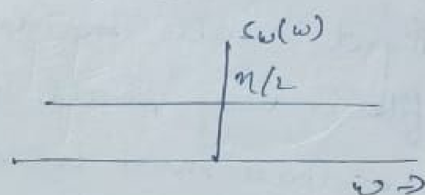
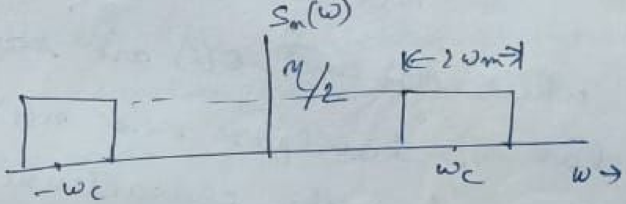
White Noise

White noise denoted by $n_w(t)$, is a random signal having a flat PSD over the frequency range $(-\infty, \infty)$. It has a uniform two sided PSD $S_{n_w}(\omega) = \eta/2$ watt/Hz.



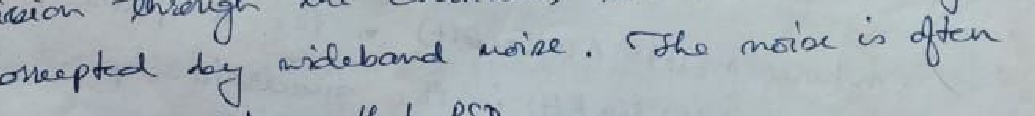
When white noise is passed through a bandpass filter, we obtain bandpass white noise. For example, if the filter has bandwidth $2\omega_m$ ~~over~~ centred about ω_c, we have bandpass white noise $n(t)$, whose PSD can be represented as



Bandpass noise.

In the process of modulation, a baseband signal is converted to a modulated signal with bandpass spectrum. During transmission through the channel, the modulated signal gets ~~converted~~ corrupted by wideband noise. The noise is often assume to be additive with a flat PSD.

At the receiver, the signal is first filtered to remove the out of band noise. ~~We assume that the centre frequency~~ of the Hence the output of the bandpass filter is the modulated signal contaminated by bandpass noise. We assume that the centre frequency of the filter is equal to the carrier frequency f_c and the bandwidth of the filter is equal to the signal bandwidth.



The modulated signal gets through the ideal bandpass filter unaltered, while the wideband noise at the filter input results in bandpass noise. If the bandwidth of the filter is very small in comparision to f_c, then the resulting bandpass noise is considered to be narrow band in nature.

The bandpass noise is modelled as a sinusoid with random time-varying amplitude and phase. i.e.

$$n(t) = A(t) \cos[\omega_c t + \theta(t)]$$

where $A(t)$ and $\theta(t)$ are randomly varying envelope and phase of bandpass noise $n(t)$. Therefore the bandpass noise, has the characteristics of both amplitude and angle modulation.

$$\text{Again } n(t) = A(t) \cos\theta(t) \cos\omega_c t - A(t) \sin\theta(t) \sin\omega_c t \quad \text{--- } *$$

$$= n_c(t) \cos\omega_c t - n_s(t) \sin\omega_c t$$

where $n_c(t) = A(t)\cos\theta(t)$ is the in-phase or I-component and $n_s(t) = A(t)\sin\theta(t)$ is the quadrature phase or Q-component.

The eq^n * is the quadrature representation of the bandpass noise. It can be shown that both $n_c(t)$ and $n_s(t)$ are lowpass signals, each bandlimited to ω_m rad/sec, and the powers of $n(t)$, $n_c(t)$ and $n_s(t)$ are identical.

$$\overline{n^2(t)} = \overline{n_c^2(t)} = \overline{n_s^2(t)}$$

Furthermore, both $n_c(t)$ and $n_s(t)$ have same PSD, which is related to the PSD of the bandpass noise $S_n(\omega)$ as

$$S_{n_c}(\omega) = S_{n_s}(\omega) = \begin{cases} S_n(\omega - \omega_c) + S_n(\omega + \omega_c), & |\omega| \leq \omega_m \\ 0, & \text{otherwise.} \end{cases}$$

Another important property of $n_c(t)$ and $n_s(t)$ is that they are uncorrelated with each other

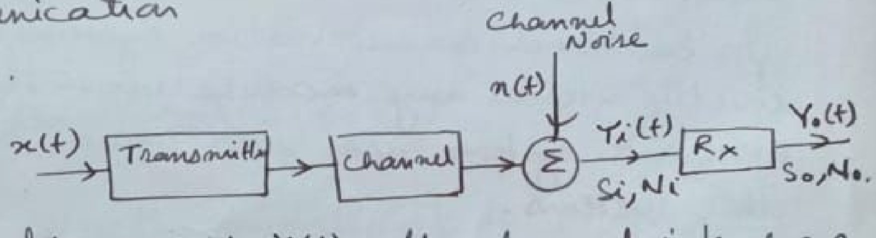
$$E[n_c(t)\, n_s(t)] = 0.$$

Noise in Analog Communication System

Introduction: The presence of noise degrades the performance of communication systems. The extent to which noise affects the performance of communication systems is measured by the output signal to noise power ratio or the probability of error.

