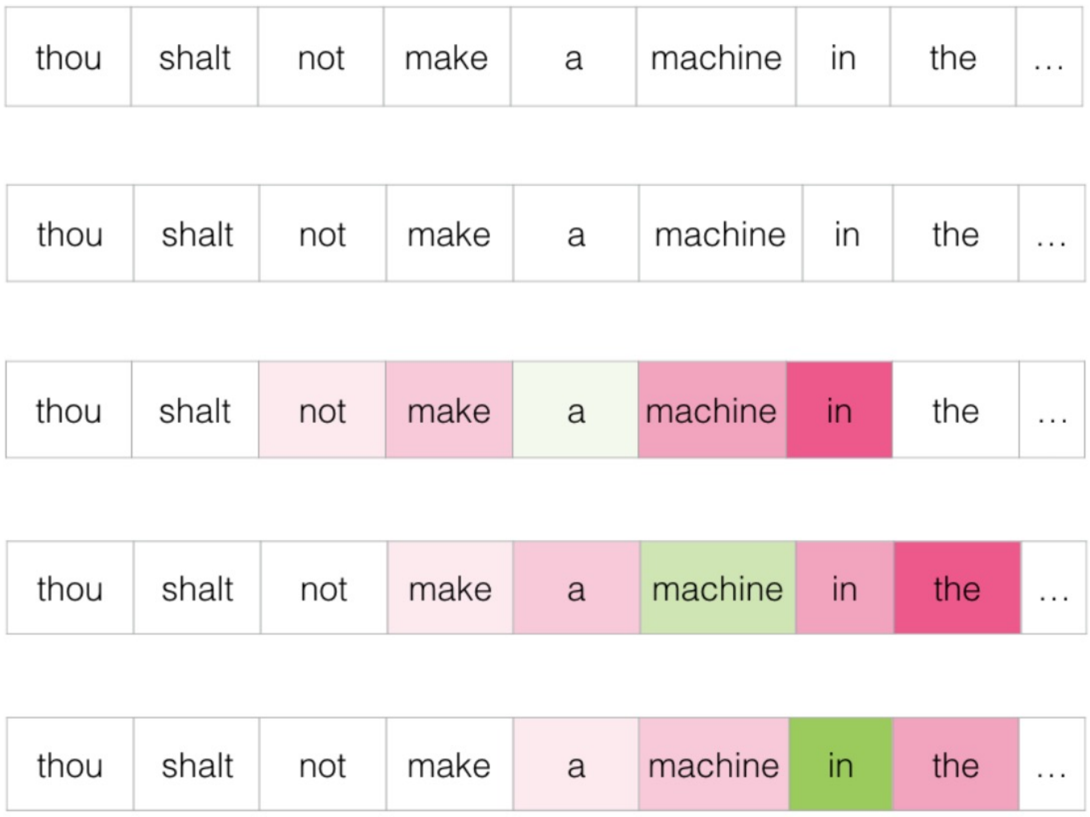
- But we'll take the learned classifier weights as the word embeddings

Big idea: **self-supervision**:

- A word c that occurs near apricot in the corpus cats as the gold "correct answer" for supervised learning
- No need for human labels
- Bengio et al. (2003); Collobert et al. (2011)

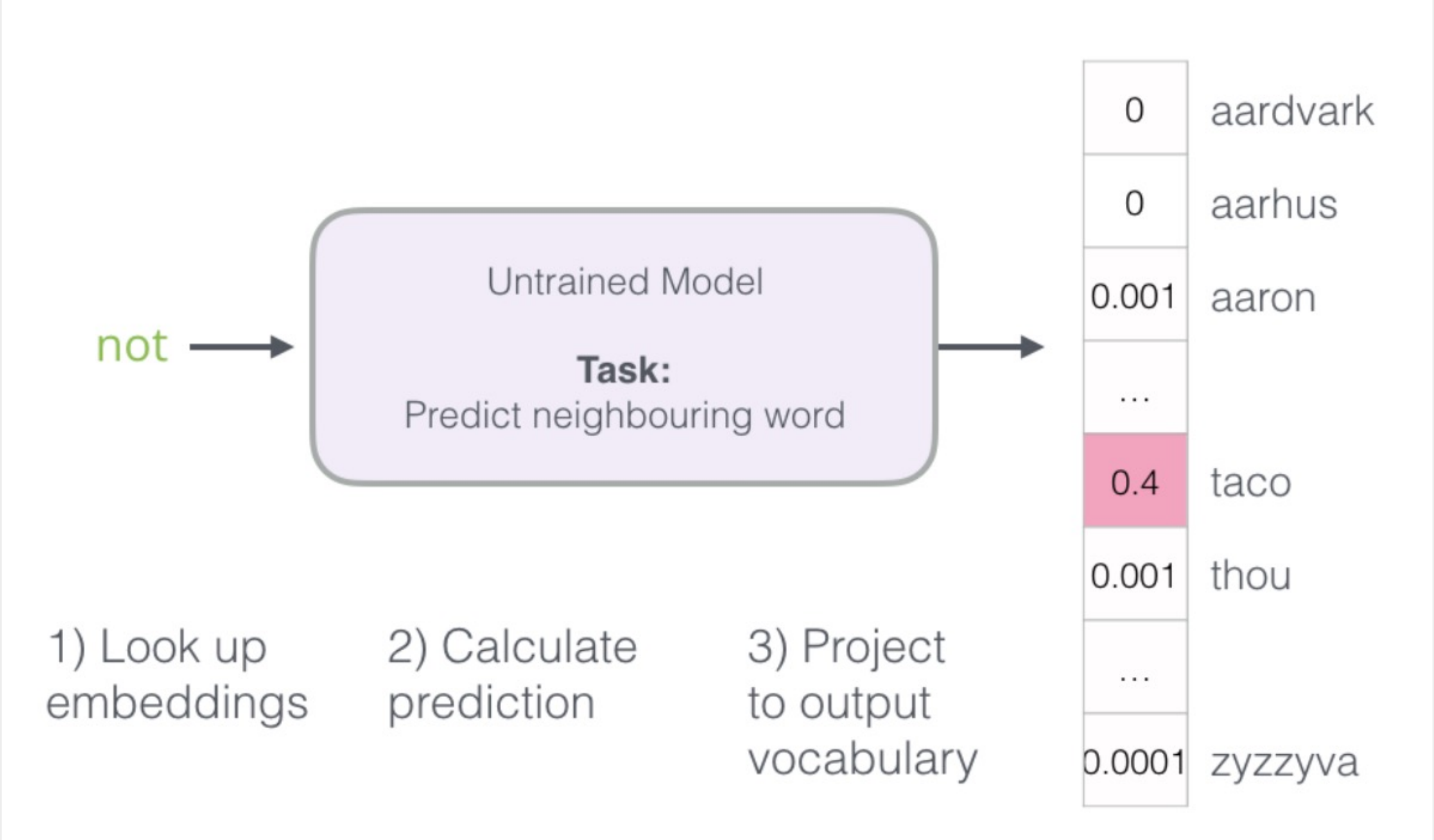
Collecting Training Examples; Slide due to Jay Alammar: https://jalammar.github.io/illustrated-word2vec/

