The isolated community evacuation problem with mixed integer programming

Introduction

The new model formulation presented in this paper was motivated by the rising need to prepare for and mitigate the effects of disasters caused by natural hazards on the populations in remote communities:

- Small islands
- Coastal communities
- Remote valley hamlets
- Mountain towns

Problem description

1. *During an emergency, how can resources be optimally routed to evacuate the entire community as quickly as possible?*

2. *During evacuation planning, which resources need to be secured to prepare for quick evacuation over a variety of disaster scenarios?*

Gaps in literature

No previous research has provided any solutions for optimizing the evacuation of isolated communities.
This includes:

- *partial incompatibility between resources and access points*
- *asymmetric travel time matrices*
- *heterogeneous speed and capacity capabilities of recovery resources*

Problem assumptions

- *All road connections out of the disaster area are considered disrupted.*
- *The evacuee populations are distributed between different locations of the affected area.*
- *A central planning entity has full authority over planning and coordination of a fleet of recovery resources*
- *All recovery resources considered are located within reasonable distance to the affected area and may differ in their capabilities in terms of :*
  - *their contracting cost*
  - *variable operating cost*
  - *carrying capacity*
  - *loaded and unloaded travel speeds*
  - *loading times*
  - *time to availability*
  - *compatibility with potential pick-up and drop-off points in the affected area*
  - *initial locations*

Problem assumptions

- *All recovery resources start from their initial positions and travel to a pick-up location in the affected area, and they alternate in between pick-up locations and shelter locations until the number of evacuees is zero*
- *The sets of initial resource positions, evacuation pick-up points, and shelters are known. Shelter and pick-up point identification is thus not part of this problem.*
- *Population of evacuees will be at the pick-up locations upon arrival of resources (arrival rates of evacuees not considered).*
- *Evacuees are considered safe once they have been dropped off at a shelter location*
- *Recovery resources are operating continuously without downtime*

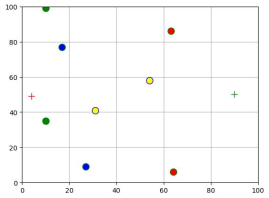
Model design: Resources

Resources are rescue transportation vehicles that will extract people from affected area:

Properties:

- *Loading capacity*
- *Loading operation timings*
- *Time to availability*
- *Loaded and unloaded travel speed*
- *Initial location*
- *Class of vehicle*
- *Variable and fixed cost*

Model design: Nodes



S: Source node, represent the entire community

A: Evacuation areas: evacuees reach pick up points traveling from here

- *Each area has his specific 'evacuation demand'*

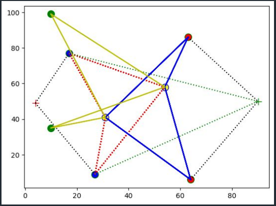
B: Pick up points: where resources pick up evacuees

C: Shelters: people are considered safed when are dropped off to a shelter

T: Sink node: represents the fact that rescued people leave the rescued zone

H: Resource initial location

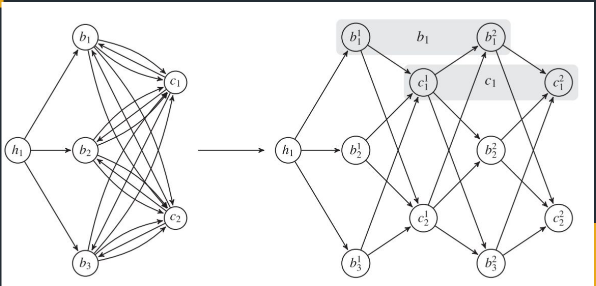
Model design: Arcs



Arcs represent the link between nodes:

- *Distance between nodes and consequently a cost in time terms*
- *The flow of rescued people*
- *Compatibility between each resource's class and arc's end node, since possibly not all rescue vehicles are allowed to reach every location due to physical reasons (eg. a ferry cannot reach a parking lot or a mountain community)*

Model: time expansion



Mathematical formulation: Notation and parameters

Notation key for D-ICEP.

