Recursion

Reading:
Savitch ch. 11

Introduction

- Sometimes it is useful to define a method in terms of itself.
- A method definition is *recursive* if it contains a call to itself.
- The method continues to call itself with ever simpler cases, until a *base case* is reached which can be resolved without any recursive calls.

Example – Searching a phone book

- Open the book to the middle.
- If the name is on that page, you're done.
- If the name comes before the names on the page, use the same approach to search for the name in the first half of the phone book.
- Otherwise use the same approach to search for the name in the second half of the phone book.

Case Study – Digits to Words
Savitch p 783

- Write a method definition **`displayAsWords`** that accepts a single **`int`** and produces words representing its digits.
- Example:

  Input: **`223`**

  Output: **`two two three`**
- Recursive algorithm

  - output all but the last digit as words

  - output the word for the last digit

Case Study – Digits to Words

- Use a helper method **`getWordFromDigit`** to get the word for a number < 10 (“one”, “two”,…).
- To “chop off” the last digit, use integer division by 10

get rid of

```
(327 / 10)      // 32
```

- To get just the last digit, use % 10

```
(327 % 10)      // 7
```

- See **`RecursionDemo.java`** under SavitchSrc on the course website

Case Study – Digits to Words

- The ***base case*** is when number is a single digit **`(number < 10)`**.
- In this case, we just print the String returned by **`getWordFromDigit`**

```
System.out.print(getWordFromDigit(number) + " ");
```

Case Study – Digits to Words

- If **`(number >= 10)`** we call **`displayAsWords`** with all except the last digit, then print the last digit.

```
displayAsWords(number/10); // last digit removed
System.out.print(getWordFromDigit(number%10) + “ “);
```

How Recursion Works

- Nothing special is required to handle a call to a recursive method.
- At each call, the needed arguments are provided and the code is executed.
- When the method completes, control returns to the statement following the call to the method.
- See example (987) on the following slides

```
public void displayAsWords(int 987) {
   if (987 < 10) {
      System.out.print(getWordFromDigit(987) + “ “);
   } else {                    98
      displayAsWords(987/10);
      System.out.print(getWordFromDigit(987%10) + “ “);
   }
}
```

```
public void displayAsWords(int 987) {
   if (987 < 10) {
      System.out.print(getWordFromDigit(987) + " ");
   } else {                 98
      displayAsWords(987/10);
      System.out.print(getWordFromDigit(987%10) + " ");
   }
}
public void displayAsWords(int 98) {
   if (98 < 10) {
      System.out.print(getWordFromDigit(98) + " ");
   } else {                 9
      displayAsWords(98/10);
      System.out.print(getWordFromDigit(98%10) + " ");
   }
}
```

```
public void displayAsWords(int 987) {
   if (987 < 10) {
      System.out.print(getWordFromDigit(987) + " ");
   } else {                        98
      displayAsWords(987/10);
      System.out.print(getWordFromDigit(987%10) + " ");
   }
}
public void displayAsWords(int 98) {
   if (98 < 10) {
      System.out.print(getWordFromDigit(98) + " ");
   } else {                        9
      displayAsWords(98/10);
      System.out.print(getWordFromDigit(98%10) + " ");
   }
}
public void displayAsWords(int 9) {
   if (9 < 10) {
      System.out.print(getWordFromDigit(9) + " ");
   } else {
      displayAsWords(9/10);    nine
      System.out.print(getWordFromDigit(9%10) + " ");
   }
}
```

```
public void displayAsWords(int 987) {
    if (987 < 10) {
        System.out.print(getWordFromDigit(987) + " ");
    } else {                          98
        displayAsWords(987/10);
        System.out.print(getWordFromDigit(987%10) + " ");
    }
}
public void displayAsWords(int 98) {
    if (98 < 10) {
        System.out.print(getWordFromDigit(98) + " ");
    } else {                          9
        displayAsWords(98/10);                              8
        System.out.print(getWordFromDigit(98%10) + " ");
    }                                          eight
}
public void displayAsWords(int 9) {
    if (9 < 10) {
        System.out.print(getWordFromDigit(9) + " ");
    } else {
        displayAsWords(9/10);       nine
        System.out.print(getWordFromDigit(9%10) + " ");
    }
}
```

```
public void displayAsWords(int 987) {
    if (987 < 10) {
        System.out.print(getWordFromDigit(987) + “ “);
    } else {                        98
        displayAsWords(987/10);                        7
        System.out.print(getWordFromDigit(987%10) + “ “);
    }
}                                         seven
public void displayAsWords(int 98) {
    if (98 < 10) {
        System.out.print(getWordFromDigit(98) + “ “);
    } else {                        9
        displayAsWords(98/10);                        8
        System.out.print(getWordFromDigit(98%10) + “ “);
    }
}                                         eight
public void displayAsWords(int 9) {
    if (9 < 10) {
        System.out.print(getWordFromDigit(9) + “ “);
    } else {
        displayAsWords(9/10);     nine
        System.out.print(getWordFromDigit(9%10) + “ “);
    }
}
```

Recursion Guidelines

- The definition of a recursive method typically includes an `if-else` statement.
    - One branch represents a **base case** which can be solved directly – without recursion.
    - Another branch includes a **recursive call** to the method, but with simpler or smaller arguments.
- The base case must be reached eventually.

Recursion Guidelines

- If the recursive call inside the method does not use a smaller or simpler argument, the base case may never be reached.
- The method then calls itself forever (or until resources run out).
- This is called *infinite recursion*.

Recursion vs. Iteration

- Any recursive method can be written without using recursion.
- Typically this is done with a loop.
- The resulting method is called the *iterative version*.
- See **`RecursionDemo.java`** and **`IterativeDemo.java`** under SavitchSrc on the course website

Recursion vs. Iteration

- A recursive version of a method typically executes <u>less efficiently</u> than the iterative version.
- This is because <u>the computer must keep track of the recursive calls and the suspended computations</u>.
- However, sometimes it is <u>much easier to write</u> a method recursively than iteratively.

Exercise 1

Self-Test Question 1, page 792. What is the output?

```
public class RecursionExercise {
    public static void main(String[] args) {
        methodA(3);
    }

    public static void methodA(int n) {
        if (n < 1)
            System.out.println(“B”);
        else {
            methodA(n – 1);
            System.out.println(“R”);
        }
    }
}
```

```
public static void methodA(int 3) {
    if (3 < 1) System.out.println("B");
    else {
        methodA(3 – 1);
        System.out.println("R");
    }
}
public static void methodA(int 2) {
    if (2 < 1) System.out.println("B");
    else {
        methodA(2 – 1);
        System.out.println("R");
    }
}
public static void methodA(int 1) {
    if (1 < 1) System.out.println("B");
    else {
        methodA(1 – 1);
        System.out.println("R");
    }
}
public static void methodA(int 0) {
    if (0 < 1) System.out.println("B");  ──► B
    else {
        methodA(0 – 1);
        System.out.println("R");
    }
}
```

Output:
B

recursion

```
public static void methodA(int 3) {
     if (3 < 1) System.out.println(“B”);
     else {
          methodA(3 – 1);
          System.out.println(“R”);
     }
}
public static void methodA(int 2) {
     if (2 < 1) System.out.println(“B”);
     else {
          methodA(2 – 1);
          System.out.println(“R”);
     }
}
public static void methodA(int 1) {
     if (1 < 1) System.out.println(“B”);
     else {
          methodA(1 – 1);
          System.out.println(“R”);  ──► R
     }
}
public static void methodA(int 0) {
     if (0 < 1) System.out.println(“B”);  ──► B
     else {
          methodA(0 – 1);
          System.out.println(“R”);
     }
}
```

Output:
B
R

unwing

```
public static void methodA(int 3) {
     if (3 < 1) System.out.println("B");
     else {
          methodA(3 – 1);
          System.out.println("R");
     }
}
public static void methodA(int 2) {
     if (2 < 1) System.out.println("B");
     else {
          methodA(2 – 1);
          System.out.println("R");  → R
     }
}
public static void methodA(int 1) {
     if (1 < 1) System.out.println("B");
     else {
          methodA(1 – 1);
          System.out.println("R");  → R
     }
}
public static void methodA(int 0) {
     if (0 < 1) System.out.println("B");  → B
     else {
          methodA(0 – 1);
          System.out.println("R");
     }
}
```

Output:

B

R

R

unwing

```
public static void methodA(int 3) {
     if (3 < 1) System.out.println("B");
     else {
          methodA(3 – 1);
          System.out.println("R");        R
     }
}
public static void methodA(int 2) {
     if (2 < 1) System.out.println("B");
     else {
          methodA(2 – 1);
          System.out.println("R");        R
     }
}
public static void methodA(int 1) {
     if (1 < 1) System.out.println("B");
     else {
          methodA(1 – 1);
          System.out.println("R");        R
     }
}
public static void methodA(int 0) {
     if (0 < 1) System.out.println("B");        B
     else {
          methodA(0 – 1);
          System.out.println("R");
     }
}
```

Output:

B

R

R

R

unwing

Exercise 2

What is the output?

```
public class RecursionExercise2 {
    public static void main(String[] args) {
        methodB(3);
    }

    public static void methodB(int n) {
        if (n < 1)
            System.out.println(“done”);
        else {
            System.out.println(n);
            methodB(n – 1);
        }
    }
}
```

```
public static void methodB(int 3) {
    if (3 < 1) System.out.println("done");
    else {
        System.out.println(3); ──► 3
        methodB(3 – 1);
    }
}
public static void methodB(int 2) {
    if (2 < 1) System.out.println("done");
    else {
        System.out.println(2); ──► 2
        methodB(2 – 1);
    }
}
public static void methodB(int 1) {
    if (1 < 1) System.out.println("done");
    else {
        System.out.println(1); ──► 1
        methodB(1 – 1);
    }
}
public static void methodB(int 0) {
    if (0 < 1) System.out.println("done"); ──► done
    else {
        System.out.println(0);
        methodB(0 – 1);
    }
}
```

在这个递归过程中，没有需要
在递归返回后执行的代码，
因此每次递归调用完成后就直
接返回到上一层调用，
而不会执行额外的步骤。

Output:

```
3
2
1
done
```

only recursion.
no unwing.

