AC CIRCUITS & TIME VARYING SIGNALS

01205479 IoT for EE

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- Capacitors
- Inductors
- Periodic Signals
- Phasor Method
- System concepts
- LPF and other noise filtering techniques

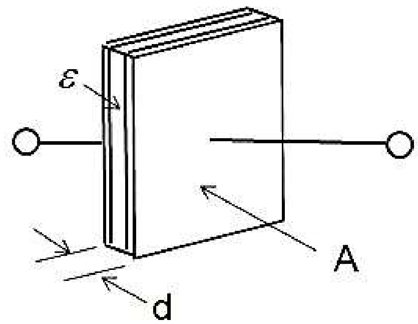
OUTLINE

CAPACITORS

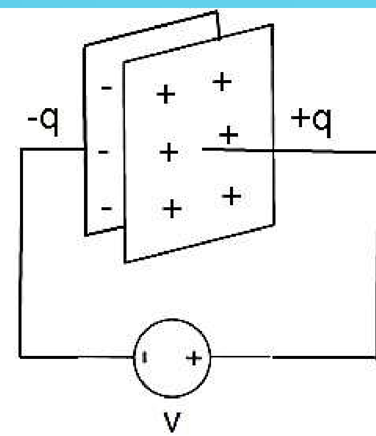
capacitance

$$C = \frac{\varepsilon A}{d}$$

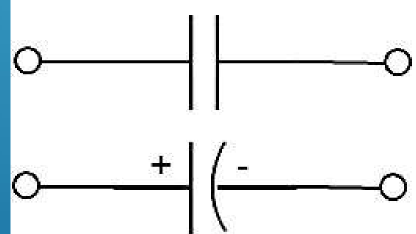
Farad



Physical structure



Capacitor when voltage is applied


Symbol

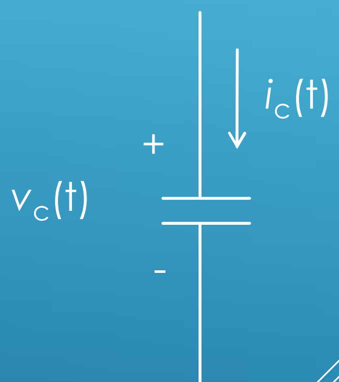

Different types of capacitors

VOLTAGE AND CURRENT

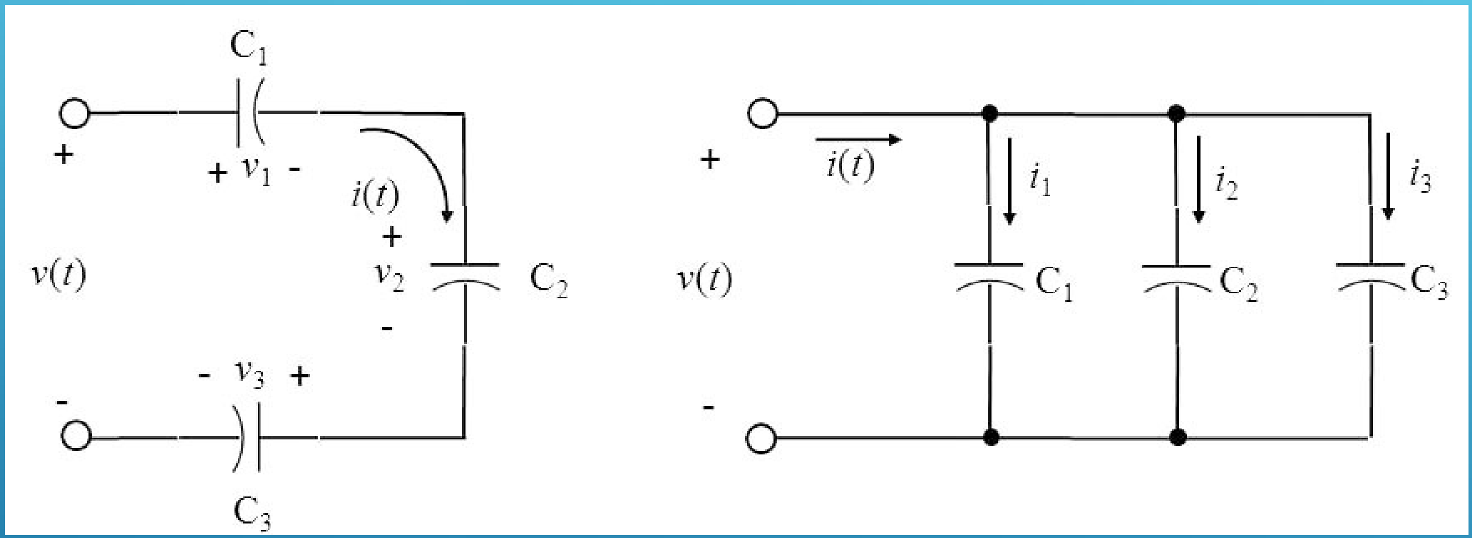
Charge : $$q(t) = Cv_c(t)$$

Current : $$i_c(t) = \frac{dq(t)}{dt} = C\frac{dv_c(t)}{dt}$$

Voltage : $$v_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(\tau)\,d\tau$$



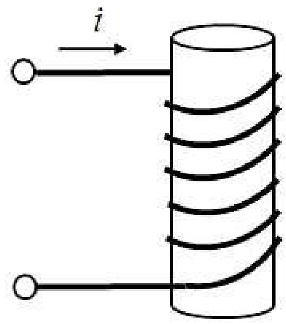
TOTAL CAPACITANCE



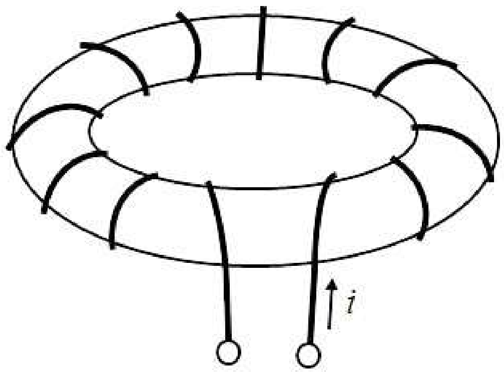
$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

