Signals

13th January

independent var.
dependent var.

Unit Step Signal

$$u(t) = 1 \quad \text{for } t \geq 0$$
$$= 0 \quad \text{otherwise}$$

Used for defining other ~~functs~~ signals (intervals)

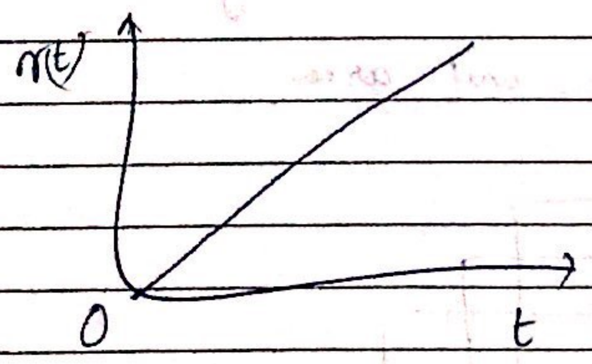
$$x(t) = 2^t u(t) \qquad (0, \infty)$$

$$x(t) = 2^t \; u(t-5)$$
$$(5, \infty)$$

Ramp Signal

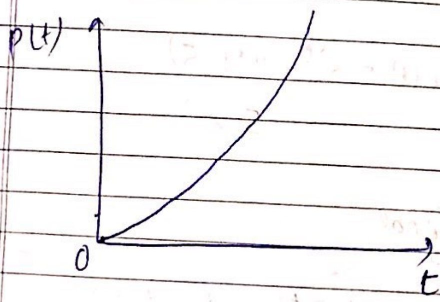
~~$x(t)$~~ $r(t) = t \quad \text{for } t \geq 0$

$$= 0 \quad \text{otherwise}$$



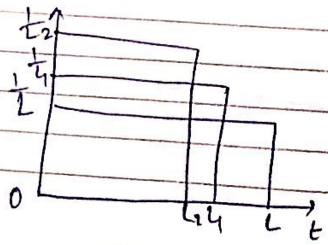
Relation
int. of $u(t)$ is $r(t)$.
dif. of $r(t)$ is $u(t)$.

Parabolic Signal
$$P(t) = \int t\,dt$$
$$= \frac{t^2}{2} \quad \text{for } t \geq 0.$$



Singularity function

Unit Impulse function (a singularity function)
we manage unit area



$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1. \quad \text{(otherwise } = 0)$$
$t \to 0$.

Q. $\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = ?$

unit impulse
defined only when $t \to 0$.

so,
$$\int_{-\infty}^{\infty} x(0)\,\delta(t)\,dt$$
$$= x(0)\int_{-\infty}^{\infty} \delta(t)\,dt$$
$$= x(0).$$

Q. $\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = ?$

$= x(t_0)$ ← Shifting Property

* $x(t)$
$$x(t_0) = \int_{-\infty}^{\infty} x(t_0)\,\delta(t-t_0)\,dt$$
$x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau$ ← convolution Integral

At 1,

$$x(1) = \int_{-\infty}^{\infty} x(\,)\ \delta(1-\tau)\,d\tau.$$

$\therefore \tau = 1$

$$x(1) = \int_{-\infty}^{\infty} x(1)\ \delta(1-\tau)\,d\tau.$$

Any signal can be expressed in terms of unit impulse as a weighted sum.

Property

1) $x(t)\ \delta(t) = x(0)\,\delta(t)$ (Multiplication Property)

$$\int_{-\infty}^{\infty} x(t)\ \delta(t)\ \psi(t)\,dt$$

$$= \int_{-\infty}^{\infty} x(0)\ \psi(0)\ \underset{t\to 0}{\delta(t)\,dt}$$

$$= x(0)\ \psi(0) \int_{-\infty}^{\infty} \delta(t)\,dt$$

$$= x(0) \int_{-\infty}^{\infty} \psi(t)\ \delta(t)\,dt$$

Q. $x(t)\ \delta(t-t_0) = x(t_0)\,\delta(t-t_0)$

$$\int_{-\infty}^{\infty} x(t)\ \delta(t-t_0)\ \psi(t)\,dt$$

$$= \int_{-\infty}^{\infty} x(t_0)\ \psi(t_0)\ \underset{t\to t_0}{\delta(t-t_0)\,dt}$$

$$= x(t_0)\ \psi(t_0) \int_{-\infty}^{\infty} \underset{t\to t_0}{\delta(t-t_0)\,dt}.$$

$$= x(t_0) \int_{-\infty}^{\infty} \psi(t)\ \delta(t-t_0)\,dt$$

$$= \int_{-\infty}^{\infty} x(t_0)\ \psi(t)\ \delta(t-t_0)\,dt$$

II) $\delta(at) = \frac{1}{|a|}\,\delta(t)$ (Scalling Property)

$$\int_{-\infty}^{\infty} \delta(at)\ x(t)\,dt$$

$at = \tau.$ $\quad a > 0.$

$t = \frac{\tau}{a}$

$$= \frac{1}{a} \int_{-\infty}^{\infty} \delta(\tau)\, x(\tau/a)\, d\tau$$

$$= \frac{1}{a} \int_{-\infty}^{\infty} x(\tau/a) \quad \delta(\tau) d\tau$$

$$\tau \to 0.$$

$$= \frac{1}{a} x(0)$$

$$\int_{-\infty}^{\infty} \delta(at)\, x(t)\, dt$$

$$at = \tau$$

$$= \frac{1}{a} \int_{-\infty} \delta(-\tau)\, x\left(\frac{\tau}{a}\right) d\tau$$

$$\Rightarrow \quad \delta(\tau) = \delta(-\tau)$$

$$= \frac{1}{a} \int_{-\infty}^{\infty} \delta(\tau)\, x\left(\frac{\tau}{a}\right) d\tau.$$

$$= \frac{1}{a} x(0).$$

$$\therefore \int_{-\infty}^{\infty} \delta(at)\, x(t)\, dt = \frac{1}{|a|} x(0)$$

$$= \frac{1}{|a|} \int_{-\infty}^{\infty} x(t)\, \delta(t)\, dt.$$

$$= \int_{-\infty}^{\infty} x(t) \frac{1}{|a|} \delta(t)\, dt$$

$$\delta(at) = \frac{1}{|a|} \delta(t).$$

20th January.

Q. $x(t) = 1 - \frac{|t|}{a} \qquad |t| \le a$

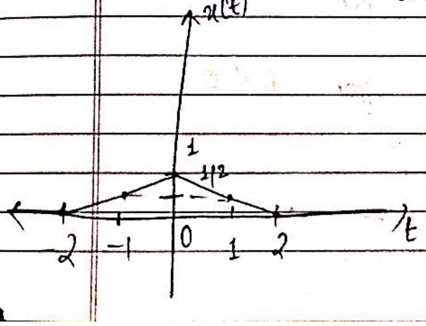
$\quad = 0 \qquad |t| > a$

$a > 0.$

$$\boxed{-a \le t \le a \qquad 1 - \frac{|t|}{a}}$$

$$\boxed{\begin{array}{ll} -\infty < t < -a & 0 \\ a < t < \infty & \end{array}}$$

$a = 2$



Triangular wave form

Exponential

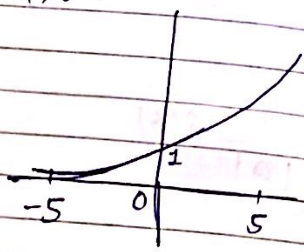
$x(t) = e^{at}$.

$a>0$

$a<0$.

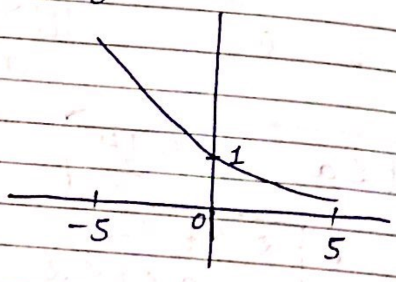
$a>0$.

$-5 \le t \le 5$



$a<0$

$-5 \le t \le 5$.



Signum $(t) = 1$ for $t \geq 0$.

$= -1$ for $t < 0$.

~~signum(t) = u(t)~~ for $t \geq 0$.

~~= -u(t)~~ for $t<0$.

~~u(t) · u(-t)~~

1 ~~-t0~~

$\text{signum}(t) = u(t) - u(-t)$ for $t \geq 0$

$= u(t) - u(-t)$ for $t<0$.

Or

$\text{signum}(t) = 2u(t) - 1$ for $t \geq 0$.

$= 2u(t) - 1$ for $t<0$.

Q. $x(t) = e^{st}$

$s = \sigma + j\Omega$

$x(t) = e^{(\sigma + j\Omega)t}$

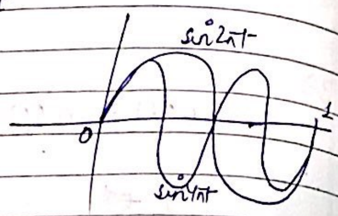
$= e^{\sigma}(\cos \Omega t + j \sin \Omega t)$.

