Demodulation of SSB-SC Signals.

SSB-SC signals can be demodulated coherently.

$$\varphi_{SSB}(t) = m(t)\cos\omega_c t \mp m_h(t)\sin\omega_c t$$

$$\therefore \varphi_{SSB}(t)\cos\omega_c t = [m(t)\cos\omega_c t \mp m_h(t)\sin\omega_c t]\cos\omega_c t$$

$$= \frac{1}{2}m(t)[1+\cos 2\omega_c t] \mp \frac{1}{2}m_h(t)\sin 2\omega_c t$$

$$= \frac{1}{2}m(t) + \frac{1}{2}m(t)\cos 2\omega_c t \mp m_h(t)\sin 2\omega_c t.$$

Thus, the product $\varphi_{SSB}(t)\cos\omega_c t$ yields the baseband signal and another SSB signal with a carrier $2\omega_c$. A ~~low~~ LPF will suppress the unwanted SSB terms, giving the desired baseband signal $m(t)/2$. Hence, the demodulator is identical to the synchronous demodulators used for ~~a~~ DSB-SC.

Envelope detection of SSB signal with carrier (SSB+C)

Such a signal can be represented as

$$\varphi_{SSB+C}(t) = A\cos\omega_c t + [m(t)\cos\omega_c t \mp m_h(t)\sin\omega_c t]$$

Although $m(t)$ can be recovered by synchronous detection, if A, the carrier amplitude, is large enough, $m(t)$ can also be recovered from φ_{SSB+C} by envelope or rectifier detection. This can be shown by rewriting φ_{SSB+C} as

$$\varphi_{SSB+C}(t) = [A + m(t)]\cos\omega_c t \mp m_h(t)\sin\omega_c t$$

$$= E(t)(\cos\omega_c t + \theta)$$

where $E(t)$, the envelope of φ_{SSB+C} is given by

$$E(t) = \{[A+m(t)]^2 + m_h^2(t)\}^{1/2}$$

$$= \{A^2 + 2Am(t) + m^2(t) + m_h^2(t)\}^{1/2}$$

$$= A\left\{1 + \frac{2m(t)}{A} + \frac{m^2(t)}{A^2} + \frac{m_h^2(t)}{A^2}\right\}^{1/2}$$

$(1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3$

If $A \gg m(t)$, then in general $A \gg m_h(t)$, and the terms $\frac{m^2(t)}{A^2}$ and $\frac{m_h^2(t)}{A^2}$ can be ignored. Thus

$$E(t) = A\left[1 + \frac{2m(t)}{A}\right]^{1/2}$$

Using binomial expansion and discarding higher order terms [because $\frac{m(t)}{A} \ll 1$] we have.

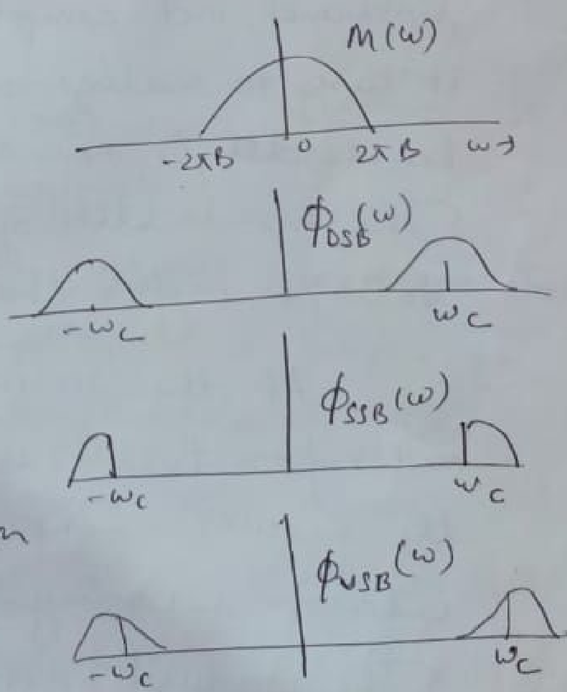
$$E(t) \simeq A\left[1 + \frac{m(t)}{A}\right]$$

$$= A + m(t)$$

It is evident that for a large carrier, the SSB-SC can be demodulated by an envelope detector.

