Angle Modulation

In frequency modulation (FM), the carrier frequency would be varied proportional to the massage $m(t)$.

The carrier frequency $\omega(t)$ would be varied with time so that $\omega(t) = \omega_c + km(t)$, where k is an arbitary constant. If the peak amplitude of $m(t)$ is m_p, then the maximum and minimum values of the carrier frequency would be $\omega_c + km_p$ and $\omega_c - km_p$, respectively. Hence, the spectral components would remain within this band with a bandwidth $2km_p$ centered at ω_c. The bandwidth is controlled by the arbitary constant k, whose value can be selected as we please. By using an arbitary small k, we could make the information bandwidth arbitarily small. But experimental result showed that FM bandwidth was found to be always greater ~~that~~ than (at best equal to) the AM bandwidth. Let us find where is the fallacy in this reasoning.

Concept of instantenous frequency

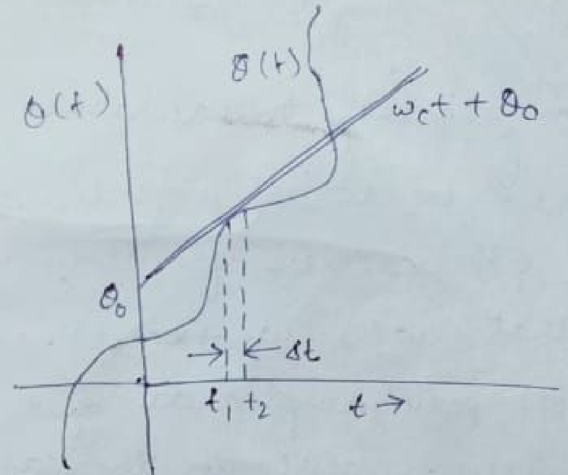
In FM we wish to vary the carrier frequency in proportion to the modulating signal $m(t)$. This means the carrier frequency is changing continuously every instant. Prima facie, this does not make much sense because to define a frequency, we must have a sinusoidal signal at least over one cycle with the same frequency.

Let us consider a generalized sinusoidal signal

$$\varphi(t) = A \cos \theta(t) \quad \text{—(A)}$$

where $\theta(t)$ is the generalized angle and is a function of t.

The figure shows a hypothetical case of $\theta(t)$. The generalized angle for a conventional sinusoid $A\cos(\omega_c t + \theta_0)$ is $\omega_c t + \theta_0$. This is a straight line with a slope ω_c and intercept θ_0. The plot for $\theta(t)$ for the hypothetical case happens



to be tangential to the angle $(\omega_c t + \theta_0)$ at some instant t. The crucial point is that over a small interval $\Delta t \to 0$, the signal $\varphi(t) = A\cos\theta(t)$ and the sinusoid $A\cos(\omega_c t + \theta_0)$ are identical; i.e.

$$\varphi(t) = A\cos(\omega_c t + \theta_0) \quad t_1 < t < t_2$$

Over this small interval Δt, the frequency of $\varphi(t)$ is ω_c. Because $(\omega_c t + \theta_0)$ is tangent to $\theta(t)$, the frequency of $\varphi(t)$ is the slope of its angle $\theta(t)$ over this small interval. We can generalize this concept at every instant and say that the instantaneous frequency ω_i at any instant t is the slope of $\theta(t)$ at t. Thus, for $\varphi(t)$, in eqn – A,

$$\omega_i(t) = \frac{d\theta(t)}{dt}$$

$$\theta(t) = \int_{-\infty}^{t} \omega_i(\alpha)\, d\alpha.$$

We can see the possibility of transmitting information of $m(t)$ by varying the angle θ of a carrier. Such techniques of modulation is known as angle modulation or exponential modulation. Two simple possibilities are phase modulation (PM) and frequency modulation (FM).

In PM, the angle $\theta(t)$ is varied linearly with $m(t)$:

$$\theta(t) = \omega_c t + \theta_0 + k_p m(t)$$

where k_p is a constant and ω_c is the carrier frequency. Assuming $\theta_0 = 0$, without loss of generality,

$$\theta(t) = \omega_c t + k_p m(t)$$

The resulting PM wave is

$$\varphi_{PM}(t) = A \cos [\omega_c t + k_p m(t)] \quad \text{——— (B)}$$

The instantaneous frequency $\omega_i(t)$ in this case is given by,

$$\omega_i(t) = \frac{d\theta}{dt} = \frac{d}{dt}[\omega_c t + k_p m(t)]$$

$$= \omega_c + k_p \dot{m}(t)$$

Hence, in PM, the instantaneous frequency ω_i varies linearly with the derivative of modulating signal.

In FM, the instantaneous frequency ω_i is varied linearly with the modulating signal. Thus,

$$\omega_i(t) = \omega_c + k_f m(t)$$, where k_f is a constant.

The angle $\theta(t)$ is now

$$\theta(t) = \int_{-\alpha}^{t} \omega_i(\alpha)\, d\alpha$$

$$= \int_{-\alpha}^{t} [\omega_c + k_f m(\alpha)]\, d\alpha$$

$$= \omega_c t + k_f \int_{-\alpha}^{t} m(\alpha)\, d\alpha$$

∴ The FM wave is

$$\varphi_{FM}(t) = A \cos \left[\omega_c t + k_f \int_{-\alpha}^{t} m(\alpha)\, d\alpha\right] \quad \text{——— (C)}$$

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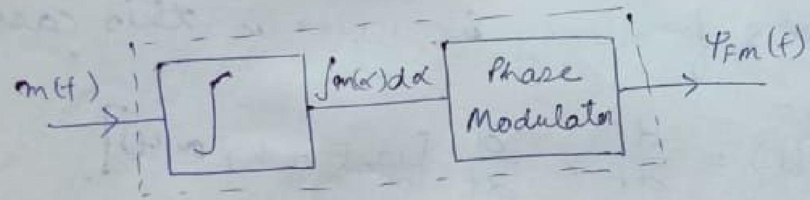
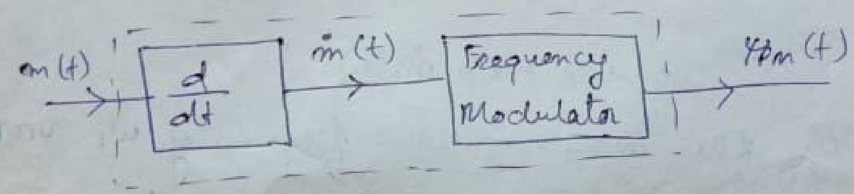
Generalized Concept of Angle Modulation

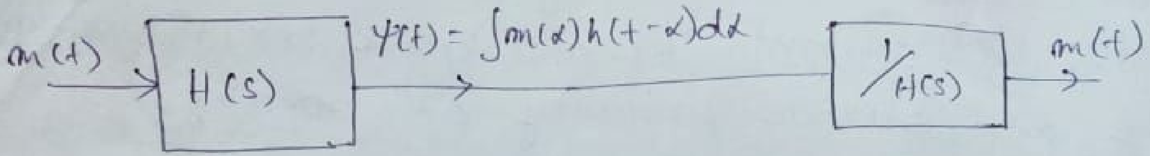
From the representation of $\varphi_{PM}(t)$ and $\varphi_{FM}(t)$ ~~we know~~ it is apparent that PM and FM are not very similar but are inseperable. Replacing $m(t)$ in eqn (B) with $\int m(t)$ changes PM to FM. Thus, a signal that is an FM wave corresponding to $m(t)$ is also the PM wave corresponding to $\int m(\alpha)d\alpha$. Similarly, a PM wave corresponding to $m(t)$ is FM wave corresponding to $\dot{m}(t)$. Therefore, by looking at an angle-modulated carrier, there is no way of telling whether it is FM or PM.





We know that in both PM and FM the angle of a carrier is varied in proportion to some measure of $m(t)$. In PM, it is directly proportional to $m(t)$, whereas in FM it is directly proportional to the integral ~~to~~ of $m(t)$.