$x(t) = \sin(\Omega t + \phi)$

$\uparrow$ ang. freq. $= 2\pi f$

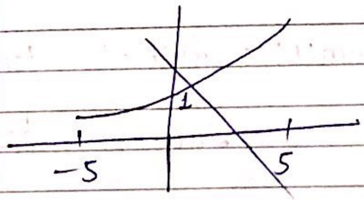
$x(t) = \sin 2\pi t$

$x_1(t) = \sin 2\pi \times 2t$
$= \sin 4\pi t$

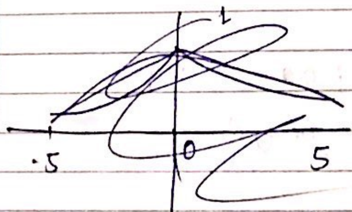


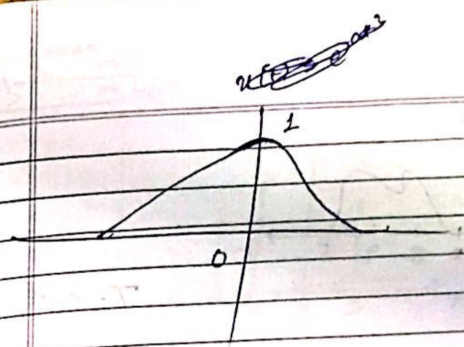
$x(t) = e^{-at^2}$ $\quad -5 \le t \le 5$

$a > 0$.



$\frac{1}{e^{2\times5}}$





Gaussian function

Q. $x(t) = e^{-at^2}$ $\quad -10 \le t \le 10$.
$a = 1.5$

Q. $x(t) = \sin \omega t$ $\quad \omega = 2\pi f$
$f = 1$
$f = 2$
$t \to 0 \to 1sec$.

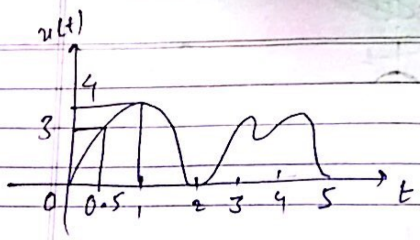
Q. $x(t) = \sin\left(\omega t + \frac{\pi}{4}\right)$. $\quad \omega = 2\pi f$
$f = 1$
$f = 2$

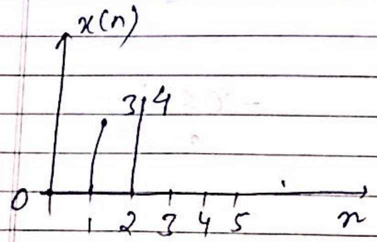
Discrete time signals

$x(n) \to$ representation.

The process of discretisation called as sampling.

$x(n) = x(nT)$
Sampling period





$T = 0.5$

$x(n) = x(nT)$

$x(0) = x(0 \times 0.5)$

$x(1) = x(1 \times 0.5)$

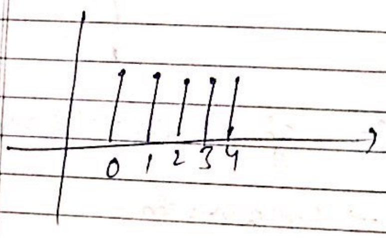
$= x(0.5)$

$= 3$

Sampling + Quantisation = Digital

$u(3) = 1$ for $n \geq 0$.

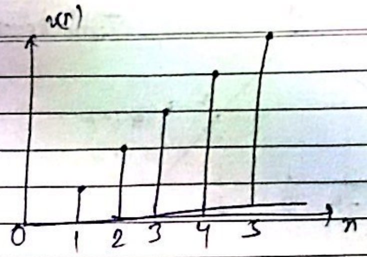
0 otherwise



Ramp

$r(t) = t$ for $t \geq 0$

$= 0$ otherwise



Unit Impulse

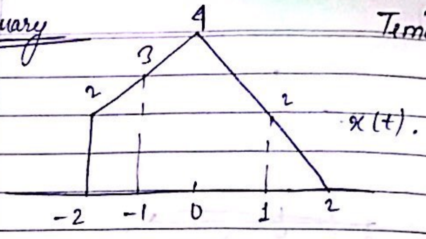
$\delta(n) = 1$ for $n = 0$.

0 otherwise

22nd January

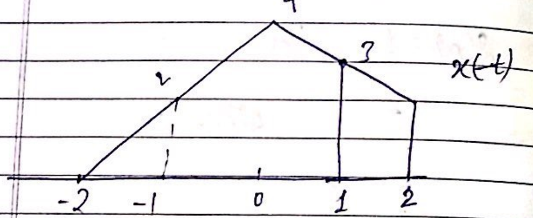
Time reversion

Q. $x(t)$.



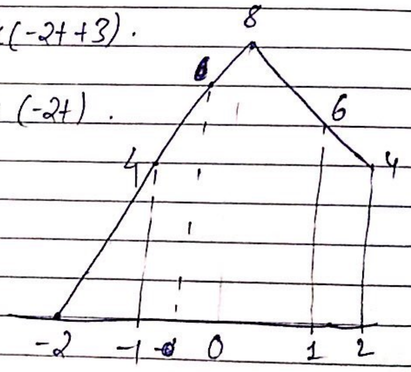
$x(-t)$

$x(at)$
$a=-1$.



Q. $x(-2t+3)$.

$x(-2t)$.



$x(-2t+3)$

$t=0$ $\quad x(3)$

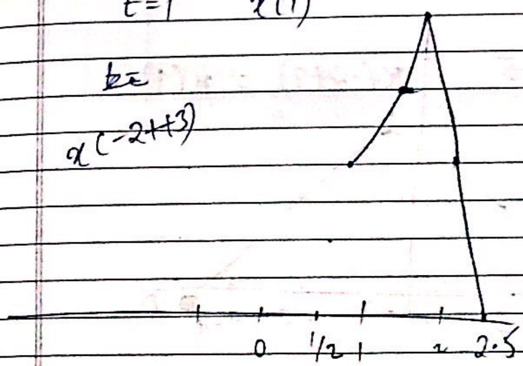
$t=-1$ $\quad x(2+3)=x(5)$

$t=2$ $\quad x(-1)$

$t=1$ $\quad x(1)$

~~t=~~

$x(-2t+3)$



$-2t+3=-1$
$-2t=-4$
$t=2$

$-2t+3=1$
$-2t=-2$
$t=1$

$+4+3$
$+1$
$-2t+3=0$
$t=\frac{+3}{2}$

2.5

$-2t+3=-2$
$-2t=-5$
$t=\frac{5}{2}$

$-2t+3=2$
$-2t=-1$
$t=\frac{1}{2}$

Scanned with CamScanner

$= 2(t+3)$

$2t+3$

$\cancel{x(t+3)}$

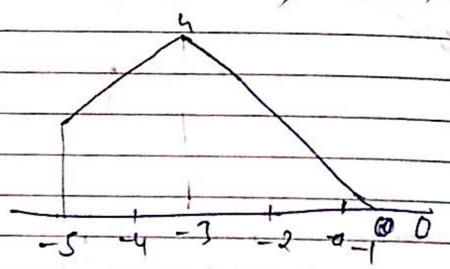
$\cancel{(t+3)}$ $\left(-2\left(t-\frac{3}{2}\right)\right)$

$t+3$

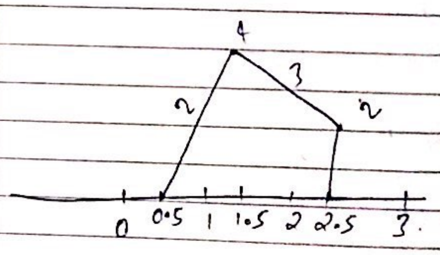
$-2t-6$

$x(t+3)$

$t=-5$ $x(-2+3) = x(1)$



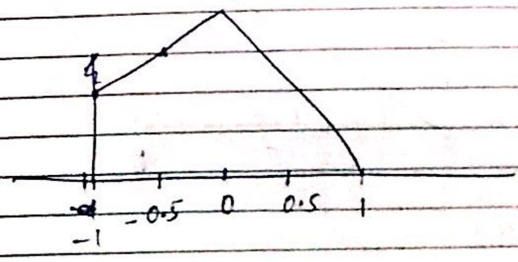
$x(-2t+3)$



$x\left(-2\left(t-\frac{3}{2}\right)\right)$

$x(-2t)$

$t=$

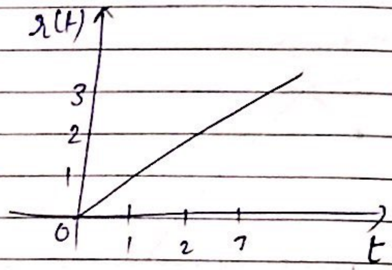


Scanned with CamScanner

Ramp Signal

$r(t) = t$ for $t \geq 0$

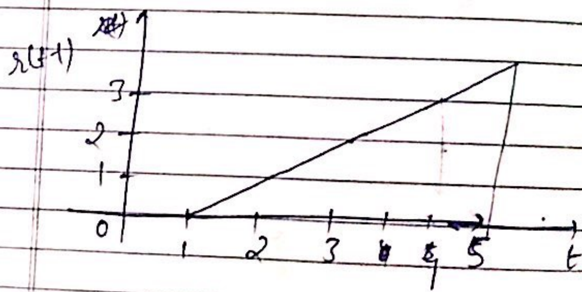
$= 0$ otherwise



$r(t-1)$

$t-1=0$

$t=1$



Q. $x(t) = r(t) - r(t-1)$

$t-1 \geq 0$

$t \geq 1$

$t < 0$

$x(t) =$

$t=1$

$r(1) - r(0)$

$= 1-0$

$= 1$

$t=2$

$r(2) - r(1)$

$= 2-1$

$= 1$

$t=3$

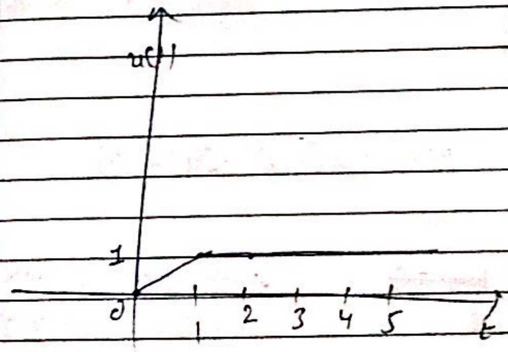
$r(3) - r(2)$

$= 1$.

