Channel → Medium of transmission. 10th January

Date / /
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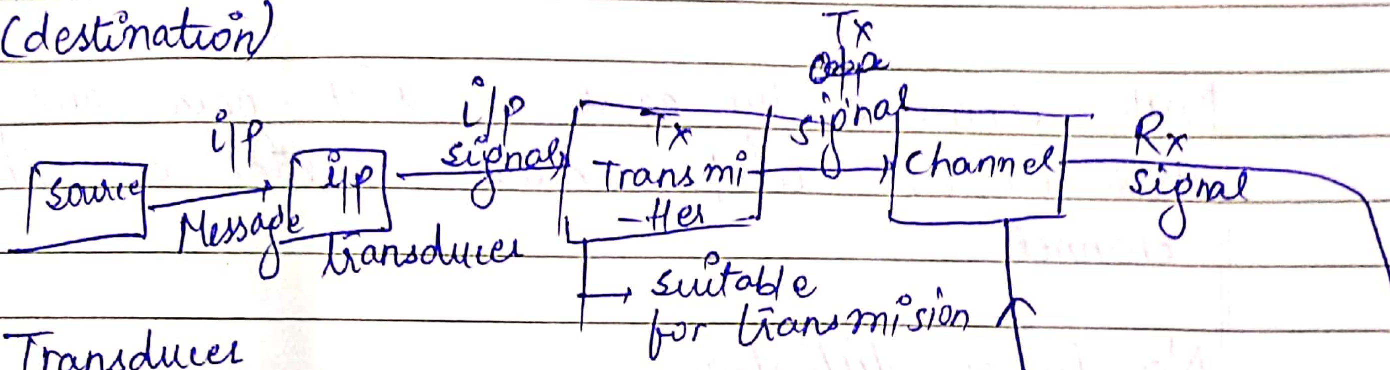
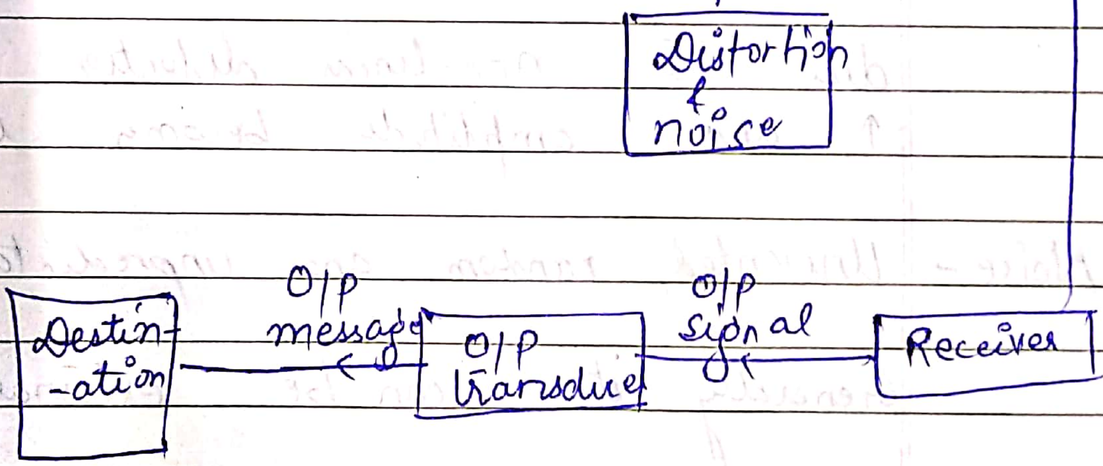
Communication → Send information from one place to another (source)
(destination)



Transducer
Any device or system which converts one form of energy to other.

Purely electrical form of message is signal.



Channels → Wire, space, air, optical fibre, wave guides, coaxial cables, etc
(if signals are in the form of wave)

channel behaves like a filter

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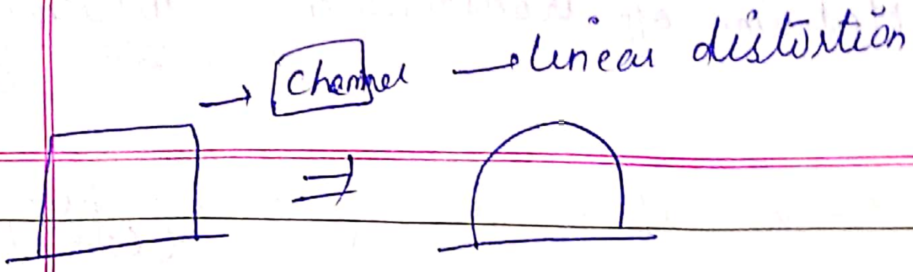
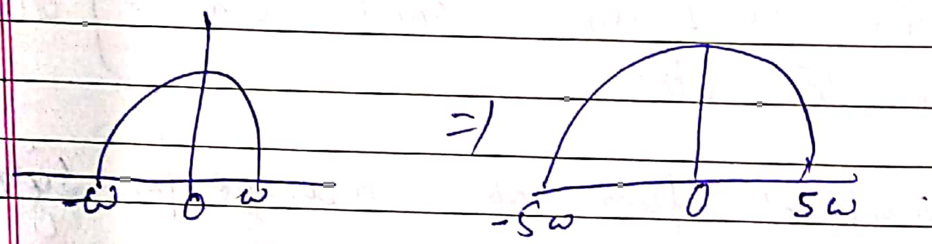


Partly connected by equalizer with gain and phase characteristics complementary to that of the channel.

Non-linear distortion.



due to non-linear distortion the bandwidth ↑ and amplitude becomes low.

Noise → Unwanted, random and unpredictable signal.

Generally it can be external or internal.

1) External → human made such as faulty contact switch, automobile ignition, flourescent light, etc

Natural source → lightning, electrical storm, solar radiations, etc.

11) Internal → thermal noise motion of e^- in conductor random emission, diffusion and recombination of charge carrier.

We can never totally remove internal noise

of communication channel
With the ↑ in the length, the noise will also increase and signal power amount of will keep on decreasing.

SNR. $\left(\frac{S}{N}\right)$
How much the signal power ~~error~~ comparison to noise.

Modulation
Shifting of the ~~signal~~ frequency band of base band signal

Baseband signal
signal produced by the source.
They are low pass signal.

Voice freq. → 300Hz to 3000Hz
$\boxed{0 - 4000\,Hz}$ → for comm.

Carrier sinusoidal signal

$$x(t) = \underbrace{A}_{\text{Amp.}}\cos(\underbrace{\omega_c}_{\text{freq.}} t + \underbrace{\theta}_{\text{phase}})$$

1) Amp. Mod. ~~acc~~ Amp. modulated acc. to m(t).
ii) freq. Mod. freq. " " " " ".
iii) Phase Mod phase " " " ".

Why do we modulate?
Modulation makes the signal more suitable for transmission.

$L \gg \frac{\lambda}{10}$

wavelength → 10km to 1000km

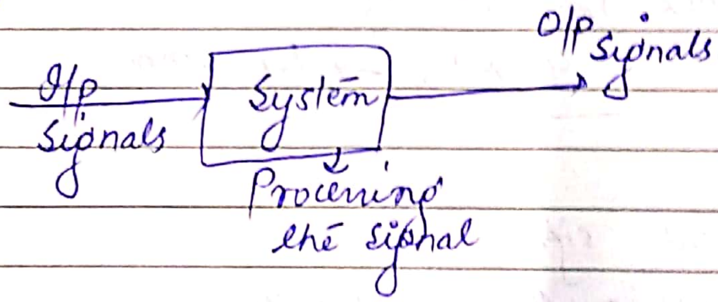
$L \gg$ 10km

~~temp~~

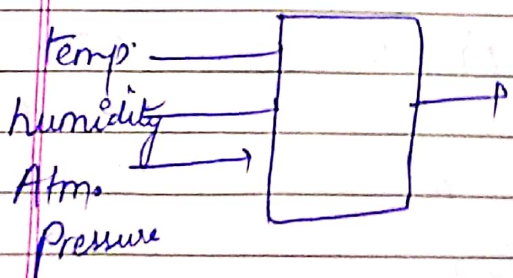
Multiplexing is the imp. ~~reason~~ cause of modulation.
↓
sharing of resources

14th January. **Signals:**
Electrical form of some information or data.

System



Weather forecasting



Signal can either have energy or Power.

RADAR

Energy / Power } signals are measured in terms of

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 \, dt$$

$$P_g = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 \, dt$$

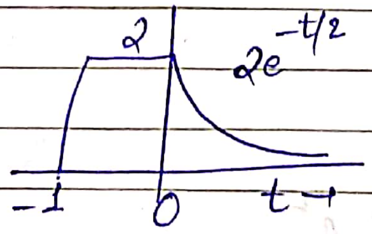
Unit → $(\text{Volt})^2 \text{sec}$ or $A^2 \text{sec}$ ← Energy.

V^2 or A^2 ← Power

P_g → Squared mean Power

R.M.S → $\sqrt{P_g}$

Q. Indicate the size of the signal.



$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 \, dt$$

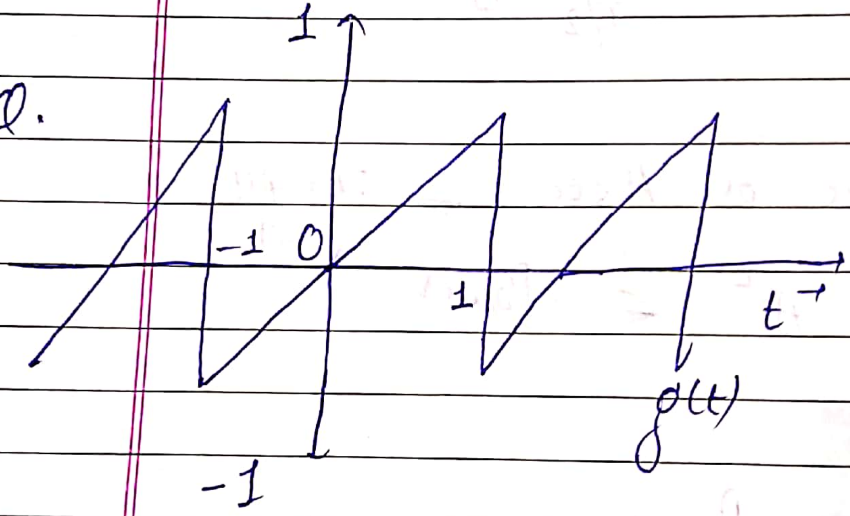
$$= \int_{-1}^{0} 4\,dt + 4\int_{0}^{\infty} e^{-t}\,dt$$

$$= 4\,[t]_{-1}^{0} + 4\,[-e^{-t}]_{0}^{\infty}.$$

$$= 4 + 4$$

$$= 8\,V^2 s.$$

Q.



$$P_g = \frac{1}{2}\int_{-1}^{1} t^2\,dt$$

$$= \left|\frac{t^3}{6}\right|_{-1}^{1}$$

$$= \frac{1}{6} + \frac{1}{6}$$

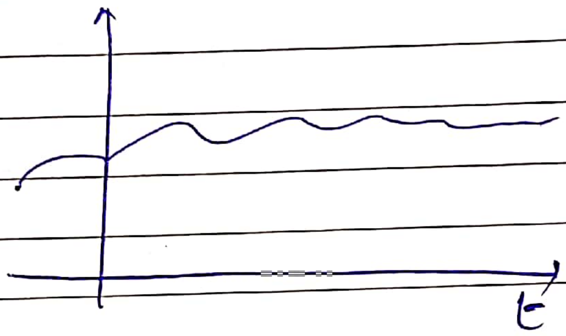
$$= \frac{1}{3}\,V^2 \quad \text{or} \quad \frac{1}{3}A^2$$

Book
Ex-2.2

Classification of Signals:-

1) continuous time & descrete time signal.

Continuous time signal
A signal which is defined for every instant of time.



Descrete time signal
A signal which is specified for descrete value of time.

Ex → Stock exchange data, humidity Graph, etc

II) Analog & Digital Signal

Analog Signal
A signal whose amplitude can take on any value in continuous range.

Digital
A signal which have only a finite no. of values.

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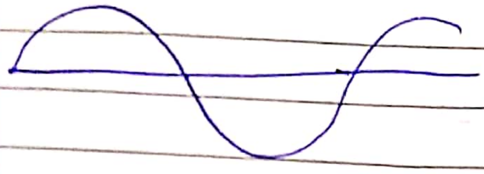
AS



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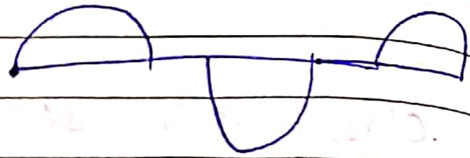
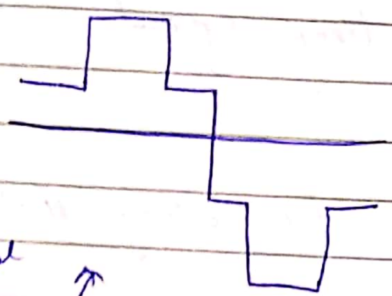


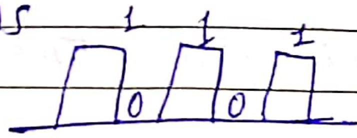
DS



4 any digital signal

level of amplitude is finite.

DS



binary

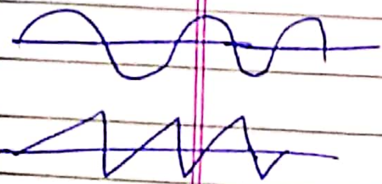
$\boxed{\text{If no.} = M. \quad M\text{ ary.}}$

(iii) Periodic & Aperiodic Signal

Periodic Signal

$$g(t) = g(t - T_0)$$

Smallest value of T which satisfy the above eqn is called time period of the signal.



which repeats after certain interval of time.

Aperiodic

Any signal which does not follow above eqn.

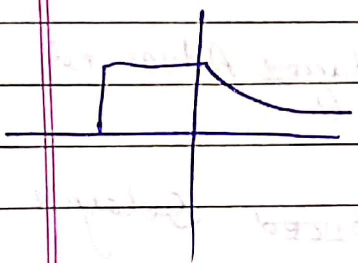
iv) Energy and Power Signal

Energy Signals which have finite energy.

Power Signals which have non-zero and finite Power.

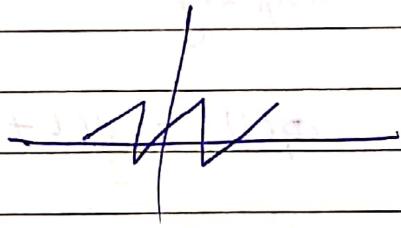
$$P_g = \lim_{T \to \infty} \frac{E_g}{T}.$$

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt$$



$E_g = 8$

$P_g = 0$



$P_g = \frac{1}{3}$

$E_g = \infty$

$\therefore$ it can be either energy or power but can't be both.

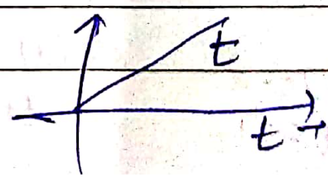
$0 < P_g < \infty$.

v) Ramp Signal

$E_g \to \infty$

$P_g \to \infty$

neither ES nor PS.



vii) Deterministic & Random Signal

A signal whose physical description is known completely in mathematical or graphical form.

Random

If a signal is known only in terms of probabilistic discription such as mean square values etc then the signal is called Random Signal.

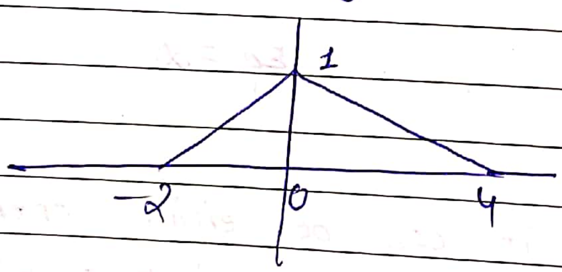
Ex→ Noise.

Signal Operations

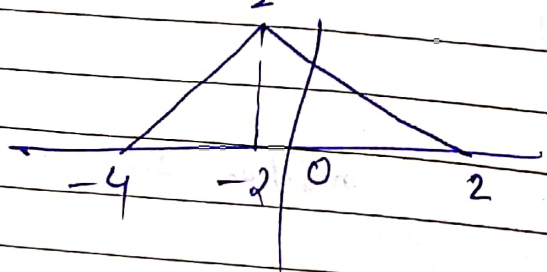
1) Time shifting

$\phi_1(t) = g(t+T)$ ← ~~Ad~~ ~~Delayed~~ Advanced

$\phi_2(t) = g(t-T)$ ← ~~Advanced~~ Delayed



$g(t)$



Advanced.

$t+2 = 4$

$t = 2.$

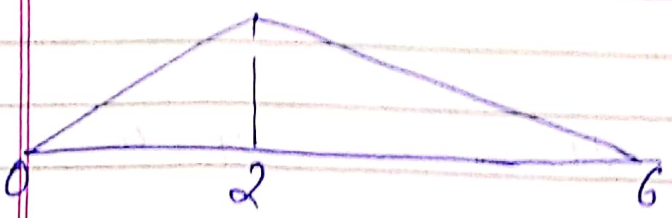
$t+2=-2$

$t=-4$

$\phi_1(t)$

$t+2=0$

$t=-2$



Delayer

$\phi_2(t)$

$t-2=-2$

$t=0$

$t-2=0$

$t=2$

$t-2=4$

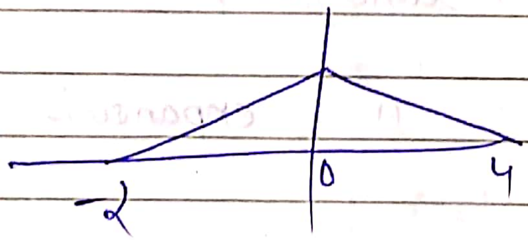
$t=6$

ii) Time Scalling

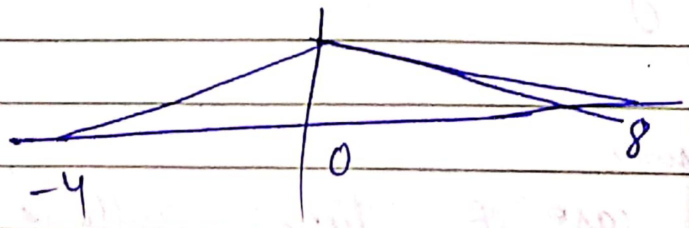
The compression or expansion in time domain is called time scalling of the signal.

$\phi_1(t) = g(2t)$

$\phi_2(t) = g(t/2)$



$g(t)$



Expansion

$\phi_2(t)$

$\frac{t}{2} = -2$

$t = -4.$

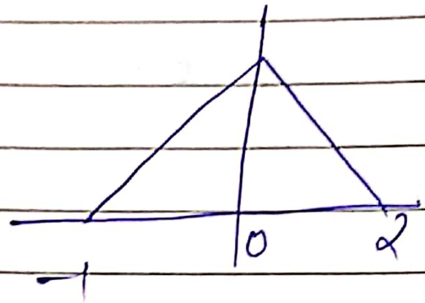
$\frac{t}{2} = 4$

$t = 8$

$\phi_1(t)$

$2t = -2$ $\quad$ $2t = 0$ $\quad$ $2t = 4$

$t = -1$ $\quad$ $t = 0$ $\quad$ $t = 2$



compression.

