Binary Trees

Reading:
Lewis & Chase 12.1, 12.3
Eck 9.4.1

Objectives

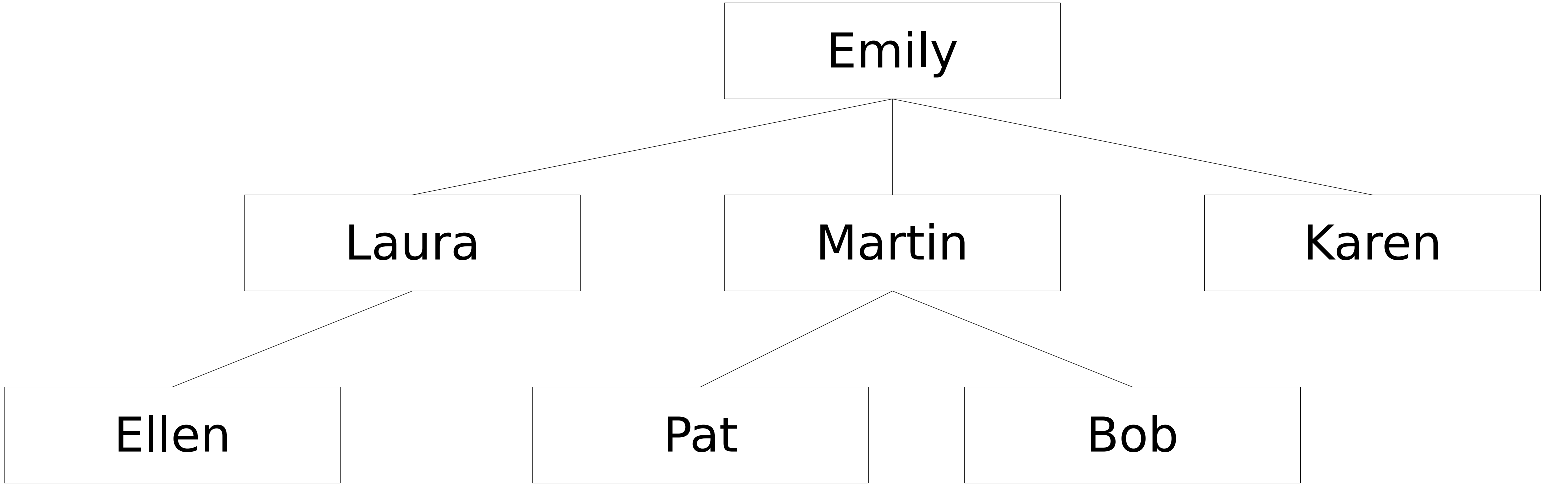
- Tree basics
- Learn the terminology used when talking about trees
- Discuss methods for traversing trees
- Discuss a possible implementation of trees with nodes
- Examine a binary tree example

Tree Basics

- So far, all the data structures that we have encountered were linear. Objects in an array, list, stack, or queue are placed one after the other in a line.
- Sometimes it is useful to organize data into groups and subgroups.
- This type of organization of data is hierarchical, or non-linear, since the data appears at various levels.

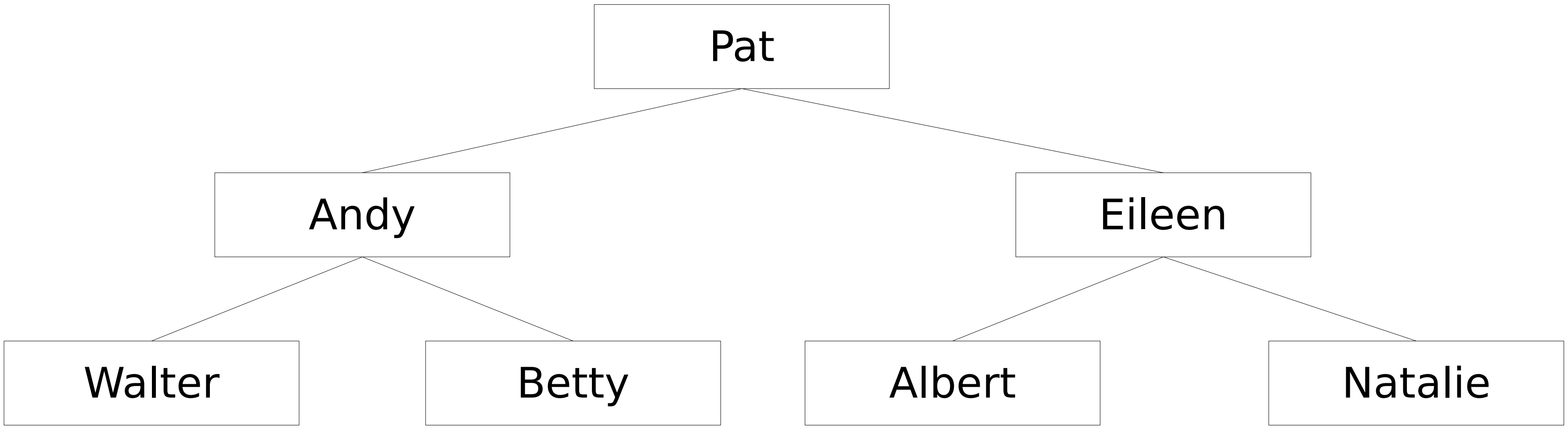
Hierarchical Organizations
Family Trees

- Family trees can be arranged in various ways.
- This diagram shows Emily's children and grandchildren.



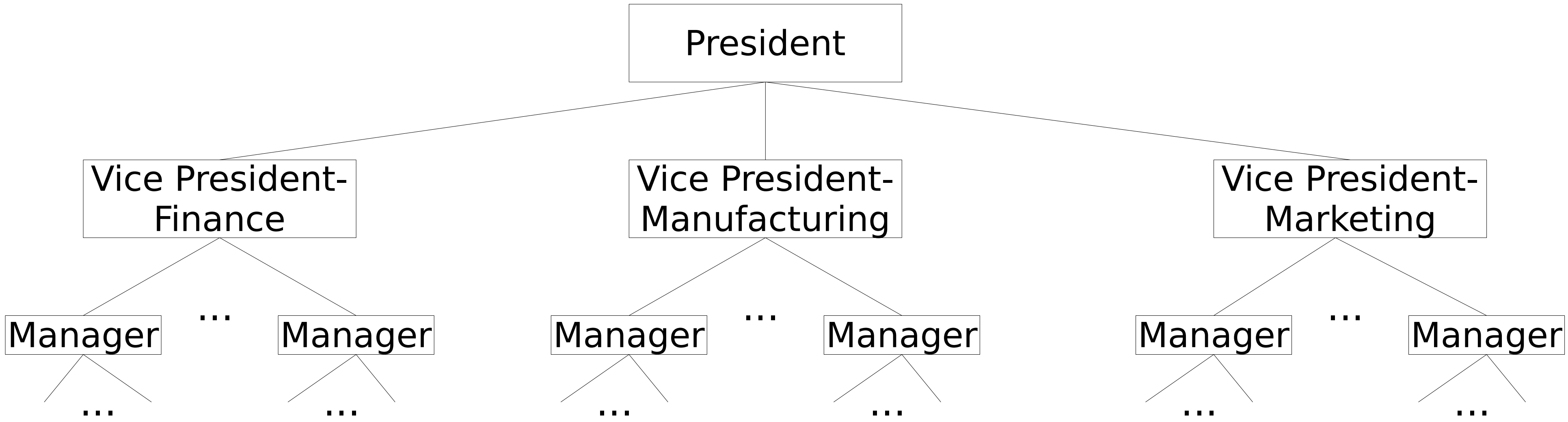
Hierarchical Organizations
Family Trees

- This diagram shows Pat's parents and grandparents.



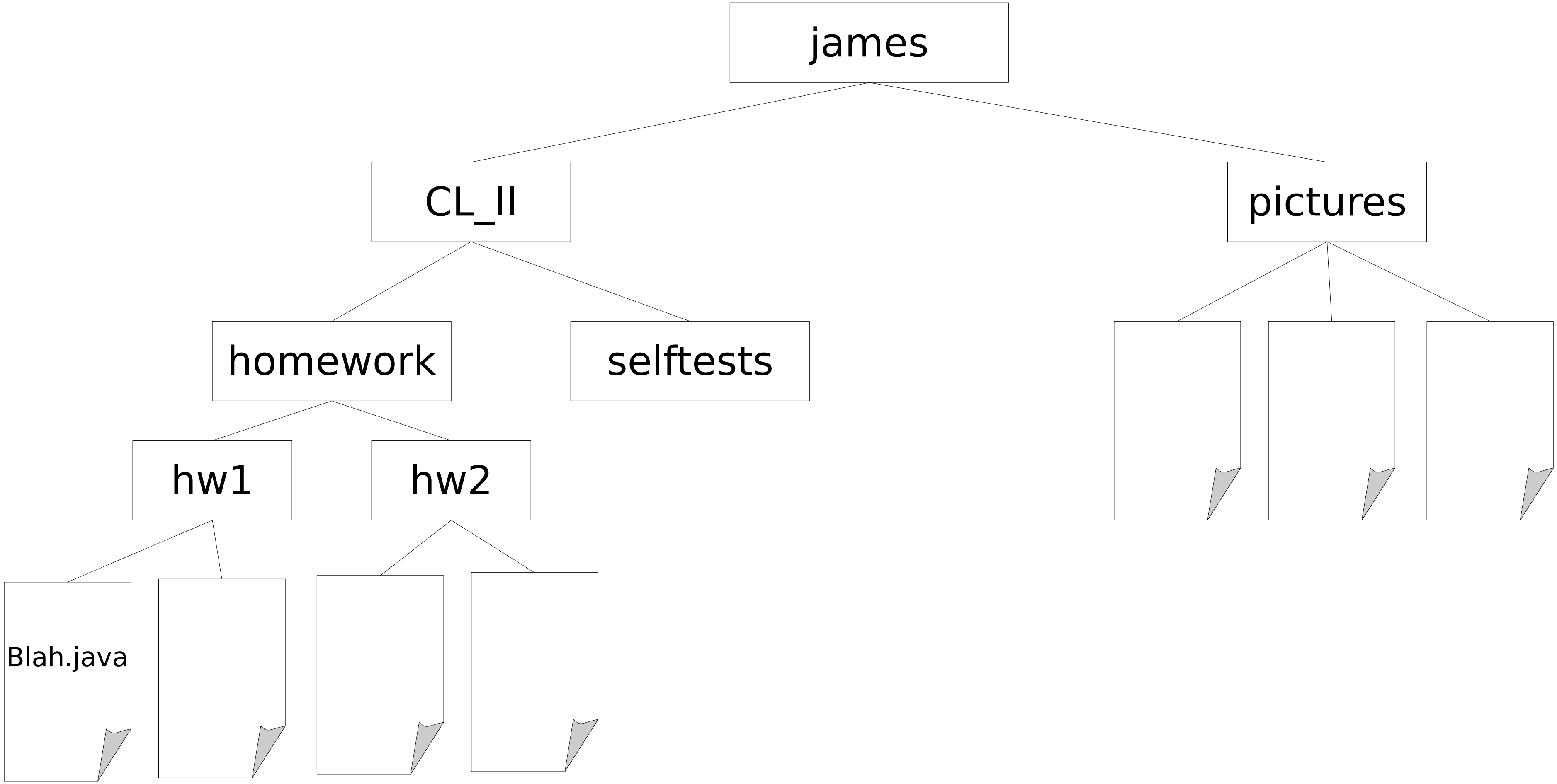
Hierarchical Organizations Businesses

- A hierarchical diagram of a business:

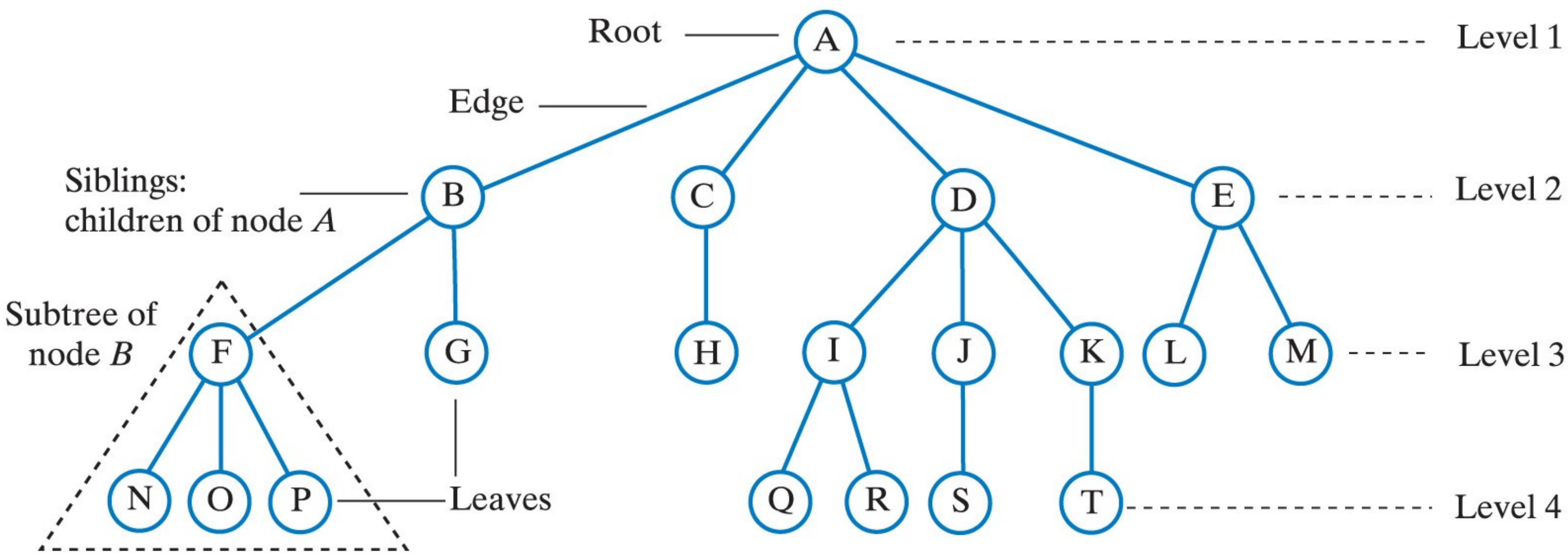


Hierarchical Organizations
Files and Directories

- Files and directories on a computer:

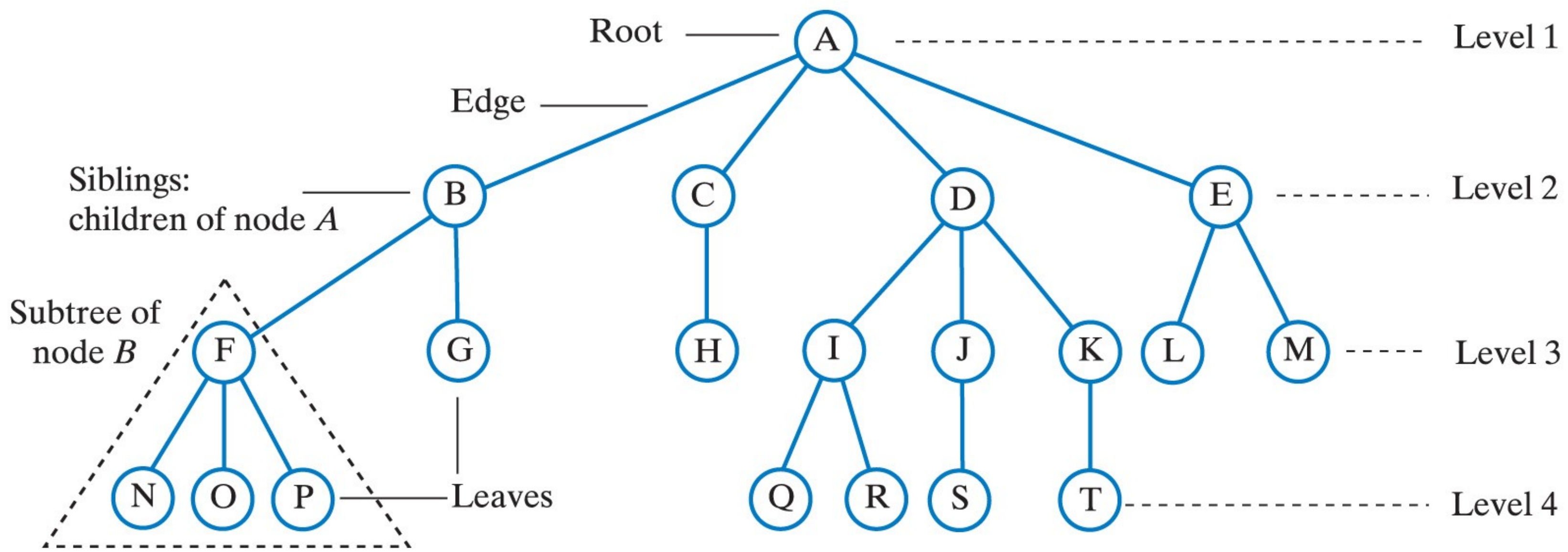


Terminology



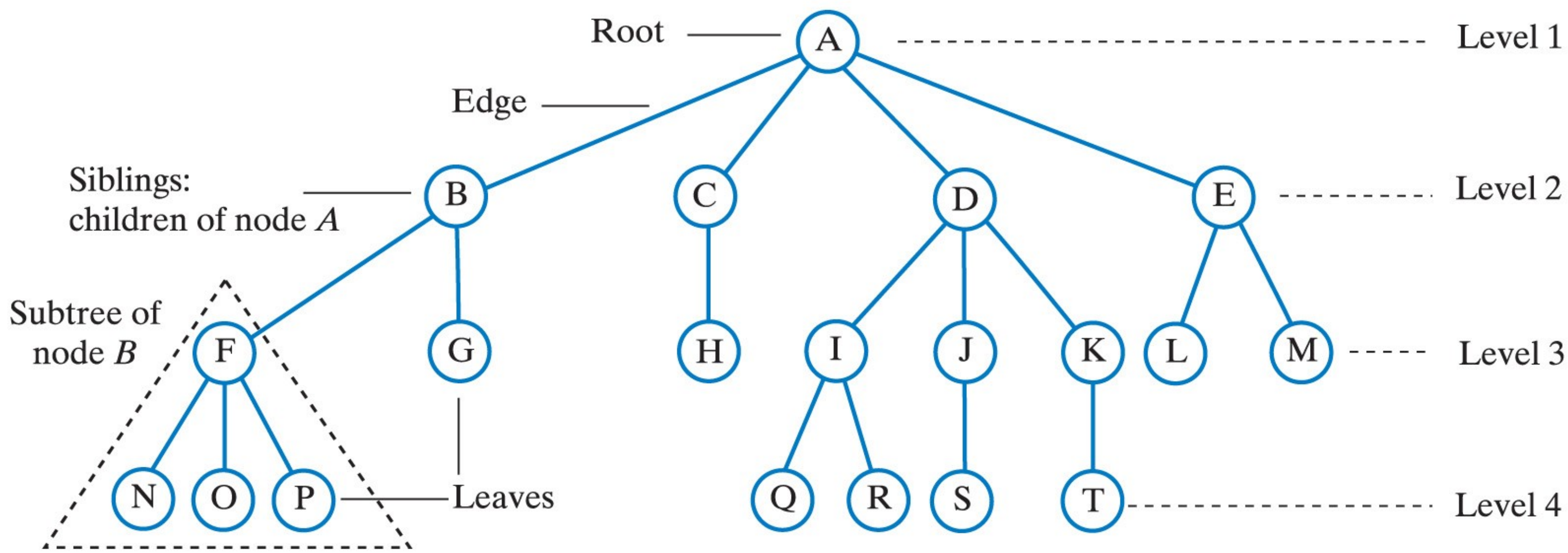
- A **tree** is a set of **nodes** connected by **edges** that show a relationship between the nodes.
- The nodes are arranged in levels that indicate the hierarchy of the nodes. The top level is a single node called the **root**.

Terminology



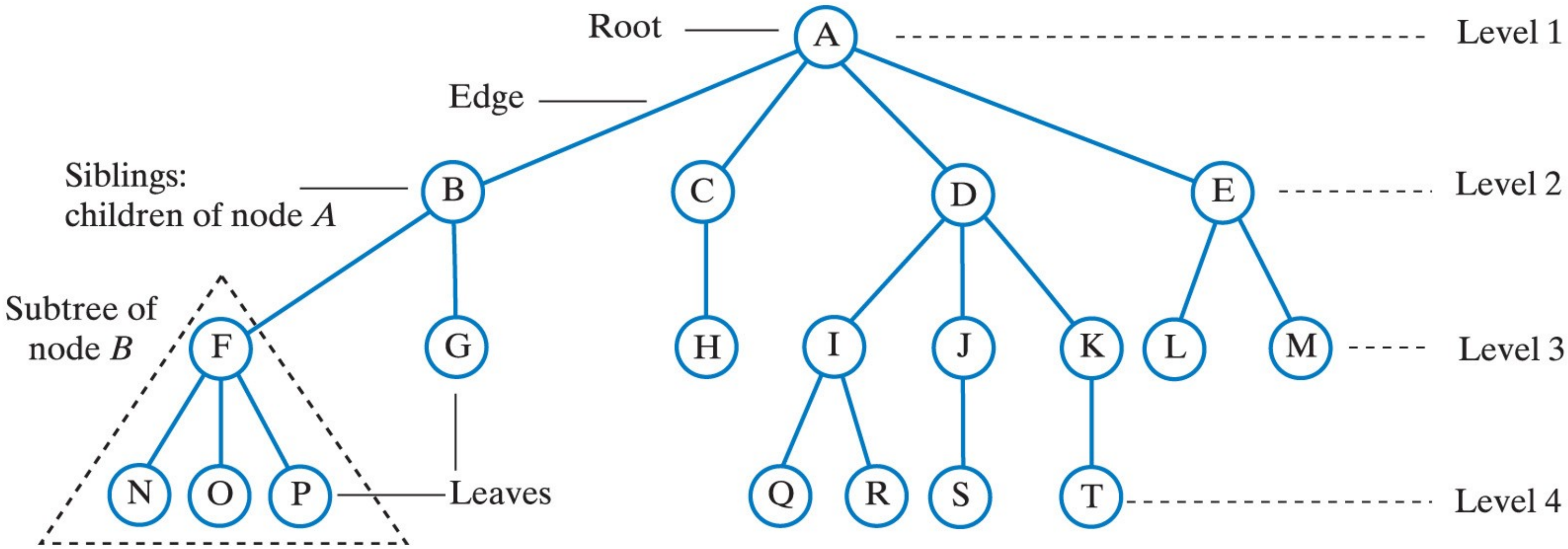
- The **children** of a node are those directly below it. *A* has 4 children: *B, C, D, E*
- The **parent** of a node is the node directly above it. *C*'s parent is *A*

Terminology



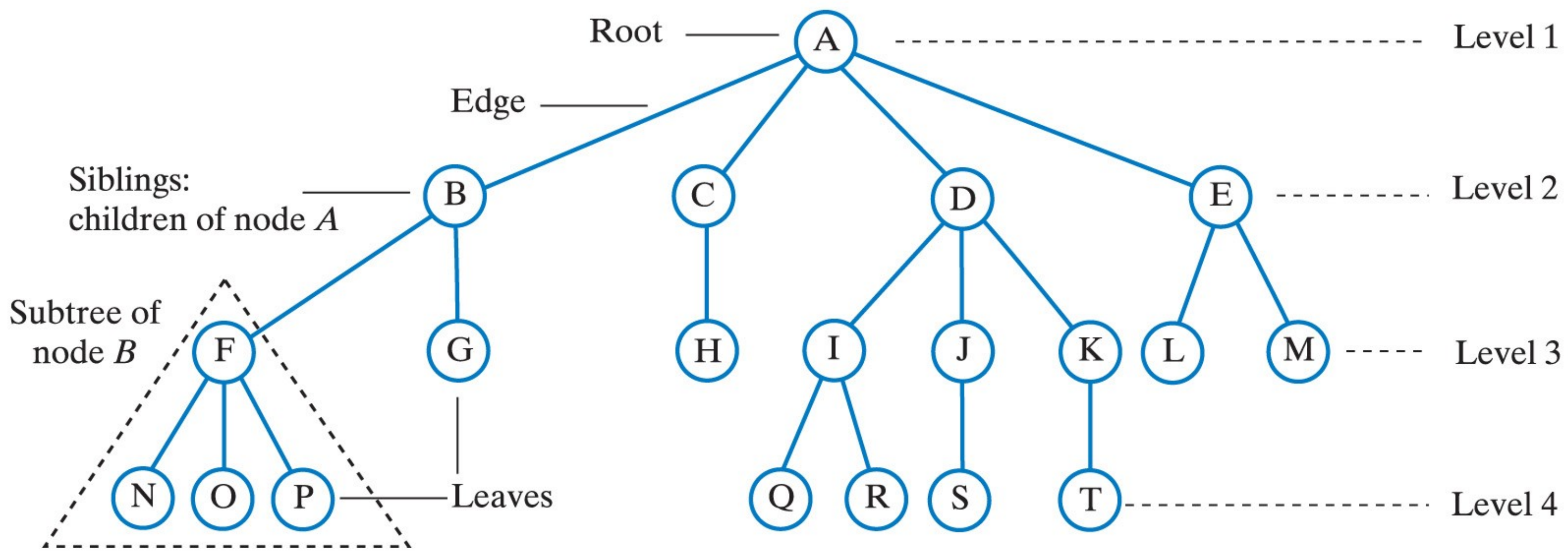
- All nodes have exactly 1 parent, except the root, which has no parent. *H*'s parent is *C*
- Nodes that share a parent are **siblings**. *B, C, D,* and *E* are siblings.

Terminology



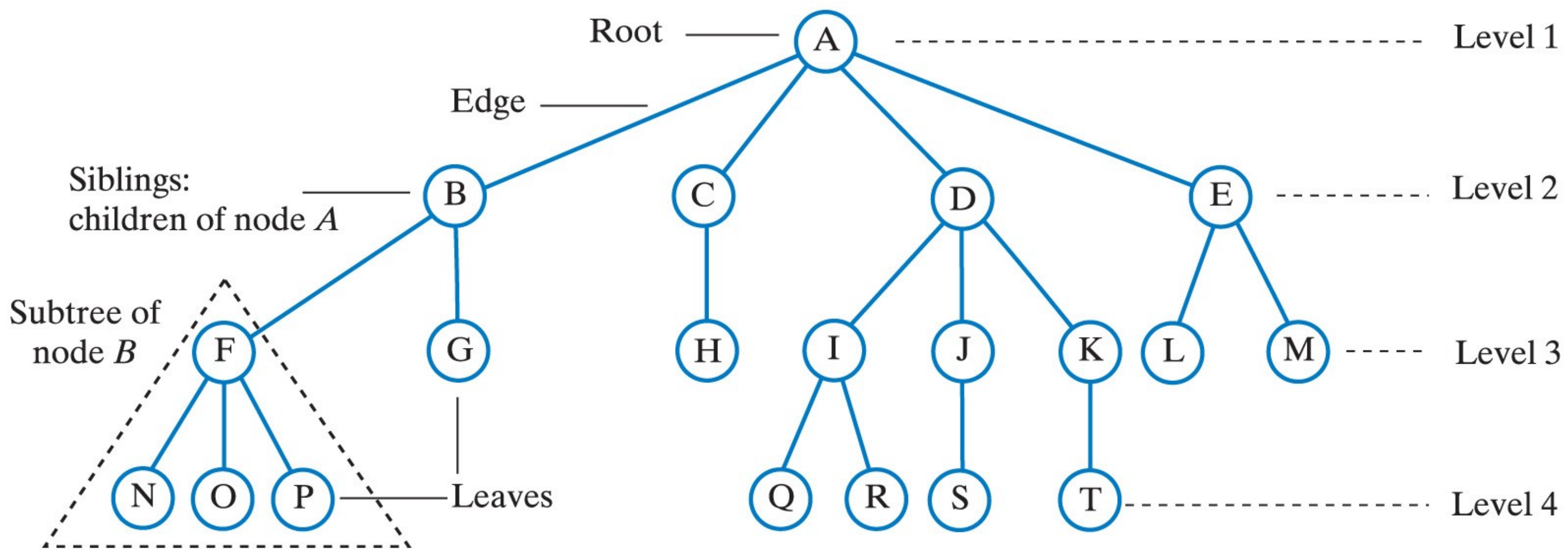
- The nodes below a given node (on the downward paths to the leaves) are its **descendants**.
- *D*'s descendants are *I, J, K, Q, R, S* and *T.*

Terminology



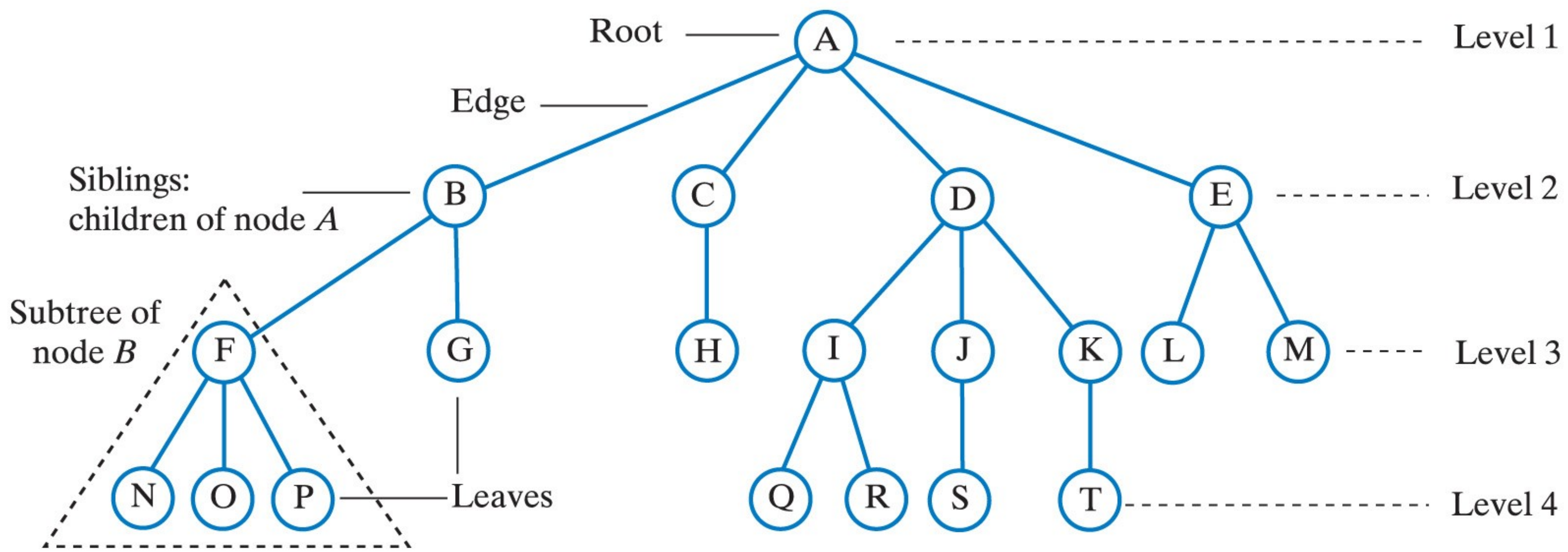
- The nodes above a given node (on the upward path towards the root) are its **ancestors**.
- *Q*'s ancestors are *I*, *D*, and *A*

Terminology



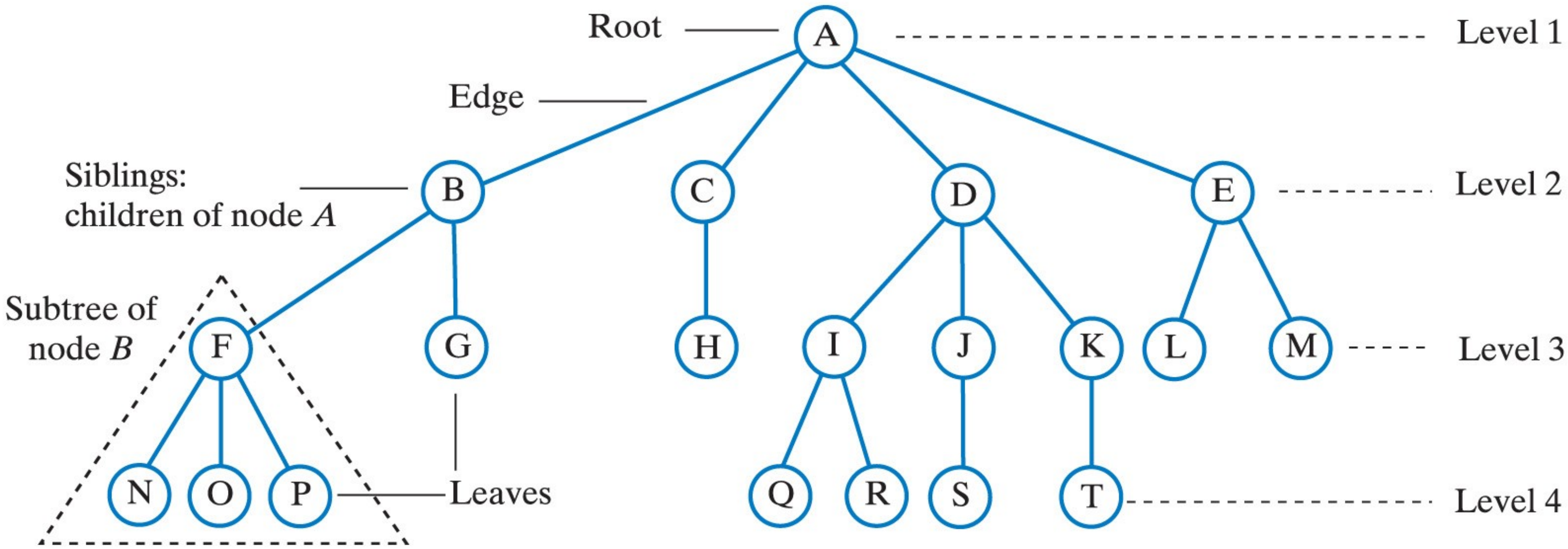
- A **leaf** is a node that has no children.
- Any node that is not a leaf is an **interior node** (or **non-leaf** node).

Exercise



- What are the leaf nodes?
- What are the siblings of J?
- What are the children of E?
- What are the descendants of B?
- What are the ancestors of P?

Exercise



- What are the leaf nodes? N O P G H Q R S T L M
- What are the siblings of J?
- What are the children of E?
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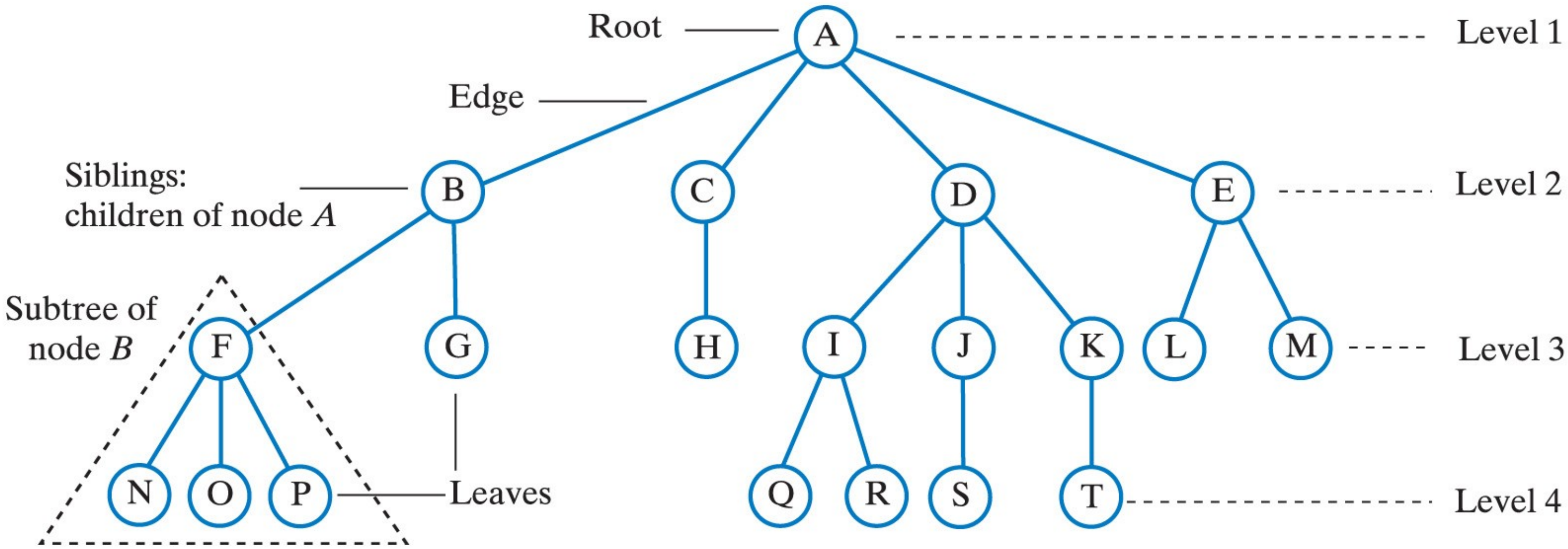
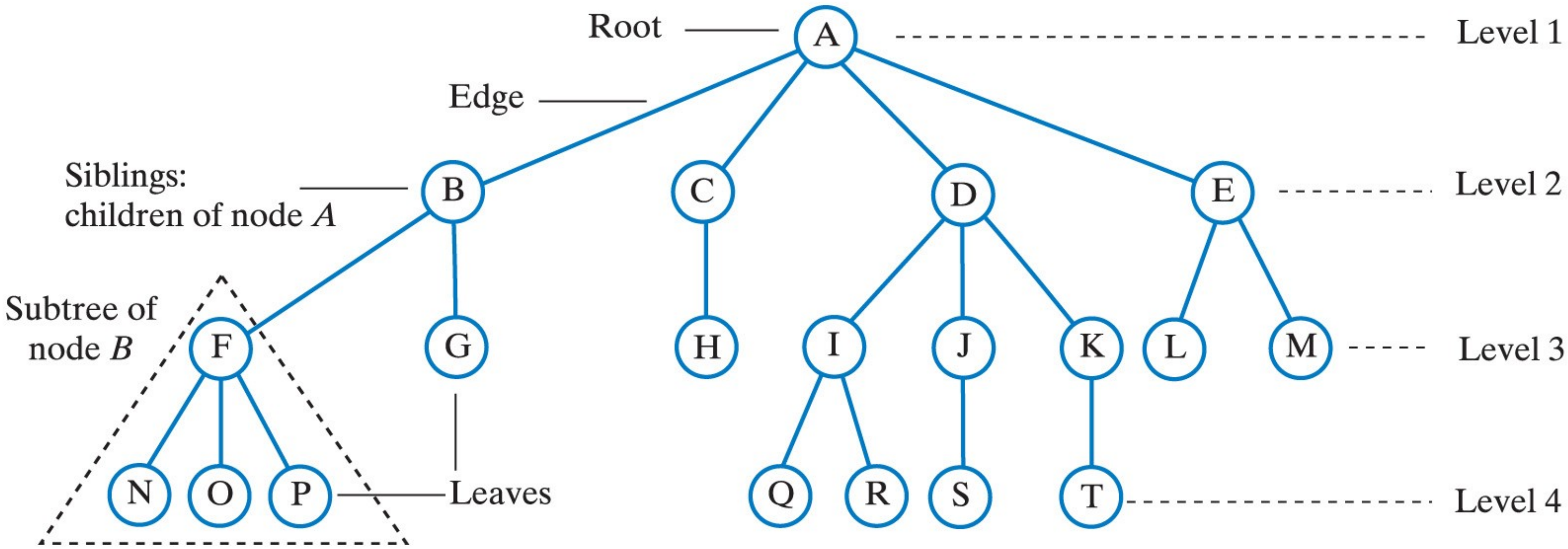
- What are the ancestors of P?

Exercise



- What are the leaf nodes? N O P G H Q R S T L M
- What are the siblings of J? I K
- What are the children of E?
- What are the descendants of B?
- What are the ancestors of P?