Additive noise and Signal to noise ratio:

A schematic of a communication is shown in the diagram.



It is assume that the input of the transmitter is modeled by the random process $X(t)$, the channel introduces no distortion other than additive random noise, and the reciever is linear. At the riciever input, we have a signal mixed with noise. The signal and noise power at the receiver input are S_i and N_i, respectively. Since the reciever is linear, the reciever output $Y_o(t)$ is given by

$$Y_o(t) = X_o(t) + n_o(t)$$

where $X_o(t)$ and $n_o(t)$ are the signal and noise component at the reciever output, respectively.

We further take two assumptions about additive noise:

1) The noise is a zero mean white gaussian noise with power spectral density $S_{nn}(\omega) = \eta/2$

2) The noise is uncorrelated with $X(t)$.

Then we have,

$$E\left[Y_o^2(t)\right] = E\left[X_o^2(t)\right] + E\left[n_o^2(t)\right] = S_o + N_o$$

where $S_o = E\left[X_o^2(t)\right]$ and $N_o = E\left[n_o^2(t)\right]$ and they are average signal and noise power at the reciever output.

The output signal to Noise ratio $(S/N)_o$ is defined as

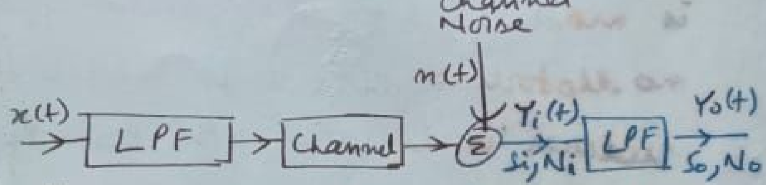
$$\left(\frac{S}{N}\right)_o = \left(\frac{S_o}{N_o}\right) = \frac{E[X_o^2(t)]}{E[n_o^2(t)]}$$

In analog communication systems, the quality of the received signal is determined by this parameter. This ratio is meaningful only when the receiver is linear.

Noise in baseband Communication System:

In baseband communication systems, the signal is transmitted directly without any modulation. This results obtained for baseband systems are serve as a basis for comparing with other systems.

For a baseband system, the receiver is a low pass filter that passes the message while reducing the noise at the output. Obviously, the filter should reject all noise frequency components that fall outside the message band. We assume that the ~~low~~ low pass filter is ideal with bandwidth $W (= 2\pi B)$.



It is assume that the message signal $x(t)$ is a zero-mean ergodic random ~~proce~~ process band-limited to W with power spectral density $S_{xx}(\omega)$. The channel is assumed to be distortionless over the message band so that

$$x_o(t) = x(t - t_d)$$

where t_d is the time delay of the system. The average output signal power S_o, is

$$S_o = E[x_o^2(t)] = E[x^2(t - t_d)]$$

$$= \frac{1}{2\pi}\int_{-W}^{W} S_{xx}(\omega)\, d\omega = S_x = S_i$$

where S_x is the average signal power and S_i is the signal power at the input of the receiver. The average ~~out~~ output noise power N_o is

$$N_o = E[n_o^2(t)] = \frac{1}{2\pi}\int_{-W}^{W} S_{nn}(\omega)\, d\omega$$

For the case of additive ~~noise~~ white noise, $S_{nn}(\omega) = \eta/2$

$$\therefore N_o = \frac{1}{2\pi}\int_{-W}^{W} \eta/2 \, d\omega = \eta \frac{W}{2\pi} = \eta B.$$