$t=0$

$r(0)$

$= 0$.



Q. ~~$x(t) = r(t)$~~

Q. $x(t) = r(t) - r(t-1) - r(t-1)$.

$= r(t) - 2r(t-1)$

$t \geq 0$

$x(1) = 1-0 = +1$

$x(3) = 3-4$

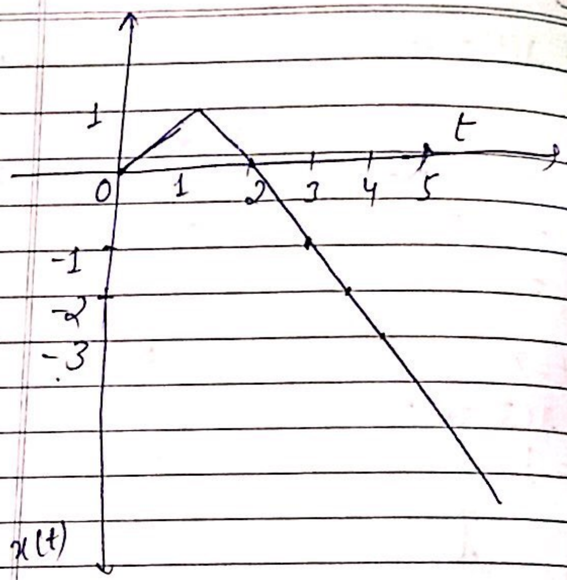
$= -1$

$x(4) = 4-6$

$= -2$

$x(2) = 2-2$

$= 0$.



Q. $x(t) = r(t) - r(t-2) - 2u(t-3)$.

$\cancel{t-3=0}$ $\cancel{t-3 \geq 0}$ $t-3 \geq 0$

$\cancel{t \geq 3}$ $t \geq 3$.

$t-2 \geq 0$

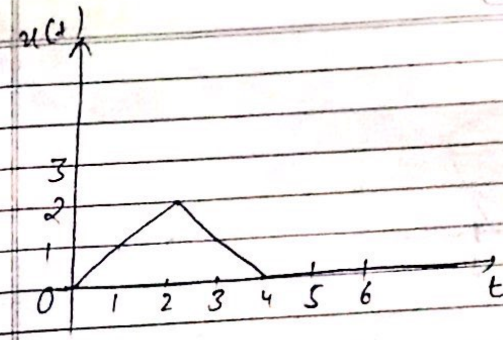
$t \geq 2$.

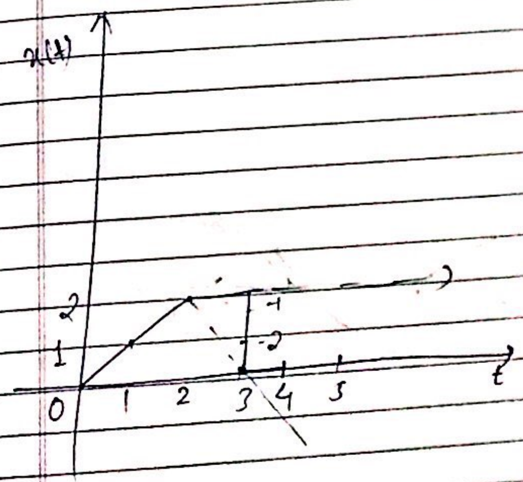
$x(3) - x(1)$
$= 3-2-1$
$= 0$

$4-2-2$
$= 0$

$5-3-2$
$= 0$

$6-4-2$

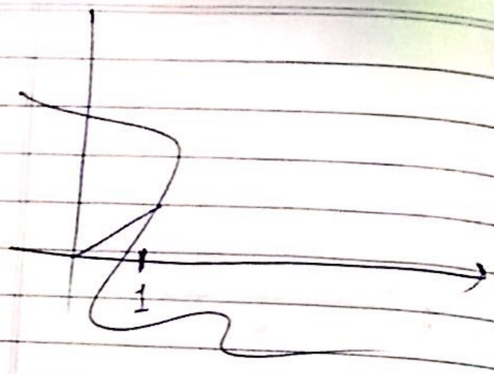


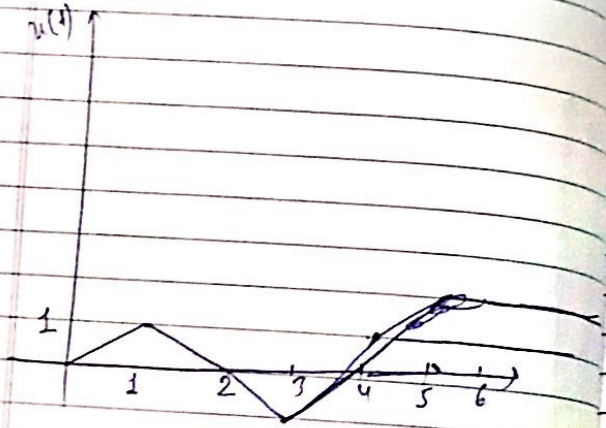


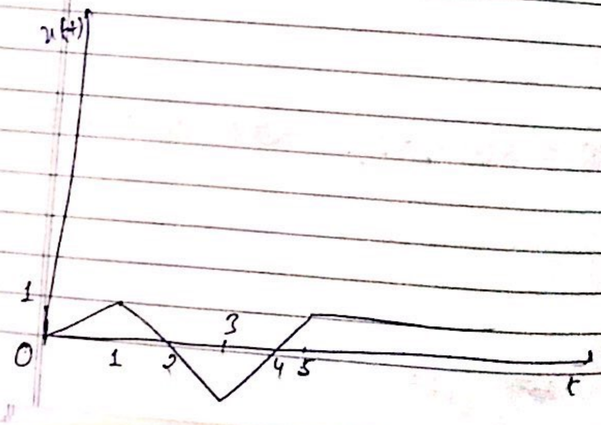
Q. $x(t) = r(t) - 2r(t-1) + 2r(t-3) - r(t-5)$.

+2

r(4)
4-6+2







27th January.

Periodicity

$x(t) = x(t+T)$.

Sine wave $T = 2\pi$

cosine wave $T = 2\pi$.

- $x(t) = \cos(\Omega_0 t + \Omega_0 T + \phi)$.

  $\Omega_0 T = 2\pi$

- $x(t) = \cos(0.2\pi t + \pi/2)$.

  $T = \frac{2\pi}{0.2\pi} = 10$.

- $x(t) = \cos(3\pi t)$

  $T = \frac{2\pi}{3\pi} = \frac{2}{3}$

- $x(t) = e^{j\Omega_0 t}$.

  $x(t+T) = e^{j\Omega_0 (t+T)}$

  $= e^{j\Omega_0 t} e^{j\Omega_0 T} \rightarrow 1$

  $e^{j\Omega_0 T} = 1$

  $\cos \Omega_0 T + j \sin \Omega_0 T = 1$

  $\Omega_0 T = 2\pi \Rightarrow T = \frac{2\pi}{\Omega_0}$

continuous complex ... periodic

- $x(t) = e^{j5t}$

~~$x(t+T) =$~~

- $x(t) = e^{5t} \rightarrow$ Aperiodic
- $x(t) = e^{j5t}$

$= \cancel{\cos 5t + j\sin 5t}$

$\cancel{5t = 2\pi}$

$\cancel{t = \frac{2\pi}{5}}$.

$x(t+T) = x(t)$

$e^{j5(t+T)} = e^{j5t} \cdot e^{j5T}$

$e^{j5T} = 1.$

$\cos 5T + j\sin 5T = 1.$

$5T = 2\pi$

$T = \frac{2\pi}{5}$.

- $x(t) = \sin(\Omega_0 t + \phi)$

$x(t+T) = \sin(\Omega_0 t + \phi + T\Omega_0)$

$T = \frac{2\pi}{\Omega_0}$

Q. $x(t) = \cos 0.3\pi t + \sin 0.2\pi t$.

$x(t+T) = \cos(0.3\pi t + 0.3\pi T_1)$

$+ \sin(0.2\pi t + 0.2\pi T_2)$.

~~$\cos(0.3\pi t) = \cos(0.3\pi t + 0.3\pi T_1)$~~

$T_1 = \frac{2\pi}{0.3\pi}$

$T_2 = \frac{2\pi}{0.2\pi}$

$\frac{T_1}{T_2} = \frac{20/3}{10} = \frac{2}{3} \left(\frac{a}{b}\right)$

$T_1 = \frac{20}{3} \times 3 = 20$. $T = T_1 \times b$

$T = T_2 \times a$

$T = 10 \times 2 = 20$

Q. $x(n) = \cos(\Omega_0 n + \phi)$.

$x(n+N) = \cos(\Omega_0 n + \Omega_0 N + \phi)$.

$\Omega_0 N = 2\pi m$

$N = \frac{2\pi m}{\Omega_0}$.

rational
no → multiple of π

for integral values of N.

- $x(n) = \cos(0.123\pi n)$.

$N = \frac{2\cancel{\pi} m}{0.123\cancel{\pi}}$

$= \frac{2m}{0.123} \times 1000$.

$m = 123$

$N = 2000$.

Q. $x(n) = e^{j0.45\pi n}$

$x(n+N) = e^{j0.45\pi n} e^{j0.45\pi N}$

$0.45\cancel{\pi} N = 2\cancel{\pi} m$

$N = \frac{2}{45} \times 100 \times m$

$= \frac{40 \times m}{9}$

$m = 9$ $N = 40$.

- $x(n) = e^{j(2n + \frac{\pi}{2})}$.