Exercise



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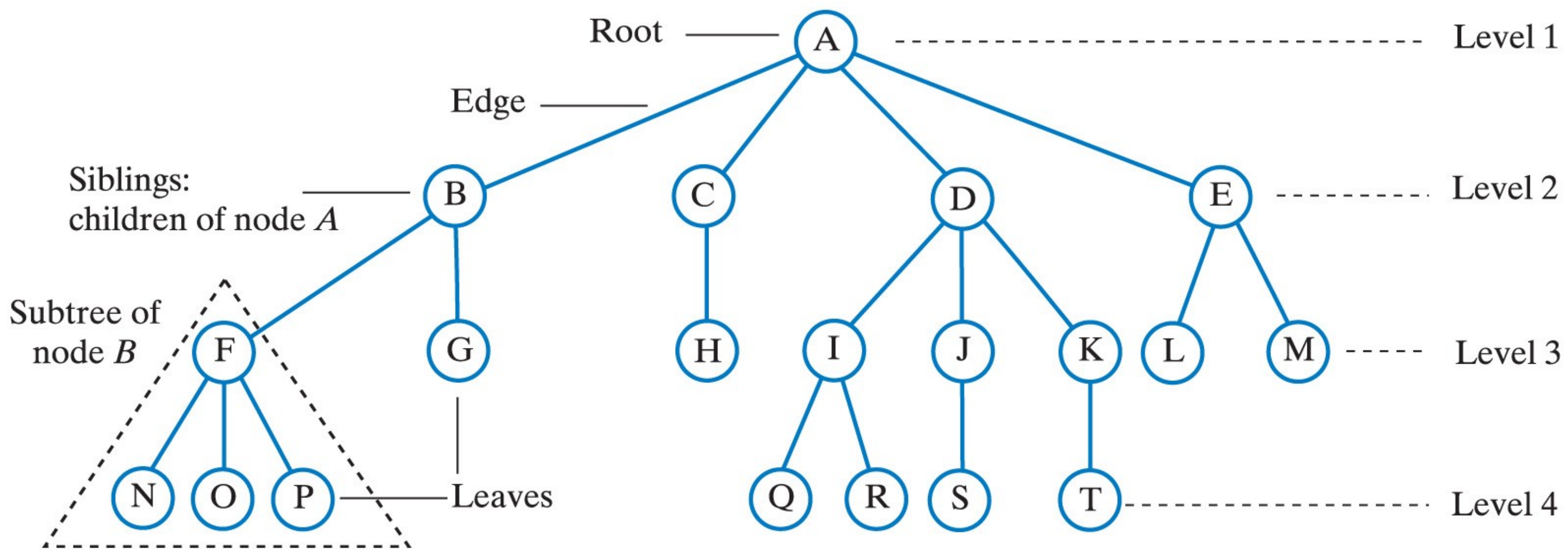
Exercise



- What are the leaf nodes? N O P G H Q R S T L M
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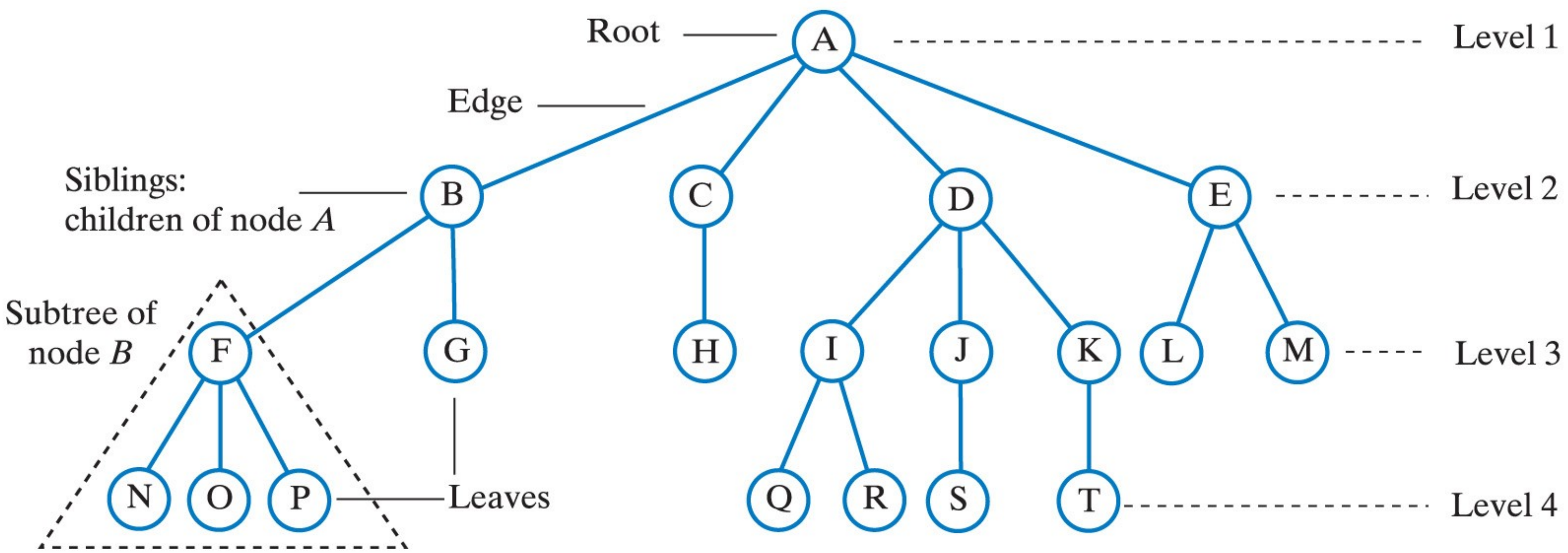
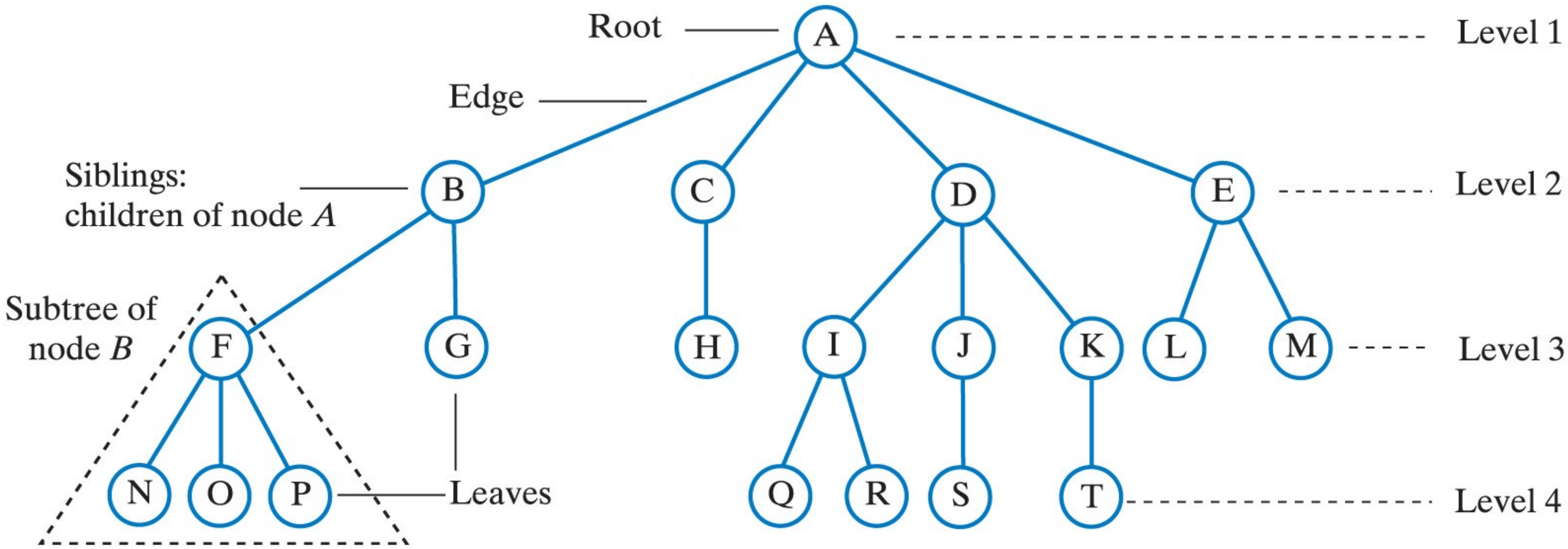
- What are the descendants of B? F G N O P
- What are the ancestors of P?

Exercise



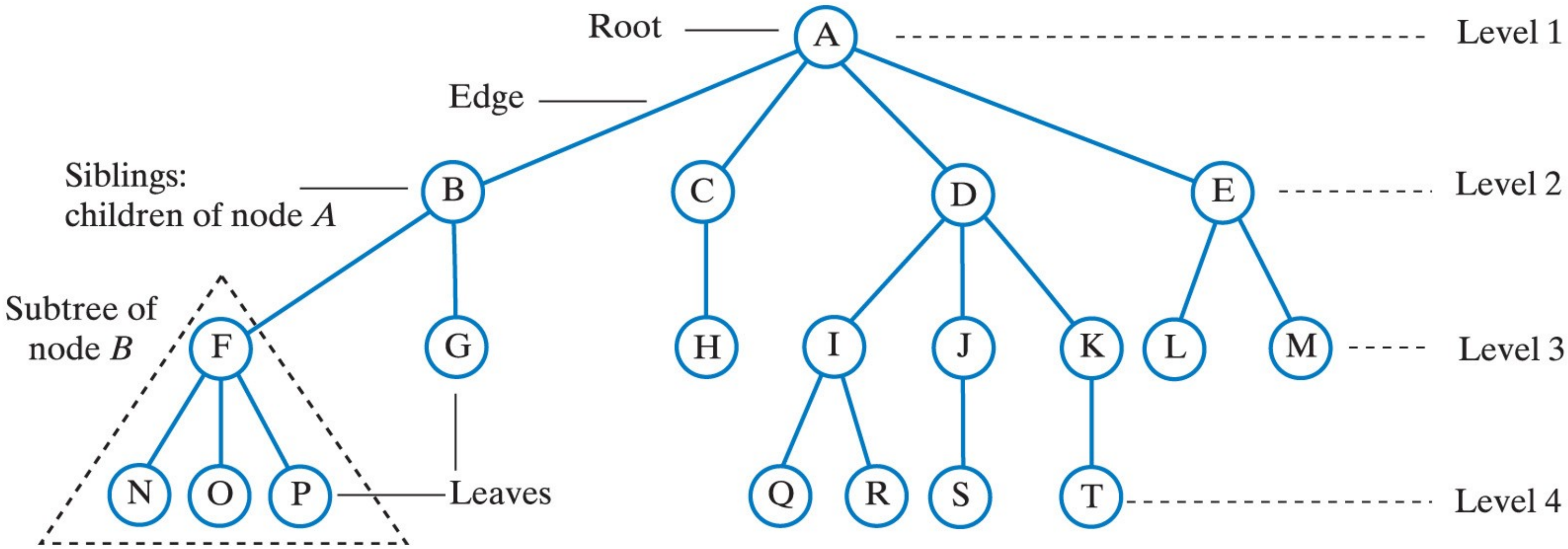
- What are the leaf nodes? N O P G H Q R S T L M
- What are the siblings of J? I K
- What are the children of E? L M
- What are the descendants of B? F G N O P
- What are the ancestors of P? F B A

More Terminology



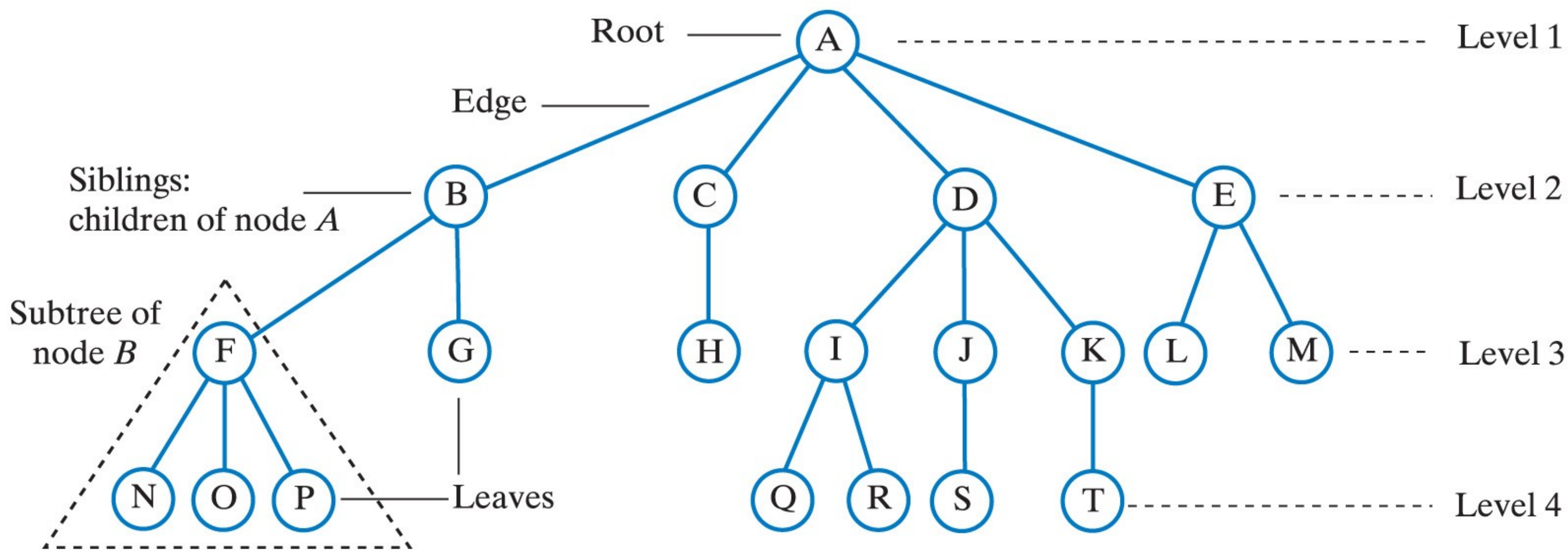
- A **subtree of a node** is a tree rooted at one of that node's children.
- Node *B* has 2 subtrees.
- Node *A* has 4 subtrees.

More Terminology



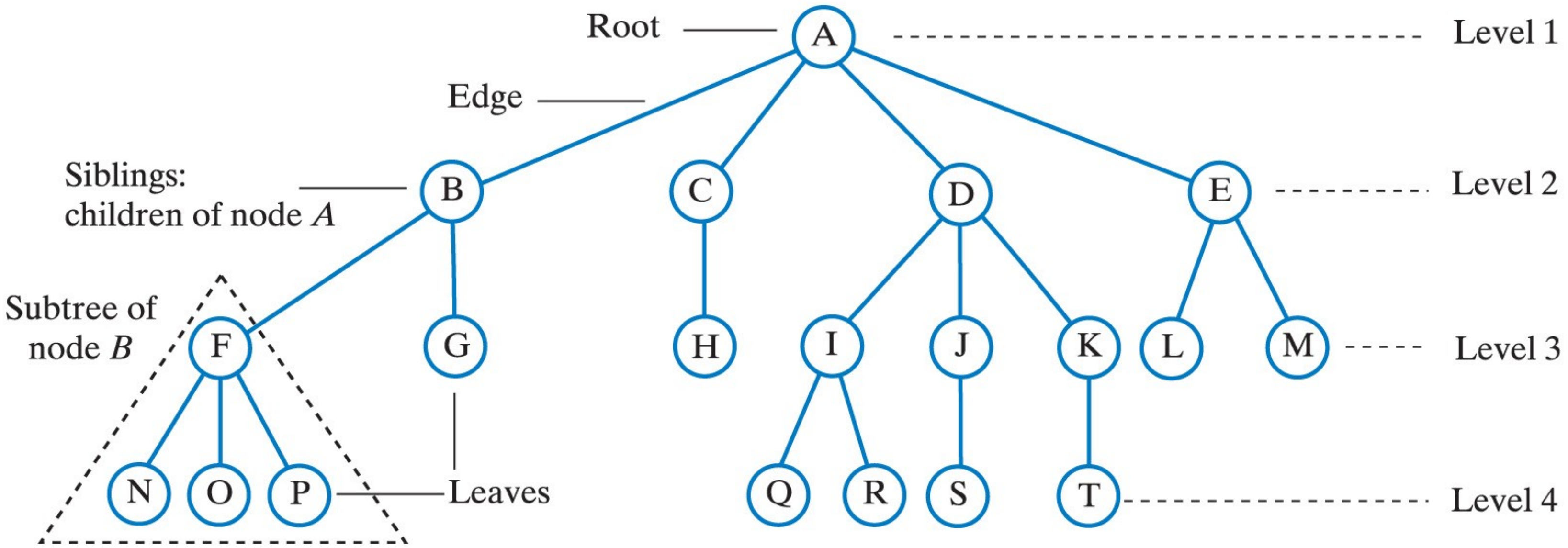
- We can reach any node in a tree by following a **path** that begins at the root and goes from node to node along the edges that connect them.
- The path to *R* is *A D I R*

More Terminology



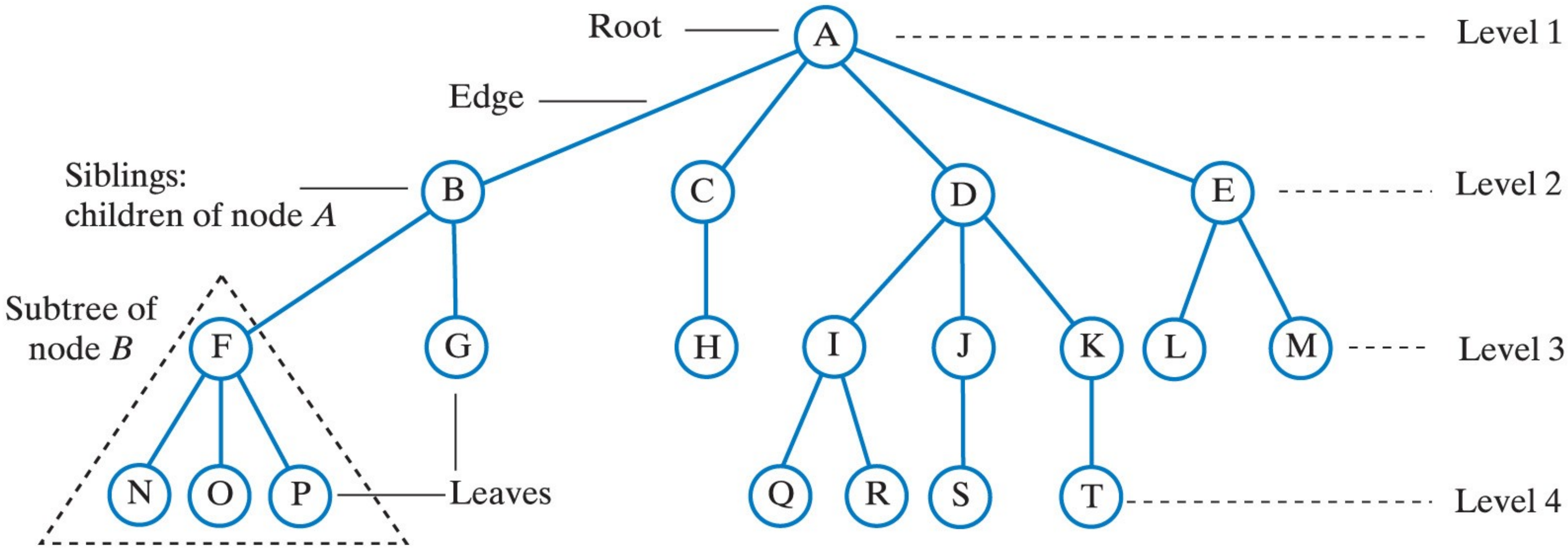
- The **length of a path** is the number of edges that compose it.
- The length of the path to *R* is 3

More Terminology



- The **height** of a tree is the number of levels in the tree, or the number of nodes along the longest path between the root and a leaf.
- This example tree has height 4.

More Terminology



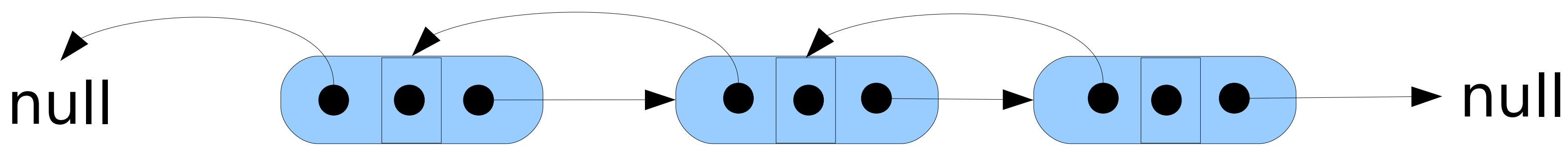
- The path between the root and any other node is unique – there is no circularity in a tree.
- A tree-like data structure that has circularity is called a **graph**.

More Terminology

- In a **general tree**, each node can have any number of children.
- A tree in which the nodes have at most *n* children is called an **n-ary** tree.
- In particular, a tree whose nodes have at most 2 children is called a **binary tree**.

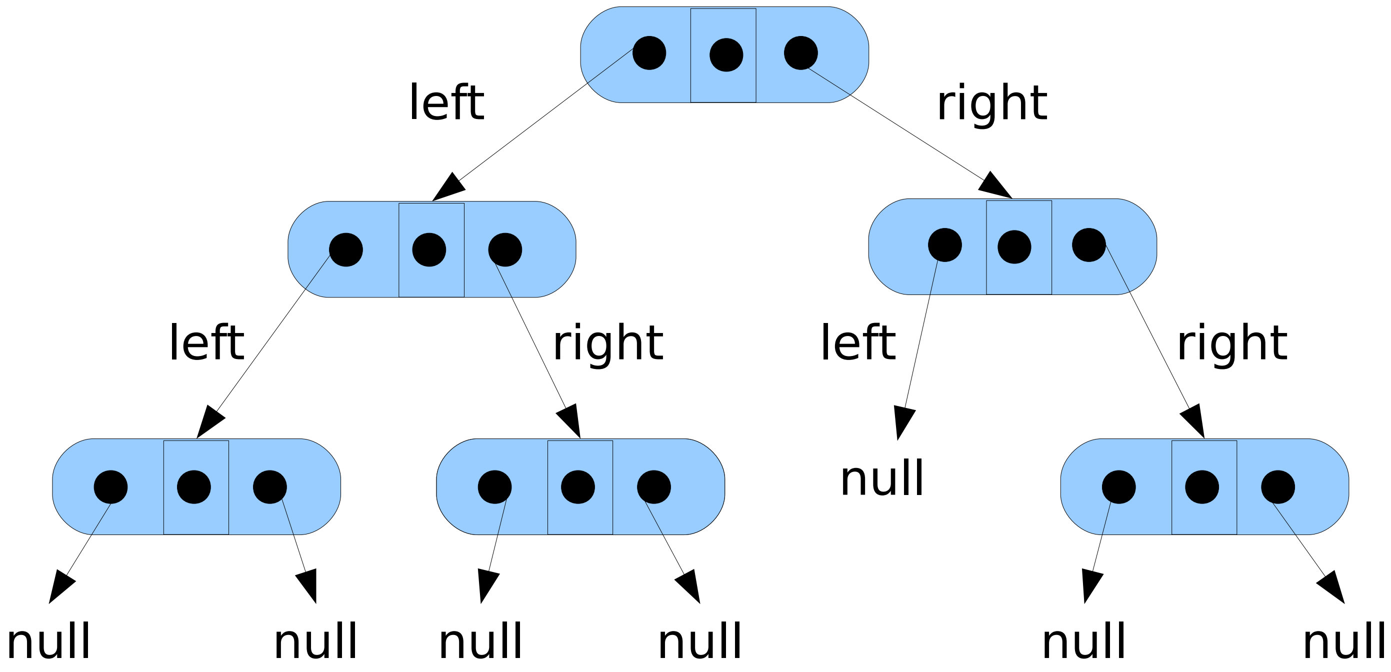
Binary Trees

- We know how to create and link nodes together to form a list.
- A doubly-linked list can be formed by using two references in the node object – one to point to the next node and one to point to the previous node in the list.



Binary Trees

- By using a similar node class with 2 references, we can form binary trees.
- Each node contains a reference to its *left* and its *right* subtree.



Tree Traversals

- When we iterate a linear data structure, such as a list, the order in which to process the data is clear.
- When we iterate a tree it is called a **traversal**, and the order of processing the nodes is not unique.
- Each node must be **visited** (i.e. processed in some way) exactly once.

Tree Traversals

- We know that the subtrees of the root of a binary tree are also binary trees.
- We can use the recursive nature of a binary tree to define its traversal.
- To visit all the nodes in a binary tree we have to
  - visit the root
  - visit all nodes in the left subtree
  - visit all nodes in the right subtree

Tree Traversals

- Visiting the left subtree before the right subtree is simply a convention.
- The root can be visited before, between, or after its two subtrees.

root	left subtree	left subtree
left subtree	**root**	right subtree
right subtree	right subtree	**root**

Tree Traversals

- The order in which the root node and its subtrees are visited allows us to define the 3 most common tree traversals
  - preorder
  - inorder
  - postorder
- The *pre-*, *in-*, and *post-* refer to the visitation of the root node.

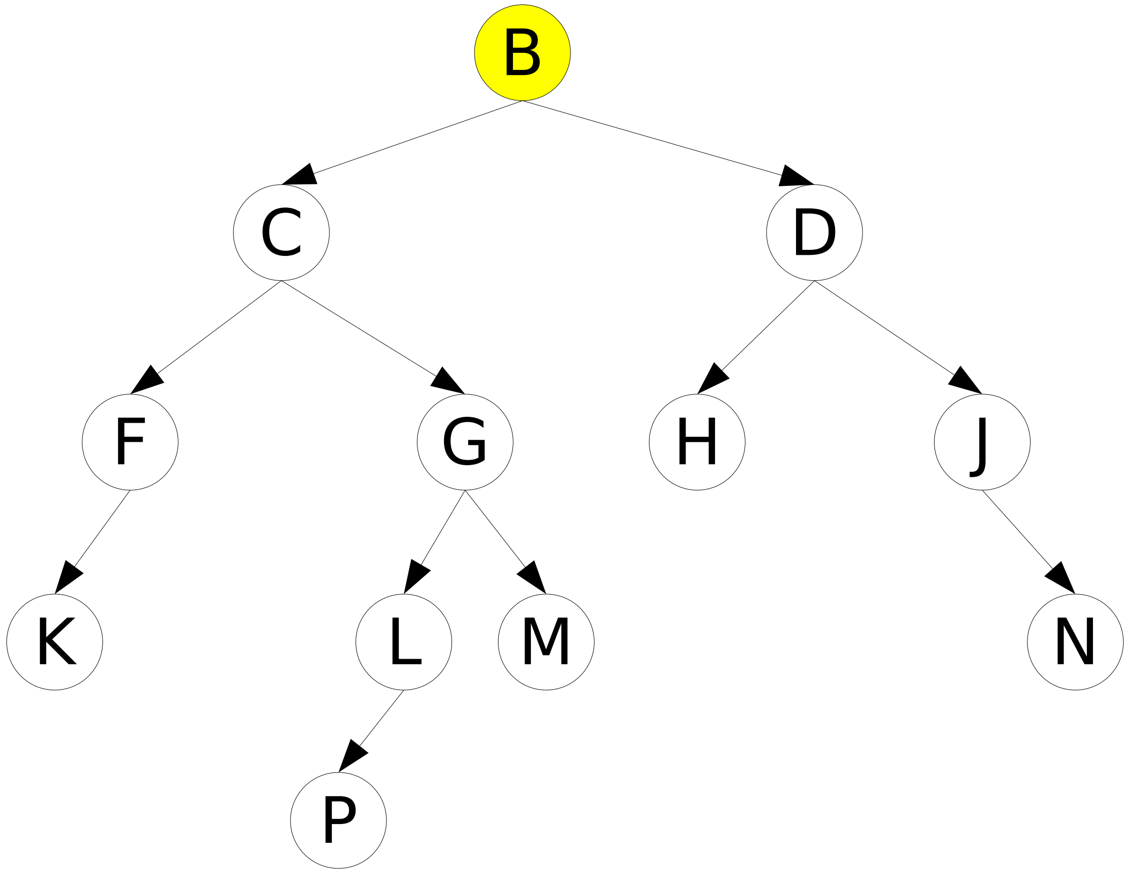
pre-order	**in-order**	**post-order**
root	left subtree	left subtree
left subtree	**root**	right subtree
right subtree	right subtree	**root**

Preorder Traversal

- In a preorder traversal, the root node is visited <u>before</u> its subtrees:
  - **visit the root**
  - visit all nodes in the left subtree
  - visit all nodes in the right subtree