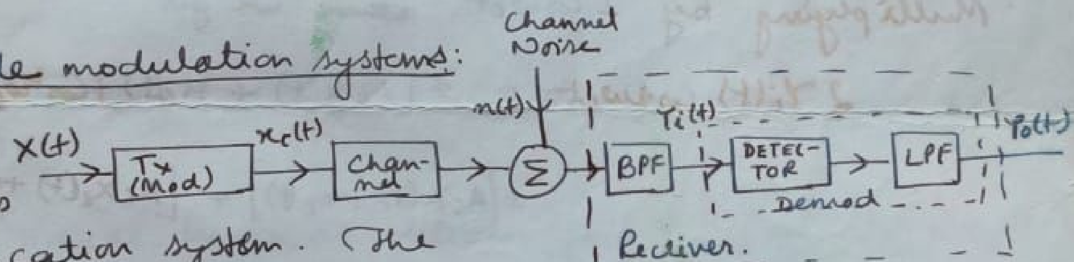
The output signal to Noise ratio is

$$\left(\frac{S}{N}\right)_o = \frac{S_o}{N_o} = \frac{S_i}{\eta B}$$

Let $\frac{S_i}{\eta B} = \gamma$ Then $\left(\frac{S}{N}\right)_o = \gamma$

The parameter γ is directly proportional to S_i. Hence, comparing various systems for the output SNR for a given S_i is the same as comparing these systems for the output SNR for a given γ.

Noise in amplitude modulation systems:



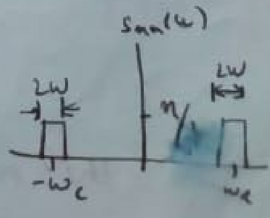
The diagram is of a continuous wave communication system. The reciever front end (RF/IF stage) is modeled as an ideal bandpass filter with a bandwidth $2W$ centered at ω_c. This bandpass filter, also known as a predetection filter, limits the ~~am~~ amount of noise outside the band that reaches the detector. The predetection bandpass filter produces

$$Y_i(t) = X_c(t) + n_i(t) \text{ where } n_i(t) \text{ is the narrow band noise.}$$

$$\therefore n_i(t) = n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t$$

If the power spectral density of $n(t)$, $S_{nn}(\omega) = \eta/2$ Then

$$E[n_c^2(t)] = E[n_s^2(t)] = E[n_i^2(t)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\eta}{2}\, d\omega$$



$$= \frac{2}{2\pi}\int_{\omega_c - W}^{\omega_c + W} \frac{\eta}{2}\, d\omega = \frac{\eta}{2\pi}\left[\omega_c + W - \omega_c + W\right]$$

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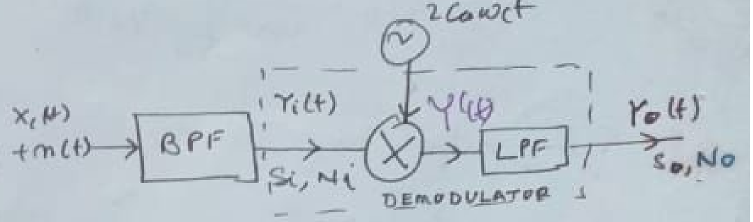
$$\Rightarrow E[m_i^2(t)] = \frac{\eta}{2\pi}\, 2W$$

$$= 2\eta\, \frac{W}{2\pi} = 2\eta B.$$ → ~~No~~ Input noise power at the detector.

A. Synchronous Detection

1. DSB System



In a DSB system, the transmitted signal $X_c(t) = A_c X(t) \cos \omega_c t$

~~It~~ The demodulator portion is shown in the figure. The input of the receiver is

$$Y_i(t) = A_c X(t) \cos\omega_c t + m_i(t)$$

$$= A_c X(t) \cos\omega_c t + m_c(t) \cos\omega_c t - m_s(t) \sin\omega_c t$$

$$= [A_c X(t) + m_c(t)] \cos\omega_c t - m_s(t) \sin\omega_c t$$

Multiplying by $2\cos\omega_c t$ we get

$$2Y_i(t) \cos\omega_c t = 2[A_c X(t) + m_c(t)] \cos^2\omega_c t - m_s(t)\, 2\cos\omega_c t \sin\omega_c t$$

$$= [A_c X(t) + m_c(t)] + [A_c X(t) + m_c(t)] \cos 2\omega_c t$$

$$- m_s(t) \sin 2\omega_c t$$

After passing through the LPF

$$Y_o(t) = A_c X(t) + m_c(t) = X_o(t) + m_o(t)$$

where $X_o(t) = A_c X(t)$ and $m_o(t) = m_c(t)$

We see that the output signal and noise are additive and quadrature noise component $m_s(t)$ has been rejected by the demodulator. Now.

$$S_o = E[X_o^2(t)] = E[A_c^2 X^2(t)] = A_c^2 E[X^2(t)] = A_c^2 S_x$$

$$N_o = E[m_c^2(t)] = E[m_s^2(t)] = E[m_i^2(t)] = 2\eta B.$$

$\therefore$ the output SNR is $\left(\frac{S}{N}\right)_o = \frac{S_o}{N_o} = \frac{A_c^2 S_x}{2\eta B}$