$x(n+N) = e^{j(2n + 2N + \frac{\pi}{2})}$.

Aperiodic

Energy of a signal $x(t)$.

$E = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 dt$

$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$

Q. $x(t) = A\cos(\Omega_0 t + \phi)$.

Ans → $E = \lim_{T \to \infty} A^2 \int_{-T}^{T} \cos^2(\Omega_0 t + \phi) dt$

$= \lim_{T \to \infty} \frac{A^2}{2} \int_{-T}^{T} 1 + \cos(2\Omega_0 t + 2\phi) \, dt$

$= \lim_{T \to \infty} \frac{A^2}{2} \times \left[(t)_{-T}^{T} \right]$ $= \lim_{T \to \infty} \frac{A^2}{2} \times 2T$

$= \frac{A^2}{2} \times \infty = \infty$

$P = \cancel{\frac{1}{2T}}$

$=$

$\cancel{P = \lim_{T\to\infty} \frac{A^2}{2} \times \frac{2T}{2T}}$

$= \frac{A^2}{2}.$

$P = \frac{1}{2T} E$

$= \frac{1}{2T} \lim_{T\to\infty} \frac{A^2}{2} \times 2T$

$= \frac{A^2}{2}.$

Q. $x(t) = u(t) - u(t-3).$

$E = \lim_{T\to\infty} \int_{-T}^{T} [u(t) - u(t-3)]^2 dt$

$= \lim_{T\to\infty} \int_{0}^{T} u^2(t)\, dt - \int_{3}^{T} u^2(t-3)$

$= \lim_{T\to\infty} [T]_0^T - [T]_3^T$

$= \lim_{T\to\infty} T - T + 3$

$= \lim_{T\to\infty} 3$

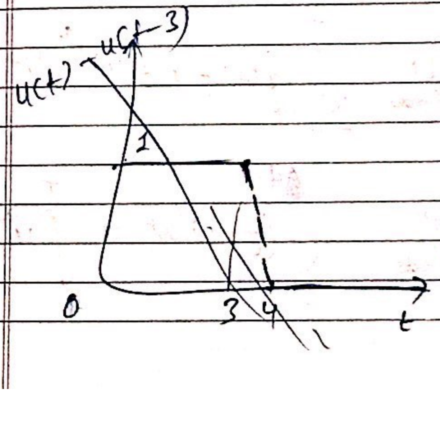
$= 3$

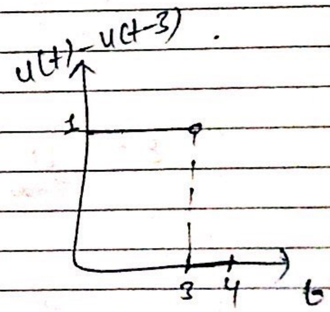
$P = \lim_{T\to\infty} \frac{1}{2T} E$

$= \lim_{T\to\infty} \frac{1}{2T} \times 3$

$= \lim_{T\to\infty} \frac{3}{2T}.$

$= 0.$



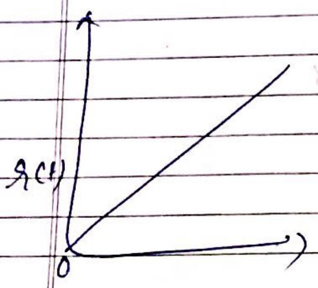


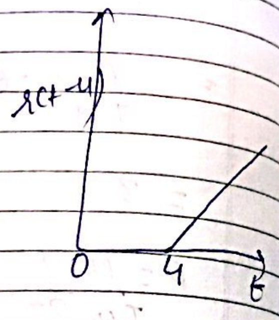
~~$4\,t^4$~~

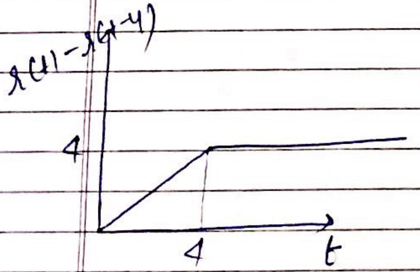
~~$\mathcal{E} = \lim_{T\to\infty} t^4$~~

$r(t) = t \qquad r(t-4) = \ldots$

Q. $x(t) = r(t) - r(t-4)$







$$\mathcal{E} = \lim_{T\to\infty} \int_{-T}^{T} \left(r(t) - r(t-4)\right)^2 dt$$

$$= \lim_{T\to\infty} \int_0^4 (t-0)^2 dt + \int_4^T 4^2 dt$$

$$= \lim_{T\to\infty} \frac{t^3}{3}\Big|_0^4 + 16T\Big|_4^T$$

$$= \lim_{T\to\infty} \left(\frac{64}{3} + 16\times T - 64\right)$$

$$= \lim_{T\to\infty} \left[16T - \frac{2\times 64}{3}\right].$$

$$= \infty$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \left[16T - \frac{2}{3}\times 64\right].$$

$$= 8 - \cancel{\frac{2\times 64}{3}}^{\,0}$$

~~$= \frac{24 - 128}{3}$~~

~~$= -\frac{40}{3}$~~

3rd February

for Discrete

$x(n)$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

Q. $x(n) = \left(\frac{1}{3}\right)^n u(n)$.

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{9}\right)^n$$

$$= \frac{1}{1-\frac{1}{9}}.$$

$$= \frac{1}{9}^0 + \frac{1}{9}^1 + \frac{1}{9}^2 + \frac{1}{9}^3 + \dots + \frac{1}{9}^\infty.$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} \left(\frac{1}{9}\right)^n$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \left(\frac{1}{9}^0 + \frac{1}{9}^1 + \frac{1}{9}^2 + \frac{1}{9}^3 + \dots + \frac{1}{9}^N \right).$$

$N \to \infty$

$2N+1 \to \infty$

$\frac{1}{2N+1} \to 0$

$P = 0.$

Q. $x(n) = e^{j\left(\frac{\pi}{4}n + \frac{\pi}{3}\right)}$.

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=-\infty}^{\infty} \left| e^{j\left(\frac{\pi}{4}n + \frac{\pi}{3}\right)} \right|^2$$

$$= \sum_{n=-\infty}^{\infty} \sqrt{\cos^2 \frac{\pi}{4} +}$$

$$\left| e^{j\left(\frac{\pi}{4}n + \frac{\pi}{3}\right)} \right| = 1$$

$$= \sum_{n=-\infty}^{\infty} (1)^{2n}$$

$$= \infty .$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \times \sum_{n=-N}^{N} (1)^2$$

$$= \lim_{N \to \infty} \frac{2N+1}{2N+1}$$

$$= 1 .$$

Discrete Time Sequence

$$x(n) = \cos(\Omega_0 n + \phi)$$

$$x(n+N) = x(n) .$$

$$x(n+N) = \cos(\Omega_0 (n+N) + \phi) .$$

$$= \cos(\Omega_0 n + \Omega_0 N + \phi) .$$

$$\Omega_0 N = 2\pi m$$

$$N = \frac{2\pi m}{\Omega_0}$$

for discrete time

Ω_0 = rational multiple of 2π

Q. $x_1(n) = \cos(\Omega_0 n + \phi)$

$$x_2(n) = \cos(\Omega_0 n + 2\pi k n + \phi) .$$

$$\cancel{\Omega = \frac{2\pi m}{2\pi k}}$$

$$= \cos(\Omega_0 n + \phi) .$$

Eg → $\cos(0.2\pi n)$.

$$= \cos[(0.2\pi \pm 2\pi) n]$$

$$= \cos 2.2\pi n \quad \text{or} \quad \cos 1.8\pi n .$$

are identical sinusoids

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for discrete time sinusoids,

$|\omega| < \pi$ identical with $|\omega| > \pi$

frequency ~~range~~ oscillates between $-\pi$ to π.

highest freq at π.

for cont it is from $-\infty$ to ∞.

Causal Signal
defined only for time interval $(0 \text{ to } \infty)$.

$x(t) = e^{at} u(t)$.

$x(t) = \{2, 5, 11, 42\}$.
$\uparrow$

$x(t) = e^{at} u(t-3)$.

Non-Causal

$x(n) = \{2, 5, 11, 42\}$.
$n=-2$ $n=-1$ $\uparrow$ $n=1$
$n=0$

$x(t) = e^{at} u(t+3)$.

Anti-Causal
defined only for -ve intervals $(-\infty \text{ to } 0)$.

$x(t) = e^{at} u(-t)$.

Causal System
i) $y(t) = x(t) + x(t-1) + y(t-2)$
ii) $y(t) = x(t) + x(t-1)$.

which depend on ~~present~~ present & past i/p & o/p not on future i/p & o/p.

~~No-n~~

Non-causal System

$y(t) = x(t) + x(t+1)$.

Q. $y(t) = 3x^2(t) + 4x(t-1)$

Causal System

Q. $y(t) = 3x(t^2) + 4x(t-1)$

Non-causal System.

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4th february.