Returning a Value

- So far, our recursive methods have been void methods.
- Recursive methods can also return a value.
- The value returned by the recursive method call is typically used to compute the return value of the method.
- Example: Consider a method that takes an **`int`** argument and returns the number of zeros in the argument.

Returning a Value

- Do this by getting the number of zeros contained in all but the last digit, then add 1 if the last digit is a zero.
- For example, the number of zeros in 50020 is the number of zeros in 5002 plus 1 for the last zero.
- The number of zeros in 50022 is the number of zeros in 5002 without adding 1 because the last digit is not zero.

Returning a Value

- Algorithm for determining the number of zeros in an int
  - If `n < 10`, return the number of zeros in n (base case)
    - if `n == 0`, return `1`
    - Otherwise return `0`
  - Otherwise (recursive invocation)
    - Get the number of zeros in `n` with the last digit removed (`numberOfZeros(n/10)`)
    - Determine if the last digit is zero (`n%10 == 0`)
      - If yes, return `numberOfZeros(n/10) + 1`
      - If no, return `numberOfZeros(n/10)`

Returning a Value

```
public static int numZeros(int n) {
   if (n == 0) {
      //n has one digit that is 0 (base case)
      return 1;
   } else if (n < 10) {
      //n has one digit that is not 0 (base case)
      return 0;
   } else if (n%10 == 0) {
      //last digit is 0 (recursive call)
      return (numZeros(n/10) + 1);
   } else {
      //last digit is not 0 (recursive call)
      return (numZeros(n/10));
}
```

- See example n = 300 on the following slides

Returning a Value

```
public static int numZeros(int 300) {
   if (300 == 0) return 1;
   else if (300 < 10) return 0;                   30
   else if (300%10 == 0) return (numZeros(300/10) + 1);
   else return (numZeros(300/10));
}
```

Returning a Value

```
public static int numZeros(int 300) {
   if (300 == 0) return 1;
   else if (300 < 10) return 0;                44
   else if (300%10 == 0) return (numZeros(300/10) + 1);
   else return (numZeros(300/10));
}

public static int numZeros(int 30) {
   if (30 == 0) return 1;
   else if (30 < 10) return 0;
   else if (30%10 == 0) return (numZeros(30/10) + 1);
   else return (numZeros(30/10));
}
```

Returning a Value

```
public static int numZeros(int 300) {
   if (300 == 0) return 1;
   else if (300 < 10) return 0;                30
   else if (300%10 == 0) return (numZeros(300/10) + 1);
   else return (numZeros(300/10));
}

public static int numZeros(int 30) {
   if (30 == 0) return 1;
   else if (30 < 10) return 0;                3
   else if (30%10 == 0) return (numZeros(30/10) + 1);
   else return (numZeros(30/10));
}

public static int numZeros(int 3) {
   if (3 == 0) return 1;
   else if (3 < 10) return 0;
   else if (3%10 == 0) return (numZeros(3/10) + 1);
   else return (numZeros(3/10));
}
```

Returning a Value

```
public static int numZeros(int 300) {
   if (300 == 0) return 1;
   else if (300 < 10) return 0;                   30
   else if (300%10 == 0) return (numZeros(300/10) + 1);
   else return (numZeros(300/10));
}

public static int numZeros(int 30) {
   if (30 == 0) return 1;
   else if (30 < 10) return 0;                 3
   else if (30%10 == 0) return (numZeros(30/10) + 1);
   else return (numZeros(30/10));

                                       0                + 1
public static int numZeros(int 3) {
   if (3 == 0) return 1;
   else if (3 < 10) return 0;
   else if (3%10 == 0) return (numZeros(3/10) + 1);
   else return (numZeros(3/10));
}
```

Returning a Value

```
public static int numZeros(int 300) {
    if (300 == 0) return 1;
    else if (300 < 10) return 0;                               30
    else if (300%10 == 0) return (numZeros(300/10) + 1);
    else return (numZeros(300/10));
}
                                                         1            + 1

public static int numZeros(int 30) {
    if (30 == 0) return 1;
    else if (30 < 10) return 0;                              3
    else if (30%10 == 0) return (numZeros(30/10) + 1);
    else return (numZeros(30/10));
}
                                                       0            + 1

public static int numZeros(int 3) {
    if (3 == 0) return 1;
    else if (3 < 10) return 0;
    else if (3%10 == 0) return (numZeros(3/10) + 1);
    else return (numZeros(3/10));
}
```

Exercise 3

Write a recursive method to reverse the input string:

```
public static String reverse(String s) {
    // base case



    // reverse all but the first character
    // then add the first character to the end


}
```

Exercise 3

Write a recursive method to reverse the input string:

```
public static String reverse(String s) {
  // base case: string is length 0 or 1
  if (s.length() <= 1)
       return s;

  // reverse all but the first character
  // then add the first character to the end
  return reverse(s.substring(1))
         + s.charAt(0);
}
```

- See example s = “step” on the following slides

```
public static String reverse(String “step”) {
   if (“step”.length() <= 1)
        return “step”;
   return reverse(“step”.substring(1)) + “step”.charAt(0);
}
```

$$\underbrace{\texttt{“step”.substring(1)}}_{\texttt{“tep”}}$$

```
public static String reverse(String "step") {
   if ("step".length() <= 1)
        return "step";
   return reverse("step".substring(1)) + "step".charAt(0);
}
                            "tep"
public static String reverse(String "tep") {
   if ("tep".length() <= 1)
        return "tep";
   return reverse("tep".substring(1)) + "tep".charAt(0);
}
                            "ep"
```

```
public static String reverse(String “step”) {
   if (“step”.length() <= 1)
        return “step”;
   return reverse(“step”.substring(1)) + “step”.charAt(0);
}
```

“tep”

```
public static String reverse(String “tep”) {
   if (“tep”.length() <= 1)
        return “tep”;
   return reverse(“tep”.substring(1)) + “tep”.charAt(0);
}
```

“ep”

```
public static String reverse(String “ep”) {
   if (“ep”.length() <= 1)
        return “ep”;
   return reverse(“ep”.substring(1)) + “ep”.charAt(0);
}
```

“p”

```
public static String reverse(String "step") {
   if ("step".length() <= 1)
        return "step";
   return reverse("step".substring(1)) + "step".charAt(0);
}
```

"tep"

```
public static String reverse(String "tep") {
   if ("tep".length() <= 1)
        return "tep";
   return reverse("tep".substring(1)) + "tep".charAt(0);
}
```

"ep"

```
public static String reverse(String "ep") {
   if ("ep".length() <= 1)
        return "ep";
   return reverse("ep".substring(1)) + "ep".charAt(0);
}
```

"p"

```
public static String reverse(String "p") {
   if ("p".length() <= 1)
        return "p";
   return reverse("p".substring(1)) + "p".charAt(0);
}
```

```
public static String reverse(String "step") {
   if ("step".length() <= 1)
        return "step";
   return reverse("step".substring(1)) + "step".charAt(0);
}
```

"tep"

```
public static String reverse(String "tep") {
   if ("tep".length() <= 1)
        return "tep";
   return reverse("tep".substring(1)) + "tep".charAt(0);
}
```

"ep"

```
public static String reverse(String "ep") {
   if ("ep".length() <= 1)
        return "ep";
   return reverse("ep".substring(1)) + "ep".charAt(0);
}
```