Thou shalt not make a machine in the likeness of a human mind

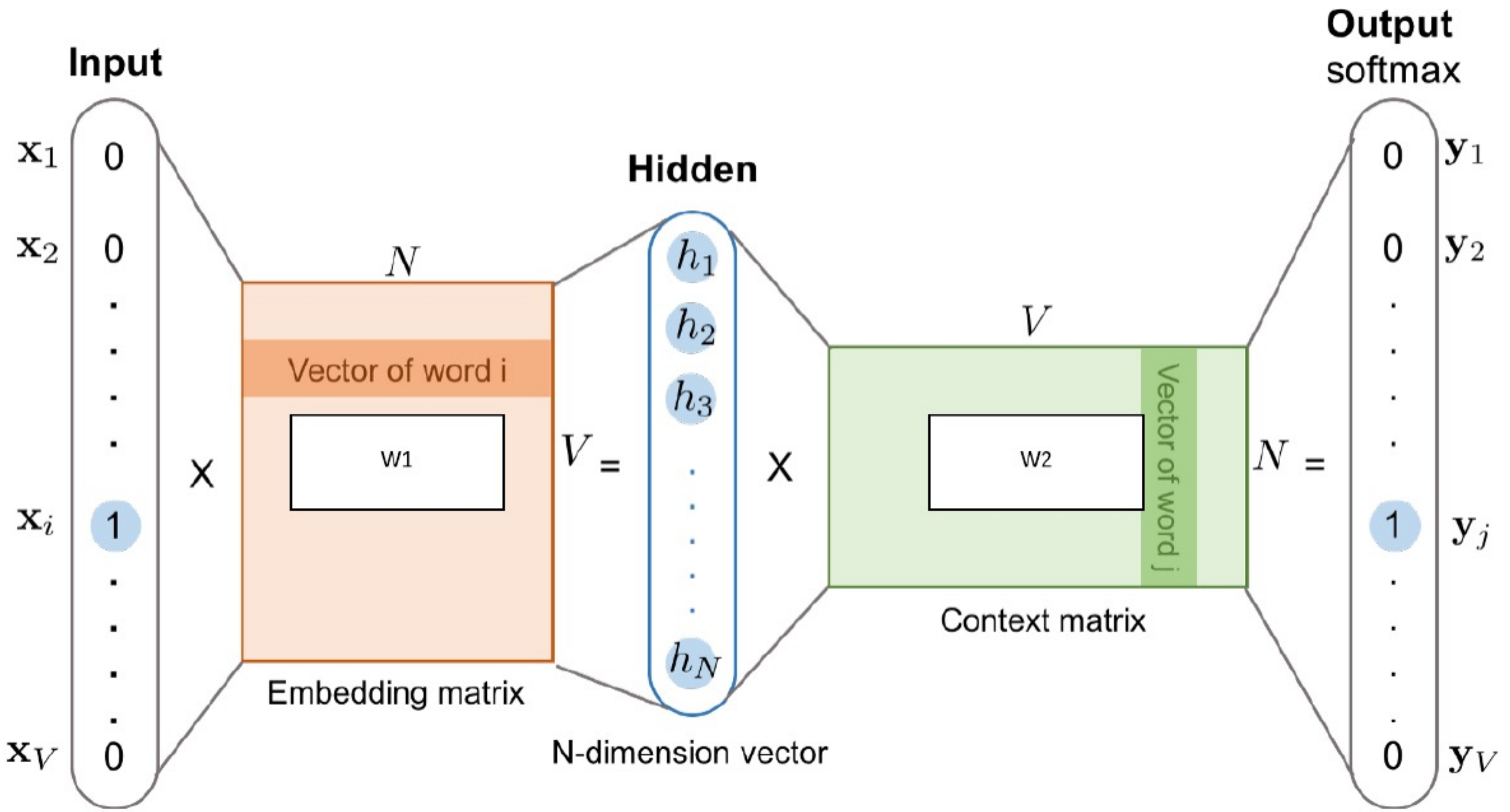


input word	target word
not	thou
not	shalt
not	make
not	a
make	shalt
make	not
make	a
make	machine
a	not
a	make
a	machine
a	in
machine	make
machine	a
machine	in
machine	the
in	a
in	machine
in	the
in	likeness

Training an Embedding Model ; Slide due to Jay Alammar: https://jalammar.github.io/illustrated-word2vec/