The input signal power

$$S_i = E\left[x_c^2(t)\right] = \cancel{A_c^2}\ \tfrac{1}{2} A_c^2 S_x$$

$$\Rightarrow \cancel{\alpha_d}\ S_x = 2\, S_i / A_c^2$$

$$\therefore \left(\frac{S}{N}\right)_o = \frac{A_c^2 \times \frac{2 S_i}{A_c^2}}{2\eta B} = \frac{S_i}{\eta B} = \gamma$$

$S_i = \cancel{\tfrac{1}{2}} E\left[A_c^2 x^2(t) \cos^2 \omega_c t\right]$

$= \frac{A_c^2}{2} E\left[x^2(t)\right] + \frac{A_c^2}{2} E\left[x^2(t) \cos 2\omega_c t\right]$

$= \frac{A_c^2}{2} S_x + \frac{A_c^2}{2} \times \frac{1}{2\pi} \int_{-\infty}^{\infty} S_x(\omega) \cos 2\omega_c t \, d\omega$

$= \frac{A_c^2}{2} S_x$

which indicates that in so far as noise is concerned, DSB with ideal synchronous detection has the same performance as the baseband system.

The SNR at the input of the detector is

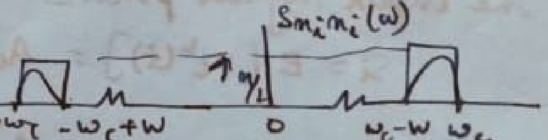
$$\left(\frac{S}{N}\right)_i = \frac{S_i}{N_i} = \frac{S_i}{2\eta B}$$

$$\therefore \frac{(S/N)_o}{(S/N)_i} = \alpha_d = 2$$

The ratio α_d is known as detector gain and is often used as a figure of merit for the demodulation.

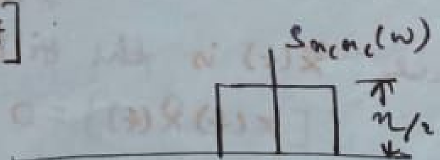
2) SSB-system:

The SSB signal $x_c(t)$ can be expressed as



$$x_c(t) = A_c\left[x(t) \cos \omega_c t + \hat{x}(t) \sin \omega_c t\right]$$

where $\hat{x}(t)$ denotes the Hilbert transform of $x(t)$.



The figures represent the lower sideband SSB signal and that the minimum bandwidth of the predetection filter is W for a ~~single side~~ single band signal.

The input to the receiver is

$$Y_i(t) = x_c(t) + n_i(t)$$

$$\cancel{= x(t)\cos\omega_c t}$$

$$= A_c[x(t)\cos\omega_c t + \hat{x}(t)\sin\omega_c t] + [n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t]$$

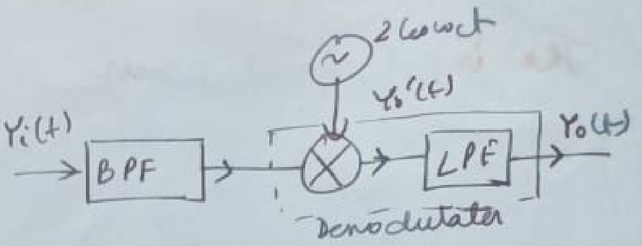


Fig: Receiver

$$= [A_c x(t) + n_c(t)]\cos\omega_c t + [A_c\hat{x}(t) - n_s(t)]\sin\omega_c t$$

Synchronous detection will give

$$Y_o'(t) = 2[A_c x(t) + n_c(t)]\cos^2\omega_c t + 2[A_c\hat{x}(t) - n_s(t)]\cos\omega_c t\sin\omega_c t$$

$$= [A_c x(t) + n_c(t)] + [A_c x(t) + n_c(t)]\cos 2\omega_c t + [A_c\hat{x}(t) - n_s(t)]\sin 2\omega_c t$$

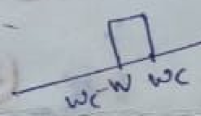
The output of the LPF.

$$Y_o(t) = A_c x(t) + n_c(t) = x_o(t) + n_o(t)$$

where $x_o(t) = A_c x(t)$ and $n_o(t) = n_c(t)$

The output signal power is

$$S_o = E[x_o^2(t)] = A_c^2 E[x^2(t)] = A_c^2 S_x$$



The output noise power $N_o = E[n_c^2(t)]$

$$= 2\frac{1}{2\pi}\frac{\eta}{2}\int_{\omega_c - W}^{\omega_c} d\omega = \eta\frac{W}{2\pi} = \eta B$$