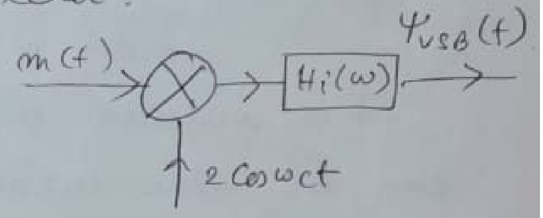
In AM, envelope detection requires $A \geq |m(t)|$, while in SSB+C the condition is $A \gg |m(t)|$. Hence, in SSB case, the required carrier amplitude is much larger than that in AM, and, consequently the efficiency of SSB+C is very low.

Amplitude Modulation: Vestigial Sideband. (VSB)



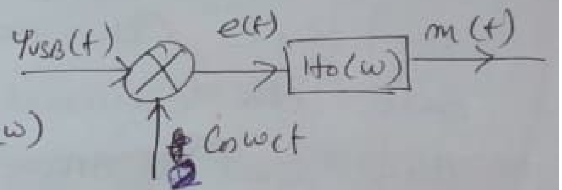
The generation of SSB signals is rather difficult. The selective filtering demands dc null in the modulating signal spectrum. A phase shifter required in phase shift method is unrealizable, or realizable only approximately. The generation of DSB-SC signal is much simpler, but requires twice the signal bandwidth. A vestigial-sideband (VSB), also called asymmetric sideband system is a compromise between DSB and SSB. It inherits the advantages of DSB and SSB but avoids ~~disad~~ disadvantages at a small cost. VSB are relatively easy to generate, and, at the same time, their bandwidth is slightly (25%) greater than SSB. If a large carrier is transmitted along with the VSB signal, the baseband signal can be recovered by an envelope (or rectifier) detector.

If the vestigial shaping filter that produces VSB from DSB is $H_i(\omega)$, then the resulting VSB signal spectrum is



Transmitter

$$\Phi_{VSB}(\omega) = [M(\omega+\omega_c) + M(\omega-\omega_c)]H_i(\omega)$$



Reciever.

VSB shaping filter $H_i(\omega)$ allows the transmission of one sideband, but suppresses other sideband, not completely, but gradually. This makes it easy to realize such a filter, but the transmission bandwidth is now somewhat higher than that of SSB. The bandwidth of the VSB signal is typically 25% to 33% higher than that of SSB.

At the reciever side, it is require to recover $m(t)$ from $\varphi_{VSB}(t)$ using synchronous demodulation at the reciever. This is done by multiplying the ~~incoming~~ incoming VSB signal $\varphi_{VSB}(t)$ by $2\cos\omega_c t$. The product $e(t)$ is given by.

$$\cancel{M(\omega)}\; e(t) = 2\varphi_{VSB}(t)\cos\omega_c t \Leftrightarrow [\Phi_{VSB}(\omega+\omega_c) + \Phi_{VSB}(\omega-\omega_c)]$$

The signal $e(t)$ is passed through the low-pass equalizer filter of transfer function $H_o(\omega)$. The output of the equalizer filter is $m(t)$. Hence, the output signal spectrum is given by

$$M(\omega)\cdot \cancel{E(\omega)} = \left[\phi_{VSB}(\omega+\omega_c) + \phi_{VSB}(\omega-\omega_c)\right] H_o(\omega)$$

$$= \Big[\{M(\omega-\omega_c+\omega_c) + M(\omega+\omega_c+\omega_c) + M(\omega-\omega_c-\omega_c) + M(\omega+\omega_c-\omega_c)\}\{H_i(\omega+\omega_c) + H_i(\omega-\omega_c)\}\Big] H_o(\omega)$$

$$= \Big[M(\omega)\{H_i(\omega+\omega_c) + H_i(\omega-\omega_c)\} + \{M(\omega+2\omega_c) + M(\omega-2\omega_c)\}\{H_i(\omega+\omega_c) + H_i(\omega-\omega_c)\}\Big] H_o(\omega)$$

After passing through the LPF, the higher frequency terms are suppressed and,

$$M(\omega) = M(\omega)\left[H_i(\omega+\omega_c) + H_i(\omega-\omega_c)\right] H_o(\omega)$$

$$\Rightarrow H_o(\omega) = \frac{1}{H_i(\omega+\omega_c) + H_i(\omega-\omega_c)}$$

- Envelope Detection of VSB+C Signals.