We have infinite number of possible ways of generating a measure of $m(t)$. If we restrict the choice to a linear operator, then a measure of $m(t)$ can be obtained as the output of a suitable linear system with $m(t)$ as its ~~output~~ input. The system transfer function is $H(s)$ and its impulse response is $h(t)$. The output of this system ~~is~~ $\psi(t)$, is a measure of $m(t)$. This is a reversible operation; i.e. $m(t)$ can be recovered from $\psi(t)$ by passing through a system of the transfer function $1/H(s)$



The generalized angle modulated carrier $\psi_{EM}(t)$ is

$$\psi_{EM}(t) = A \cos[\omega_c t + \psi(t)]$$
$$= A \cos\left[\omega_c t + \int_{-\infty}^{t} m(\alpha) h(t-\alpha) d\alpha\right]$$

If $\underline{h(t) = k_p \delta(t)}$, then

$$\psi_{EM}(t) = A \cos\left[\omega_c t + k_p \int_{-\infty}^{t} m(\alpha)\, \delta(t-\alpha)\, d\alpha\right]$$
$$= A \cos[\omega_c t + k_p m(\alpha)]$$ which is the

conventional PM.

If $\underline{h(t) = k_f u(t)}$,

$$\psi_{EM}(t) = A \cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\, u(t-\alpha)\, d\alpha\right]$$
$$= A \cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha) d\alpha\right]$$ which is

the conventional FM.

So, FM and PM are just two possibilities ~~at~~ of an infinite number.

The generalized angle modulation concept is useful as it shows the convertibility of one type of angle modulation to another. (PM to FM and vice versa).

The bandwidth of FM is approximately $\underline{2k_f m_p}$, where m_p is the peak amplitude of $m(t)$.

Power of Angle Modulated waves

Although the instantaneous frequency and phase of an angle modulated wave can vary with time, the amplitude A always remain constant. Hence the power of angle modulated wave (PM or FM) is always $A^2/2$, regardless of the value of k_p or k_f.

Bandwidth of angle modulated waves

In order to ~~define the~~ determine the bandwidth of an FM wave, let us define

$$a(t) = \int_{-\alpha}^{t} m(\alpha)\, d\alpha$$

and
$$\hat{\varphi}_{FM}(t) = A e^{j[\omega_c t + k_f a(t)]}$$
$$= A e^{+jk_f a(t)} e^{j\omega_c t}$$

Now
$$\varphi_{FM}(t) = Re\, \hat{\varphi}_{FM}(t)$$

Expanding the exponential $e^{jk_f a(t)}$ in power series we have

$$\hat{\varphi}_{FM}(t) = A\left[1 + jk_f a(t) - \frac{k_f^2}{2!} a^2(t) + \dots + j^n \frac{k_f^n}{n!} a^n(t) + \dots\right] e^{j\omega_c t}$$

$\cos\omega_c t - j\sin\omega_c t$

and
$$\varphi_{FM}(t) = Re\, \hat{\varphi}_{FM}(t)$$
$$= A\left[\cos\omega_c t - k_f a(t)\sin\omega_c t - \frac{k_f^2}{2!} a^2(t)\cos\omega_c t + \frac{k_f^3}{3!} a^3(t)\sin\omega_c t + \dots\right] \quad \text{——— (1)}$$

The modulated wave consists of an unmodulated carrier plus various amplitude modulated terms, such as $a(t)\sin\omega_c t$, $a^2(t)\cos\omega_c t$, $a^3(t)\sin\omega_c t$, ... The signal $a(t)$

is integral of $m(t)$. If $M(\omega)$ is band-limitted to B, $A(\omega)$ is also band limitted to B. The spectrum $a^2(t)$ is simply $\left(A(\omega) * \frac{A(\omega)}{2\pi}\right)/2\pi$ and is band limitted to $2B$. Similarly, the spectrum of $a^n(t)$ is bandlimitted to nB. Hence, the spectrum consists of modulated carrier plus spectra of $a(t)$, $a^2(t)$, ... $a^n(t)$ ... centered at ω_c. Clearly, the modulated wave is not band-limitted. It has an infinite bandwidth and is not related to the modulating signal spectrum in any simple way, as was the case in AM.

Although the theoritical bandwidth of a FM wave is infinite, we shall see that most of the modulated-signal power resides in a finite bandwidth. There are two distinct possibilities in terms of bandwidths - ~~nar~~ narrow band FM and wide band FM.

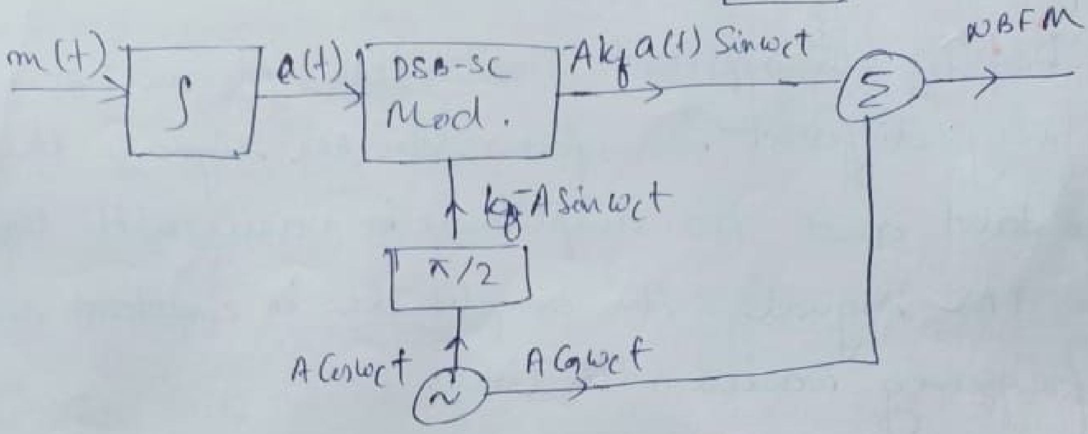
Narrow band FM

Unlike AM, angle modulation is nonlinear. The principle of superposition does not apply. This may be varified from the fact that

$$A\cos\left[\omega_c t + k_f[a_1(t) + a_2(t)]\right] \neq A\cos\left[\omega_c t + k_f a_1(t)\right] + A\cos\left[\omega_c t + k_f a_2(t)\right]$$

The principle of superposition does not hold. If, however, k_f is very small, i.e. $k_f a(t) \ll 1$, then all the terms except the first two are negligible, and we have

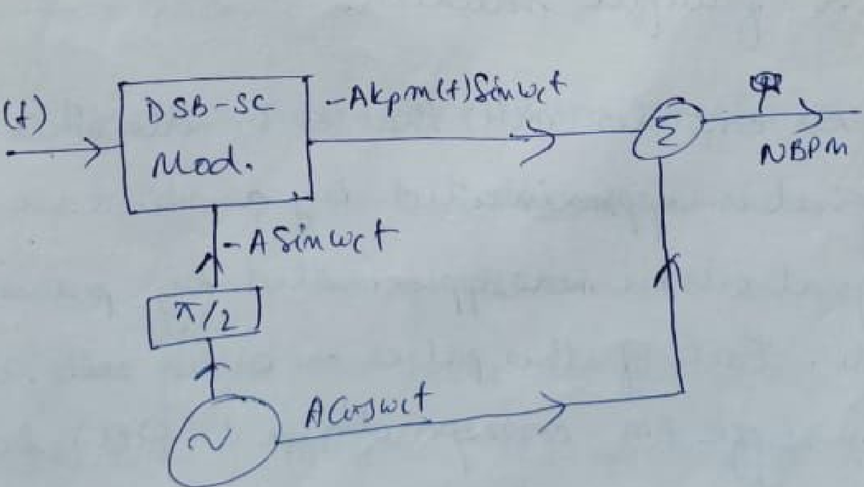
$$\varphi_{FM}(t) \simeq A[\cos\omega_c t - k_f\, a(t) \sin\omega_c t]$$



This is linear modulation. This expression is similar to that of AM wave. $[A\cos\omega_c t + \mu\, m(t)\cos\omega_c t]$. Since the bandwidth of $a(t)$ is B, the bandwidth of $\varphi_{FM}(t)$ is only 2B. For this reason, this case $|k_f a(t)| \ll 1$, is called NBFM.