$\phi_1(t)$

$$\phi(t) = g(at)$$

where "a" is scaling factor.

if $a > 1$ → time compression

$a < 1$ → " expansion

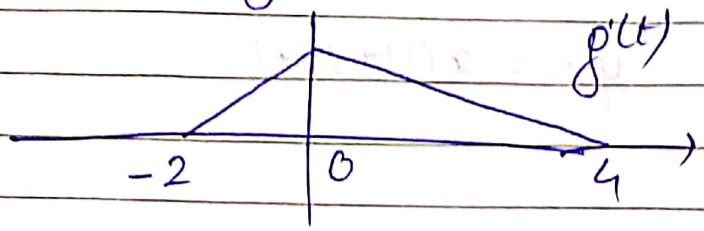
15th January. Time Scalling

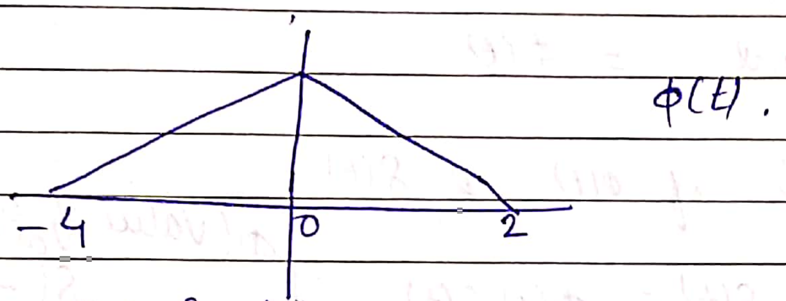
$$\phi(t) = g(at)$$

(III) Time Inversion

Special case of time scalling $a = -1$.

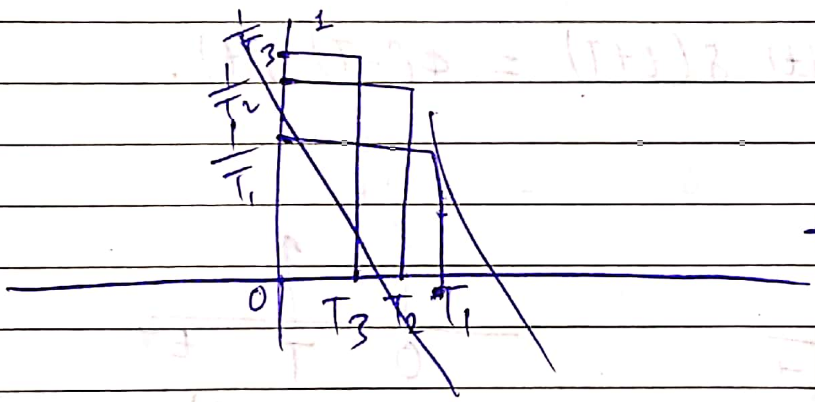
$\phi(t) = g(-t).$

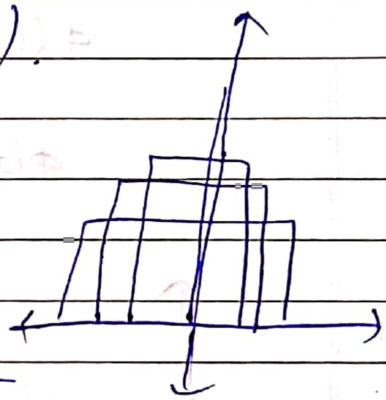


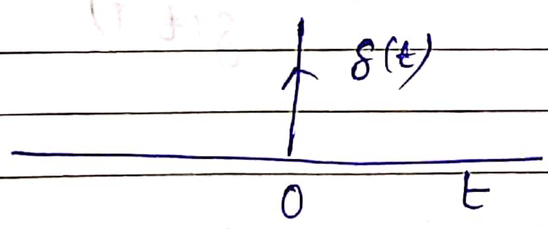


* Special functions

a) Unit Impulse function. $(\delta(t))$.

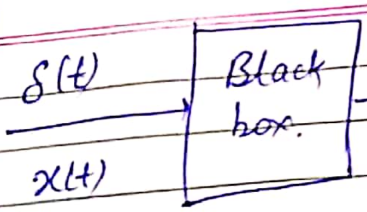






$$\delta(t) = 0, \quad t \neq 0.$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1.$$



$y(t) = h(t)$

$y(t) = x(t) * h(t)$

Property

1) General signal $= \phi(t)$

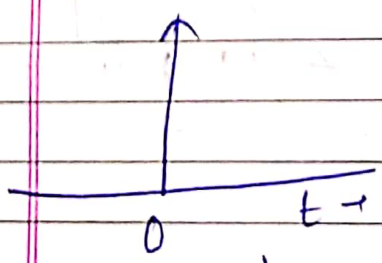
Multiplication of $\phi(t)$ & $\delta(t)$

ϕ(value of location at which δ is located)
$\delta(\text{—})$

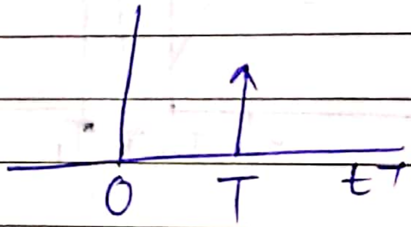
$$\phi(t)\,\delta(t) = \phi(0)\,\delta(t)$$

$$\phi(t)\,\delta(t-T) = \phi(T)\,\delta(t)$$

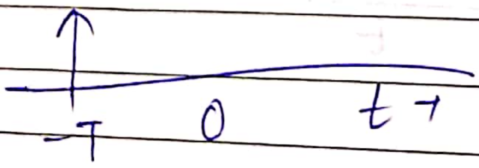
$$\phi(t)\,\delta(t+T) = \phi(-T)\,\delta(t).$$



$\delta(t)$



$\delta(t-T)$



$\delta(t+T)$

II) $$\int_{-\infty}^{\infty} \phi(t)\,\delta(t)\,dt = \phi(0)\int_{-\infty}^{\infty}\delta(t)\,dt$$

$$= \phi(0).$$

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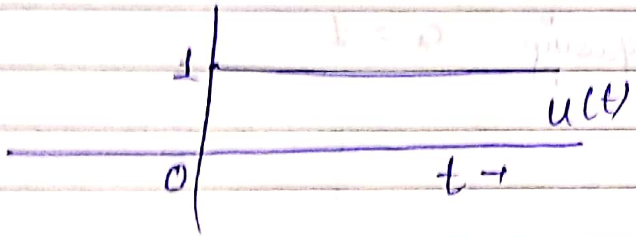
$$\int_{-\infty}^{\infty} \phi(t)\, \delta(t-T)\, dt = \phi(T)$$

$$\int_{-\infty}^{\infty} \phi(t)\, \delta(t+T)\, dt = \phi(-T)$$

Sampling or Shifting

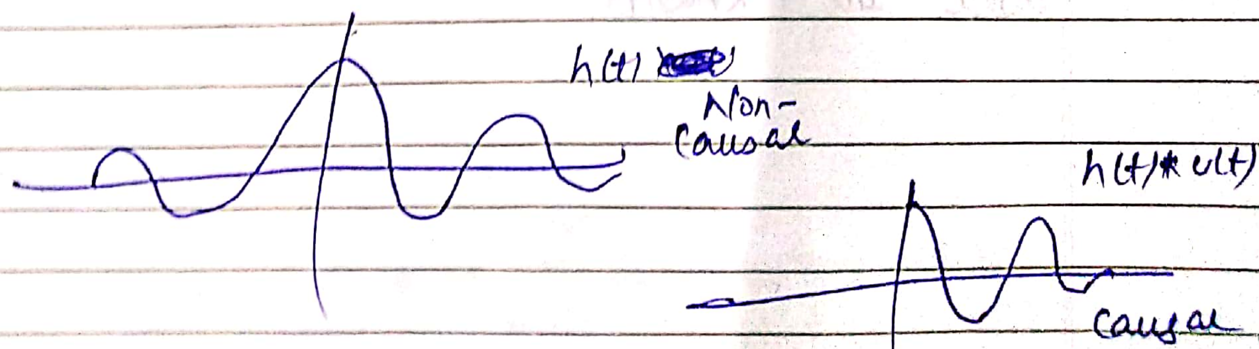
b) Unit Step function (-to remove unwanted portions of signals).
defined as u(t)



$$u(t) = \begin{cases} 1 & , t \geq 0 \\ 0 & , t < 0. \end{cases}$$

Non-causal
When i/p is not given, then also we get an o/p

To make system causal we use unit step funcn.



Signal Corelation

$$C_x = \frac{1}{\sqrt{E_g E_x}} \int_{-\infty}^{\infty} g(t)\, x(t)\, dt.$$

Coefficient of corelation

$$-1 \leq C_x \leq 1.$$

when orthogonal $C_x = 0$.

facing $C_x = 1$

opposing $C_x = -1$

Corelation functions

Cross Corelation

$$\Psi_{gz}(\tau) = \int_{-\infty}^{\infty} g(t)\, z(t+\tau)\, dt.$$

Autocorelation

$$\Psi_g(t) = \int_{-\infty}^{\infty} g(t)\, g(t+\tau)\, dt$$

Used in RADAR.

Fourier Series

for a periodic signal $g(t)$

a) Trigonometric FS

$$g(t) = a_0 + \sum_{\cancel{n=1}}^{\infty} \left[a_n \cos n\omega_0 t + b_n \sin n\omega_0 t \right]$$
$$n=1$$

where

$$a_0 = \frac{1}{T_0} \int_{t_1}^{t_1+T_0} g(t)\, dt$$

$$a_n = \frac{2}{T_0} \int_{t_1}^{t_1+T_0} g(t) \cos n\omega_0 t\, dt$$

$$b_n = \frac{2}{T_0} \int_{t_1}^{t_1+T_0} g(t) \sin n\omega_0 t\, dt.$$

$\omega_0 \rightarrow$ fundamental freq.

$\cos \omega_0 t$ & $\sin \omega_0 t \rightarrow$ fundamental Harmonic

$\cos 2\omega_0 t$, $\cos 3\omega_0 t \rightarrow$ simply Harmonics.

b) Cosine or compact FS

$$g(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$$

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where $C_0 = a_0$

$$C_n = \sqrt{a_n^2 + b_n^2}$$

$$\theta_n = \tan^{-1} \frac{-b_n}{a_n}$$

c) Exponential FS

$$g(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

where

$$D_n = \frac{1}{T_0} \int_{-\infty}^{\infty} g(t) e^{-jn\omega_0 t} \, dt$$

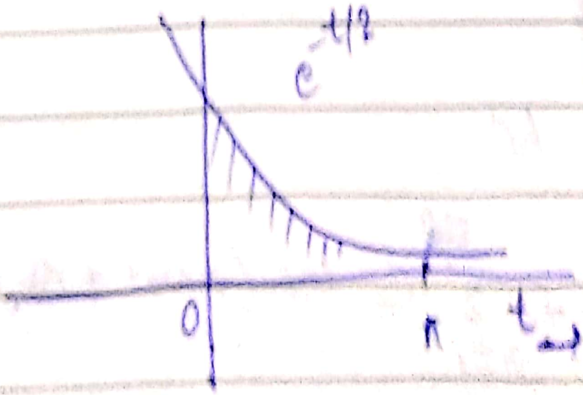
Or,

$$D_0 = C_0$$

$$D_n = \frac{1}{2} C_n e^{j\theta_n}$$

$$D_{-n} = \frac{1}{2} C_n e^{-j\theta_n}$$

Q. find the FS of $e^{-t/2}$ over the interval $0 \le t \le \pi$.



let $g(t) = e^{-t/2} \quad 0 \le t \le \pi$

$\cancel{a_0 + \sum^{\infty}}$

Here, $T_0 = \pi$

$$\omega_0 = \frac{2\pi}{T_0} = 2.$$

$$g(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos 2nt + b_n \sin 2nt]$$

where

$$a_0 = \frac{1}{T_0} \int_t^{t+T_0} g(t)\, dt$$

$$= \frac{1}{\pi} \int_0^{\pi} e^{-t/2} dt$$

$$= \frac{-2}{\pi} \left[e^{-t/2}\right]_0^{\pi}$$

$$= \frac{-2}{\pi} \left[e^{-\pi/2} - 1\right]$$

$$= \frac{2}{\pi} \left[1 - e^{-\pi/2}\right].$$

$$= 0.504$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} e^{-t/2} \cos 2nt\, dt$$

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let $f_1 = \int_0^{\pi} e^{-t/2} \cos 2nt \, dt$

$$= -2\cos 2nt \; e^{-t/2}\Big|_0^{\pi}$$

$$- \int_0^{\pi} -\sin 2nt \times 2n \times -2e^{-t/2} \, dt$$

$$= 2\cancel{\cos 2nt}\left(1 - e^{-\pi/2}\right)$$

$$- 4n \int_0^{\pi} \sin 2nt \; e^{-t/2} \, dt$$

$$= 2\cancel{\cos 2nt}\left(1 - e^{-\pi/2}\right)$$

$$\cancel{- 4n \int_0^{\pi} 2n \cos 2nt \times -2e^{-t/2} \, dt}$$

$$\cancel{- 4n \left[\sin 2nt \times -2 e^{-t/2}\right]_0^{\pi}} \rightarrow 0$$

$$+ 4n \int_0^{\pi} \cos 2nt \times 2n \times -2 \times e^{-t/2} \, dt$$

$$= 2\cancel{\cos 2nt}\left(1 - e^{-\pi/2}\right) - 16n^2 \int_0^{\pi} \cos 2nt \; e^{-t/2} \, dt.$$

$$= 2 - 2e^{-\pi/2} - 16n^2 f_1$$

$$\therefore \quad I_1 = \frac{2(1-e^{-\pi/2})}{1+16n^2}$$

$$a_n = \frac{2(1-e^{-\pi/2})}{\pi}\left[\frac{2}{1+16n^2}\right]$$

$$= 0.504\left(\frac{2}{1+16n^2}\right).$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} e^{-t/2}\sin 2nt\,dt.$$

$$I_2 = \int_0^{\pi} e^{-t/2}\sin 2nt\,dt$$

$$= \cancel{\Big[\sin 2nt\, e^{-t/2} \times -2\Big]_0^{\pi}} \rightarrow 0$$

$$- \int_0^{\pi} \cos 2nt \times 2n \times -2\, e^{-t/2}\,dt$$

$$\cancel{= \Big[4n \cos 2nt \times \int -2\, e^{-t/2}\Big]_0^{\pi}}$$

$$\cancel{- 4n \int_0^{\pi} -\sin 2nt \times 2n \times -2 \times e^{-t/2}\,dt}$$

$$= 4n\int_0^{\pi} \cos 2nt\, e^{-t/2}\,dt = 4n I_1.$$

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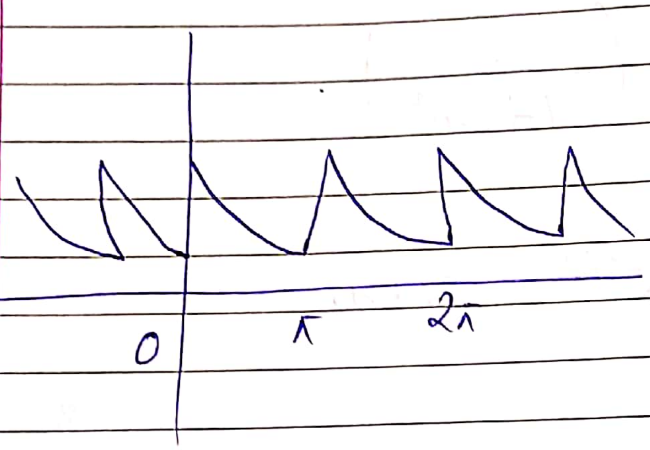
$$\therefore b_n = 0.504 \left[\frac{8n}{1+16n^2} \right].$$

$$g(t) = 0.504 \left[1 + \sum_{n=1}^{\infty} \frac{2}{1+16n^2} \left(\cos 2nt + 4n \sin 2nt \right) \right].$$



16th January

$$g(t) = C_0 + \sum_{n=1}^{\infty} (\cos 2nt + \theta_n)$$

$C_0 = a_0$

$C_n = \sqrt{a_n^2 + b_n^2}$

$$= 0.504 \sqrt{\frac{4}{(1+16n^2)^2} + \frac{64n^2}{(1+16n^2)^2}}$$

$$= 0.504 \times \frac{2}{\sqrt{1+16n^2}}$$

$$\theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right)$$

$$= \tan^{-1} -4n$$

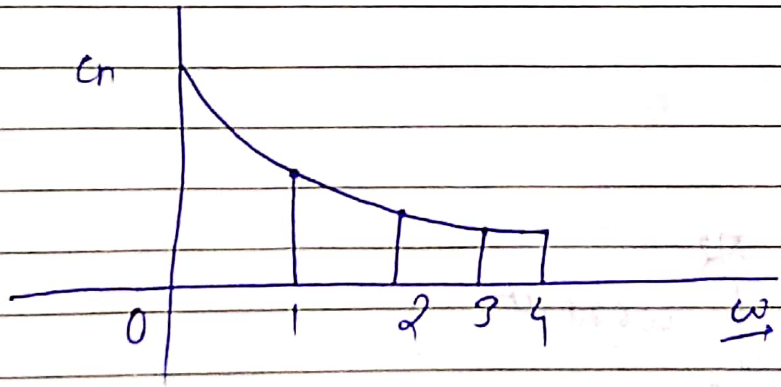
$$= -\tan^{-1} 4n.$$

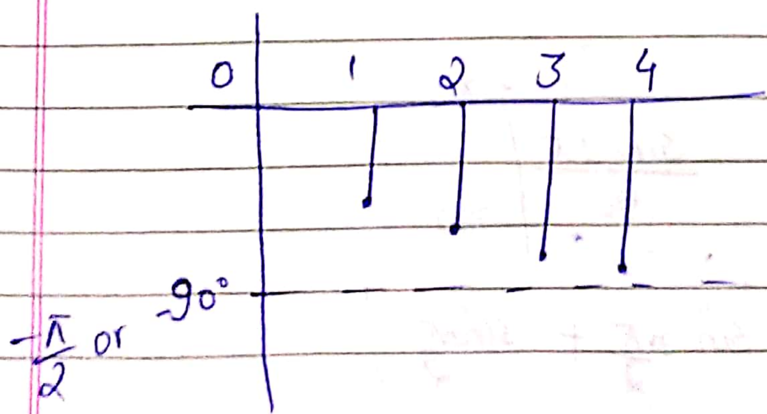
$$g(t) = 0.504 + 0.504 \sum_{n=1}^{\infty} \frac{2}{\sqrt{1+16n^2}} \left(\cos 2nt - \tan^{-1} 4n\right)$$

$$\cancel{= 0.504 \left[1 + \right.}$$

$$= 0.504 + 0.244 \cos(2t - 75.96^\circ) + 0.125 \cos(4t - 82.87^\circ)$$
$$+ 0.084 \cos(6t - 85.24^\circ) + 0.063 \cos(8t - 86.42^\circ) + \dots$$

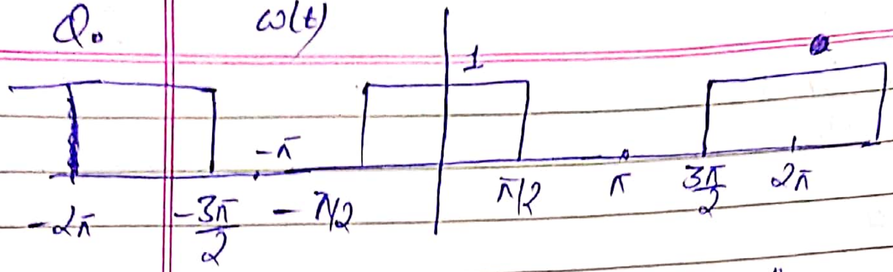
fourier Spectrum.