$$C_{eq} = C_1 + C_2 + C_3$$

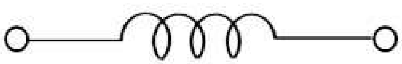
INDUCTORS



Physical structure



Toroid type



Symbol

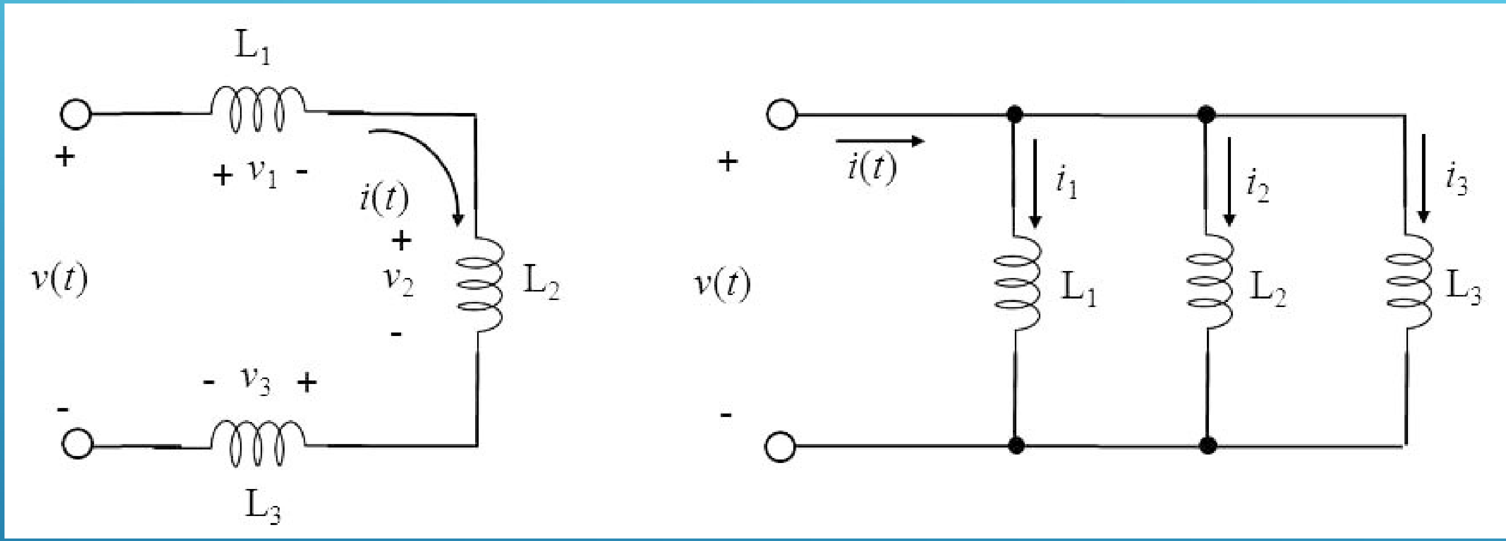
Different types of inductors

VOLTAGE AND CURRENT

Current $$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(\tau)\,d\tau$$

Voltage $$v_L(t) = L\frac{di_L(t)}{dt}$$

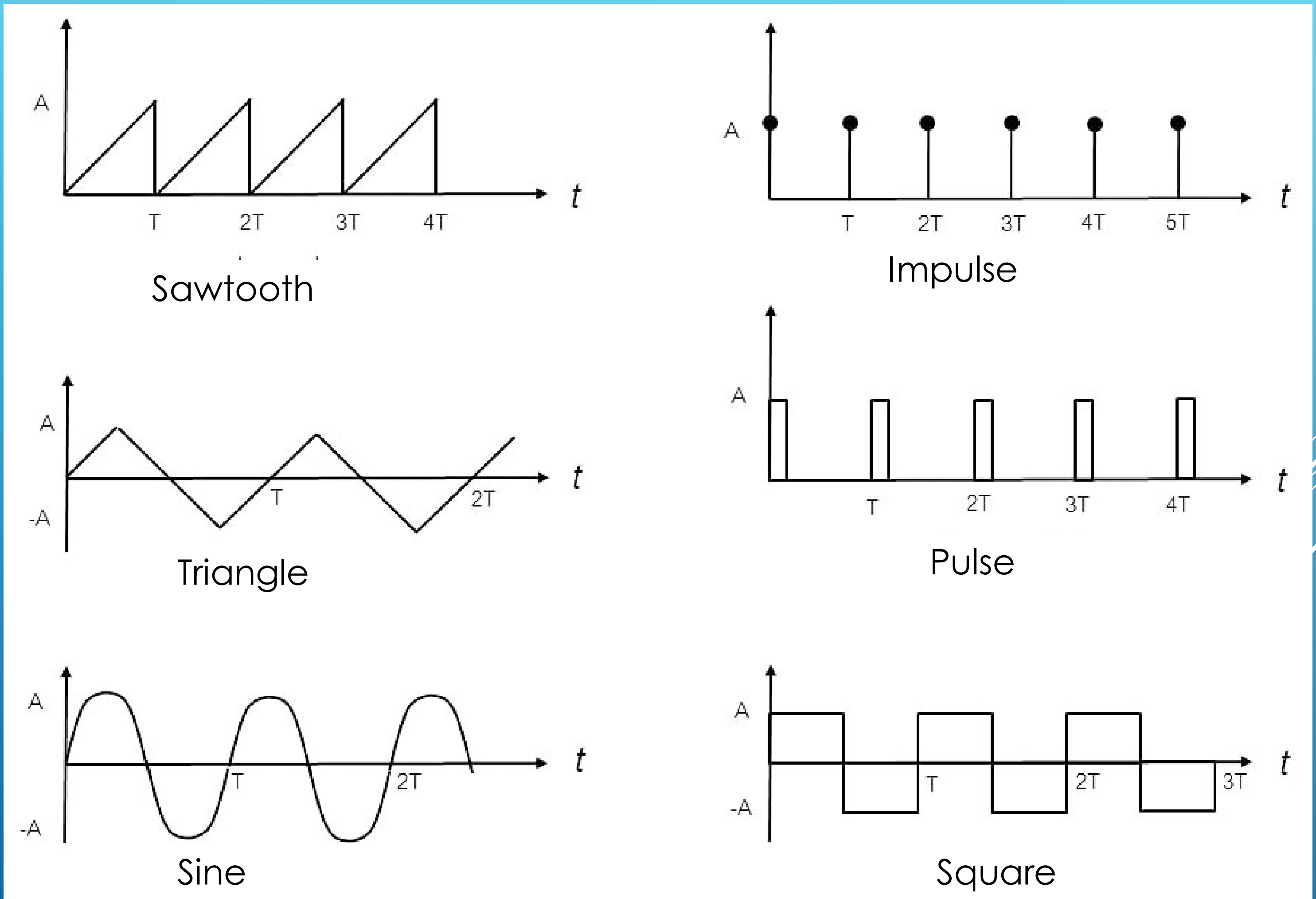