Preorder Traversal Example

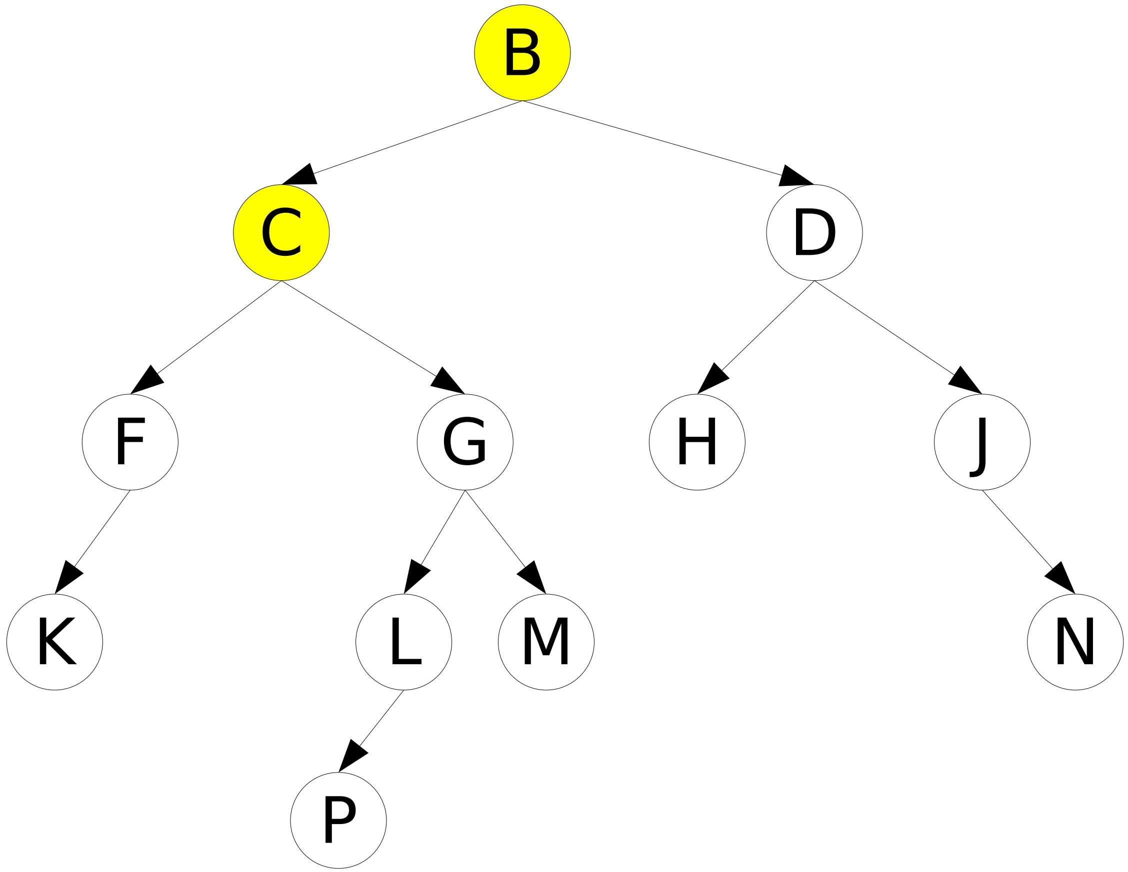
root
left
right



B

Preorder Traversal Example

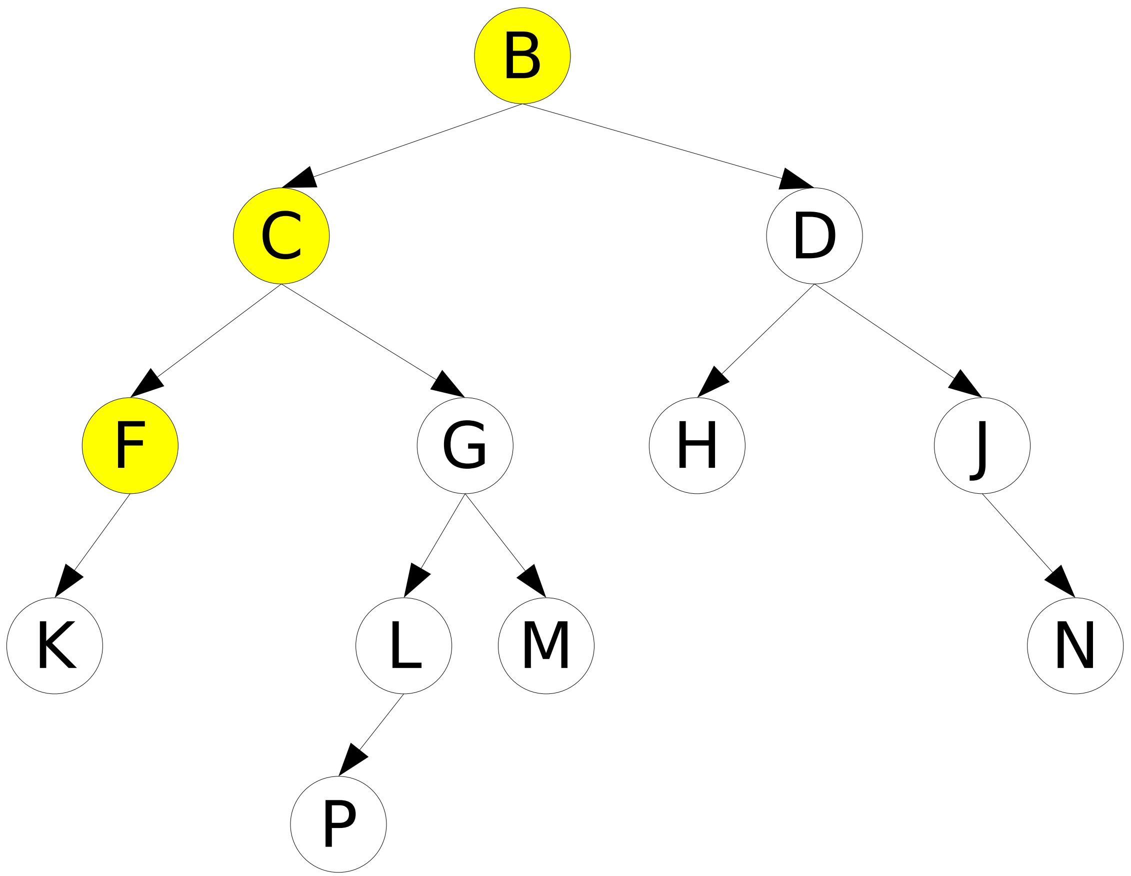
root
left
right



B C

Preorder Traversal Example

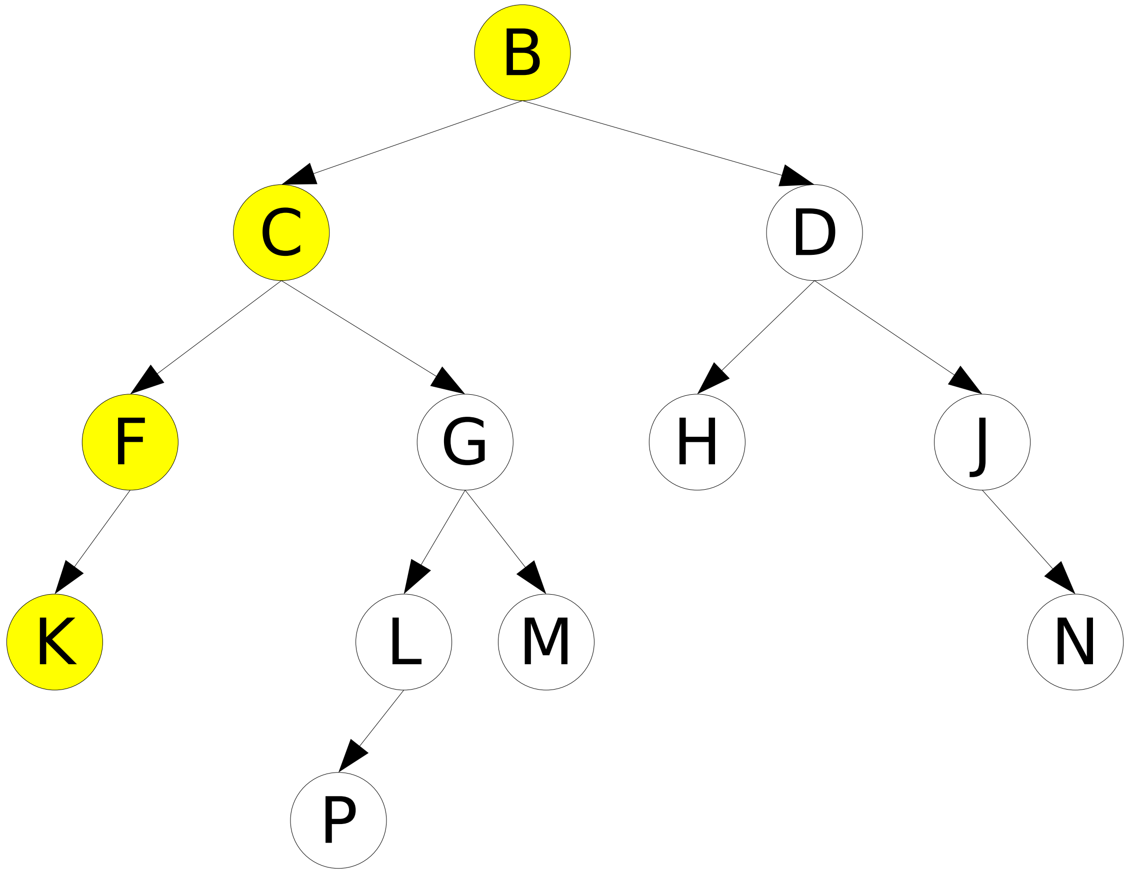
root
left
right



B C F

Preorder Traversal Example

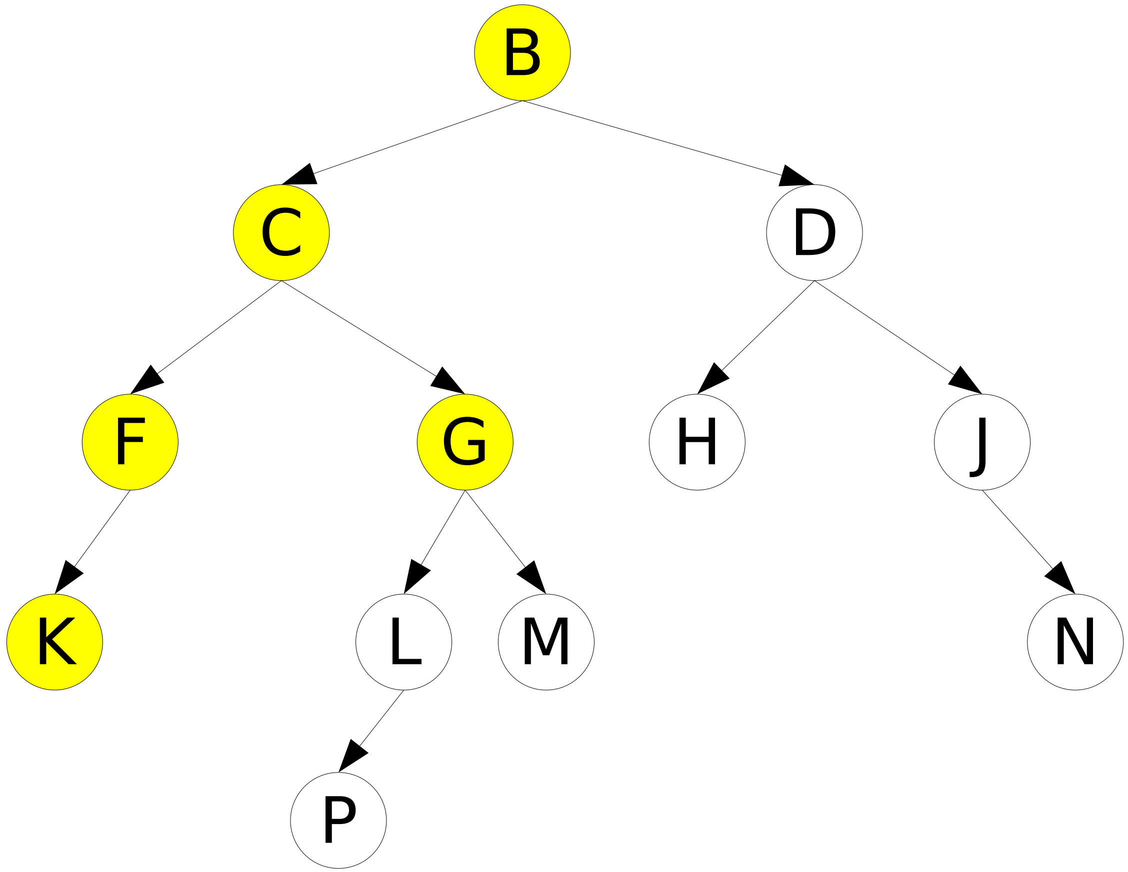
root
left
right



B C F K

Preorder Traversal Example

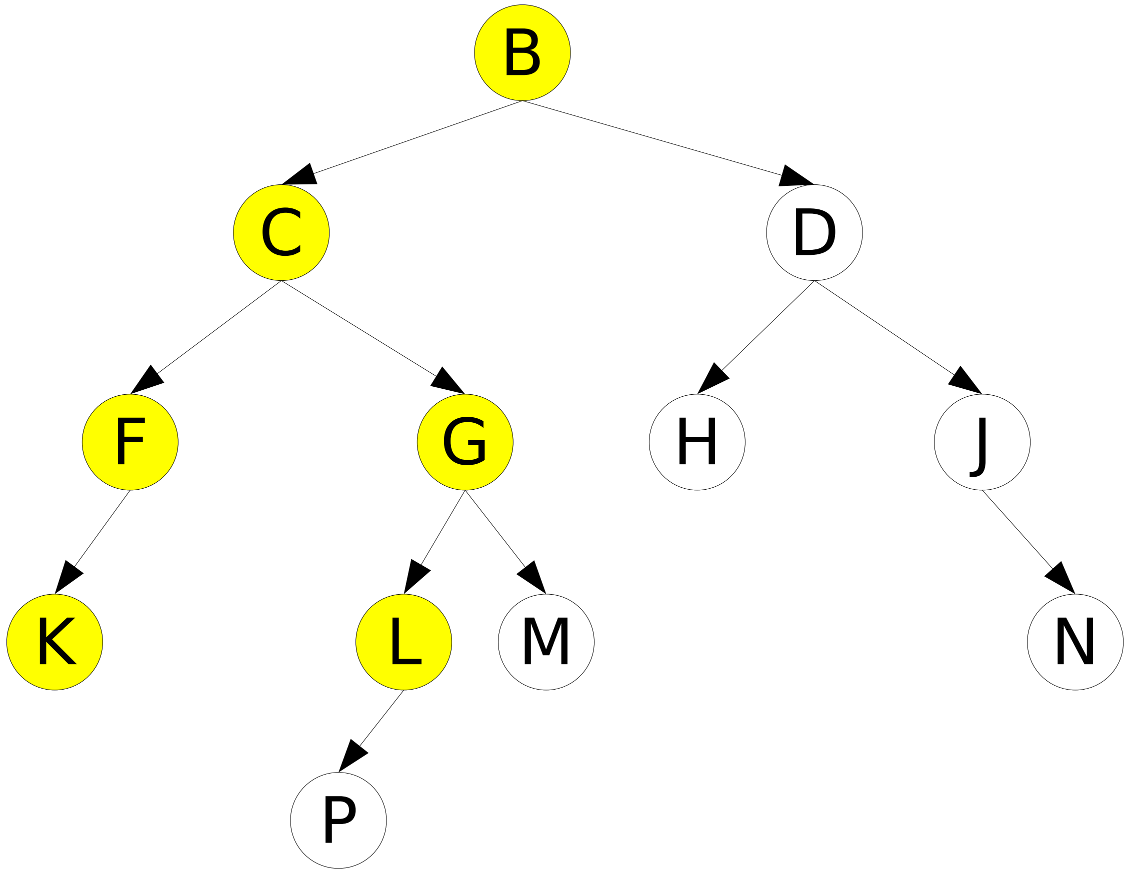
root
left
right



B C F K G

Preorder Traversal Example

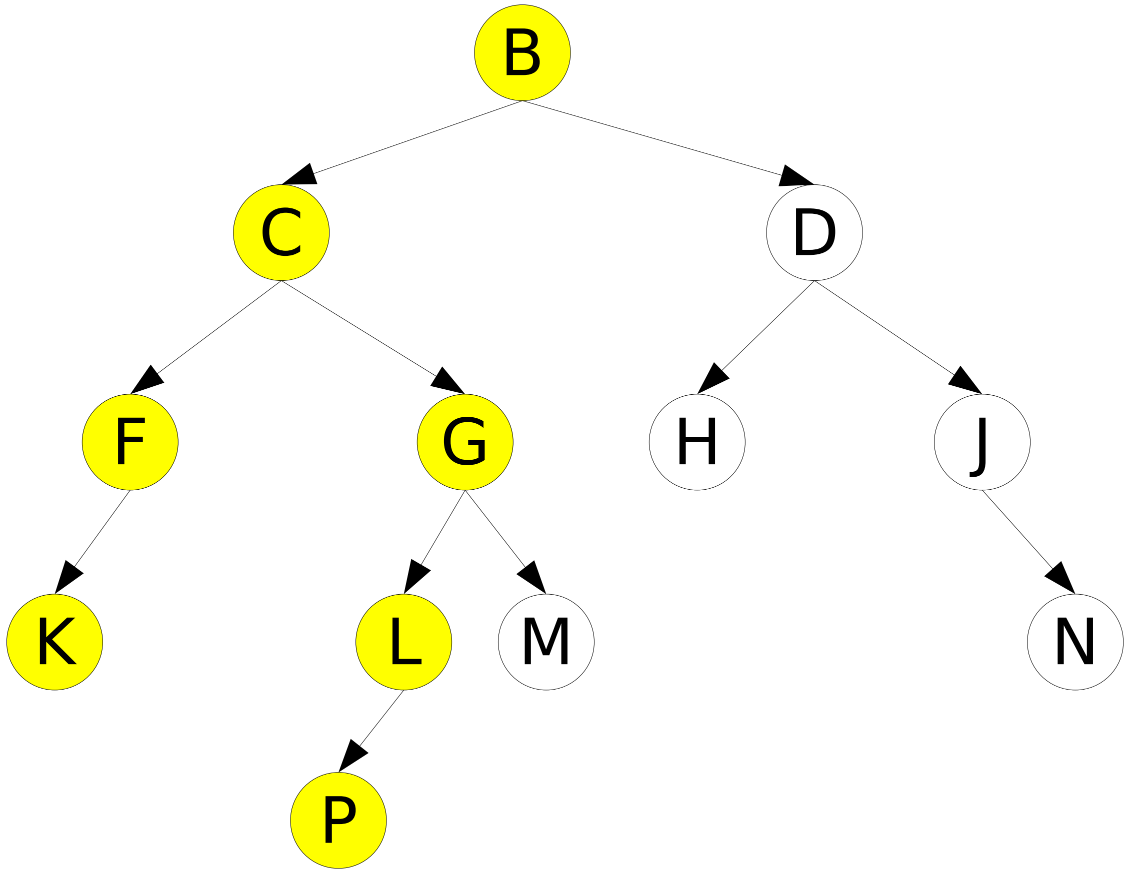
root
left
right



B C F K G L

Preorder Traversal Example

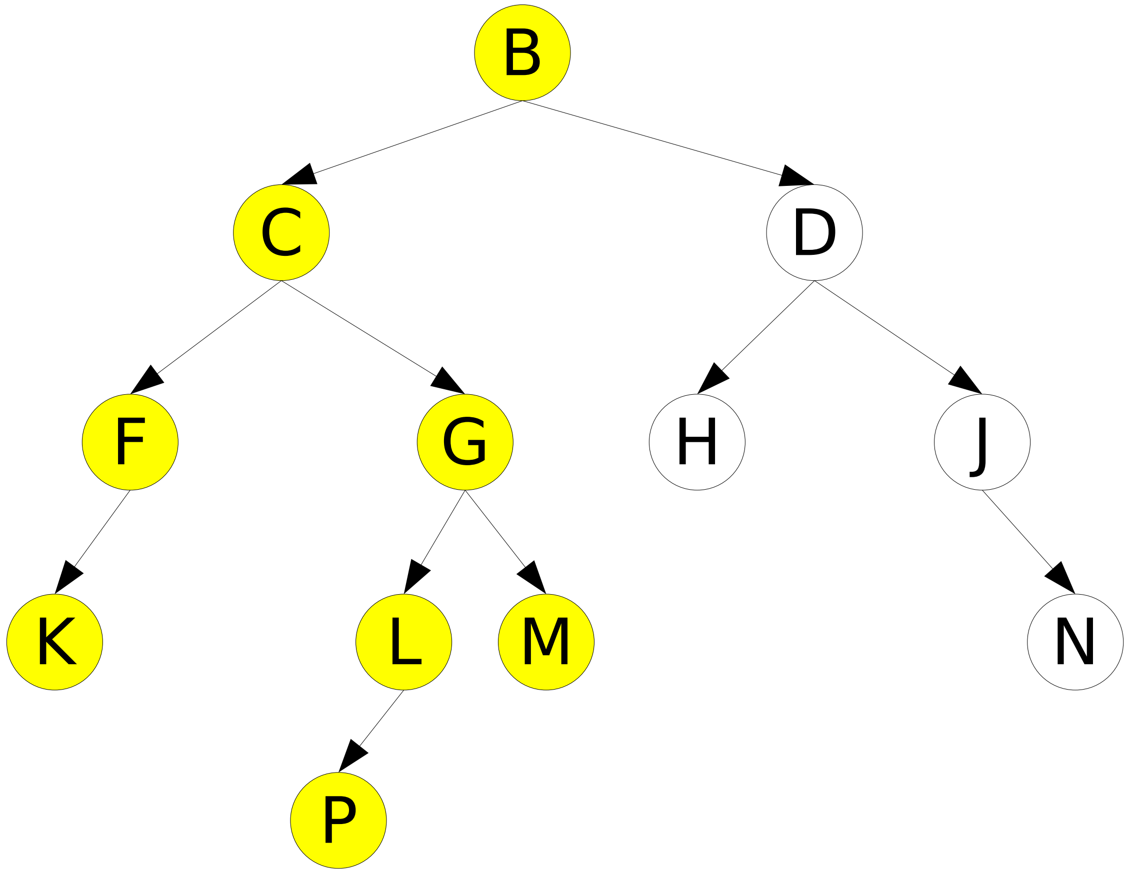
root
left
right



B C F K G L P

Preorder Traversal Example

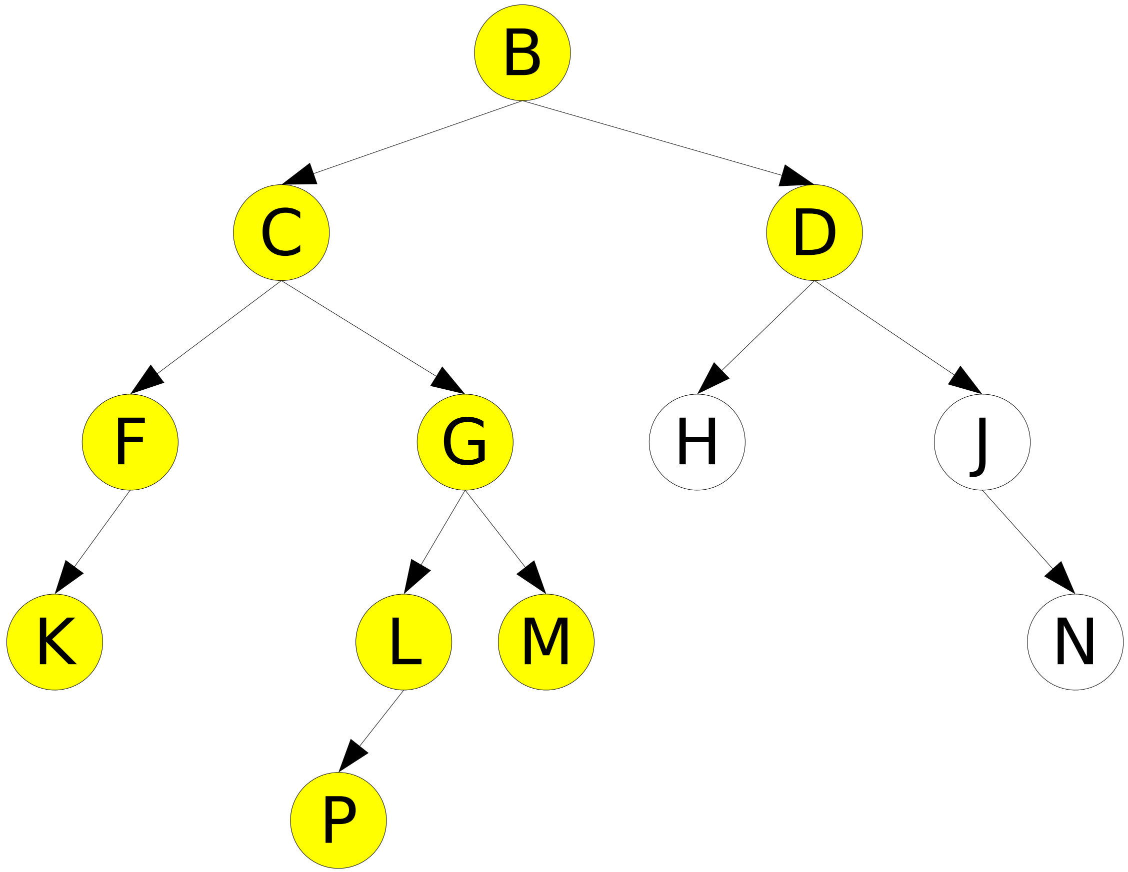
root
left
right



B C F K G L P M

Preorder Traversal Example

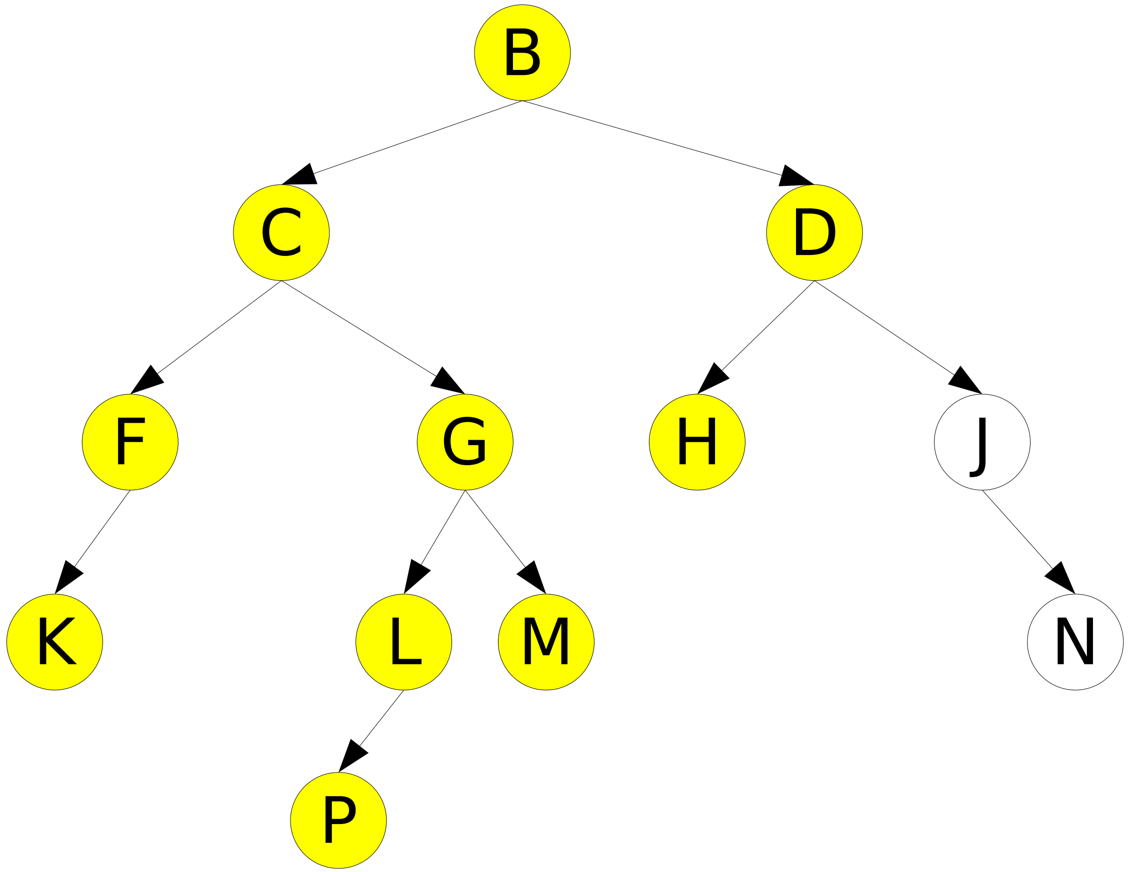
root
left
right



B C F K G L P M D

Preorder Traversal Example

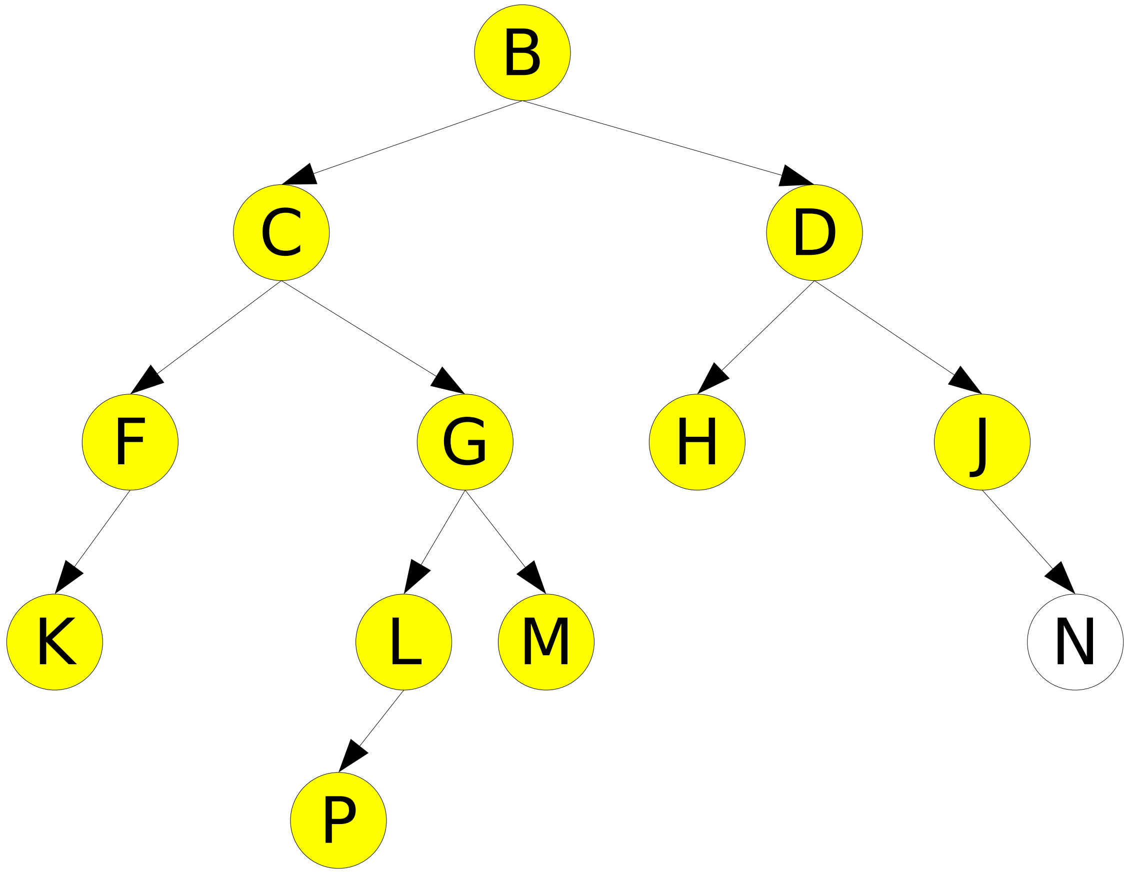
root
left
right



B C F K G L P M D H

Preorder Traversal Example

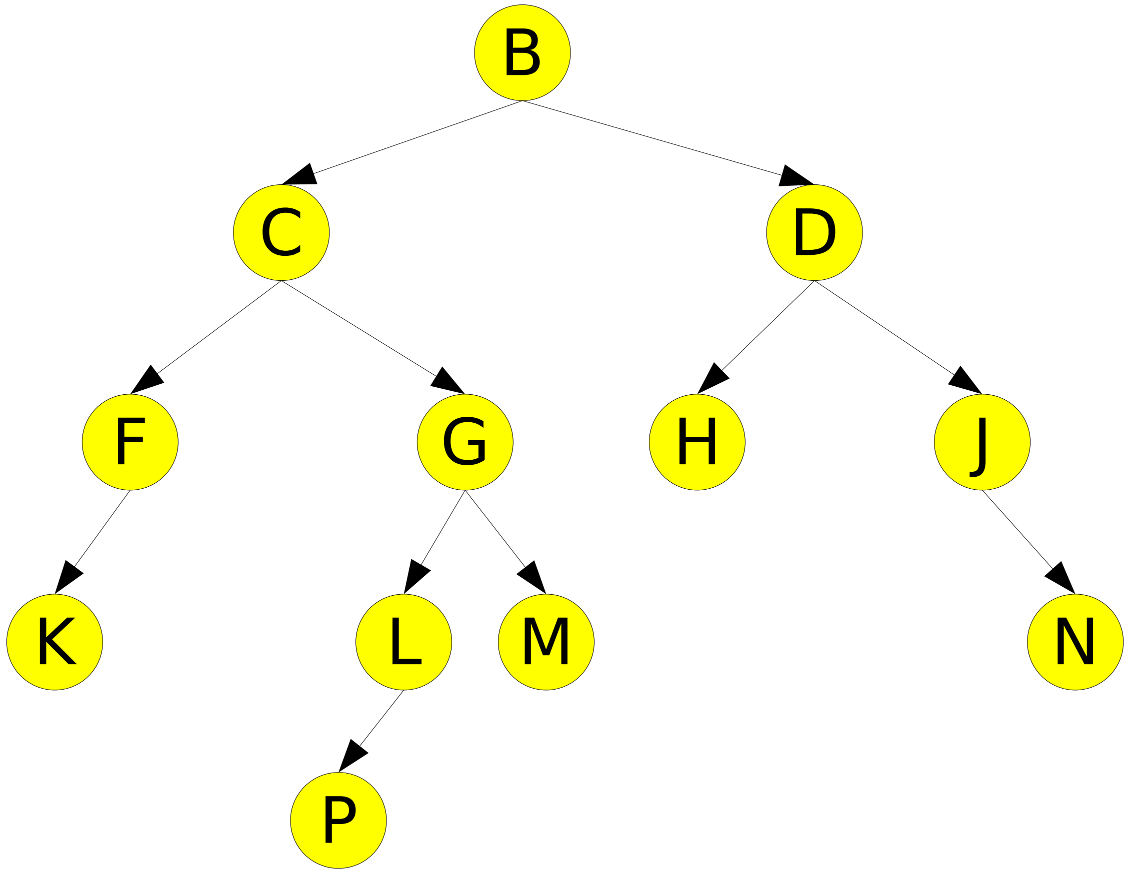
root
left
right



B C F K G L P M D H J

Preorder Traversal Example

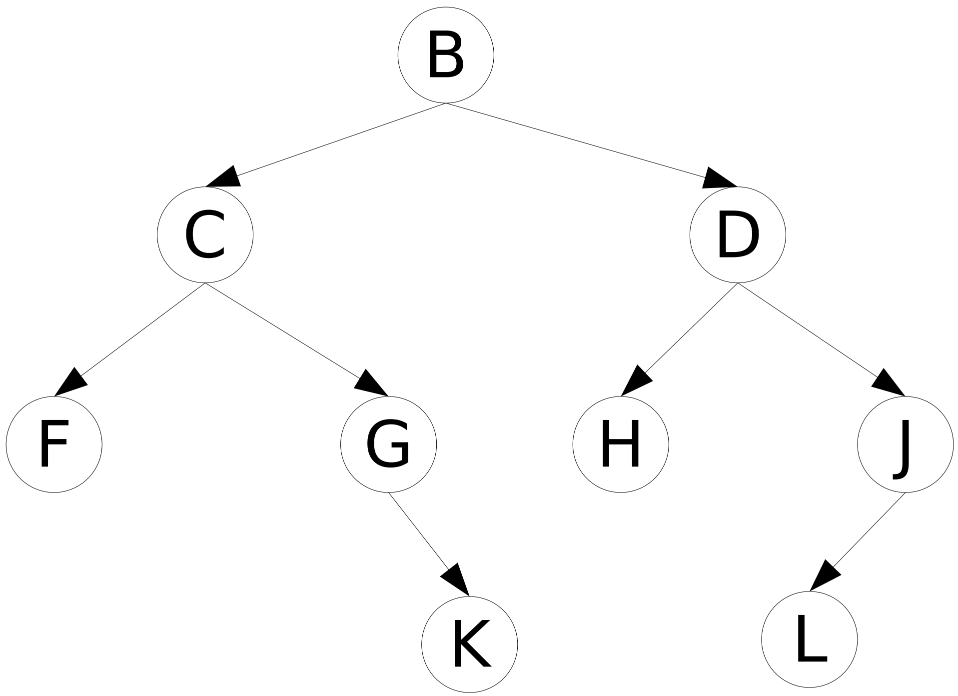
root
left
right



B C F K G L P M D H J N

Preorder Traversal Exercise

What is the preorder traversal of this tree?



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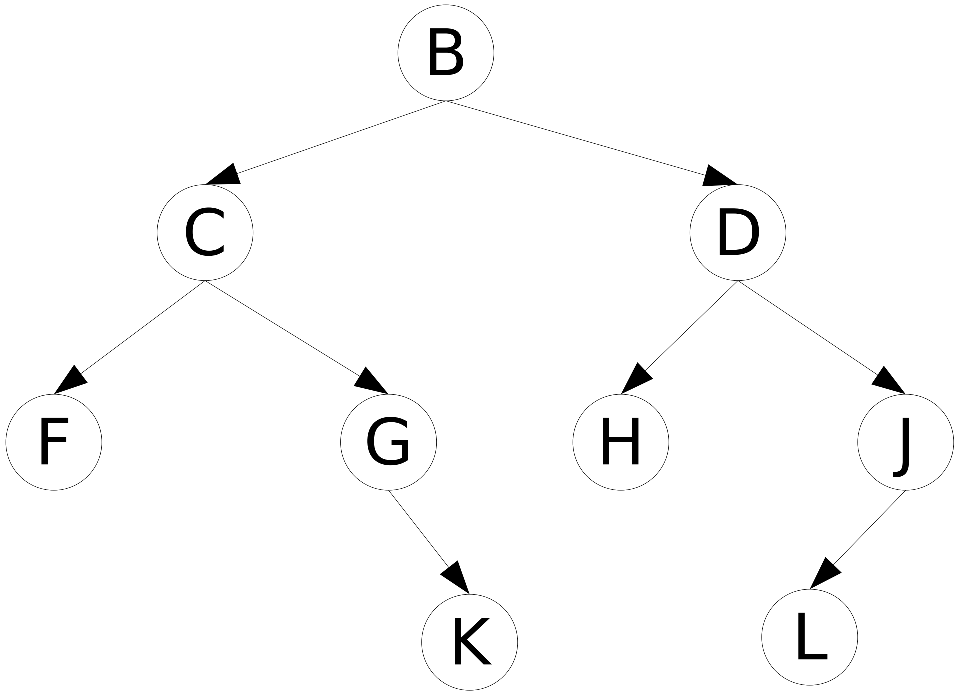


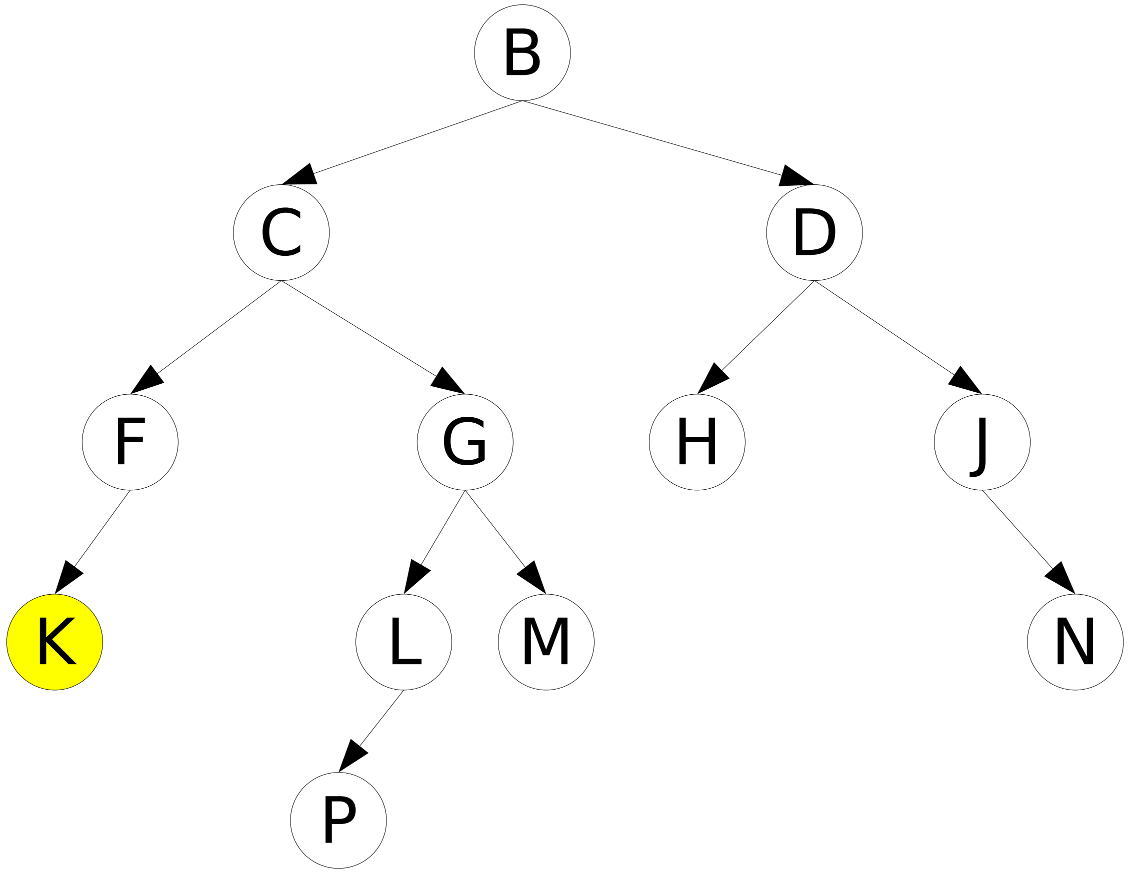
B C F G K D H J L

Inorder Traversal

- In an inorder traversal, the root node is visited <u>between</u> its subtrees:
  - visit all nodes in the left subtree
  - **visit the root**
  - visit all nodes in the right subtree