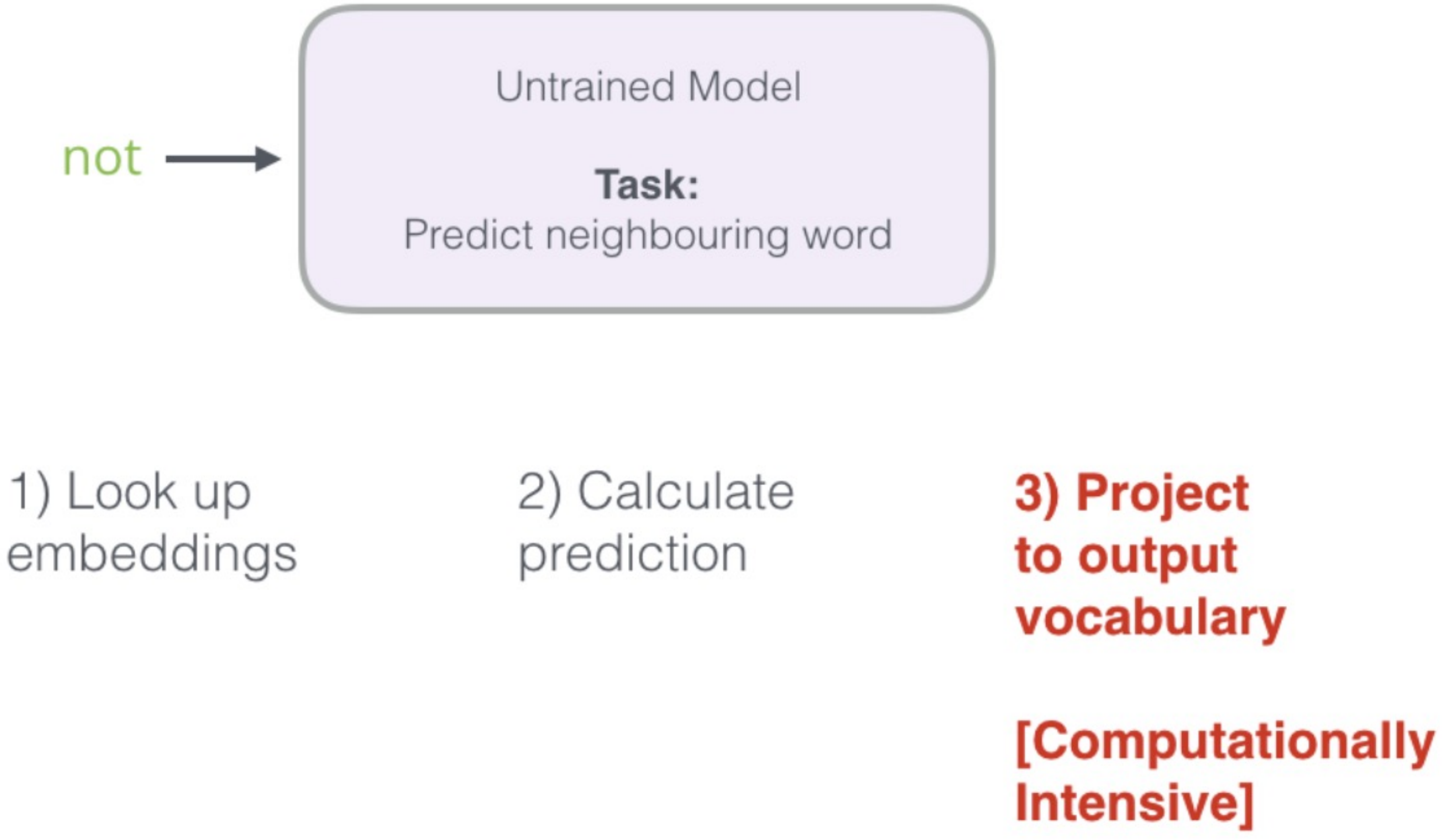


Word2Vec Architecture with a Hidden Layer



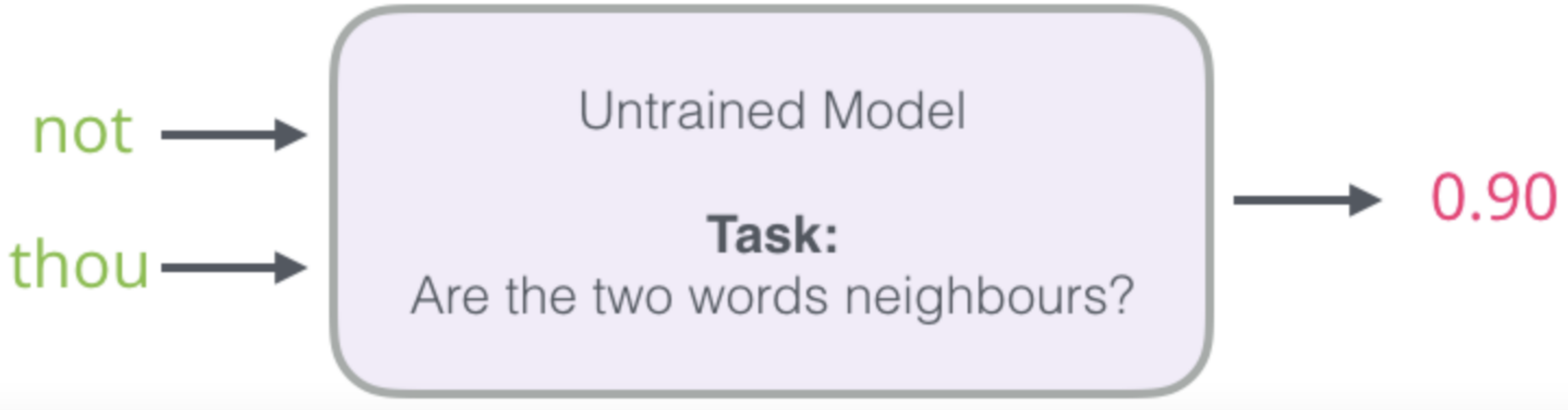
Change task for training; Slide due to Jay Alammar: https://jalammar.github.io/illustrated-word2vec/

From:



Change task for training; Slide due to Jay Alammar: https://jalammar.github.io/illustrated-word2vec/

To:



Approach: predict if candidate word *c* is a "neighbor"

1. Treat the target word *t* and a neighboring context word *c* as **positive examples**.
2. Randomly sample other words in the lexicon to get negative examples
3. Use logistic regression to train a classifier to distinguish those two cases
4. Use the learned weights as the embeddings

Skip-Gram Training Data

Assume a +/- 2 word window, given training sentence:

…lemon, a [tablespoon of apricot jam, a] pinch…

c1 c2 [target] c3 c4

Skip-Gram Training data

…lemon, a [tablespoon of apricot jam, a] pinch…

c1 c2 [target] c3 c4

positive examples +

t	c
apricot	tablespoon
apricot	of
apricot	jam
apricot	a

Skip-Gram Training data

…lemon, a [tablespoon of apricot jam, a] pinch…

c1 c2 [target] c3 c4

positive examples +

t	c
apricot	tablespoon
apricot	of
apricot	jam
apricot	a

For each positive example we'll grab k negative examples, sampling by frequency

Skip-Gram Training data

…lemon, a [tablespoon of apricot jam, a] pinch…

c1 c2 [target] c3 c4

positive examples +

t	c
apricot	tablespoon
apricot	of
apricot	jam
apricot	a

negative examples -

t	c	t	c
apricot	aardvark	apricot	seven
apricot	my	apricot	forever
apricot	where	apricot	dear
apricot	coaxial	apricot	if