TOTAL INDUCTANCE



$$L_{eq} = L_1 + L_2 + L_3$$

$$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}}$$

PERIODIC SIGNALS



SINUSOID

$$x(t) = A\cos(\omega t + \phi)$$

magnitude

frequency: radian

phase

$$A\sin(\omega t) = A\cos\left(\omega t - \frac{\pi}{2}\right)$$

Exercise: write Python code to plot sine and cosine waveforms

PHASOR METHOD

sinusoid signal $$v(t) = A\cos(\omega t + \theta)$$

phasor $$V(j\omega) = Ae^{j\theta} = A\angle\theta$$

frequency dependent

magintude

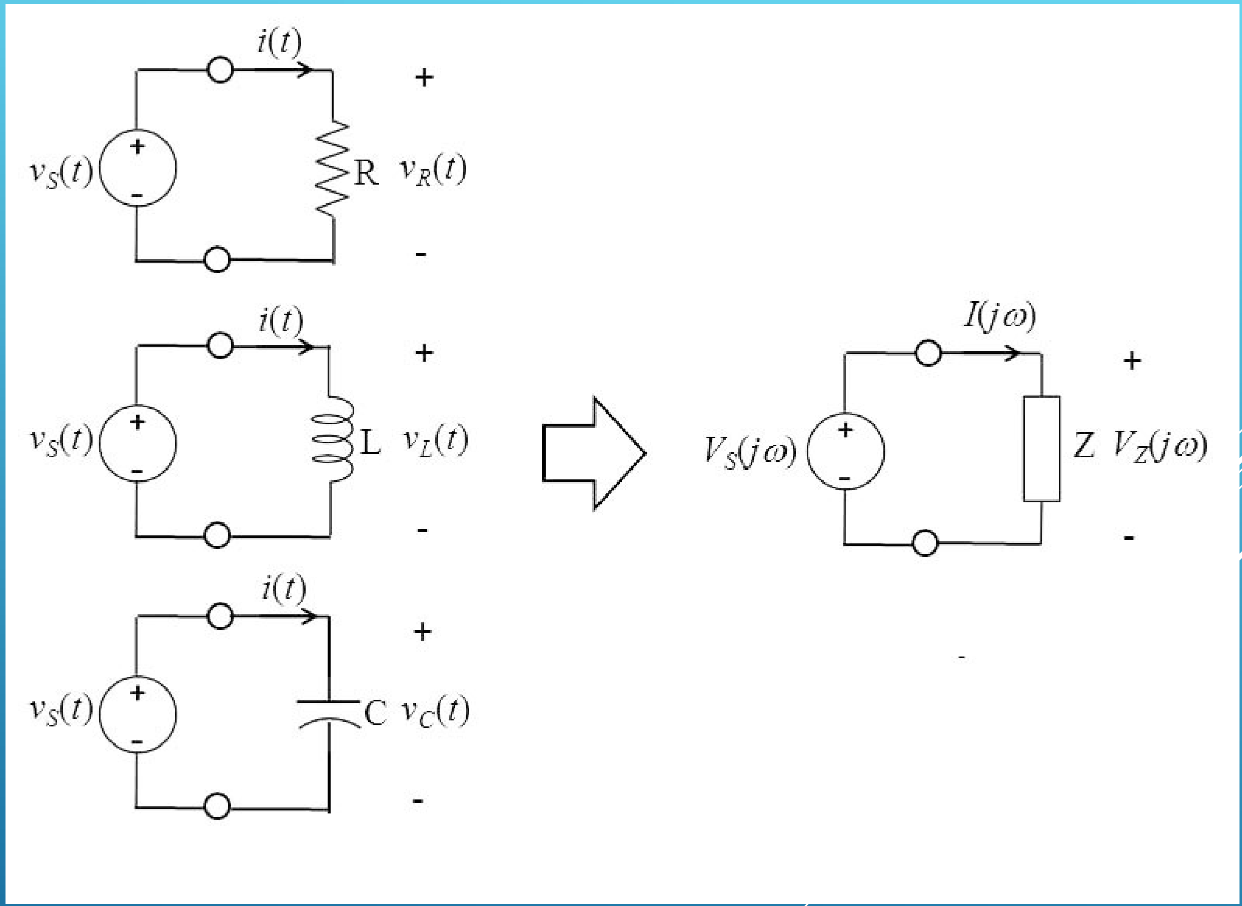
phase

Applicable when all signals in a circuit have only one common frequency

IMPEDANCE

$$v_S(t) = A\cos\omega t$$

$$V_S(j\omega) = Ae^{j\theta} = A\angle\theta$$



RESISTORS

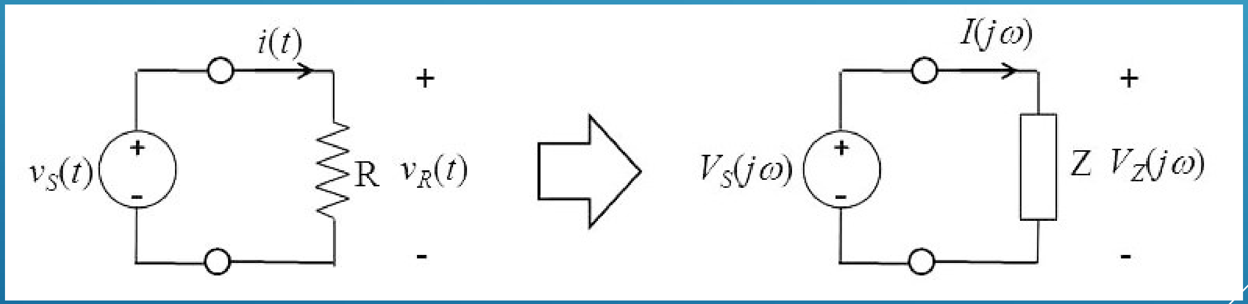