Thus, the output SNR is

$$\left(\frac{S}{N}\right)_o = \frac{A_c^2 S_x}{\eta B}$$

The input signal power

$$S_i = E[x_c^2(t)] = A_c^2\left\{\frac{1}{2}E[x^2(t)] + \frac{1}{2}E[\hat{x}^2(t)]\right\}$$

$$= A_c^2 E[x^2(t)] = A_c^2 S_x$$

Since $\hat{x}(t)$ is the Hilbert transform of $x(t)$

$$E[x(t)\hat{x}(t)] = 0 \quad \text{and} \quad E[x^2(t)] = E[\hat{x}^2(t)]$$

∴ We get

$$\left(\frac{S}{N}\right)_o = \frac{S_i}{\eta B} = \gamma$$

This shows that, as noise is concern, SSB with ideal synchronous detection has the same performance as both DSB and base band system.

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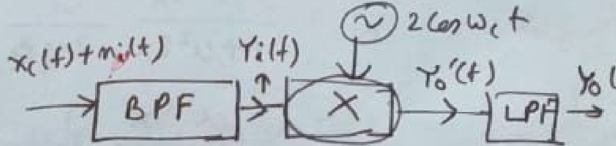
AM Systems:

In ordinary AM (or simply AM) systems, AM signals can be demodulated by synchronous detector or by envelope detector. The modulated signal in an AM system has the form

$x_c(t) = A_c[1+\mu x(t)]\cos\omega_c t$ where μ is the modulation index of AM signal and $\mu \le 1$ and $|x(t)| \le 1$.

The output of the synchronous detector is



$$Y_o'(t) = [x_c(t) + n_i(t)]\, 2\cos\omega_c t$$

$$= 2[A_c + \mu A_c x(t) + n_c(t)]\cos^2\omega_c t \mp 2\, n_s(t)\sin\omega_c t\cos\omega_c t$$

$$= [A_c + \mu A_c x(t) + n_c(t)] + [A_c + \mu A_c x(t) + n_c(t)]\cos 2\omega_c t - n_s(t)\sin 2\omega_c t$$

The detector has an ~~di~~ ideal dc suppressor and the output of the LPF is

$$Y_o(t) = \mu A_c x(t) + n_c(t) = x_o(t) + n_o(t)$$

where $x_o(t) = \mu A_c x(t)$ and $n_o(t) = n_c(t)$

The output signal power $S_o = E[x_o^2(t)]$

$$= \mu^2 A_c^2\, E[x^2(t)]$$

$$= \mu^2 A_c^2\, S_x$$

~~and~~ and output noise power

$$N_o = E[n_o^2(t)] = 2\eta B.$$

$\therefore$ the output signal to noise ratio

$$\left(\frac{S}{N}\right)_o = \frac{S_o}{N_o} = \frac{\mu^2 A_c^2 S_x}{2\eta B}$$

E is a linear operator

The input signal power

$$S_i = E[x_c^2(t)] = E[A_c^2\cos^2\omega_c t + A_c^2\mu^2 x^2(t)\cos^2\omega_c t]$$

$$= E[A_c^2\{1+\mu x(t)\}^2\cos^2\omega_c t]$$

$$= \frac{1}{2} E[A_c^2\{1+\mu^2 x^2(t)\}^2] + \frac{1}{2} A_c^2 E[\{1+\mu^2 x^2(t)\}^2 \cos 2\omega_c t]$$

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$\Rightarrow S_i = \frac{1}{2} A_c^2 E\left[1+\mu^2 x^2(t)\right]$

$\quad = \frac{1}{2} A_c^2 (1+\mu^2 S_x)$

$\Rightarrow A_c^2 = \dfrac{2 S_i}{1+\mu^2 S_x}$

We have $S_o = A_c^2 \mu^2 S_x = \dfrac{2\mu^2 S_x}{1+\mu^2 S_x} S_i$

$\therefore \dfrac{S_o}{N_o} = \dfrac{2\mu^2 S_x}{1+\mu^2 S_x} \dfrac{S_i}{2\eta B}$

$\quad = \dfrac{\mu^2 S_x}{1+\mu^2 S_x} \gamma$

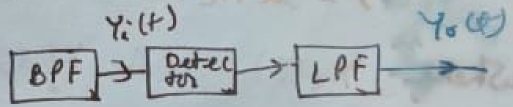
Because $\mu^2 S_x \leq 1$, we have $\left(\frac{S}{N}\right)_o \leq \frac{\gamma}{2}$

which indicates the the output SNR in AM is atleast 3 dB worse than that in DSB and SSB system.