Skip-Gram Classifier

(assuming a +/- 2 word window)

…lemon, a [tablespoon of apricot jam, a] pinch…

c1 c2 [target] c3 c4

Goal: train a classifier that is given a candidate (**w**ord, **c**ontext) pair

(apricot, jam)

(apricot, aardvark)

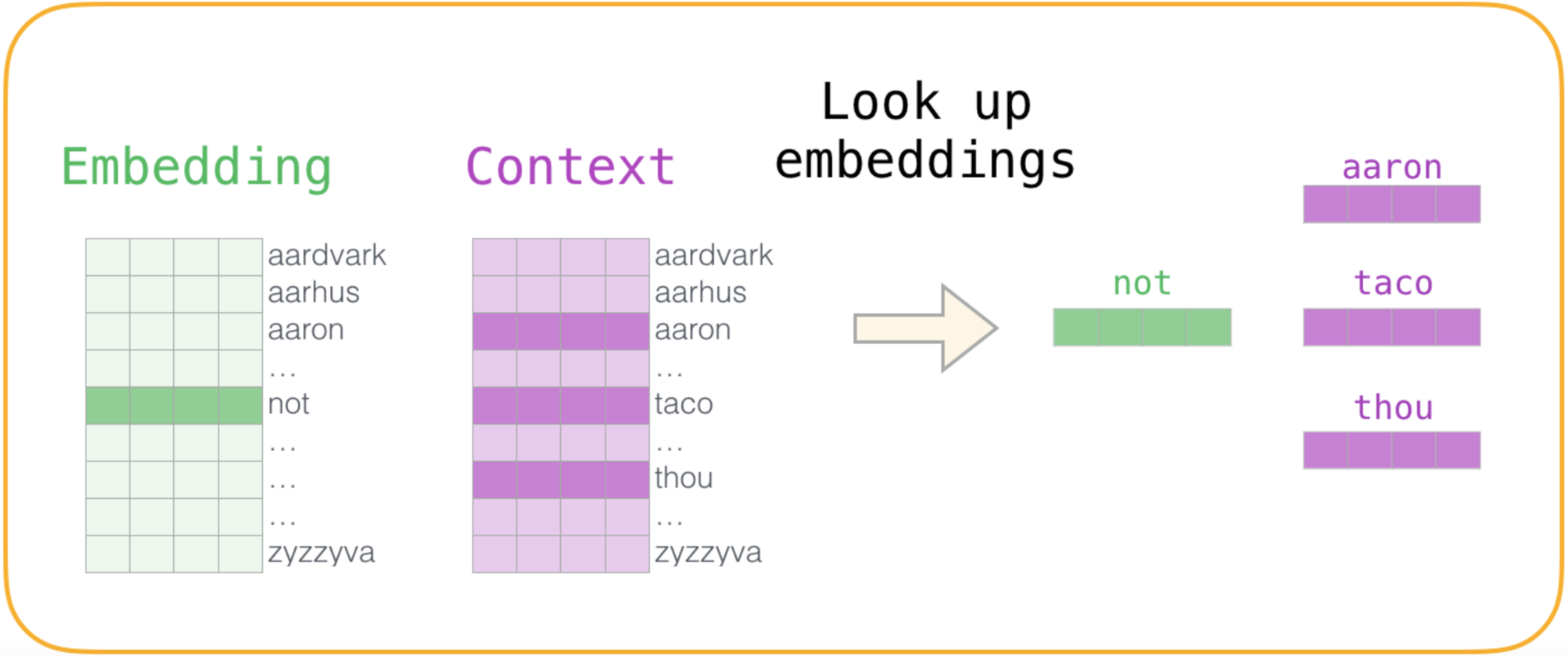
…

And assigns each pair a probability:

$P(+|w, c)$

$P(-|w, c) = 1 - P(+|w, c)$

Word2Vec Training Process



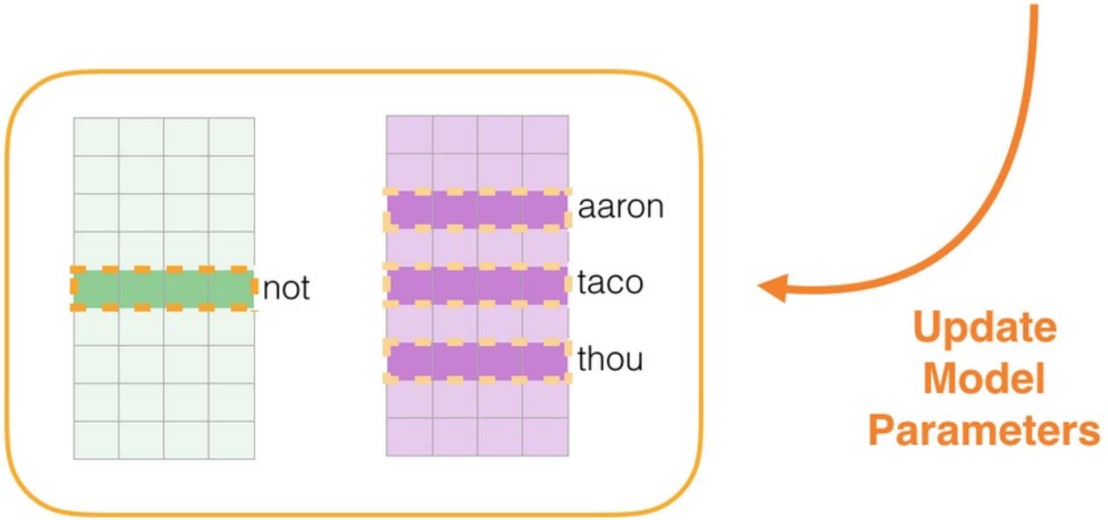
Skigram with Negative Sampling (SGNS); Slide due to Jay Alammar: https://jalammar.github.io/illustrated-word2vec/

input word	output word	target	input • output	sigmoid()	Error
not	thou	1	0.2	0.55	0.45
not	aaron	0	-1.11	0.25	-0.25
not	taco	0	0.74	0.68	-0.68

error = target - sigmoid_scores

Training

input word	output word	target	input • output	sigmoid()	Error
not	thou	1	0.2	0.55	0.45
not	aaron	0	-1.11	0.25	-0.25
not	taco	0	0.74	0.68	-0.68