$v = iR$

$$i(t) = \frac{v_S(t)}{R} = \frac{A}{R}\cos\omega t$$

$$V_S(j\omega) = A\angle 0$$

$$I(j\omega) = \frac{A}{R}\angle 0$$

$$\boxed{Z_R(j\omega) = \frac{V_S(j\omega)}{I(j\omega)} = R}$$



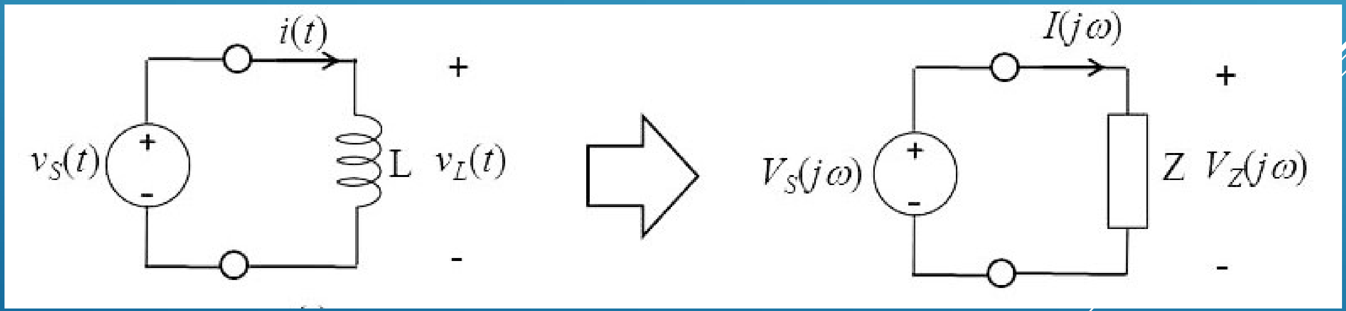
INDUCTORS

$$i(t) = \frac{1}{L}\int v_S(\tau)d\tau$$
$$= \frac{1}{L}\int A\cos\omega\tau d\tau$$
$$= \frac{A}{\omega L}\sin\omega t$$