If ac & dc area is same the avg. will be 0, so value of dc will be zero.

Q. $w(t)$



find compact fourier series

Soln:- $T_0 = 2\pi$

$$\omega_0 = \frac{2\pi}{2\pi} = 1.$$

$$w(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt + b_n \sin nt$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} 1\,dt$$

$$= \frac{1}{2\pi}\left(\frac{\pi}{2}+\frac{\pi}{2}\right)$$

$$= \frac{1}{2}.$$

$$a_n = \frac{2}{2\pi}\int_{-\pi/2}^{\pi/2} \cos nt\,dt$$

$$= \frac{1}{\pi}\left.\frac{\sin nt}{n}\right|_{-\pi/2}^{\pi/2}.$$

$$= \frac{1}{n\pi}\left(\sin\frac{n\pi}{2} + \sin\frac{n\pi}{2}\right)$$

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$$= \frac{2}{n\pi} \sin\frac{n\pi}{2} = \begin{cases} 0 & n = 2, 4, 6, \ldots \\ \frac{2}{n\pi} & n = 1, 5, 9, \ldots \\ -\frac{2}{n\pi} & n = 3, 7, 11, \ldots \end{cases}$$

$$b_n = \frac{2}{2\pi} \int_{-\pi/2}^{\pi/2} \sin nt\, dt$$

$$= \frac{1}{n\pi} \left. -\cos nt \right|_{-\pi/2}^{\pi/2}$$

$$= \frac{-1}{n\pi} \left(\cos \frac{n\pi}{2} - \cos \frac{n\pi}{2} \right)$$

$$= 0.$$

[If the waveform is even then b_n will be zero.]

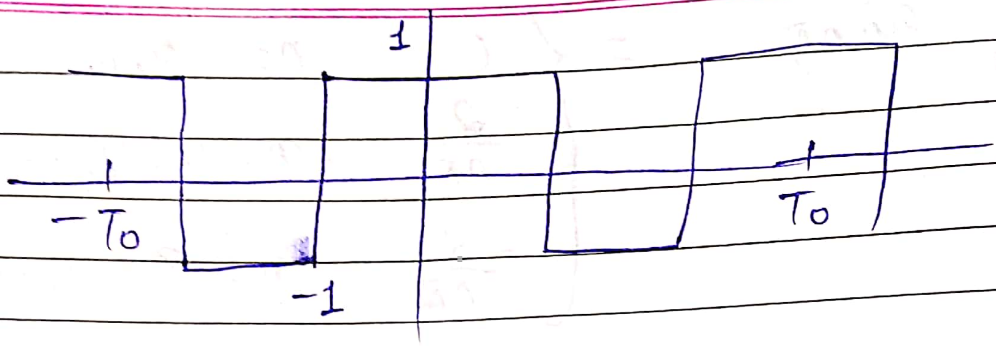
$$w(t) = 0.5 + \frac{2}{\pi} \left[\cos t - \frac{1}{3} \cos 3t + \frac{1}{5} \cos 5t - \frac{1}{7} \cos 7t + \ldots \right].$$

Compact fourier series

$$w(t) = 0.5 + \frac{2}{\pi} \left[\cos t + \frac{1}{3} \cos (3t - \pi) + \frac{1}{5} \cos 5t + \frac{1}{7} \cos (7t - \pi) + \ldots \right].$$

Ex-2.9

~~Exer~~ Q.



Find compact FS

Ans→ $a_0 = 0.$

$\omega_0 = \frac{2\pi}{T_0}$

$a_n = \cancel{\frac{2T_0}{2\pi}} \quad \frac{2}{T_0}.$

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Condition for existence of fourier series
Dirichlet's condition

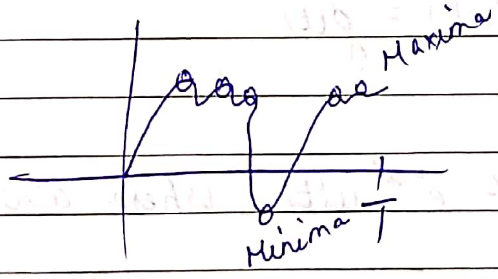
1) for a fourier series to exist, the coefficients should be finite $|a_0| < \infty$, $|a_n| < \infty$ and ~~$b_n < \infty$~~ $|b_n| < \infty$

$$\int_{T_0} g(t)\,dt < \infty.$$

→ gurantees the existance of f.s.
→ the series may or may not converse.
→ weak Dirichlet's condition.

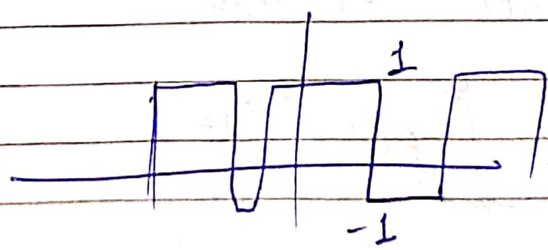
Converging series: $n \to \infty$, $a_n \to 0$ & $b_n \to 0$

II) a) The waveform should have finite no. of maxema and minima over a period



b) The waveform may have finite no. of finite discontinuity.

level of discontinuity is finite
as for below graph, it is -1 to 1.



Strong Dirichlet's condition

Series will always converse.

21st January

Fourier Integral / Transform

$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) e^{j\omega t} d\omega.$$

where

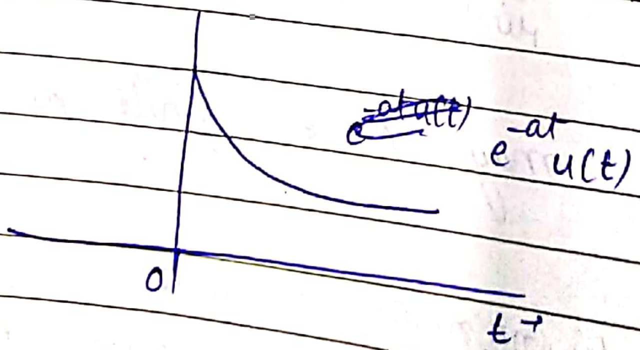
$$G(\omega) = \int_{-\infty}^{\infty} g(t) \cdot e^{-j\omega t} dt$$

$G(\omega)$ is called FT of $g(t)$.

$$F[g(t)] = G(\omega).$$

$$F^{-1}[G(\omega)] = g(t)$$

1. Find FT of $e^{-at} u(t)$ where $a>0$.



Let $g(t) = e^{-at} u(t)$ for $a>0$.

$$F[g(t)] = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt$$

$$G(\omega) = \int_{-\infty}^{\infty} e^{-at} u(t)\, e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-at} e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

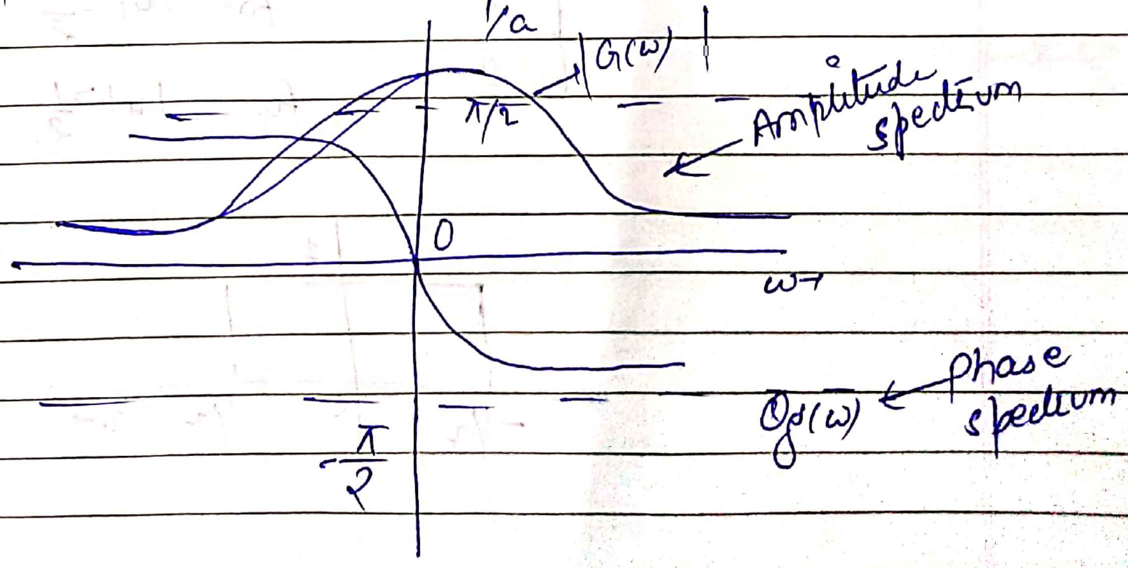
$$G(\omega) = \left. \frac{-e^{-(a+j\omega)}}{(a+j\omega)} \right|_{0}^{\infty}$$

$$= \frac{1}{a+j\omega}$$

$$|G(\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}$$

$$\theta = \tan^{-1}\left(-\frac{\omega}{a}\right).$$

Amplitude spectrum



For any real signal the amplitude spectrum → even

phase spectrum → odd

The cond^n for existence is same for FS & FT.

1) Linearity of fourier Transform

$$F[g_1(t)] = G_1(\omega)$$

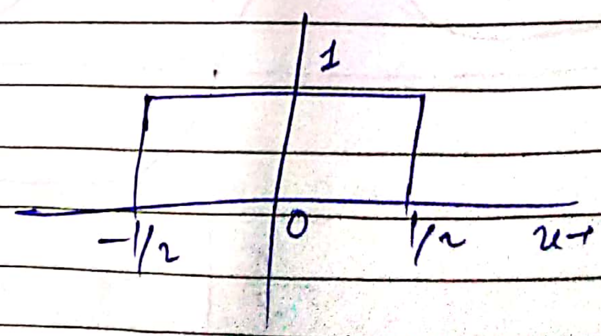
$$F[g_2(t)] = G_2(\omega).$$

$$a_1 g_1(t) + a_2 g_2(t) \overset{F}{\Longleftrightarrow} a_1 G_1(\omega) + a_2 G_2(\omega).$$

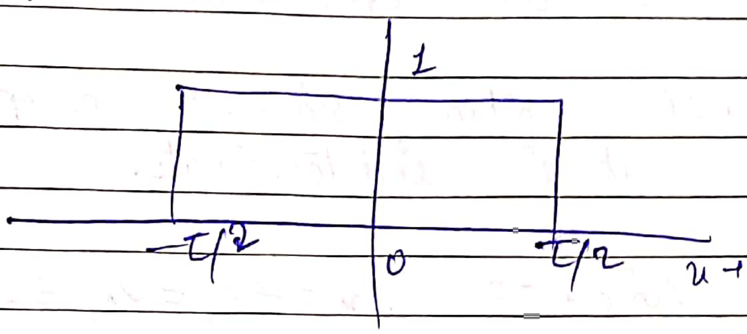
functions:

1) Unit Gate function.

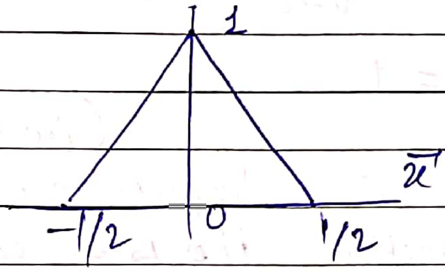
$$\text{rect}(x) = \begin{cases} 1 & \text{for } |x| < \frac{1}{2} \\ \frac{1}{2} & \text{for } |x| = \frac{1}{2} \\ 0 & \text{for } |x| > \frac{1}{2}. \end{cases}$$



rect(x/τ).



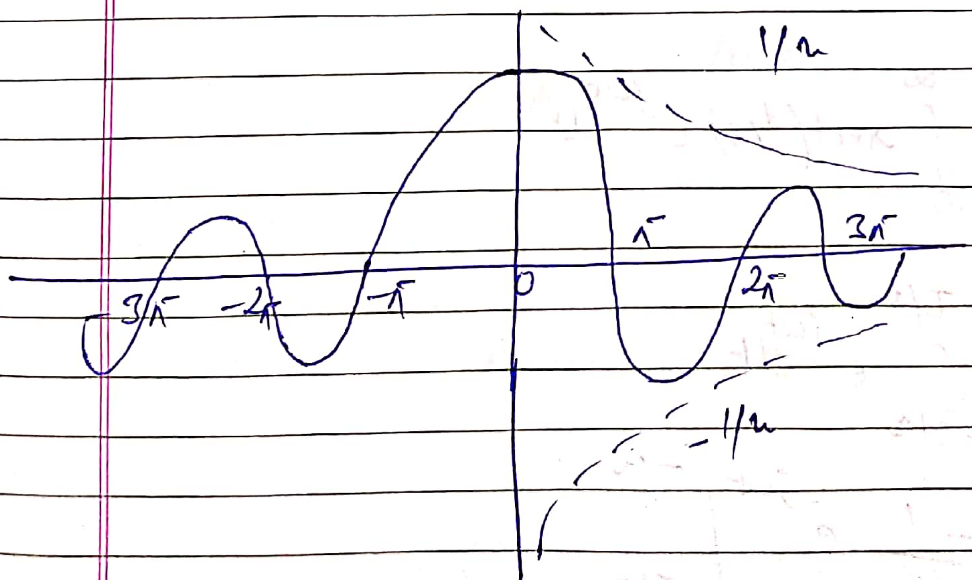
II) Unit Triangle function.



$$\Delta(x) = \begin{cases} 0 & \text{for } |x| > \frac{1}{2}. \\ 1 - 2|x| & \text{for } |x| \leq \frac{1}{2}. \end{cases}$$

III) Interpolation function:

$$\operatorname{sinc}(x) = \frac{\sin x}{x}.$$



a) sinc x is an even function of x.

b) sinc x = 0 when sin x = 0 except at x = 0, where it is indeterminate.

i.e sinc x = 0 , x = ±π, ±2π, ±3π, ...

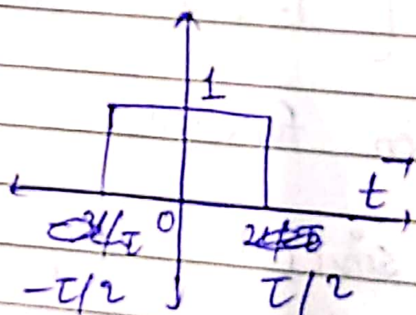
$$\text{sinc}(0) = \lim_{x \to 0} \frac{\sin x}{x}$$

$$= 1$$

(i.e from dig. to analog)

The value of the graph can be derived from various functions therefore called Interpolation function. sinc

Q. FT of ~~rect~~ rect (t/τ).



Ans. $$G(\omega) = \int_{-\infty}^{\infty} \text{rect}(t/\tau)\, e^{-j\omega t}\, dt$$

$$= \int_{-\tau/2}^{\tau/2} e^{-j\omega t}\, dt$$

$$= \frac{-1}{j\omega} e^{-j\omega t}\Big|_{-\tau/2}^{\tau/2}$$

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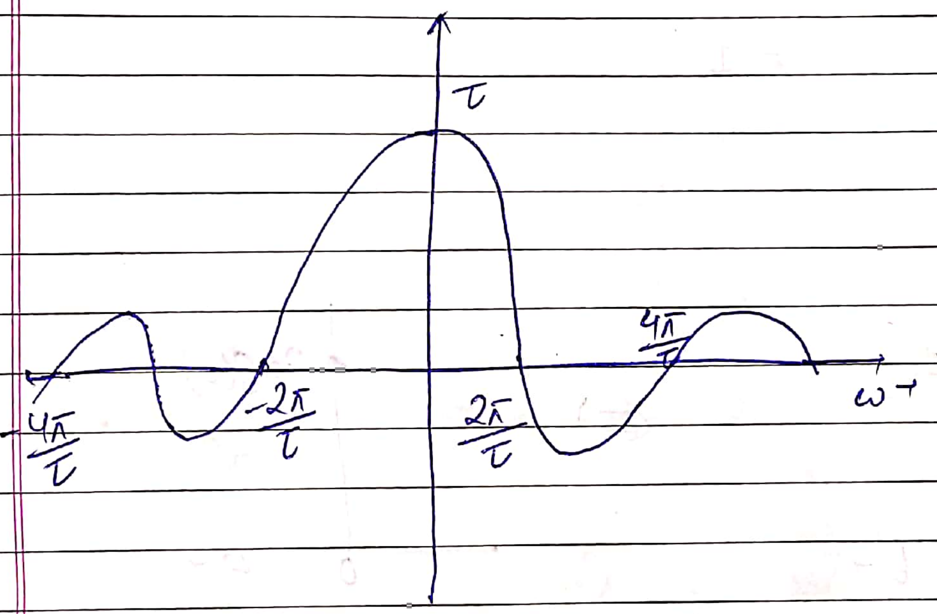
$$= \frac{e^{j\omega\tau/2} - e^{-j\omega\tau/2}}{j\omega}$$

$$= \frac{2}{\omega} \sin \omega\tau/2$$

$$= \tau \frac{\sin \omega\tau/2}{\omega\tau/2}$$

$$= \tau \operatorname{sinc}(\omega\tau/2).$$



$\tau > 1$.
wave form expanding in time
" contracting " freq.

~~So the freq. domain will contract and~~

$\tau < 1$
waveform contracting in time
" expanding " freq.

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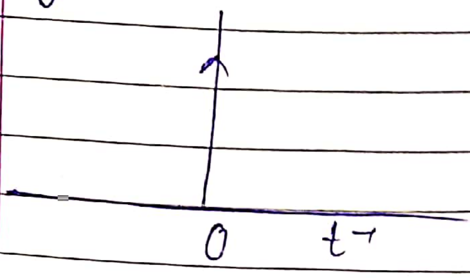
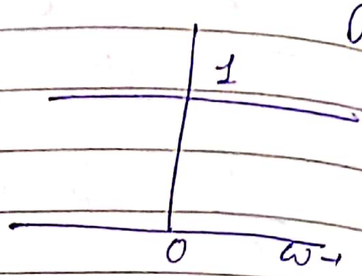
Q.