Sets	Set description	Parameters	Parameter description
$i \in I$	Recovery resources	q_i	Passenger capacity of resource i
$k \in K$	Potential round trips per resource	u_i	Time to availability of resource i
s	Source node	o_i	Loading time of resource i
$a \in A$	Evacuation areas	p_i	Unloading time of resource i
$b \in B$	Pick-up points in evacuation area	d_a	Evacuation demand at location a
$c \in C$	Drop-off points in safe locations	g_a	Max. no. of self-evacuations from area a
t	Sink node	t^i_{hb}	$\frac{distance(h \to b)}{\text{empty travel speed of resource } i}$: cost of arc ζ^{1i}_{hb}
$h \in H$	Initial resource locations	t^i_{bc}	$\frac{distance(b \to c)}{\text{loaded travel speed of resource } i}$: cost of arc γ^{ki}_{bc}
		t^i_{cb}	$\frac{distance(c \to b)}{\text{empty travel speed of resource } i}$: cost of arc δ^{ki}_{cb}, Only $k = 1, \dots, K-1$

Mathematical formulation: Arcs and variables description

Arcs	Arc description	Variables	Variable description
$\alpha_{sa} \in \bar{A}$	Source s to area a	fl_{at}	Flow on arc λ_{at}
$\beta^{ki}_{ab} \in \bar{B}$	Area a to pick-up b of trip k for resource i	fl^{ki}_{ab}	Flow on arc β^{ki}_{ab}
$\gamma^{ki}_{bc} \in \bar{\Gamma}$	Pick-up b to drop-off c of trip k for resource i	fl^{ki}_{bc}	Flow on arc γ^{ki}_{bc}
$\delta^{ki}_{cb} \in \bar{\Delta}$	Drop-off c to pick-up b of trip k to trip $k+1$	fl^{ki}_{ct}	Flow on arc ϵ^{ki}_{ct}
	For resource i, for $k = 1, \ldots, K-1$	w^{1i}_{hb}	{**1**: if route on ζ^{1i}_{hb} selected, **0**: otherwise}
$\epsilon_{ct} \in \bar{E}$	Drop-off c to sink node t	x^{ki}_{bc}	{**1**: if route on γ^{ki}_{bc} selected, **0**: otherwise}
$\zeta^{1i}_{hb} \in \bar{Z}$	Initial resource location h to pick-up b	y^{ki}_{cb}	{**1**: if route on δ^{ki}_{cb} selected, **0**: otherwise}
	For resource i, on trip 1	r	Total evacuation time
$\lambda_{at} \in \bar{\Lambda}$	Area a to sink node t, for private evacuations	s_i	Route completion time of resource i

Mathematical formulation: constraints

Total time lower bound and route completion time constraints:

$$\min \quad r \tag{1}$$

$$s.t. \quad r \geq s_i \qquad \forall i \in I \tag{2}$$

$$s_i = \sum_{\zeta_{hb}^{1i} \in \bar{Z}} \left(t_{hb}^{i} w_{hb}^{1i}\right) + \sum_{\gamma_{bc}^{ki} \in \bar{\Gamma}} \left(t_{bc}^{i} x_{bc}^{ki}\right) + \sum_{\delta_{cb}^{ki} \in \bar{\Delta}} \left(t_{cb}^{i} y_{cb}^{ki}\right) + \sum_{\zeta_{hb}^{1i} \in \bar{Z}} \left(u_i w_{hb}^{1i}\right) + \sum_{\zeta_{hb}^{1i} \in \bar{Z}} \left(o_i w_{hb}^{1i}\right) + \sum_{\delta_{cb}^{ki} \in \bar{\Delta}} \left(o_i y_{cb}^{ki}\right) + \sum_{\gamma_{bc}^{ki} \in \bar{\Gamma}} \left(p_i x_{bc}^{ki}\right) \qquad \forall i \in I \tag{3}$$