For envelope detection of VSB signals we need to transmit a carrier whose amplitude is ~~having~~ higher than that of the amplitude of the carrier used in ~~SSB+C~~ ~~but~~ DSB but lower ~~thatn~~ than the amplitude of the carrier used in SSB+C.

Carrier Acquisition:

In the suppressed-carrier amplitude-modulated system (DSB-SC, SSB-SC and VSB-SC), one must generate a local carrier at the receiver for the purpose of synchronous demodulation. Ideally, the local carrier must be in frequency and phase synchronism with the incoming carrier. Any discrepancy in the frequency or phase of the local carrier gives rise to distortion in the detector output.

Consider a DSB-SC case where a received signal is $m(t)\cos\omega_c t$ and local carrier is $2\cos[(\omega_c+\Delta\omega)t+\delta]$. The local carrier frequency and phase errors in this case are $\Delta\omega$ and δ respectively. The product of the local carrier and the received signal is $e(t)$, given by

$$e(t) = 2m(t)\cos\omega_c t \cdot \cos[(\omega_c+\Delta\omega)t+\delta]$$

$2\cos A\cos B = \cos(A-B)+\cos(A+B)$

$$= m(t)\left[\cos\{(2\omega_c+\Delta\omega)t+\delta\} + \cos\{\Delta\omega t+\delta\}\right]$$

$$= m(t)\cos(\Delta\omega t+\delta) + m(t)\cos[(2\omega_c+\Delta\omega)t+\delta]$$

The second term is filtered out by the LPF, leaving the output $e_0(t)$,

$$e_0(t) = m(t)\cos(\Delta\omega t+\delta)$$

If both $\Delta\omega$ and δ are 0, then

$$e_0(t) = m(t)$$

Lett us consider two special cases. If $\Delta\omega = 0$, then

$$e_0(t) = m(t)\cos\delta$$

The output is proportional to m(t) when δ is constant. The output will be maximum when $\delta = 0$ and minimum when $\delta = \pm\pi/2$. Thus, the phase error in the local carrier causes the attenuation of the output signal without causing any distortion, as long as δ is constant. But, the phase error δ may vary randonly with time. This may occur because of variation in the propogation path. This caruses the gain factor $\cos\delta$ at the reciever to vary randonly and is undesirable.

Next we consider the case where $\delta = 0$ and ~~$\delta \neq 0$~~ $\Delta\omega \neq 0$, then

$$e_0(t) = m(t)\cos(\Delta\omega)t$$

The output here is not merely an attenuated replica of the original signal but ~~at~~ is also distorted. Because $\Delta\omega$ is usually small, the output is the signal m(t) multiplied by a low-frequency sinusoid. This causes the amplitude of the desired signal m(t) to vary from maximum to zero periodically at twice the period of the beat frequency $\Delta\omega$.

To ensure identical carrier freqeencies at the transmitter and reciever, we can use quartz crystal oscillators, which generally are very stable. Identical crystals are cut to yield the same frequency at the transmitter and the reciever. At very high carrier frequencies, where the crystal dimension become too small to match exactly, quartz-crystal performance may not be adequate.

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In such a case, a carrier, or pilot, is transmitted at a reduced level along with the sidebands. The ~~pito~~ pilot is separated at the receiver by a very narrow-band filter tuned to the pilot frequency. It is amplified and used to synchronize the local oscillator.

The phase lock loop (PLL) plays an important role in carrier acquisition.

Phase-Locked Loop (PLL)

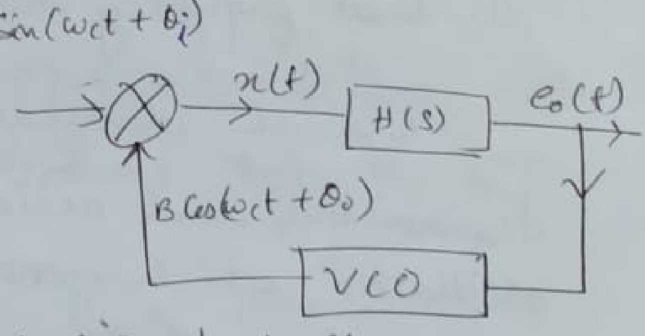
The phase-locked loop can be used to track the phase and the frequency of the carrier component of an incoming signal. It is, therefore, a useful device for synchronous demodulation of AM signal with suppressed carrier or with a little carrier. It can ~~a~~also be used for demodulation of angle modulated signals, specially under low SNR conditions.