Similarity is computed from dot product

Remember: two vectors are similar if they have a high dot product

- Cosine is just a normalized dot product

So:

- Similarity(w,c) $\propto w \cdot c$

We'll need to normalize to get a probability

- (cosine isn't a probability either)

Turning dot products into probabilities

$\mathrm{Sim}(\mathrm{w},\mathrm{c}) \approx \mathrm{w} \cdot c$

To turn this into a probability

We'll use the sigmoid from logistic regression:

$$P(+|w,c) = \sigma(c \cdot w) = \frac{1}{1+\exp(-c \cdot w)}$$

$$\begin{aligned} P(-|w,c) &= 1 - P(+|w,c) \\ &= \sigma(-c \cdot w) = \frac{1}{1+\exp(c \cdot w)} \end{aligned}$$

How Skip-Gram Classifier computes $P(+|w, c)$

$$P(+|w,c) = \sigma(c \cdot w) = \frac{1}{1+\exp(-c \cdot w)}$$

This is for one context word, but we have lots of context words.
We'll assume independence and just multiply them:

$$P(+|w,c_{1:L}) = \prod_{i=1}^{L} \sigma(c_i \cdot w)$$

$$\log P(+|w,c_{1:L}) = \sum_{i=1}^{L} \log \sigma(c_i \cdot w)$$

Skip-gram classifier: summary

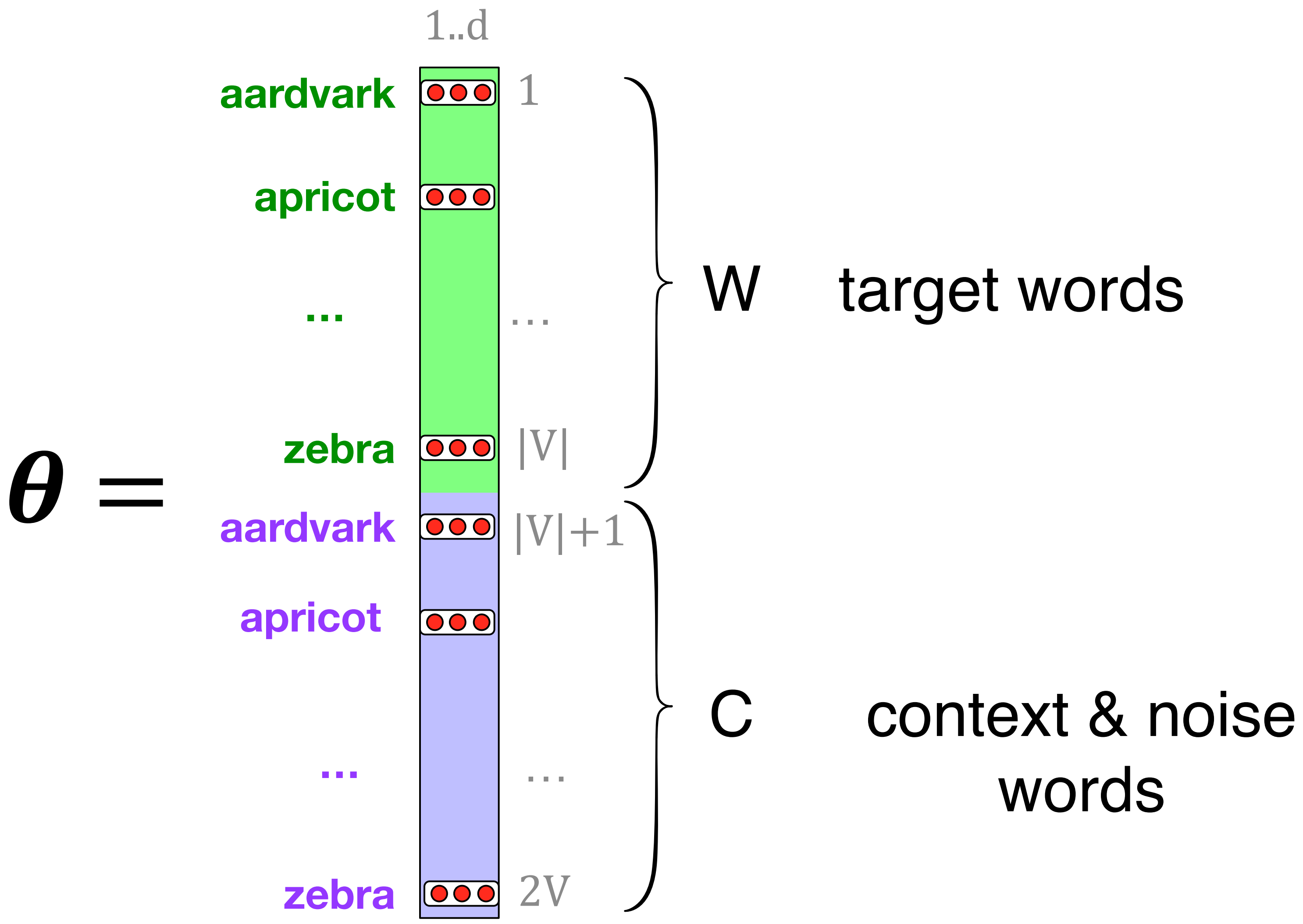
A probabilistic classifier, given

- a test target word *w*
- its context window of *L* words $c_{1:L}$

Estimates probability that w occurs in this window based on similarity of w (embeddings) to $c_{1:L}$ (embeddings).

To compute this, we just need embeddings for all the words.