Similarly NBPM is given by

$$\varphi_{PM}(t) \simeq A[\cos\omega_c t - k_p m(t) \sin\omega_c t]$$



Comparision of NBFM with AM brings out clearly the similarities and differences between the two types of modulation. Both cases have a carrier term and sidebands centered at $\pm\omega_c$. The modulated-signal bandwidths are identical

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The sideband spectrum of FM ~~are~~ has a phase shift of $\pi/2$ with respect to the carrier, whereas that of AM is in the same phase with the carrier.

Despite the apparent similarities, AM and FM signals have very different waveforms. In AM signal, the frequency is constant and the amplitude ~~is~~ varies with time, whereas in an FM signal, the amplitude is constant and the frequency varies with time.

<u>WBFM</u> (Bandwidth)

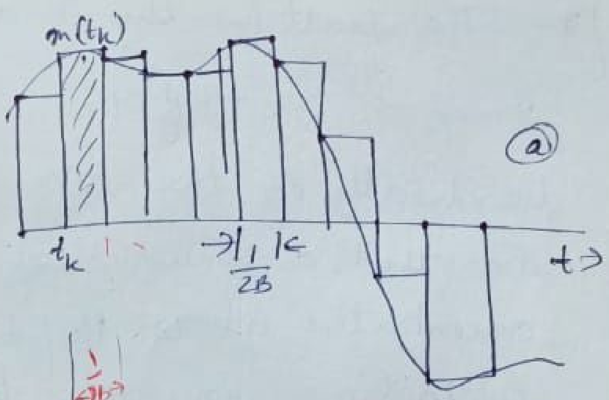
If the deviation in ~~very~~ carrier frequency is large i.e. if $|k_f a(t)| \ll 1$ does not satisfy, we cannot ignore the order terms in the eqn

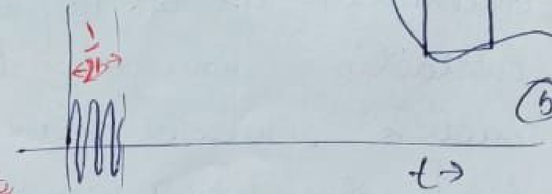
$$\varphi_{FM}(t) = A\left[\cos\omega_c t - k_f a(t)\sin\omega_c t - \frac{k_f^2}{2!}a^2(t)\cos\omega_c t + \frac{k_f^3}{3}a^3(t)\sin\omega_c t + \ldots\right].$$

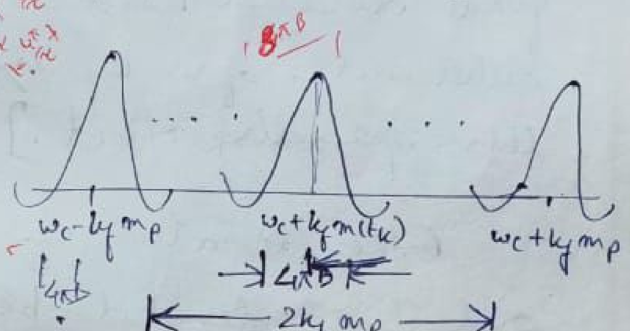
And hence the analysis becomes too complicated to lead to a fruitful solution.

Let us consider $m(t)$ that is bandlimited to B Hz. This signal is approximated by a stair case signal $\hat{m}(t)$. The signal $m(t)$ is now approximated by pulses of constant amplitude. Each of this pulses are called cells. It is relatively easy to analyze FM corresponding to $\hat{m}(t)$ because it has constant amplitudes. To ensure $\hat{m}(t)$ has all the informations of $m(t)$, the cell width in $\hat{m}(t)$ must not be greater than the Nyquist interval $1/2B$ seconds. Thus, $m(t)$ is approximated by a constant amplitude cell of width $T = \frac{1}{2B}$ seconds.

Let us consider a typical cell starting at $t = t_k$. This cell has a constant amplitude $m(t_k)$. Hence the FM corresponding to this cell is a sinusoid of frequency $\omega_c + k_f m(t_k)$ and a duration of $T = \frac{1}{2B}$. The FM signal for $\hat{m}(t)$ consists of a sequence of such sinusoidal pulses corresponding to various cells of $\hat{m}(t)$.







So, the FM spectrum of $\hat{m}(t)$ is the sum of all such sinusoids corresponding to all the cells. The Fourier transform of a sinusoidal pulse is a sinc function. The spectrum of this pulse is spread out on either side of its frequency $\omega_c + k_f m(t_k)$ by $\frac{2\pi}{T} = \frac{2\pi}{1/2B} = 4\pi B$.

The minimum and maximum amplitudes of the cells are $-m_p$ and m_p, hence the ~~maximum~~ minimum and maximum frequencies of the sinusoidal pulse corresponding to the FM signal for all the cells are $\omega_c - k_f m_p$ and $\omega_c + k_f m_p$ respectively. Moreover, the spectrum of each sinusoid spread out on either side of its frequency by $4\pi B$ rad/sec. Hence the maximum and minimum significant frequencies in this spectrum are $\omega_c + k_f m_p + 4\pi B$ and $\omega_c - k_f m_p - 4\pi B$ respectively. The spectrum bandwidth difference is

$$2k_f m_p + 8\pi B.$$

[* The fault in ~~the a~~ our previous reasoning is that $f_{max} = \omega_c + k_f m_p$. and $f_{min} = \omega_c - k_f m_p$. Hence the bandwidth of the FM is $2k_f m_p$. The implicit assumption was that a sinusoid of frequency ω has its entire spectrum concentrated at ω. Unfortunately, this is true only for the everlasting sinusoid because the Fourier Transform of such a sinusoid is an impulse at ω. For a sinusoid of finite duration T seconds, the spectrum spreads out on either side of ω by $2\pi/T$. The pioneers has missed this spreading effect.]

The deviation of the carrier frequency is $\pm k_f m_p$, and it is denoted by

$$\Delta\omega = k_f m_p$$

∴ Frequency deviation,

$$\Delta f = \frac{k_f}{2\pi} m_p$$

∴ The estimated bandwidth of AM is (in Hz)

$$B_{FM} = \frac{1}{2\pi}\left[2 k_f m_p + 8\pi B\right] \qquad 2\Delta\omega + 8\pi B$$

$$= \frac{1}{2\pi}\left[4\pi\Delta f + 8\pi B\right]$$

$$= 2\Delta f + 4B = 2(\Delta f + 2B).$$

The bandwidth thus estimated is somewhat higher than the actual value because this is the bandwidth corresponding to the staircase approximation of $m(t)$, not the actual $m(t)$ which is considerably smoother.

Hence the actual BW is somewhat smaller than this value. Therefore we need to readjust our BW estimation.

- We observe that in NBFM, k_f is very small, hence Δf is very small as compared to B. So, we can ~~rewrit~~ rewrite the above eq^n as

$$B_{FM} \simeq 4B$$

But we have seen that for a NBFM, BW is 2B Hz.

$$\therefore \; B_{FM} = 2(\Delta f + B)$$

$$= 2\left(\frac{k_f}{2\pi} m_p + B\right)$$

This is obtained by Carlson and is known as Carlson's rule.

Observe that for a truly WBFM, $\Delta f \gg B$, and BW can be approximated as

$$B_{FM} \approx 2\Delta f \quad , \text{if } \Delta f \gg B.$$

Since $\Delta f = k_f m_p$, this is the BW ~~what~~ of FM, what we have thought earlier. The only mistake is ~~that~~ the thinking that it will hold for all cases, especially for narrow band case, where $\Delta f \ll B$.