A' PLL has three basic components:

(a) A voltage controlled oscillator (VCO)

(b) A multiplier, serving as a phase detector (PD) or a phase comparator.

(c) A loop filter $H(s)$.



PLL is basically a feedback system. In a feedback system, the signal feedback tends to follow the input signal. If the signal feedback is not equal to the input signal, the difference will change the signal fed back until it is closed to the input signal. A. PLL works in in similar principle, except that the quantity fed back and compared is not the amplitude, but the phase. The VCO adjust its own frequency until it is equal to that of the input sinusoid. At this point, the frequency and phase of the two signals are in synchronism.

<u>Voltage Controlled Oscillator</u> (VCO): An oscillator whose frequency can be controlled by an external voltage is known as voltage controlled oscillator (VCO). In a VCO, the oscillation frequency varies linearly with the input voltage. If a VCO input voltage is $e_0(t)$, its output is a sinusoid of frequency ω, given by

$$\omega(t) = \omega_c + c\,e_0(t)$$

where c is a constant and ω_c is the free running frequency of the VCO. The multiplier o/p is further passed to LPF and then applied to the input of VCO. This voltage changes the frequency of the oscillator and keeps the loop locked.

<u>Working of PLL</u>: Let the input to PLL be $A\sin(\omega_c t + \theta_i)$ and let the VCO o/p be $B\cos(\omega_c t + \theta_0)$. The multiplier o/p is

$$x(t) = AB\sin(\omega_c t + \theta_i)\cos(\omega_c t + \theta_0)$$

$$= \frac{AB}{2}\left[\sin(2\omega_c t + \theta_i + \theta_0) + \sin(\theta_i - \theta_0)\right]$$

The first term is suppressed by loop filter as it is a high frequency component. So,

$$\underline{x(t)} = \frac{AB}{2}\sin(\theta_i - \theta_0)$$

$e_i(t)$

Loop filter is <u>lowpass narrow band</u> filter. 4

Hence, $e_0(t)$, the input to the VCO is

$$e_0(t) = \frac{AB}{2} \sin \theta_e \quad , \theta_e = \theta_i - \theta_0$$

θ_e is the phase error.

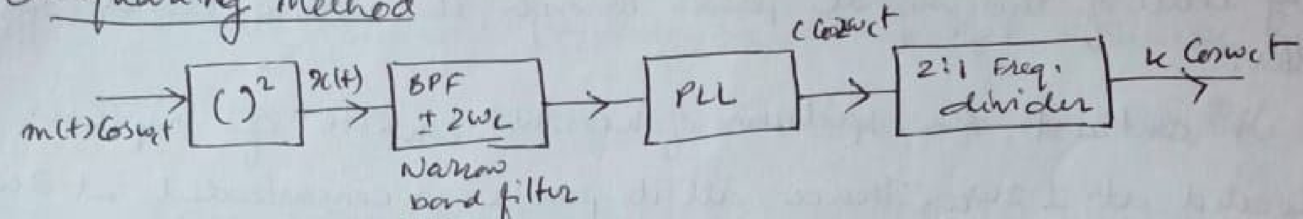
Suppose the loop is locked, meaning that the frequencies of ~~th~~ both the input and output sinusoids are identical. This means things are in the steady state, and θ_i, θ_0 and θ_e are constant.

Suppose the input sinusoid frequency suddenly increases from ω_c to $\omega_c + k$. This means the incoming signal is $A \cos[(\omega_c + k)t + \theta_i] = A \cos(\omega_c t + \hat{\theta}_i)$, where $\hat{\theta}_i = kt + \theta_i$. Thus the increase in frequency cause the increase in θ_i and thereby ~~cin~~ increasing θ_e. Now, the frequency of ~~voc~~ VCO will also increase to match the input frequency. This also happens if the input frequency decreases. Thus the PLL tracks the input sinusoid. The two signals are said to be mutually phase coherent or in phase.