$\overset{f}{\Longleftrightarrow}$



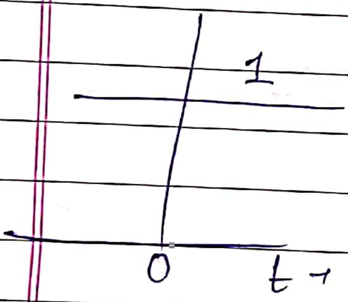
Frequency doma

$$f[\delta(t)] = \int_{-\infty}^{\infty} \delta(t)\, \cancel{dt}\, e^{-j\omega t}\, dt$$

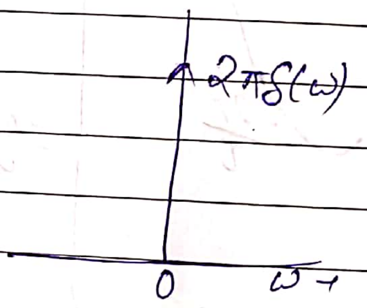
$$= e^{0} \int_{-\infty}^{\infty} \delta(t)\, dt$$

$$= 1$$

Q. FT of 1



$\Longleftrightarrow$



$f=0$

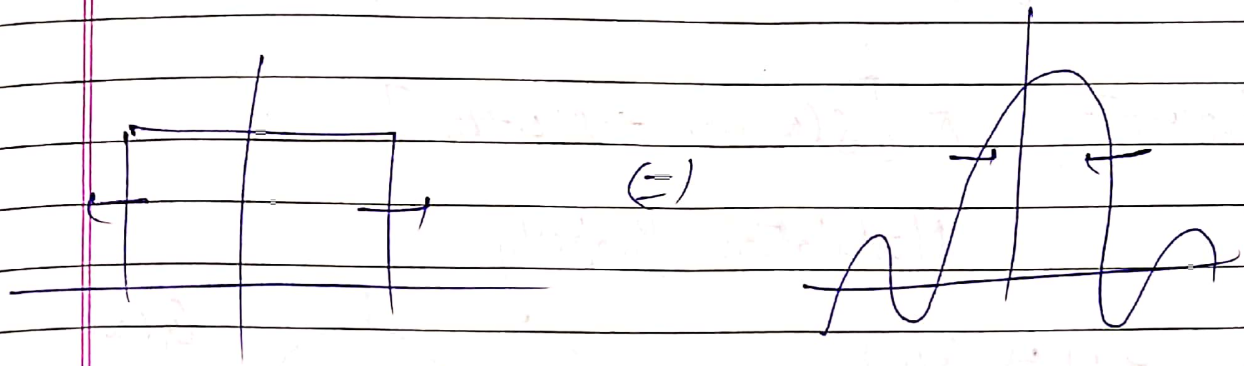
$$f^{-1}\left[\delta(\omega)\right] = \frac{1}{2\pi} \int_{-\infty}^{\infty} \delta(\omega)\, e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi} e^{0} \int_{-\infty}^{\infty} \delta(\omega)\, d\omega$$

$$= \frac{1}{2\pi}$$

$$\therefore \frac{1}{2\pi} \overset{f}{\Longleftrightarrow} \delta(\omega)$$

$$1 \overset{F}{\Longleftrightarrow} 2\pi\delta(\omega).$$



Q. find IFT $\delta(\omega-\omega_0)$.

$$F^{-1}(\delta(\omega-\omega_0)) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega-\omega_0) e^{j\omega t} d\omega$$

$$= \frac{e^{j\omega_0 t}}{2\pi}$$

$$\therefore \frac{e^{j\omega_0 t}}{2\pi} \overset{f}{\Longleftrightarrow} \delta(\omega-\omega_0)$$

$$\Rightarrow e^{j\omega_0 t} \Longleftrightarrow 2\pi\,\delta(\omega-\omega_0).$$

$$e^{-j\omega_0 t} \Longleftrightarrow 2\pi\,\delta(\omega+\omega_0)$$

Frequency shifting

Q. FT of $\cos\omega_0 t$

$$(\cos\omega_0 t) = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$$

$$f(\cos\omega_0 t) = \frac{1}{2}\left[\int_{-\infty}^{\infty} e^{\,j t(\omega+\omega_0)}\,dt + \int_{-\infty}^{\infty} e^{\,j t(\omega-\omega_0)}\,dt\right]$$

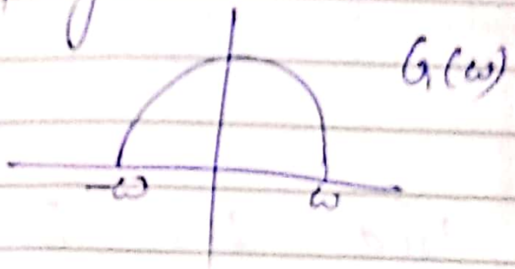
$$= \cancel{\frac{1}{2} e^{j t(\omega+\omega_0)}}$$

$$= \frac{1}{2}\left[2\pi\, \delta(\omega-\omega_0) + 2\pi\, \delta(\omega+\omega_0)\right]$$

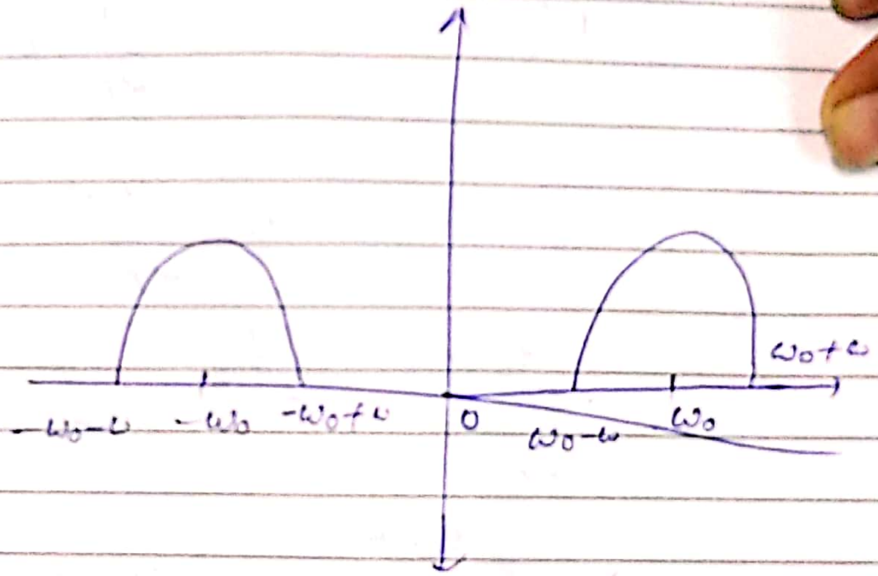
$$F[\cos\omega_0 t] = \pi\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right].$$

Modulation Property

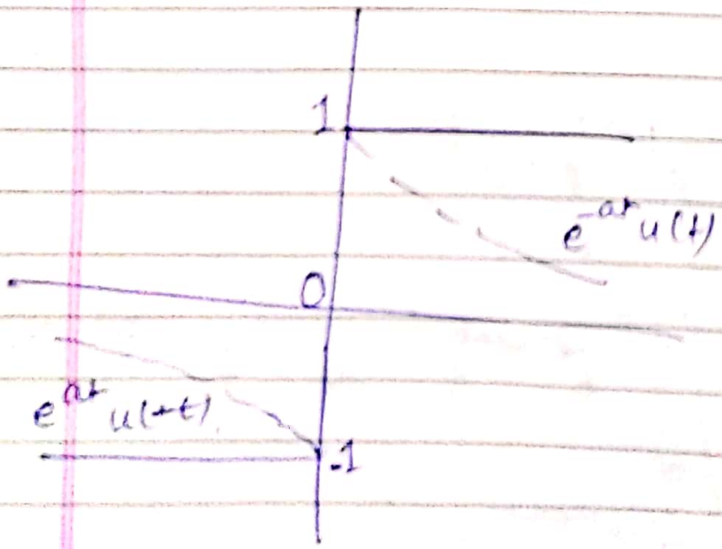
$g(t) \Leftrightarrow G(\omega)$



$g(t)\cos\omega_0 t$



Q. FT of signum(t).



$t = 1 \quad t > 0$

$= -1 \quad t < 0$

$$= 2(2\pi\,\delta(\omega))$$

$$= 4\pi\,\delta(\omega).$$

$$\boxed{\operatorname{sgn}(t) = \lim_{a \to 0}\left[e^{-at}u(t) - e^{at}u(-t)\right]}$$

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$$f[e^{-at}u(t)] = \int_0^{\infty} e^{-at} e^{-j\omega t} dt$$

$$= \int_0^{\infty} e^{-(a+j\omega)t} dt$$

$$= \frac{1}{a+j\omega}.$$

$$f[e^{at}u(-t)] = \int_{-\infty}^{0} e^{at} e^{-j\omega t} dt$$

$$= e^{-(j\omega-a)t} \Big]_{-\infty}^{0}$$

$$= \frac{-1}{j\omega-a}.$$

$$= \frac{1}{a-j\omega}.$$

$$\therefore f[\mathrm{sgn}(t)] = \lim_{a\to 0} \left[\frac{+1}{a+j\omega} - \frac{1}{a-j\omega} \right].$$

$$= \lim_{a\to 0} \left[\frac{\cancel{a}-j\omega-\cancel{a}-j\omega}{a^2+\omega^2} \right].$$

$$= \frac{-2j\cancel{\omega}}{\omega^{\cancel{2}}} = \frac{-2j}{\omega}. = \frac{2}{j\omega}$$

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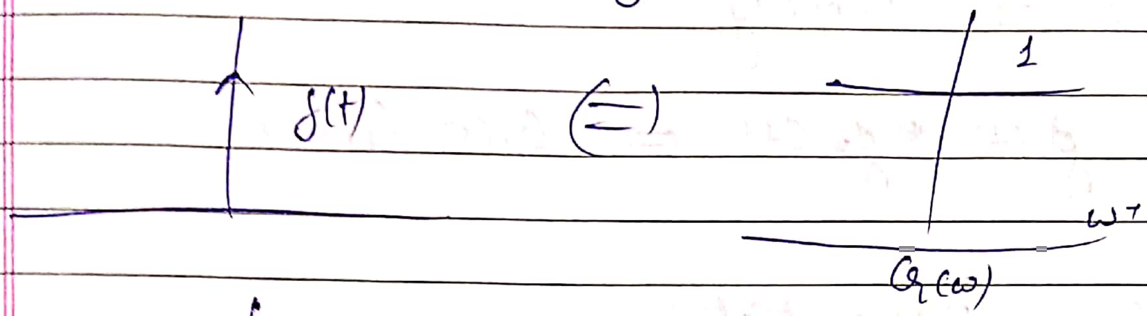
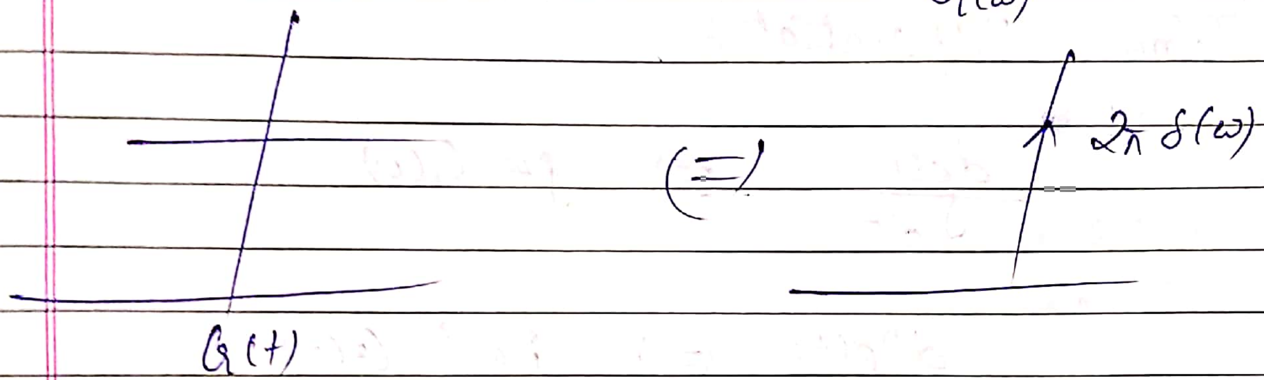
Properties of FT.

1. Symmetric Property

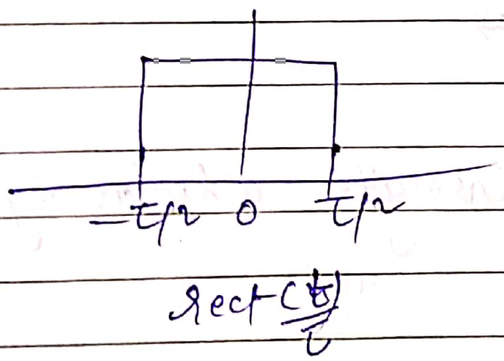
$$G(t) \Leftrightarrow 2\pi\, g(-\omega).$$





2. Scaling Property

$$g(at) \Leftrightarrow \frac{1}{|a|}\, G(\omega/a).$$



$$\Rightarrow \tau \operatorname{sinc}\left(\frac{\omega\tau}{2}\right)$$

3. Time shifting Property

$$g(t-t_0) \Leftrightarrow G(\omega)\, e^{-j\omega t_0}.$$

4. Frequency Shifting Property

$$g(t)\, e^{j\omega_0 t} \Longleftrightarrow G(\omega-\omega_0).$$

5. Convolution Property

$$g_1(t) * g_2(t) \Longleftrightarrow G_1(\omega)\, G_2(\omega).$$

6. Time Differentiation

$$\frac{dg(t)}{dt} \Longleftrightarrow j\omega\, G(\omega)$$

$$\frac{d^n g(t)}{dt^n} \Longleftrightarrow (j\omega)^n\, G(\omega).$$

7. Time Integration

$$\int_{-\infty}^{t} g(u)\,du \Longleftrightarrow \frac{G(\omega)}{j(\omega)} + \pi\, G(0)\, \delta(\omega).$$

23rd January Signal transmission through a Linear System

$$g(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$y(t) = g(t) * h(t).$$

If $y(t) \Longleftrightarrow Y(\omega)$, $g(t) \Longleftrightarrow G(\omega)$, $h(t) \Longleftrightarrow H(\omega)$,

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$Y(\omega) = G(\omega)\ H(\omega).$

$$\Rightarrow |Y(\omega)|\ e^{j\theta_y(\omega)} = |G(\omega)|\ e^{j\theta_g(\omega)}\ |H(\omega)|\ e^{j\theta_h(\omega)}.$$

$$= |G(\omega)|\ |H(\omega)|\ e^{j(\theta_g(\omega) + \theta_h(\omega))}$$

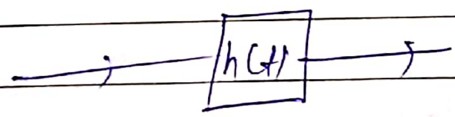
$$|Y(\omega)| = |G(\omega)|\ |H(\omega)| \longrightarrow \text{Amplitude factor}$$

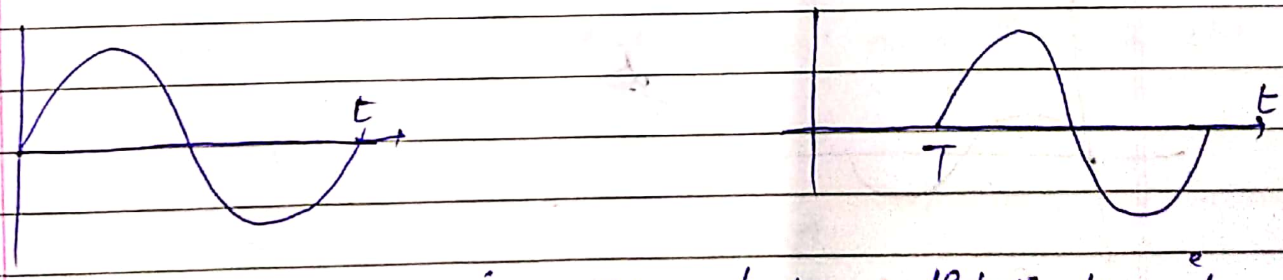
$$\theta_y(\omega) = \theta_g(\omega) + \theta_h(\omega) \longrightarrow \text{shifted phase.}$$

Plot of $|H(\omega)|$ and $\theta_h(\omega)$ as a function of ω shows how the system modifies the amplitude and phase of various sinusoidal ~~funct~~ components of the i/p signal.

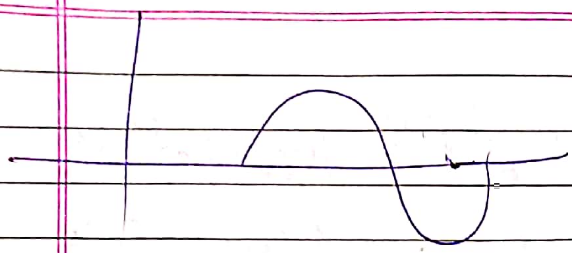
$|H(\omega)| \rightarrow$ frequency response of the system
Transfer function of the system.

Distortion less transmission



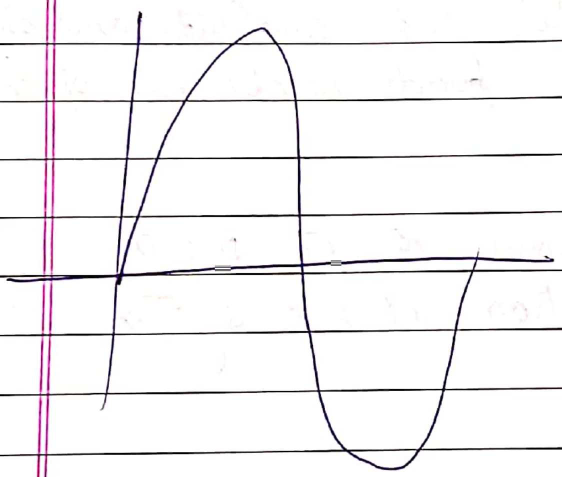


A transmission is said to be distortion less if the i/p and o/p have identical waveshapes within a multiplication constants.

