The Language Model Zoo

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Seminar für Sprachwissenschaft

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> *A language model is a machine learning model that predicts upcoming words. More formally, a language model assigns a probability to each possible next word, or equivalently gives a probability distribution over possible next words.*
>
> — *Jurafsky and Martin* (2025)

What is a language model?

- Historically, language models used to have important, but rather limited applicability in NLP, e.g., in machine translation and speech recognition
- Until recently, 'language model' meant '*n-gram* language model'
- With successful application of *neural* language models, the application areas of language models also grew

What is in this course

- Review of a relatively large body of literature: some (few) historical, many modern language models
- Lots of reading & discussion – not much practice

The topics (tentative)

- Basics, n-gram language models (1 week)
- RNN language models (2 weeks)
- Encoder-only Transformer models (2 weeks)
- Encoder–decoder Transformer models (1 week)
- Decoder only Transformer models (2 weeks)
- Smaller, parameter-/compute-/data-efficient models (1 week)
- Fine-tuning for downstream applications efficiently (Adapters, LORA, quantization - 1 week)
- Efficient, smaller models (1 week)
- Multilinguality (1 week)
- Speech and Vision-language models (1 week)

Participation

- You need to read the paper for the session (or week)
- Post two questions or discussion points *before 8am* on the day of the lecture
- Participate in the class discussion

Credits, grading

6CP Participation during the class:

- Reading all the papers timely
- Posting your discussion points for papers to be discussed (at least for 80%)
- Participating in the in-class discussions

Graded:

- A short (2-4 pages) survey / summary of a subfield

9CP Besides all of the above

- In addition to all above, a term paper either on an application, or an extended survey of a particular area

3CP Better not, but the same as 6CP

The aim

We want to solve two related problems:

- Given a sequence of words $\boldsymbol{w} = (w_1 w_2 \dots w_m)$,

  what is the probability of the sequence
  $P(\boldsymbol{w})$?

  (machine translation, automatic speech recognition, spelling correction)
- Given a sequence of words $w_1 w_2 \dots w_{m-1}$,

  what is the probability of the next word $P(w_m \mid w_1 \dots w_{m-1})$?

  (predictive text)

Assigning probabilities to sentences

count and divide?

How do we calculate the probability of a sentence like
$P(\text{I like pizza with spinach})$

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- Can we count the occurrences of the sentence, and divide it by the total number of sentences (in a large corpus)?

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- Short answer: No.

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- Short answer: No.
  - Many sentences are not observed even in very large corpora

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- Can we count the occurrences of the sentence, and divide it by the total number of sentences (in a large corpus)?
- Short answer: No.
  - Many sentences are not observed even in very large corpora
  - For the ones observed in a corpus, probabilities will not reflect our intuitions, or will not be useful in most applications

Assigning probabilities to sentences

applying the chain rule

- The solution is to *decompose*

  We use probabilities of parts of the sentence (words) to calculate the probability of the whole sentence

- Using the chain rule of probability (without loss of generality), we can write

$$\begin{aligned} P(w_1, w_2, \ldots, w_m) = \; & P(w_2 \mid w_1) \\ & \times P(w_3 \mid w_1, w_2) \\ & \times \ldots \\ & \times P(w_m \mid w_1, w_2, \ldots w_{m-1}) \end{aligned}$$

Example: applying the chain rule

$$
\begin{aligned}
P(\text{I like pizza with spinach}) = \quad & P(\text{like} \mid \text{I}) \\
\times\ & P(\text{pizza} \mid \text{I like}) \\
\times\ & P(\text{with} \mid \text{I like pizza}) \\
\times\ & P(\text{spinach} \mid \text{I like pizza with})
\end{aligned}
$$

- Did we solve the problem?

Example: applying the chain rule

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$$

- Did we solve the problem?
- Not really, the last term is equally difficult to estimate

Example: bigram probabilities of a sentence

$$
\begin{aligned}
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\end{aligned}
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Example: bigram probabilities of a sentence

with first-order Markov assumption

$$
\begin{aligned}
P(\text{I like pizza with spinach}) = \; & P(\text{like} \mid \text{I}) \\
\times\; & P(\text{pizza} \mid \text{like}) \\
\times\; & P(\text{with} \mid \text{pizza}) \\
\times\; & P(\text{spinach} \mid \text{with})
\end{aligned}
$$

- Now, hopefully, we can count them in a corpus

Maximum-likelihood estimation (MLE)