A PLL can track the incoming frequency over a finite range of frequency shift. This range is called the hold in or lock range. Moreover, if initially the i/p ad o/p frequencies are not closed enough, the loop may not acquired lock. The range of frequency over which the input will cause the loop to lock is called the pull in or capture range

Carrier Acquisition in DSB-SC

① Squaring Method



The incoming signal is squared and then passed through a narrow (high Q) bandpass filter tuned to $2\omega_c$. The output of this filter is the sinusoid $k\cos 2\omega_c t$, with some residual unwanted signal. This signal is applied to a PLL to obtain a cleaner sinusoid of twice the carrier frequency, which is passed through a 2:1 frequency divider to obtain a local carrier in phase and frequency synchronism with the incoming signal.

The squarer output $x(t)$ is

$$x(t) = [m(t)\cos\omega_c t]^2$$

$$= \frac{1}{2}m^2(t) + \frac{1}{2}m^2(t)\cos 2\omega_c t$$

$m^2(t)$ is a nonnegative signal, and therefore has a nonzero average value. Let the average value, which is dc component of $\frac{m^2(t)}{2}$ be k. Then we can now express $m^2(t)/2$ as

$$\frac{m^2(t)}{2} = k + \phi(t)$$

where $\phi(t)$ is a zero mean baseband signal. Thus.

$$x(t) = \frac{1}{2}m^2(t) + \frac{1}{2}k\cos 2\omega_c t + \frac{1}{2}\phi(t)\cos 2\omega_c t$$

The bandpass filter is a narrow band (high-Q) filter tuned to frequency $2\omega_c$. It completely suppress the signal $m^2(t)$, whose spectrum is centered at $\omega = 0$.

It also suppressed most of the signal $\phi(t)\cos 2\omega_c t$. This is because although this signal spectrum is centered at $2\omega_c$, it has infinitesimal (zero) power centered at $2\omega_c$ since $\phi(t)$ has a zero dc value. Moreover this component is 6 distributed over the band $4B$ Hz centered at $2\omega_c$. Hence, very little of this signal passes through the narrow-band filter.

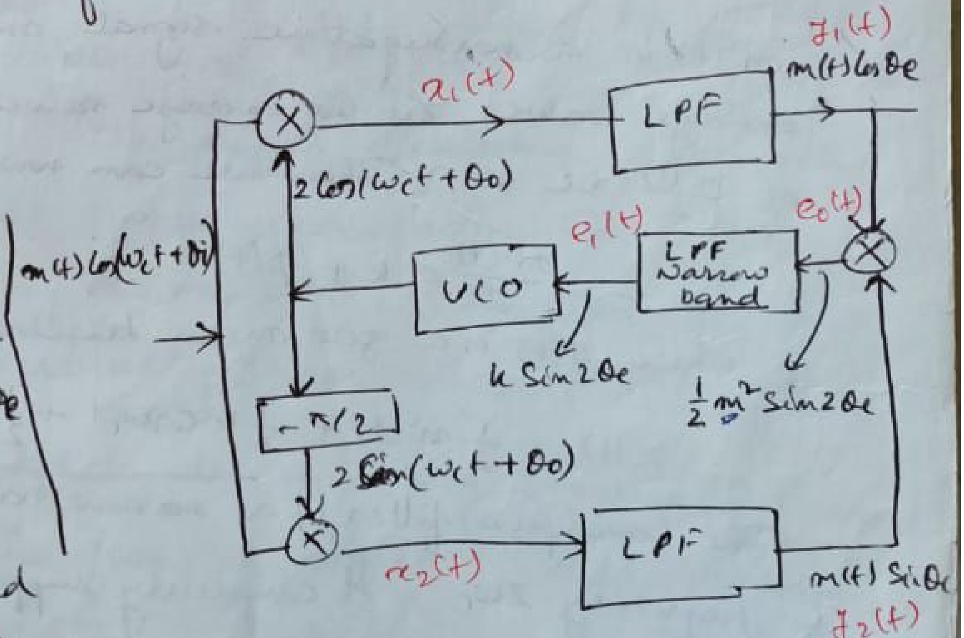
In contrast, the spectrum of $k\cos 2\omega_c t$ consists of impluses located at $\pm 2\omega_c$. Hence, all its power is concentrated at $2\omega_c$, and will pass through.