Static and dynamic System

do not require memory — Memory Required

Static System

$$y(t) = 3x(t) + x^2(t).$$

Dynamic System

$$y(t) = 3x(t-1) + x(t^2) + 4x(t).$$

$$y(t) = 3x(t-1) + 4x(t).$$

Linear System

$$T[a_1 x_1(t) + a_2 x_2(t)] = \underbrace{T[a_1 x_1(t)] + T[a_2 x_2(t)]}_{\text{Additivity Principle}}$$

$$= \underbrace{a_1 T[x_1(t)] + a_2 T[x_2(t)]}_{\text{Homogeneity Principle}}$$

$$= a_1 y_1(t) + a_2 y_2(t)$$

Q. $y(t) = x(t) + 3x(t-1)$

Ans: $x_1(t)$ $x_2(t)$

$$y_1(t) = x_1(t) + 3x_1(t-1)$$

$$a_1 y_1(t) = a_1[x_1(t) + 3x_1(t-1)] \quad \text{— (i)}$$

$$a_2 y_2(t) = a_2[x_2(t) + 3x_2(t-1)]. \quad \text{— (ii)}$$

$$x_3(t) = a_1 x_1(t) + a_2 x_2(t).$$

$$y_3(t) = x_3(t) + 3x_3(t-1).$$

$$= a_1 x_1(t) + a_2 x_2(t) + 3[a_1 x_1(t-1) + a_2 x_2(t-1)].$$

(i) + (ii).

$$a_1 y_1(t) + a_2 y_2(t) = a_1 x_1(t) + a_2 x_2(t) + 3[a_1 x_1(t-1) + a_2 x_2(t-1)].$$

$$= y_3(t)$$

∴ Linear System.

Q. $y(t) = 3x^2(t) + x(t-2)$

$y_1(t) = 3x_1^2(t) + x_1(t-2)$

$a_1 y_1(t) = 3a_1 x_1^2(t) + a_1 x_1(t-2)$ — (I)

$a_2 y_2(t) = 3a_2 x_2^2(t) + a_2 x_2(t-2)$ — (II)

$x_3(t) = a_1 x_1(t) + a_2 x_2(t)$

$y_3(t) = 3x_3^2(t) + x_3(t-2)$.

(I) + (II)

$a_1 y_1(t) + a_2 y_2(t) = 3[a_1 x_1^2(t) + a_2 \ldots$

$+ a_1 x_1(t-2) + a_2 x_2 \ldots$

$y_3(t) = 3[a_1^2 x_1^2(t) + a_2^2 x_2^2(t) + 2a_1 a_2 x_1 x_2(t)]$

$+ a_1 x_1(t-2) + a_2 x_2(t-2)$.

∴ Non-linear

Q. $y(t) = x(t+4) + x(t^2)$ → Linear System

Q. $y(t) = 3x^3(t) + 5x(t+9)$ → N.L

Q. $y(t) = e^{x(t)}$.

Q. $y(t) = \frac{dx(t)}{dt} + x(t)$.

$a_1 y_1(t) = a_1 \frac{dx_1(t)}{dt} + a_1 x_1(t)$. — (I)

$a_2 y_2(t) = a_2 \frac{dx_2(t)}{dt} + a_2 x_2(t)$. — (II)

$y_3(t) = \frac{dx_3(t)}{dt} + x_3(t)$

$x_3(t) = a_1 x_1(t) + a_2 x_2(t)$.

$= a_1 \frac{dx_1(t)}{dt} + a_2 \frac{dx_2(t)}{dt}$.

$y_3(t) = a_1 \frac{dx_1(t)}{dt} + a_2 \frac{dx_2(t)}{dt} + a_1 x_1(t) + a_2 x_2(t)$

(I) + (II).

$a_1 y_1(t) + a_2 y_2(t) = a_1 \frac{dx_1(t)}{dt} + a_2 \frac{dx_2}{dt} + a_1 x_1(t) + a_2 x_2(t)$.

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$= y_3(t)$

Linear

Q. $y(t) = e^{x(t)}$.

$a_1 y_1(t) = a_1 e^{x_1(t)}$

$a_2 y_2(t) = a_2 e^{x_2(t)}$

$a_1 y_1(t) + a_2 y_2(t) = a_1 e^{x_1(t)} + a_2 e^{x_2(t)}$

$y_3(t) = e^{x_3(t)}$

$= e^{a_1 x_1(t)} \cdot e^{a_2 x_2(t)}$.

$\therefore$ Non-linear

5th feb

Linear System
(for discrete signal).

$T[a_1 x_1(n) + a_2 x_2(n)] = a_1 T[x_1(n)] + a_2 T[x_2(n)]$.

$y_3(n) = a_1 y_1(n) + a_2 y_2(n)$.

Q. $y(n) = x(n) + \frac{1}{x(n-1)}$.

$a_1 y_1(n) = a_1 x_1(n) + \frac{a_1}{x_1(n-1)}$.

$a_2 y_2(n) = a_2 x_2(n) + \frac{a_2}{x_2(n-1)}$.

$y_3(n) = x_3(n) + \frac{1}{x_3(n-1)}$

$x_3(n) = a_1 x_1(n) + a_2 x_2(n)$.

$y_3(n) = a_1 x_1(n) + a_2 x_2(n) + \frac{1}{a_1 x_1(n-1) + a_2 x_2(n-1)}$.

$\therefore$ Non-linear

Q. $y(n) = \cos[x(n)]$

Ans: $a_1 y_1(n) = a_1 \cos(a_1 x_1(n))$

$a_2 y_2(n) = a_2 \cos(x_2(n))$.

$x_3(n) = a_1 y_1(n) + a_2 y_2(n)$

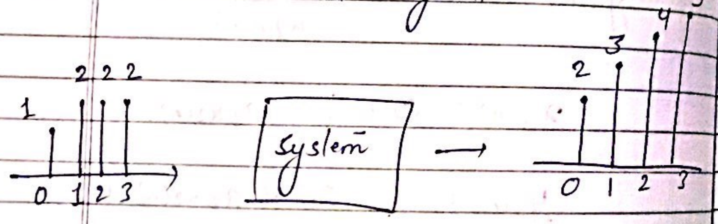
$y_3(n) = \cos(x_3(n))$

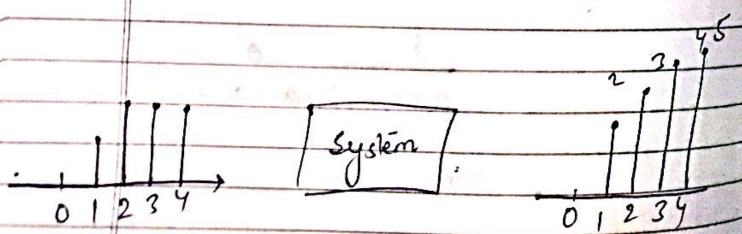
$= \cos(a_1 y_1(n) + a_2 y_2(n))$.

∴ Non-linear

Time Variant and Invariant System

Time IN-variant system





$x(t) \rightarrow \boxed{T} \rightarrow y(t)$

$y(t,\tau) = T[x(t-\tau)]$.

$y(t-\tau) = T[x(t-\tau)]$.

T is Time-invariant system if above eqn satisfies otherwise time-variant.

Eg: $y(t) = 3x(t) + x(t-1)$.

$y(t,\tau) = 3x(t-\tau) + x[(t-1)-\tau]$

$y(t-\tau) = 3x(t-\tau) + x(t-\tau-1)$.

∴ Time Invariant

Eg: $y(t) = x(t) + 2x(t^2)$.

$y(t,\tau) = x(t-\tau) + 2x(t^2-\tau)$

$y(t-\tau) = x(t-\tau) + 2x((t-\tau)^2)$

$= x(t-\tau) + 2x(t-\tau)^2$

∴ Time Variant.

Q. $y(t) = x^2(t) + x(-t-2)$.

$$y(t,\tau) = x^2(t-\tau) + x[(-t-2)-\tau]$$

$$y(t-\tau) = x^2(t-\tau) + x[-t+\tau-2]$$

∴ Time ~~in~~variant

LTI (Linear Time Invariant) Systems

Impulse Response $(h(t))$

$I/p \rightarrow$ Unit Impulse

Eg. $h(n) = 2^n u(n)$.

Eg. $x(t) = \int_{-\infty}^{\infty} x(\tau)\, \delta(t-\tau) d\tau$.

$$= \int_{-\infty}^{\infty} x(t)\, \delta(t-\tau)\, d\tau. \quad \tau \to t$$

$$= x(t) \int_{-\infty}^{\infty} \delta(t-\tau) d\tau \quad \tau \to t$$

$$= x(t)$$

Convolution is commutative.

$$\delta(t) \longrightarrow \boxed{T} \longrightarrow h(t)$$

$$h(t,\tau) = T[\delta(t-\tau)] = h(t-\tau). \quad \text{(I)}$$

$$x(t) \longrightarrow \underset{h(t)}{\boxed{LTI}} \longrightarrow y(t)$$

$$y(t) = T[x(t)].$$

$$= T\left[\int_{-\infty}^{\infty} x(\tau)\, \delta(t-\tau)\, d\tau\right].$$

$$\cancel{= x(t)\, T\left[\int_{-\infty}^{\infty} \delta(t-\tau) d\tau\right]} \quad \tau \to t$$

$$\cancel{= x(t)\, T[1]}.$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, T[\delta(t-\tau)]\, d\tau. \quad \text{(II)}$$

from (I) & (II)

$$y(t) = \underbrace{\int_{-\infty}^{\infty} x(\tau)\, h(t-\tau) d\tau}_{\text{convolution Integral}}.$$

$$= \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau) d\tau$$

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$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$x(t) = u(t)$$

$$h(t) = 2u(t)$$

$$h(t-\tau) = 2u(t-\tau)$$

$$y(t) = \int_{-\infty}^{\infty} u(\tau) \cdot 2u(t-\tau)\, d\tau$$

$$= \int_{0}^{\infty} 2u(t-\tau)\, d\tau.$$

$$= 2 \int_{t}^{\infty} u(t-\tau)\, d\tau.$$

$$= 2 \cdot \left[\tau \right]_{t}^{\infty}$$

$$= 2 \int_{0}^{t} d\tau$$

$$= 2\, t\, u(t).$$

Q. $x(t) = e^{-5t} u(t)$

$h(t) = e^{-3t} u(t)$

$$y(t) = \int_{-\infty}^{\infty} e^{-5\tau} u(\tau)\, e^{-3(t-\tau)} u(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} e^{-3t} \cdot e^{-5\tau+3\tau}\, u(\tau)\, u(t-\tau)\, d\tau$$

$$= e^{-3t} \int_{0}^{t} e^{-2\tau}\, d\tau$$

$$= e^{-3t} \left. \frac{e^{-2\tau}}{-2} \right|_{0}^{t}$$

$$= e^{-3t} \left[\frac{e^{-2t} - 1}{-2} \right]$$

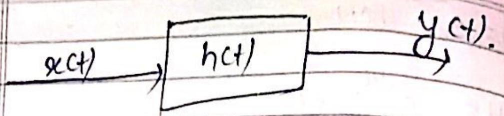
$$= e^{-3t} \cdot \left(\frac{1 - e^{-2t}}{2} \right).$$

Q. $x(t) = t e^{-5t} u(t)$

$h(t) = e^{-3t} u(t)$

10^{th} Feb

LTI.