B. Envelope Detection and Threshold Effect

An ordinary AM signal is demodulated by envelope detection.

The input to the detector

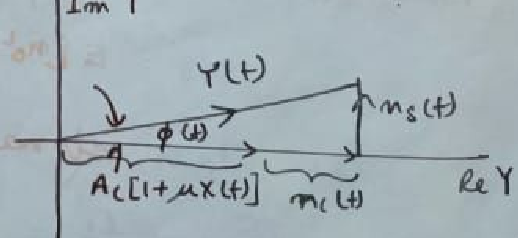


is $\quad Y_i(t) = x_c(t) + n_i(t)$

$\quad = A_c[1+\mu x(t)] \cos\omega_c t + n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t$

$\quad = \left[A_c\{1+\mu x(t)\} + n_c(t)\right]\cos\omega_c t - n_s(t)\sin\omega_c t$

The effect of the noise is analyzed by considering the phasor representation of $Y_i(t)$



$Y_i(t) = \mathrm{Re}\left[Y(t) e^{j\omega_c t}\right]$

Phasor representation of $Y_i(t)$

where $Y(t) = A_c[1+\mu x(t)] + n_c(t) + j n_s(t)$

From the figure we can write

$Y_i(t) = v(t)\cos[\omega_c t + \phi(t)]$ where

$v(t) = \sqrt{\{A_c[1+\mu x(t)] + n_c(t)\}^2 + n_s^2(t)}$ and $\phi(t) = \tan^{-1}\dfrac{n_s(t)}{A_c[1+\mu x(t)]+n_c(t)}$

Case - I Large SNR (Signal Dominance) case

When $(S/N)_i \gg 1$, $A_c[1+\mu X(t)] \gg n_i(t)$, hence $A_c[1+\mu X(t)] \gg n_c(t)$ and $n_s(t)$ for almost all t.

Under this condition the envelope ~~is~~ $v(t)$ is approximated as

$$v(t) \approx \sqrt{\{A_c[1+\mu X(t)] + n_c(t)\}^2 + 0^2}$$

$$\approx A_c[1+\mu X(t)] + n_c(t)$$

An ideal envelope detector reproduce the envelope $v(t)$ minus the dc component, so

$$Y_0(t) = A_c \mu X(t) + n_c(t)$$

which is ~~ideal to~~ identical to synchronous detector.

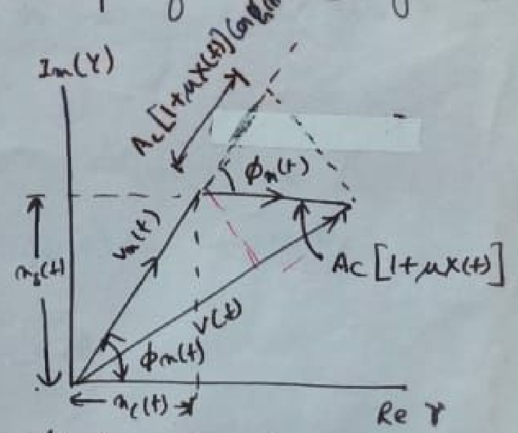
The output SNR is then given by

$$\left(\frac{S}{N}\right)_o = \frac{\mu^2 S_x}{1+\mu^2 S_x} \gamma$$

Therefore, for AM, when $(S/N)_i \gg 1$, the performance of the envelope detector is identical to that of the synchronous detector

Small - SNR (Noise Dominance) Case

~~Then When SNR~~ when $(S/N)_i \ll 1$, the envelope of the resultant signal is primarily dominated by the envelope of noise signal.



The envelope of the resultant signal is approximated by

$$v(t) \approx V_n(t) + A_c[1+\mu X(t)]\cos\phi_n(t)$$

where $V_n(t)$ and $\phi_n(t)$ are the envelope and the phase of noise $n_i(t)$.

The above eqn indicates that the output contains no term proportional to $X(t)$ and that noise is multiplicative. The signal is multiplied by noise in the form of $\cos\phi_n(t)$, which is random. Thus, the message signal is badly multilated, and its information has been lost. Under these ~~circum~~ circumstances it is meaningless to talk about output SNR.

The loss or multilation of the signal message at low predetection SNR is called the threshold effect. The name comes about because there is some value of $(S/N)_i$, ~~above~~ ~~above~~ below which system performance deteriorates rapidly. The threshold occurs when ~~α~~ $(S/N)_i$ is about 10dB or less.