Inorder Traversal Example

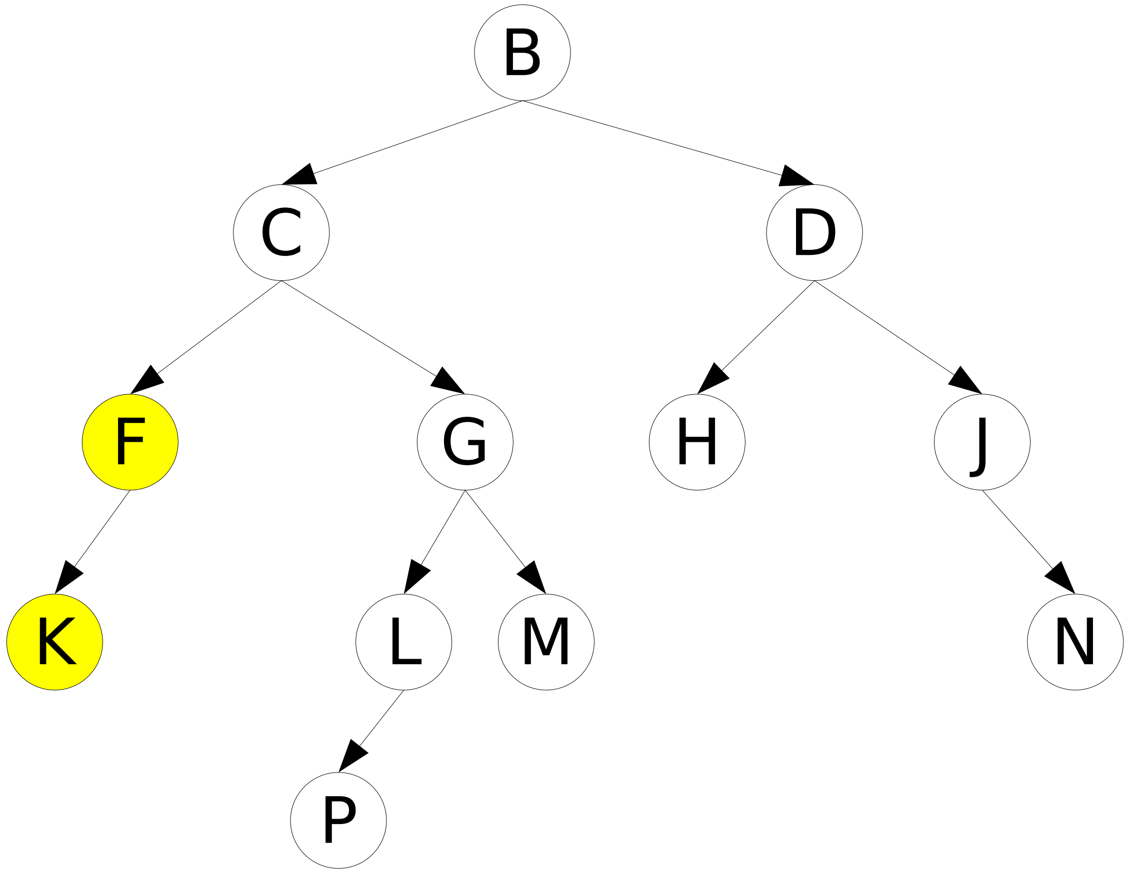
left
root
right



K

Inorder Traversal Example

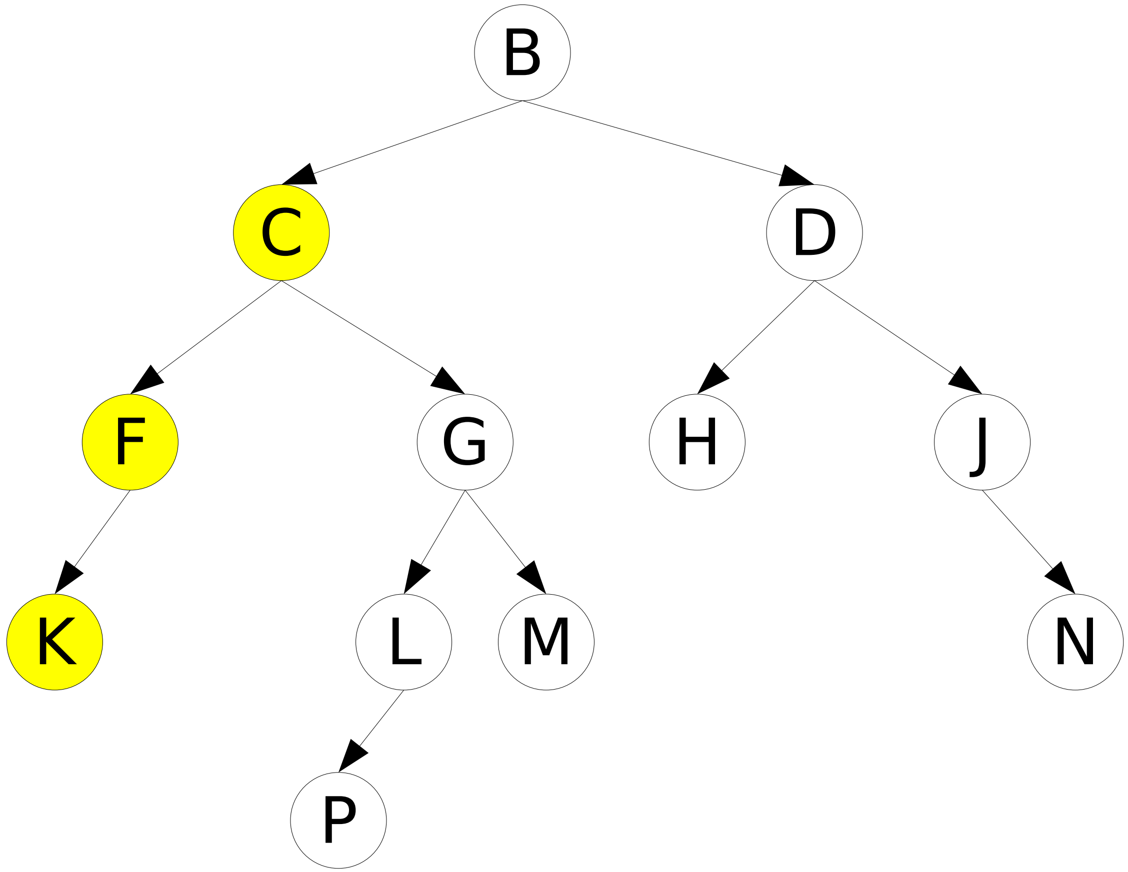
left
root
right



K F

Inorder Traversal Example

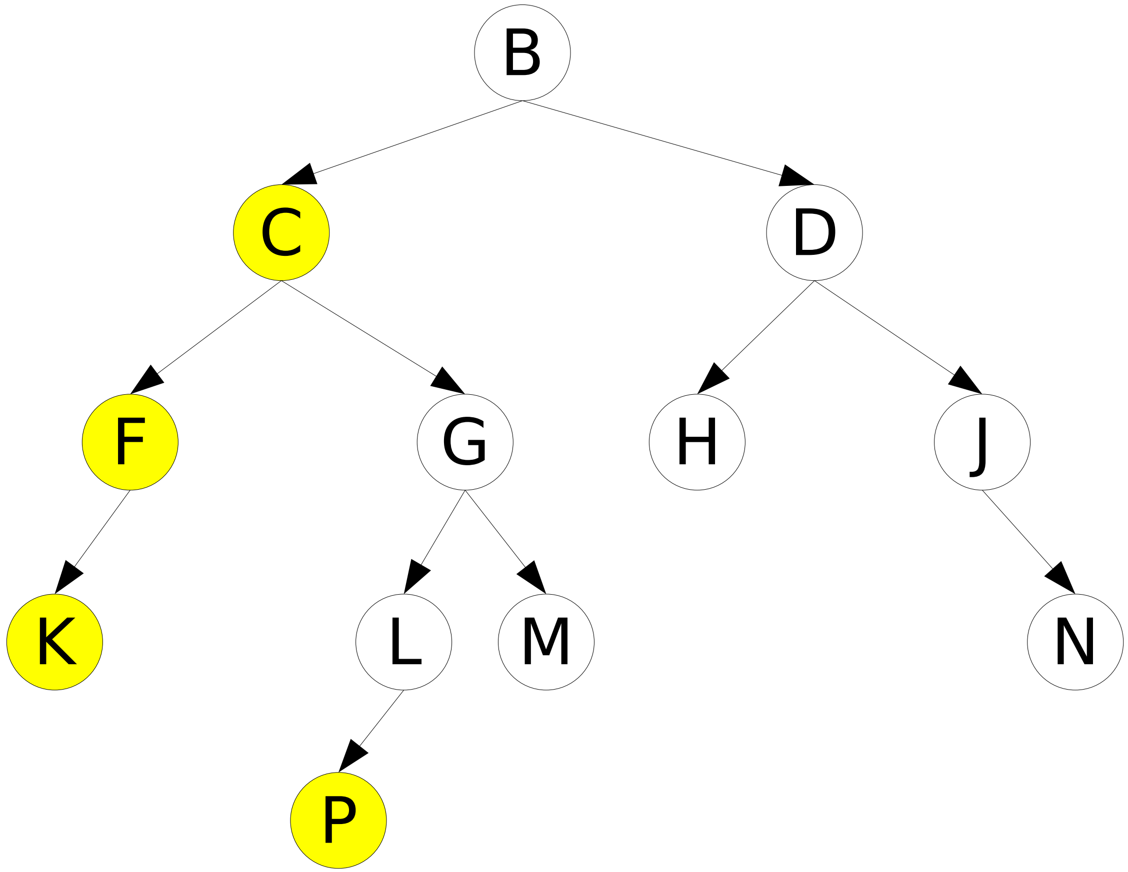
left
root
right



K F C

Inorder Traversal Example

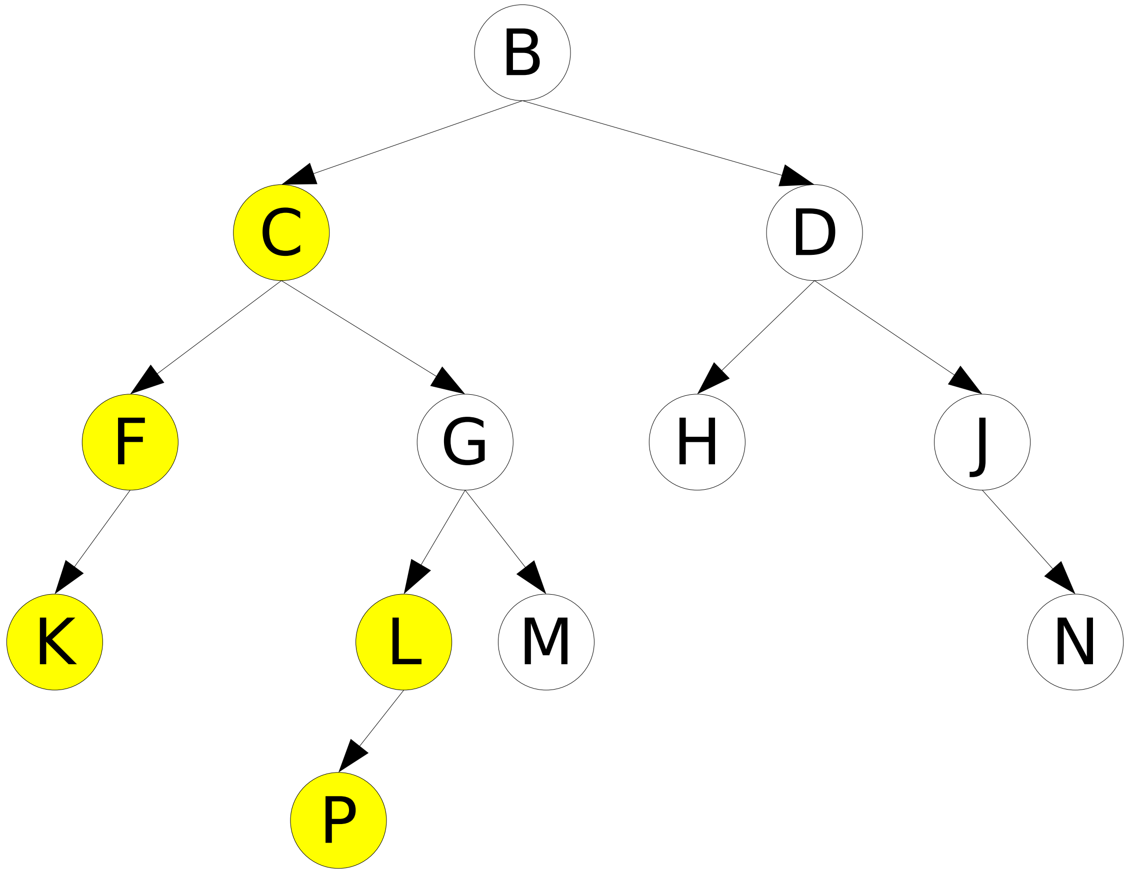
left
root
right



K F C P

Inorder Traversal Example

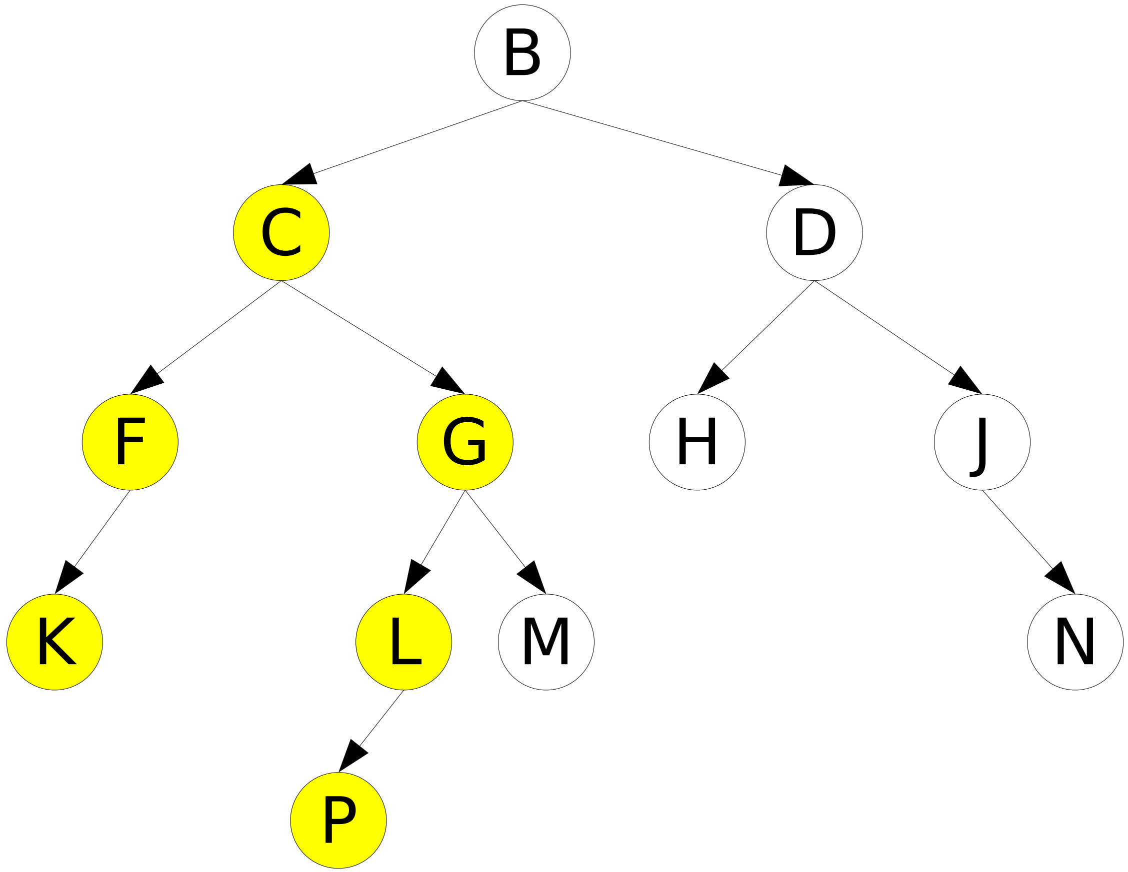
left
root
right



K F C P L

Inorder Traversal Example

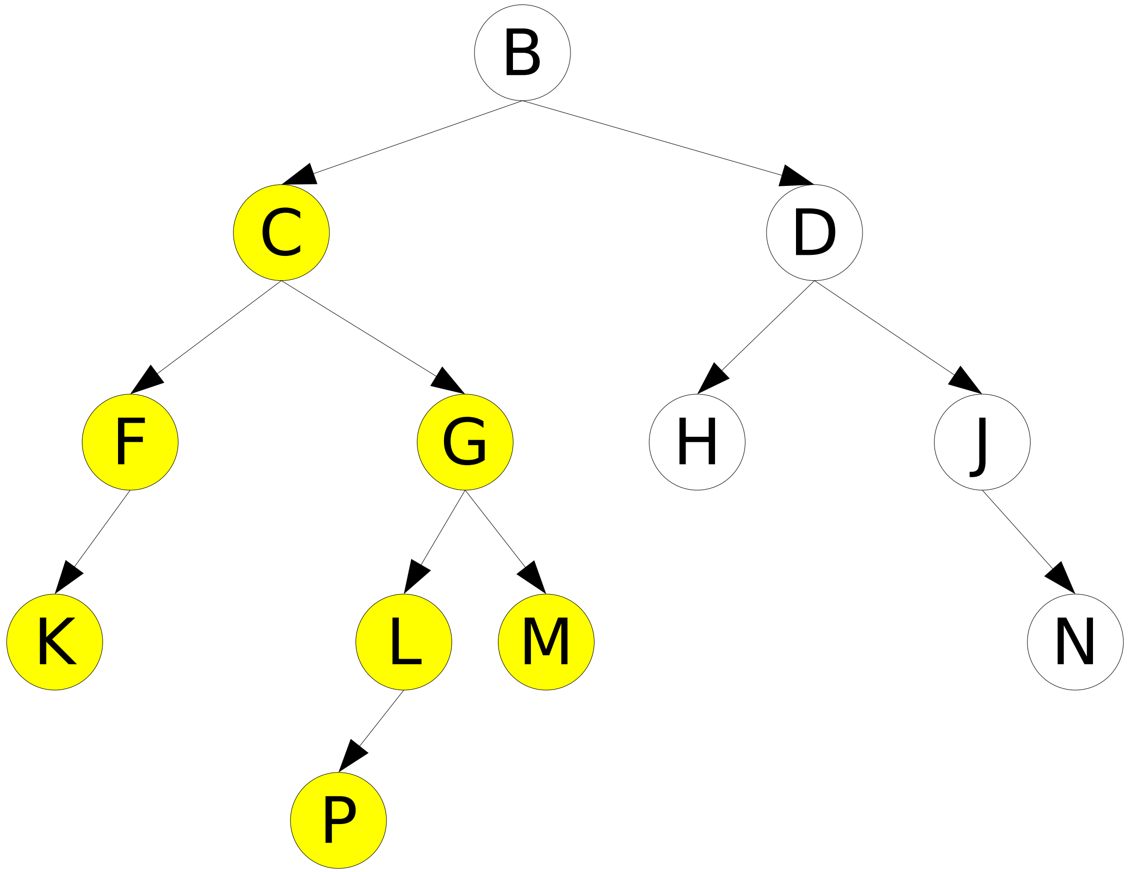
left
root
right



K F C P L G

Inorder Traversal Example

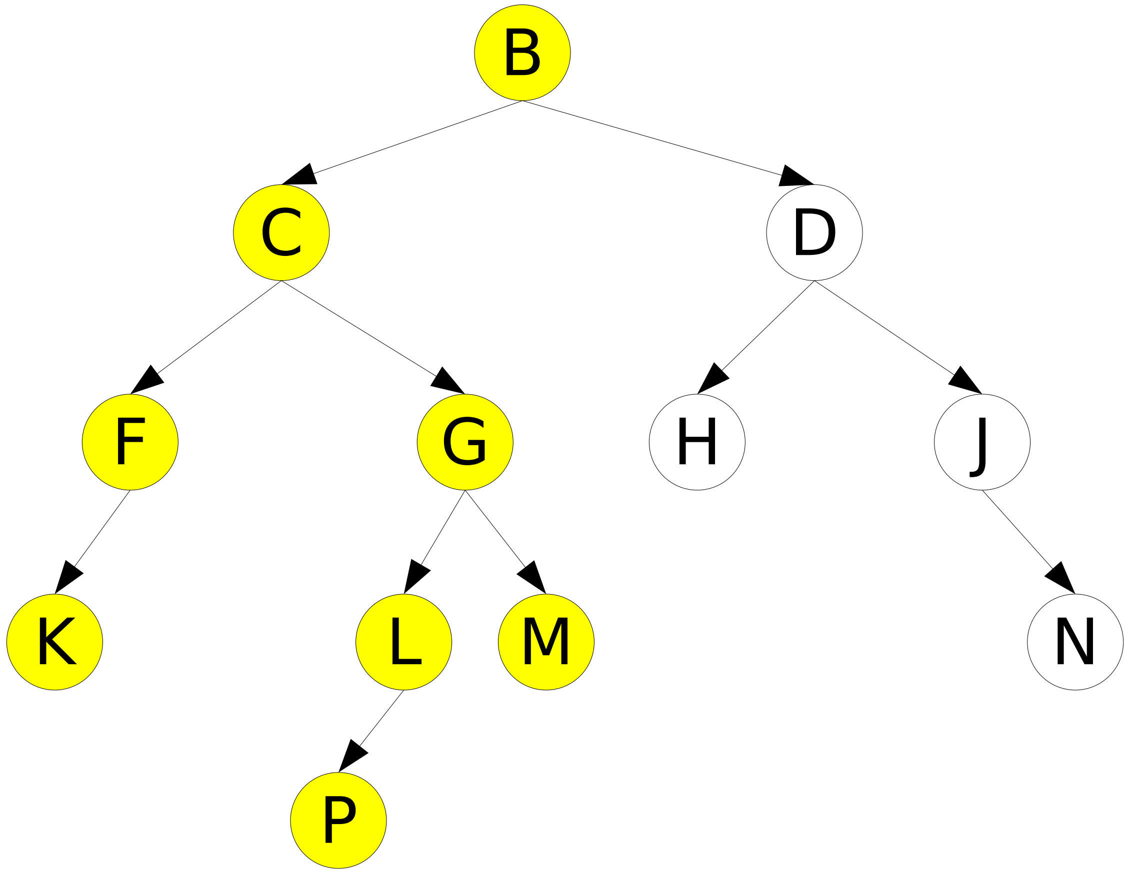
left
root
right



K F C P L G M

Inorder Traversal Example

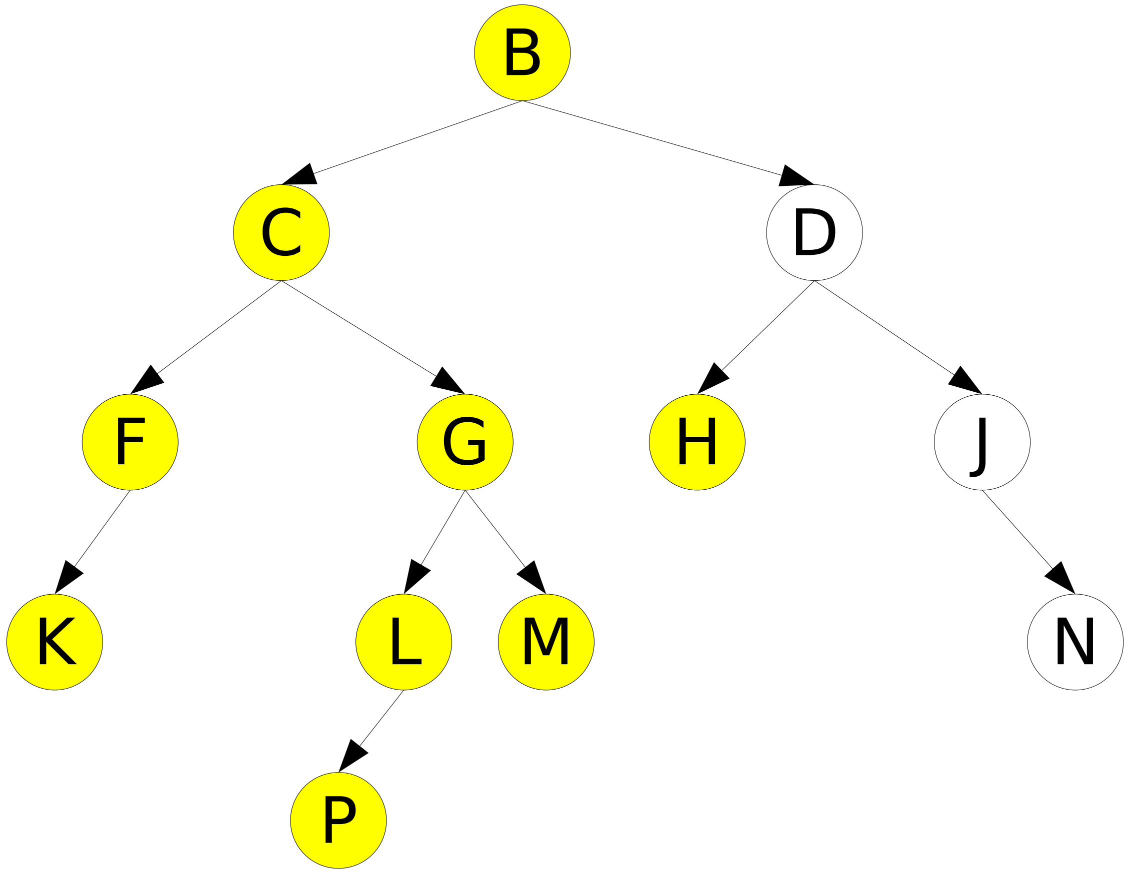
left
root
right



K F C P L G M B

Inorder Traversal Example

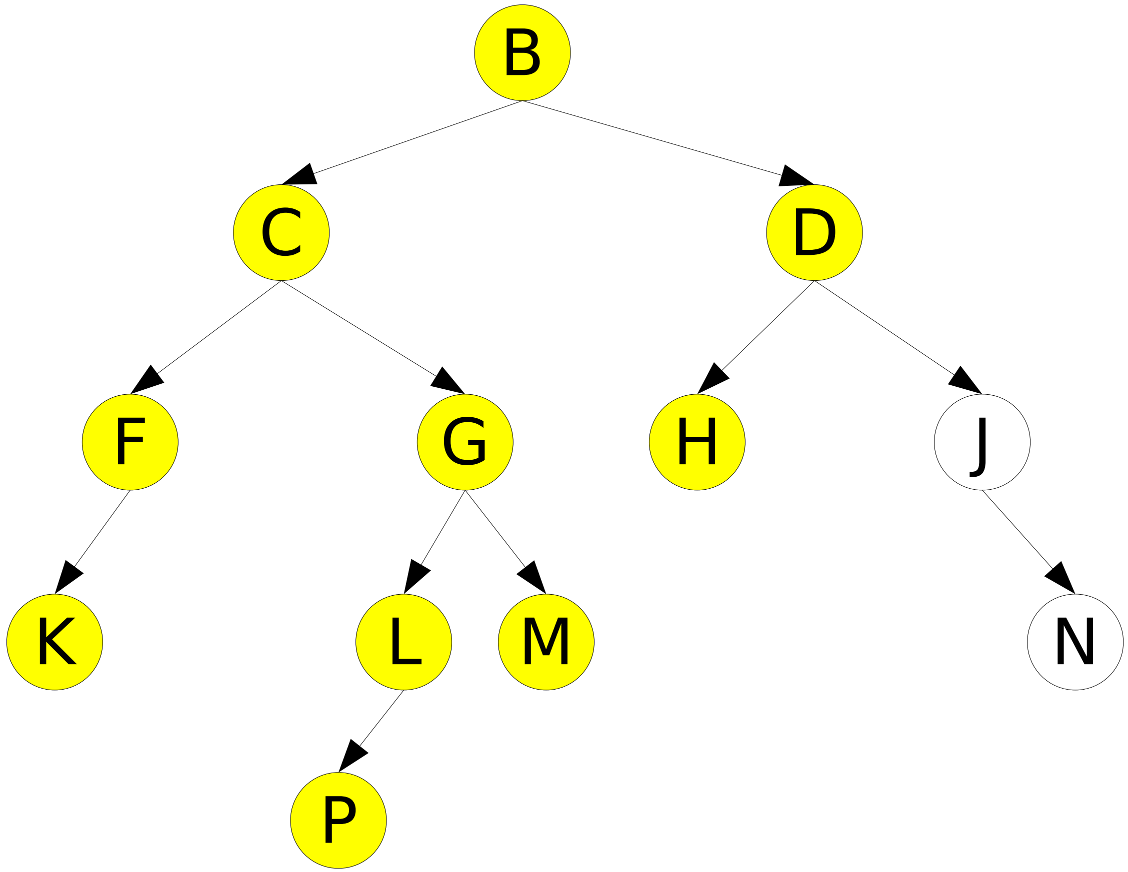
left
root
right



K F C P L G M B H

Inorder Traversal Example

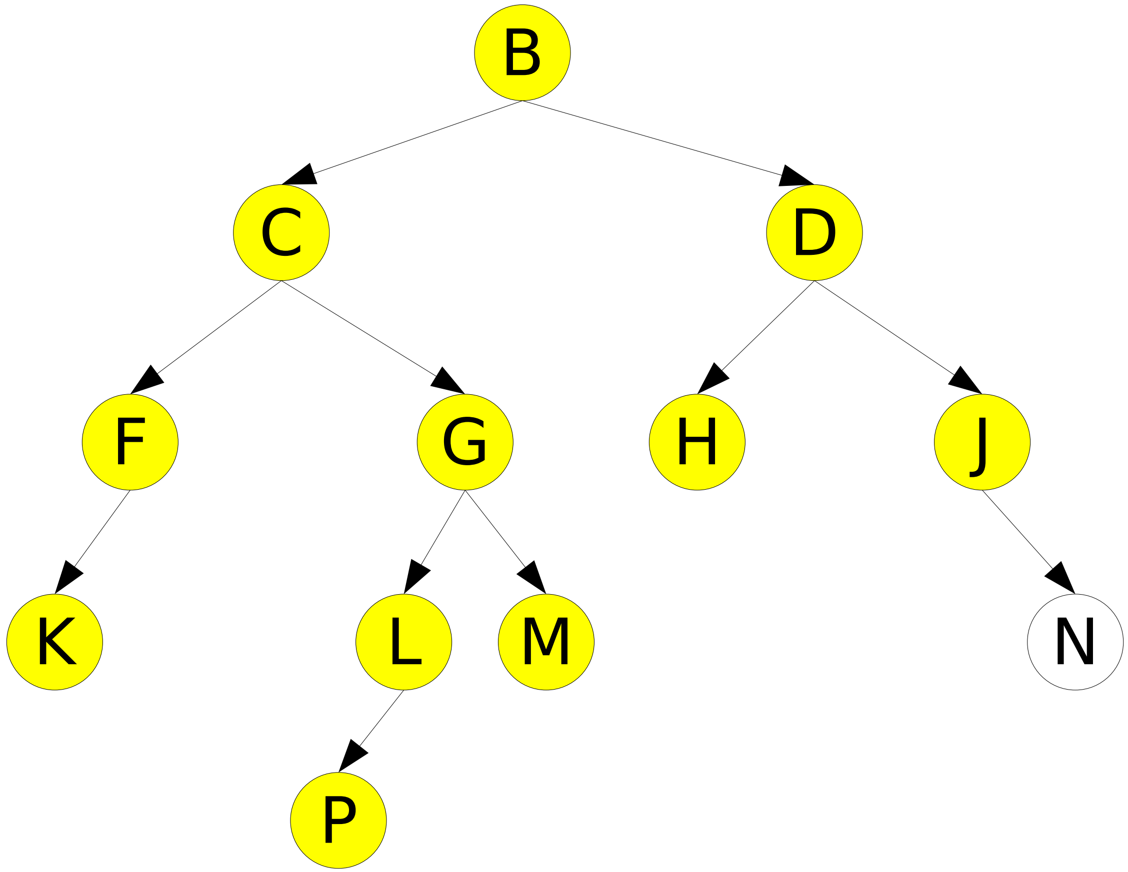
left
root
right



K F C P L G M B H D

Inorder Traversal Example

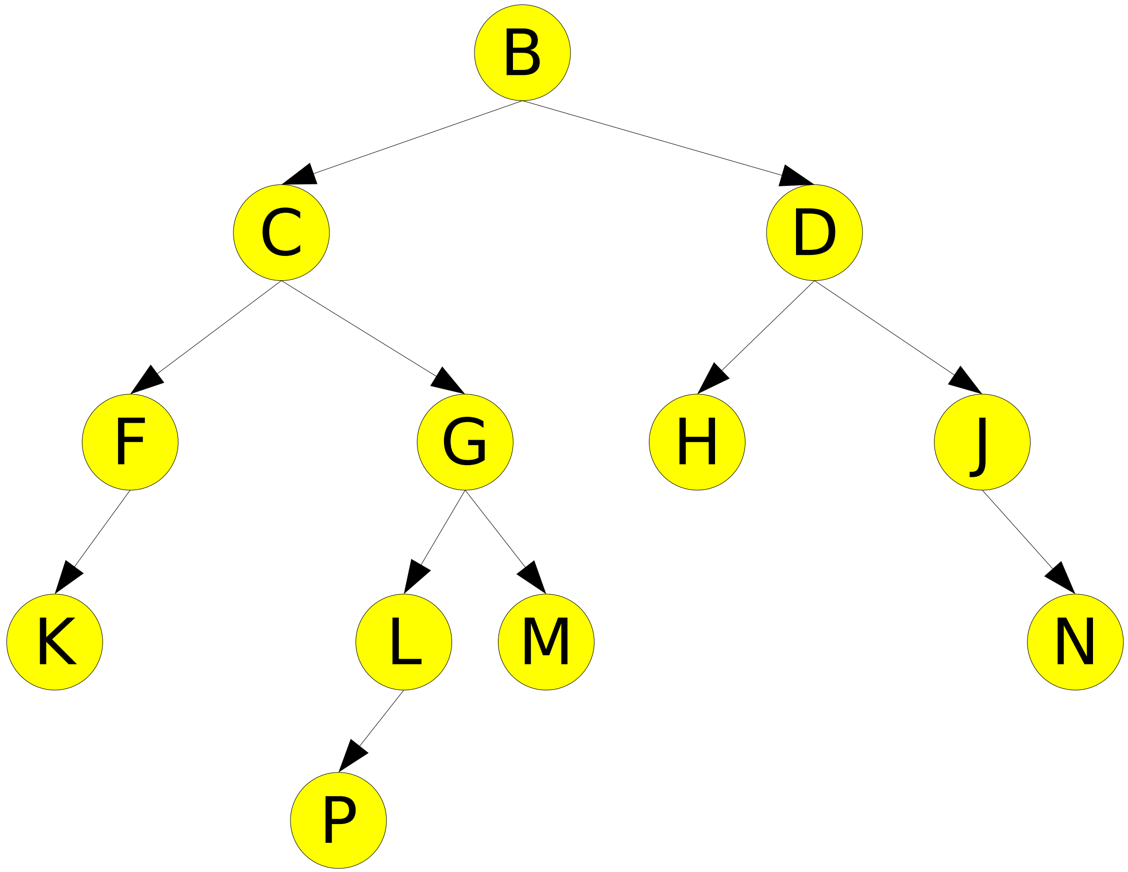
left
root
right



K F C P L G M B H D J

Inorder Traversal Example

left
root
right



K F C P L G M B H D J N

Inorder Traversal Exercise

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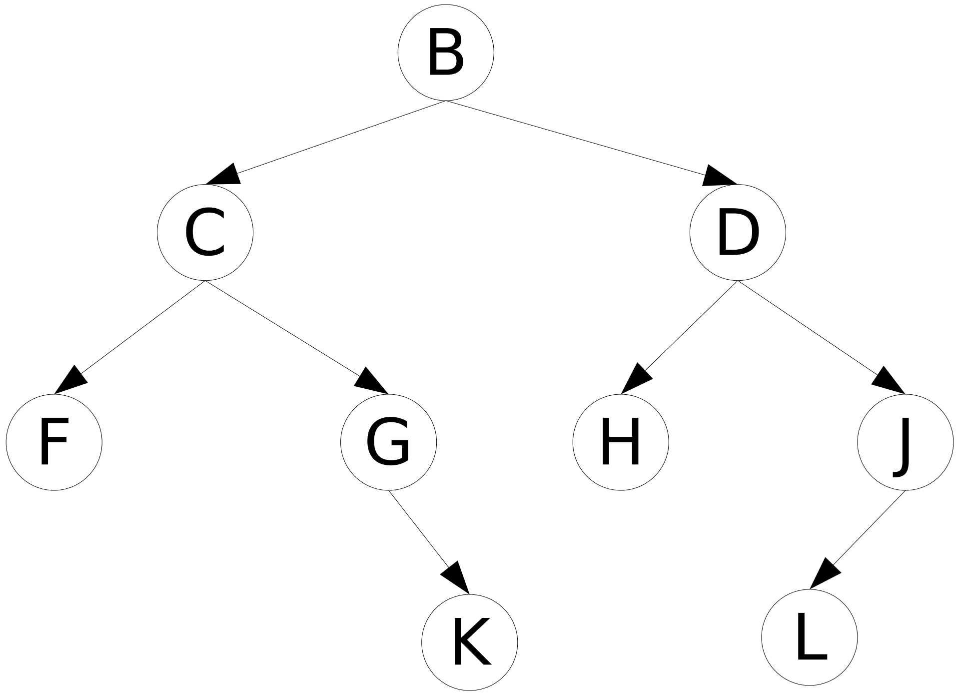

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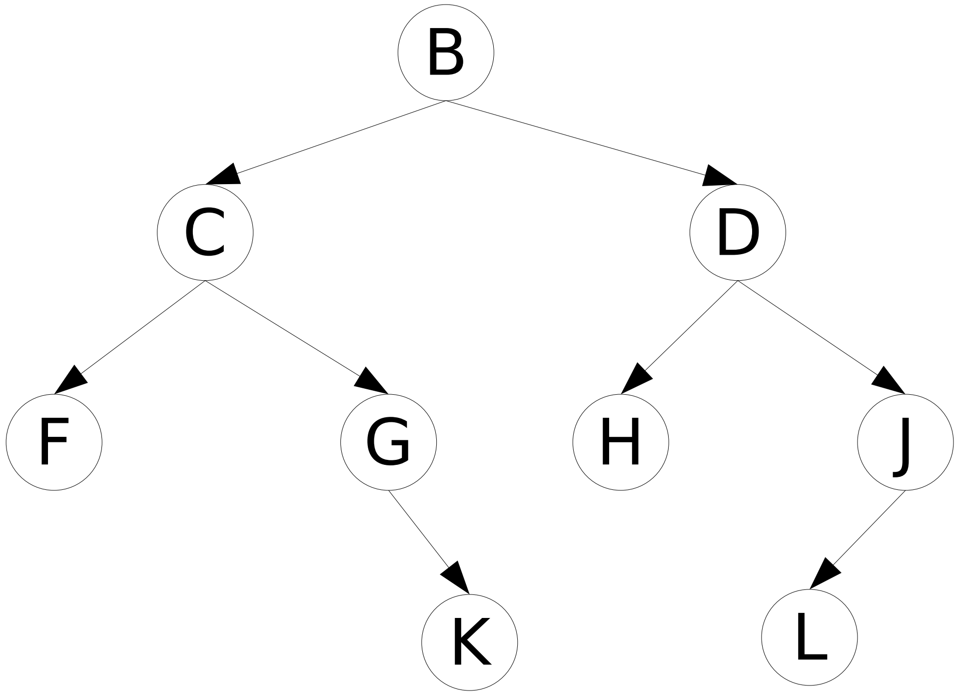


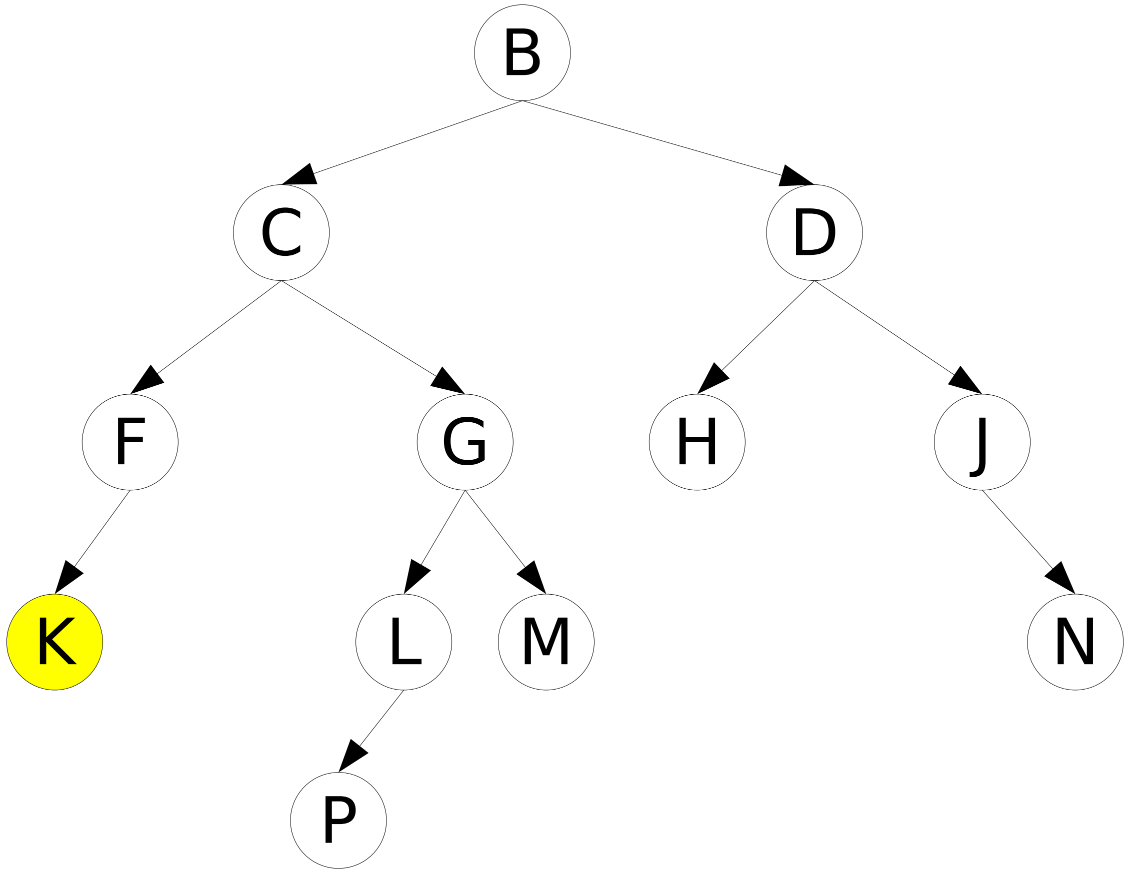
F C G K B H D L J

Postorder Traversal

- In a postorder traversal, the root node is visited after its subtrees:
  - visit all nodes in the left subtree
  - visit all nodes in the right subtree
  - **visit the root**