We ~~f~~ define a deviation ratio as

$$\beta = \frac{\Delta f}{B}$$

And Carlson's rule gives

$$B_{FM} = 2B(\beta + 1) \quad \text{— which is the reqd BW.}$$

The deviation ratio controls the amount of modulation. For tone modulation ~~s~~, β is known as modulation index of FM

Phase Modulation.

All the results ~~of~~ derived for FM can be directly applied to PM. Thus, for PM, the instantaneous frequency is given by $\omega_i = \omega_c + k_p \dot{m}(t)$

Therefore, the frequency deviation $\Delta\omega$ is given by

$$\Delta\omega = k_p m_p'$$

where m_p' is $[\dot{m}(t)]_{max}$

$$\therefore \quad B_{PM} = 2(\Delta f + B)$$

$$= 2\left(\frac{k_p m_p'}{2\pi} + B\right)$$

One interesting aspect of FM is that $\Delta\omega = k_f m_p$ depends only on the peak value of $m(t)$. It is independent of the spectrum of $m(t)$. On the other hand, in PM, $\Delta\omega = k_p m_p'$, depends on the peak value of $\dot{m}(t)$. But $\dot{m}(t)$ depends strongly on the frequency spectrum of $m(t)$. The presence of higher frequency components in $m(t)$ implies rapid variations resulting in higher value of m_p'. Similarly, predominance of lower frequency components will result in lower value of m_p'.

~~As so~~ Thus the WBFM carrier bandwidth is practically independent of the spectrum of $m(t)$, the WB PM carrier bandwidth strongly depends on the spectrum of $m(t)$.

For $m(t)$ spectrum concentrated at lower frequencies B_{PM} will be smaller than ~~the~~ ~~is~~ when the spectrum of $m(t)$ is concentrated at higher frequencies.

(5.3, 5.4, 5.5)

5

(1) Features of Angle Modulation:

The transmission of bandwidth of AM system cannot be changed. Because of this AM systems do not have the features of exchanging signal power for transmission bandwidth. In angle modulation the transmission bandwidth can be adjusted by adjusting Δf.

$$\cancel{BW = (}\ B_{FM} = 2B(\Delta f + 1)$$

of FM $\Delta f = \frac{k_f}{2\pi} m_p$

for PM $\Delta f = \frac{k_p}{2\pi} m'_p$

(2) Immunity ~~of~~ of angle modulation to Nonlinearities:

The amplitude of angle modulation is constant, this makes it less susceptible to non-linearities. Let us consider a second order nonlinear device whose input is $x(t)$ and output is $y(t)$, then

$$y(t) = a_1 x(t) + a_2 x^2(t)$$

If $x(t) = \cos[\omega_c t + \psi(t)]$

$$y(t) = a_1 \cos[\omega_c t + \psi(t)] + a_2 \cos^2[\omega_c t + \psi(t)]$$

$$= a_1 \cos[\omega_c t + \psi(t)] + \frac{a_2}{2} + \frac{a_2}{2}\cos[2\omega_c t + 2\psi(t)]$$

$$= \frac{a_2}{2} + a_1 \cos[\omega_c t + \psi(t)] + \frac{a_2}{2}\cos[2\omega_c t + 2\psi(t)]$$

For FM wave, $\psi(t) = k_f \int m(\alpha)\, d\alpha$ and.

$$y(t) = \frac{a_2}{2} + a_1 \cos\left[\omega_c t + k_f \int m(\alpha)\, d\alpha\right] + \frac{a_2}{2}\cos\left[2\omega_c t + 2k_f \int m(\alpha)\, d\alpha\right]$$

The dc term is filtered out to give the output that contains the original signal plus an additional FM signal, whose carrier frequency as well as frequency deviation are multiplied by 2. However, in both the ~~thar~~ terms the information $m(t)$ is intact. Thus, the nonlinearity has not distorted the information in any way. Because of the property of multiplying the carrier frequency, such nonlinear devices are called frequency multipliers.

In the preceeding case, as the device was of second order, it multiplied the frequency by 2. we can generalize this result for an n-th order multiplier. Any nonlinear device, such as a diode or a transistor, can be used for this purpose. The characteristics of these device can be expressed as

$$y(t) = a_0 + a_1 x(t) + a_2 x^2(t) + \dots + a_n x^n(t)$$

If $x(t) = A\cos\left[\omega_c t + k_f \int m(\alpha)d\alpha\right]$, then we can write

$$y(t) = c_0 + c_1 \cos\left[\omega_c t + k_f \int m(\alpha)d\alpha\right] + c_2 \cos\left[2\omega_c t + 2k_f \int m(\alpha)d\alpha\right]$$
$$+ \dots + c_n \left[n\omega_c t + n k_f \int m(\alpha)d\alpha\right]$$

The output will be a spectra at $\omega_c, 2\omega_c, \dots n\omega_c$ with frequency deviations $\Delta f, 2\Delta f, \dots, n\Delta f$, respectively. Hence, the nonlinearity generates components at unwanted frequencies. But the desired term $\cos\left[\omega_c t + \psi(t)\right]$ is undistorted, and by using a bandpass filter centered at ω_c, we can suppress all the unwanted terms in $y(t)$ and obtain the desired signal component without distortion.

Note that even the unwanted terms have the desired information intact, and any one of the unwanted terms can be used to extract information. The term $\cos\left[2\omega_c t + 2k_f \int m(\alpha)d\alpha\right]$ has twice the original carrier frequency and twice the original frequency deviation. Hence, such a nonlinear device can be used to increase the carrier frequency as well as the carrier deviation.

The immunity form nonlinearity is the reason why angle modulation is used in microwave radio relay systems, where power levels are high. This requires highly efficient non-linear class-C amplifier. In addition, the constant amplitude of FM gives it a kind of immunity against rapid fading. The effect of amplitude variations caused by rapid fading can be eliminated by using automatic gain control and bandpass limiting. This features make FM attractive for microwave radio relay systems.

Angle modulation is also less vulnerable to small signal interference from adjacent channels than AM. Finally FM is capable of exchanging SNR for the transmission bandwidth.

In telephone systems, several channels are multiplexed using SSB signals. The multiplexed signal is frequency modulated and transmitted over a microwave radio relay system with ~~may~~ many links in tandem. In this application FM is not used to realize the noise reduction but to realize other advantages of constant amplitude. Hence NBFM is used rather than WBFM.

WBFM is used in space and satellite communication system. The large bandwidth expansion reduces the required SNR and thus reduces the transmitter power requirement, which is very important because of weight considerations in space. WBFM is also used for high-fidelity radio transmission.

Generation of FM Waves

There are two ways of generating FM waves: indirect generation and direct generation.

Indirect method of Armstrong:

In this method, NBFM is generated by integrating $m(t)$ and using it to phase modulate a carrier. The NBFM is then converted to WBFM by using frequency multiplier. If we want a 12 fold increase in frequency deviation, we can used a 12^{th} order nonlinear device or two second order on one third order device in cascade. The output has a bandpass filter at $12\omega_c$ so that it selects only the appropriate term, whose carrier frequency as well as frequency deviation Δf are 12 times the original values. Generally, we require to increase Δf by a very large factor n. This increases the ~~frequency~~ carrier frequency also by n.

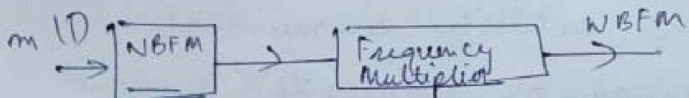
The NBFM generated by Armstrong's method has some distortion because of the following approximation.

$$\varphi_{FM}(t) = A\left[\cos\omega_c t - k_f a(t)\sin\omega_c t - \frac{k_f^2}{2}a^2(t)\cos\omega_c t + \dots\right]$$

$$\text{where } a(t) = \int_{-\alpha}^{t} m(\alpha)\,d\alpha$$

and $\varphi_{FM}(t) \simeq A[\cos\omega_c t - k_f\, a(t)\sin\omega_c t]$

The output of Armstrong NBFM modulator has ~~ant~~ some amplitude modulation. Amplitude limitting in frequency multipliers removes most of this distortion.