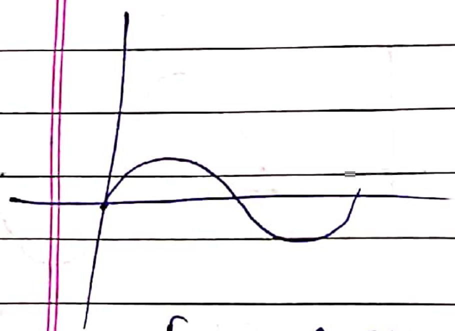


DL

D



DL



D

for a DL transmission, the o/p can be written as

$$y(t) = K g(t - t_d).$$

FT $$Y(\omega) = K\, G(\omega)\, e^{-j\omega t_d}$$

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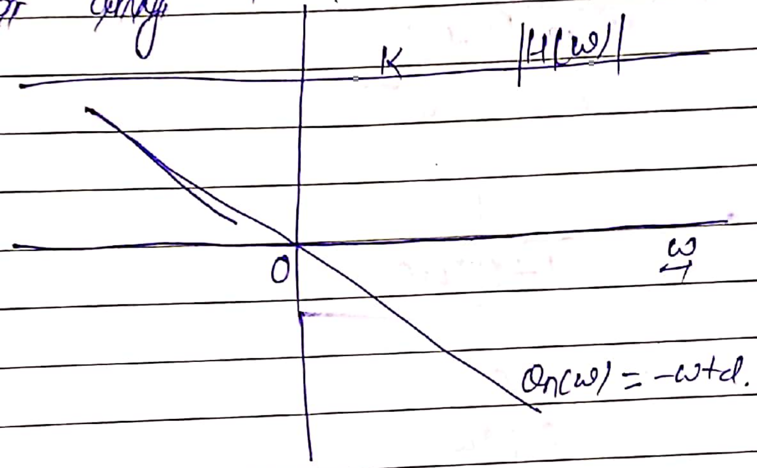
$Y(\omega) = G(\omega) H(\omega)$

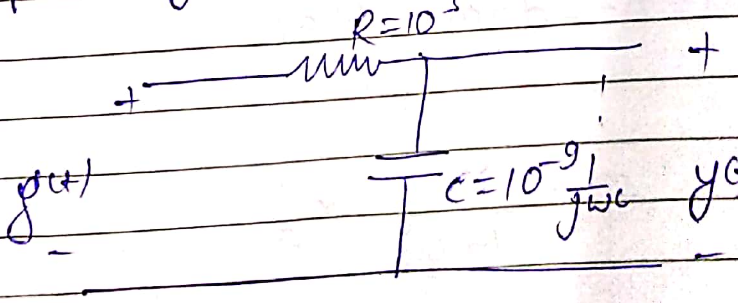
$H(\omega) = K e^{-j\omega t_d}$

$|H(\omega)| = K \qquad \theta_h(\omega) = -\omega t_d.$

~~for any~~ for distortion-less ~~transmission, transfer func^n :-~~



Q. If $g(t)$ and $y(t)$ are i/p and o/p, respectively of a simple RC low pass filter, determine the transfer func^n $H(\omega)$ and sketch $|H(\omega)|$ and $\theta_h(\omega)$ and $t_d(\omega)$. for distortion-less trans. through this filter, what is the requirement on the band width of $g(t)$ if amp. variation within 2% and time delay variation within 5% are tolerable? what is the trans. delay? Find the output $y(t)$.



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$$a = \frac{1}{RC} = \frac{1}{10^3 \times 10^{-9}} = 10^6.$$

By applying Voltage Divider ~~Rat~~ law,

$$H(\omega) = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

$$= \frac{1}{1 + Rj\omega C}$$

$$= \frac{a}{a + j\omega}$$

$$|H(\omega)| = \frac{|a|}{\sqrt{a^2 + \omega^2}}$$

$$\theta_h(\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right).$$

$$t_d(\omega) = -\frac{d\theta_h(\omega)}{d\omega}.$$

$$= -\frac{d \tan^{-1}(\omega/a)}{d\omega}$$

$$= -\frac{1}{1 + \frac{\omega^2}{a^2}} \times \frac{1}{a}.$$

$$= \frac{a^2}{a^2 + \omega^2} \times \frac{1}{a} = \frac{a}{a^2 + \omega^2}$$

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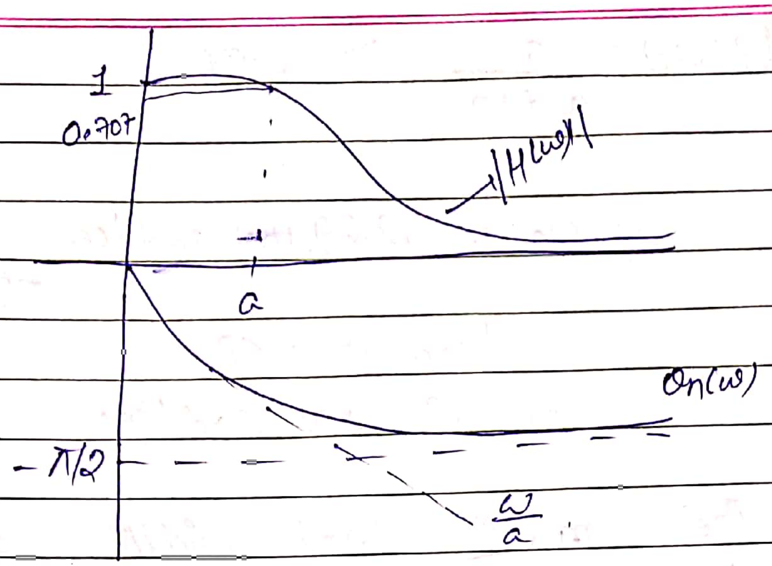
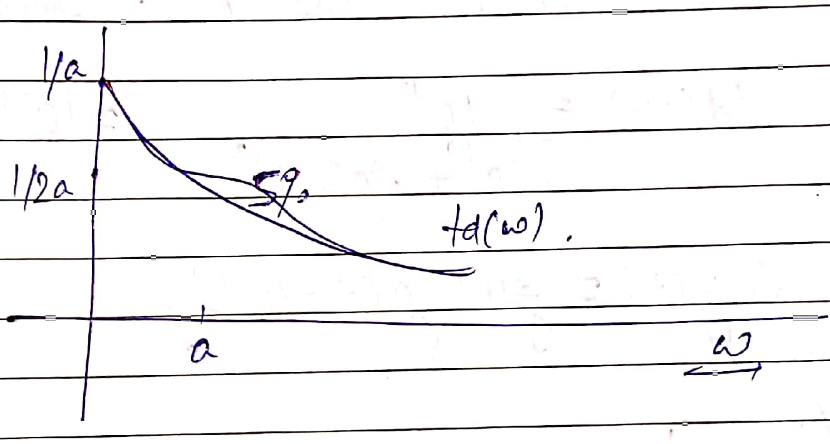




$|H(0)| = 1$. $\quad td(0) = \frac{1}{a}$

Let ω_0 be the highest band width of the signal that can be transmitted within the speci given in the equn.

i.e. $|H(\omega_0)| = \frac{a}{\sqrt{a^2+\omega_0^2}} \geq 0.98$

$\Rightarrow \frac{1}{\sqrt{1+\left(\frac{\omega_0}{a}\right)^2}} \geq 0.98$ $\qquad \omega_0 = 203{,}000$ — (1)

$\frac{1}{(0.98)^2} \geq 1 + \frac{\omega_0^2}{a^2}$

$$t_d(\omega_0) = \frac{a}{a^2+\omega_0^2} \geq \frac{0.95}{a}.$$

$\frac{1}{a}\times\frac{\pi}{180}\times 0.05$

$$\Rightarrow \omega_0 = 229,400 \text{ rad/sec.} \quad \text{— (II)}$$

from (I) & (II)

$\therefore \omega_0 = 203,000$ rad/sec

$\frac{1}{a}(1-0.05)$

$\therefore$ The max^m band width of $g(t)$ should be $203,000$ rad/sec $= 32.31$ KHz.

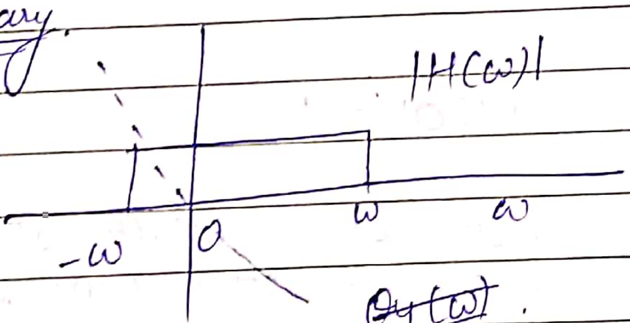
$\frac{0.95}{a}$

$$t_d(\omega_0) = \frac{10^6}{10^{12}+(203,000)^2} \approx 10^{-6}$$

$$\therefore y(t) = \cancel{g(t)}\; g(t-10^{-6}).$$

6th February

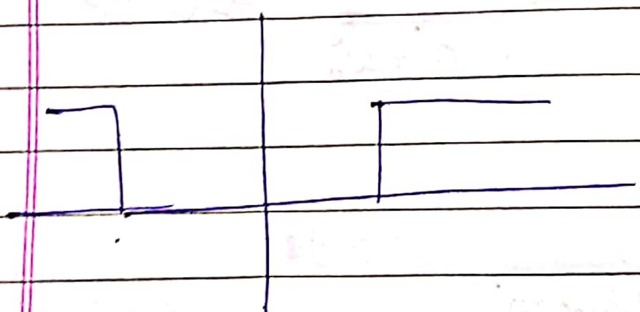
LPF



~~$\theta_y(\omega)$~~.

$\theta_h(\omega) = -\omega t_d$

HPF



$$g(t) \longrightarrow \boxed{LPF} \longrightarrow y(t)$$

If $g(t)$ is bandlimited to W rad/sec.

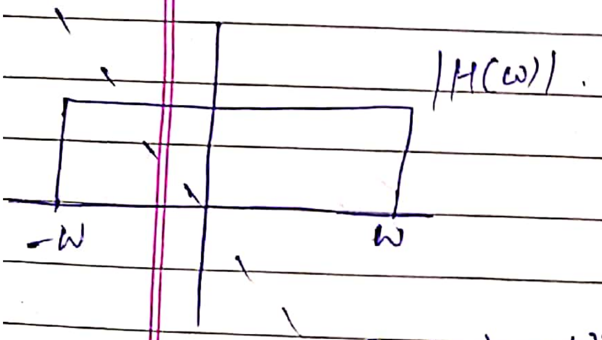
$$y(t) = g(t-td).$$

for this filter $|H(\omega)| = rect\left(\frac{\omega}{2W}\right)$.

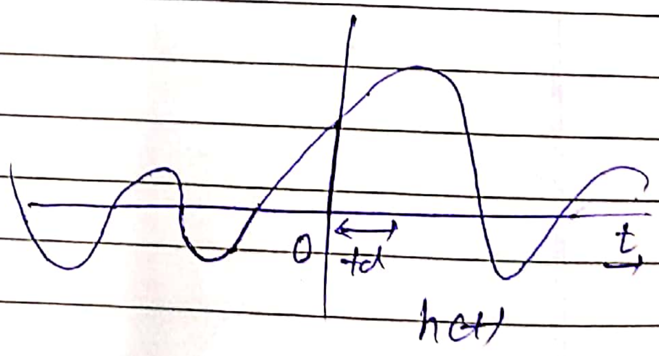
$$\theta_h(\omega) = -\omega td.$$

$$H(\omega) = rect\left(\frac{\omega}{2W}\right) e^{-j\omega td}.$$

$$F^{-1}[H(\omega)] = h(t) = \frac{W}{\pi} sinc[W(t-td)]$$



$\iff$



$\theta_h(\omega) = -\omega td.$

Non-causal system

Ideal LPF is not physically-realisable.

To be realisable, $h(t)$ should be causal

i.e $h(t) = 0, \ t < 0$.

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In frequency domain,

Paley Wiener Criterion
It states that the necessary and sufficient condⁿ for the amp. response $|H(\omega)|$ to be ~~selfi~~ realisable is

$$\int_{-\infty}^{\infty} \frac{|\ln\{H(\omega)\}|}{1+\omega^2}\, d\omega < \infty.$$

$\boxed{\text{Zero for a } \cancel{\text{def integral}} \text{ certain range of frequency} \rightarrow \text{Non-realisable}}$

$\hat{h}(t) = h(t)\,u(t)$

If t_d is $\uparrow$ $\hat{h}(t) \simeq h(t)$.

Cost of realising ideal filter is higher time delay.

Process of making a system causal is called Turncation Operation.

Problems
- Spectral leakage.
- Spectral Spreading

$$g(t) = \int_{-\infty}^{\infty} \frac{1}{2\pi} G(\omega)\, e^{j\omega t}\, dt$$

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$$E_g = \int_{-\infty}^{\infty} g(t)\, g^*(t)\, dt$$

$$= \int_{-\infty}^{\infty} g(t) \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} G^*(\omega)\, e^{-j\omega t}\, d\omega \right] dt$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} G^*(\omega) \left[\int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt \right] d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega)\, G^*(\omega)\, d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} |G(\omega)|^2\, d\omega$$

Energy Spectral density

unit : E/freq.

Essential Bandwidth

Most of the signal bandwidth is contained within a certain band of 'B' Hz and the energy contain of the components of frequencies > B Hz is negligible. We can supress the signal spectrum beyond B Hz with little effect on the signal shape and energy.

B → essential Bandwidth.

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strictly bandlimited $\rightarrow$ $-\infty$ to ∞ as BW

Q. Estimate the essential Bandwidth W rad/sec of the signal $e^{-at}u(t)$ if the necessary bw contain 95% of the signal energy

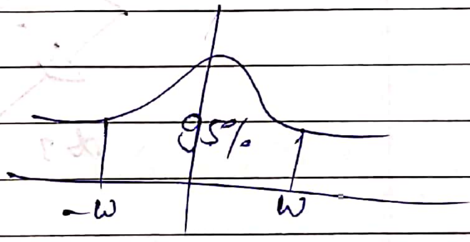
Ans- $g(t) = e^{-at}u(t)$.

$$G(\omega) = \frac{1}{a+j\omega}.$$

$$|G(\omega)|^2 = \frac{1}{a^2+\omega^2}$$

$$\therefore E_g = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{a^2+\omega^2}\, d\omega.$$

$$= \frac{1}{2\pi a} \tan^{-1}\frac{\omega}{a}\Big|_{-\infty}^{\infty}$$



$$= \frac{1}{2\pi a}\left(\frac{\pi}{2}+\frac{\pi}{2}\right)$$

$$= \frac{1}{2a}$$

$\frac{\pi}{2}+\frac{\pi}{2}$

Let EB to be $-W$ to W

$$\therefore \frac{0.95}{2a} = \frac{1}{2\pi}\int_{-W}^{W} |G(\omega)|^2\, d\omega.$$

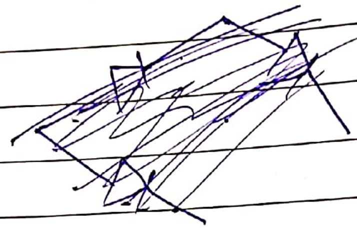
$$= \frac{1}{2\pi}\int_{-W}^{W} \frac{1}{a^2+\omega^2}\, d\omega.$$

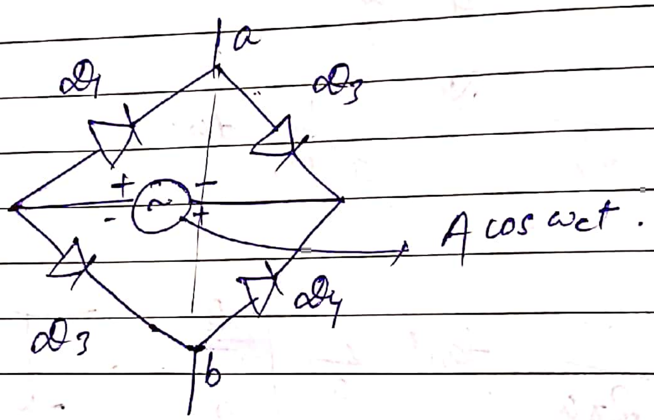
Power Spectral Density

=) $\omega = 12.706a$ rad/sec

Read from Notes

12th Feb ~~Di~~ Diode Bridge





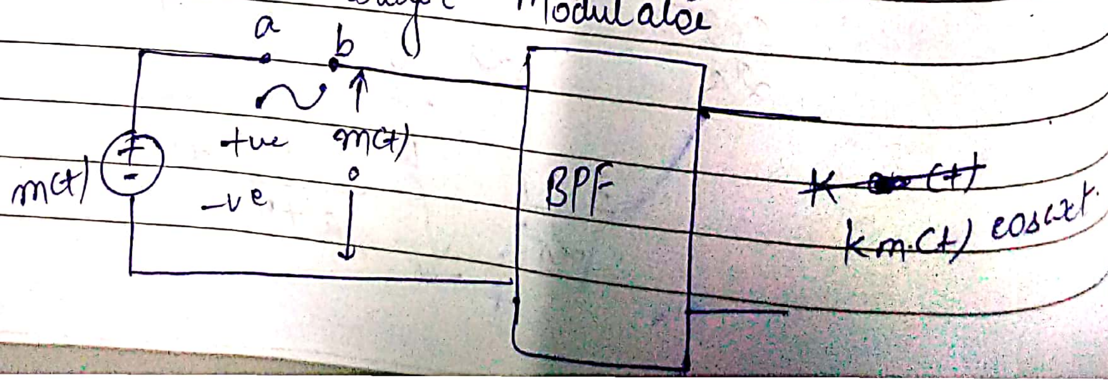
D_1, D_2 and D_3, D_4 are matched pairs.