Postorder Traversal Example

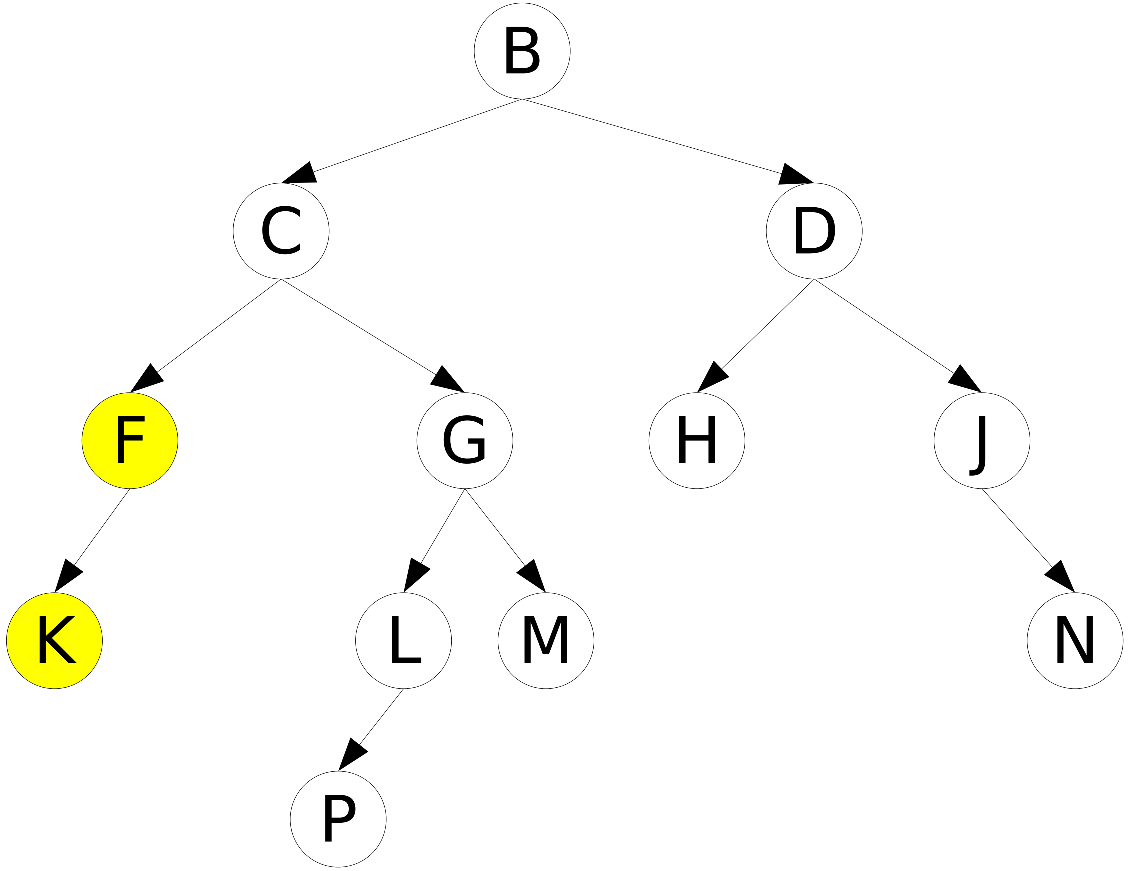
left
right
root



K

Postorder Traversal Example

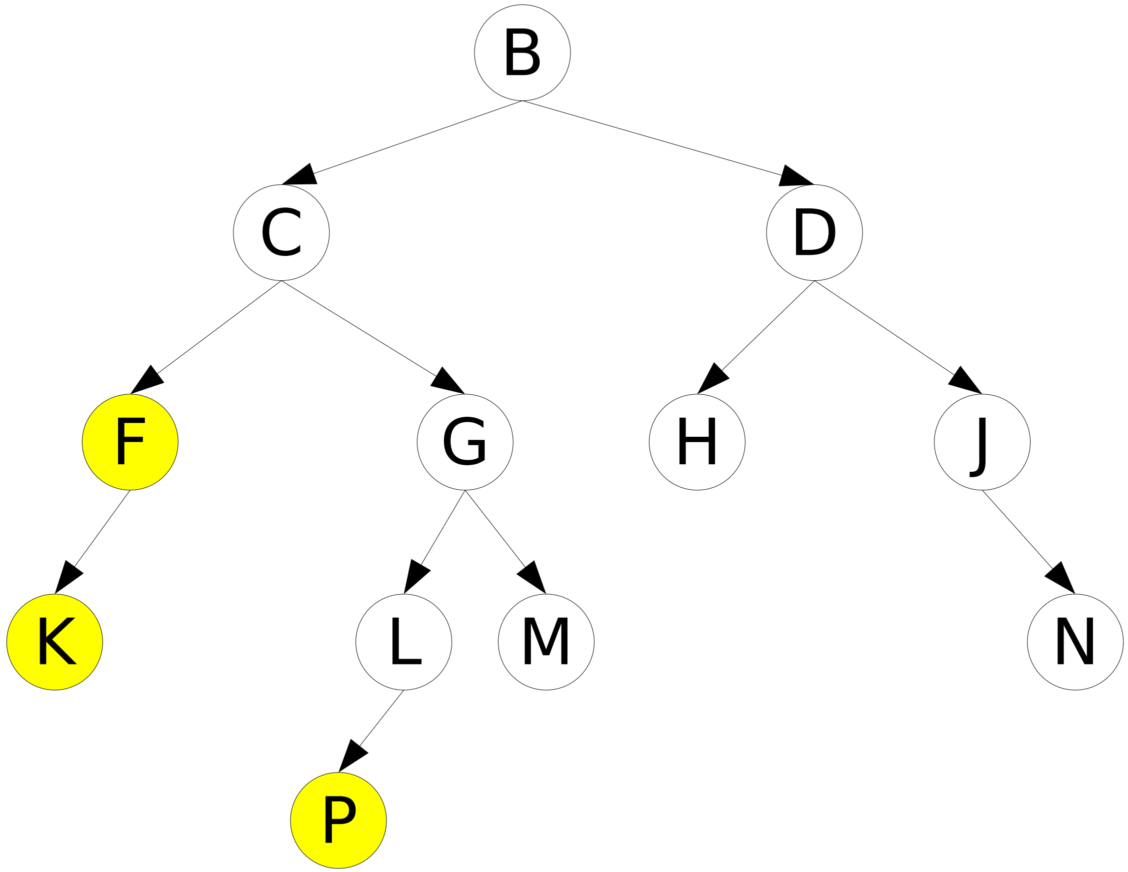
left
right
root



K F

Postorder Traversal Example

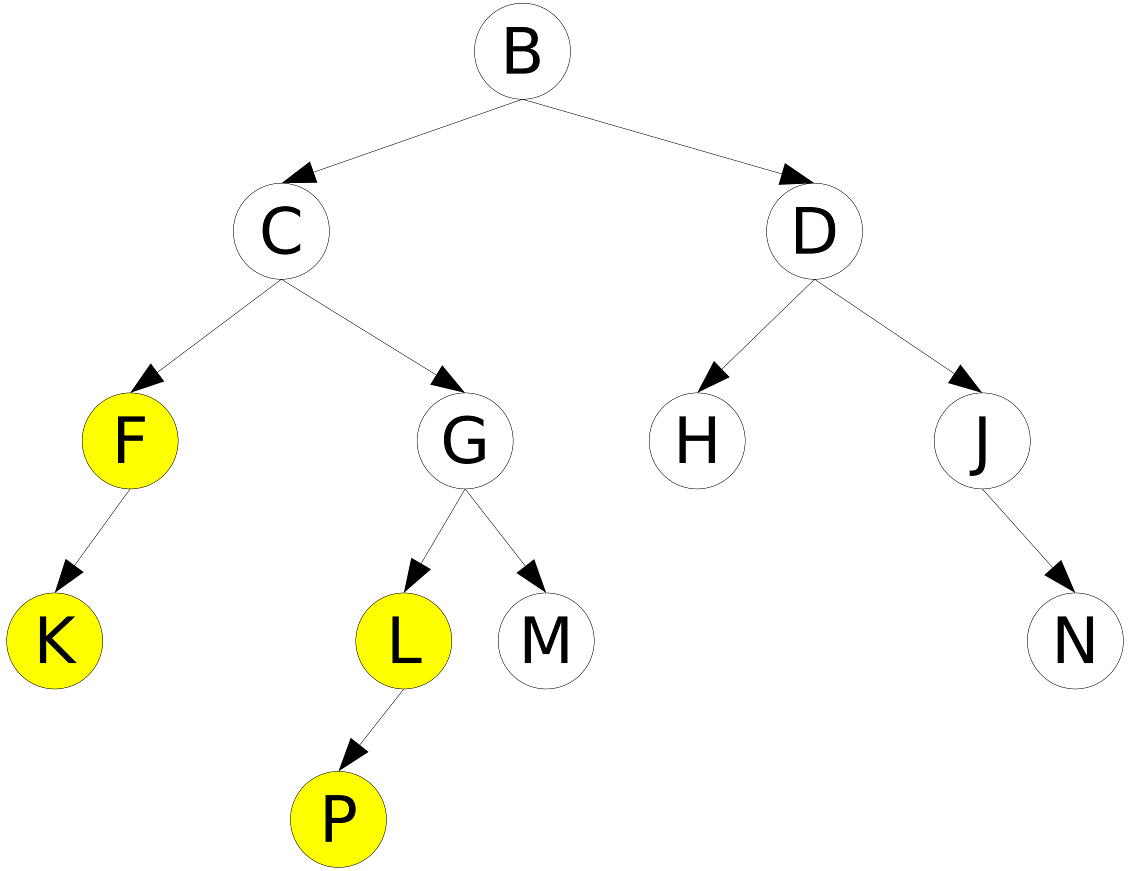
left
right
root



K F P

Postorder Traversal Example

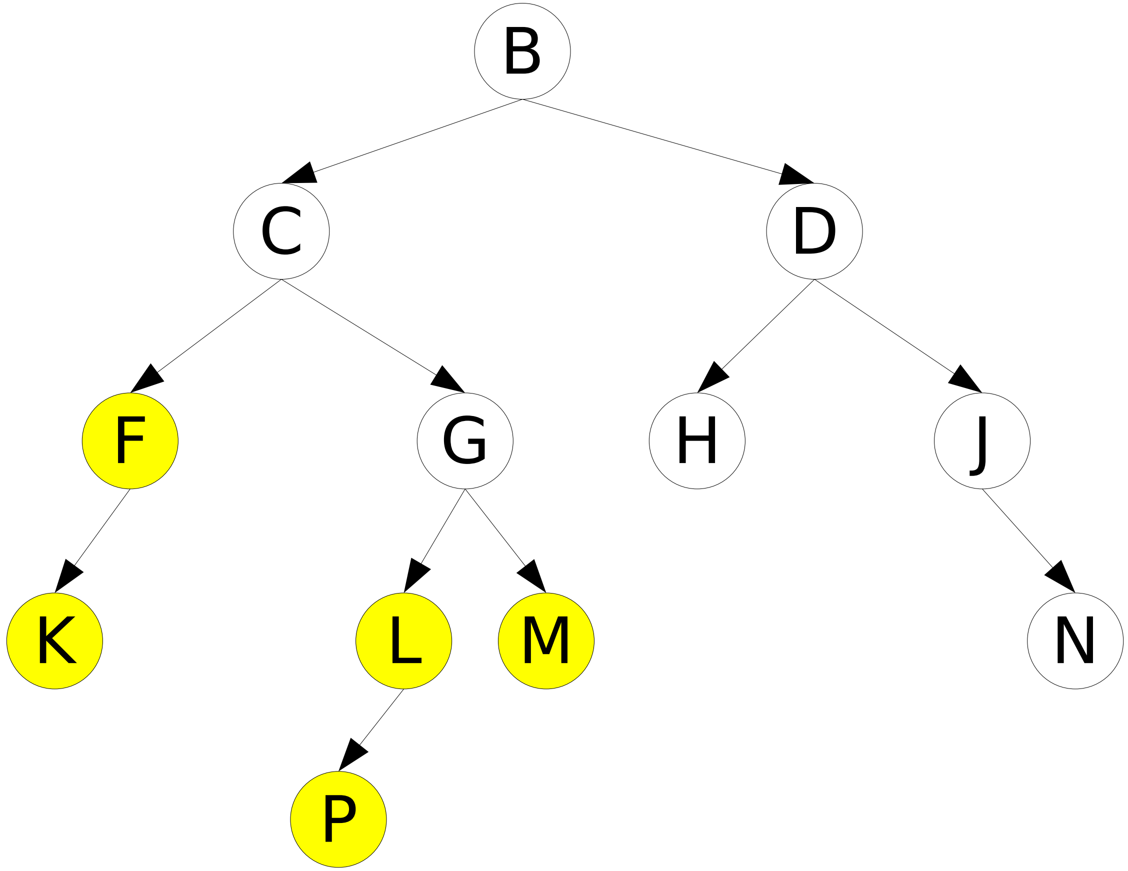
left
right
root



K F P L

Postorder Traversal Example

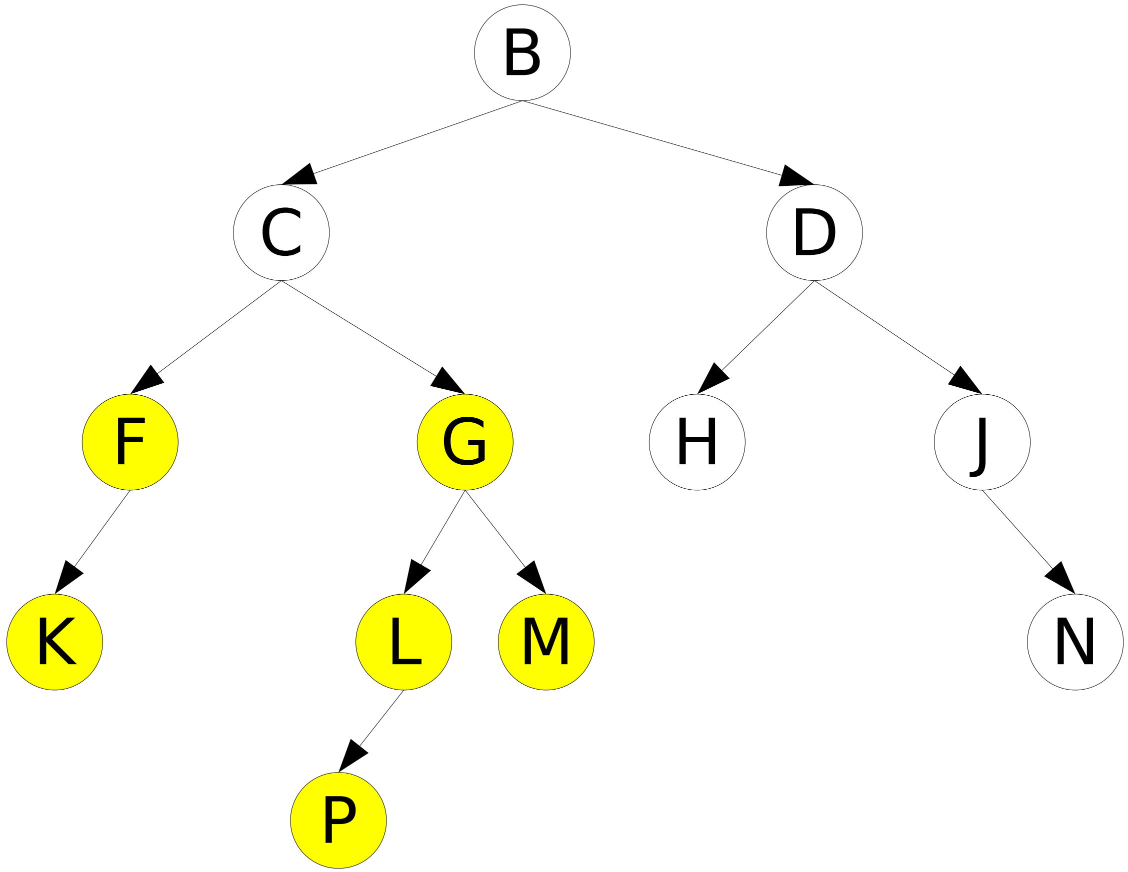
left
right
root



K F P L M

Postorder Traversal Example

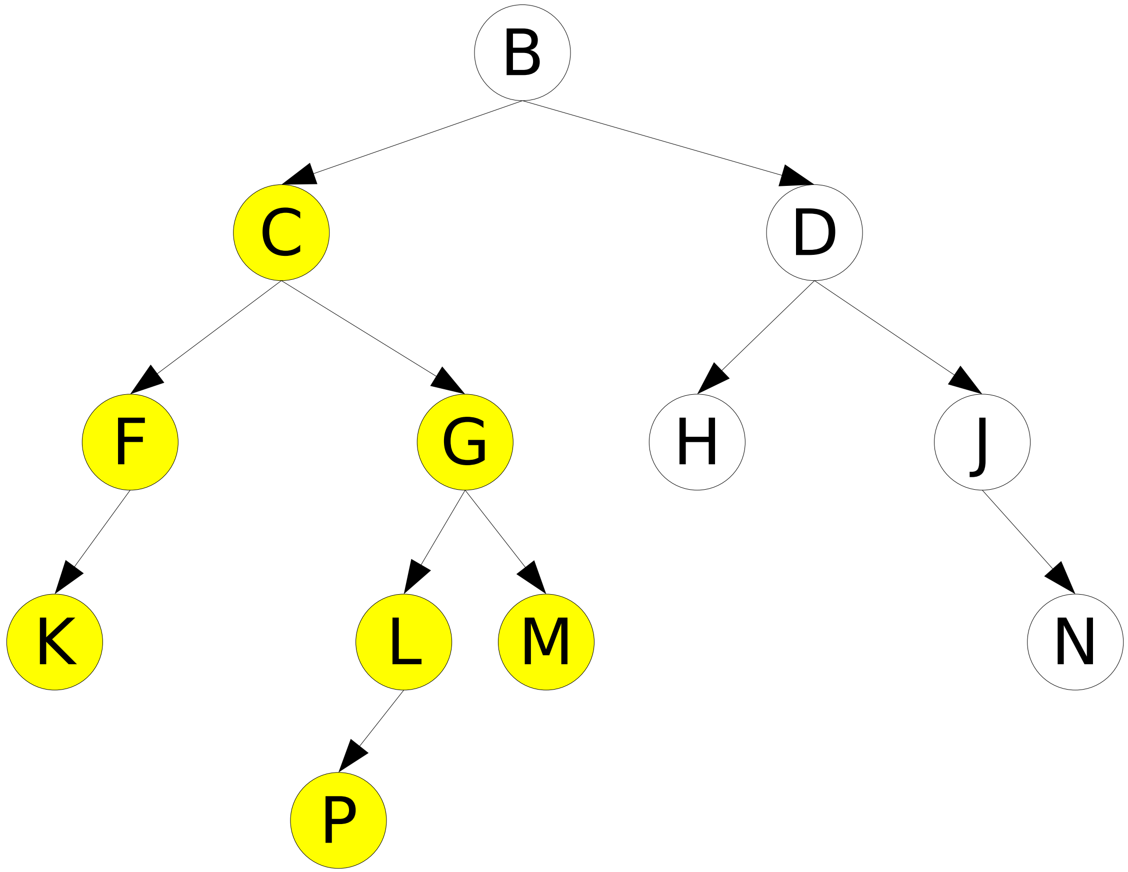
left
right
root



K F P L M G

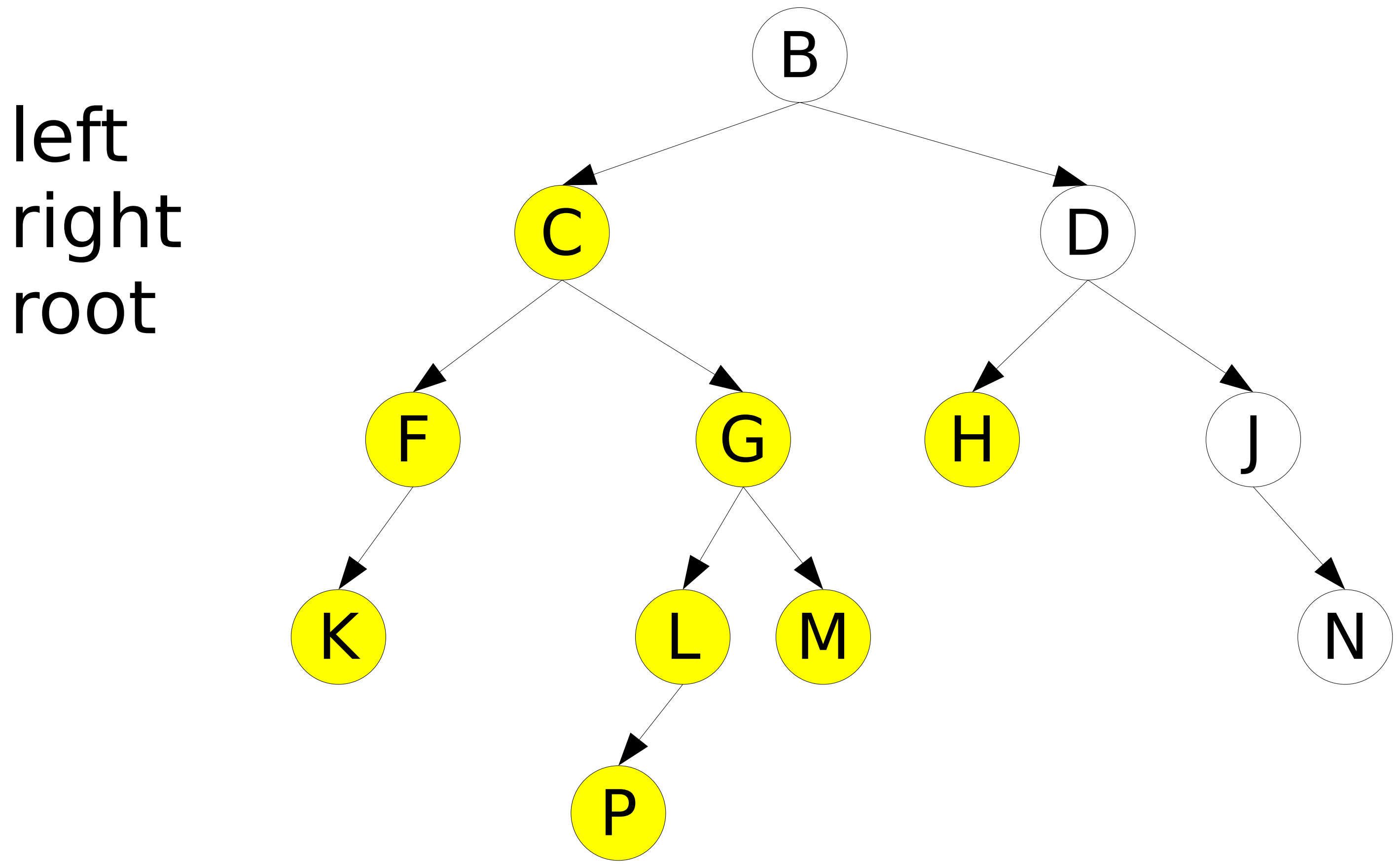
Postorder Traversal Example

left
right
root



K F P L M G C

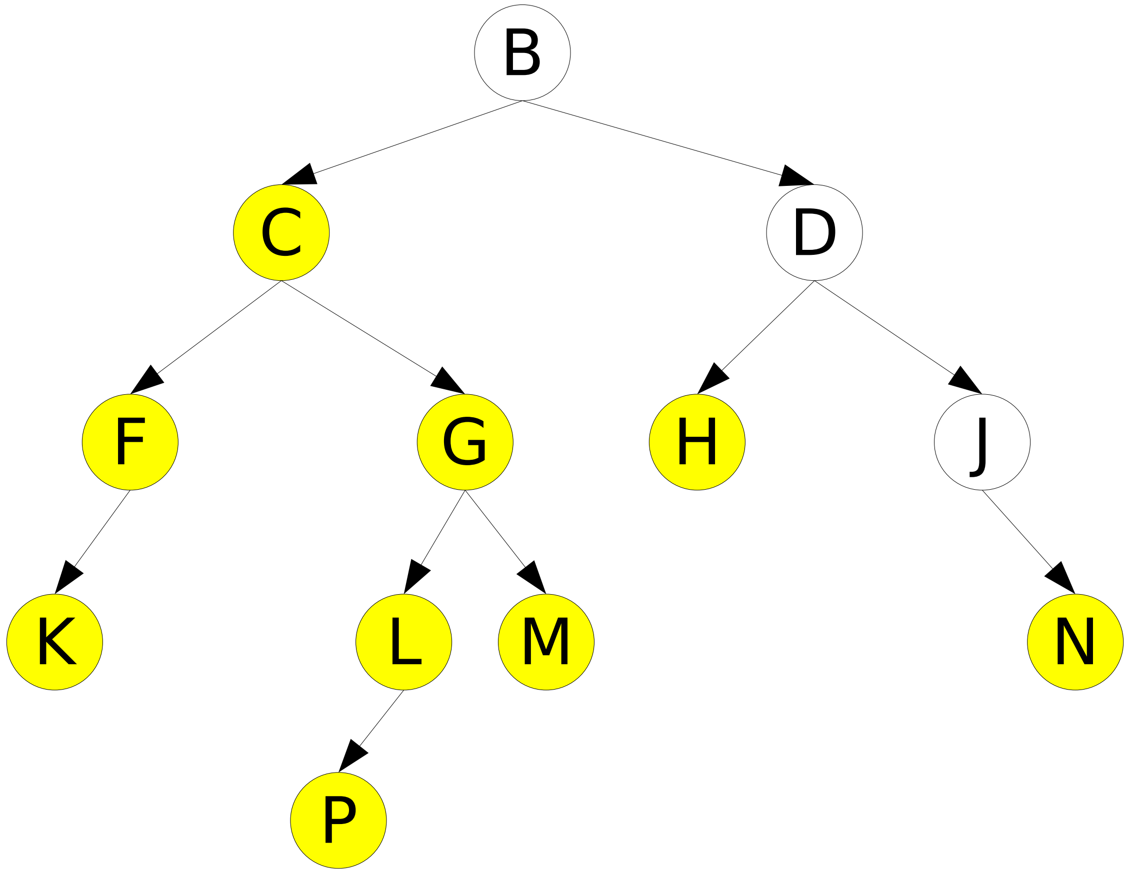
Postorder Traversal Example



K F P L M G C H

Postorder Traversal Example

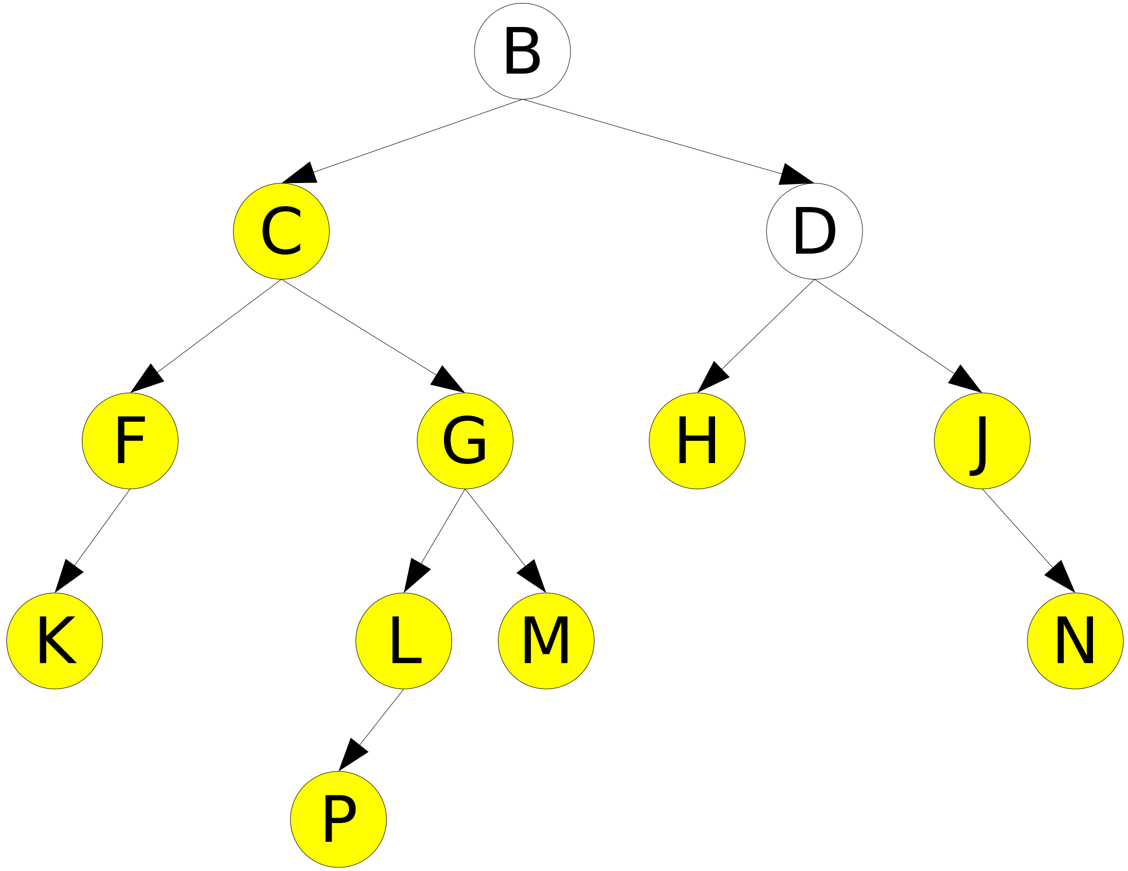
left
right
root



K F P L M G C H N

Postorder Traversal Example

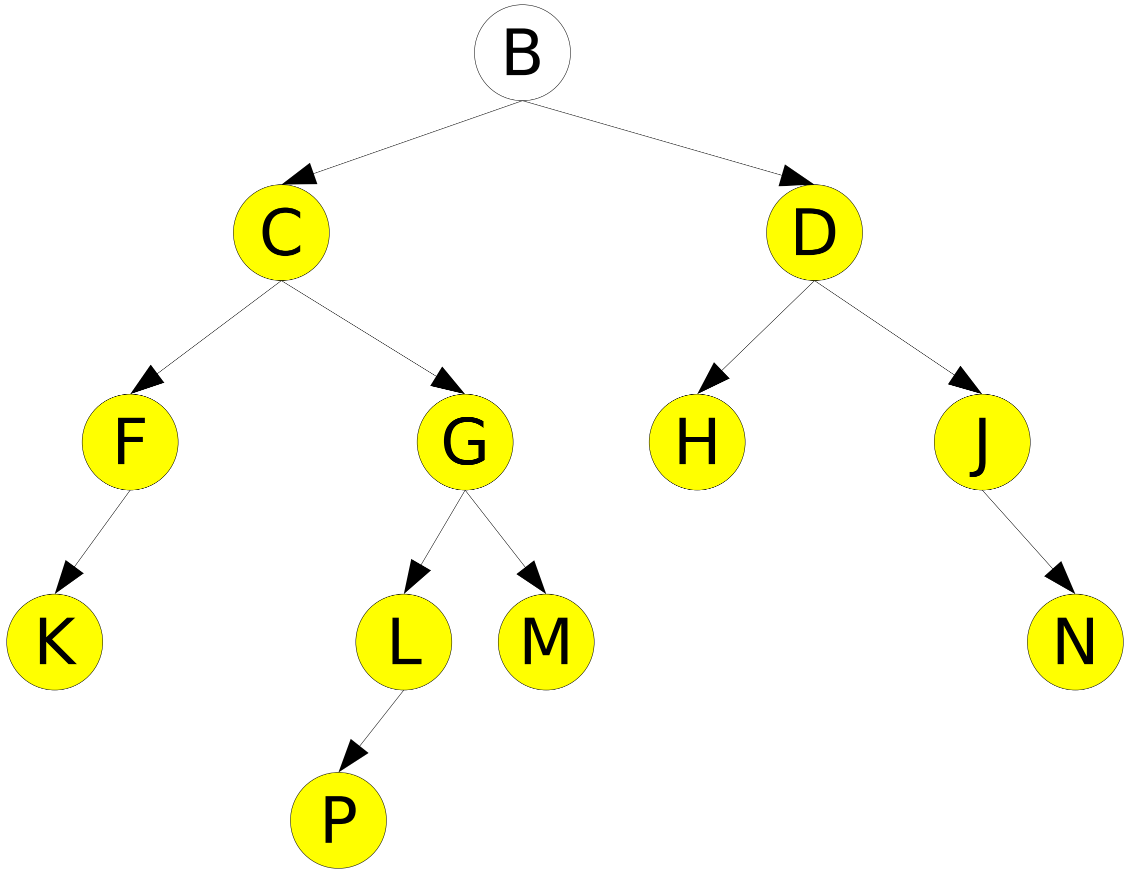
left
right
root



K F P L M G C H N J

Postorder Traversal Example

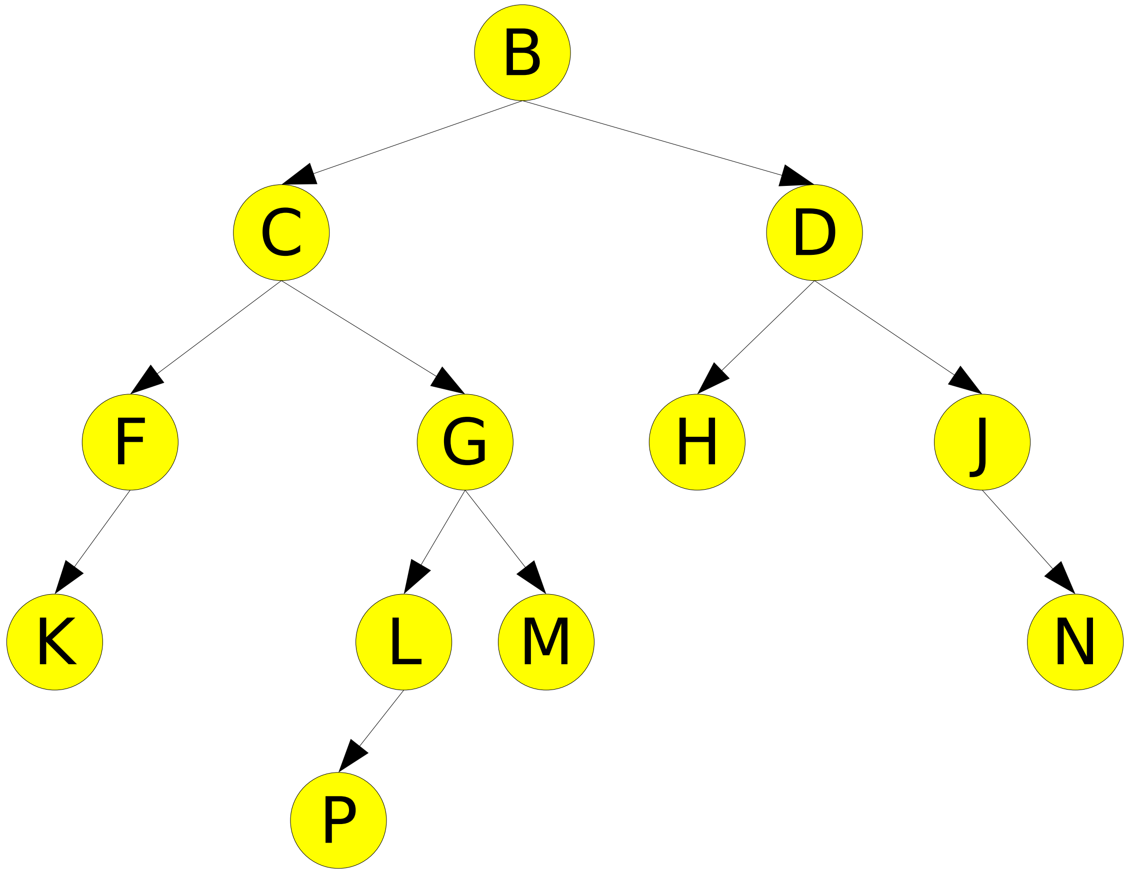
left
right
root



K F P L M G C H N J D

Postorder Traversal Example

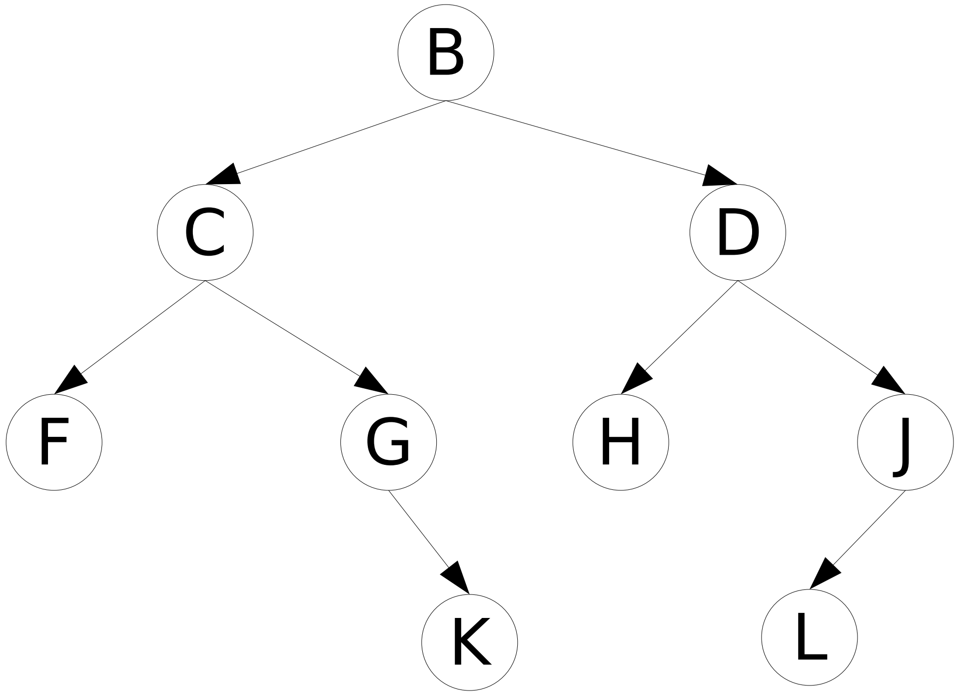
left
right
root



K F P L M G C H N J D B

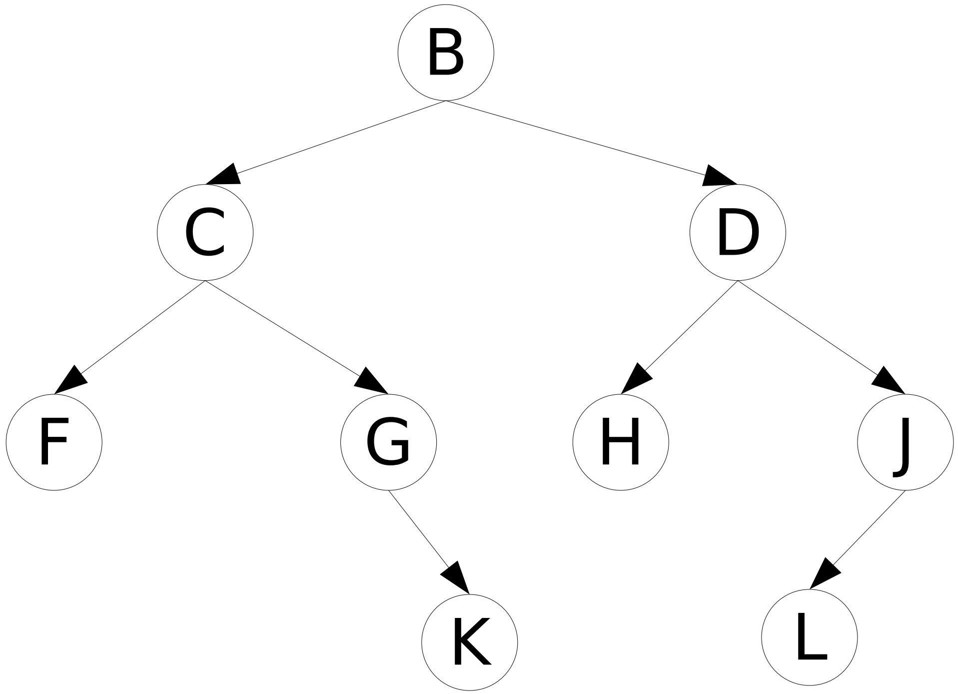
Postorder Traversal Exercise

What is the postorder traversal of this tree?



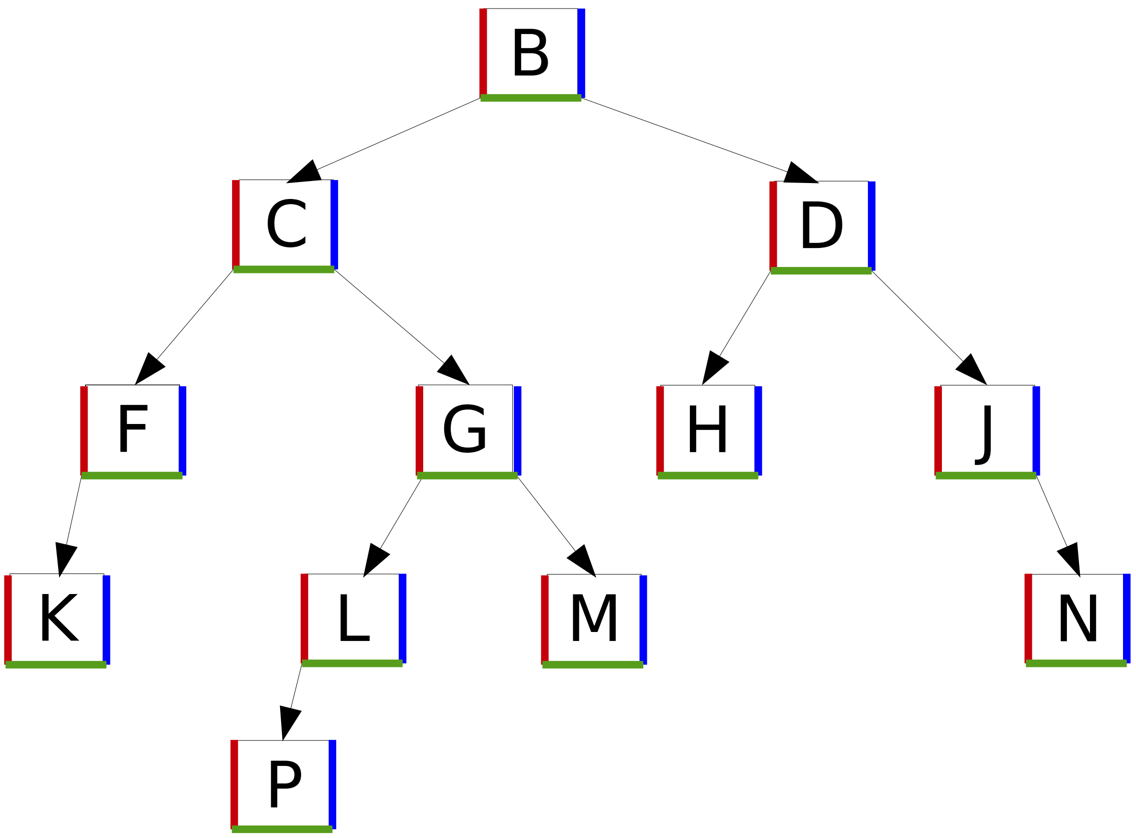
Postorder Traversal Exercise

What is the postorder traversal of this tree?

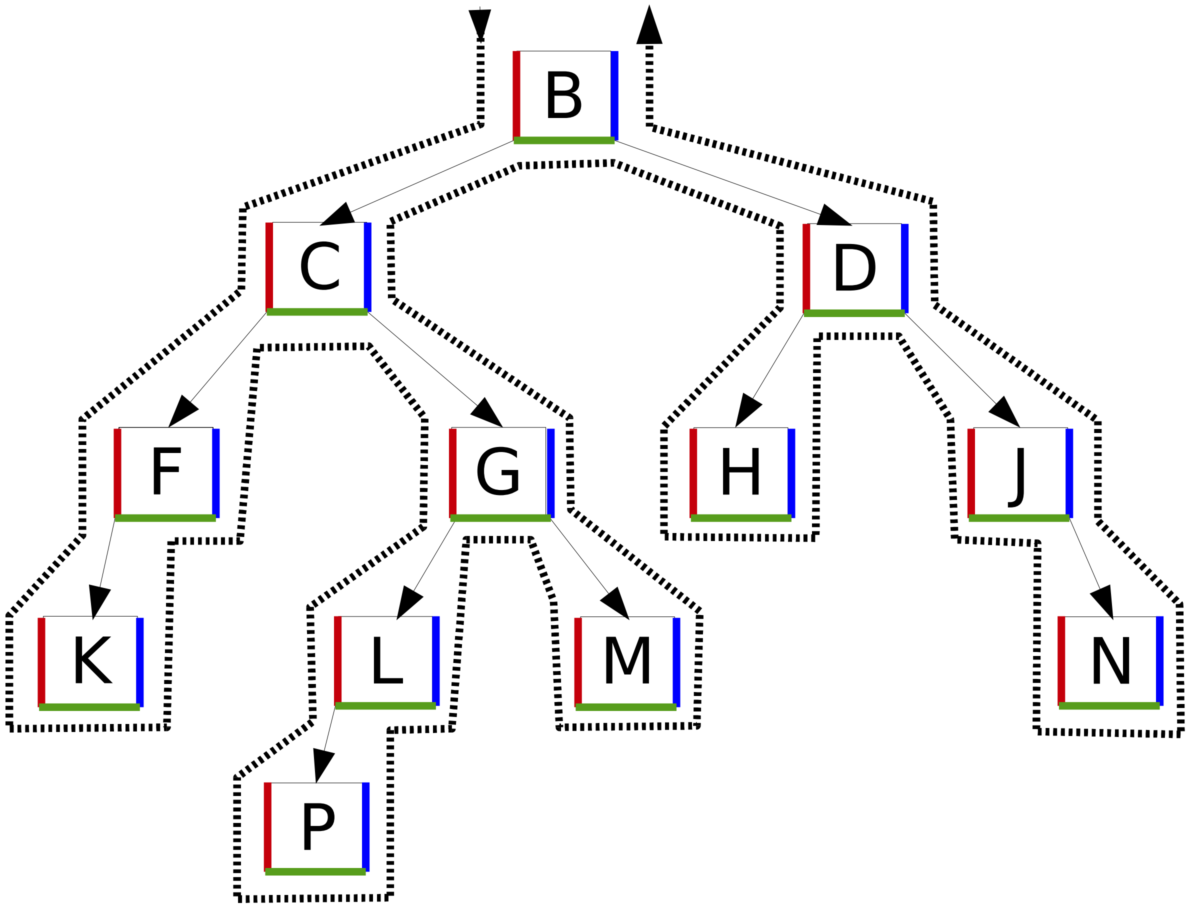


F K G C H L J D B

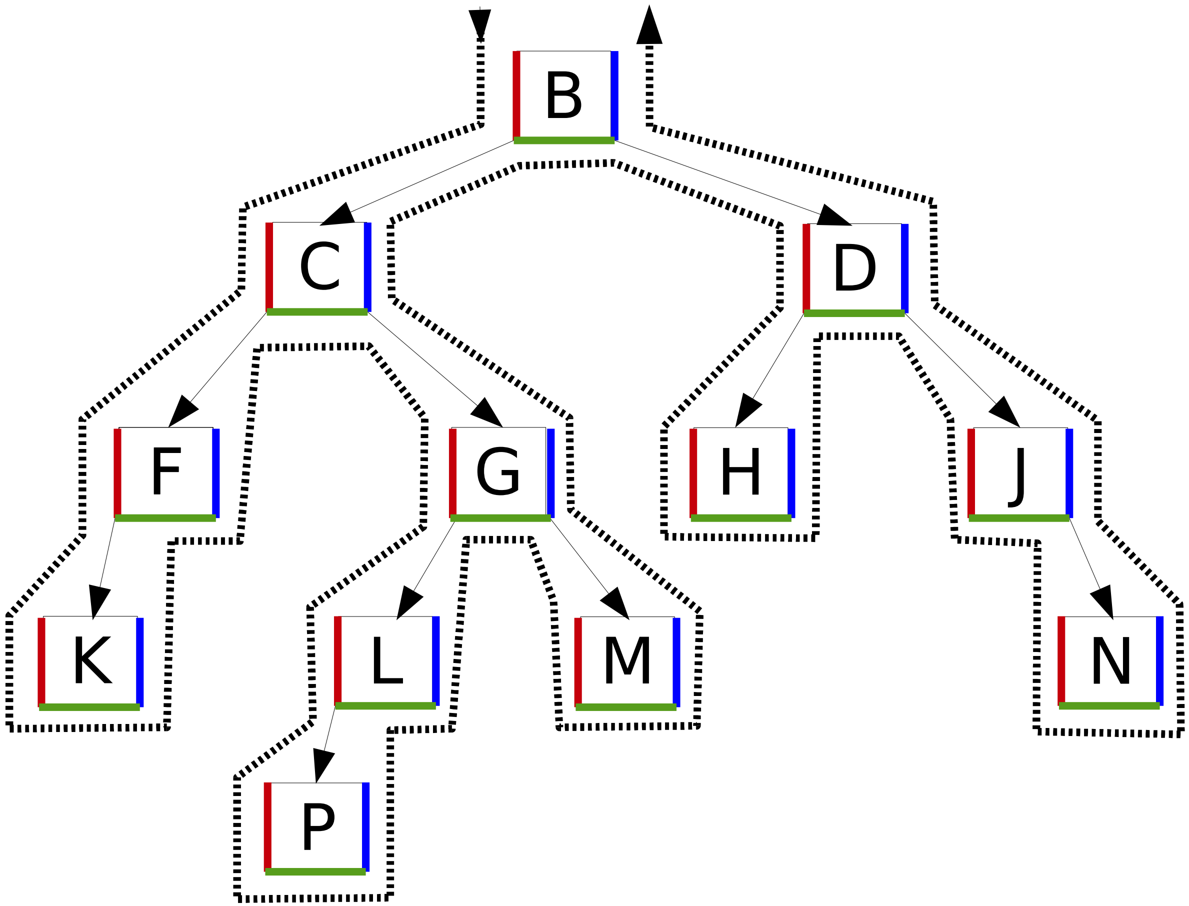
Distinguishing the Traversal Types



Distinguishing the Traversal Types



Distinguishing the Traversal Types



Preorder (left/red window): B C F K G L P M D H J N

Inorder (bottom/green window): K F C P L G M B H D J N

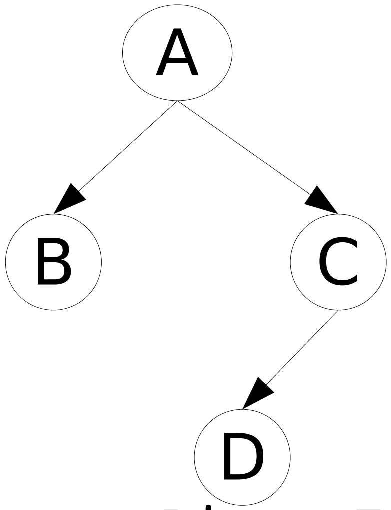
Postorder (right/blue window): K F P L M G C H N J D B

Building a Binary Tree

- Unlike classes that represent lists, stacks, or queues, tree classes often do **not** have methods to add or remove elements.
- There is no obvious place to add a new element.
- Removing a node is even less clear
  - how would you indicate which node should be removed? - can't number them like in a list
  - what happens to the children of the removed node?

Building a Binary Tree

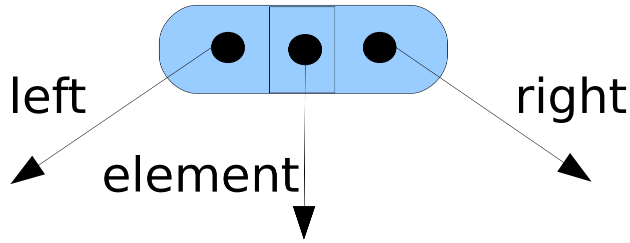
- Trees are built up, node by node.
- Let's build this tree:




```
BinaryTree<String> b = new BinaryTree<String>("B");
BinaryTree<String> d = new BinaryTree<String>("D");
BinaryTree<String> c = new BinaryTree<String>("C", d, null);
BinaryTree<String> a = new BinaryTree<String>("A", b, c);
```

BinaryTreeNode<T> Instance Variables

```
public class BinaryTreeNode<T> {
    T element;
    BinaryTreeNode<T> left;
    BinaryTreeNode<T> right;

    ...

}
```



BinaryTreeNode<T> Constructors

```
public class BinaryTreeNode<T> {
    T element;
    BinaryTreeNode<T> left;
    BinaryTreeNode<T> right;

    public BinaryTreeNode(T dataObj) {
        element = dataObj;
        left = right = null;
    }

    public BinaryTreeNode(T dataObj,
                          BinaryTreeNode<T> l,
                          BinaryTreeNode<T> r) {
        element = dataObj;
        left = l;
        right = r;
    }
}
```

BinaryTreeNode<T> toString

```
public class BinaryTreeNode<T> {
    T element;
    BinaryTreeNode<T> left;
    BinaryTreeNode<T> right;

    <Constructors>

    public String toString() {
        return element.toString();
    }
}
```

BinaryTreeNode<T> isLeaf - Exercise

```
public class BinaryTreeNode<T> {
    T element;
    BinaryTreeNode<T> left;
    BinaryTreeNode<T> right;

    <Constructors>

    public boolean isLeaf() {

        //************  To Do *********

    }
}
```

BinaryTreeNode<T>
isLeaf

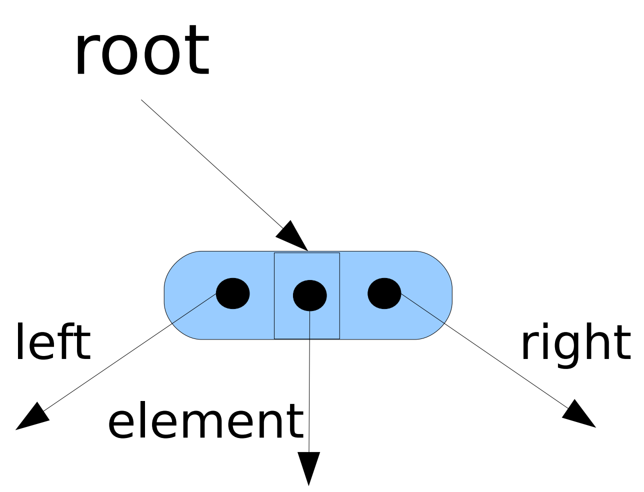
```
public class BinaryTreeNode<T> {
    T element;
    BinaryTreeNode<T> left;
    BinaryTreeNode<T> right;

    <Constructors>

    public boolean isLeaf() {

        return (left == null && right == null);

    }
}
```

BinaryTree<T>

- Now that we have a class to represent a node in the tree, we can define a class to represent the tree itself.
- The only node that we need a reference to is the root node. All other nodes are accessible from the root.
- We will declare a reference to the root node as an instance variable.