$y(t) = x(t) * h(t)$

$= h(t) * x(t).$

Commutative law:

$x_1(t) * x_2(t) = x_2(t) * x_1(t)$

Distributive law:

$x_1(t) * [x_2(t) + x_3(t)].$

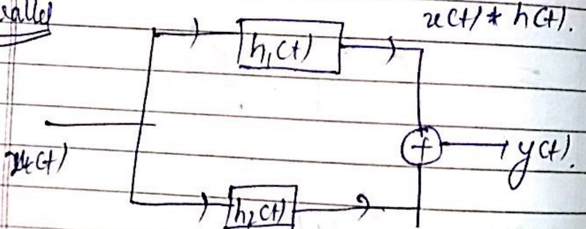
$= x_1(t) * x_2(t) + x_1(t) * x_2(t)$

Associative law.

$x_1(t) * [x_2(t) * x_3(t)].$

$= [x_1(t) * x_2(t)] * x_3(t).$

Parallel



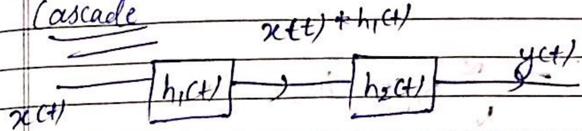
$h(t) = h_1(t) + h_2(t)$

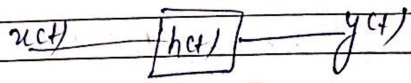
$\therefore y(t) = x(t) * h_1(t) + x(t) * h_2(t)$

$= x(t) * [h_1(t) + h_2(t)].$

$= x(t) * h(t).$

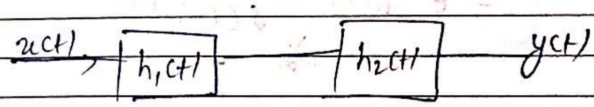
Cascade





$h(t) = h_1(t) * h_2(t).$

Q.



$h_1(t) = e^{-2t} u(t) \qquad x(t) = e^{-4t} u(t)$

$h_2(t) = \delta(t)$

$y(t) = ?$

$y(t) = (h_1(t) * h_2(t)). x(t).$

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$$= e^{-4t}\left(\int_{-\infty}^{\infty} e^{-2\tau} u(\tau)\, \delta(t-\tau)\, d\tau\right)\delta(t)$$

$$= e^{-4t}\left(\int_{-\infty}^{\infty} e^{-2\tau} u(\tau)\, \delta(t-\tau)\, \delta(t)\, d\tau\right)$$

$$h(t) = \int_{-\infty}^{\infty} h_1(\tau)\, h_2(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} e^{-2\tau} u(\tau)\, \delta(t-\tau)\, d\tau$$

$\tau \to t$

$$= e^{-2t} u(t) \int_{-\infty}^{\infty} \delta(t-\tau)\, d\tau$$

$\tau \to t$

$$= e^{-2t} u(t)$$

$$y(t) = h(t) * u(t)$$

$$= \int_{-\infty}^{\infty} e^{-2\tau} u(\tau)\, e^{-4(t-\tau)} u(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} e^{-2\tau-4t+4\tau} u(\tau)\, u(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} e^{2\tau-4t} u(\tau)\, u(t-\tau)\, d\tau$$

$$= e^{-4t} \int_{0}^{t} e^{2\tau}\, d\tau$$

$$= e^{-4t} \left.\frac{e^{2\tau}}{2}\right]_0^t$$

$$= e^{-4t}\left[\frac{e^{2t}}{2} - \frac{1}{2}\right]$$

$$= \frac{e^{-4t}}{2}\left(e^{2t}-1\right)$$

$$= \left(\frac{e^{-2t}}{2} - \frac{e^{-4t}}{2}\right)$$

Discrete

$x_1(n)$, $x_2(n)$

$\cancel{y_1(n) = \sum x_1(n)\, x_2(n)}$

$$y_1(n) = \sum_{k=-\infty}^{\infty} x_1(k)\, x_2(n-k)$$

$x_1(n) = \{\underset{\uparrow}{2}, 3, 6\}$

$x_2(n) = \{\underset{\uparrow}{1}, 1, 5\}$

$\cancel{y_1(n) = 1\times 2 + 3\times 1}$

$\cancel{y_1(n) =}$

			2	3	6		
-3	-2	-1	0	1	2	3	4
			1	1	5		
	5	1	1				

$$y_1(n) = \sum_{k=-\infty}^{\infty} x_1(k)\, x_2(n-k)$$

$$y(0) = \sum_{k=-\infty}^{\infty} x_1(k)\, x_2(-k).$$

$= x_1(0)\, x_2(-0).$

$= 2\times 1 = 2$

$$y(1) = \sum_{k=\infty}^{\infty} x_1(k)\, x_2(1-k).$$

			2	3	6
-3	-2	-1	0	1	2
		5	1	1	

$= x_1(0)\, x_2(1-0) + x_1(1)\, x_2(1-1)$

$= 2\times 1 + 3\times 1$

$= 5.$

$$y(2) = \sum_{k=\infty}^{\infty} x_1(k)\, x_2(2-k)$$

			2	3	6
-3	-2	-1	0	1	2
			5	1	1

$= x_1(0)\, x_2(2) + x_1(1)\, x_2(1)$

$+ x_1(2)\, x_2(0)$

$= 2\times 5 + 3\times 1 + 6\times 1$

$= 10+3+6 = 19.$

$$y(3) = \sum_{k=-\infty}^{\infty} x_1(k)\, x_2(3-k).$$

$$\cancel{= x_1(0)\, x}$$

$$= x_1(1)\, x_2(2) + x_2(2)\, x_2(1)$$

$$= 3 \times 1 + 6 \times 1$$

$$= 9$$

$$y(4) = \sum_{k=-\infty}^{\infty} x_1(k)\, x_2(4-k)$$

$$= x_1(2)\, x_2(2)$$

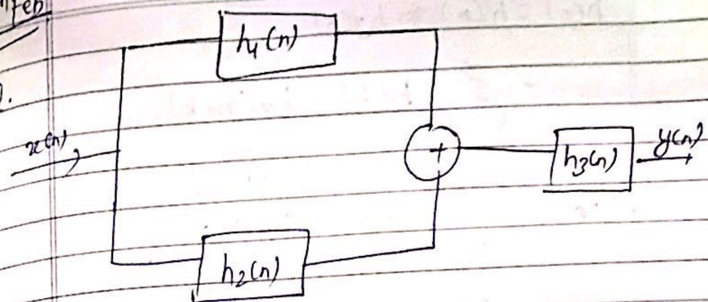
$$= 3 \times 1$$

$$= 3$$

~~11 Feb~~

11th Feb

Q.



$h_1(n) = \{\underset{\uparrow}{2}, 1, 1\}$ $\qquad$ $h_3(n) = \delta(n)$.

$h_2(n) = \{\underset{\uparrow}{1}, 3\}$

$x(n) = \{3, \underset{\uparrow}{2}, 1, 4\}$.

$\cancel{y(n) =} \ \cancel{h(n) = h_1(n) * h_2(n)}$

$\cancel{= \sum_{k=-\infty}^{\infty}}$

$h'(n) = h_1(n) + h_2(n)$

$= \{\underset{\uparrow}{3}, 4, 1\}$

$\cancel{h'(n) + h_3} \ \cancel{h(n) = h'(n) * h_3(n)}$

$\cancel{= \sum_{k=-\infty}^{\infty} h'(}$

$h(n) = h'(n) * h_3(n)$

$= \sum_{k=-\infty}^{\infty} h(k)\ h_3(n-k).$

$= h(k).$

$\cancel{h(n) = \sum_{k=-\infty}^{\infty} h(k)\ \delta(n-k)}$

$\cancel{= \sum_{k=-\infty}^{\infty}}$

$\cancel{= h(k)}$

$\cancel{y(n) = \sum_{k=-\infty}^{\infty} h(k) * x(n)\, x(k-n)}$

$\cancel{y(0) = \sum_{k=-\infty}^{\infty} h(k)\ x(k).}$

$\cancel{= h(0)\ x(0)}$

$y(n) = x(n) * h(n).$

$= \sum_{k=-\infty}^{\infty} x(k)\ h(n-k).$

$y(0) = \sum_{k=-\infty}^{\infty} x(k)\ h(-k).$

$= x(0)\ h(0) + x(-1)\ h(1) = 2\times3 + 3\times4$

$= 18$

$y(1) = \sum_{k=-\infty}^{\infty} x(k)\ h(1-k)$

$= x(0)\ h(1) + x(1)\ h(0) + x(-1)\ h(2)$

$= 2\times3 + 1\times3 + 3\times1$

$= 12$

$\cancel{y(2) =}$

$y(-1) = \sum_{k=-\infty}^{\infty} x(k)\ h(-1-k).$

$= x(-1)\ h(0)$

$= 3\times3$

$= 9$

$y(2) = \sum_{k=-\infty}^{\infty} x(k)\ h(2-k)$

$= x(0)\ h(2) + x(1)\ h(1)$

$\cancel{+ x(-1)\ h(3)}$

$= 2\times1 + 1\times3$

$= 5$

$y(3) = \sum_{k=-\infty}^{\infty} x(k)\ h(3-k)$

$= x(1)\ h(2) + x(2)\ h(1) = 1\times4 + 4\times4$
$= 20$

$$y(4) = \sum_{k=\infty}^{\infty} x(k)\, h(4-k)$$

$= x(2)\, y(2)$

$= 4 \times 1$

$= 4$

$y(n) = \{9, \underset{\uparrow}{18}, 5, 20, 4\}$.