$$i(t) = \frac{A}{\omega L}\cos\left(\omega t - \frac{\pi}{2}\right)$$

$$V_S(j\omega) = A\angle 0$$
$$I(j\omega) = \frac{A}{\omega L}\angle -\frac{\pi}{2}$$

$$\boxed{Z_L(j\omega) = \frac{V_S(j\omega)}{I(j\omega)} = \omega L\angle\frac{\pi}{2} = j\omega L}$$



CAPACITORS

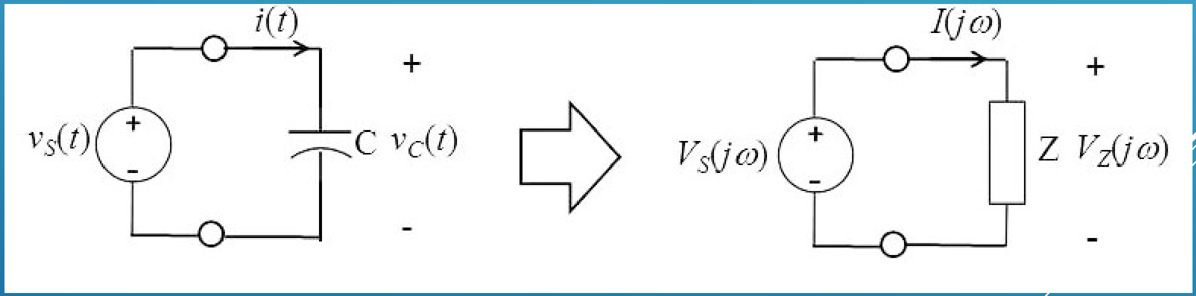
$$i(t) = C\frac{dv_S(t)}{dt}$$
$$= C\frac{d}{dt}(A\cos\omega t)$$
$$= -C(A\omega\sin\omega t)$$

$$i(t) = \omega CA\cos\left(\omega t + \frac{\pi}{2}\right)$$

$$V_S(j\omega) = A\angle 0$$

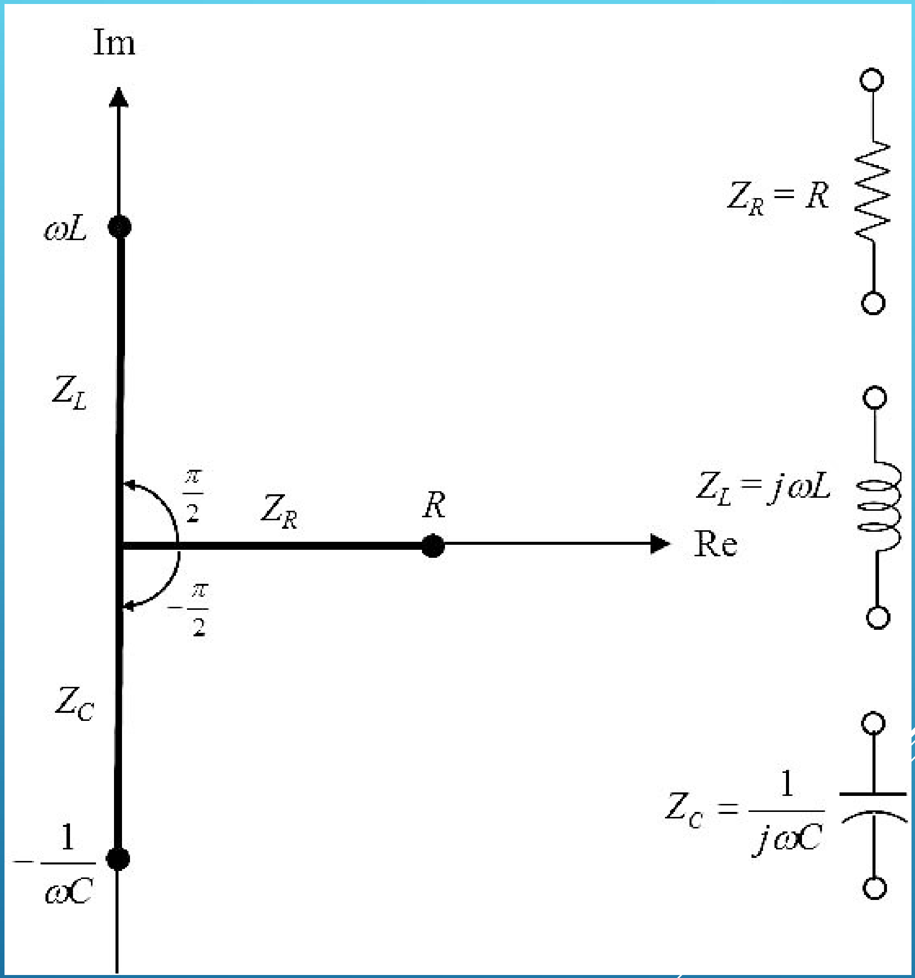
$$I(j\omega) = \omega CA\angle\frac{\pi}{2}$$