BinaryTree<T> Instance Variable

```
public class BinaryTree<T> {
    BinaryTreeNode<T> root;

    ...

}
```



BinaryTree<T> Constructors

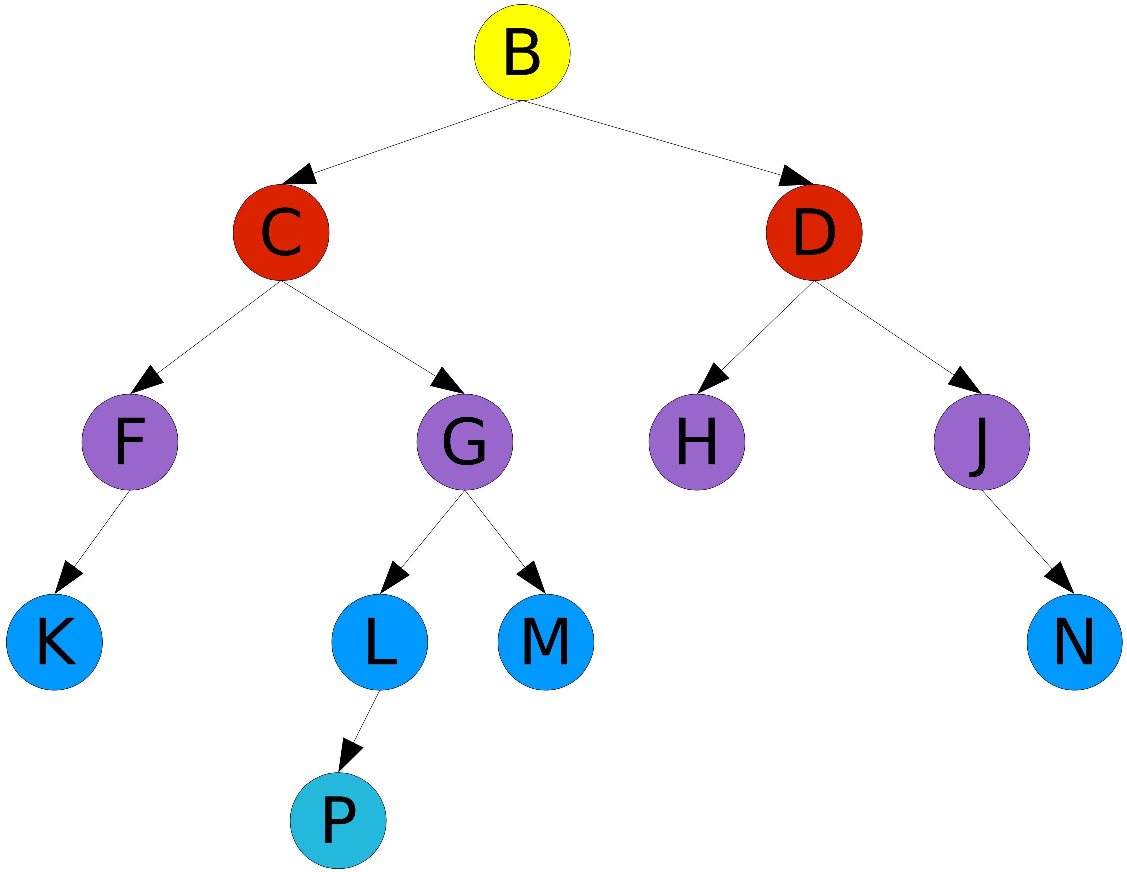
```
public class BinaryTree<T> {
    BinaryTreeNode<T> root;

    public BinaryTree() {
        root = null;
    }

    public BinaryTree(T element) {
        root = new BinaryTreeNode<T> (element);
    }

    ...
}
```

BinaryTree<T> Constructors, cont.

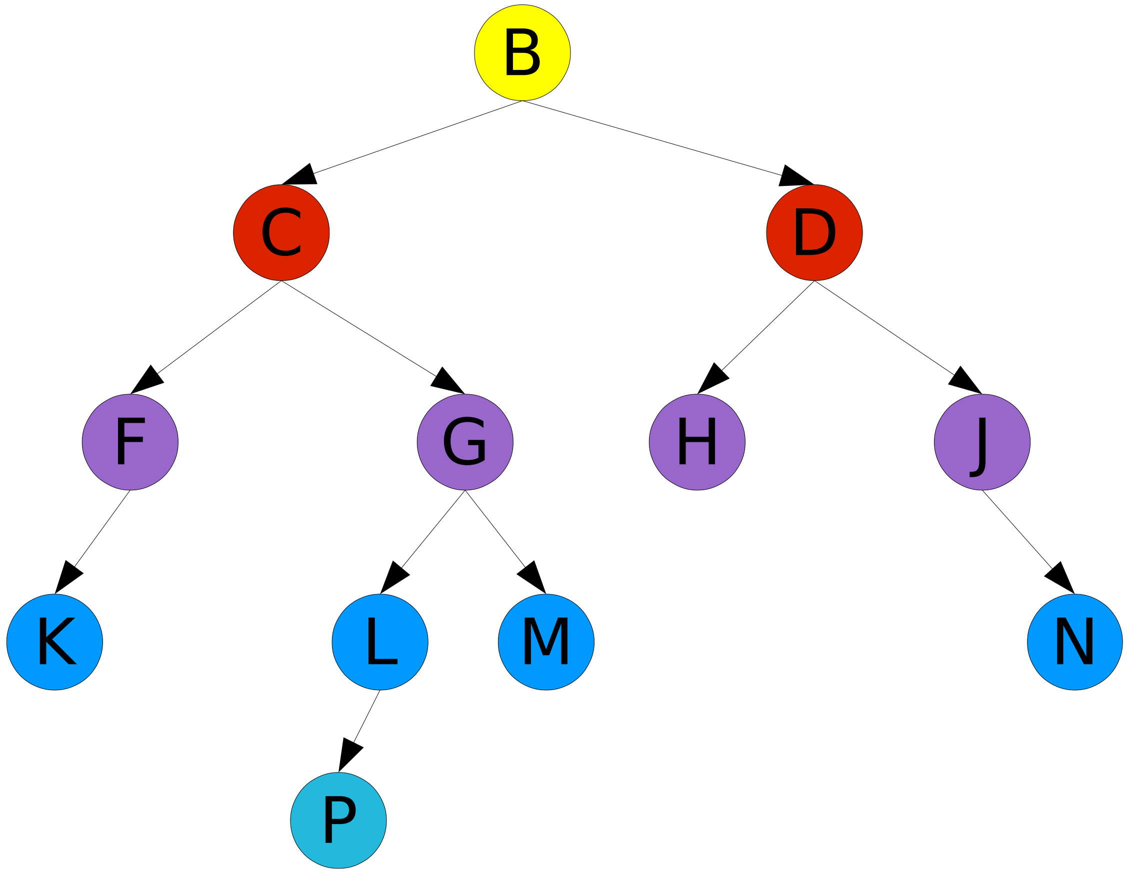
```
public class BinaryTree<T> {
   ...

   public BinaryTree(T element, BinaryTree<T> leftSubtree,
                                        BinaryTree<T> rightSubtree) {
       root = new BinaryTreeNode<T> (element);

       if (leftSubtree != null) {
           root.left = leftSubtree.root;
       } else {
           root.left = null;
       }

       if (rightSubtree != null) {
           root.right = rightSubtree.root;
       } else {
           root.right = null;
       }
   }
}
```

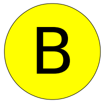
Levelorder traversal of a binary tree

Levelorder tree traversal

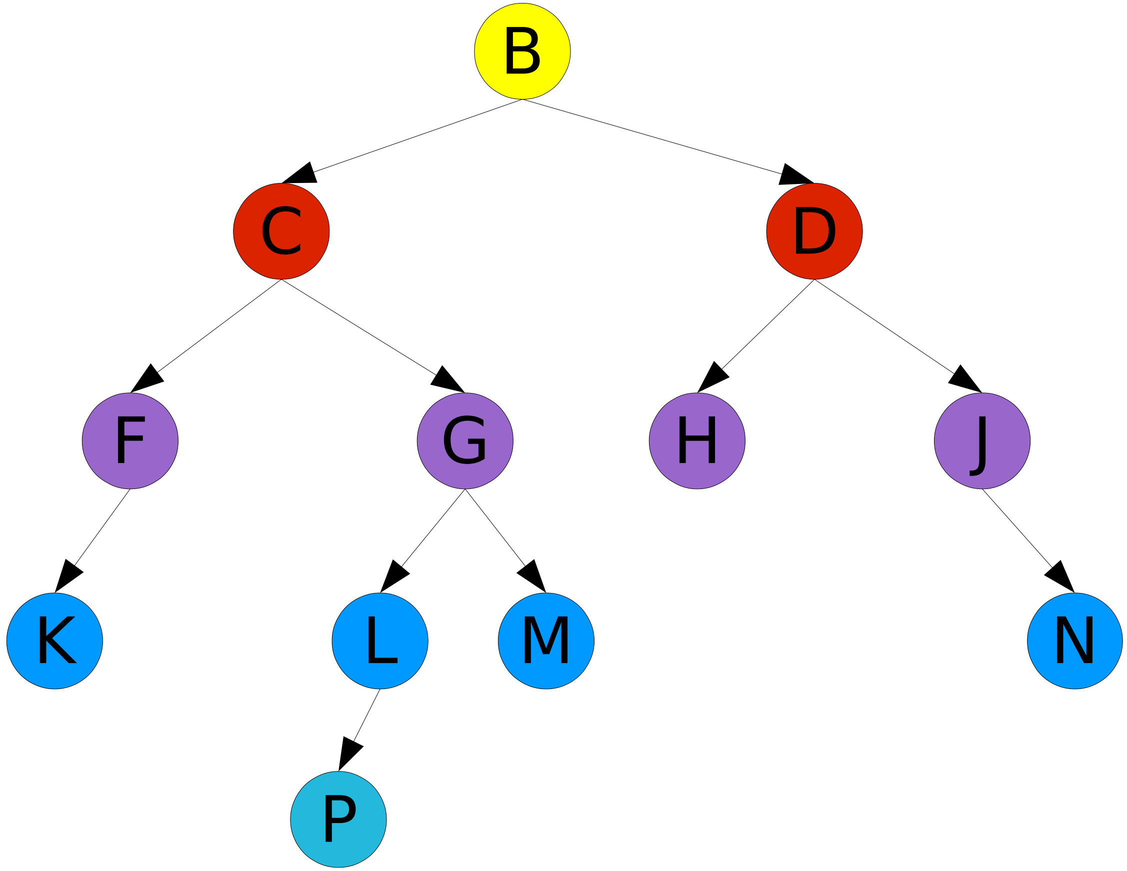


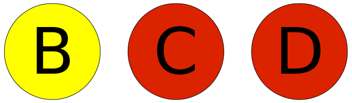
Levelorder tree traversal



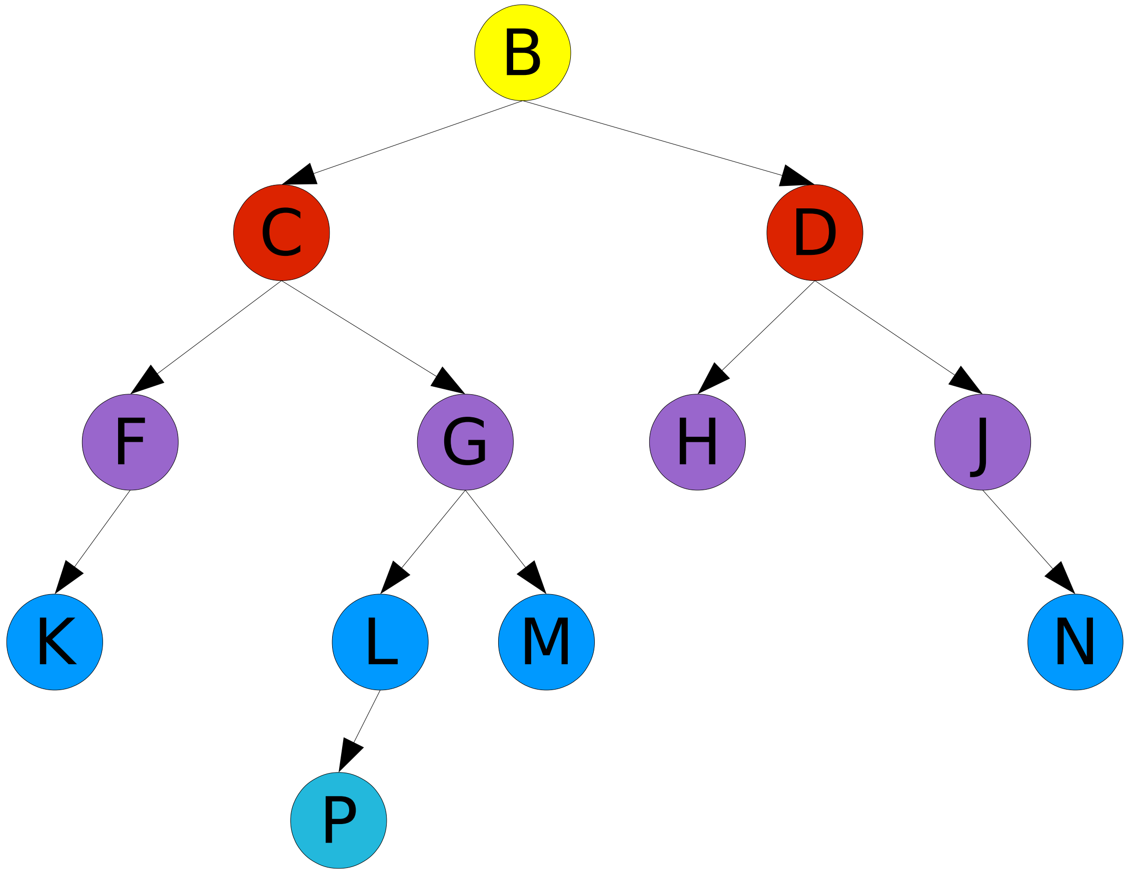


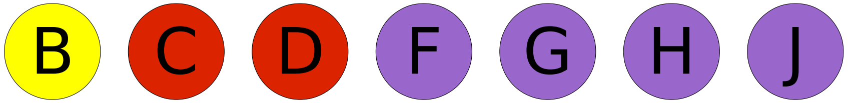
Levelorder tree traversal



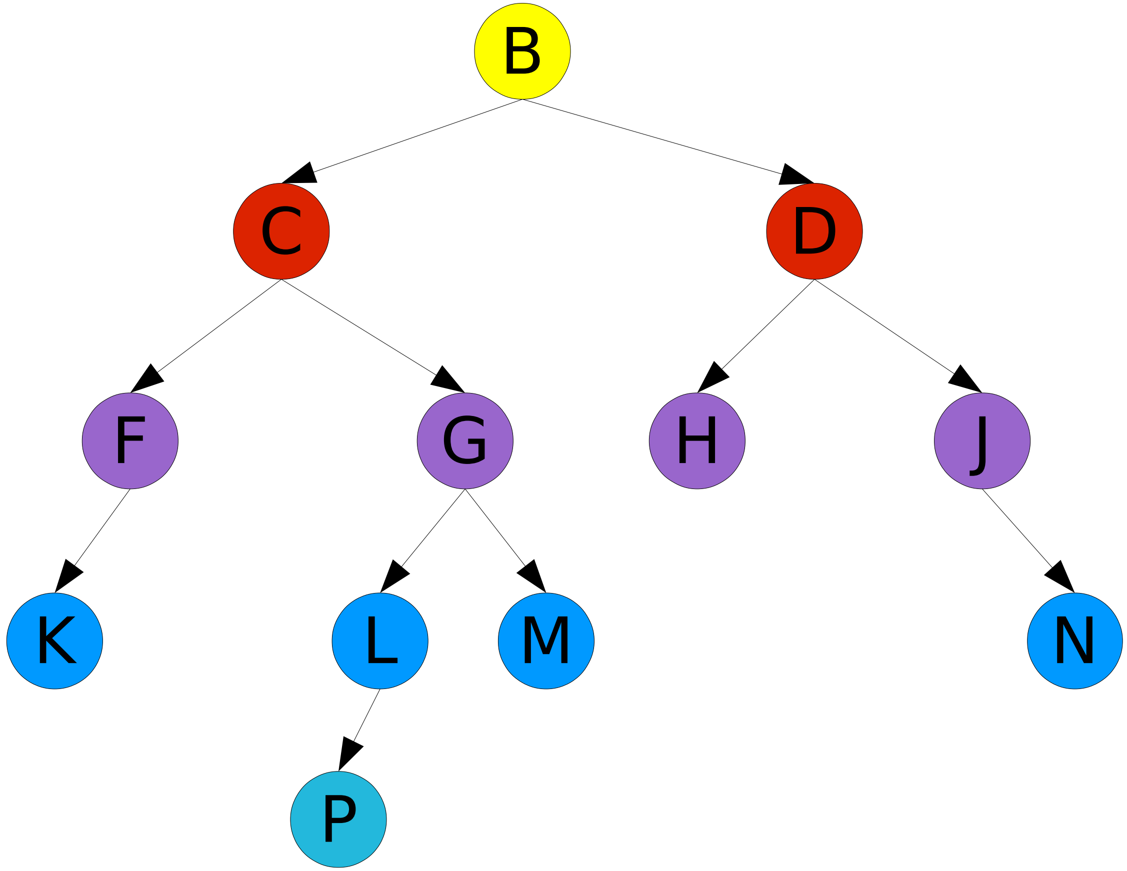


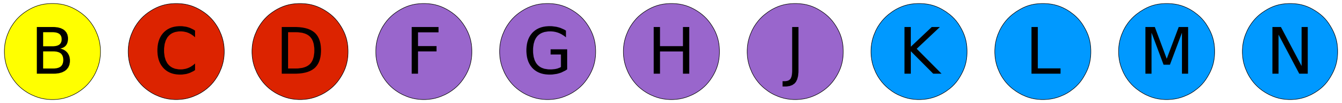
Levelorder tree traversal



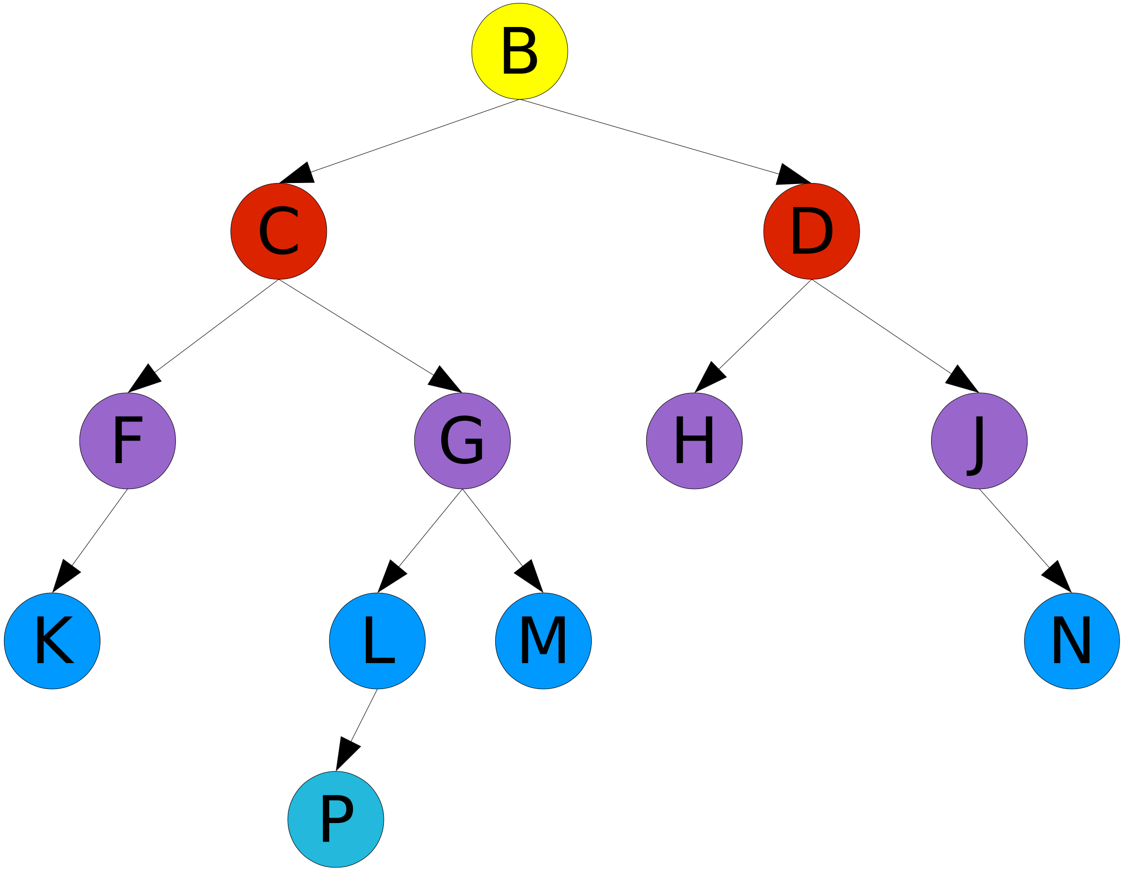


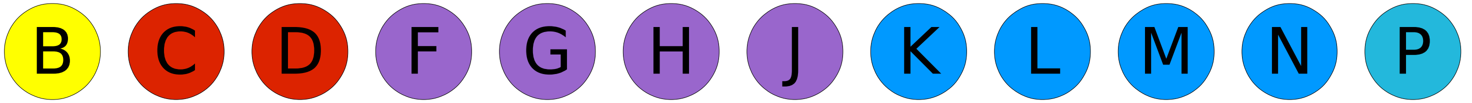
Levelorder tree traversal





Levelorder tree traversal

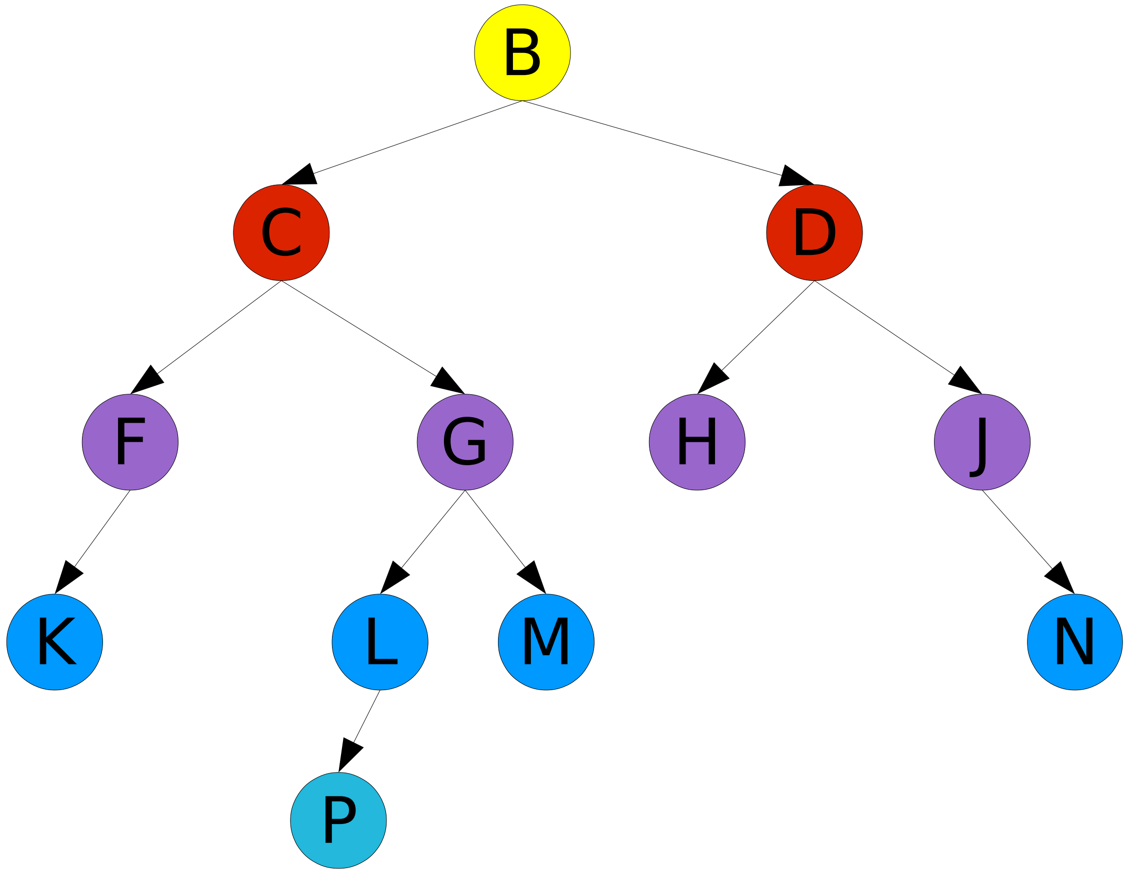




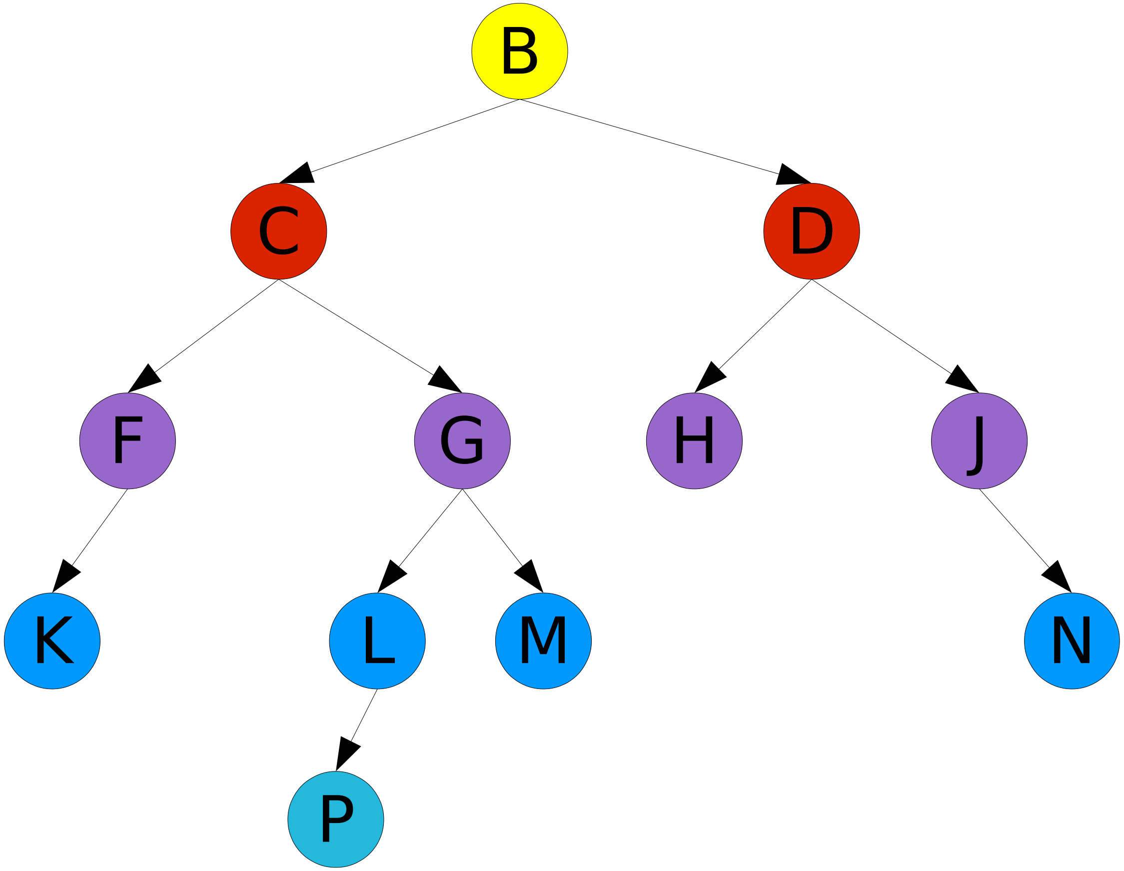
Recursive level order traversal

```
For d = 1 to height of tree
     levelorder(root node of tree, d)

levelorder(node, level)
   if node is NULL then return
   if level is 1 then
      do something with node
   else if level greater than 1 then
     levelorder(leftChild of node, level-1)
     levelorder(rightChild of node, level-1)
```