Noise in Angle modulation system

In an angle ~~do~~ modulated system the transmitted signal $x_c(t)$ has the form

$$X_c(t) = A_c \cos[\omega_c t + \phi(t)]$$

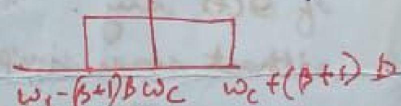
where $\phi(t) = \begin{cases} k_p\, x(t) & \text{for PM} \\ k_f \int_{-\infty}^{t} x(t) & \text{for FM} \end{cases}$

$$X_c(t) + m(t) \rightarrow \boxed{BPF} \rightarrow \boxed{\text{Limiter}} \xrightarrow[S_i,\,N_i]{Y_i(t)} \boxed{\text{Discrimi-nator}} \rightarrow \boxed{LPF} \xrightarrow[S_o,\,N_o]{Y_o(t)}$$

Fig: Analog demodulation system.

The bandwidth of the predetection filter $B_T \approx 2(\beta+1)B$ where β is the deviation ratio and B is the bandwidth of the message signal. The detector input is

Narrow band



$$Y_i(t) = x_c(t) + m_i(t)$$
$$= A_c \cos[\omega_c t + \phi(t)] + m_i(t)$$

The carrier amplitude remains constant, therefore

$$S_i = E[x_c^2(t)] = \frac{1}{2}A_c^2$$

and $N_i = E[m_i^2(t)]$ ~~$2\eta B_T$~~ $= 2\eta B$ $= 2\frac{1}{2\pi}\int_{-W}^{W}\frac{\eta}{2}\,d\omega$

$2\times\frac{1}{2\pi}\times\frac{\eta}{2}\int_{\omega_c-(\beta+1)B}^{\omega_c+(\beta+1)B} d\omega = \eta B_T = 2\eta B$

Hence $\frac{S_i}{N_i} = \frac{A_c^2}{2\eta B_T}$ which is independent of $x(t)$. if β is small

The above expression is known as carrier to noise ratio (CNR)

Because $m_i(t)$ is narrowband, we write

$$m_i(t) = v_n(t)\cos[\omega_c t + \phi_n(t)]$$

where $v_n(t)$ is Rayleigh distributed and $\phi_n(t)$ is uniformly distributed in $(0, 2\pi)$. Then $Y_i(t)$ can be written as.

~~$Y_i(t) = v(t)\cos[\omega_c t + \theta(t)]$~~

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$$Y_i(t) = A_c \cos[\omega_c t + \phi(t)] + v_n(t)[\cos\omega_c t + \phi_n(t)]$$

$$= A_c \cos\omega_c t \cos\phi(t) - A_c \sin\omega_c t \sin\phi(t)$$
$$+ v_n(t)\cos\omega_c t \cos\phi_n(t) - v_n(t)\sin\omega_c t \sin\phi_n(t)$$

$$= [A_c \cos\phi(t) + v_n(t)\cos\phi_n(t)]\cos\omega_c t - [A_c \sin\phi(t) + v_n(t)\sin\phi_n(t)]\sin\omega_c t$$

$$= V(t)\cos[\omega_c t + \theta(t)]$$

where $V(t) = \sqrt{[A_c \cos\phi(t) + v_n(t)\cos\phi_n(t)]^2 + [A_c \sin\phi(t) + v_n \sin\phi_n(t)]^2}$

and $\theta(t) = \tan^{-1} \dfrac{A_c \sin\phi(t) + v_n(t)\sin\phi_n(t)}{A_c \cos\phi(t) + v_n(t)\cos\phi_n(t)}$

The limiter suppresses any amplitude variation $v(t)$. Hence in angle modulation, SNR's are derived from consideration of $\theta(t)$ only. The expression for $\theta(t)$ is too complicated for analysis without some simplification. The detector is assumed to be ideal. The output of the detector is

$$Y_o(t) = \begin{cases} \theta(t) & \text{for PM} \\ \dfrac{d\theta(t)}{dt} & \text{for FM} \end{cases}$$

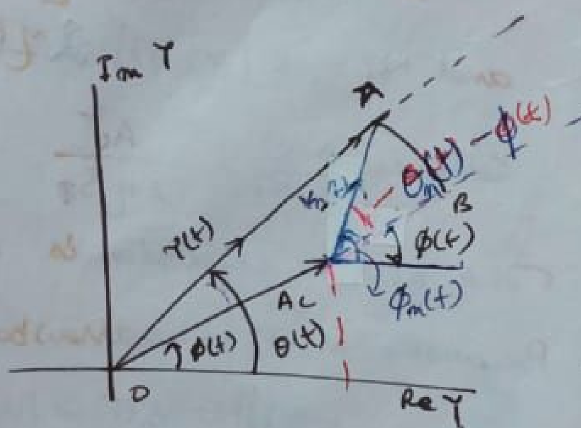
A. Signal Dominance Case

A phasor diagram is obtained from

$Y_i(t) = \mathrm{Re}[Y(t)e^{j\omega_c t}]$

where $Y(t) = A_c e^{j\phi(t)} + v_n(t)e^{j\phi_n(t)}$

and $v_n(t) << A_c$ for almost all t.