These embeddings we'll need: a set for w, a set for c



Vector Semantics & Embeddings

Word2vec

Vector Semantics & Embeddings

Word2vec: Learning the embeddings

Vector Semantics & Embeddings

A quick reminder about binary logistic regression

Summary of Logistic regression

- Logistic regression is a supervised machine learning classifier that extracts real-valued **features from the input, multiplies each by a weight, sums them, and passes the sum through a sigmoid function** to generate a probability. A threshold is used to make a decision.

- Logistic regression can be used **with two classes** (e.g., positive and negative sentiment) **or with multiple classes** (multinomial logistic regression, for example for n-ary text classification, part-of-speech labeling, etc.).

Summary of Logistic regression

- Multinomial logistic regression uses the **softmax function** to compute probabilities.
- The weights (vector w and bias b) are learned from a labeled training set via a **loss function**, such as the **cross-entropy loss**, that **must be minimized**.
- Minimizing this loss function is a convex optimization problem, and iterative algorithms like **gradient descent** are used to find the optimal weights.

Word2vec: how to learn vectors

Given the set of positive and negative training instances, and an initial set of embedding vectors

The goal of learning is to adjust those word vectors such that we:

- **Maximize** the similarity of the target word, context word pairs (w , c_{pos}) drawn from the positive data
- **Minimize** the similarity of the (w , c_{neg}) pairs drawn from the negative data.

Loss function for one *w with* c_{pos}, c_{neg1} ...c_{negk}

Maximize the similarity of the target with the actual context words, and minimize the similarity of the target with the *k* negative sampled non-neighbor words.

$$\begin{aligned}
L_{\mathrm{CE}} &= -\log\left[P(+|w,c_{pos})\prod_{i=1}^{k}P(-|w,c_{neg_i})\right] \\
&= -\left[\log P(+|w,c_{pos}) + \sum_{i=1}^{k}\log P(-|w,c_{neg_i})\right] \\
&= -\left[\log P(+|w,c_{pos}) + \sum_{i=1}^{k}\log\left(1-P(+|w,c_{neg_i})\right)\right] \\
&= -\left[\log\sigma(c_{pos}\cdot w) + \sum_{i=1}^{k}\log\sigma(-c_{neg_i}\cdot w)\right]
\end{aligned}$$

Learning the classifier

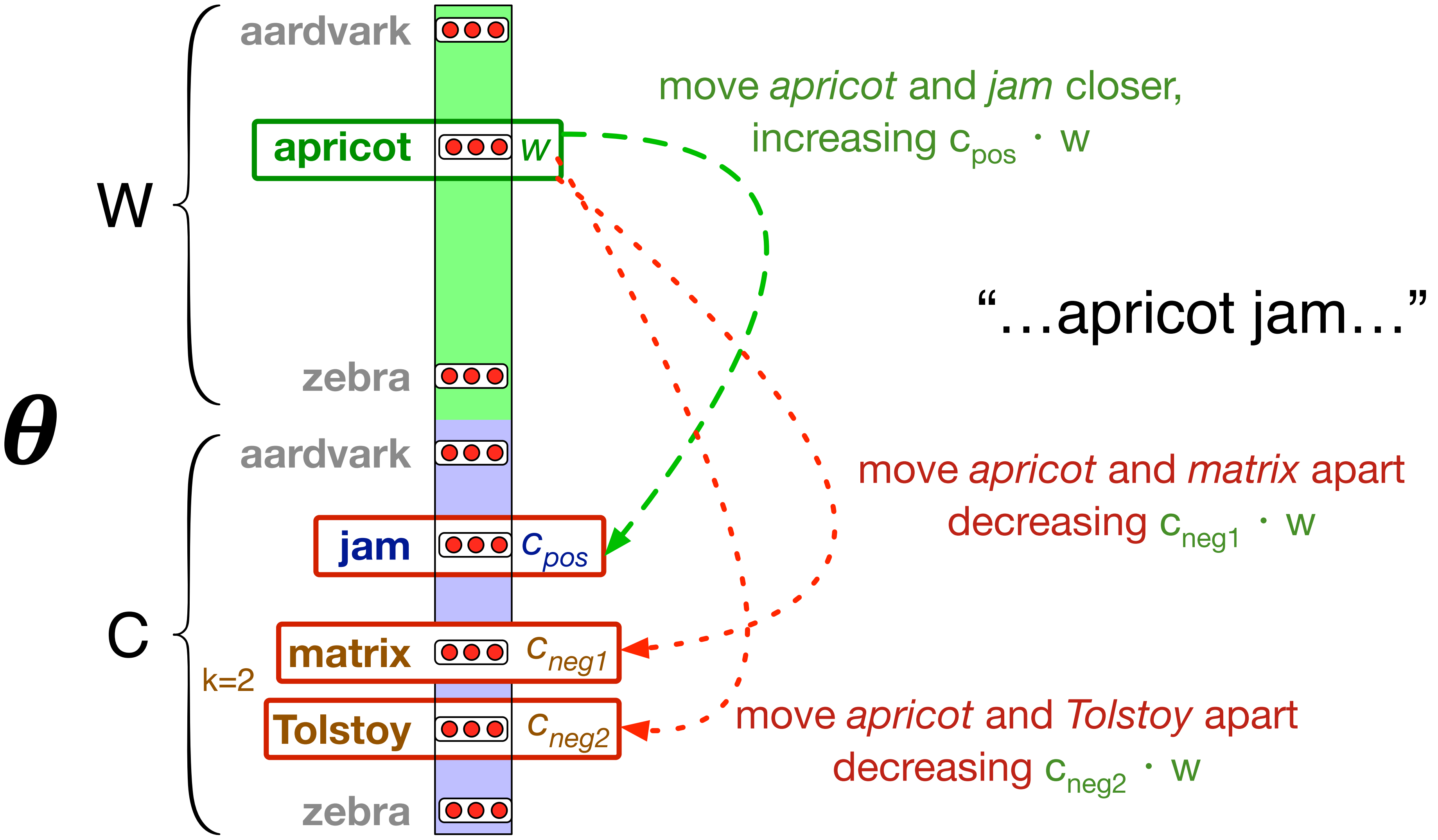
How to learn?

- Stochastic gradient descent!

We'll adjust the word weights to

- make the positive pairs more likely
- and the negative pairs less likely,
- over the entire training set.