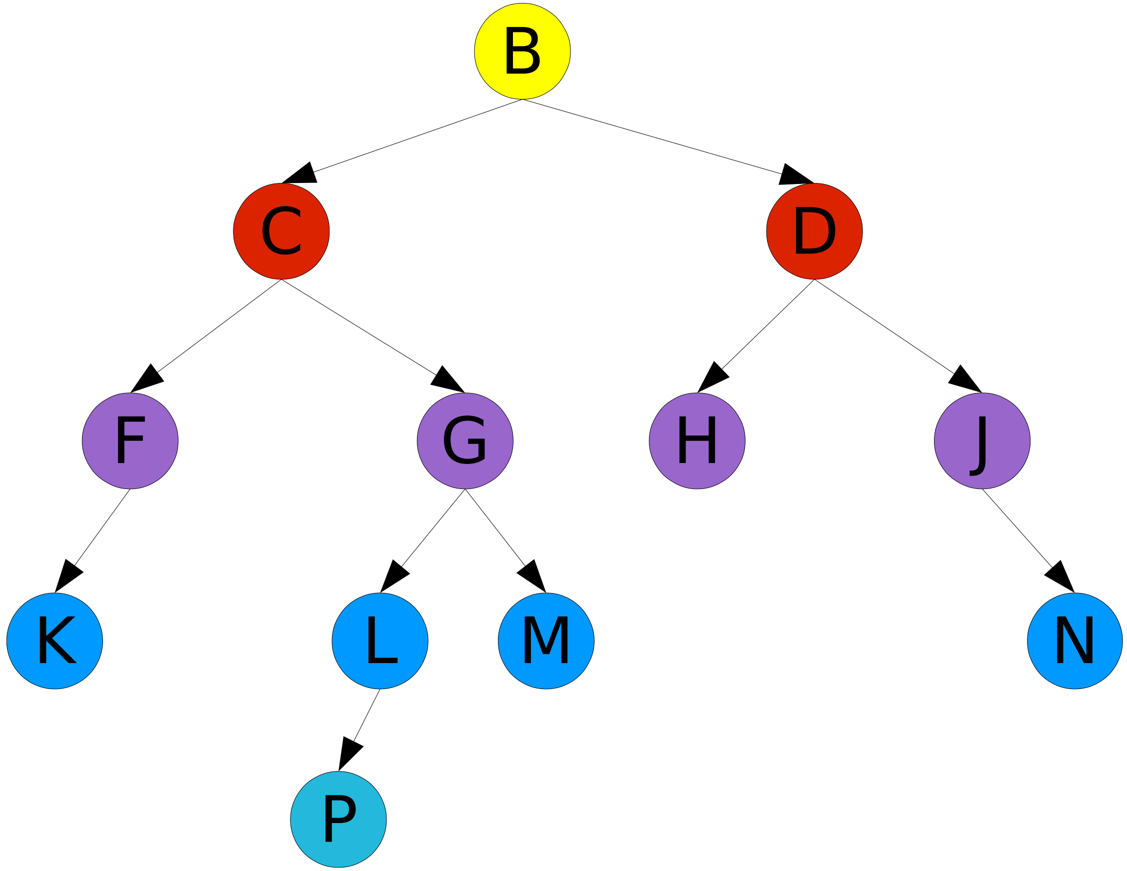
Levelorder traversal recursive

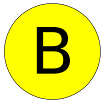


Levelorder traversal recursive

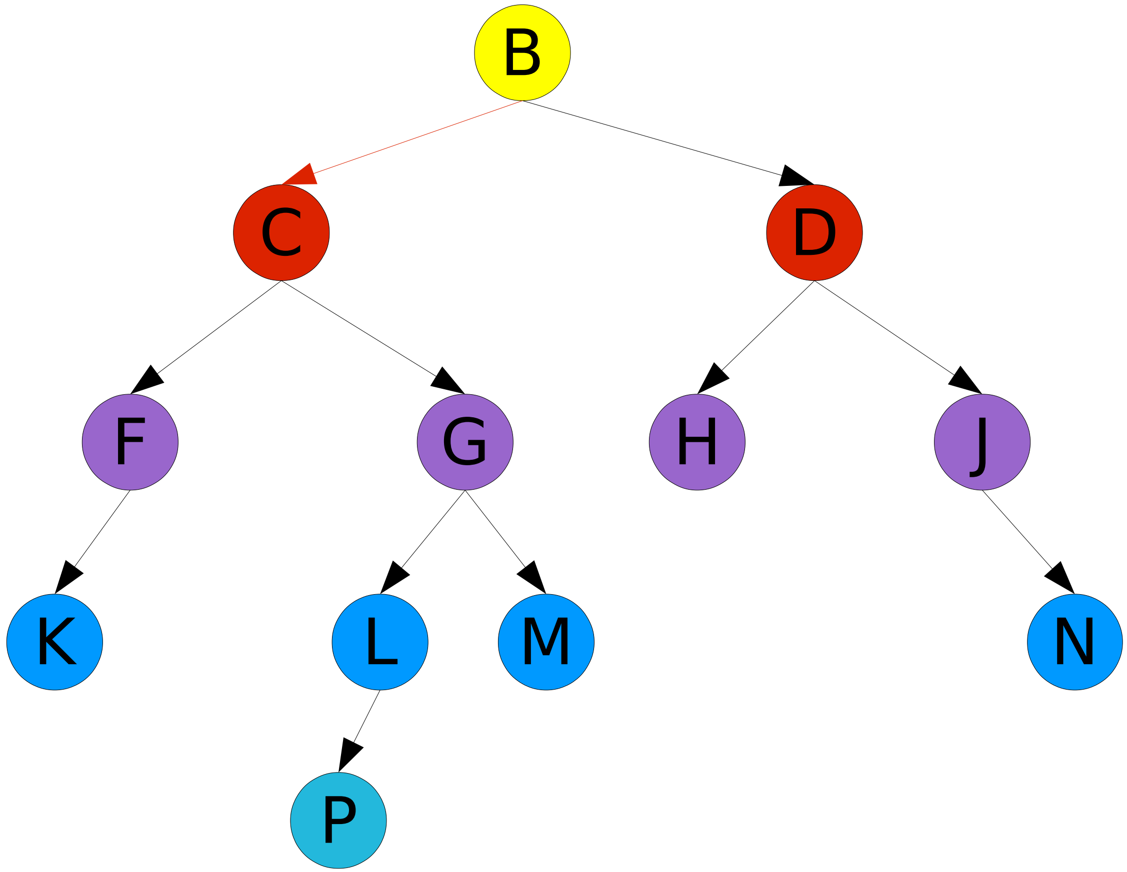


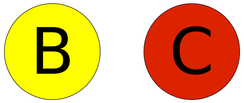
Levelorder traversal recursive



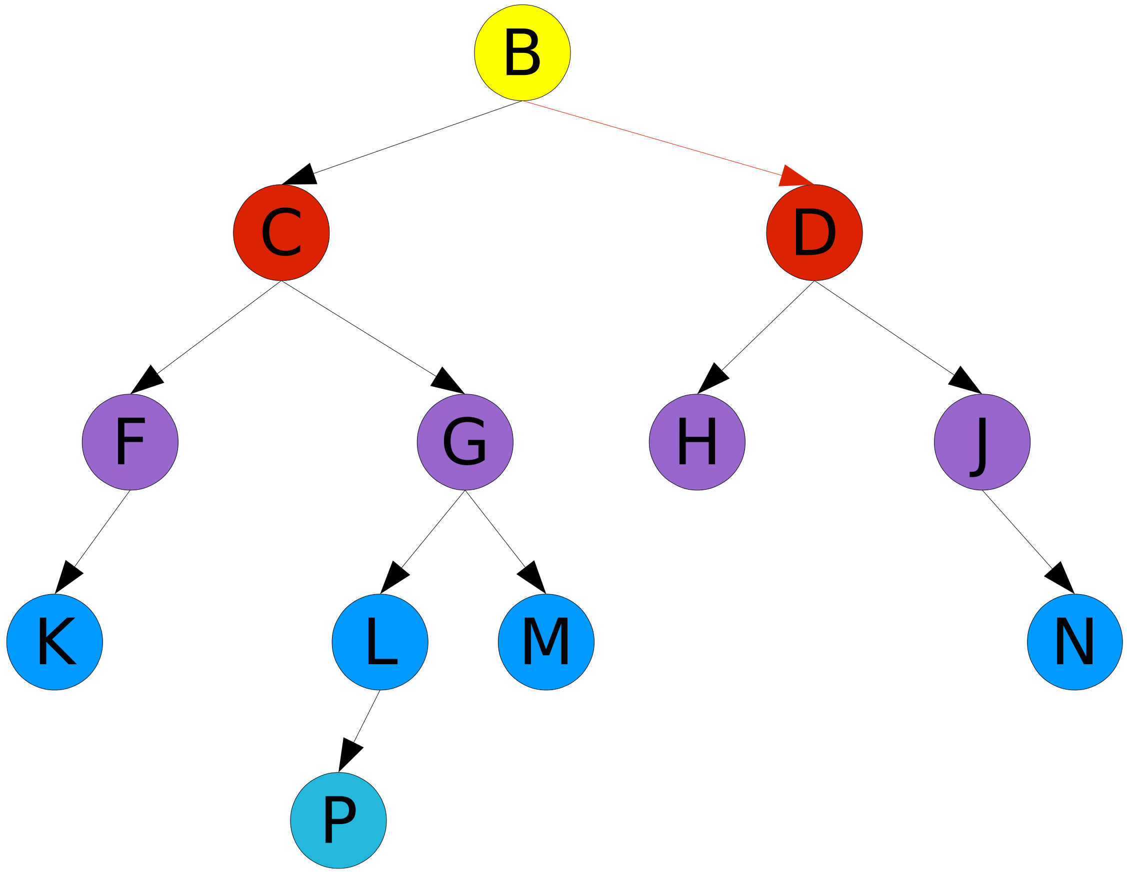


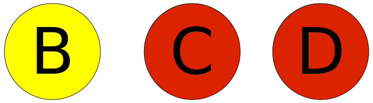
Levelorder traversal recursive



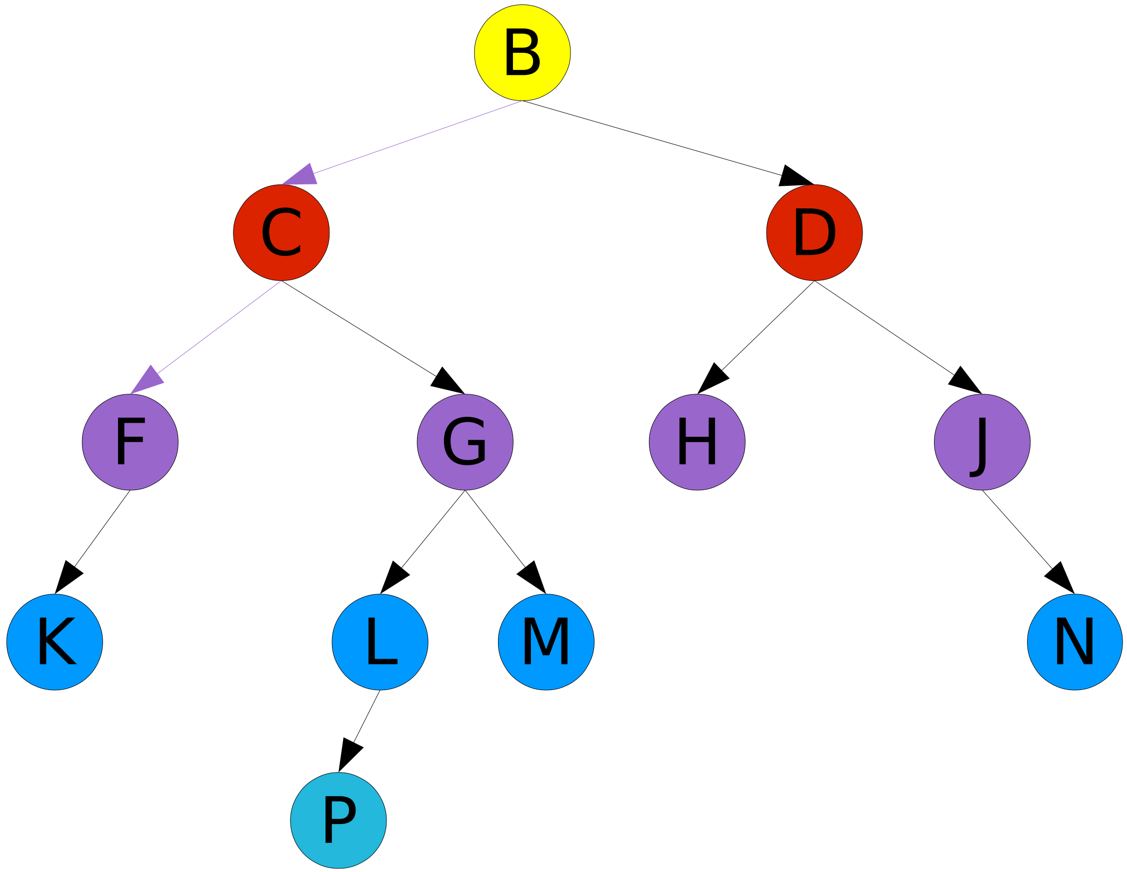


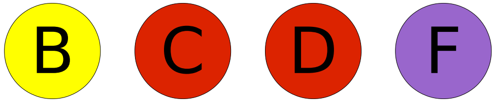
Levelorder traversal recursive



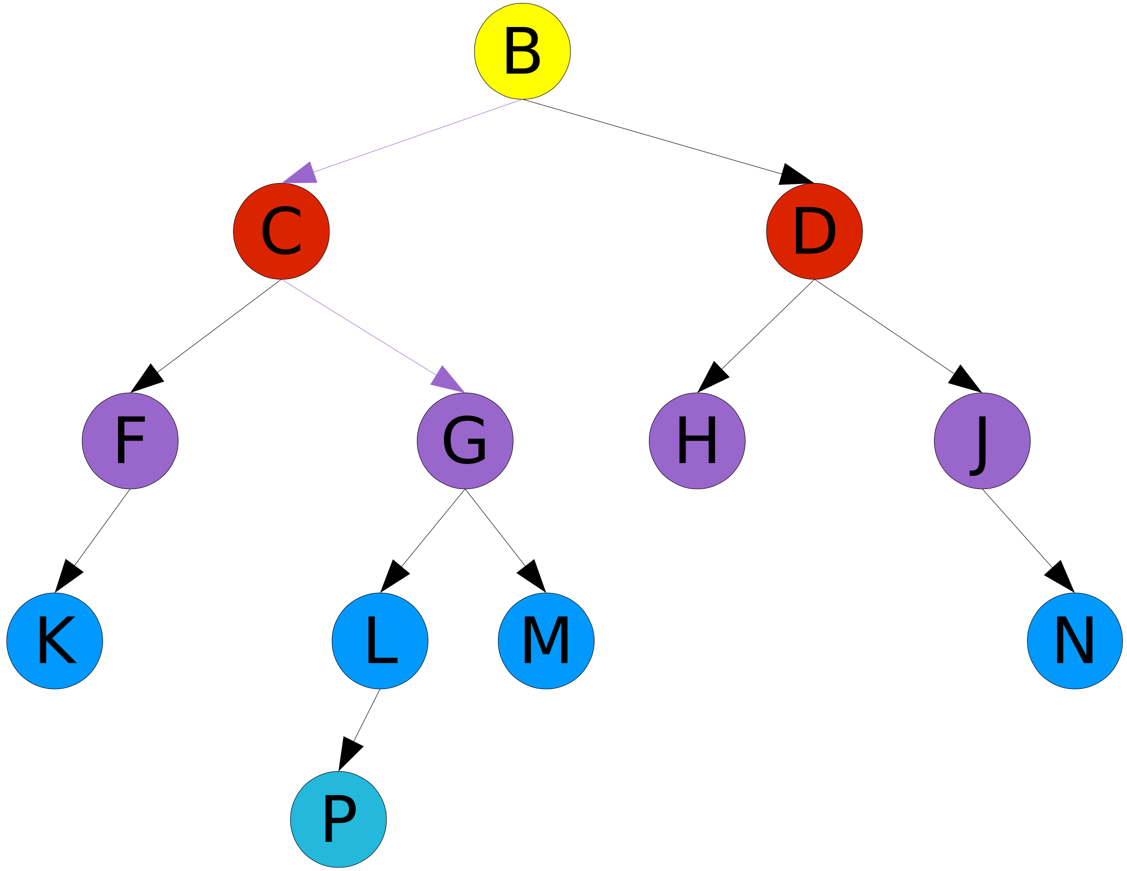


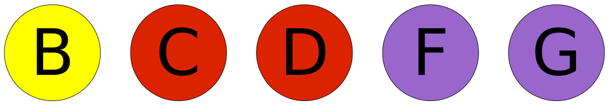
Levelorder traversal recursive



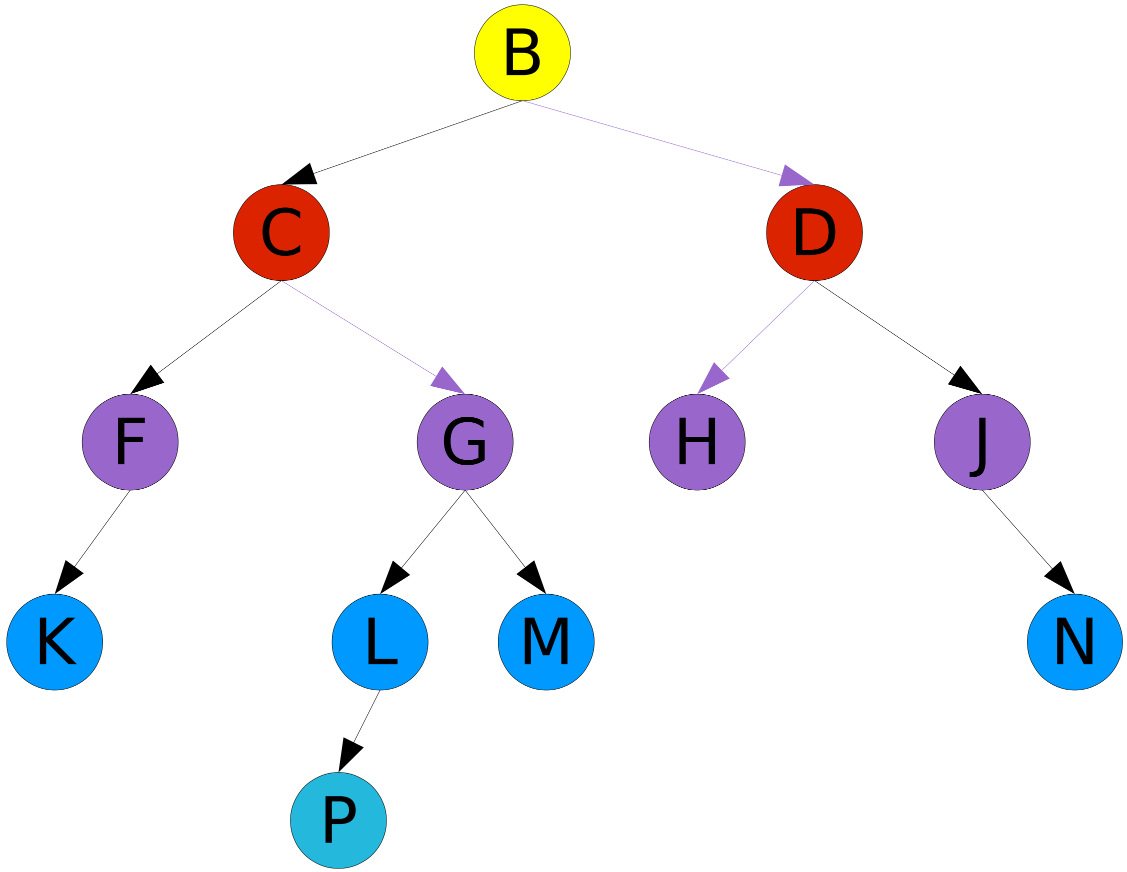


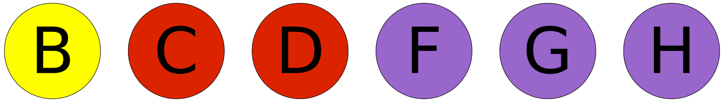
Levelorder traversal recursive



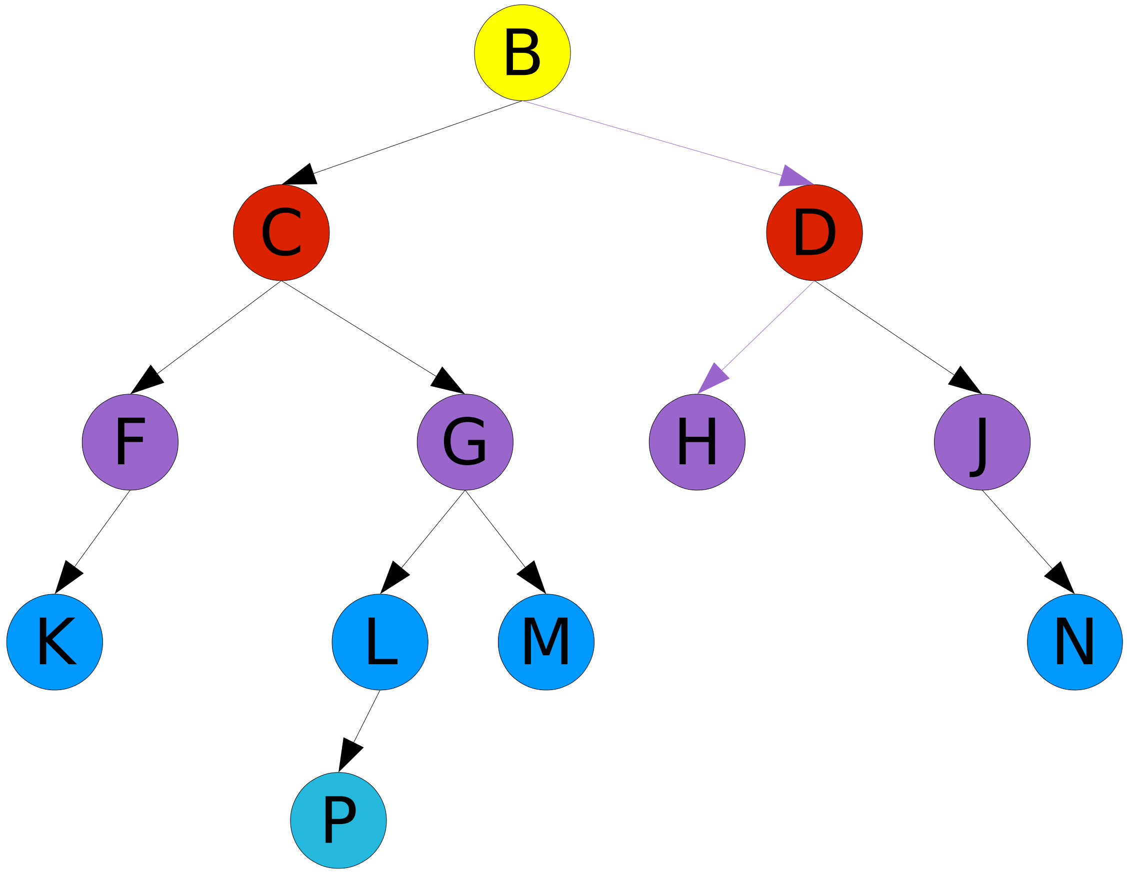


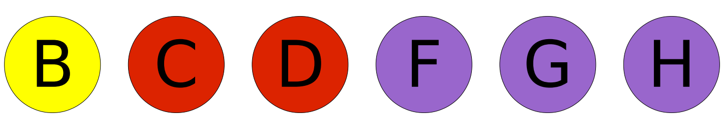
Levelorder traversal recursive



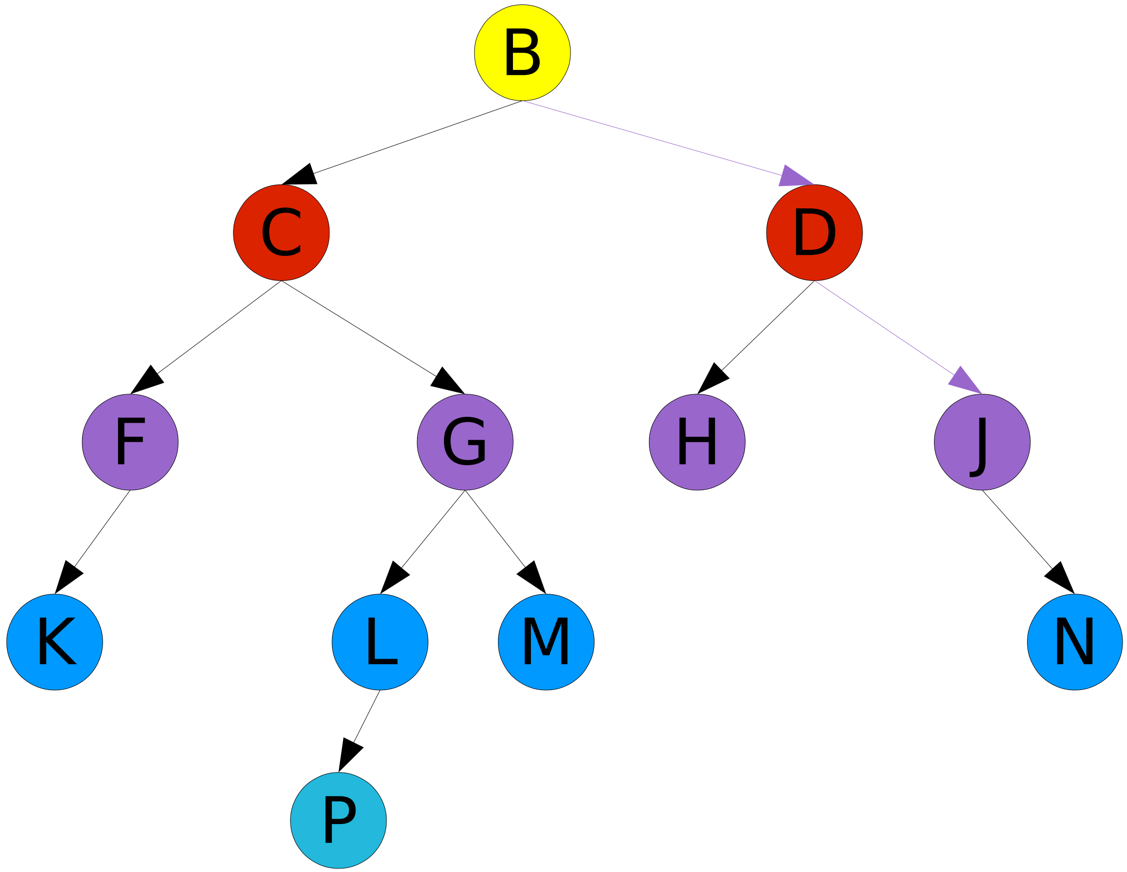


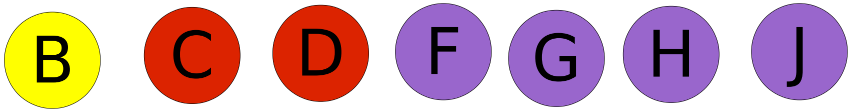
Levelorder traversal recursive



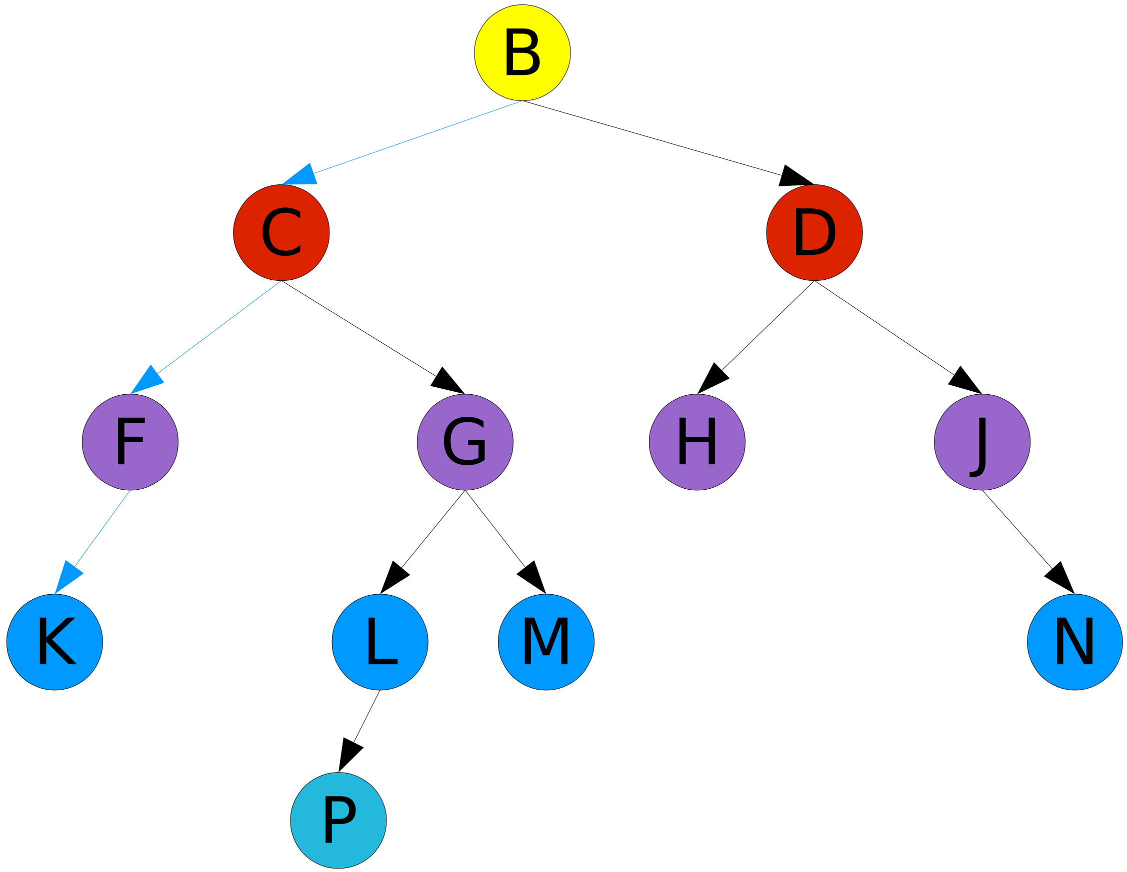


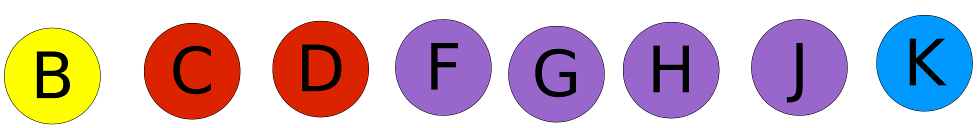
Levelorder traversal recursive



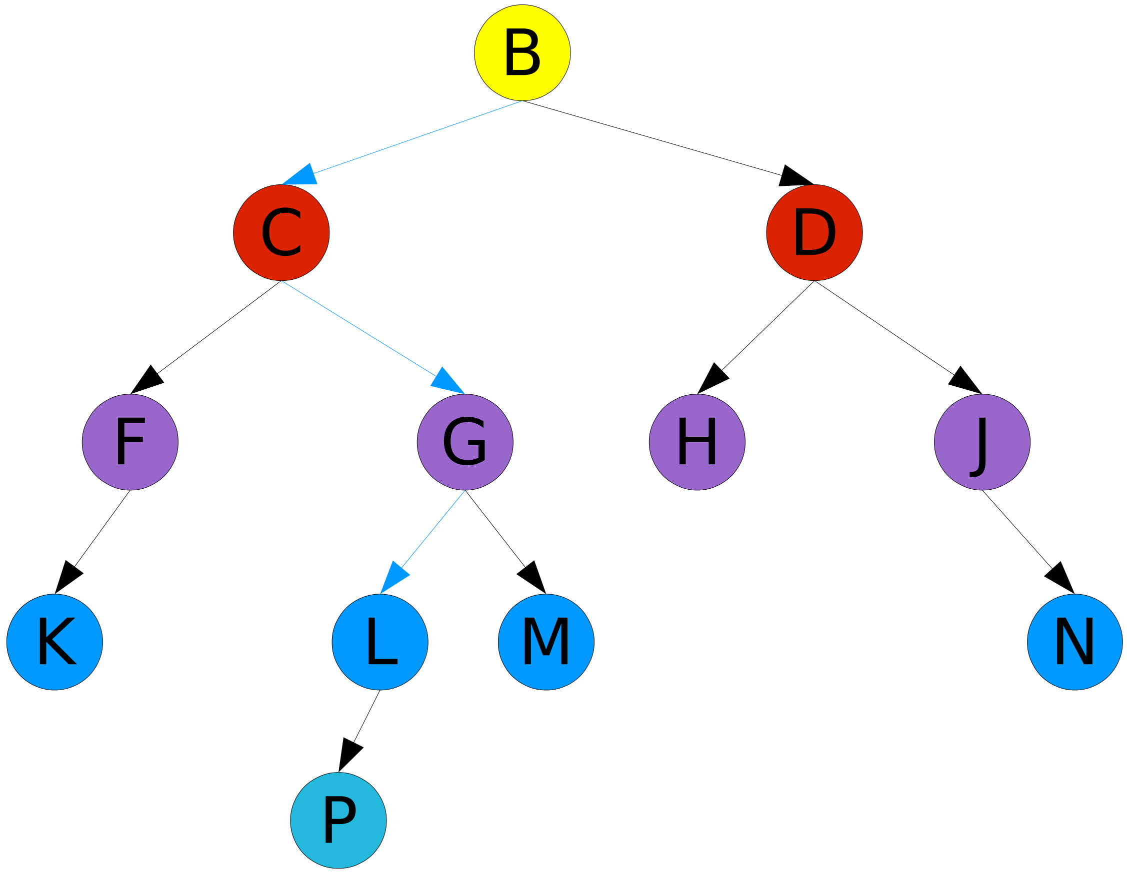


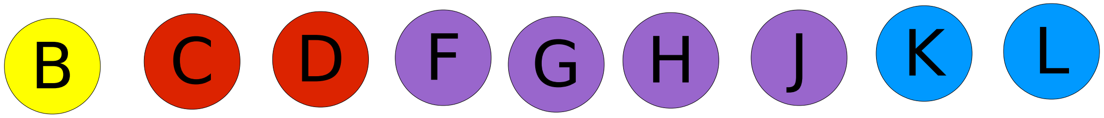
Levelorder traversal recursive



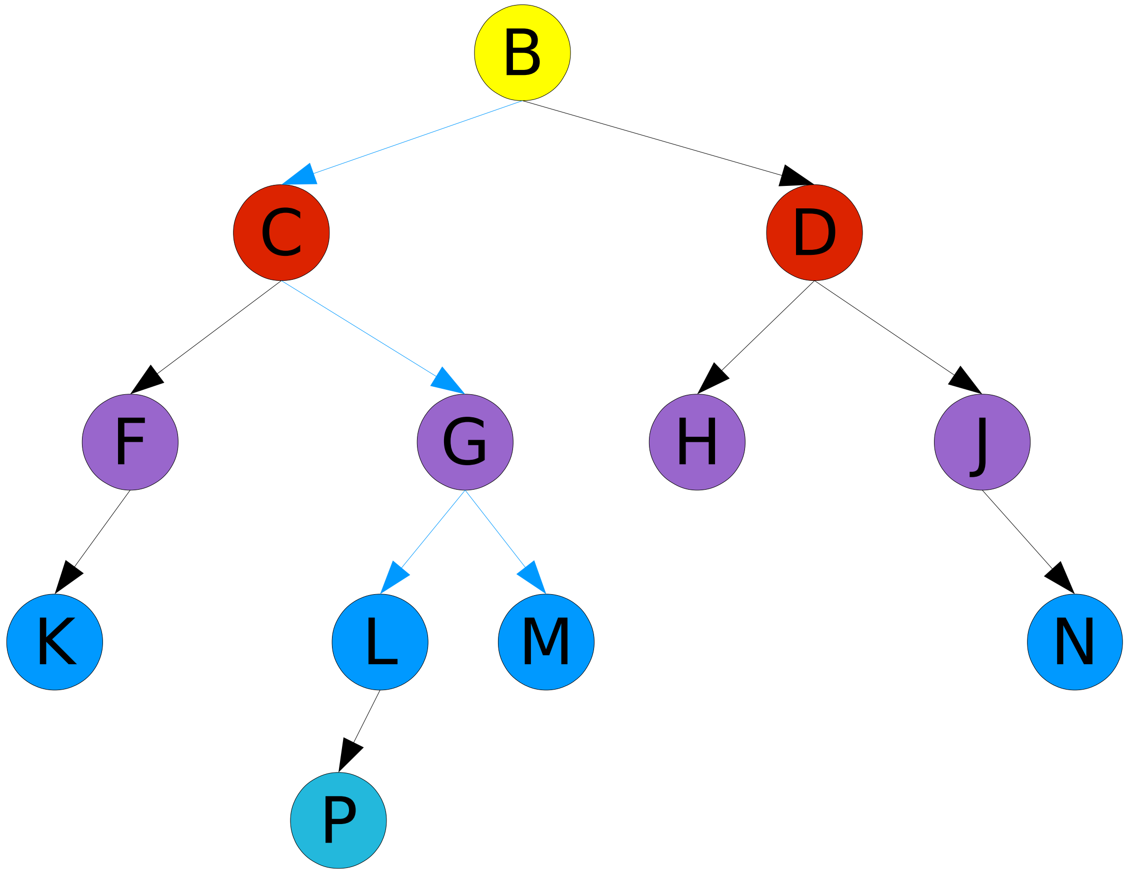


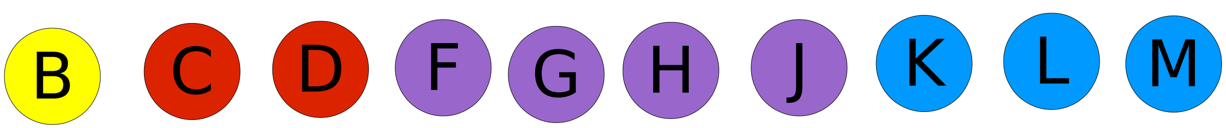
Levelorder traversal recursive



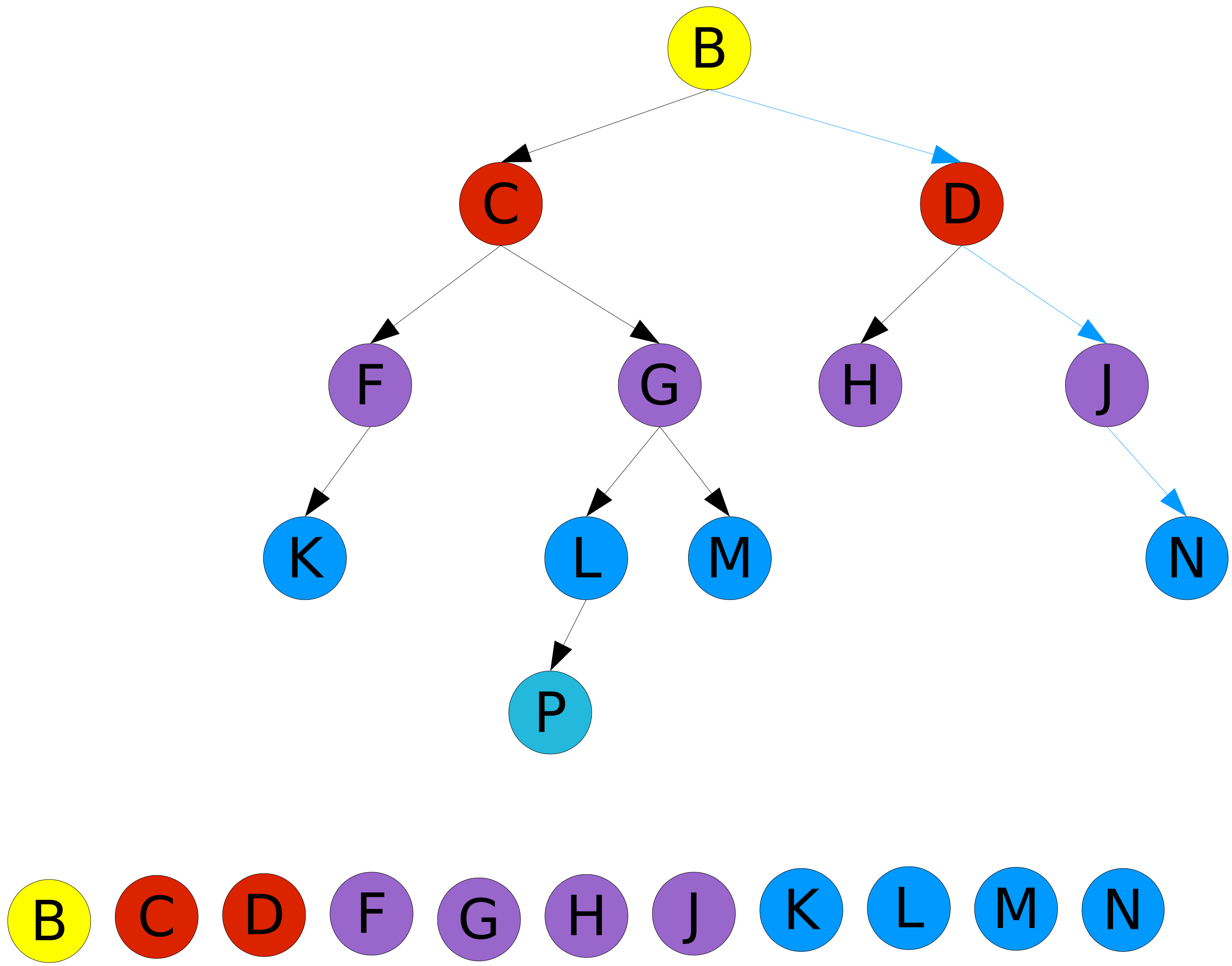


Levelorder traversal recursive

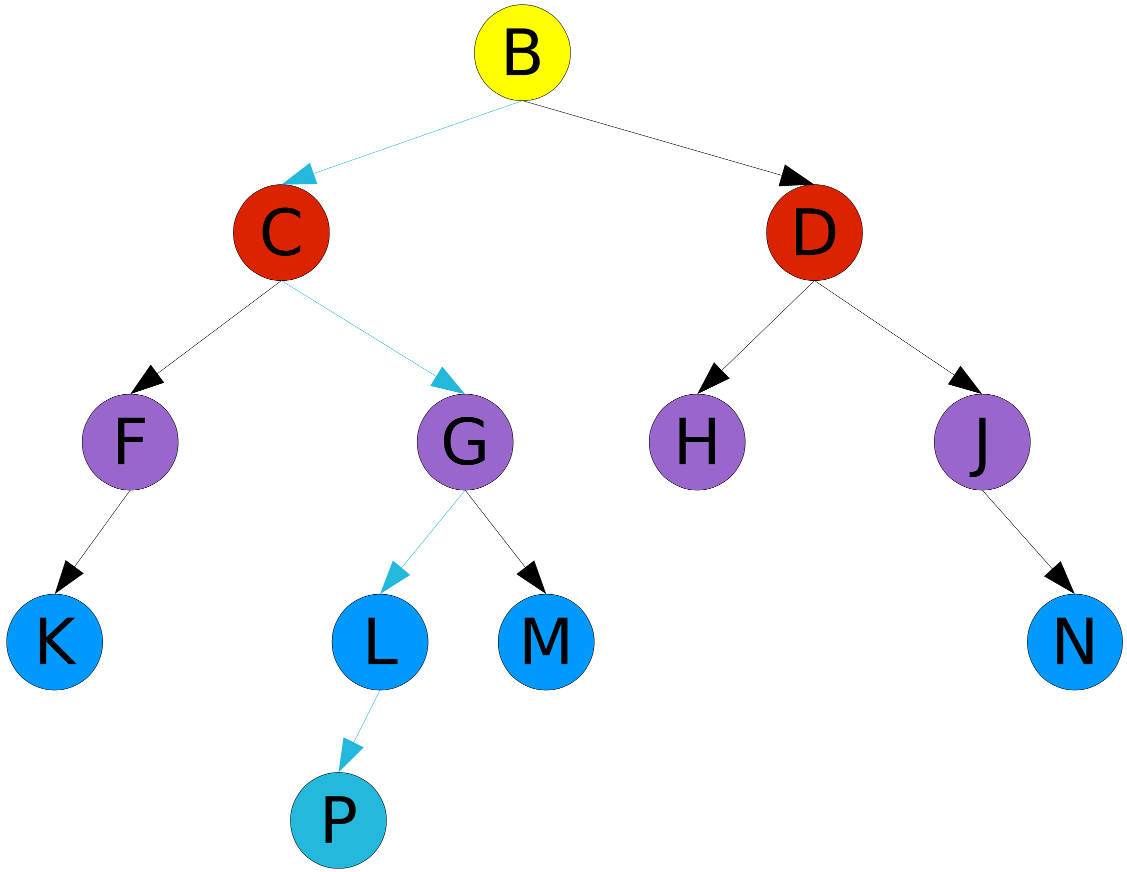


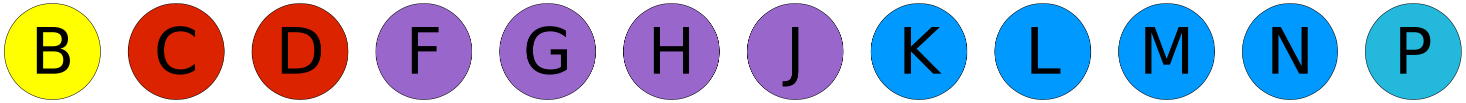


Levelorder traversal recursive

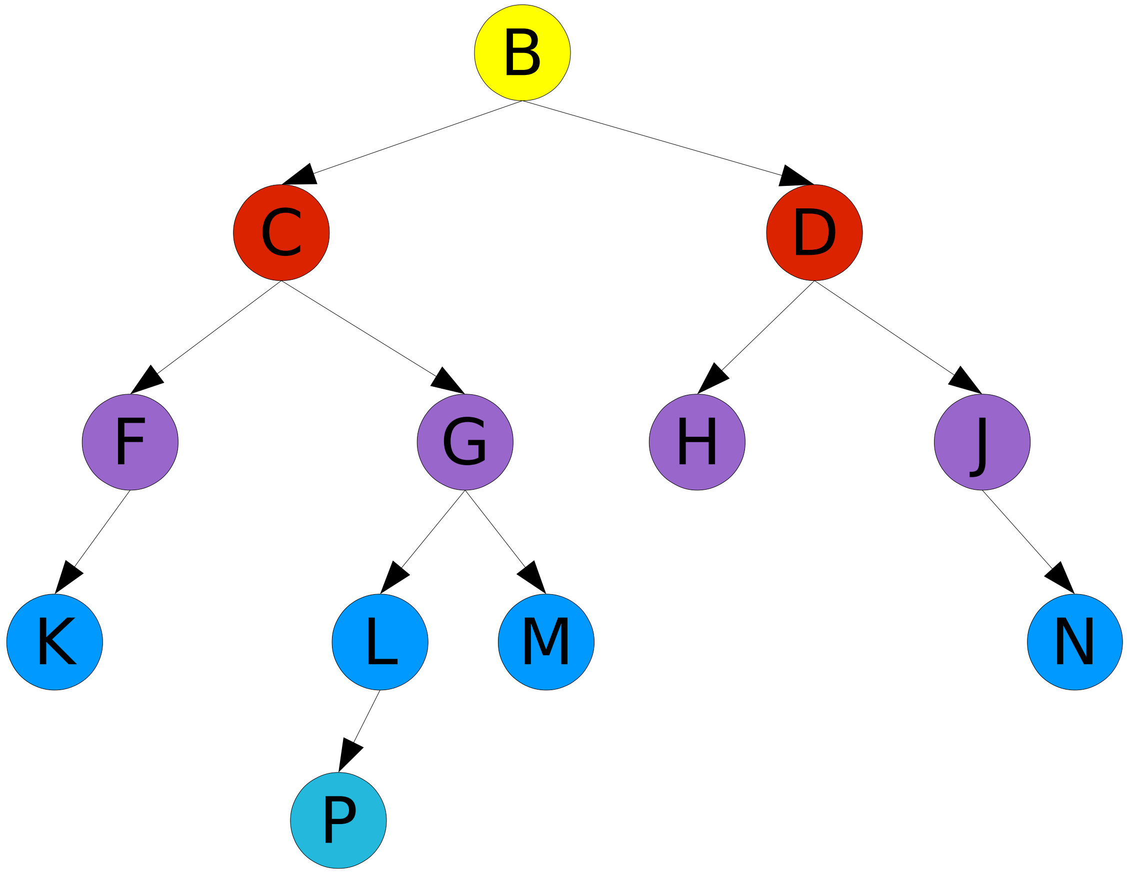


Levelorder traversal recursive





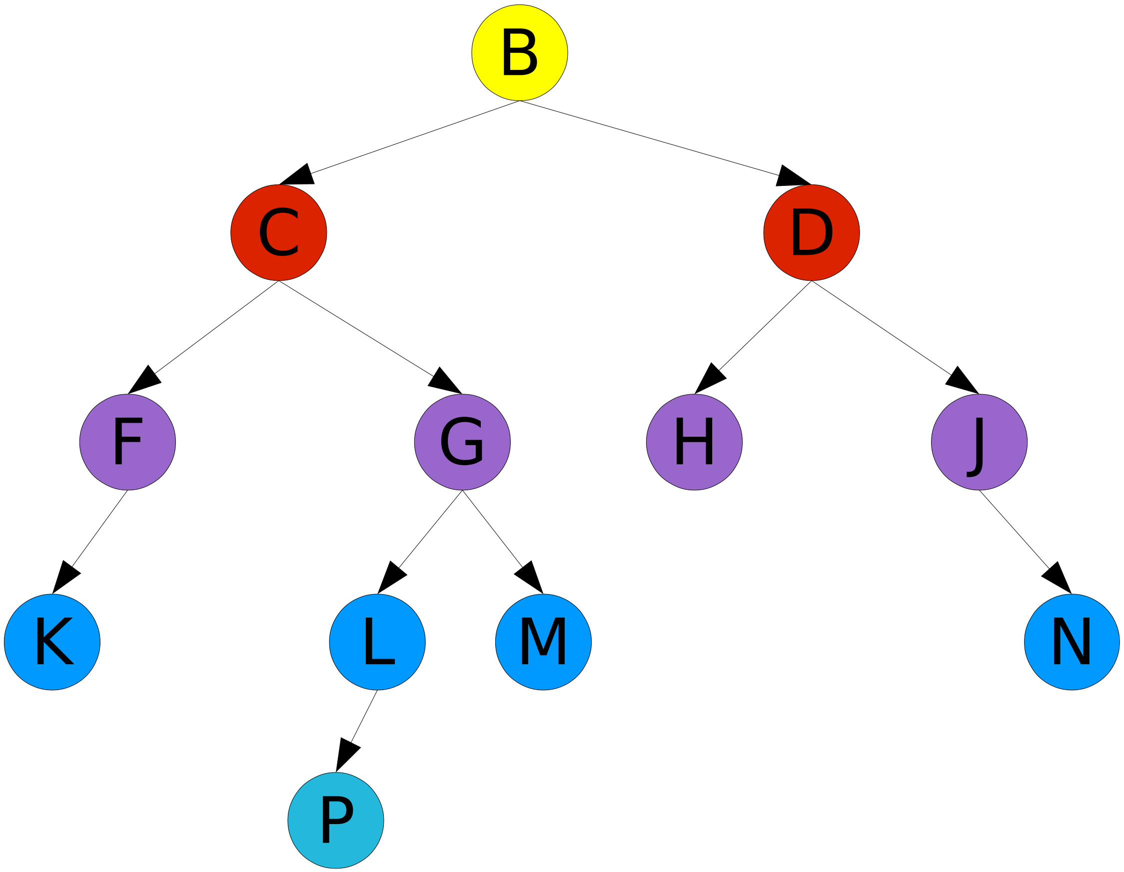
Levelorder traversal using a queue



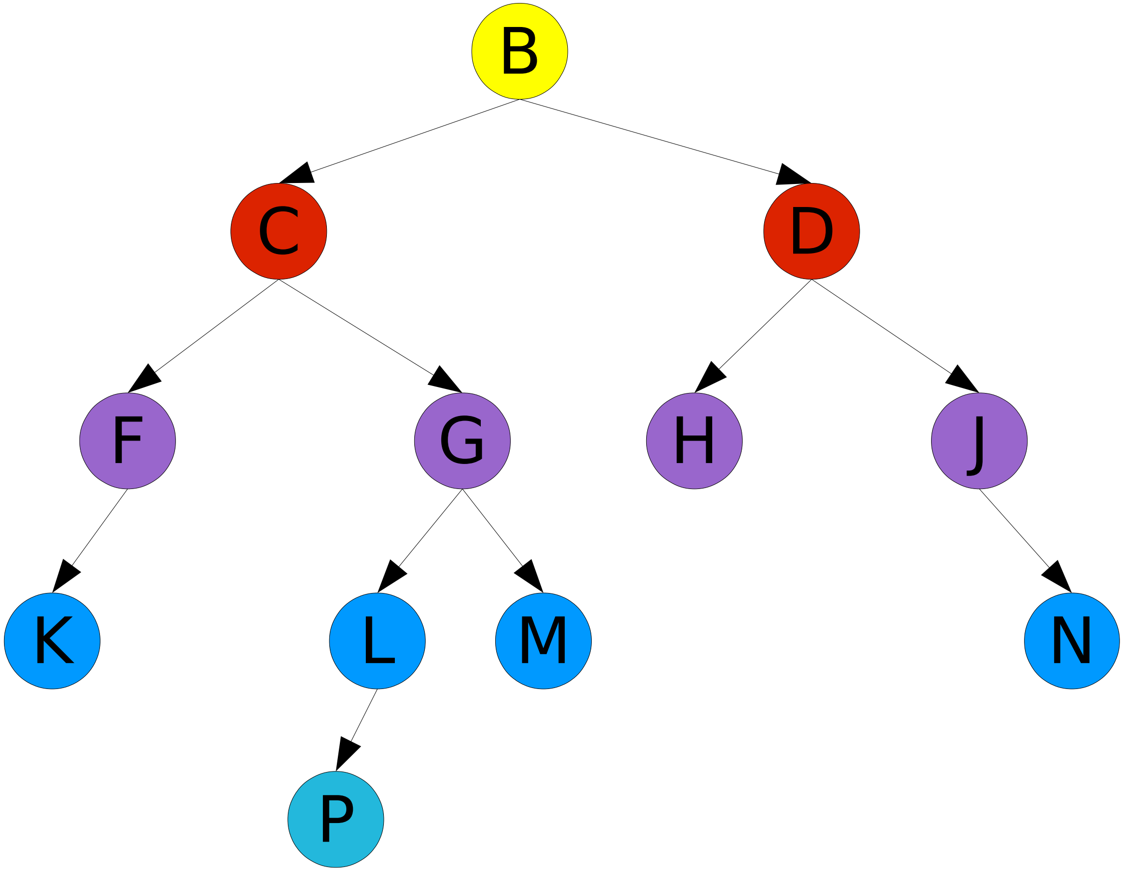
Level order traversal using a queue

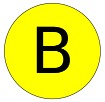
```
Create queue aQueue
Add root node to aQueue

While aQueue is not empty
    Get node from queue into aNode
    Do something with aNode
    If exists add left child of aNode to aQueue
    If exists add right child of aNode to aQueue
```