Mathematical formulation: constraints

Capacity constraints:

$$fl_{at} \leq g_a \qquad \forall \lambda_{at} \in \bar{\Lambda} \qquad (4)$$

$$fl^{ki}_{bc} \leq q_i(x^{ki}_{bc}) \qquad \forall \gamma^{ki}_{bc} \in \bar{\Gamma} \qquad (5)$$

Flow conservation constraints:

$$d_a = fl_{at} + \sum_{\beta^{ki}_{jb} \in \bar{B} : j=a} fl^{ki}_{ab} \qquad \forall a \in A \qquad (6)$$

$$\sum_{\beta^{ki}_{aj} \in \bar{B} : j=b} fl^{ki}_{ab} = \sum_{\gamma^{ki}_{jc} \in \bar{\Gamma} : j=b} fl^{ki}_{bc} \qquad \forall b \in B, \forall k \in K, \forall i \in I \qquad (7)$$

$$\sum_{\gamma^{ki}_{bj} \in \bar{\Gamma} : j=c} fl^{ki}_{bc} = fl^{ki}_{ct} \qquad \forall c \in C, \forall k \in K, \forall i \in I \qquad (8)$$

Mathematical formulation: constraints

Single route selection constraints:

$$\sum_{\zeta_{hb}^{1i} \in \bar{Z}} w_{hb}^{1i} \leq 1 \qquad \forall i \in I \qquad (9)$$

$$\sum_{\gamma_{bc}^{ki} \in \bar{\Gamma}} x_{bc}^{ki} \leq 1 \qquad \forall i \in I, k \in K \qquad (10)$$

$$\sum_{\delta_{cb}^{ki} \in \bar{\Delta}} y_{cb}^{ki} \leq 1 \qquad \forall i \in I, k \in K \setminus \{k = K\} \qquad (11)$$

Mathematical formulation: constraints

Routes adjacency constraints:

$$\sum_{h\in H} w_{hb}^{1i} = \sum_{c\in C} x_{bc}^{1i} \qquad \forall b \in B, \forall i \in I \qquad (12)$$

$$\sum_{c\in C} y_{cb}^{(k-1)i} = \sum_{c\in C} x_{bc}^{ki} \qquad \forall b \in B, \forall i \in I, \forall k \in K \setminus \{k = 1\} \qquad (13)$$

$$\sum_{b\in B} x_{bc}^{ki} \geq \sum_{b\in B} y_{cb}^{ki} \qquad \forall c \in C, \forall i \in I, \forall k \in K \setminus \{k = K\} \qquad (14)$$

Mathematical formulation: constraints

Non negative constraints:

$$fl_{at} \geq 0 \qquad \forall \lambda_{at} \in \bar{\Lambda} \qquad (15)$$

$$fl_{ab}^{ki} \geq 0 \qquad \forall \beta_{ab}^{ki} \in \bar{B} \qquad (16)$$

$$fl_{bc}^{ki} \geq 0 \qquad \forall \gamma_{bc}^{ki} \in \bar{\Gamma} \qquad (17)$$

$$fl_{ct}^{ki} \geq 0 \qquad \forall \epsilon_{ct}^{ki} \in \bar{E} \qquad (18)$$

$$s_i \geq 0 \qquad \forall i \in I \qquad (19)$$

$$r \geq 0 \qquad (20)$$

$$w_{hb}^{1i} \in \{0,1\} \qquad \forall \zeta_{hb}^{1i} \in \bar{Z} \qquad (21)$$

$$x_{bc}^{ki} \in \{0,1\} \qquad \forall \gamma_{bc}^{ki} \in \bar{\Gamma} \qquad (22)$$

$$y_{cb}^{ki} \in \{0,1\} \qquad \forall \delta_{cb}^{ki} \in \bar{\Delta} \qquad (23)$$