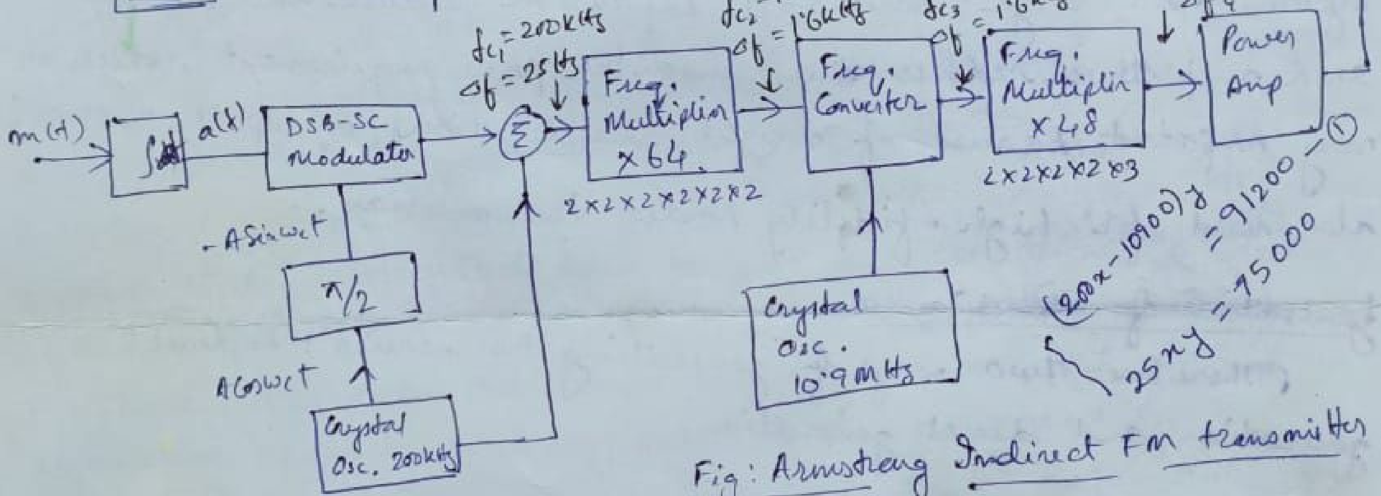


Fig: Armstrong Indirect FM transmitter.

This is the simplified diagram of a commercial FM transmitter using Armstrong's method. The final o/p is ~~reqd~~ required to have a carrier frequency of 91.2 MHz and $\Delta f = 75$ kHz. We begin with a carrier frequency $f_{c_1} = 200$ kHz generated by a crystal oscillator. This frequency is choosen because it is easy to construct stable crystal oscillators as well as balanced modulators at this frequency. The deviation Δf is chosen to be 25 Hz in order to maintain $\beta \ll 1$, as required in NBPM. For tone modulation $\beta = \Delta f / f_m$. The baseband required for a high fidelity system ranges from 50 Hz to 15 kHz. So, the choice of $\Delta f = 25$ Hz is reasonable as it gives $\beta = 0.5$.

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In order to achieve $\Delta f = 75\,kHz$, we need a multiplication of $75{,}000/25 = 3000$. This can be done by two multipliers stages of 64 and 48 giving a total multiplication of 3072 and $\Delta f = 76.8\,kHz$. The multiplication of 64 can be obtained by 6 doublers in cascade and the multiplication of 48 can be obtained by four doubler and a tripler in cascade.

Multiplication of $f_c = 200\,kHz$ by 3072, ~~you~~ would yield a final carrier of about 600MHz. This ~~fift~~ difficulty is avoided by using a frequency translator after first multiplier. The first multiplication ~~f~~ by 64 results in ~~25×6~~

$\Delta f_2 = 25 \times 64 = 1.6\,kHz$ and $f_{c_2} = 200\,kHz \times 64 = 12.8\,MHz$.

~~The entire spectrum~~ is We can now shift the entire spectrum using a carrier converter with carrier frequency 10.9MHz. This results in a new carrier frequency $f_{c_3} = 12.8 - 10.9 = 1.9\,MHz$. The frequency converter shifts the entire spectrum without altering Δf. Hence $\Delta f_3 = 1.6\,kHz$. Further multiplication by 48 yields $f_{c_4} = 1.9 \times 48 = 19.2\,MHz$ and $\Delta f_4 = 1.6 \times 48 = 76.8\,kHz$.

This scheme has a advantage of frequency stability, but it suffers from inherent noise caused by excessive multiplication and distortion at lower modulating frequency where $\Delta f / f_m$ is not small enough

5

~~85~~ 68

Direct Generation:

In a voltage controlled oscillator (VCO), the frequency is controlled by an external voltage. The oscillation frequency ~~vat~~ varies linearly with the controlle voltage. We can generate an FM wave by using the modulating signal $m(t)$ as a controlled signal. This gives.

$$\omega_i(t) = \omega_c + k_f\, m(t)$$

We can construct a VCO using an operational amplifier and hysteric comparator such as a Schmitt trigger circuit.

Another way of accomplishing the same goal is to vary one of the reactive parameter C or L of the resonant circuit of an oscillator. We can used a varicap, whose capacitance varies with the bias voltage, ~~can b~~ and can be approximated over a limitted range of ~~m~~ say $m(t)$.

In Hartley or Colpit oscillators, the frequency of oscillation is given by

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

If the capacitance C varies linearly with $m(t)$, then

$$C = C_0 - k\, m(t)$$

$$\therefore \omega_0 = \frac{1}{\sqrt{L[C_0 - k m(t)]}} = \frac{1}{\sqrt{LC_0\left[1 - \frac{k}{C_0} m(t)\right]}}$$

$$= \frac{1}{\sqrt{LC_0}\left[1 - \frac{k\, m(t)}{C_0}\right]^{1/2}}$$

$$\approx \frac{1}{\sqrt{LC_0}}\left[1 + \frac{k\, m(t)}{2C_0}\right] \quad \text{by Binomial expansion if } \frac{k\, m(t)}{C_0} \ll 1.$$

$$\therefore \omega_0 = \omega_c \left[1 + \frac{k}{2C_0} m(t)\right]$$

$$= \omega_c + k_f\, m(t)] \quad , k_f = \frac{k\omega_c}{2C_0} \Rightarrow k = \frac{2k_f C_0}{\omega_c}$$

As $C = C_0 - km(t)$, the maximum capacitance deviation is

$$\Delta C = km_p = \frac{2k_f C_0}{\omega_c} m_p$$

Hence $\frac{\Delta C}{C_0} = \frac{2k_f m_p}{\omega_c} = \frac{2\Delta f}{f_c}$

In practice, $\Delta f/f_c$ is usually small, and, hence ΔC is a small fraction of C_0, which helps limit the harmonic distortion that arises because of the approximation used in this derivation.

Direct FM generation generally produces sufficient frequency deviation and requires little frequency multiplication. But this method has poor frequency ~~deviation~~ stability. In practice, feedback is used to stabilize the frequency. The output frequency is compared with a constant frequency generated by a stable crystal oscillator. An error signal is detected and fed back to the oscillator to correct the error.

Demodulation of FM

The information in an ~~AM~~ FM signal resides in the instantaneous frequency $\omega_i = \omega_c + k_f m(t)$. Hence a frequency selective network with a transfer function of the form $|H(\omega)| = a\omega + b$ over the FM band would yield an output proportional to the instantaneous frequency. One of such simplest network is ideal differentiator with transfer function $j\omega$.