Thus the output of the filter is $k\cos 2\omega_c t$ ~~plus~~ plus a small undesired residue from $\phi(t)\cos 2\omega_c t$. The residue can be suppressed by using a PLL, which will track $k\cos 2\omega_c t$. The PLL output, after passing through a 2:1 frequency divider, yields the desired carrier.

In this method we have phase ambiguity of π or sign ambiguity because the incoming signal sign is lost in the squarer. This is immaterial for analog signals. For a digital baseband signal, the carrier sign is essential, and this method, therefore, cannot be used.

Costas Loop

The incoming signal is $m(t)\cos(\omega_c t+\theta_i)$. At the receiver, a VCO generates the carrier $\cos(\omega_c t+\theta_o)$. The phase error ~~θ~~ is θ_e, $\theta_e=\theta_i-\theta_o$. The two LPF suppress high frequency terms to yield $m(t)\cos\theta_e$ and $m(t)\sin\theta_e$.



The outputs are further multiplied to ~~get~~ give $m^2(t)\sin 2\theta_e$. When this signal is passed ~~that~~ through a narrow band LPF, the output is $k\sin 2\theta_e$, where k is the dc component of $m^2(t)/2$. The signal $k \sin 2\theta_e$ is applied to the input of a VCO with quiescent frequency ω_c. The input $k \sin 2\theta_e$ increases the output frequency, which in turn reduces θ_e.

Carrier Acquisition in SSB-SC

For the purpose of synchronization at the SSB receiver, one may use highly stable & crystal oscillators, with crystals cut for the same frequency at the transmitter and the receiver. At very high frequencies, where ~~eq~~ even quartz crystals may have inadequate performance, a pilot carrier may be transmitted. These are same the methods used for ~~B~~ DSB-SC. However, the received signal squaring technique as well as the ~~Cot~~ Costas loop used in DSB-SC cannot be used for SSB-SC. This can bee seen by expressing the SSB signal as

$$\varphi_{SSB}(t) = m(t)\cos\omega_c t \mp m_h(t)\sin\omega_c t$$

$$= E(t)\cos[\omega_c t + \theta(t)]$$

where $E(t) = \sqrt{m^2(t) + m_h^2(t)}$

$$\theta(t) = \tan^{-1}\left[\frac{\pm m_h(t)}{m(t)}\right]$$

Squaring yields.

$$\varphi_{SSB}^2(t) = E^2(t)\cos^2[\omega_c t + \theta(t)]$$

$$= \frac{E^2(t)}{2}\left[1 + \cos 2[\omega_c t + \theta(t)]\right]$$

The signal $\frac{E^2(t)}{2}$ is eliminated by a bandpass filter. Unfortunately the ~~s~~ remaining signal is not a pure sinusoid of frequency $2\omega_c$. There is nothing

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we can do to remove the time varying phase $\angle\theta(t)$ from this sinusoid. Hence, for SSB, the squaring technique does not work. The same argument can be used to show that the Costas loop will not work either. These conclusions also apply for VSB.

Costas loop:

$$x_1(t) = E(t)\cos[2\omega_c t \mp \theta(t) + \theta_0] + E(t)\cos(\mp\theta(t) - \theta_0)$$

$$\therefore y_1(t) = E(t)\cos(\mp\theta(t) - \theta_0)$$

$$x_2(t) = E(t)\sin[2\omega_c t \mp \theta(t) + \theta_0] + E(t)\sin[\mp\theta(t) - \theta_0]$$

$$\therefore y_2(t) = E(t)\sin(\mp\theta(t) - \theta_0)$$

$$\therefore e_0(t) = \frac{E^2(t)}{2}\sin(\mp 2\theta(t) - 2\theta_0)$$

$$= k\sin 2(\mp\theta(t) - \theta_0) + \phi(t)\sin 2(\mp\theta(t) - \theta_0)$$

$$\therefore e_i(t) = k\sin 2(\mp\theta(t) - \theta_0)$$

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