"p"   "e"

```
public static String reverse(String "p") {
   if ("p".length() <= 1)
        return "p";
   return reverse("p".substring(1)) + "p".charAt(0);
}
```

```
public static String reverse(String "step") {
   if ("step".length() <= 1)
        return "step";
   return reverse("step".substring(1)) + "step".charAt(0);
}
                          "tep"
public static String reverse(String "tep") {
   if ("tep".length() <= 1)
        return "tep";
   return reverse("tep".substring(1)) + "tep".charAt(0);
}
                       "pe"                       "t"
public static String reverse(String "ep") {
   if ("ep".length() <= 1)
        return "ep";
   return reverse("ep".substring(1)) + "ep".charAt(0);
}
                       "p"                       "e"
public static String reverse(String "p") {
   if ("p".length() <= 1)
        return "p";
   return reverse("p".substring(1)) + "p".charAt(0);
}
```

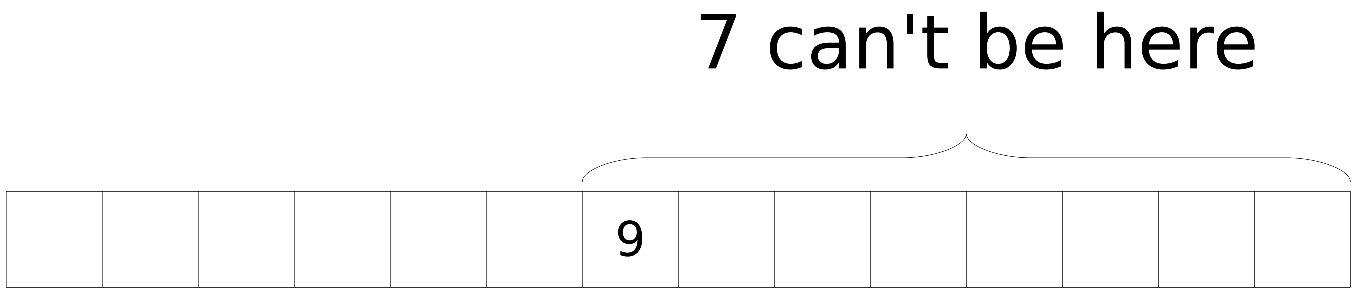
```
public static String reverse(String "step") {
   if ("step".length() <= 1)
        return "step";
   return reverse("step".substring(1)) + "step".charAt(0);
}
                     "pet"                       "s"
public static String reverse(String "tep") {
   if ("tep".length() <= 1)
        return "tep";
   return reverse("tep".substring(1)) + "tep".charAt(0);
}
                     "pe"                      "t"
public static String reverse(String "ep") {
   if ("ep".length() <= 1)
        return "ep";
   return reverse("ep".substring(1)) + "ep".charAt(0);
}
                     "p"                     "e"
public static String reverse(String "p") {
   if ("p".length() <= 1)
        return "p";
   return reverse("p".substring(1)) + "p".charAt(0);
}
```

Binary Search
Savitch p 800 (case study)

- Let's design a recursive method that determines if a given number is in a <u>sorted</u> array of ints.
- If the number is in the array, return the index of the number in the array.
- If the number is not in the array, return -1
- Instead of searching the array *linearly/sequentially* (starting at index 0, comparing each element to number), we will search for the number *recursively*.

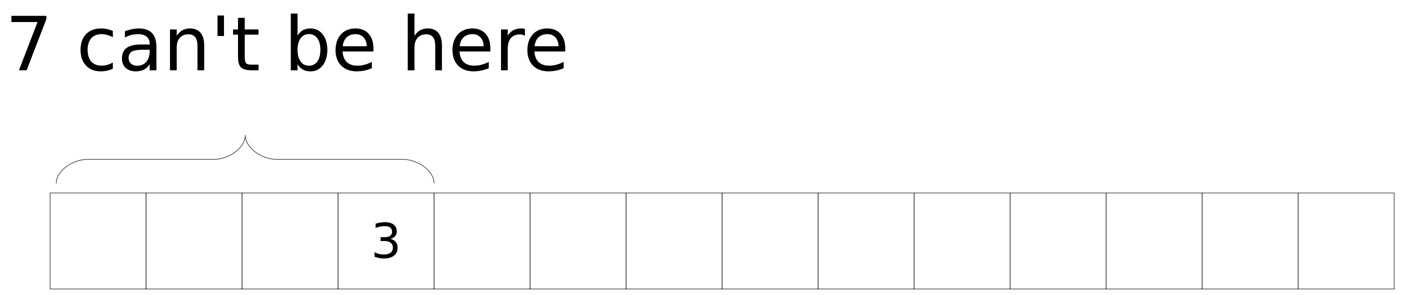
Binary Search

- Since the array is **sorted**, we can rule out entire sections of the array as we search.
- For example, if we are looking for a 7 and we encounter an element of the array containing a 9, we know that the 7 will not be at any index >= to the location of the 9.



Binary Search

- Similarly, if we are looking for a 7 and we encounter an element of the array containing a 3, we know that the 7 will not be at any index <= to the location of the 3.



- Of course, if we are looking for a 7 and find it, then we stop searching.

Binary Search

- Binary search is an example of a *divide and conquer* algorithm.
- We divide the problem into smaller and smaller problems until it can be solved.
- We use a binary search algorithm when looking up a number in the phone book.
- Flip open the phone book to a page near the middle.
- To find the middle of our array, use integer division: **`mid = (first + last)/2`**

Binary Search

- Base case: If our number is at index **`mid`** we are done
- Recursive case: If our number is smaller than the number at index **`mid`**, search the first half (from **`first`** to **`mid-1`**).
- Recursive case: If our number is greater than the number at index `mid`, search the second half (from **`mid+1`** to **`last`**).
- See **`ArraySearcher.java`** under SavitchSrc on the course website

Binary Search – Number Found

- Example: search target 33 in sorted array of length 10 (index 0-9)

```
search(33, 0, 9)      mid=4
```

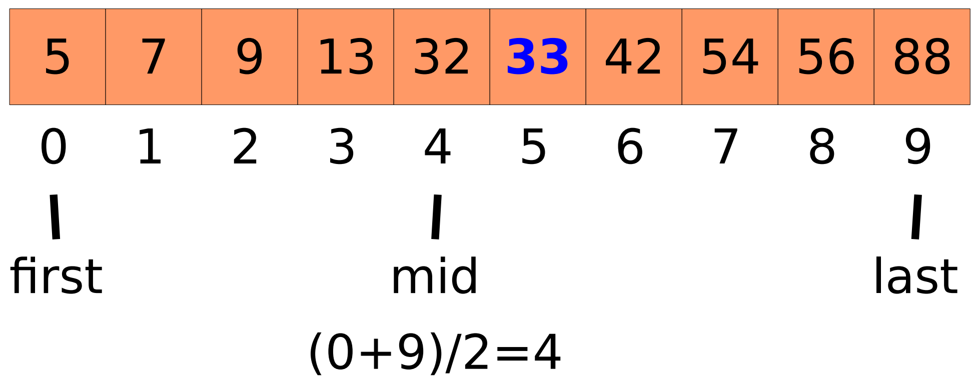
5	7	9	13	32	**33**	42	54	56	88
0	1	2	3	**4**	5	6	7	8	9

```
search(33, 5, 9)      mid=7
```

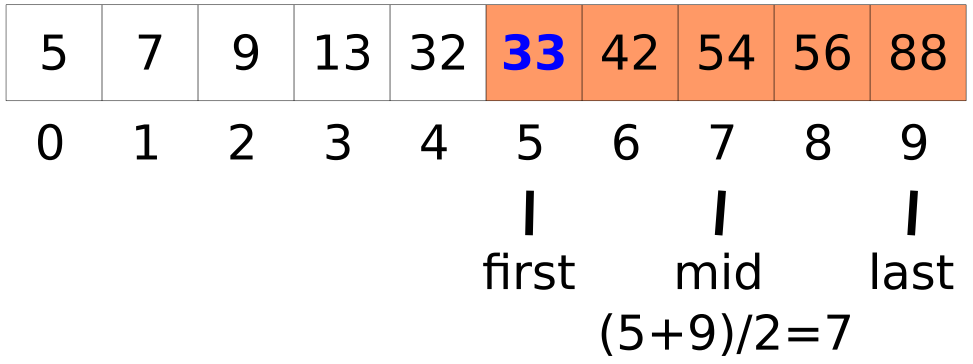
5	7	9	13	32	**33**	42	54	56	88
0	1	2	3	4	5	6	**7**	8	9

```
search(33, 5, 6)      mid=5
found at index 5
return 5;
```