Intuition of one step of gradient descent



Reminder: gradient descent

- At each step
  - Direction: We move in the reverse direction from the gradient of the loss function
  - Magnitude: we move the value of this gradient $\frac{d}{dw}L(f(x;w),y)$ weighted by a **learning rate** η
  - Higher learning rate means move *w* faster

$$w^{t+1} = w^t - \eta \frac{d}{dw} L(f(x;w),y)$$

The derivatives of the loss function

$$L_{\mathrm{CE}} = -\left[\log \sigma(c_{pos} \cdot w) + \sum_{i=1}^{k} \log \sigma(-c_{neg_i} \cdot w)\right]$$

$$\frac{\partial L_{CE}}{\partial c_{pos}} = [\sigma(c_{pos} \cdot w) - 1]w$$

$$\frac{\partial L_{CE}}{\partial c_{neg}} = [\sigma(c_{neg} \cdot w)]w$$

$$\frac{\partial L_{CE}}{\partial w} = [\sigma(c_{pos} \cdot w) - 1]c_{pos} + \sum_{i=1}^{k} [\sigma(c_{\mathrm{neg}_i} \cdot w)]c_{neg_i}$$

Update equation in SGD

Start with randomly initialized C and W matrices, then incrementally do updates

$$
\begin{aligned}
c_{pos}^{t+1} &= c_{pos}^{t} - \eta[\sigma(c_{pos}^{t} \cdot w^t) - 1]w^t \\
c_{neg}^{t+1} &= c_{neg}^{t} - \eta[\sigma(c_{neg}^{t} \cdot w^t)]w^t \\
w^{t+1} &= w^t - \eta\left[[\sigma(c_{pos} \cdot w^t) - 1]c_{pos} + \sum_{i=1}^{k}[\sigma(c_{\mathrm{neg}_i} \cdot w^t)]c_{neg_i}\right]
\end{aligned}
$$

Two sets of embeddings

SGNS learns two sets of embeddings

Target embeddings matrix W

Context embedding matrix C

It's common to just add them together, representing word *i* as the vector $\mathrm{w_i} + \mathrm{c_i}$

Summary: How to learn word2vec (skip-gram) embeddings

Start with V random d-dimensional vectors as initial embeddings

Train a classifier based on embedding similarity

- Take a corpus and take pairs of words that co-occur as positive examples
- Take pairs of words that don't co-occur as negative examples
- Train the classifier to distinguish these by slowly adjusting all the embeddings to improve the classifier performance
- Throw away the classifier code and keep the embeddings.

Vector Semantics & Embeddings

Word2vec: Learning the embeddings

Vector Semantics & Embeddings

Properties of Embeddings

The kinds of neighbors depend on window size

Small windows (C= +/- 2) : nearest words are syntactically similar words in same taxonomy

- *Hogwarts* nearest neighbors are other fictional schools
  - *Sunnydale, Evernight, Blandings*

Large windows (C= +/- 5) : nearest words are related words in same semantic field

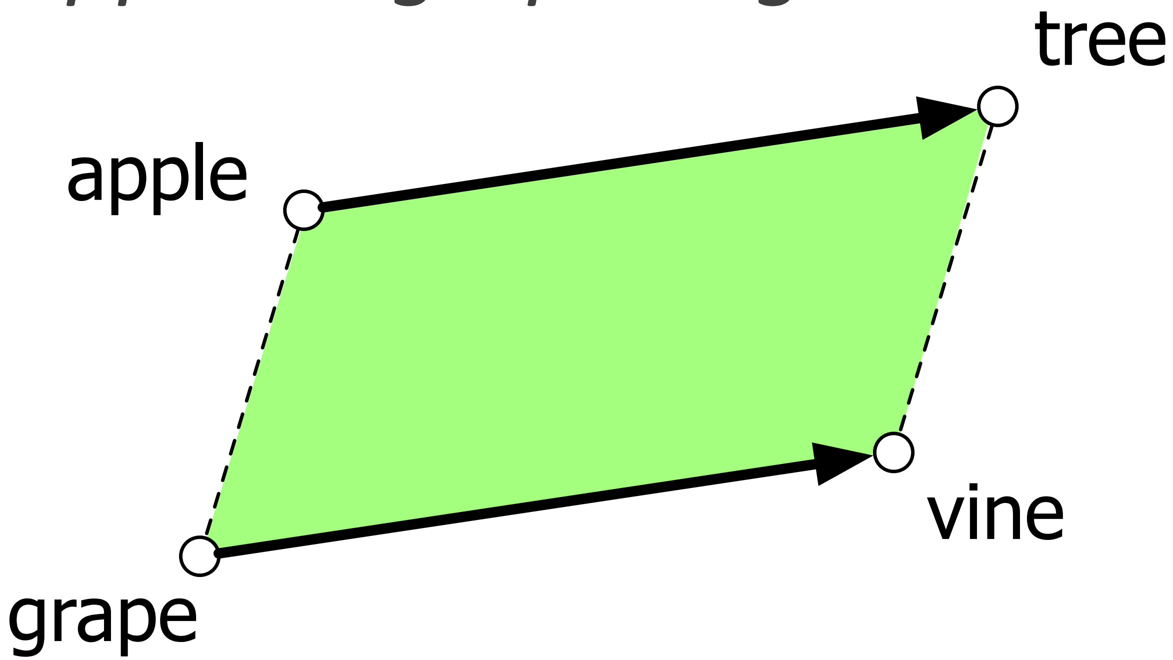
- *Hogwarts* nearest neighbors are Harry Potter world:
  - *Dumbledore, half-blood, Malfoy*

Analogical relations

The classic parallelogram model of analogical reasoning (Rumelhart and Abrahamson 1973)

To solve: *"apple is to tree as grape is to _____"*

Add $\overrightarrow{tree} - \overrightarrow{apple}$ *to* $\overrightarrow{grape}$ *to get* $\overrightarrow{vine}$



Analogical relations via parallelogram

The parallelogram method can solve analogies with both sparse and dense embeddings (Turney and Littman 2005, Mikolov et al. 2013b)