+ve $V_a = V_b$. Close ~~flow~~ switch

-ve $V_a \neq V_b$. Open Switch

Two types

1) Series diode bridge Modulator



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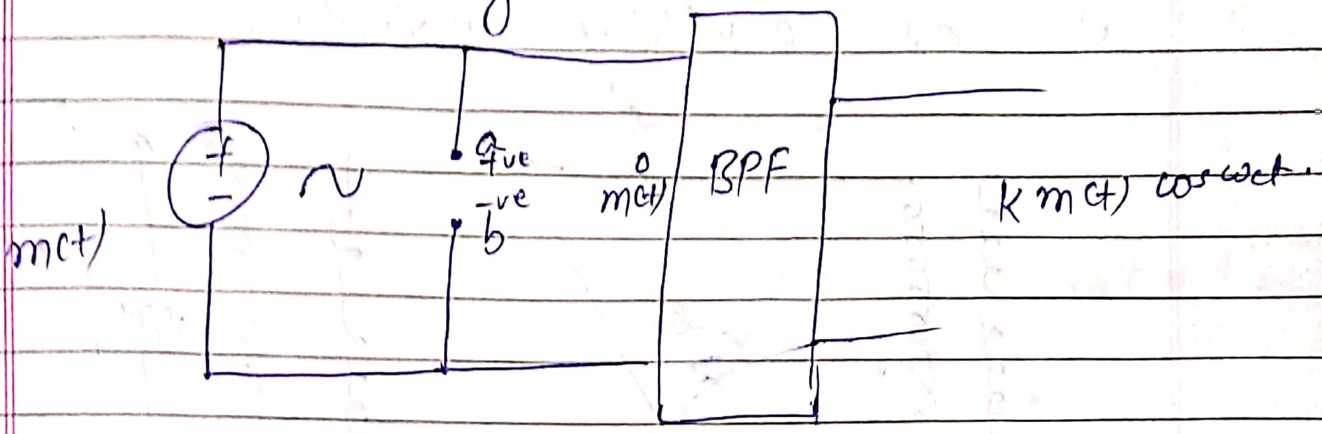
11) Shunt diode bridge modulator

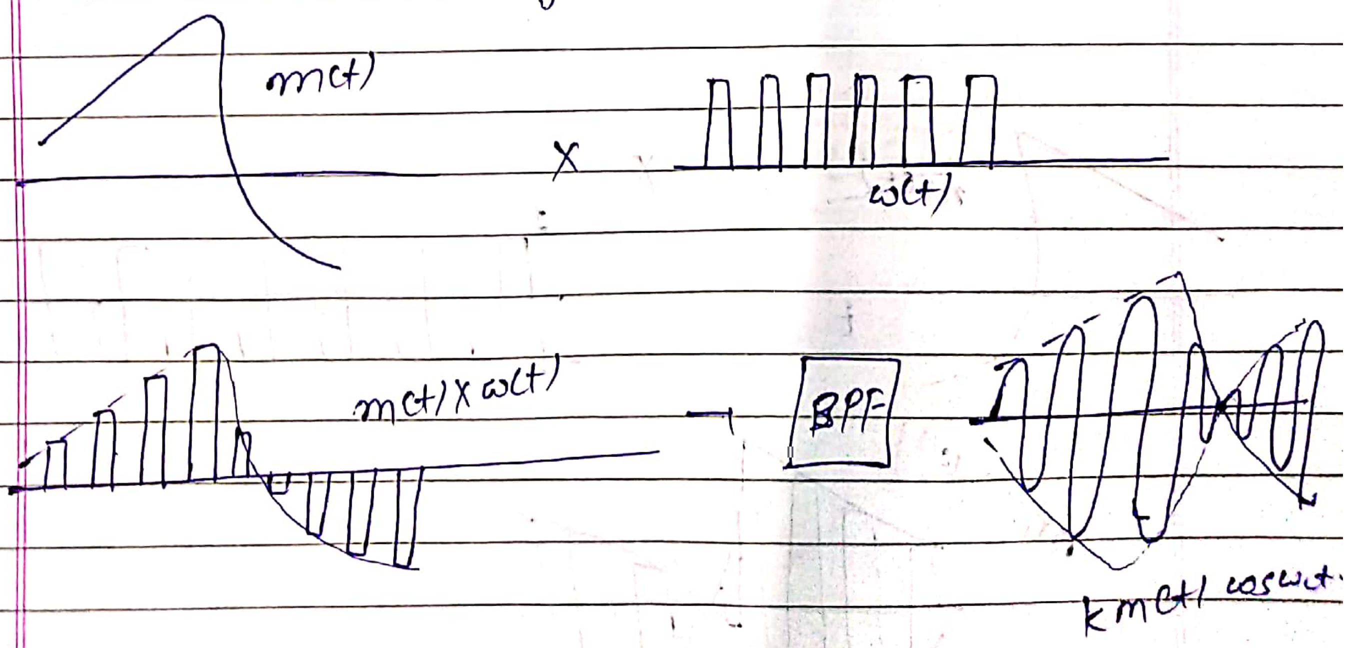


$m(t) \times w(t)$

$$= m(t) \times \left[\frac{1}{2} \times \frac{2}{\pi}\left(\cos w_c t - \frac{1}{3}\cos 3w_c t + \frac{1}{5}\cos 5w_c t - \dots\right)\right].$$

$$= \frac{1}{2} m(t) + \frac{2}{\pi} m(t)\cos w_c t - \frac{2}{3\pi} m(t)\cos 3w_c t - \frac{2}{5\pi} m(t) \cos 5w_c t + \dots$$

single balance



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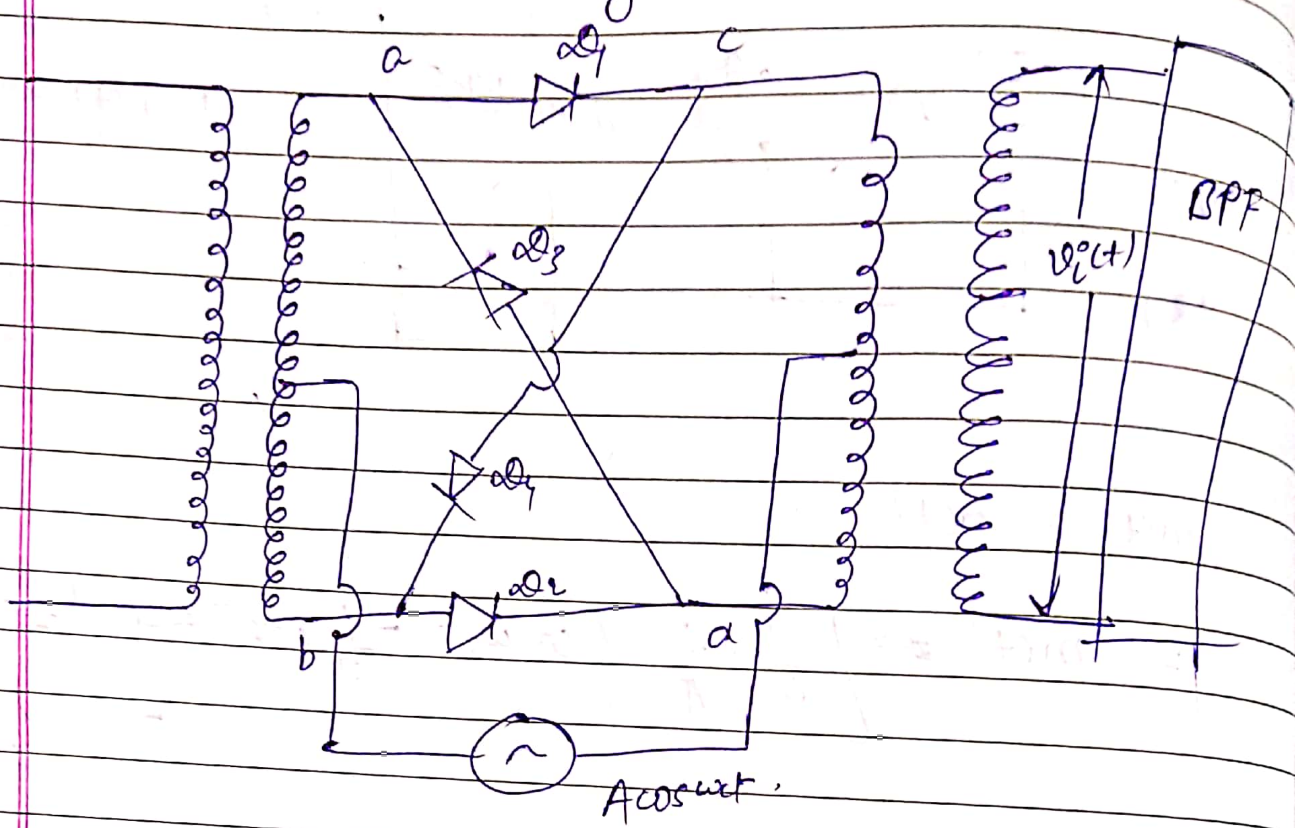
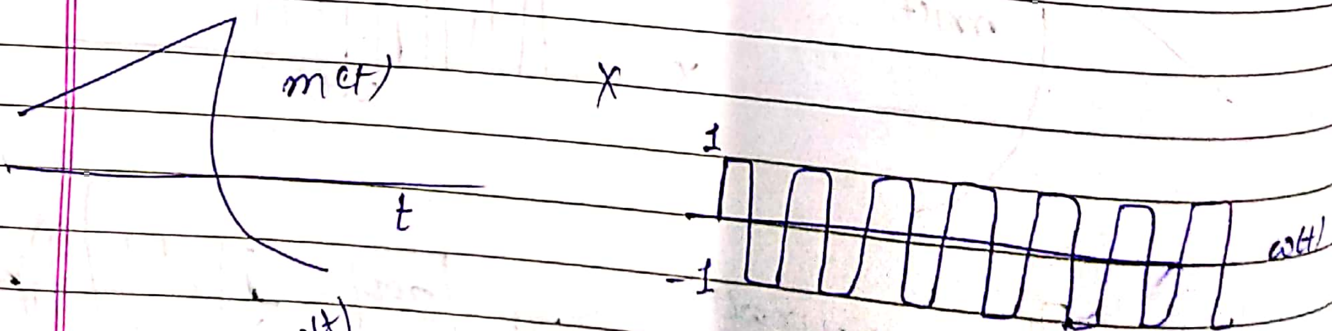
Ring Modulator

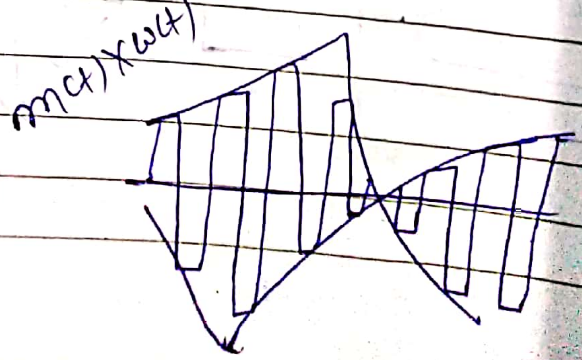
also a switching modulator



+ve : D_1 & D_2 are f biased ∴ a→c , b→d.

−ve : D_3 & D_4 " " ∴ a→d , b→c.

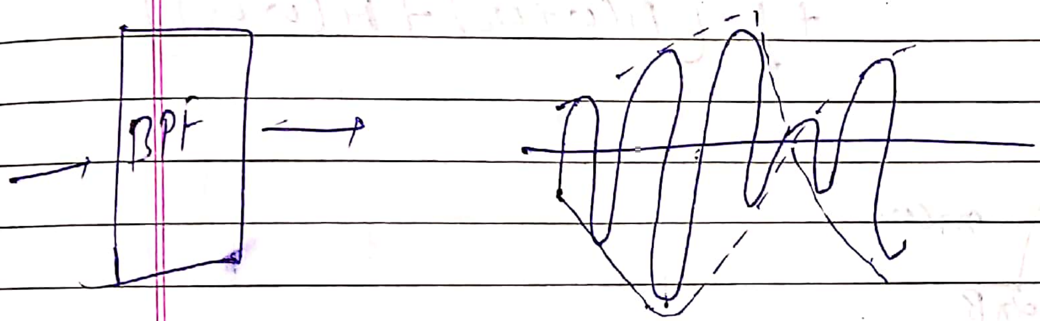


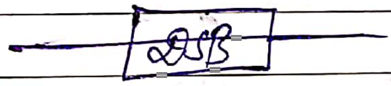


$$\omega(t) = \frac{4}{\pi}\left[\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \frac{1}{7}\cos 7\omega_c t + \dots\right]$$

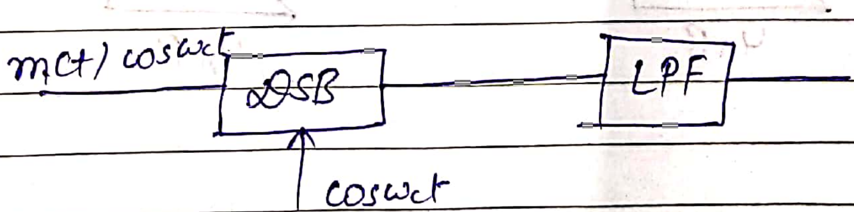
$$m(t)\,\omega(t) = \frac{4}{\pi} m(t)\cos\omega_c t - \frac{4}{3\pi} m(t)\cos 3\omega_c t + \frac{4}{5\pi} m(t)\cos 5\omega_c t - \frac{4}{7\pi} m(t)\cos 7\omega_c t + \dots$$

Double balanced





Demodulator



Coherent / Synchronous demodulation.

DSB is not used for broadcasting

used for one to one communication.

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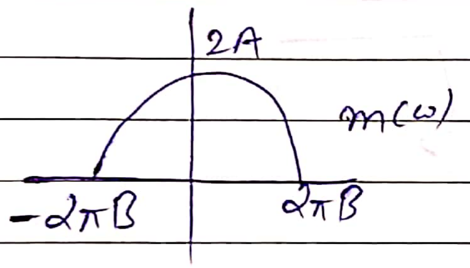
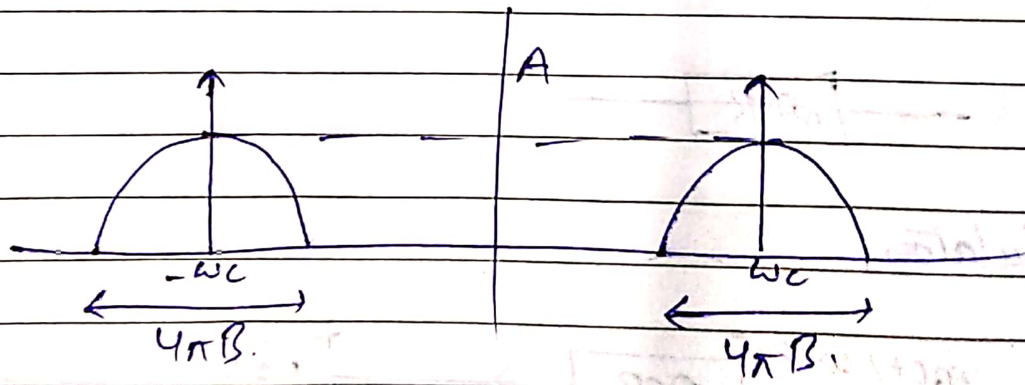
Amplitude Modulation (AM)

$$\varphi_{AM}(t) = \underbrace{A\cos\omega_c t}_{\text{carrier}} + \underbrace{m(t)\cos\omega_c t}_{DSB}$$

$$= [A + m(t)] \cos\omega_c t$$

$$F[\varphi_{AM}(t)] = A\pi[\delta(\omega+\omega_c) + \delta(\omega-\omega_c)] + \frac{1}{2}[M(\omega+\omega_c) + M(\omega-\omega_c)]$$





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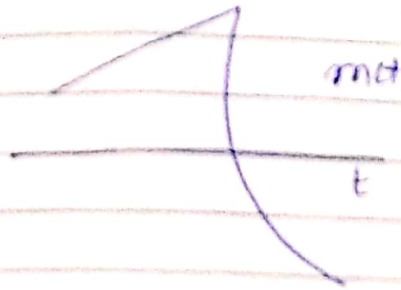
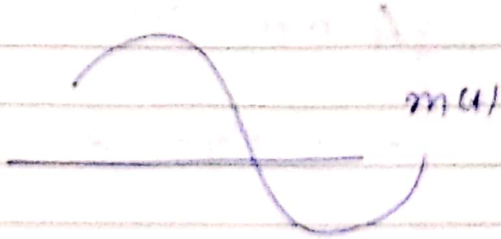
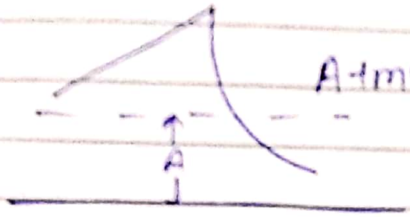
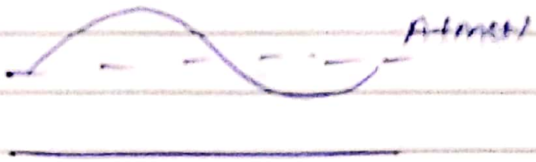
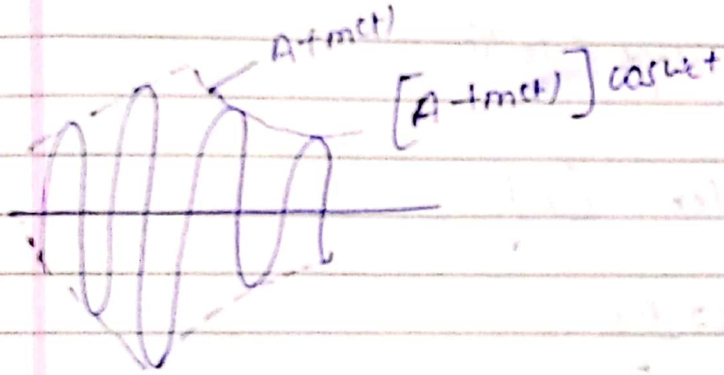
13th Feb

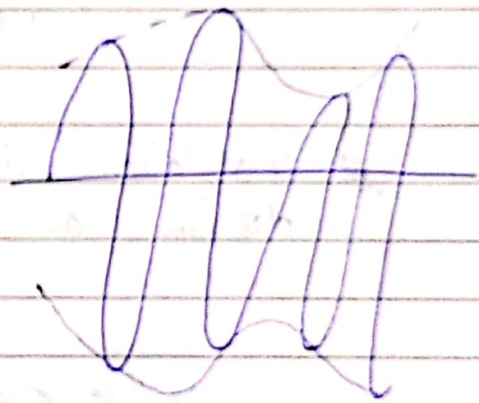






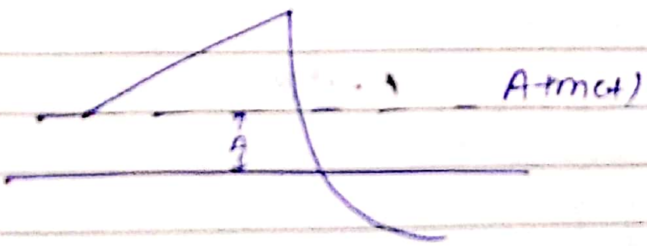


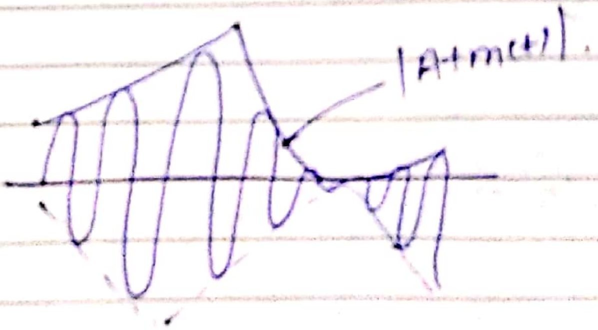




$A + m_t \geq 0$ for all t

$A + m(t) \ngtr 0$ for all t

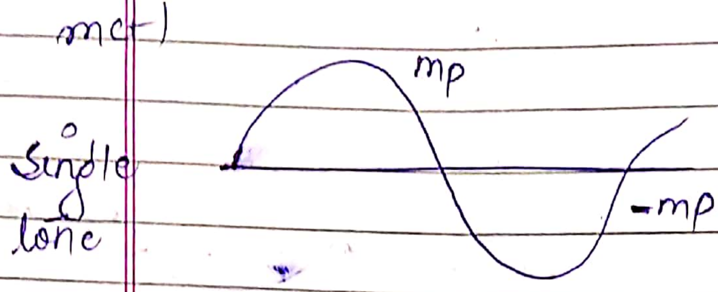




single frequency sinusoids → single tone
multiple " " → Multi" tone

$A + m(t) \geq 0$ for all t

→ envelope detection.