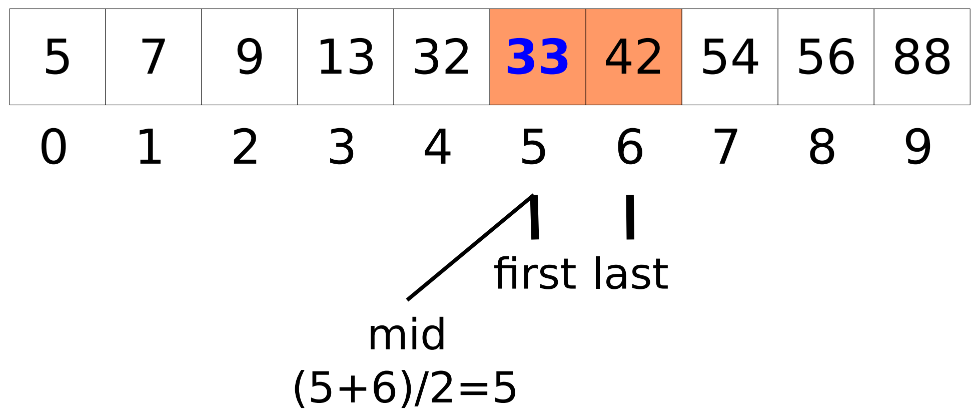
5	7	9	13	32	**33**	42	54	56	88
0	1	2	3	4	**5**	6	7	8	9



```
private int search(target 33, first 0, last 9) {
   int mid, result=-1;
   if (first <= last) {
       mid = (first + last)/2;        ⟶   mid = (0+9)/2 = 4
       if (target == a[mid]) {
           result = mid;
       } else if (target < a[mid]) {
           result = search(target, first, mid-1);
       } else {
           result = search(target, mid+1, last);
       }                           33,      5,      9
   }
   return result;
}
```



```
private int search(target 33, first 5, last 9) {
   int mid, result=-1;
   if (first <= last) {
      mid = (first + last)/2;   ⟶   mid = (5+9)/2 = 7
      if (target == a[mid]) {
         result = mid;
      } else if (target < a[mid]) {
         result = search(target, first, mid-1);
                         33,     5,     6
      } else {
         result = search(target, mid+1, last);
      }
   }
   return result;
}
```



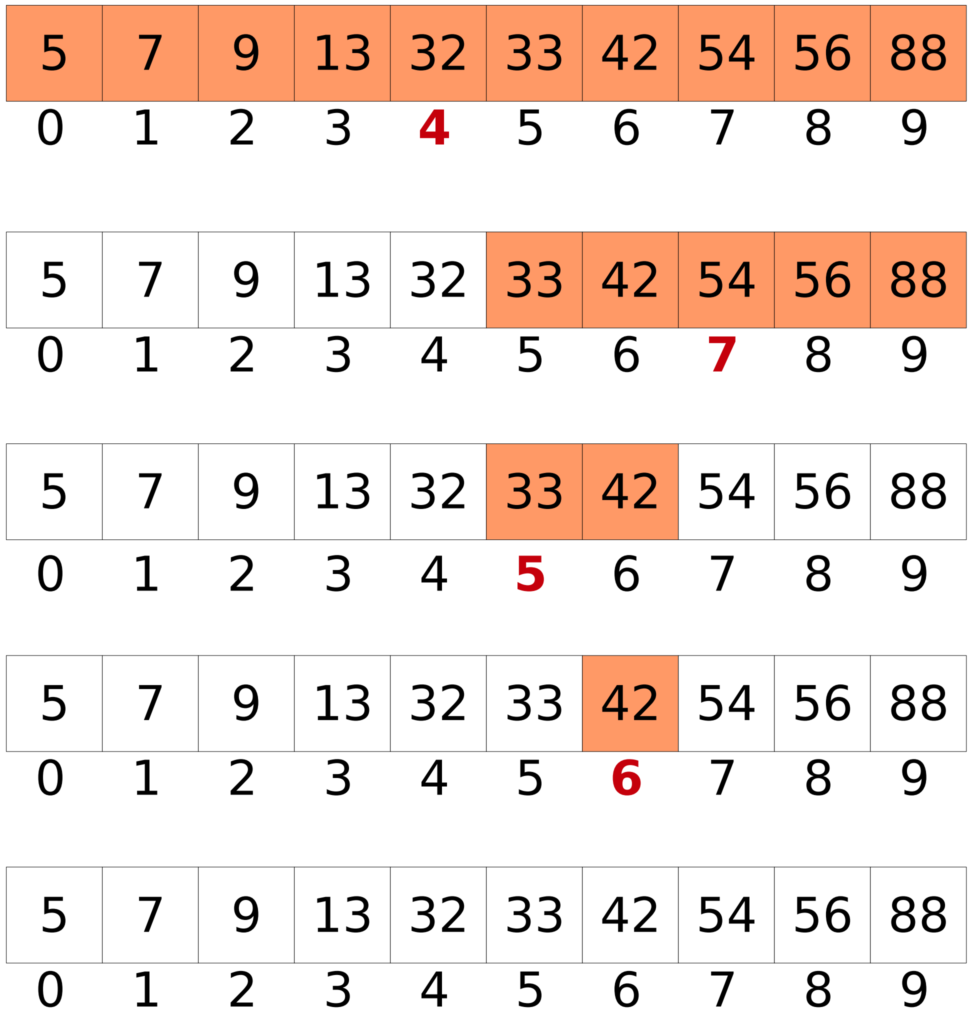
```
private int search(target 33, first 5, last 6) {
   int mid, result=-1;
   if (first <= last) {
      mid = (first + last)/2;    ⟶  mid = (5+6)/2 = 5
      if (target == a[mid]) { //base case
         result = mid;
      } else if (target < a[mid]) {
         result = search(target, first, mid-1);
      } else {
         result = search(target, mid+1, last);
      }
   }
   return result;
                5
}
```

Binary Search – Number Not Found

- What if the number is not in the array?
- Eventually, **`first`** will become greater than **`last`**.
- **`-1`** is returned, indicating that the number was not found.

Binary Search – Number Not Found

- Example: search target 35 in sorted array of length 10 (index 0-9)



```
search(35, 0, 9)    mid=4
```

5	7	9	13	32	33	42	54	56	88
0	1	2	3	**4**	5	6	7	8	9

```
search(35, 5, 9)    mid=7
```

5	7	9	13	32	33	42	54	56	88
0	1	2	3	4	5	6	**7**	8	9

```
search(35, 5, 6)    mid=5
```

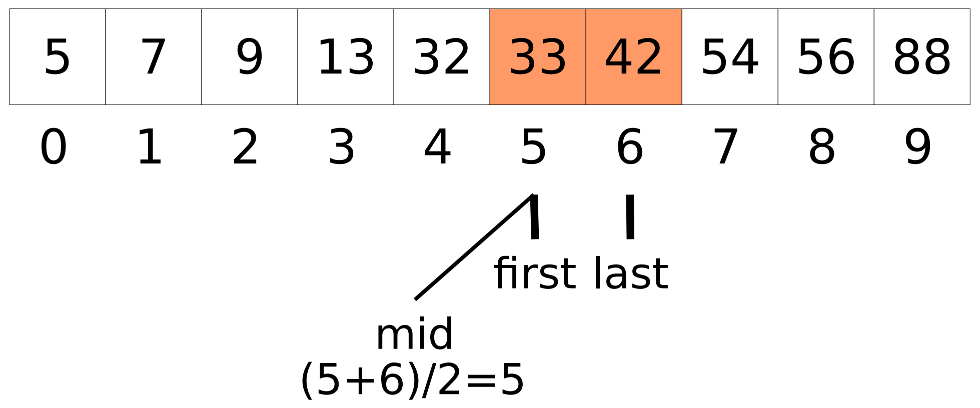
5	7	9	13	32	33	42	54	56	88
0	1	2	3	4	**5**	6	7	8	9

```
search(35, 6, 6)    mid=6
```

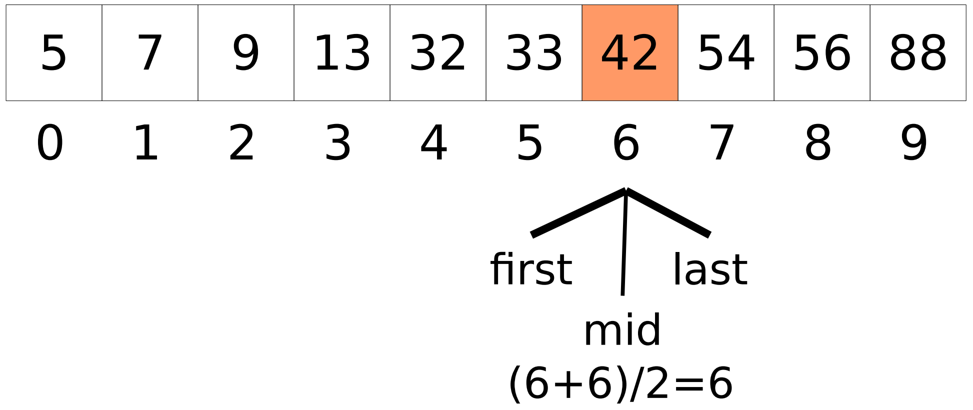
5	7	9	13	32	33	42	54	56	88
0	1	2	3	4	5	**6**	7	8	9

```
search(35, 6, 5)
first>last, return -1
```