$$\boxed{\begin{aligned} Z_C(j\omega) &= \frac{V_S(j\omega)}{I(j\omega)} = \frac{1}{\omega C}\angle -\frac{\pi}{2} \\ &= \frac{-j}{\omega C} = \frac{1}{j\omega C} \end{aligned}}$$



IMPEDANCE OF RLC CIRCUIT

$$Z(j\omega) = R(j\omega) + jX(j\omega)$$



TOTAL IMPEDANCE

Series :

$$Z_{EQ} = \sum_{n=1}^{N} Z_n$$

Parallel :

$$Z_{EQ} = \frac{1}{1/Z_1 + 1/Z_2 + \ldots + 1/Z_N}$$

VOLTAGE/CURRENT DIVIDER

Voltage divider

$$V_n = \frac{Z_n}{Z_1 + Z_2 + \ldots Z_n + \ldots + Z_N} V_S$$

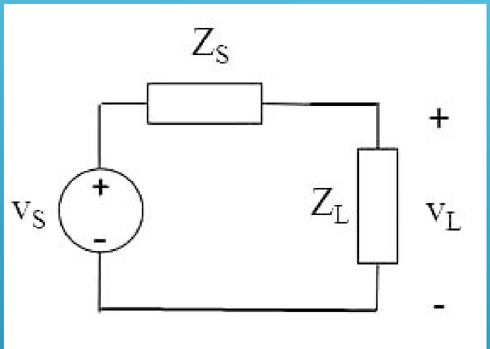
Current divider

$$I_n = \frac{1/Z_n}{1/Z_1 + 1/Z_2 + \ldots 1/Z_n + \ldots + 1/Z_N} I_S$$

SYSTEM CONCEPTS

Frequency responses & transfer functions

FREQUENCY RESPONSE OF AC CIRCUITS



$$V_L = \frac{Z_L}{Z_S + Z_L} V_S$$

$$\frac{V_L}{V_S} = \frac{Z_L}{Z_S + Z_L}$$

$$\boxed{V_L(j\omega) = H(j\omega) V_S(j\omega)}$$

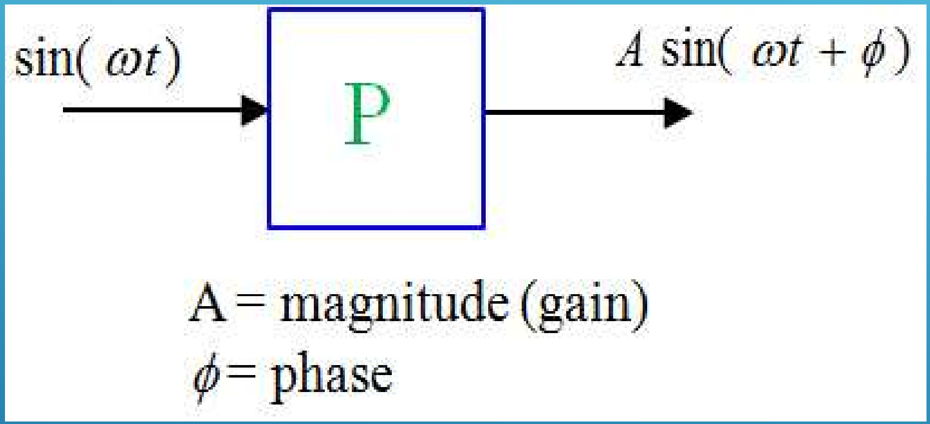
$$V_L e^{j\phi_L} = |H| e^{j\angle H} V_S e^{j\phi_S} = |H| V_S e^{j\angle H + \phi_S}$$