If we apply $\varphi_{FM}(t)$ to an ideal ~~filter~~ differentiator, the output is

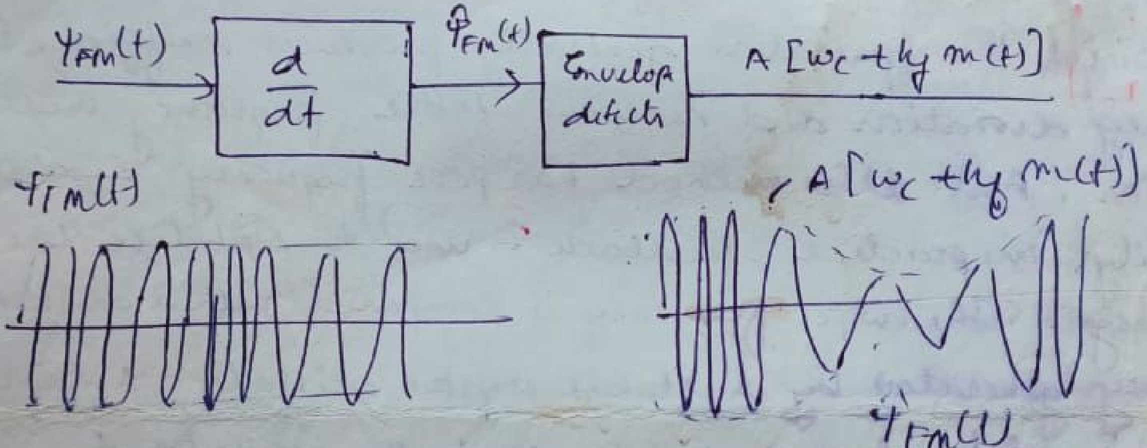
$$\dot{\varphi}_{FM}(t) = \frac{d}{dt}\left\{A\cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha\right]\right\}$$

$$= A\left[\omega_c + k_f m(t)\right] \sin\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha\right]$$

The signal $\hat{\varphi}_{FM}(t)$ is both amplitude and frequency modulated, the envelope being $A[\omega_c + k_f m(t)]$

As $\Delta\omega = k_f m_p < \omega_c$.

$\omega_c + k_f m(t) > 0$ for all t, and $m(t)$ can be obtained by envelope detection of $\hat{\varphi}_{FM}(t)$.

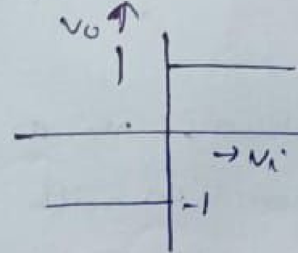


The amplitude A of the incoming FM carrier is assumed to be constant. If the amplitude A were not constant, but a function of time, there would be an ~~d~~ additional term containing $\frac{dA}{dt}$. Even if this term is neglected the envelope of $\hat{\varphi}_{FM}(t)$ will be $A(t)[\omega_c + k_f m(t)]$, and the envelope-detector output will be proportional to $A(t)m(t)$. Hence it is essential to maintain A ~~co~~ constant.

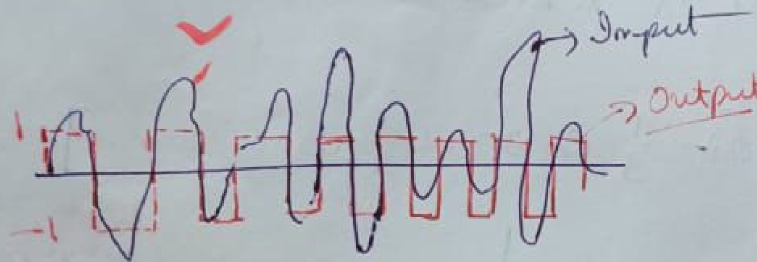
Several factors such as channel noise, fading etc cause A to vary. This variation in A should be removed ~~p~~ before applying the signal to the FM detector.

Bandpass limitter:

The amplitude variation of an angle-modulated carrier is eliminated by bandpass limitter, which consists of a hard limitter followed by a bandpass filter.



i/p ; o/p characteristics



We can see that the bandpass limitter output to a sinusoid will be a square wave of unit amplitude regardless of the incoming sinusoidal amplitude. Moreover, the zero crossings of the incoming sinusoids are preserved in the output because when the input is zero, the output is also zero. Thus if we have an angle modulated sinusoidal input $v_i(t) = A(t)\cos\theta(t)$ or $A(t)\cos[\omega_c t + k_f \int_{-\alpha}^{t} m(\alpha)d\alpha]$ then the output is an angle modulated square wave v_0 of constant amplitude. When $v_0(t)$ is passed through a bandpass filter centered at ω_c, the output is a constant amplitude, angle modulated wave. Let us consider

$$v_i(t) = A(t)\cos[\omega_c t + k_f \int_{-\alpha}^{t} m(\alpha)d\alpha]$$

$$= A(t)\cos\theta(t).$$

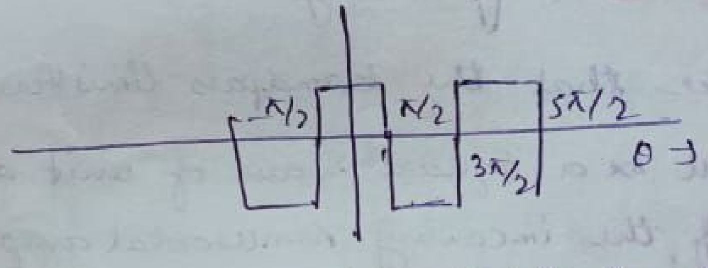
where $\theta(t) = \omega_c t + k_f \int_{-\alpha}^{t} m(\alpha)d\alpha$.

The output $v_0(t)$ of a hard limitter is +1 or -1 depending on whether $v_i(t) = A(t)\cos\theta(t)$ is positive or negative. Because $A(t) \geqslant 0$, $v_o(t)$ can be expressed as a function of θ,

$$v_o(\theta) = \begin{cases} +1, & \cos\theta > 0 \\ -1, & \cos\theta < 0 \end{cases}$$

Hence, v_o as a function of θ is a periodic square wave function with period 2π which can be expanded by Fourier series as

$$v_o(\theta) = \frac{4}{\pi}\left[\cos\theta - \frac{1}{3}\cos 3\theta + \frac{1}{5}\cos 5\theta \ldots\right]$$



This is valid for any real variable θ. At any instant t, $\theta = \omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha$ and the output is

$$v_o[\theta(t)] = v_o\left[\omega_c t + k_f \int m(\alpha)\,d\alpha\right] \quad \text{Vo functi of}$$

$$= \frac{4}{\pi}\left[\cos\{\omega_c t + k_f \int m(\alpha)\,d\alpha\} - \frac{1}{3}\cos\{3\omega_c t + 3k_f \int m(\alpha)\,d\alpha\} + \frac{1}{5}\cos\{5\omega_c t + 5k_f \int m(\alpha)\,d\alpha\} - \ldots\right]$$

The output has the original FM wave plus a frequency multiplied FM wave with multiplication factor 3, 5, 7, ... We can pass the output of the hard Limiter through a bandpass filter with center frequency at ω_c and bandwidth B_{FM}.