Mathematical formulation: constraints

Additional constraints:

$$\sum_{\zeta_{hb}^{1i}} \mathrm{w}_{hb}^{1i} \leq \mathrm{v}_{hb}$$

$$\sum_{\gamma_{bc}^{ki}} \mathrm{x}_{bc}^{ki} \leq \mathrm{v}_{bc}$$

$$\sum_{\delta_{cb}^{ki}} \mathrm{y}_{cb}^{ki} \leq \mathrm{v}_{cb}$$

$$\mathrm{v}_{**} = \{\mathbf{1} : \text{if arc is valid}, \mathbf{0} : \text{if arc is not valid}\}$$

Mathematical formulation: constraints

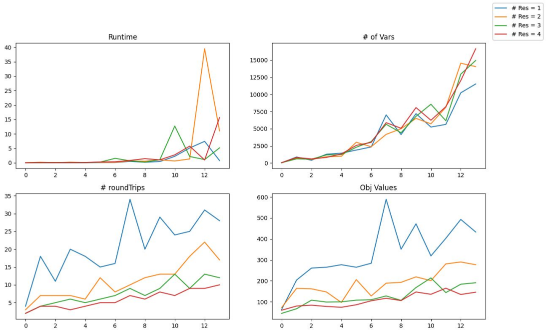
Consideration on the number of round trips

$$K \sum_{i \in I} q_i \geq \sum_{a \in A} d_a \tag{24}$$

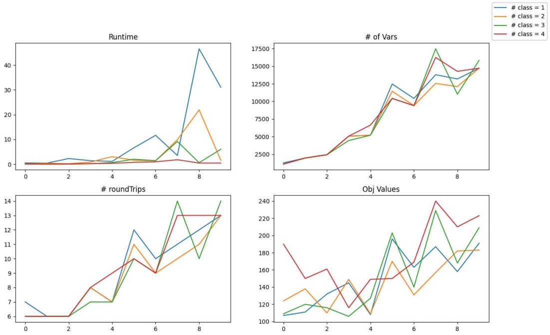
As a lower bound for the number of round trips this relation is proposed, which although does not guarantees that entire population will be saved.
A safer choice cold be:

$$K \sum_{i \in I} q_i \geq 2 \sum_{a \in A} d_a$$

Different # of resources over growing # of nodes



Different # of resource classes over growing # of nodes



Problem expansion: Stochastic - ICEP

We still want to answer to the second initial question:

- *During evacuation planning, which resources need to be secured to prepare for quick evacuation over a variety of disaster scenarios?*

To answer, we have to modify our model in order to take into account also costs of resources by modifying the objective function, add some constraints and edit others

Additional notations for S-ICEP (in addition to D-ICEP)

Sets	Set description
$\xi \in \Xi$	set of evacuation scenarios passed to the model
Parameters	**Parameter description**
cf_i	Fixed cost parameter of selecting resource i into the evacuation fleet
cv_i	Variable cost parameter for resource i
T	Upper time limit for total evacuation time r
P	Penalty cost for each person that is not successfully evacuated
Variables	**Variable description**
z_i	{**1**: if resource i gets selected into resource fleet, **0**: otherwise}
n_a	Number of non-evacuated people at area a

Model design: Scenarios

Scenarios are needed for evaluating rescue plans over a set of different conditions:

- *Seasonal fluctuation*
- *Disaster time and nature not known*
- *Weather condition can slow down travel time and loading operations*

Problem expansion: Stochastic - ICEP

The new objective becomes:

$$\min \quad \frac{\sum_{i\in I} cf_i(z_i)}{\sum_{i\in I}(cf_i + cv_i(T))} + \mathbb{E}[C(z,\xi)] \tag{26}$$

Fixes the fixed-cost component of the evacuation plan cost.

$$s.t. \quad z_i \in \{0,1\} \qquad \forall i \in I \tag{27}$$

where

$$C(z,\xi) := \min \quad r + \frac{\sum_{i\in I} cv_i(s_i)}{\sum_{i\in I}(cf_i + cv_i(T))} + P\sum_{a\in A} n_a \tag{28}$$

D-ICEP + the **variable cost** and a **penalty**. This allows for not evacuating the entire population if a scenario is extreme and has a low probability.