Stability

BIBO

A system is said to be stable if it gives bounded output when bounded i/p is given.

$|x(t)| \leq M_x$

Ex of bounded signal

$x(t) = e^{-at} u(t)$

$|y(t)| \leq M_y$.

Q. $r(t) = t$

$= 0$ otherwise.

Unbounded signal

for LTI systems

Q. Check BIBO or not

$y(t) = \cos(u(t))$

BIBO ✓

as output lies betⁿ $[-1, 1]$

~~for LTI system~~

Q. $y(t) = x(-t+3)$.

~~$y(t-t)$~~

~~$y(t-t) = x(t+t+3)$~~.

$y(t, \tau) = x(-t+3-\tau)$.

$y(t-\tau) = x(-(t-\tau)+3)$

$= x(-t+\tau+3)$

Time Variant

BIBO ✓ System

for LTI system.

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$|x(t)| \le M_x .$$

$$|y(t)| \le M_y$$

$$|y(t)| = \left| \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau \right| .$$

$$\left| \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau \right| \le \int_{-\infty}^{\infty} |h(\tau)|\, |x(t-\tau)|\, d\tau$$

$$|y(t)| \le \int_{-\infty}^{\infty} |h(\tau)|\, |x(t-\tau)|\, d\tau.$$

$$\le \int_{-\infty}^{\infty} |h(\tau)|\, M_x\, d\tau$$

$$\le M_x \int_{-\infty}^{\infty} |h(\tau)|\, d\tau.$$

Stability condn for L.T.I

$$\int_{-\infty}^{\infty} |h(t)|\, dt \le M_y$$

Q. Check $h(t) = e^{-4t} u(t)$

$$\int_{-\infty}^{\infty} e^{-4t} u(t)\, dt$$

$$= \int_{0}^{\infty} e^{-4t}\, dt$$

$$= \left. \frac{e^{-4t}}{-4} \right|_{0}^{\infty}$$

$$= \frac{1}{4}.$$

finite value

$\therefore$ ~~stable~~ BIBO stable

18th Feb.

Q. $h(t) = \frac{1}{RC} e^{-t/RC} u(t)$

$$\int_{-\infty}^{\infty} |h(t)| dt < \infty$$

$$\frac{1}{RC} \int_{0}^{\infty} e^{-t/RC} dt \leq M_y$$

$$\frac{1}{RC} \left[-e^{-t/RC} \times RC \right]_{0}^{\infty} \leq M_y$$

$$1 \leq M_y$$

Stable System

Q. $h(t) = e^{-6|t|}$

$$\int_{-\infty}^{\infty} |h(t)| dt \leq M_y$$

$$\int_{-\infty}^{\infty} |e^{-6|t|}| dt \leq M_y$$

$$\int_{-\infty}^{0} e^{6t} dt + \int_{0}^{\infty} e^{-6t} dt \leq M_y$$

$$\left[\frac{e^{6t}}{6} \right]_{-\infty}^{0} + \left(\frac{e^{-6t}}{-6} \right)_{0}^{\infty} \leq M_y$$

$$\frac{1}{6} + \left(\frac{1}{6} \right) \leq M_y$$

$$\frac{1}{3} \leq M_y$$

Stable System

for discrete

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

Q. $h(n) = 2^n u(n)$

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

$$\sum_{n=-\infty}^{\infty} |2^n \cdot u(n)| < \infty$$

$$\sum_{n=0}^{\infty} 2^n < \infty$$

$$1 + 2 + 2^2 + 2^3 + \dots + 2^{\infty} < \infty$$

Not stable

Unstable System

Q. $h(n) = 2^n u(-n)$.

$$\sum_{n=-\infty}^{\infty} 2^n u(-n)$$

$$= \sum_{n=-\infty}^{0} 2^n$$

$$= 2^{-\infty} + 2^{-\infty+1} + \dots + 2^0$$

$$\cancel{= 1 + 2 + \dots}$$

$$= 1 + \frac{1}{2} + \frac{1}{4} + \dots + 2^{-\infty}$$

converging series

$$1\left(\frac{1}{1-\frac{1}{2}}\right) < \infty$$

$$\frac{2}{1} < \infty$$

$$2 < \infty$$

stable

Q. $h(n) = e^{-2n} u(n-1)$

$n-1=0$

$n=1$

$$\sum_{n=-\infty}^{\infty} e^{-2n} u(n-1)$$

$$= \sum_{n=1}^{\infty} e^{-2n}$$

$$= e^{-2} + e^{-4} + \dots + e^{-2\times\infty}$$

$$= \frac{1}{e^2} + \frac{1}{e^4} + \dots + \frac{1}{e^{\infty}}$$

$$\cancel{= \frac{1}{4}\left(\frac{1}{1-\frac{1}{4}}\right)}$$

$$\cancel{= \frac{1}{4}\left(\frac{4}{3}\right)}$$

$$\cancel{= \frac{1}{3} < \infty}$$

~~stable~~

$$= \frac{1}{e^2}\left(\frac{1}{1-\frac{1}{e^2}}\right).$$

$$= \frac{1}{e^2}\left(\frac{e^2}{e^2-1}\right) = \frac{1}{e^2-1} < \infty.$$

stable

$\frac{5^{-1}}{5^{-3}} = 5^{-3}$

Q. $h(n) = 5^n u(3-n)$

$\sum_{n=-\infty}^{3} 5^n < \infty$

$= \sum_{n=-3}^{\infty} 5^{-n}$

$= 5^{-3} + \cancel{5^{-6}} + 5^{-9} + \ldots + 5^{-\infty}$

$= \frac{1}{5^3} + \frac{1}{5^6} + \frac{1}{5^9} + \ldots \frac{1}{5^\infty}$

$= \frac{1}{125}\left(\frac{1}{1-\frac{1}{125}}\right) \sum_{n=-3}^{\infty} 5^{-n}$

$= \frac{1}{\cancel{125}} \times \frac{\cancel{125}}{124}$ $= 5^3 + 5^2 + 5^1 + \ldots + 5^{-\infty}$

$= \frac{1}{124} < \infty$ $= 5^3\left(\frac{1}{1-\frac{1}{5}}\right)$

stable

$= \frac{5^3 \times 5}{4}$

$= \cancel{5^{+3} + 5} = \sum_{n=3}^{\infty} 5^{n}$ $= \frac{625}{4}$

$\cancel{= 5}$ $= 156.25$

$= 5^{-\infty} + 5^{-(\infty+1)} + 5^{-(\infty+2)} + \ldots + 5^2 + 5^3$

$= 5^3 + 5^2 + 5^1 + 1 + \ldots + 5^{-\infty}$

$= 5^3\left(\frac{1}{1-\frac{1}{5}}\right)$

$= \frac{125 \times 5}{4}$ $= \frac{625}{4} < \infty$

stable

Express causality in terms of impulse response

$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$

$= \ldots + h(-5)x(n+5) + h(-4)x(n+4) + \ldots + h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + \ldots$

−ve ~~entop~~ intervals → future i/p needed

+ve " → past & present i/p

for causal,
for discrete $h(n) = 0$ for $n < 0$.

// for Continuous

$h(t) = 0 \qquad t < 0$

Q. Consider a causal system $h(t)$ and process it i/p $x(t)$. Find $y(t)$.

Ans →

$$y(t) = \int_{\tau=-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau.$$

For causal $h(t) = 0 \qquad t < 0$

$$y(t) = \int_{\tau=0}^{\infty} h(\tau)\, x(t-\tau)\, d\tau.$$

but $x(t-\tau)$

$\tau \to 0 \text{ to } t$

$$y(t) = \int_{\tau=0}^{t} h(\tau)\, x(t-\tau)\, d\tau.$$

$$y(t) = \int_{0}^{t} h(\tau)\, x(t-\tau)\, dt$$

for causal system → i/p can be causal or non-causal.

$$y(t) = \int_{0}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

for i/p to be causal,

$x(t-\tau)$

$\tau \to 0 \text{ to } t$

$$y(t) = \int_{0}^{t} h(\tau)\, x(t-\tau)\, d\tau$$

for i/p to be non-causal,

$x(t-\tau)$

$$y(t) = \int_{0}^{\infty} h(\tau)\, x(t-\tau)\, d\tau.$$

Q. $h(t)$ → non-causal system

$x(t)$ → causal signal

$$y(t) = \int_{\tau=-\infty}^{\infty} h(\tau)\, x(t-\tau)\, dt$$

$$= \int_{\tau=-\infty}^{t} h(\tau)\, x(t-\tau)\, dt$$

19th feb

Unit step response

$x(t) = u(t)$.

$h(t) = t\,u(t)$.

$y(t) = u(t) * t\,u(t)$.