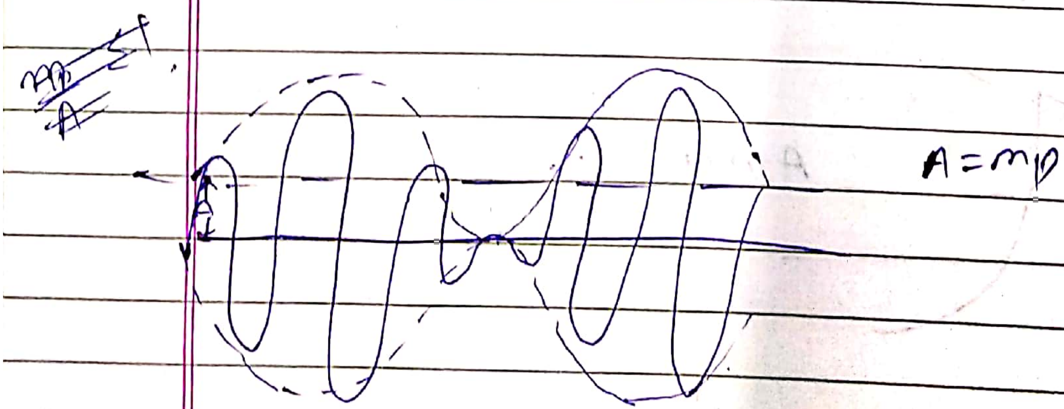


$A \geq mp$.
↳ Envelope detection.

Modulation Index (μ)
defined as

$$\mu = \frac{mp}{A}$$

$$0 \leq \mu \leq 1.$$

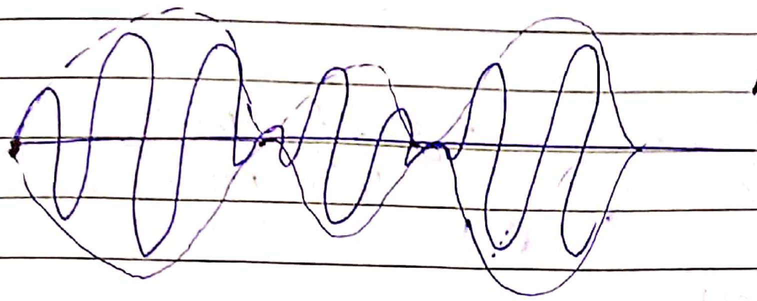


$\mu = 1$

100% modulation

$\mu > 1 \rightarrow$ coherant

overmodulation



$A < m_p$.

$$\varphi_{AM}(t) = A \cos \omega_c t + m(t) \cos \omega_c t$$

$$= [A + m(t)] \cos \omega_c t$$

If $m(t) = m_p \cos \omega_m t$

$$\varphi_{AM}(t) = A \cos \omega_c t + m_p \cos \omega_m t \cos \omega_c t$$

$$= A \cos \omega_c t + \mu A \cos \omega_m t \cos \omega_c t$$

Sideband and Carrier Power

In AM the carrier term does not carry any information hence the carrier power is wasted.

$$\varphi_{AM}(t) = \underbrace{\overset{P_c \uparrow}{A \cos \omega_c t}}_{\substack{\text{carrier} \\ \downarrow \\ \text{wasted}}} + \underbrace{\overset{P_s \uparrow}{m(t) \cos \omega_c t}}_{\text{sideband}}$$

for demodulation we add carrier (it makes demodulation easier)

$$P_c = \frac{A^2}{2}.$$

$$P_s = \frac{\overline{m^2(t)}}{2}.$$

$$\therefore \eta = \frac{P_s}{P_c + P_s}. \qquad \to \frac{\text{useful power}}{\text{total power}}$$

$$= \frac{\overline{m^2(t)}}{A^2 + \overline{m^2(t)}}$$

$$\overline{m^2(t)} = \frac{1}{T}\int_T (\mu A)^2 \cos^2\omega t \, dt$$

$$m_p = \mu A$$

$$= \frac{\mu^2 A^2}{2}.$$

$$\therefore \eta = \frac{\frac{\mu^2 A^2}{2}}{A^2 + \mu^2 A^2} = \frac{\mu^2}{2+\mu^2}$$

$$\eta\% = \frac{\mu^2}{2+\mu^2} \times 100\%$$

$\eta_{max} = 33\%$ at $\mu = 1$

$\eta_{min} = 0\%$ at $\mu = 0$.

~~Transmission~~

Transmitter → costly

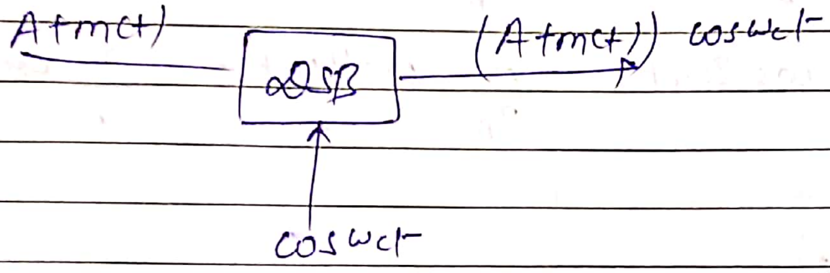
Receiver → Normal

$\eta = 25\%$ practically

Ex-Yes

Generation of A.M.

All DSB modulator can be used for Generation of AM.

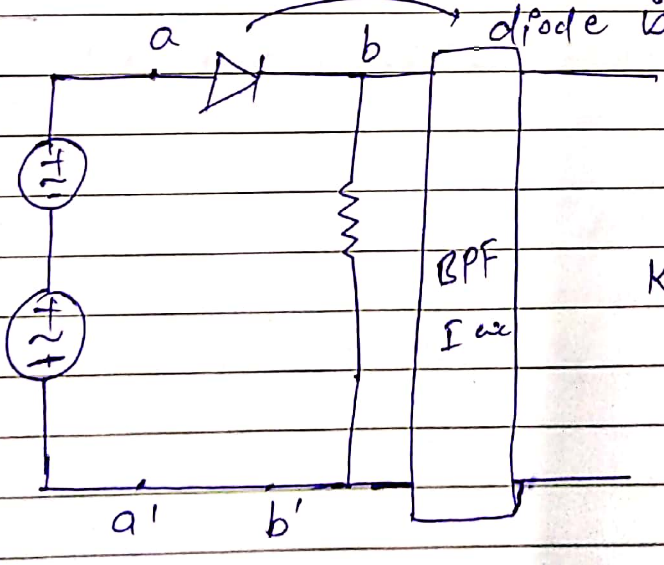


A → dc value
m(t) → signal

by using clamper A can be added to m(t).

We do not need to suppress carrier at the o/p.

diode is operated by carrier



We use PIN diode for fast switching here.

$A > m_p$.

$K \cos \omega_c t + b\, m(t) \cos \omega_c t.$

$$V_{bb'}(t) = [m(t) + A \cos \omega_c t]\, w(t)$$

$$= [m(t) + A \cos \omega_c t] \left[\frac{1}{2} + \frac{2}{\pi} [\cos \omega_c t \right.$$

$\frac{1}{3}\cos 3\omega_c t + \ldots)]$

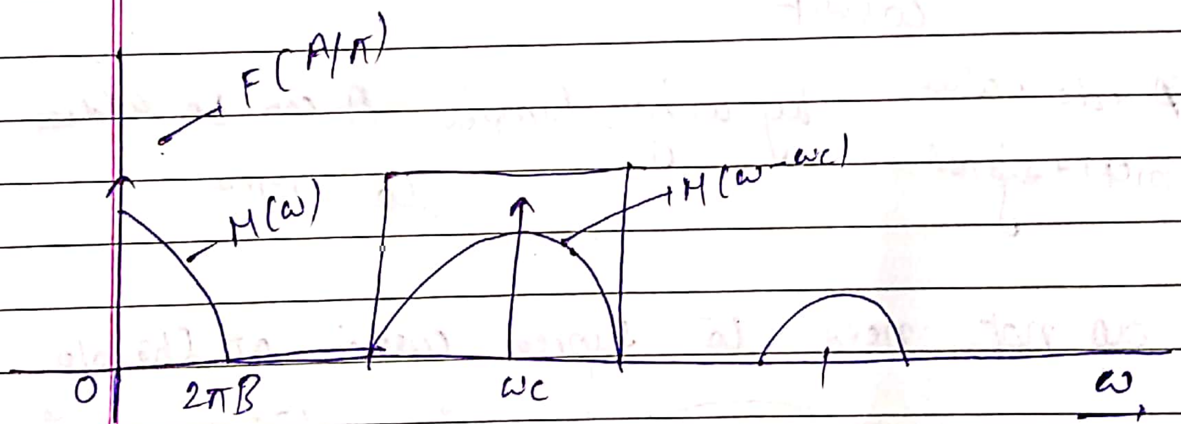
$$= \frac{1}{2} m(t) + \frac{A}{2}\cos\omega_c t + \frac{2}{\pi} m(t)\cos\omega_c t + \frac{2A}{\pi}\cos^2\omega_c t$$

$$- \frac{2}{3\pi} m(t)\cos 3\omega_c t - \frac{2A}{3\pi}\cos\omega_c t \cos 3\omega_c t + \ldots$$

LP   CS   MS   DC   ↑ freq.

$$= \frac{1}{2} m(t) + \frac{A}{2}\cos\omega_c t + \frac{2}{\pi} m(t)\cos\omega_c t + \frac{A}{\pi} + \frac{A}{\pi}\cos 2\omega_c t$$

$$- \frac{2}{3\pi} m(t)\cos 3\omega_c t - \frac{A}{3\pi}\cos 4\omega_c t - \frac{A}{3\pi}\cos 2\omega_c t + \ldots$$



Switching controlled by $A\cos\omega_c t$

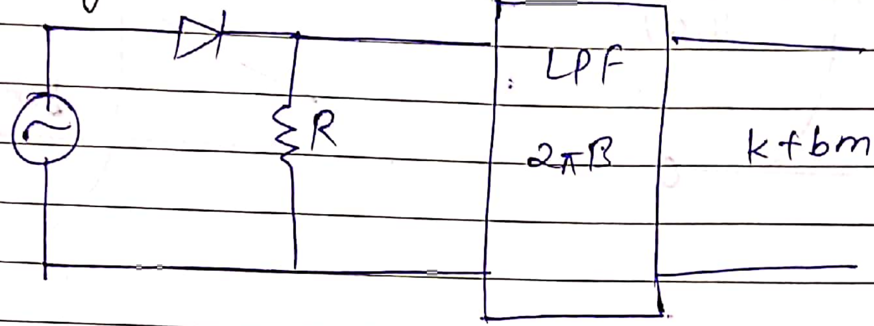
Demodulation of AM

$\text{Demodulation of AM} \to$ Rectifier type, Envelope detector

Rectifier type

signal

$[A+m(t)]\cos\omega_c t$



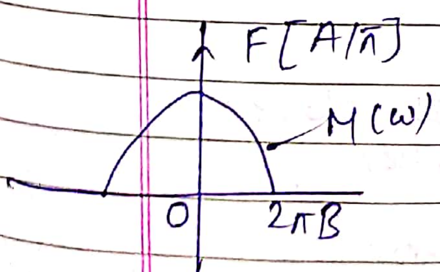
$A >> m_p$

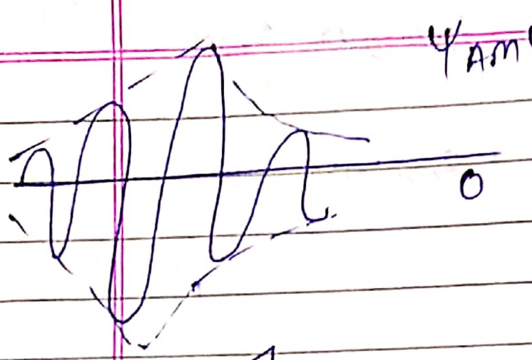
$$v_R(t) = [A+m(t)]\cos\omega_c t \times w(t)$$

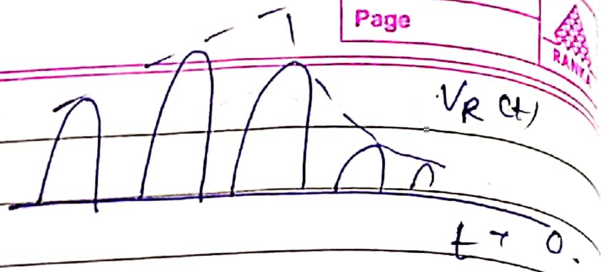
$$= [A+m(t)]\cos\omega_c t\left[\frac{1}{2}+\frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \dots\right)\right]$$

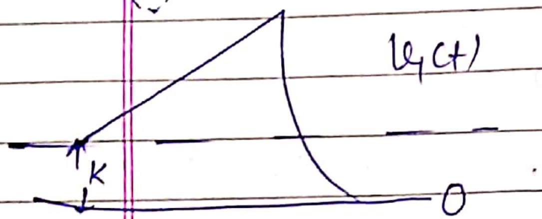
$$= \frac{1}{2}[A+m(t)]\cos\omega_c t + \frac{2}{\pi}[A+m(t)]\cos^2\omega_c t - \frac{2}{3\pi}[A+m(t)]\cos\omega_c t\cdot\cos 3\omega_c t + \dots$$

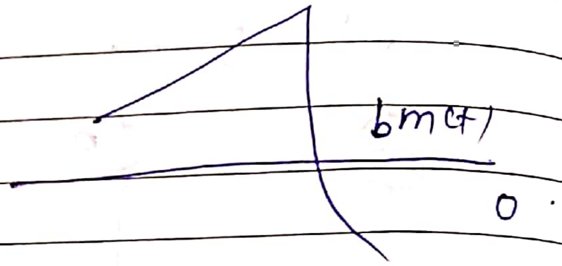
$$= \frac{[A+m(t)]}{2}\cos\omega_c t + \frac{1}{\pi}[A+m(t)] + \frac{1}{\pi}\cos 2\omega_c t - \frac{1}{3\pi}[A+m(t)]\cos 4\omega_c t - \frac{1}{3\pi}[A+m(t)]\cos 2\omega_c t + \dots$$





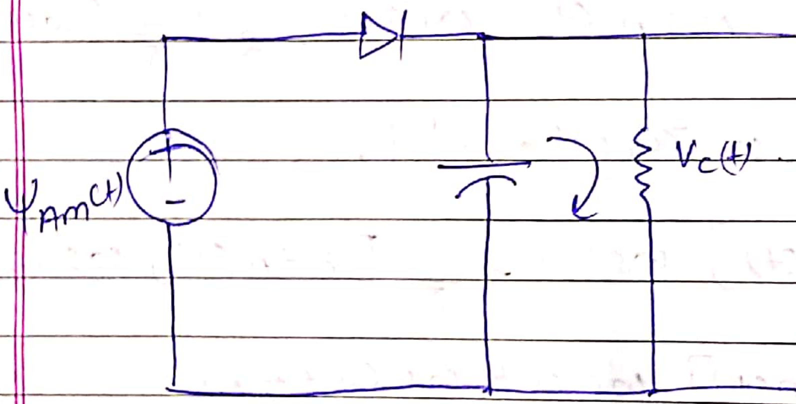


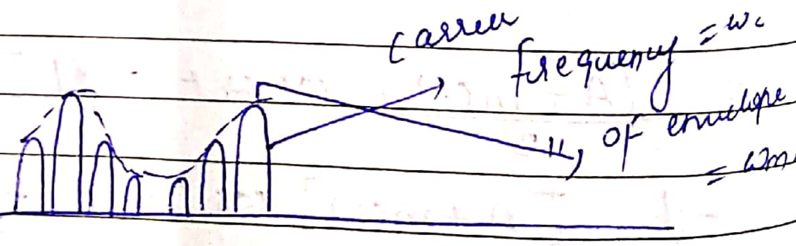




18th feb.