- The MLE of n-gram probabilities is based on their frequencies in a corpus
- We are interested in conditional probabilities of the form: $P(w_i \mid w_1, \ldots, w_{i-1})$, which we estimate using

$$P(w_i \mid w_{i-n+1}, \ldots, w_{i-1}) = \frac{C(w_{i-n+1} \ldots w_i)}{C(w_{i-n+1} \ldots w_{i-1})}$$

  where, $C()$ is the frequency (count) of the sequence in the corpus.
- For example, the probability $P(\text{like} \mid \text{I})$ would be

$$\begin{aligned} P(\text{like} \mid \text{I}) &= \frac{C(\text{I like})}{C(\text{I})} \\ &= \frac{\text{number of times I like occurs in the corpus}}{\text{number of times I occurs in the corpus}} \end{aligned}$$

MLE estimation of an n-gram language model

An n-gram model conditioned on $n - 1$ previous words.

$$
\begin{array}{ll}
\text{unigram} & P(w_i) = \dfrac{C(w_i)}{N} \\
\text{bigram} & P(w_i) = P(w_i \mid w_{i-1}) = \dfrac{C(w_{i-1}w_i)}{C(w_{i-1})} \\
\text{trigram} & P(w_i) = P(w_i \mid w_{i-2}w_{i-1}) = \dfrac{C(w_{i-2}w_{i-1}w_i)}{C(w_{i-2}w_{i-1})}
\end{array}
$$

Parameters of an n-gram model are these conditional probabilities.

N-gram models define probability distributions

- An n-gram model defines a probability distribution over words

$$\sum_{w \in V} P(w) = 1$$

- They also define probability distributions over word sequences of equal size. For example (length 2),

$$\sum_{w \in V} \sum_{v \in V} P(w)P(v) = 1$$

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- Example: probabilities in sentence
I'm sorry, Dave, I'm afraid I can't do that.

word	prob
I	0.200
'm	0.133
.	0.133
't	0.067
,	0.067
Dave	0.067
afraid	0.067
can	0.067
do	0.067
sorry	0.067
that	0.067
	1.000

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- Example: probabilities in sentence *I'm sorry, Dave, I'm afraid I can't do that.*
- What about sentences?

word	prob
I	0.200
'm	0.133
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Bigrams

Bigrams are overlapping sequences of two tokens.

I 'm sorry , Dave .

I 'm afraid I can 't do that .

Bigram counts

ngram	freq	ngram	freq	ngram	freq	ngram	freq
I 'm	2	, Dave	1	afraid I	1	n't do	1
'm sorry	1	Dave .	1	I can	1	do that	1
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- What about the bigram ‘ `. I` ’?

Sentence boundary markers

If we want sentence probabilities, we need to mark them.

⟨s⟩ I ’m sorry , Dave . ⟨/s⟩
⟨s⟩ I ’m afraid I can ’t do that . ⟨/s⟩

- The bigram ‘ ⟨s⟩ `I` ’ is not the same as the unigram ‘ `I` ’
  Including ⟨s⟩ allows us to predict likely words at the beginning of a sentence
- Including ⟨/s⟩ allows us to assign a proper probability distribution to sentences

Short detour: colorless green ideas

> *But it must be recognized that the notion 'probability of a sentence' is an entirely useless one, under any known interpretation of this term.* — *Chomsky (1968)*

- Can n-gram models assign 'useful' probabilities to sentences like
  - *Colorless green ideas sleep furiously*
- How can they be improved?

How to evaluate (n-gram) language models?

Intrinsic: the higher the probability assigned to a test set better the model. A few measures:

- Likelihood
- (cross) entropy
- perplexity

Extrinsic: improvement of the target application due to the language model:

- Speech recognition accuracy
- BLEU score for machine translation
- Keystroke savings in predictive text applications

Likelihood

- Likelihood of a model M is the probability of the (test) set $\boldsymbol{w}$ given the model

$$\mathcal{L}(M \mid \boldsymbol{w}) = P(\boldsymbol{w} \mid M) = \prod_{s \in \boldsymbol{w}} P(s)$$

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- Likelihood is sensitive to the test set size
- Practical note: (minus) log likelihood is used more commonly, because of ease of numerical manipulation

Cross entropy

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Reminder: Cross entropy is the bits required to encode the data coming from P using another (approximate) distribution $\widehat{P}$.

$$H(P, Q) = -\sum_{x} P(x) \log \widehat{P}(x)$$

Perplexity

- Perplexity is a more common measure for evaluating language models

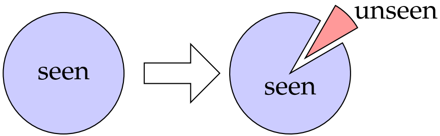
$$\mathrm{PP}(\boldsymbol{w}) = 2^{\mathrm{H}(\boldsymbol{w})} = \mathrm{P}(\boldsymbol{w})^{-\frac{1}{N}} = \sqrt[N]{\frac{1}{\mathrm{P}(\boldsymbol{w})}}$$

- Perplexity is the average branching factor
- Similar to cross entropy
  - lower better
  - not sensitive to test set size

What do we do with unseen n-grams?