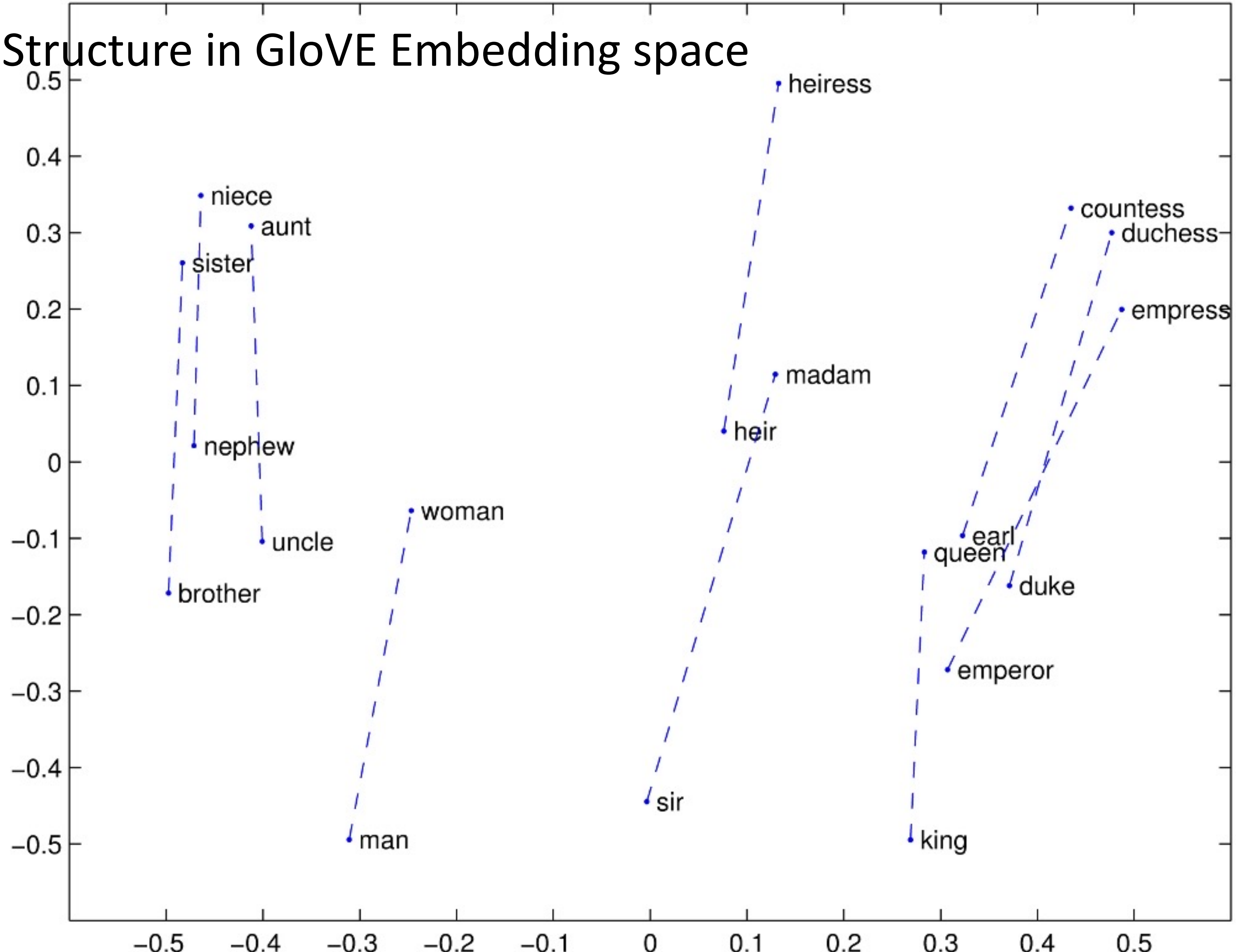
$\overrightarrow{\text{king}} - \overrightarrow{\text{man}} + \overrightarrow{\text{woman}}$ is close to $\overrightarrow{\text{queen}}$

$\overrightarrow{\text{Paris}} - \overrightarrow{\text{France}} + \overrightarrow{\text{Italy}}$ is close to $\overrightarrow{\text{Rome}}$

For a problem a:a*::b:b*, the parallelogram method is:

$$\hat{b}^* = \underset{x}{\operatorname{argmax}} \ \operatorname{distance}(x, a^* - a + b)$$

Structure in GloVE Embedding space



Caveats with the parallelogram method

It only seems to work for frequent words, small distances and certain relations (relating countries to capitals, or parts of speech), but not others. (Linzen 2016, Gladkova et al. 2016, Ethayarajh et al. 2019a)

Understanding analogy is an open area of research (Peterson et al. 2020)

Embeddings as a window onto historical semantics

Train embeddings on different decades of historical text to see meanings shift

~30 million books, 1850-1990, Google Books data

William L. Hamilton, Jure Leskovec, and Dan Jurafsky. 2016. Diachronic Word Embeddings Reveal Statistical Laws of Semantic Change. Proceedings of ACL.

Embeddings reflect cultural bias!

Bolukbasi, Tolga, Kai-Wei Chang, James Y. Zou, Venkatesh Saligrama, and Adam T. Kalai. "Man is to computer programmer as woman is to homemaker? debiasing word embeddings." In *NeurIPS*, pp. 4349-4357. 2016.

Ask "Paris : France :: Tokyo : x"

- x = Japan

Ask "father : doctor :: mother : x"

- x = nurse

Ask "man : computer programmer :: woman : x"

- x = homemaker

Algorithms that use embeddings as part of e.g., hiring searches for programmers, might lead to bias in hiring

Historical embedding as a tool to study cultural biases

Garg, N., Schiebinger, L., Jurafsky, D., and Zou, J. (2018). Word embeddings quantify 100 years of gender and ethnic stereotypes. Proceedings of the National Academy of Sciences 115(16), E3635–E3644.

- Compute a **gender or ethnic bias** for each adjective: e.g., how much closer the adjective is to "woman" synonyms than "man" synonyms, or names of particular ethnicities
  - Embeddings for **competence** adjective (*smart, wise, brilliant, resourceful, thoughtful, logical)* are biased toward men, a bias slowly decreasing 1960-1990
  - Embeddings for **dehumanizing** adjectives (barbaric, monstrous, bizarre) were biased toward Asians in the 1930s, bias decreasing over the 20th century.
- These match the results of old surveys done in the 1930s

Vector Semantics & Embeddings

Properties of Embeddings