~~$= \int_{-\infty}^{\infty} u(\tau)\, (\tau - t)\, u(\tau - t)\, dt$~~

~~$= \int_{0}^{t} \tau - t \, dt$~~

~~$= \left[(\tau - t) t \right]_0^t$~~

~~$= \tau t - t^2$~~

$= \int_{-\infty}^{\infty} u(\tau)\, (t-\tau)\, u(t-\tau)\, d\tau$

$= \int_{0}^{t} (t-\tau)\, d\tau$

~~$= \left[\frac{t^2}{2} - \tau t \right]_0^t$~~

~~$= \frac{t^2}{2} -$~~

$= \left[t\tau - \frac{\tau^2}{2} \right]_0^t$.

$= t^2 - \frac{t^2}{2}$,

$= \frac{t^2}{2}\, u(t)$.

Q. $h(t) = \delta(t) - \delta(t-1)$

$y(t) = u(t) * [\delta(t) - \delta(t-1)]$.

$= u(t) - u(t-1)$

$y(t) = u(t) * [\delta(t) - \delta(t-1)]$.

$= \int_{y=-\infty}^{\infty} u(\tau)\, \delta(t-\tau)\, d\tau$

$- \int_{y=-\infty}^{\infty} u(\tau)\, \delta(t-\tau-1)\, d\tau$.

$\Rightarrow \tau \to t$, $\tau \to t-1$.

$= u(t) \int_{-\infty}^{\infty} \delta(t-\tau)\, d\tau + u(t-1) \int_{-\infty}^{\infty} \delta(t+\tau-1)\, d\tau$

$\tau \to t$ $\tau \to t-1$

$\therefore = u(t) - u(t-1)$.

Q. $h_1(t) = u(t+2)$

Q. $h_2(t) = u(t-2)$

Q. $h_3(t) = u(t+2) + u(t-2)$

Ans 1 $h_1(t) = u(t+2)$.

$$y(t) = \int_{-\infty}^{\infty} u(\tau)\, u(t-\tau+2)\, d\tau$$

$\Rightarrow \tau = t+2$

$$= \int_0^{t+2} d\tau$$

$= (t+2) \cancel{u(t)}\, u(t+2)$

$h_2(t) = u(t-2)$

$$y_2(t) = \int_{-\infty}^{\infty} u(\tau)\, u(t-\tau-2)\, d\tau$$

$\Rightarrow \tau = t-2$

$$= \int_0^{t-2} d\tau$$

$= (t-2) \cancel{u(t)}\, u(t-2)$

$h_3(t) = u(t+2) + u(t-2)$

$$y_3(t) = \int_{-\infty}^{\infty} u(\tau)\, u(t-\tau+2)\, d\tau + \int_{-\infty}^{\infty} u(\tau)\, u(t-\tau-2)\, d\tau$$

$$= \int_0^{t+2} d\tau + \int_0^{t-2} d\tau$$

$= t+2+t-2$

$= 2t \cancel{u(t)}\, u(t+2)$

Generalised Equⁿ of Unit step Response.

$x(t) = u(t)$

$h(t)$

$y(t) = u(t) * h(t)$

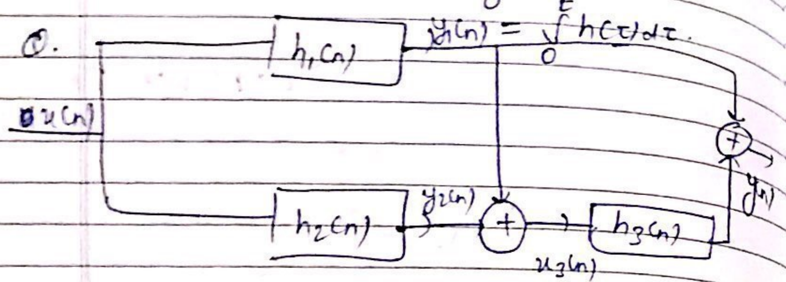
$$= \cancel{\int_{-\infty}^{\infty} u(\tau)\, h(t-\tau)\, d\tau}$$

$\int_0^t$

$$y(t) = \int_0^\infty h(t-\tau)\,d\tau = \int_{-\infty}^t h(\tau)\,d\tau$$

for causal → $y(t) = \int_0^t h(t-\tau)\,d\tau.$

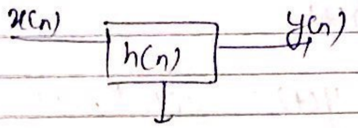
Q.



$h_1(n) = 2^n u(n)$

$h_2(n) = \frac{1}{3}^n u(n)$

$h_3(n) = \frac{2}{3}^n u(n)$



$$\left(u(n)*h_1(n) + u(n)*h_2(n)\right) * h_3(n)$$

$$+ \left(u(n)*h_1(n)\right) = y(n).$$

$$y_1(n) = u(n)*h_1(n) = \int_0^\infty 2^{k-n} u($$

$$= \sum_{k=0}^{\infty} 2^{n-k}\, u(n-k)\, \cancel{u(k-n)}.$$

$$= \sum_{k=0}^{n} 2^{n-k}\, \underbrace{u(n-k)}_{=1}.$$

$$= \sum_{k=0}^{n} 2^{n-k}$$

$$= 2^n \sum_{k=0}^{n} 2^{-k}$$

$$= 2^n \left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n}\right].$$

$$\cancel{2^n \frac{1}{1-\frac{1}{2}}}$$

$$= 2^n \frac{1\left(1-\frac{1}{2}^n\right)}{1-\frac{1}{2}} =$$

$$=\frac{(2^n-1)}{1}\times 2$$

$$=(2^{n+1}-2)$$

$$y(n)= u(n)*h_2(n) = \sum_{k=0}^{\infty} \frac{1}{3}^{n-k} u(n-k)$$

$$= \sum_{k=0}^{n} \frac{1}{3}^{n-k}$$

$$= \frac{1}{3}^n \sum_{k=0}^{n} \left(\frac{1}{3}\right)^{-k}$$

$$= \left(\frac{1}{3}\right)^n \sum_{k=0}^{n} 3^k$$

$$= 3^{-n}\left[1+3+9+\ldots+3^n\right].$$

$$= 3^{-n} \frac{1(3^n-1)}{3-1}$$

$$= \frac{1-3^{-n}}{2}$$

$$= \frac{1}{2}-\frac{1}{2\times 3^n}.$$

$$u(n)*h_1(n)+u(n)*h_2(n)$$

$$= 2^{n+1}-2+\frac{1}{2}-\frac{1}{2\times 3^n}$$

$$= 2^{n+1}-\frac{1}{2\times 3^n}-\frac{3}{2}.$$

$$h_3(n) * \left(2^{n+1}-\frac{1}{2\times 3^n}-\frac{3}{2}\right)$$

$$= \sum_{k=-\infty}^{n} \left(2^{k+1}-\frac{1}{2\times 3^k}-\frac{3}{2}\right) \left(\frac{2}{3}^{n-k} \cdot u(n-k)\right)$$

$$= \sum_{k=-\infty}^{n} \left(\frac{2}{3}^{k+1+n-k} - \frac{2^{n-k-1}}{3^{k+1}} - 2^{n-k-1}\right)$$

$$= \frac{2}{3}^{n+1} \left[\sum_{k=-\infty}^{n} \frac{2^{n-k-1}}{3^{k+1}} - \sum_{k=-\infty}^{n} 2^{n-k-1}\right].$$

RANKA DATE / / PAGE

$h_1(n) = 2^n u(n)$

$h_3(n) = \left(\frac{2}{3}\right)^n u(n)$.

$= \sum_{k=\infty}^{\infty} 2^k u(k) \left(\frac{2}{3}\right)^{n-k} u(n-k)$

$= \sum_{0}^{n} 2^k \left(\frac{2}{3}\right)^{n-k}$

$= \left(\frac{2}{3}\right)^n \sum_{0}^{n} 2^k \left(\frac{2}{3}\right)^{-k}$.

$= \left(\frac{2}{3}\right)^n \sum_{0}^{n} 3^k$

$= \left(\frac{2}{3}\right)^n \left(\frac{3^n-1}{3-1}\right)$

$= \left(\frac{2}{3}\right)^n \left(\frac{3^n-1}{2}\right)$

$= 2^{n-1}\left(1-\frac{1}{3^n}\right) u(n)$.

RANKA DATE / / PAGE

$h_2(n) = \left(\frac{1}{3}\right)^n u(n)$

$h_3(n) = \left(\frac{2}{3}\right)^n u(n)$.

$= \sum_{-\infty}^{\infty} \left(\frac{1}{3}\right)^k u(k) \left(\frac{2}{3}\right)^{n-k} u(n-k)$

$= \left(\frac{2}{3}\right)^n \sum_{0}^{n} \left(\frac{1}{3}\right)^k \left(\frac{2}{3}\right)^{-k}$

$= \left(\frac{2}{3}\right)^n \sum_{0}^{n} 2^{-k}$.

$= \left(\frac{2}{3}\right)^n \left(1+\frac{1}{2}+\cdots+\frac{1}{2^n}\right)$

$= \left(\frac{2}{3}\right)^n \left[\frac{1-\frac{1}{2^n}}{1-\frac{1}{2}}\right]$.

$= \frac{1}{3^n}\left(\frac{2^n-1}{1}\right)^2$

$= \frac{2}{3^n}(2^n-1).\ u(n)$

$$h_1(n) + h_1(n) * h_3(n) + h_2(n) * h_3(n)$$

$$= 2^n u(n) + 2^{n-1} \left(1 - \frac{1}{3^n}\right) u(n)$$

$$+ \frac{2}{3^n} (2^n - 1)\, u(n).$$