...and other issues with MLE estimates

- Words (and word sequences) are distributed according to the Zipf's law: *many words are rare*.
- MLE will assign 0 probabilities to unseen words, and sequences containing unseen words
- Even with non-zero probabilities, MLE *overfits* the training data
- One solution is smoothing: take some probability mass from known words, and assign it to unknown words



Laplace smoothing

(Add-one smoothing)

- The idea (from 1790): add one to all counts
- The probability of a word is estimated by

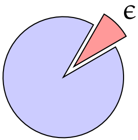
$$P_{+1}(w) = \frac{C(w)+1}{N+V}$$

N number of word tokens
V number of word types - the size of the vocabulary

- Then, probability of an unknown word is:

$$\frac{0+1}{N+V}$$

Absolute discounting



- An alternative to the additive smoothing is to reserve an explicit amount of probability mass, ϵ, for the unseen events
- The probabilities of known events has to be re-normalized
- How do we decide what ϵ value to use?

Good-Turing smoothing

- Estimate the probability mass to be reserved for the novel n-grams using the observed n-grams
- Novel events in our training set is the ones that occur once

$$p_0 = \frac{n_1}{n}$$

  where n_1 is the number of distinct n-grams with frequency 1 in the training data
- Now we need to discount this mass from the higher counts
- The probability of an n-gram that occurred r times in the corpus is

$$(r+1)\frac{n_{r+1}}{n_r n}$$

Back-off

Back-off uses the estimate if it is available, 'backs off' to the lower order n-gram(s) otherwise:

$$P(w_i \mid w_{i-1}) = \begin{cases} P^*(w_i \mid w_{i-1}) & \text{if } C(w_{i-1}wi) > 0 \\ \alpha P(w_i) & \text{otherwise} \end{cases}$$

where,

- $P^*(\cdot)$ is the discounted probability
- α makes sure that $\sum P(w)$ is the discounted amount
- $P(w_i)$, typically, smoothed unigram probability

Interpolation

Interpolation uses a linear combination:

$$P_{int}(w_i \mid w_{i-1}) = \lambda P(w_i \mid w_{i-1}) + (1 - \lambda) P(w_i)$$

In general (recursive definition),

$$P_{int}(w_i \mid w_{i-n+1}^{i-1}) = \lambda P(w_i \mid w_{i-n+1}^{i-1}) + (1 - \lambda) P_{int}(w_i \mid w_{i-n+2}^{i-1})$$

- $\sum \lambda_i = 1$
- Recursion terminates with
  - either smoothed unigram counts
  - or uniform distribution $\frac{1}{V}$

Some shortcomings of the n-gram language models

The n-gram language models are simple and successful, but …

- They cannot handle long-distance dependencies:
  `In the last race, the horse he bought last year finally _____.`
- The success often drops in morphologically complex languages
- The smoothing methods are often 'a bag of tricks'
- They are highly sensitive to the training data: you do not want to use an n-gram model trained on business news for medical texts

Summary

- A (n-gram) language model assigns probabilities to sequences (sentences)
- N-gram language models do this by
  - estimating probabilities of parts of the sentence (n-grams)
  - use the n-gram probability and a conditional independence assumption to estimate the probability of the sentence
- MLE estimate for n-gram overfit
- Smoothing is a way to fight overfitting
- Back-off and interpolation yields better 'smoothing'
- There are other ways to improve n-gram models, and language models without (explicitly) use of n-grams
- Many problems remain: n-gram language models have been very difficult to improve
- Neural language models fix many of the problems of n-gram models

Recommended reading

(not required, but highly recommended)

- Claude E. Shannon (1948). "A mathematical theory of communication". In: *Bell Systems Technical Journal* 27, pp. 379–423, 623–656
- Stanley F Chen and Joshua Goodman (1998). *An empirical study of smoothing techniques for language modeling*. Tech. rep. TR-10-98. Harvard University, Computer Science Group. URL: `https://dash.harvard.edu/handle/1/25104739`
- Joshua T Goodman (2001). "A bit of progress in language modeling". In: *Computer Speech & Language* 15.4, pp. 403–434

References / additional reading material

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- Shannon, Claude E. (1948). "A mathematical theory of communication". In: *Bell Systems Technical Journal* 27, pp. 379–423, 623–656.