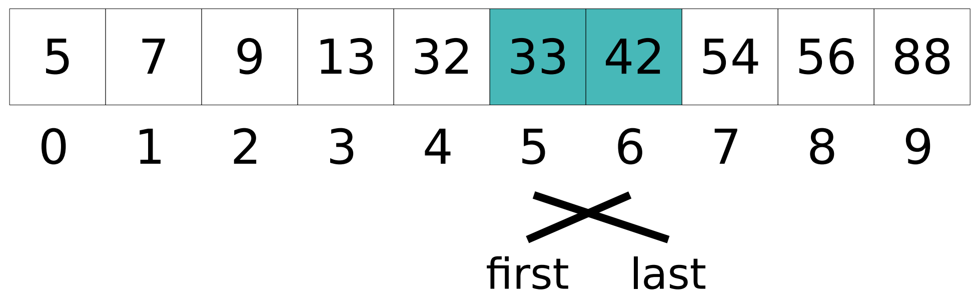
5	7	9	13	32	33	42	54	56	88
0	1	2	3	4	5	6	7	8	9



```
private int search(target 35, first 5, last 6) {
   int mid, result=-1;
   if (first <= last) {
       mid = (first + last)/2;  ───►  mid = (5+6)/2 = 5
       if (target == a[mid]) {
           result = mid;
       } else if (target < a[mid]) {
           result = search(target, first, mid-1);
       } else {                    35,      6,     6
           result = search(target, mid+1, last);
       }
   }
   return result;
}
```



```
private int search(target 35, first 6, last 6) {
   int mid, result=-1;
   if (first <= last) {
      mid = (first + last)/2;   ──►  mid = (6+6)/2 = 6
      if (target == a[mid]) {
         result = mid;
      } else if (target < a[mid]) {
         result = search(target, first, mid-1);
                          35,     6,     5
      } else {
         result = search(target, mid+1, last);
      }
   }
   return result;
}
```



```
private int search(target 35, first 6, last 5) {
   int mid, result=-1;
   if (first <= last) {
       mid = (first + last)/2;
       if (target == a[mid]) {
           result = mid;
       } else if (target < a[mid]) {
           result = search(target, first, mid-1);
       } else {
           result = search(target, mid+1, last);
       }
   }
   return result;
              -1
}
```

Binary Search - Code

- Notice that the recursive search method is **`private`**.
- A non-recursive, **`public`** method **`find`** is used to get the recursion going.
- **`find`** calls search with the first (**`0`**) and last (**`a.length-1`**) index of the array.
- Subsequent recursive calls to search are made with ever smaller portions of the array.

Binary Search - Efficiency

- With each recursive call, about half of the array is eliminated from consideration.
- The number of recursive calls required to find an element or determine that it is not in the array is **$\log_2$ n** for an array of length n.
- We say that the binary search algorithm **has order $\log_2$ n,** written **O($\log_2$ n).**
- This is also known as **big O notation**.

Binary Search - Efficiency

- For example, an array with 1024 elements will need to do only 10 comparisons.
  - $\log_2(1024) = 10$
  - $2^{10} = 1024$

Binary Search - Efficiency

- An array with 100,000 elements would only need to make about 17 comparisons.
- This is <u>much</u> better than a linear/sequential search, which would need to make 50,000 comparisons (on average).

Merge Sort
Savitch p 808 (case study)

- The binary search algorithm works only if the array is sorted.
- **Merge sort** is a very efficient, recursive algorithm to sort an array.
- Like binary search, merge sort also takes a “divide and conquer” approach.
  - The array is divided in halves and the halves are sorted recursively.
  - Sorted subarrays are merged to form a larger sorted array.

Merge Sort - Pseudocode

```
Base case: If the array a has only 1 element
  (a.length == 1)
    stop, a is of length 1, so it is already sorted
Otherwise (recursive case):
    Divide the input array a into two halves
        - copy the first half of the elements into
          an array called front
        - copy the second half of the elements into
          an array called tail
    Sort array front recursively
    Sort array tail recursively
    Merge the arrays front and tail into a
```

Merging Sorted Arrays - Pseudocode

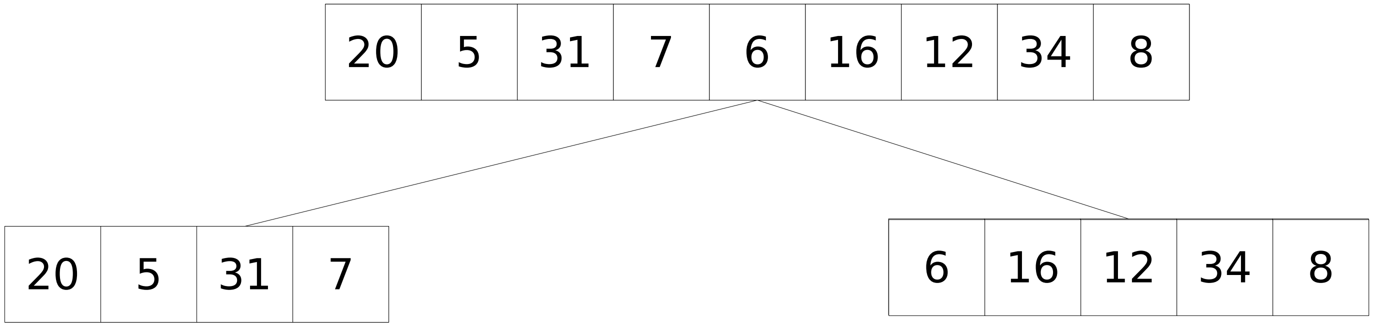
- **`front`** and **`tail`** are both sorted
- Initialize **`frontIndex=0`**, **`tailIndex=0`**, and **`aIndex=0`**
- while we haven't reached the end of either list {
    - copy the smaller of **`front`** and **`tail`** to **`a`**
    - increment **`aIndex`**
    - increment index of array copied from
- }
- copy remaining elements from **`front`**, if any
- copy remaining elements from **`tail`**, if any

Merge Sort Visualization

- Array to sort:

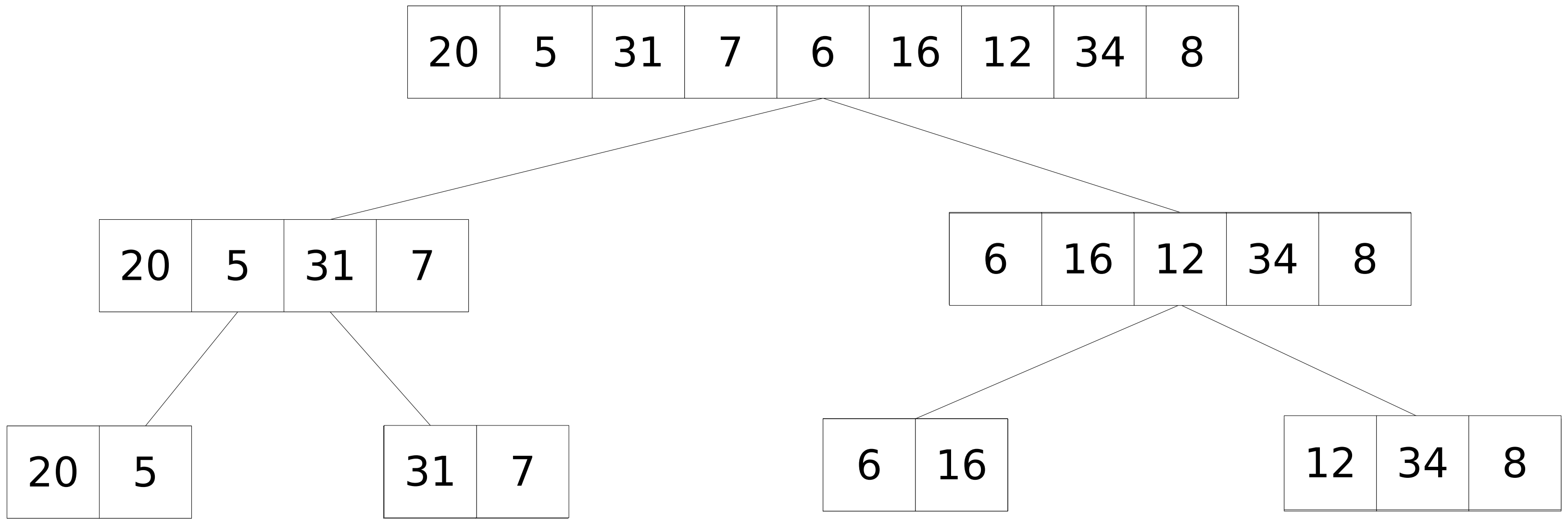
20	5	31	7	6	16	12	34	8

Merge Sort Visualization



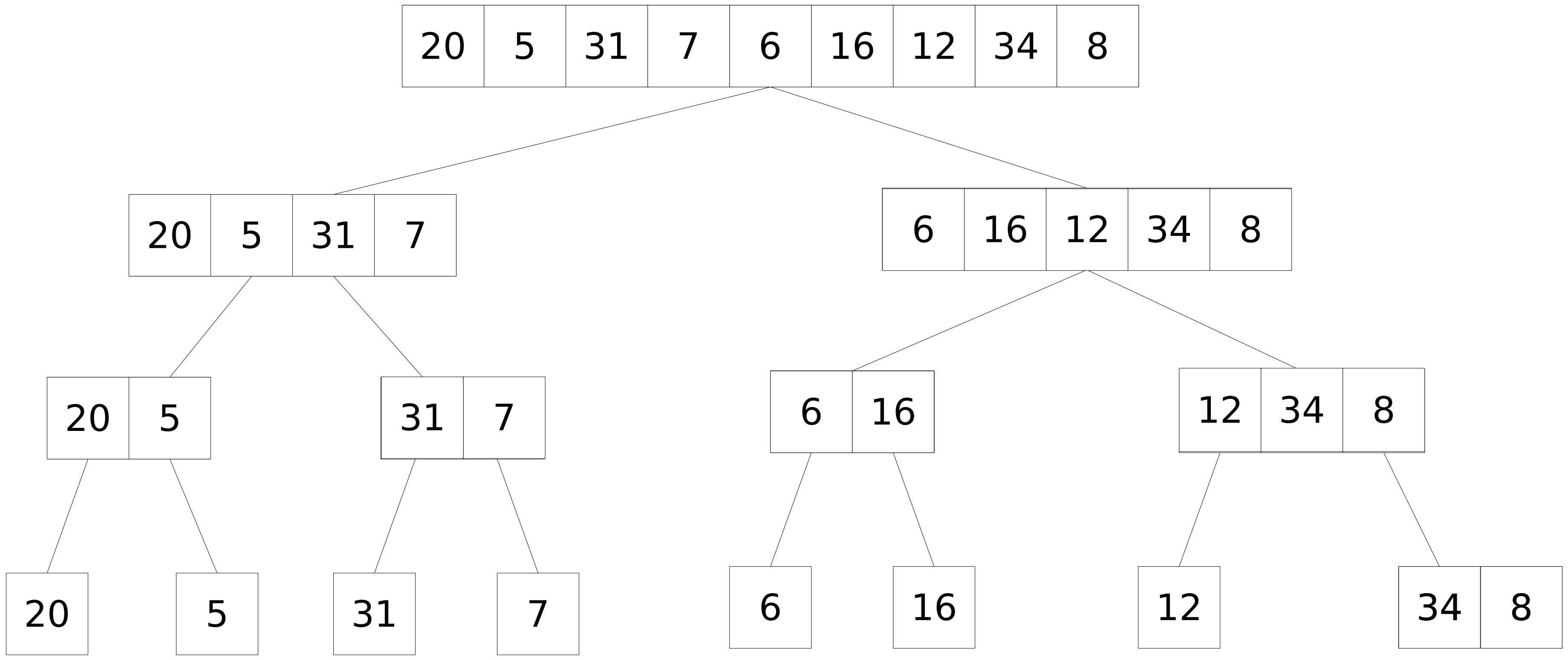
- First, split array(s) recursively into two halves

Merge Sort Visualization



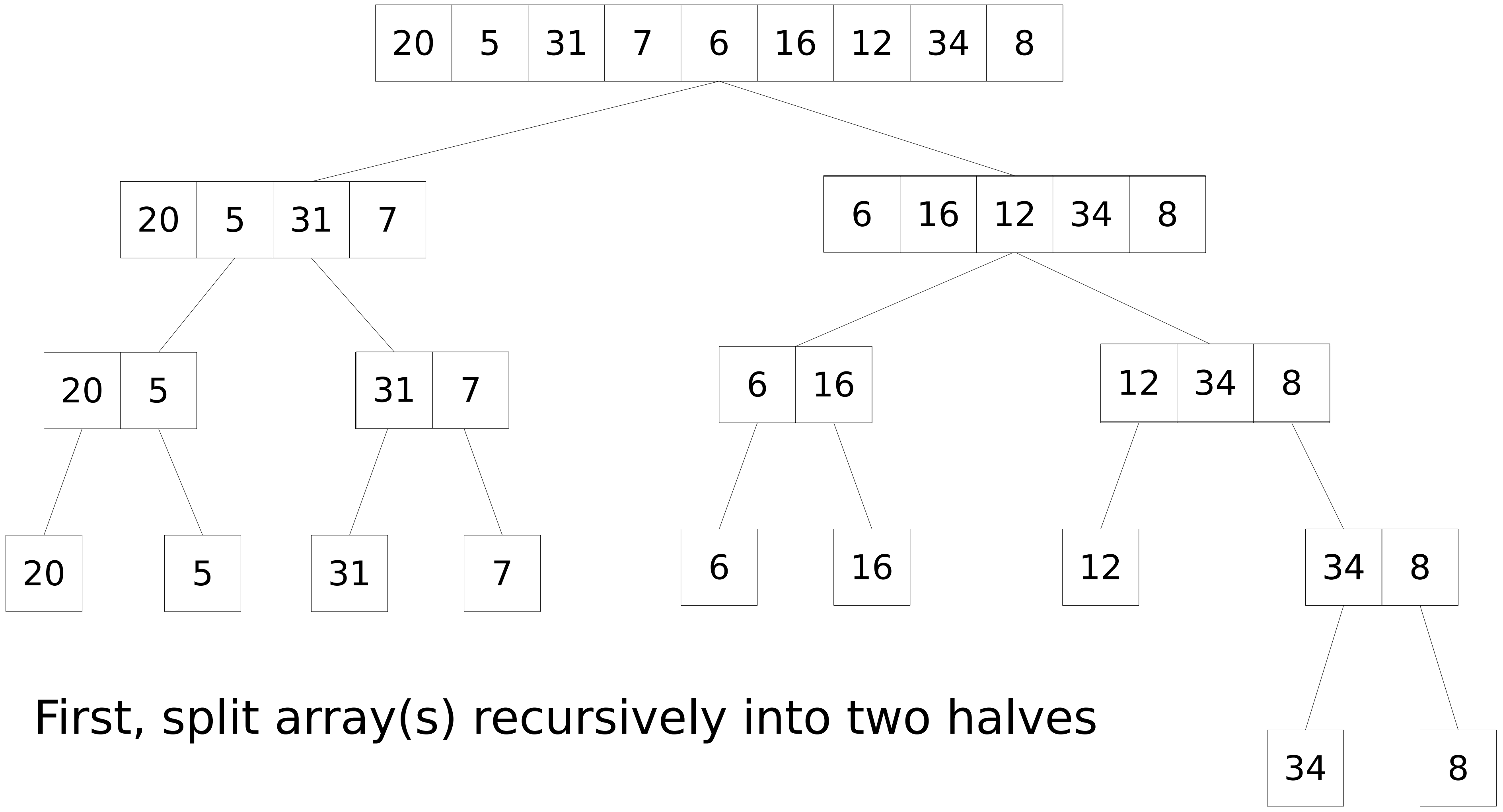
- First, split array(s) recursively into two halves

Merge Sort Visualization



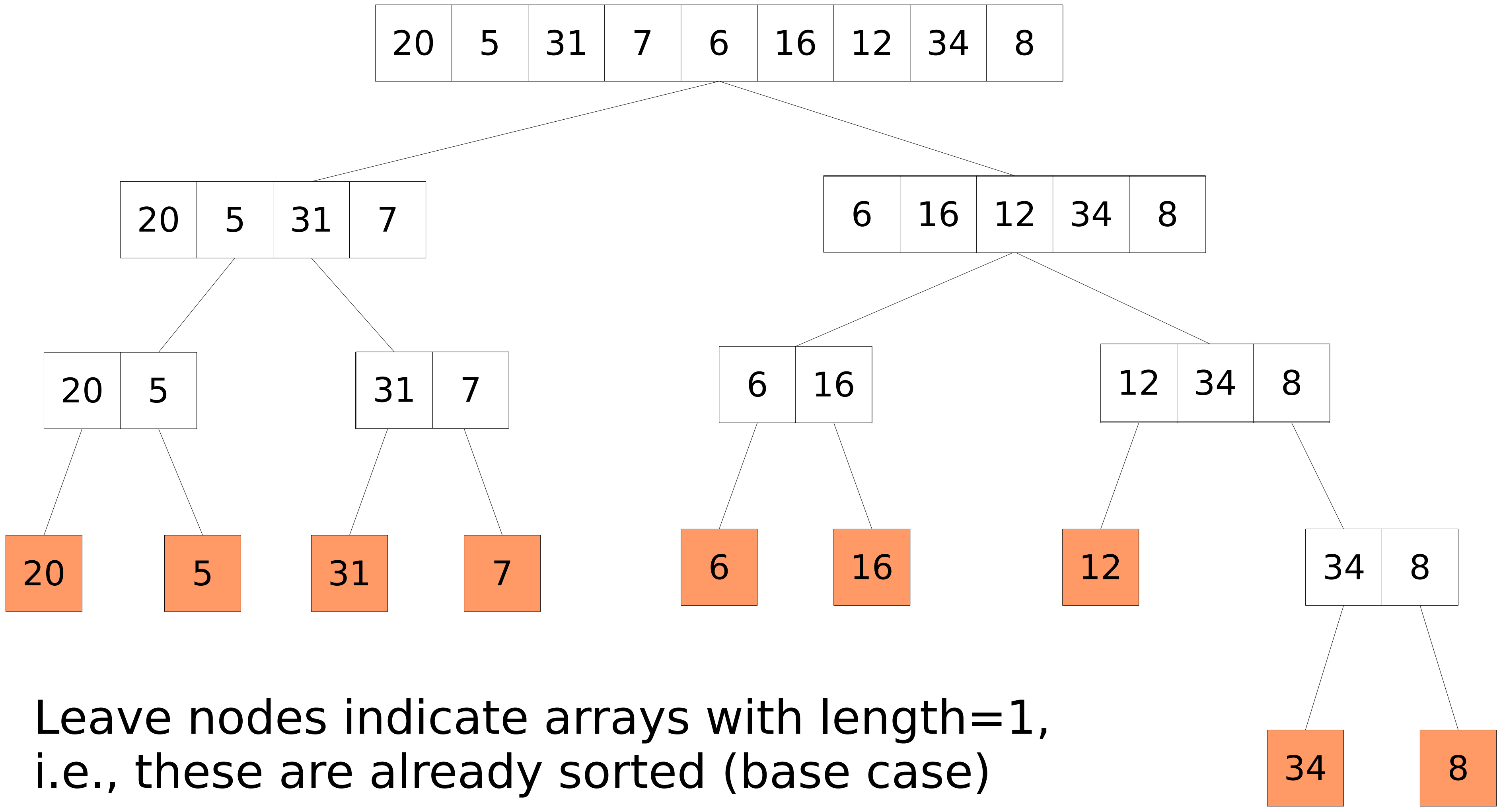
- First, split array(s) recursively into two halves