The filter output $e_o(t)$ is the desired angle modulated wave $e_o(t) = \frac{4}{\pi}\cos\left[\omega_c(t) + k_f \int m(\alpha)\,d\alpha\right]$

5

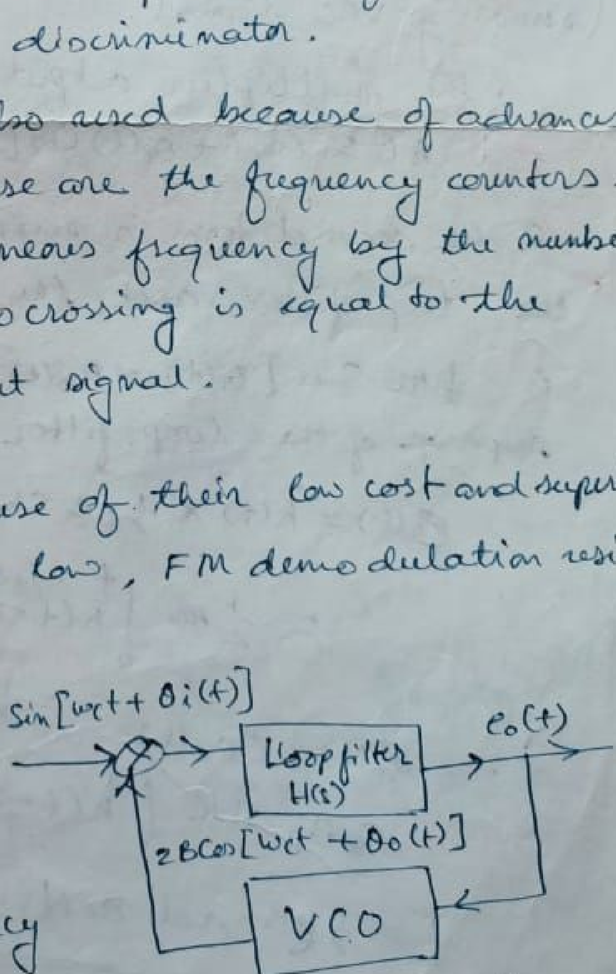
Practical frequency Demodulators

We can use operational amplifier ~~demodulator~~ differentiator as an FM demodulator. A simple tuned circuit followed by an envelope detector can also serve as a frequency detector because its frequency response $|H(\omega)|$, below or above the resonant frequency is approximately linear of the form $a\omega + b$. Since the operation is on the slope of $|H(\omega)|$, this method is also called slope detection. It suffers from the fact that the slope of $H(\omega)$ is linear over only a small band and hence causes considerable distortion in the output. This fault can be partially corrected by a ~~blan~~ balanced discriminator.

Another balanced demodulator, the ratio detector, was widely used in the past. It offers better protection against carrier amplitude variations than does the discriminator.

Zero crossing detector~~s~~ are also used because of advances in digital integrated circuits. These are the frequency counters designed to measure the instantaneous frequency by the number of zero crossings. The rate of zero crossing is equal to the instantaneous frequency of the input signal.

Phase-Locked Loop (PLL): Because of their low cost and superior performance especially when SNR is low, FM demodulation using PLL is most widely used.



Let us consider a PLL. The output $e_0(t)$ of the loop filter $H(s)$ acts as an input to VCO. The free running frequency of vco ~~is ω~~ is set at the carrier frequency ω_c. The instantaneous frequency of the VCO is given by

$$\omega_{vco} = \omega_c + c\,e_o(t)$$

$\omega_i =$

If the VCO output is $B\cos[\omega_c t + \theta_o(t)]$, then its instantaneous frequency is $\omega_c + \dot{\theta}_o(t)$. Therefore $\left[\omega_i = \frac{d}{dt}[\omega_c t + \theta_o(t)]\right]$

$$\dot{\theta}_o(t) = c\,e_o(t)$$ where c and B are constants of PLL.

Let the incoming signal be $A\sin[\omega_c t + \theta_i(t)]$. If the incoming signal is $A\sin[\omega_o t + \psi(t)]$, it can still be expressed as $A\sin[\omega_c t + \theta_i(t)]$, where $\theta_i(t) = (\omega_o - \omega_c)t + \psi(t)$

$$\left[\omega_c t + \theta_i(t) = \omega_o t + \psi(t)\right.$$
$$\left.\Rightarrow \theta_i(t) = (\omega_o - \omega_c)t + \psi(t)\right]$$

Hence the analysis that follows is general and not restricted to equal frequencies of the incoming signal and the free running VCO signal.

The multiplier output is

$$AB\sin(\omega_c t + \theta_i(t))\cos[\omega_c t + \theta_o(t)] = \frac{AB}{2}\left[\sin(\theta_i - \theta_o) + \sin[2\omega_c t + \theta_i + \theta_o]\right]$$

The second term is suppressed by the loop filter and the effective input to the loop filter is $\frac{1}{2}AB\sin[\theta_i(t) - \theta_o(t)]$. If $h(t)$ is the unit impulse response of the loop filter

$$e_o(t) = h(t) * \frac{1}{2}AB\sin[\theta_i(t) - \theta_o(t)]$$
$$= \frac{1}{2}AB\int_0^t h(t-x)\sin[\theta_i(x) - \theta_o(x)]\,dx.$$
$$= \frac{1}{2}AB\,\theta'(t). \qquad = \frac{1}{2}AB\int_0^t h(t-x)\sin\theta_e(x)\,dx$$

$$\therefore \quad \dot{\theta}_o(t) = c\,e_o(t)$$

$$\dot{\theta}_o(t) = AK\int_0^t h(t-x)\sin\theta_e(x)\,dx$$ where

$K = \frac{1}{2}cB$ and $\theta_e(t) = \theta_i(t) - \theta_o(t)$. is the phase error.

When the incoming FM is $A\sin[\omega_c t + \theta_i(t)]$

$$\theta_i(t) = k_f\int_{-\infty}^t m(\alpha)\,d\alpha$$

Hence $\theta_0(t) = k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha - \theta_e$. $\qquad \theta_e = \theta_i - \theta_0$

$\theta_0(t) = k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha$.

Assuming a small error θ_e.

$[\dot{\theta}_0(t) = c\,e_0(t)]$

$$e_0(t) = \frac{1}{c}\dot{\theta}_0(t) = \frac{1}{c}\left[k_f \frac{d}{dt}\int_{-\infty}^{t} m(\alpha)\,d\alpha - \frac{d}{dt}\theta_e\right]$$

$$\approx \frac{k_f}{c} m(t).$$

Thus, the PLL acts as an FM demodulator. If the incoming wave is a PM wave, $\theta_0(t) = \theta_i(t) = k_p\,m(t)$.

$e_0(t) = \frac{k_p}{c}\dot{m}(t)$. In this case we need to integrate $e_0(t)$ to get the desired signal.

Interference in Angle-Modulated Systems:

Let us consider the simple case of the interference of an unmodulated carrier $A\cos\omega_c t$ with another sinusoid $I\cos(\omega_c+\omega)t$. The received signal

$$r(t) = A\cos\omega_c t + I\cos(\omega_c+\omega)t$$

$$= A\cos\omega_c t + I\cos\omega_c t\cos\omega t - I\sin\omega_c t\sin\omega t$$

$$= (A + I\cos\omega t)\cos\omega_c t - I\sin\omega_c t\sin\omega t$$

$$= (A + I\cos\omega t)\left[\cos\omega_c t - \frac{I\sin\omega_c t\sin\omega t}{A + I\cos\omega t}\right]$$

$$= E_r(t)\cos[\omega_c t + \psi_d(t)]$$

$E_r = \sqrt{(A+I\cos\omega t)^2 + (I\sin\omega t)^2} = \sqrt{A^2 + 2AI\cos\omega t + I^2}$

$E_r(t) = (A + I\cos\omega(t)) \times \sqrt{1 + \left(\frac{I\sin\omega t}{A + I\cos\omega t}\right)^2}$

where $\psi_d(t) = \tan^{-1}\frac{I\sin\omega t}{A + I\cos\omega t}$

when the interfering signal is very small compared to the carrier i.e. $I \ll A$

$$\psi_d(t) \approx \frac{I}{A}\sin\omega t$$

$[\tan^{-1}\theta = \theta$ if θ small$]$

3

The phase of $E_a(t) \cos[\omega_c t + \psi_a(t)]$ is $\psi_a(t)$, and its instantaneous frequency is $\omega_c + \dot{\psi}_a(t)$. If $E_n(t) \cos[\omega_c t + \psi(t)]$ is applied to an ideal phase detector, the out put

$$y_d(t) = \frac{I}{A} \sin \omega_0 t \quad \text{for PM}$$

and if it is applied to ideal frequency demodulator

$$y_d(t) = \frac{I\omega}{A} \cos \omega t \quad \text{for FM.}$$

Observe that in either case, the interference output is proportional to the carrier amplitude A. Thus the larger the carrier amplitude A, the smaller the interference effect. This behaviour is very different from that in AM signals, where the interference output is ~~de~~ independent of the carrier amplitude. Hence, angle modulated wave suppress weak interference much better than the AM system do.