Levelorder traversal using a queue

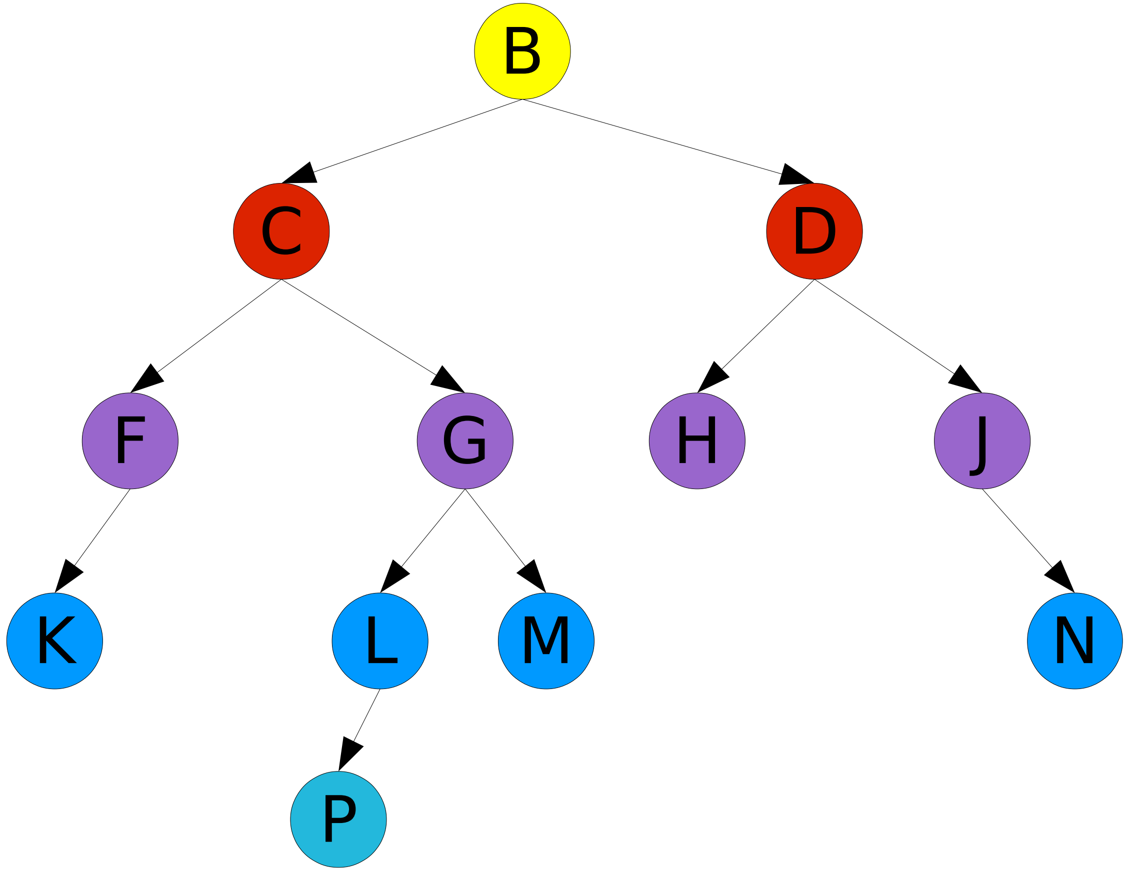


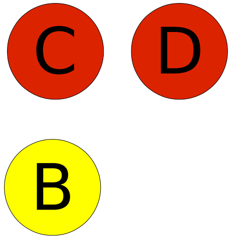
Levelorder traversal using a queue



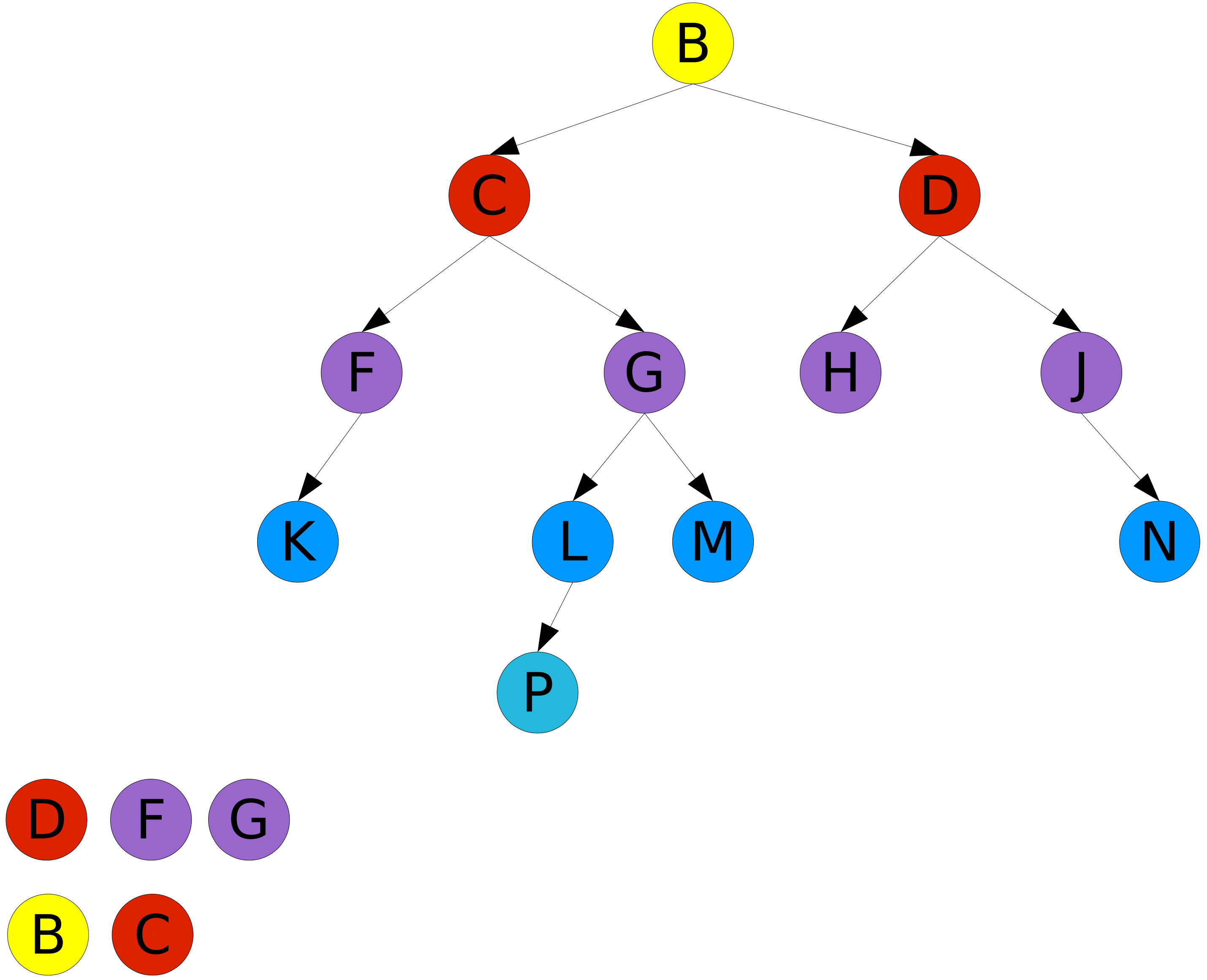


Levelorder traversal using a queue





Levelorder traversal using a queue



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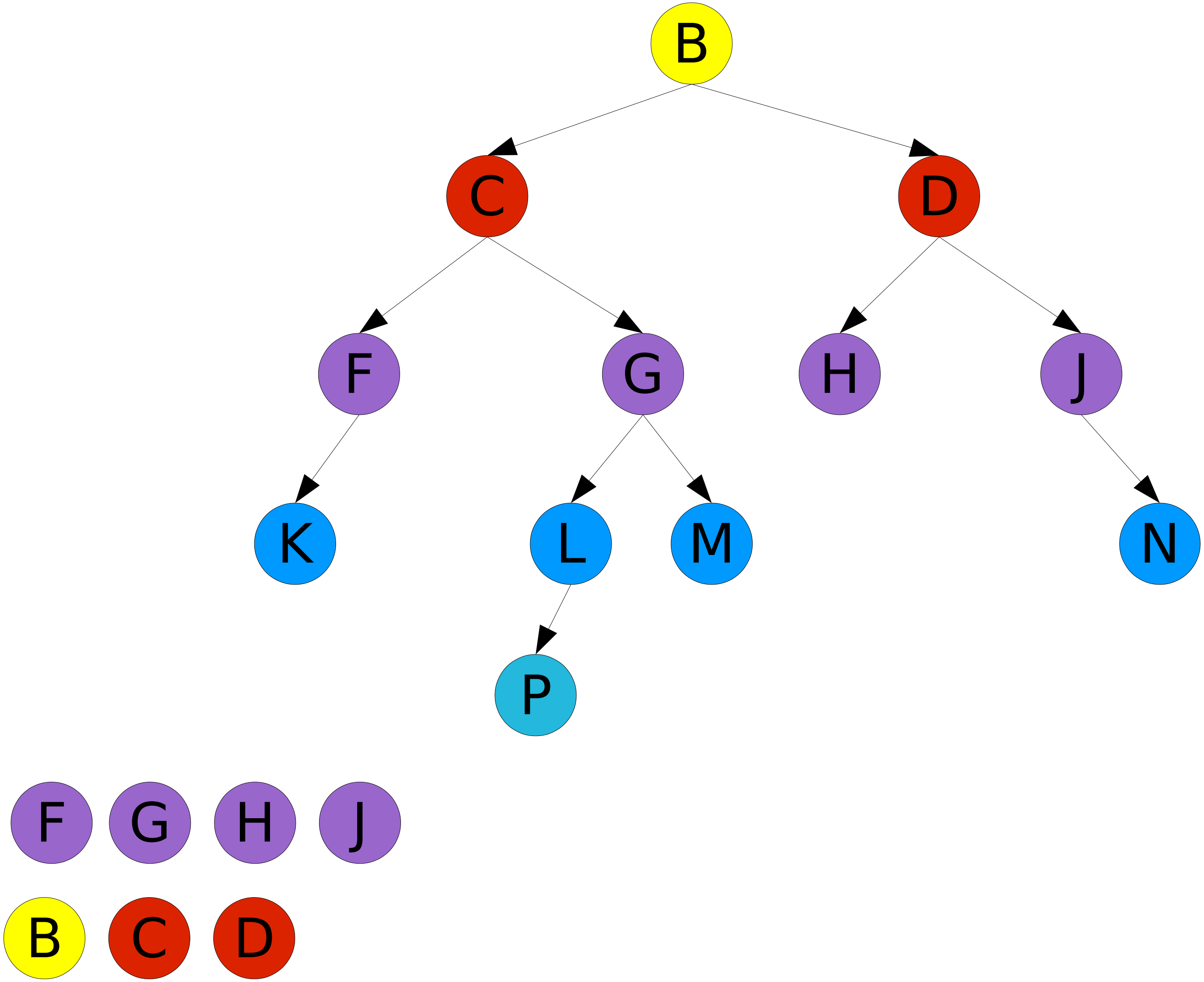

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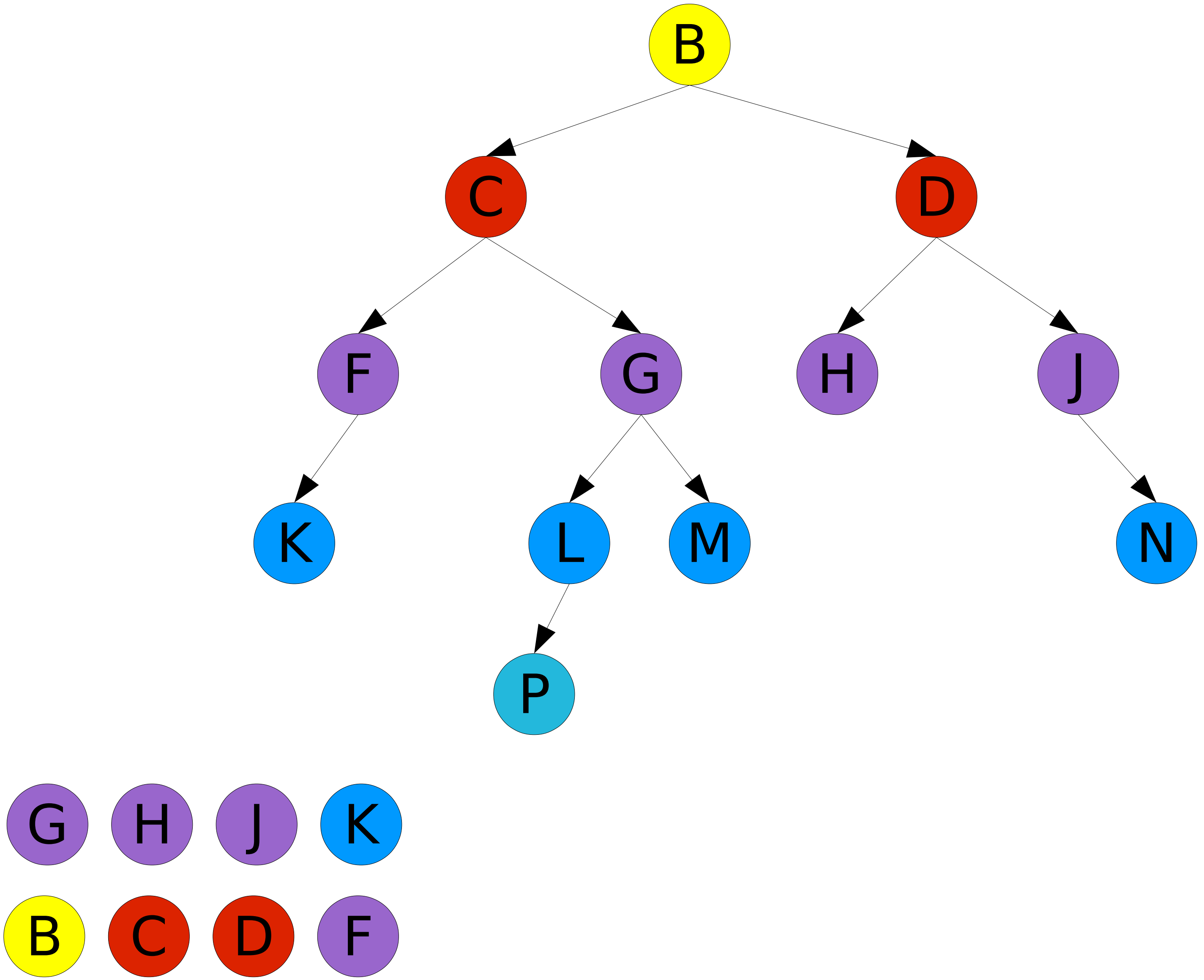

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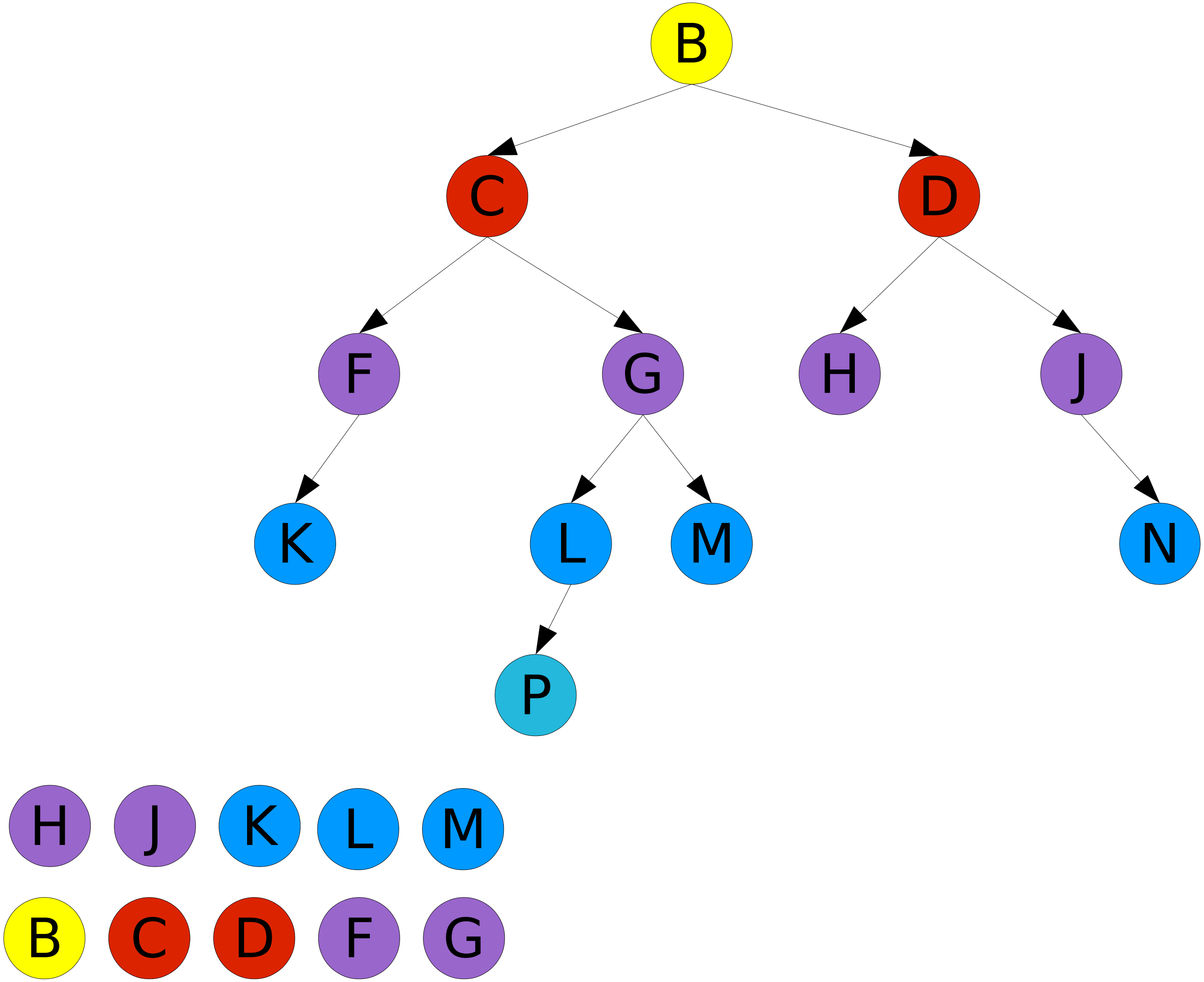

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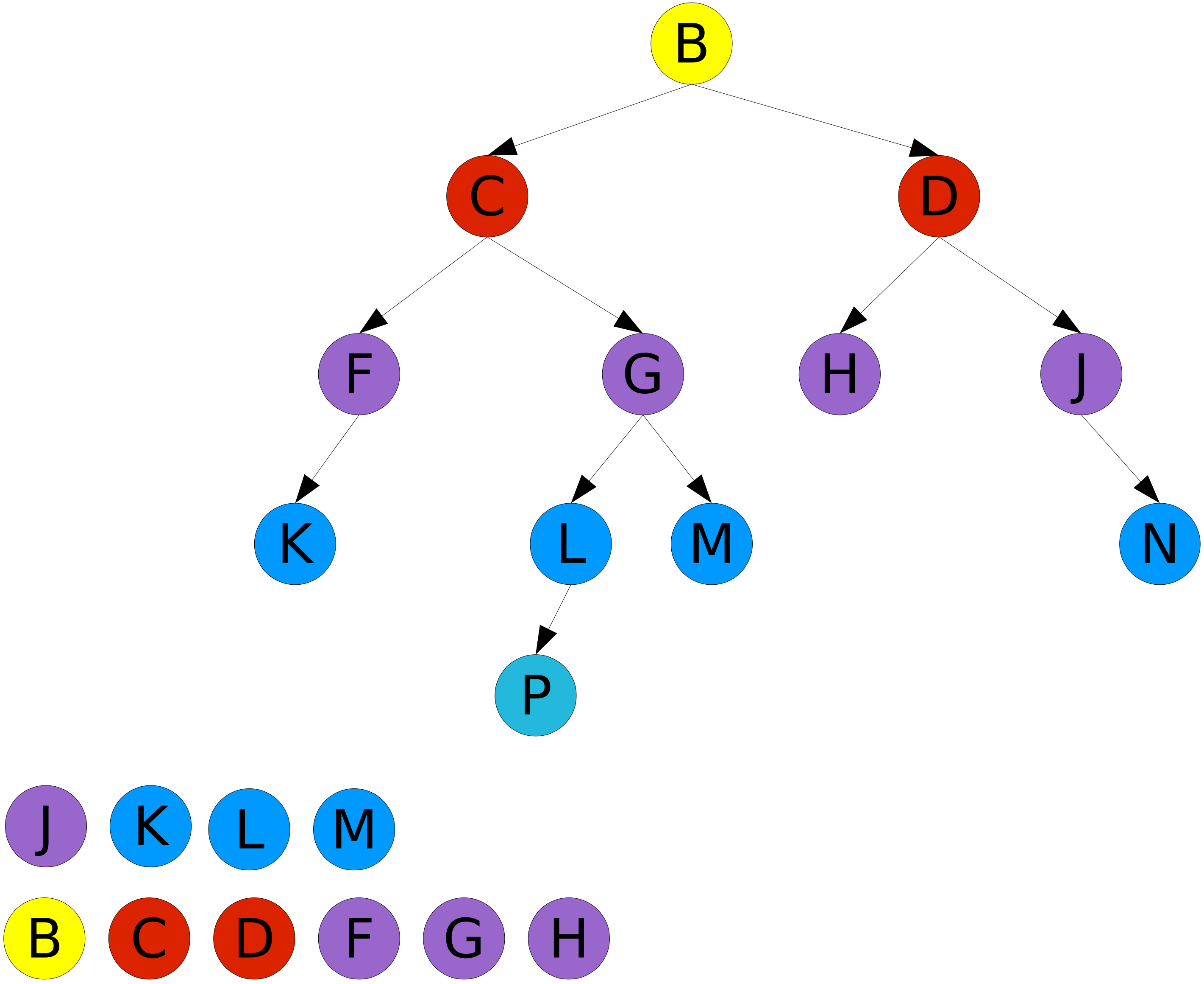
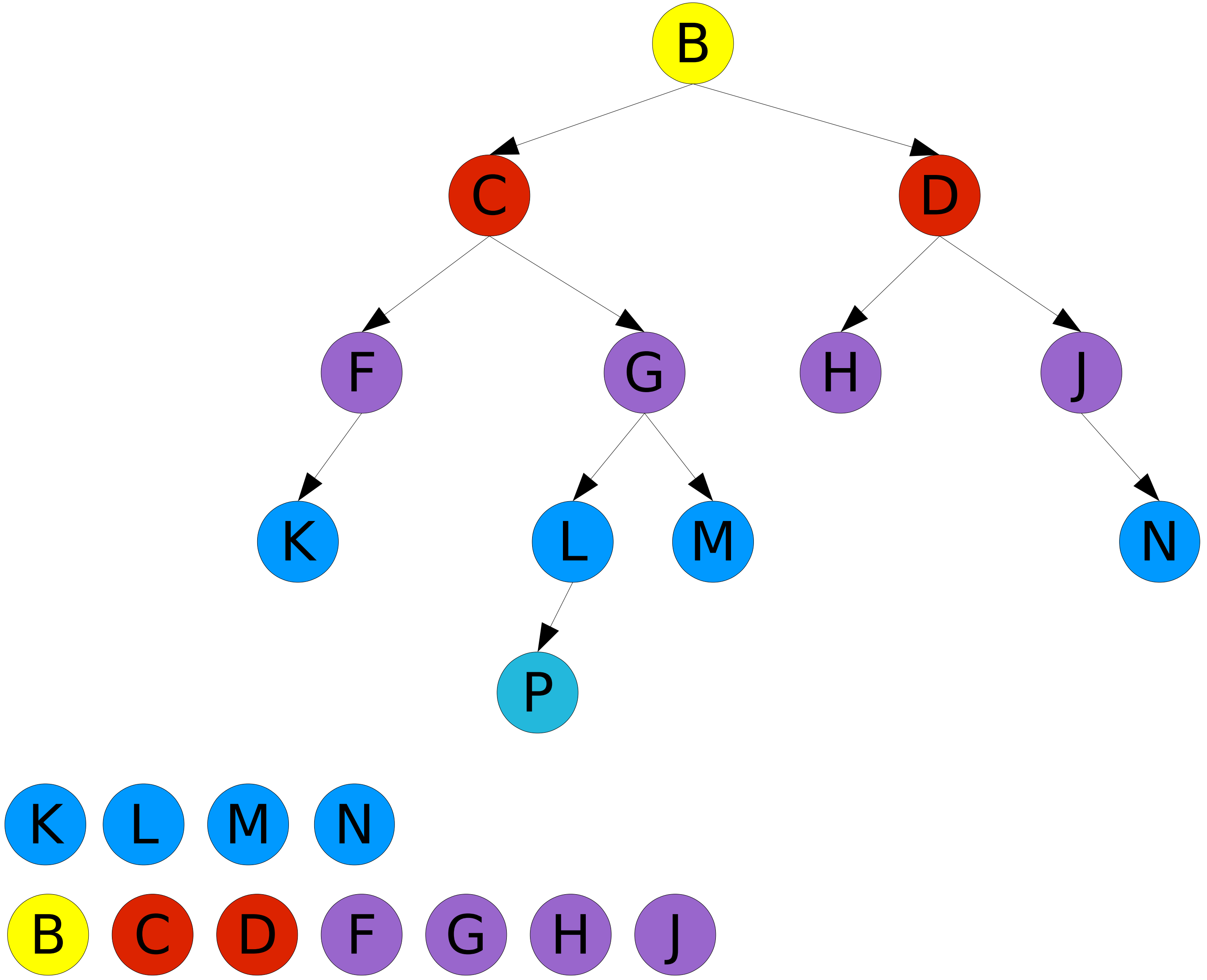
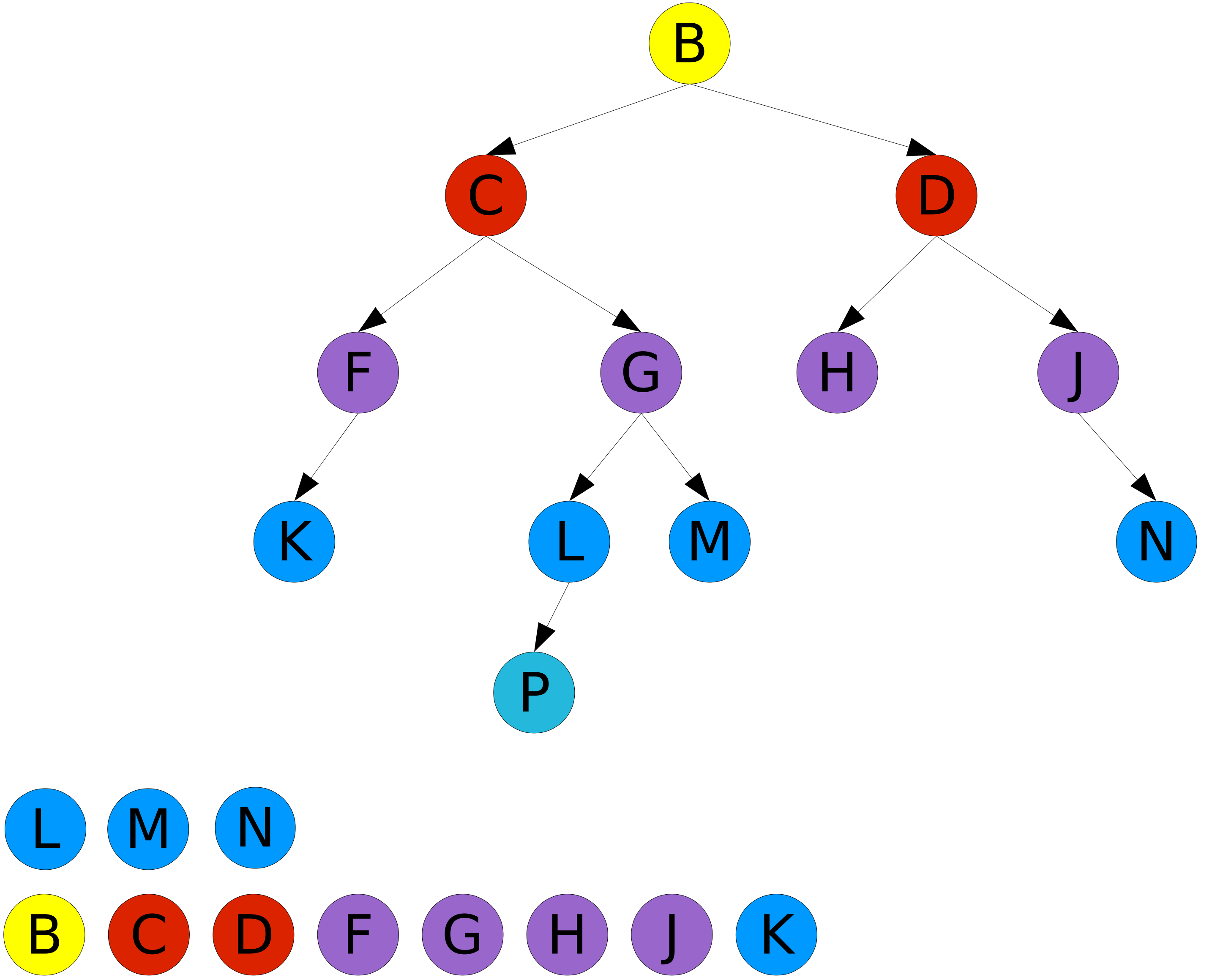


Levelorder traversal using a queue



Levelorder traversal using a queue



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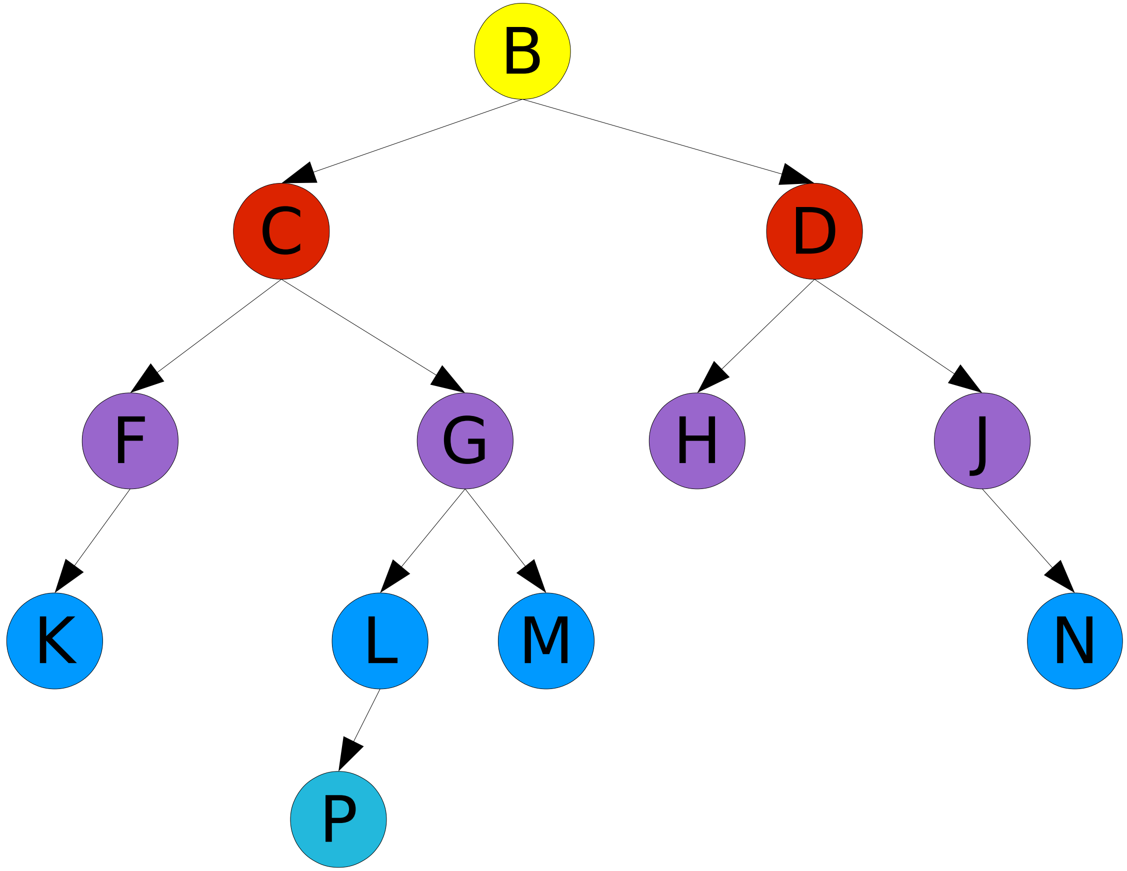

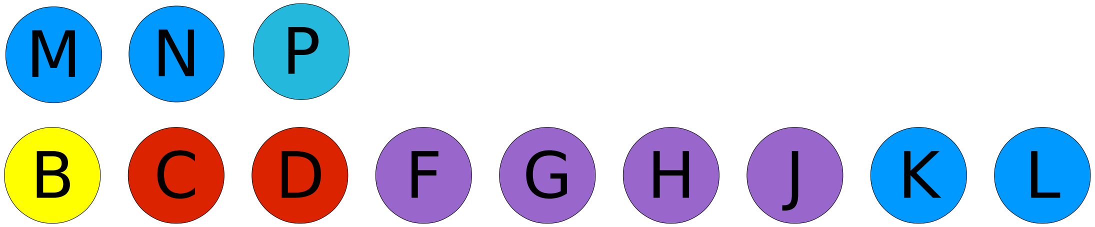

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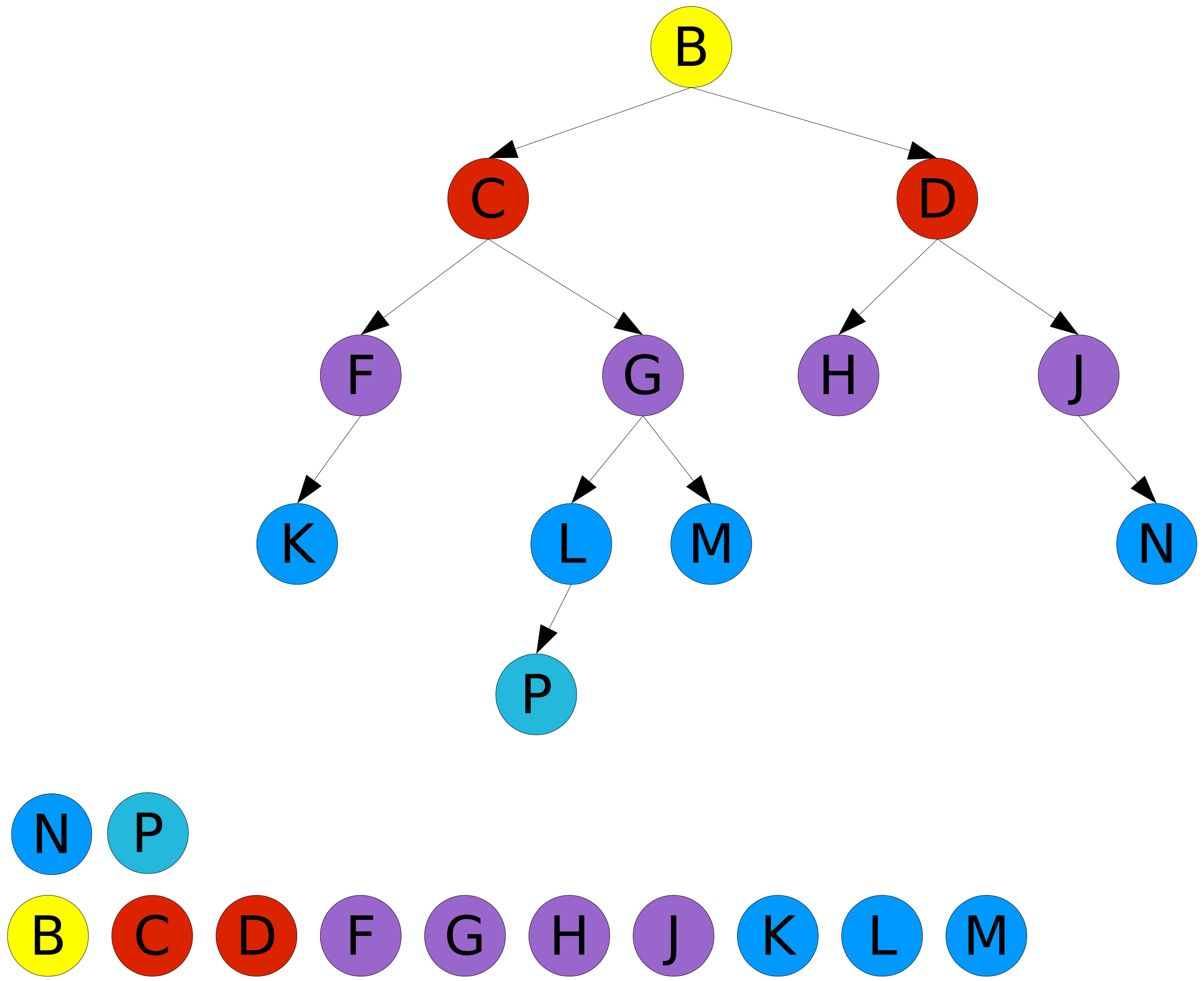

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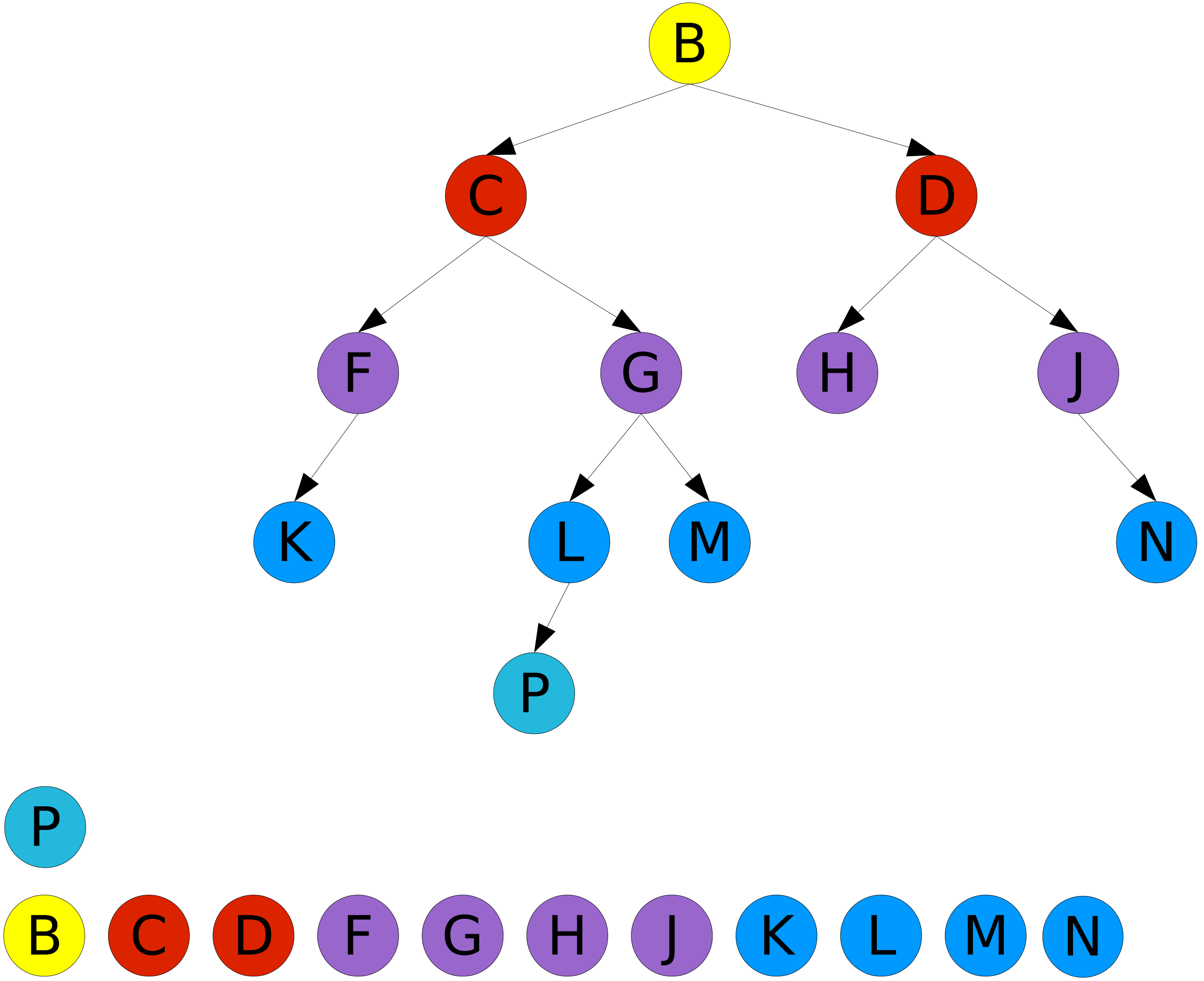

Levelorder traversal using a queue