Merge Sort Visualization



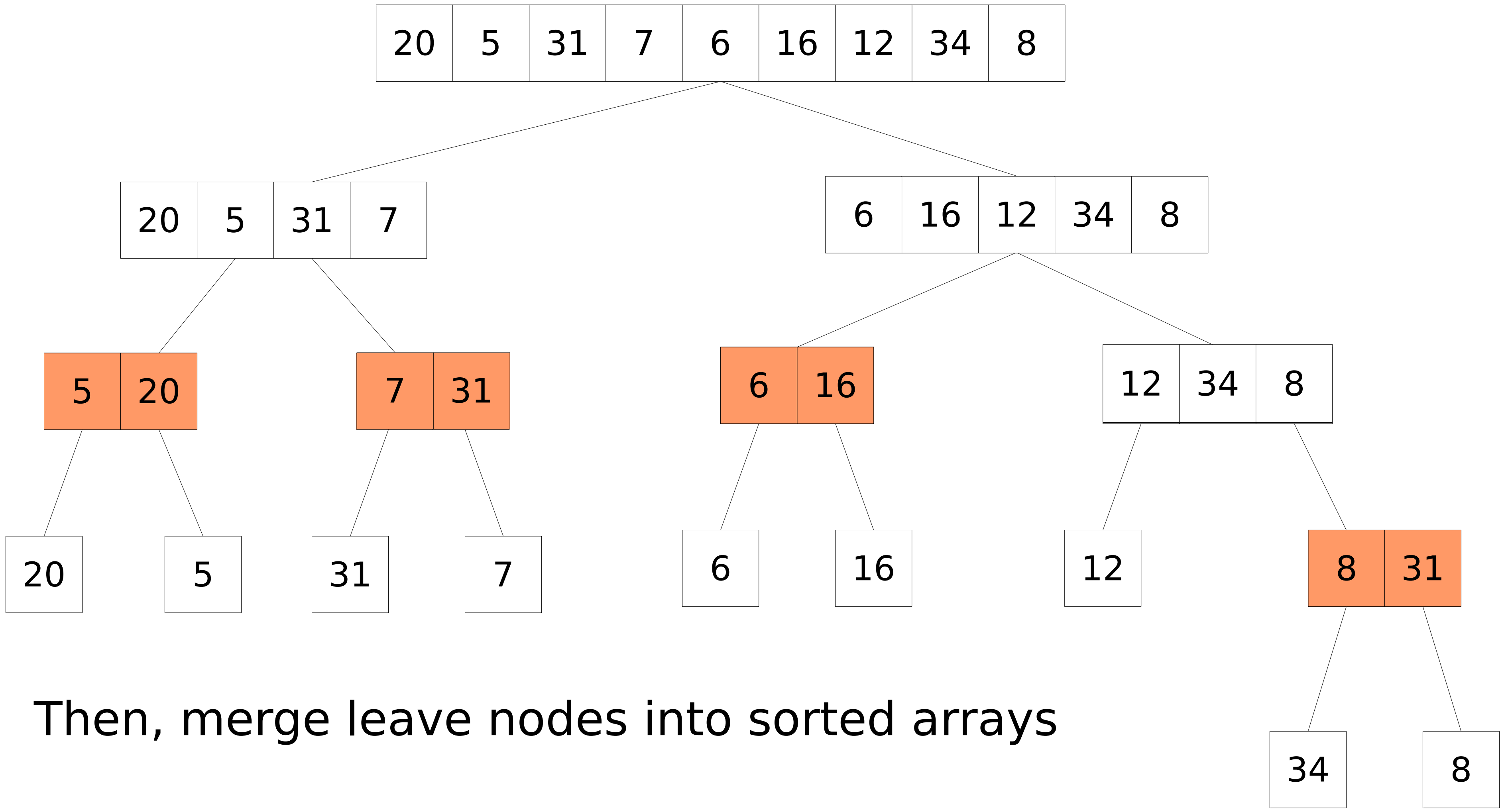
- First, split array(s) recursively into two halves

Merge Sort Visualization



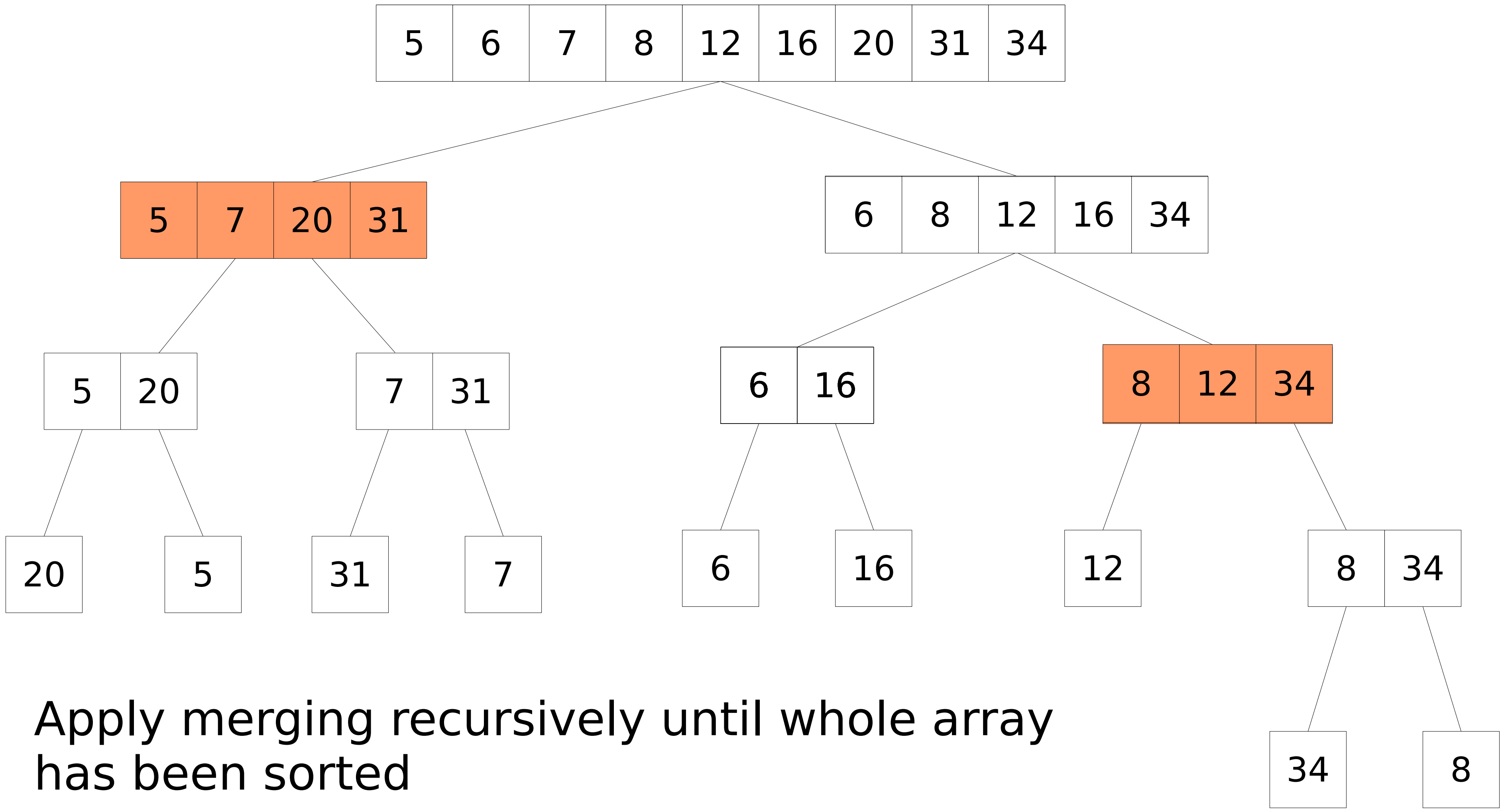
- Leave nodes indicate arrays with length=1, i.e., these are already sorted (base case)