Envelope detector





$$\psi_{Am}(t) = A[1 + \mu m(t)]\cos \omega_c t.$$

RC → time constant. (time for which charge is stored in capacitor)

R ↑ → very slow discharging

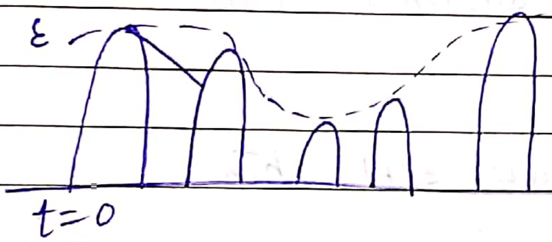
R ↓ → discharges very fast

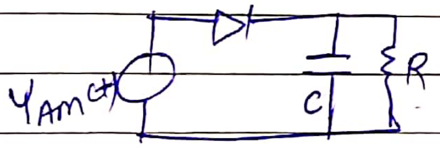
$RC > \frac{1}{\omega_c}$ (time period)

$$\boxed{\frac{1}{\omega_c} < RC < \frac{1}{\omega_m}}$$

Q. ~~Photon~~ for tone modulation, determine the upper limit of RC to ensure that the capacitor voltage follows the envelope

Ans





$\varphi_{AM}(t) = A[1+\mu\cos\omega_m t]\cos\omega_c t$

$$v_c(t) = E e^{-t/RC}$$

$$e^{-x} = \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{\underline{\lfloor n}}$$

$$e^{-t/RC} = 1 - \frac{t}{RC} + \frac{1}{2}\left(\frac{t}{RC}\right)^2 - \dots$$

$\because RC >> t$

$$e^{-t/RC} \approx 1 - \frac{t}{RC}$$

$$V_c(t) = E\left(1 - \frac{t}{RC}\right)$$

Slope of $V_c(t)$

$$\frac{d V_c(t)}{dt} = \frac{-E}{RC}$$

Envelope

$$E(t) = A[1 + \mu \cos w_m t]$$

$$\frac{dE(t)}{dt} = -A\mu \sin w_m t \times w_m$$

If $V_c(t)$ has to follow $E(t)$ then

$$\left|\frac{d V_c(t)}{dt}\right| \geq \left|\frac{dE(t)}{dt}\right|$$

$$\Rightarrow \frac{E(t)}{RC} \geq + A\mu w_m \sin w_m t$$

$$\frac{A[1 + \mu \cos w_m t]}{RC} \geq A\mu w_m \sin w_m t$$

$$\Rightarrow RC \leq \frac{1 + \mu \cos w_m t}{\mu w_m \sin w_m t}$$

Upper limit of RC.

$$\frac{d}{dt} \frac{1 + \mu \cos w_m t}{\mu w_m \sin w_m t} = 0$$

$$\mu w_m \sin w_m t (-\mu w_m \sin w_m t) - (1 + \mu \cos w_m t)(\mu w_m^2 \cos w_m t) = 0$$

carrier + used for ease of transmission

$$\mu^2 \omega_m^2 \sin^2 \omega_m t = -\mu \omega_m^2 \cos \omega_m t + -\mu \omega_m^2 \cos^2 \omega_m t$$

$$-\mu \sin^2 \omega_m t = \cos \omega_m t + \mu \cos^2 \omega_m t + \mu$$

$$\boxed{\cos \omega_m t = -\mu}$$

$$\cancel{\mu \sin^2 \omega_m t = \cos \omega_m t \left(1 + \mu \cos \omega_m t\right)}$$

$$\boxed{\cos \omega_m t = -\mu}$$

$$RC \leq \frac{1 + \mu(-\mu)}{\mu \omega_m \sqrt{1-\mu^2}}$$

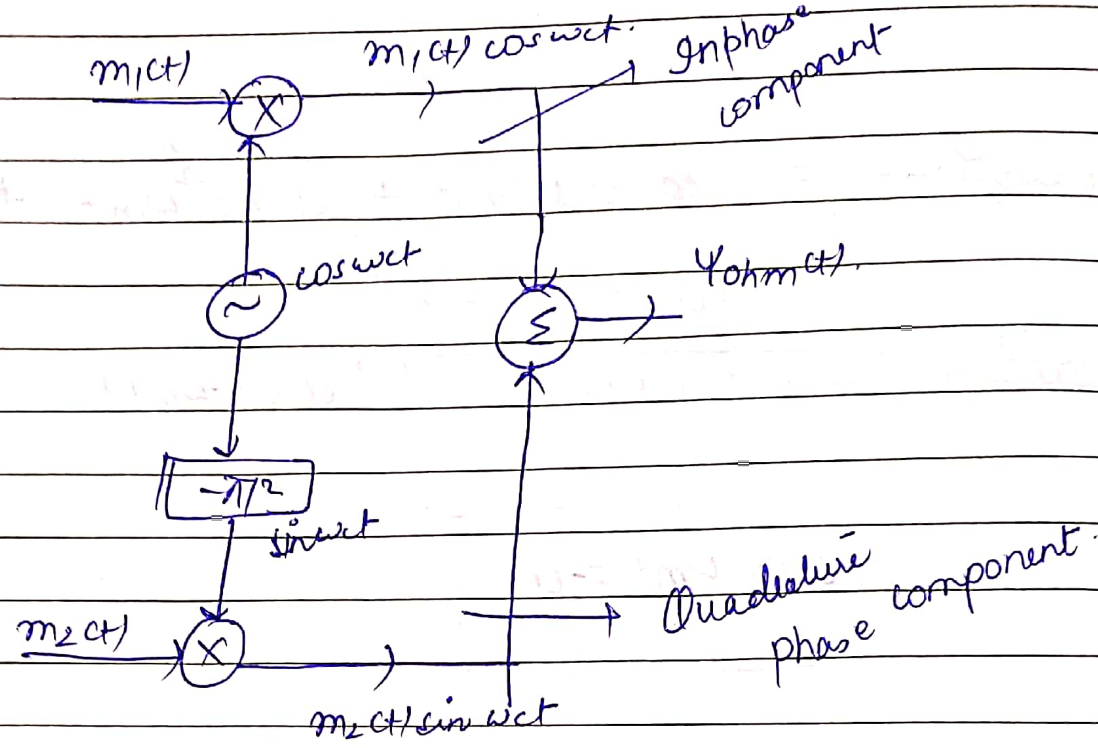
$$\leq \frac{1}{\mu \omega_m} \frac{1-\mu^2}{\sqrt{1-\mu^2}}$$

$$\Rightarrow RC \leq \frac{1}{\omega_m} \frac{\sqrt{1-\mu^2}}{\mu}$$

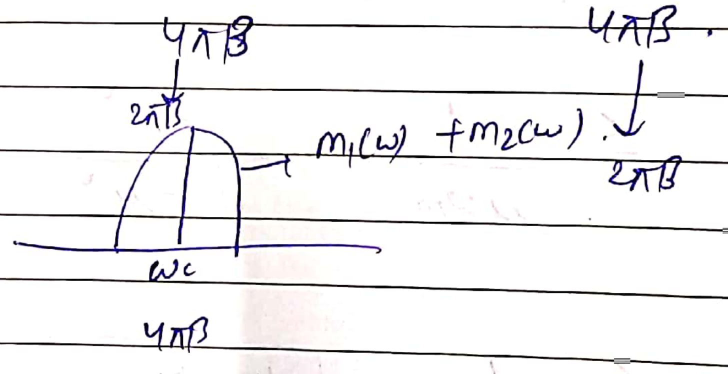
mc+1 $-2\pi\beta$

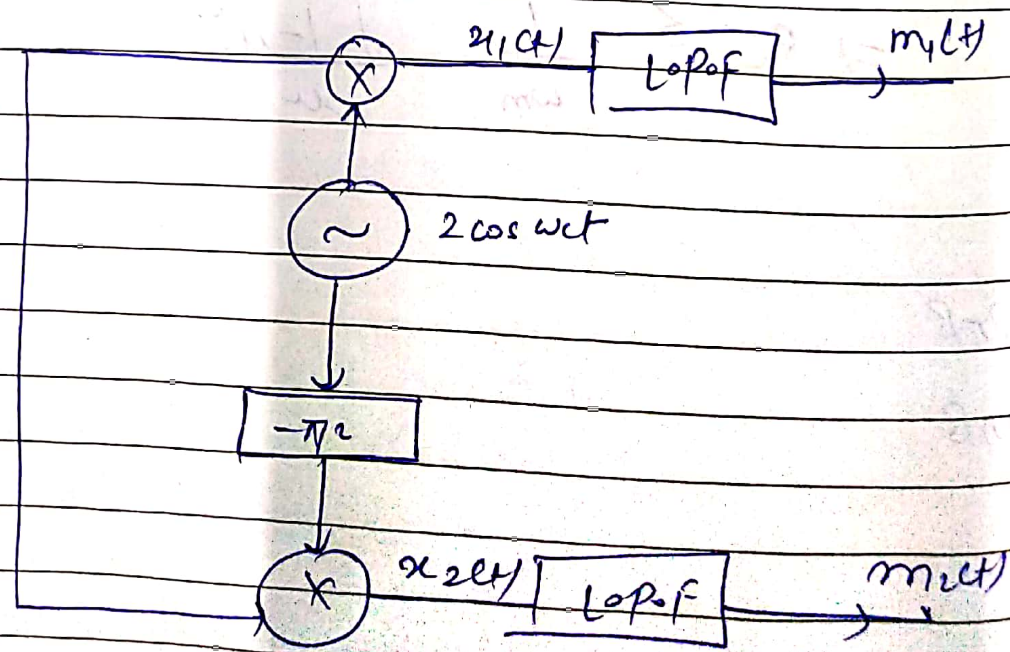
$4\pi\beta$

Quadrature Amplitude Modulation



$$\psi_{QAM}(t) = \underbrace{m_1(t)\cos\omega_c t}_{4\pi B} + \underbrace{m_2(t)\sin\omega_c t}_{4\pi B}$$





If phase error → there will be cross talk or cochannel interference

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$y_1(t) = m_1(t)\, 2\cos^2\omega_c t + m_2(t)\, 2\sin\omega_c t \cos\omega_c t$

$= m_1(t)\, [1 + \cos 2\omega_c t] + m_2(t) \sin 2\omega_c t$

$= m_1(t) + m_1(t)\cos 2\omega_c t + m_2(t)\sin 2\omega_c t$

If θ → phase error is there

$2\cos(\omega_c t + \theta)$.

$y_1(t) = m_1(t)\, 2\cos(\omega_c t + \theta)\cos\omega_c t + m_2(t)\, 2\sin\omega_c t \cos(\omega_c t + \theta)$

$\cancel{= m_1(t)[1 + \cos 2(\omega_c t + \theta)] + m_2(t)\, 2\sin 2(\omega_c t + \theta)}$

$\cancel{= m_1(t)\, 2\cos(\omega_c t + \theta)}$.

$= m_1(t)\cos\theta + m_1(t)\cos(2\omega_c t + \theta) - m_2(t)\sin\theta - m_2(t)\sin(2\omega_c t + \theta)$ → H.F component

O/P

$\therefore - m_1(t)\cos\theta - \underbrace{m_2(t)\sin\theta}$

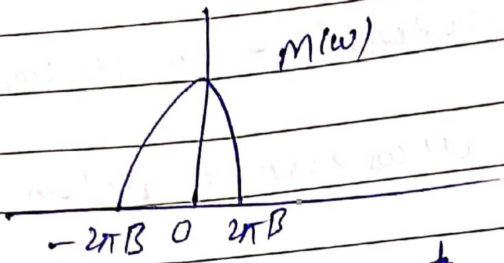
undesirable output

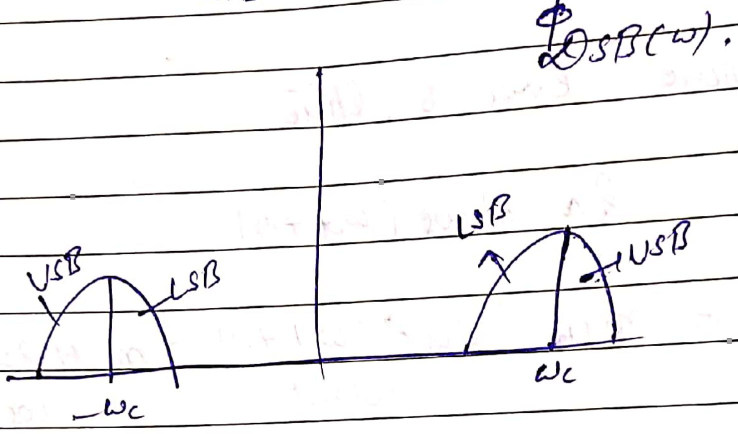
No way to suppress it

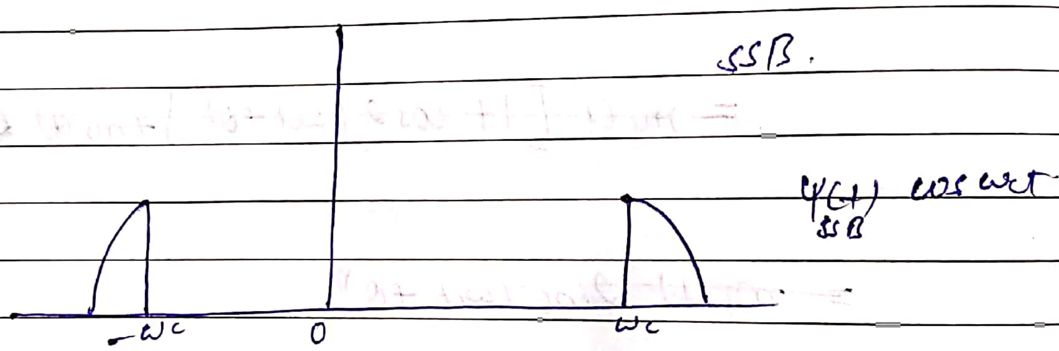
This phenomenon is known as cross talk or co-channel interference

19th feb

Single Sideband Modulation (SSB)

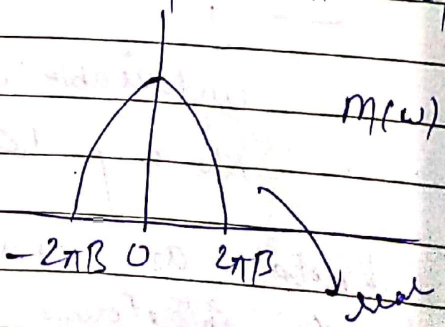


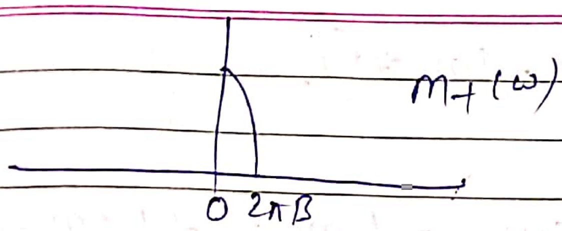




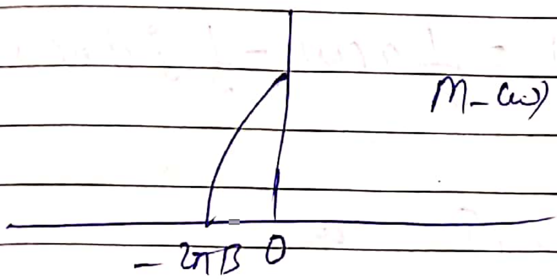
2πB → Bandwidth requirement reduced to half to that of DSB.

Time domain representation of SSB





$$M_+(\omega) = M(\omega)\,u(\omega) \quad \text{(I)}$$



$$M_-(\omega) = M(\omega)\,u(-\omega) \quad \text{(II)}$$

Let $f^{-1}[M_+(\omega)] = m_+(t)$ &

$f^{-1}[M_-(\omega)] = m_-(t)$.

Amplitude is even → real signal

otherwise complex.

∵ $|M_+(\omega)|$ & $|M_-(\omega)|$ are not even funcn of ω.

$m_+(t)$ and $m_-(t)$ are not real but complex.

∵ $M_+(\omega) + M_-(\omega) = \underset{\text{real}}{M(\omega)}$, $m_+(t)$ and $m_-(t)$ are complex conjugate.

$$m_+(t) = \frac{1}{2}\left[m(t) + j m_h(t)\right].$$

$u(\omega) = \frac{1}{2}(1 + sgn(\omega))$.

$m_-(t) = \frac{1}{2}[m(t) - j m_h(t)]$.

$\therefore F[m_+(t)] = M_+(\omega) = \frac{1}{2} M(\omega) + \frac{1}{2} j M_h(\omega)$ — (III).

$F[m_-(t)] = M_-(\omega) = \frac{1}{2} M(\omega) - \frac{1}{2} j M_h(\omega)$ — (IV).

Rewrite Equn (I) & (II) as

$$M_+(\omega) = \frac{1}{2} M(\omega) + \frac{1}{2} M(\omega) sgn(\omega). \quad \text{— (V)}$$

$$M_-(\omega) = \frac{1}{2} M(\omega) - \frac{1}{2} M(\omega) sgn(\omega). \quad \text{— (VI)}$$

from (III) & (V)

$$j M_h(\omega) = M(\omega)\, sgn(\omega).$$

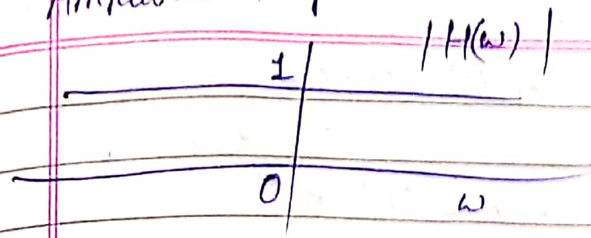
$$= -j\, M(\omega)\, sgn(\omega).$$

$$= M(\omega) [-j\, sgn(\omega)]$$

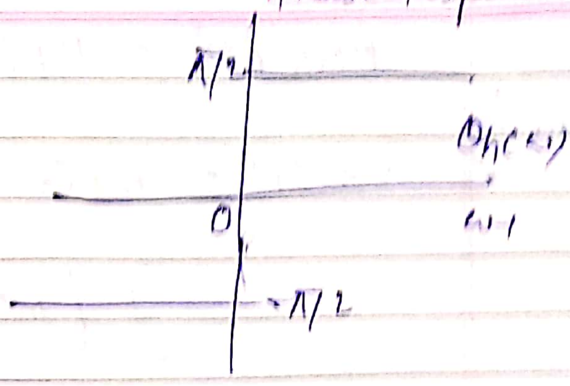
$$m(t) \longrightarrow \boxed{-j\,sgn(\omega)} \longrightarrow m_h(t)$$

$$H(\omega) = -j sgn(\omega). = \begin{cases} -j = e^{-j\pi/2} & \omega > 0 \\ j = e^{j\pi/2} & \omega < 0 \end{cases}$$

Amplitude Response



Phase Response



Phase is shifted by $\frac{\pi}{2}$ and magnitude is same

i.e. $-j\,\text{sgn}(\omega) \to$ phase shifter

$$\text{sgn}(t) \Leftrightarrow \frac{1}{j\omega}$$

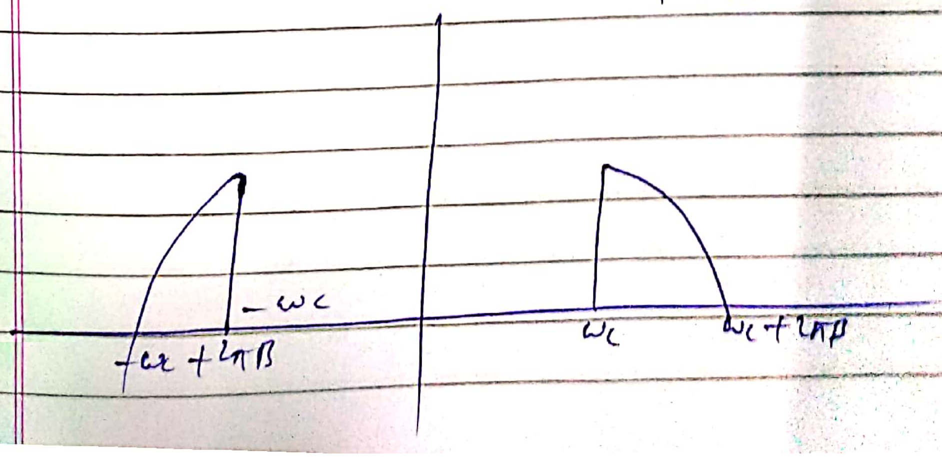
$$\frac{1}{\pi t} \Leftrightarrow -j\,\text{sgn}\,\omega$$

multiplication in ~~t~~ ω is convolution in t

$$F^{-1}[M_h(\omega)] = m_h(t) = m(t) * \frac{1}{\pi t}$$

$$= \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{m(\tau)}{t-\tau}\, d\tau$$

Hilbert Transform



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$$\Phi_{SSB}(\omega) = M_+(\omega-\omega_c) + M_-(\omega+\omega_c)$$

$$\therefore \varphi_{SSB}(t) = m_+(t)\, e^{j\omega_c t} + m_-(t)\, e^{-j\omega_c t}$$

$$= \frac{1}{2}\left[m(t) + j m_h(t)\right] e^{j\omega_c t} + \frac{1}{2}\left[m(t) - j m_h(t)\right] e^{-j\omega_c t}.$$

$$= m(t)\, \frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2} - m_h(t)\, \frac{e^{j\omega_c t} - e^{-j\omega_c t}}{2j}$$

$$= m(t)\cos\omega_c t - m_h(t)\sin\omega_c t \quad \text{— USB}$$

$$= m(t)\cos\omega_c t + m_h(t)\sin\omega_c t \quad \text{— LSB.}$$