- Note that the problem has expanded into a Two-stage stochastic program.
- Cost normalization ensures that an improvement by at least one time unit in evacuation time will always dominate the cost objective

Problem expansion: Stochastic - ICEP

Since the number of scenarios actually is finite, we can re-formulate our objective in terms of a weighted linear combination:

$$\min \quad \frac{\sum_{i \in I} cf_i(z_i)}{\sum_{i \in I}(cf_i + cv_i(T))} + p_1 \left(r(\xi_1) + \frac{\sum_{i \in I} cv_i(s_i(\xi_1))}{\sum_{i \in I}(cf_i + cv_i(T))} + P \sum_{a \in A} n_a(\xi_1) \right) + p_2 \left(r(\xi_2) + \frac{\sum_{i \in I} cv_i(s_i(\xi_2))}{\sum_{i \in I}(cf_i + cv_i(T))} + P \sum_{a \in A} n_a(\xi_2) \right)$$

Where each weight p is the probability associated to that specific scenario.

Problem expansion: Stochastic - ICEP

The new constraints are:

$$fl_{bc}^{ki} \leq q_i(z_i) \qquad \text{Capacity constraint} \qquad \forall \gamma_{bc}^{ki} \in \bar{\Gamma} \tag{30}$$

$$d_a(\xi) = fl_{at} + \sum_{\beta_{jb}^{ki} \in \bar{B}: j=a} fl_{ab}^{ki} + n_a \qquad \text{Flow conservation} \qquad \forall a \in A \tag{31}$$

$$\sum_{\zeta_{hb}^{1i} \in \bar{Z}} w_{hb}^{1i} \leq z_i \qquad \text{Route selection} \qquad \forall i \in I \tag{32}$$

$$\sum_{\gamma_{bc}^{ki} \in \bar{\Gamma}} x_{bc}^{ki} \leq z_i \qquad \forall i \in I, k \in K \tag{33}$$

$$\sum_{\delta_{cb}^{ki} \in \bar{\Delta}} y_{cb}^{ki} \leq z_i \qquad \forall i \in I, k \in K \setminus \{k = K\} \tag{34}$$

$$n_a \geq 0 \qquad \text{Non negative} \qquad \forall a \in A \tag{35}$$

Problem expansion: Stochastic - ICEP variants

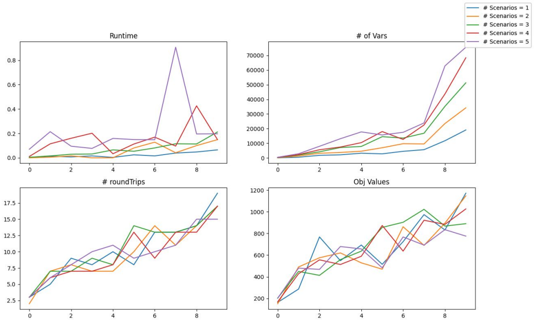
Moreover we can inspect our model by trying different objective functions:

- *Bal_1*: the one seen before. multi-objective formulation that prioritizes evacuation time over cost.
- *Bal_2*: minimize the overall sum of route times to generate higher efficiency in individual route choices

$$\frac{\sum_{i\in I} cf_i z_i}{\sum_{i\in I}(cf_i + cv_i(T))} + \mathbb{E}\left[\sum_{i\in I} s_i + \frac{\sum_{i\in I} cv_i s_i}{\sum_{i\in I}(cf_i + cv_i(T))} + P\sum_{a\in A} n_a\right]$$

Results: objective functions over # of nodes

Different scenarios over # of nodes



Thanks for your attention!