From the figure the length of the arc AB

$$L = Y(t)[\theta(t) - \phi(t)] \quad \text{—(1)}$$

and $Y(t) \approx A_c + v_n(t)\cos[\phi_n(t) - \phi(t)] \approx A_c$

$\cancel{L \approx A}$ $L \approx v_n(t)\sin[\phi_n(t) - \phi(t)]$

$\therefore v_n(t)\sin[\phi_n(t) - \phi(t)] = Y(t)[\theta(t) - \phi(t)]$ Putting in (1)

$\therefore\ \theta(t) = \phi(t) + \frac{v_n(t)}{r(t)} \sin[\phi_n(t) - \phi(t)]$, but $r(t) \approx A_c$

For the purpose of computing an output SNR, replacing $\phi_n(t) - \phi(t)$ by ~~ϕt~~ with $\phi_n(t)$ will not affect the result as the ~~ess~~ ensemble avarages $\phi_n - \phi$ differs from ϕ_n only by a shift of the mean value. Thus

$$\theta(t) \approx \phi(t) + \frac{v_n(t)}{A_c} \sin \phi_n(t)$$

$$= \phi(t) + \frac{n_s(t)}{A_c}$$

$\therefore$ The detector output is

$$Y_o(t) = \theta(t) = k_p x(t) + \frac{n_s(t)}{A_c} \text{ for PM}$$

$$Y_o(t) = \frac{d\theta(t)}{dt} = k_f x(t) + \frac{n_s'(t)}{A_c} \text{ for FM.}$$

i) $(S/N)_o$ in PM System.

$$S_o = E[k_p^2 x^2(t)] = k_p^2 E[x^2(t)] = k_p^2 S_x$$

$$N_o = E\left[\frac{n_s^2(t)}{A_c^2}\right] = \frac{1}{A_c^2} E[n_s^2(t)] = \frac{1}{A_c^2}\frac{1}{2\pi}\int_{-w}^{w} S_n \, d\omega = \frac{2\eta B}{A_c^2}$$

$$\therefore \left(\frac{S}{N}\right)_o = \frac{k_p^2 S_x A_c^2}{2\eta B} = k_p^2 S_x \times \frac{A_c^2}{2\eta B}$$

Then ~~S_{i}~~ $\gamma = \frac{S_i}{2\eta B} = \frac{A_c^2}{2\eta B}$

$$\therefore \left(\frac{S}{N}\right)_o = k_p^2 S_x \gamma$$

ii) $(S/N)_o$ in FM system.

$$S_o = E[k_f^2 x^2(t)] = k_f^2 S_x$$

$$N_o = E\left[\frac{n_s'^2(t)}{A_c^2}\right] = \frac{1}{A_c^2} E[n_s'^2(t)]$$

The power spectral density of $n_s'(t)$ is given by

$$S_{n_s' n_s'}(\omega) = \omega^2 S_{n_s n_s}(\omega) = \begin{cases} \omega^2 \eta & \text{for } |\omega| < w (= 2\pi B) \\ 0 & \text{otherwise.} \end{cases}$$

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Then,

$$N_0 = \frac{1}{A_c^2} \times \frac{1}{2\pi} \int_{-W}^{W} \omega^2 \eta \, d\omega = \frac{2}{3} \frac{\eta}{A_c^2} \frac{W^3}{2\pi} \qquad \text{— (1A)}$$

$$\therefore \left(\frac{S}{N}\right)_0 = \frac{3 A_c^2 k_f^2 (2\pi) S_x}{2\eta W^3}$$

$$= 3\left(\frac{k_f^2 S_x}{W^2}\right)\left(\frac{A_c^2}{2\eta B}\right)$$

$$= 3\left(\frac{k_f^2 S_x}{W^2}\right)\gamma$$

$$= 3\left(\frac{\Delta\omega}{W}\right)^2 S_x \gamma$$

$$= 3\beta^2 S_x \gamma$$

where β is the deviation ratio.

Equation ~~8~~ 1A shows that the output noise power is inversely proportional to the mean carrier power $A_c^2/2$ in FM. This effect of a decrease in output noise power as the carrier power increases is called noise quieting.