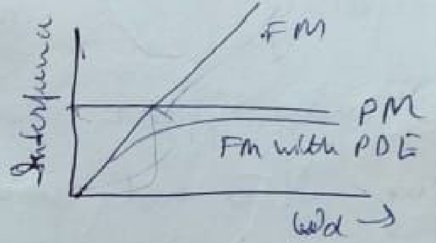
Because of the suppression of weak interference in FM, we observe a phenomenon known as capture effect. For two transmitters with carrier frequency separation less than the audio range, instead of getting interference, we observe that the stronger carrier effectively suppresses the weak carrier. Hence in AM, the interference level ~~sed~~ should be kept below 35 dB. On the other hand, for FM, because of the capture effect the interference level need only be below 6 dB.

Interference due to channel Noise:

The channel noise acts as interference in an angle modulated signal. Let us consider the most common form of noise, white noise, which has constant ~~...~~ power spectral density. Such a noise may be considered as a sum of all frequencies in the band. All components have the same amplitudes. This means I is constant for all ω. The interference amplitude spectra is constant for PM, and increases linearly with ω for FM.

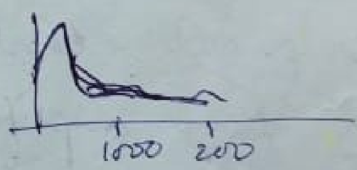
P.M = $\frac{I}{A}\sin\omega t$

FM $\frac{I\omega}{A}\cos\omega t$



Preemphasis and Deemphasis in FM Boardcasting:

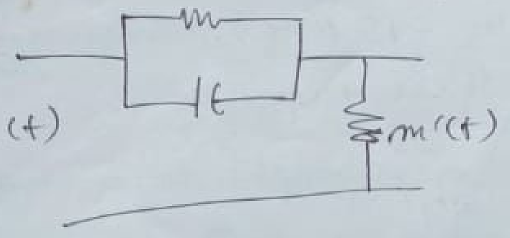
In FM, the interference (noise) increases linearly with frequency, and the noise power in the receiver is concentrated at higher frequencies. In case of audio signal m(t), the PSD is concentrated at lower frequencies below 2.1 kHz. Thus, the noise PSD is concentrated at higher frequencies, where m(t) is weakest. This may seem like disaster. But actually, this situation gives opportunity to ~~reg~~ reduce noise greatly.

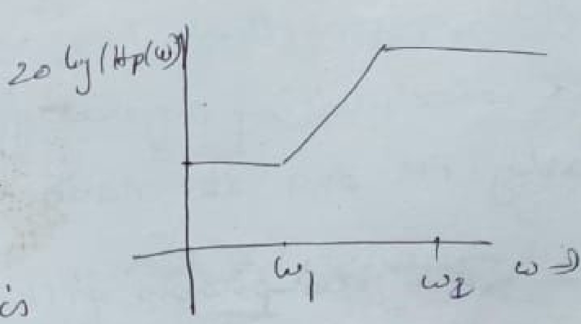


At the transmitter, the weaker high-frequency components of the audio signal m(t) are boosted before modulation by a preemphasis filter of transfer function $H_p(j\omega)$. At the receiver, the demodulator o/p is passed through a deemphasis filter of transfer function $H_d(\omega) = 1/H_p(j\omega)$. Thus, the deemphasis filter undoes the preemphasis by attenuating the ~~ig~~ higher frequency components and thereby restores the original signal $m(t)$. The noise, however, enters at the channel, and therefore has ~~to~~ not been preemphasized. However it passes through the deemphasis filter, which attenuates its higher frequency components, where most of the noise power is concentrated. Thus the process of emphasis - deemphasis leaves the desired signal untouched, but reduces the noise power considerably.

It may appear that we are gaining something for nothing. It is not so. Boosting of higher frequency components of $m(t)$ increases its peak value m_p, which in turn increases ($\Delta f = k_p m_p$). Thus, the preemphasis may seem to increase the transmission bandwidth. But the increase is minuscule because the frequency components that are boosted are so weak that even a large amplification does not increase their absolute amplitude much. Thus, preemphasis causes such a small increases in the signal power that the change in m_p is imperceptible, and we pay practically no price.

Preemphasis and Demphasis filter





We see that FM has smaller interference than PM at lower frequencies, while the opposite is true at higher frequencies. If we can make our system behave like ~~*~~ FM at lower frequencies and behave like a PM at higher frequencies, we will get the advantages of both the systems. This is acomplished by a system with the preemphasis (before modulation) ad demphasis filters $H_p(\omega)$ ad $H_d(\omega)$. The frequency f_1 is 2.1 kHz and f_2 is typically 30 kHz or more so that f_2 does not even enter into picture.

These filters can be realized by simple RC circuits. The choice of $f_1 = 2.1$ kHz is made on an experimental basis. It was found that this choice of f_1 maintain the same peak value m_p with or without preemphasis. This satisfied the constraint of a fixed transmission bandwidth.

The preemphasis transfer function is

$$H_p(\omega) = k\,\frac{j\omega + \omega_1}{j\omega + \omega_2}$$ where k is the gain $\frac{\omega_2}{\omega_1}$ is a set at ~~a~~ value of ω_2/ω_1. Thus.

$$H_p(\omega) = \left(\frac{\omega_2}{\omega_1}\right)\left(\frac{j\omega + \omega_1}{j\omega + \omega_2}\right)$$

For $\omega \ll \omega_1$ $H_p(\omega) \approx 1$

For $\omega_1 \ll \omega \ll \omega_2$, $H_p(\omega) \simeq \frac{j\omega}{\omega_1}$

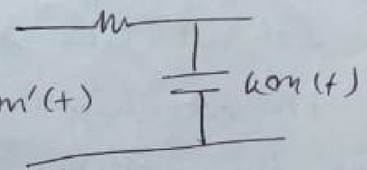
for $\omega_1 < \omega$ $H_p(\omega) = \dfrac{\frac{j\omega + \omega_1}{\omega_2} \times}{\frac{j\omega + \omega_1}{\omega_1}}$ $>$

The, the premphasizer acts as a differentiator at intermediate frequencies (2.1kHz – 15 kHz), which effectively makes the scheme ie PM over these frequencies.

This means that FM with a preemphasis is FM over the modulating signal frequency range 0 – 2.1kHz and is nearly PM over the range of 2.1 to 15kHz.

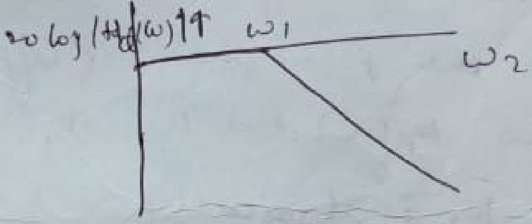
The demphasis filter $H_d(\omega)$ is given by

$$H_d(\omega) = \frac{\omega_1}{j\omega + \omega_1}$$



Note that if $\omega \ll \omega_2$ $\frac{\omega_2}{\omega_1} \times \frac{j\omega + \omega_1}{\omega_2}$

$$H_p(\omega) = \frac{j\omega + \omega_1}{\omega_1}$$



$$H_d(\omega) = \frac{\omega_1}{j\omega + \omega_1}$$

$\therefore H_p(\omega) H_d(\omega) = 1$ over the banband of 0 – 15kHz.

The PDE method of noise reduction is not limited just to FM boardcast. It is also used in audiotape recording and in phonograph recording, where the hissing noise is also concentrated at higher frequency end. Sharp hissing sound is caused by irregularities in the recording material. The Dolby noise reduction systems for audiotapes operats on the same principle of PDE.