Merge Sort Visualization



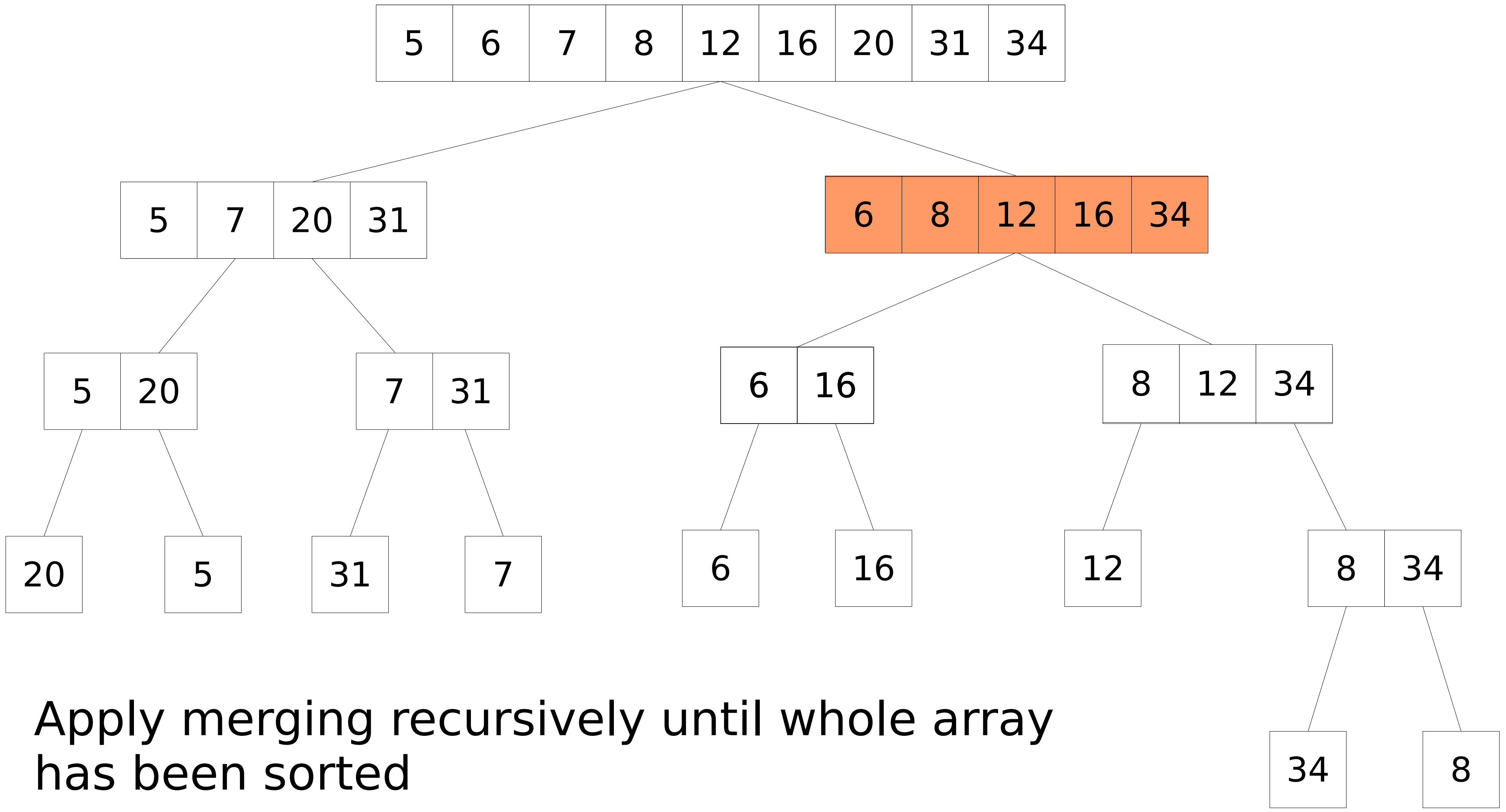
- Then, merge leave nodes into sorted arrays

Merge Sort Visualization



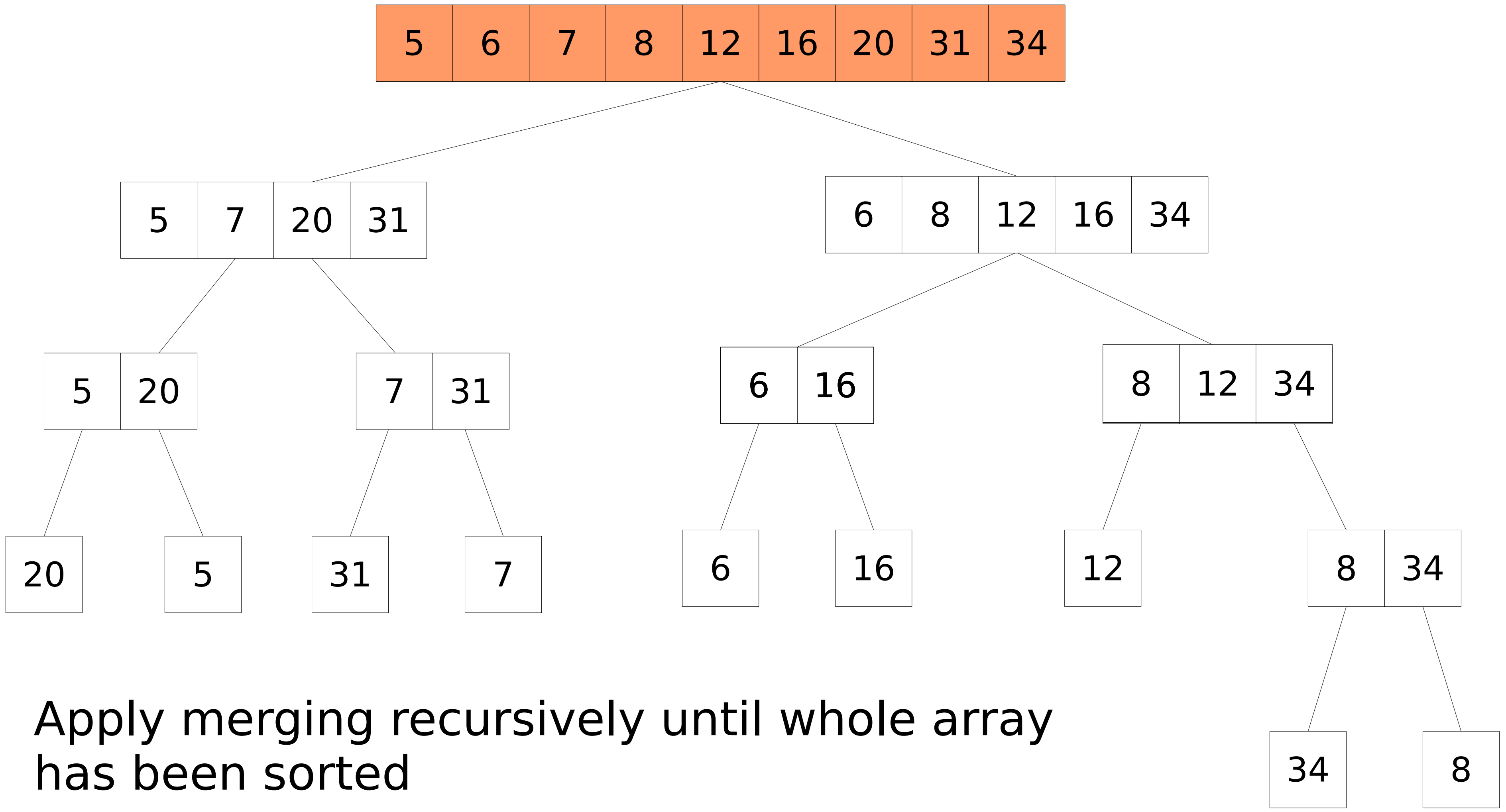
- Apply merging recursively until whole array has been sorted

Merge Sort Visualization



- Apply merging recursively until whole array has been sorted

Merge Sort Visualization



- Apply merging recursively until whole array has been sorted
- See **`MergeSort.java`** under SavitchSrc on the course website