$$V_L = |H| V_S$$ magnitude

$$\phi_L = \angle H + \phi_S$$ phase

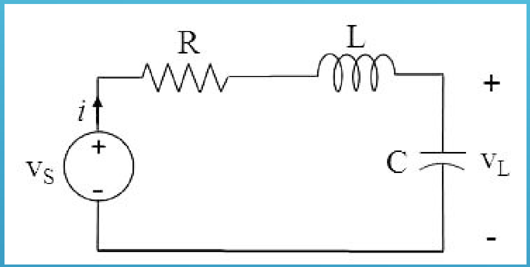
LINEAR TIME INVARIANT (LTI) SYSTEMS

transfer function



Math tool : Laplace Transform

EXAMPLE: RLC CIRCUIT



Let R = 100 Ω, C = 10 μF and L = 5 mH

KVL: $$iR + L\frac{di}{dt} + v_L = v_S$$

$$i = C\frac{dv_L}{dt} \Rightarrow RC\frac{dv_L}{dt} + LC\frac{d^2 v_L}{dt^2} + v_L = v_S$$

Laplace's $$RCsv_L(s) + LCs^2 v_L(s) + v_L(s) = v_S(s)$$

Substitution

$$P(s) = \frac{v_L(s)}{v_S(s)} = \frac{1}{LCs^2 + RCs + 1} \Rightarrow \boxed{P(s) = \frac{1}{0.05s^2 + 0.001s + 1}}$$

EXERCISE:

Create transfer function using Python control library and plot frequency response

$$\boxed{P(s) = \frac{1}{0.05s^2 + 0.001s + 1}}$$

Useful commands:

```
import control as ctl
s = ctl.tf('s')

_,_,_ = ctl.bode_plot(P,dB=True,omega_limits=(0.01,1000))
```

Relationships between transfer function and frequency response

$$\boxed{P(j\omega) = P(s)\big|_{s=j\omega}}$$

$$P(s) = \frac{1}{0.05s^2 + 0.001s + 1}$$

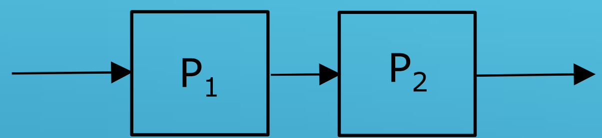
$$\Rightarrow$$

$$P(j\omega) = \frac{1}{0.05(j\omega)^2 + 0.001(j\omega) + 1}$$

$$= \frac{1}{1 - 0.05\omega^2 + j0.001\omega}$$

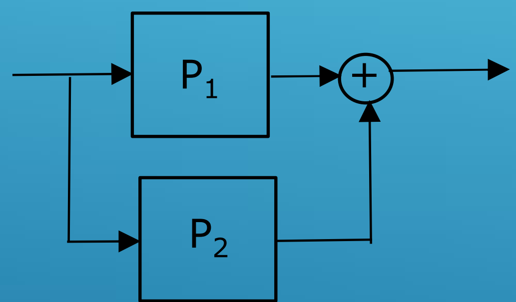
BLOCK DIAGRAM REDUCTION

Series



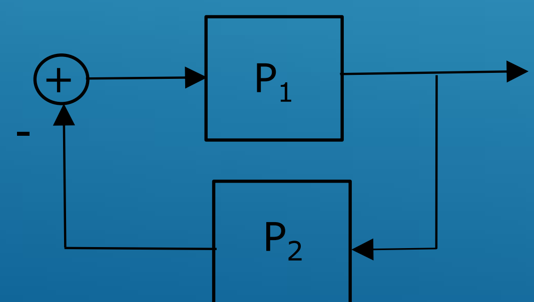
$$P = P_1 P_2$$

Parallel



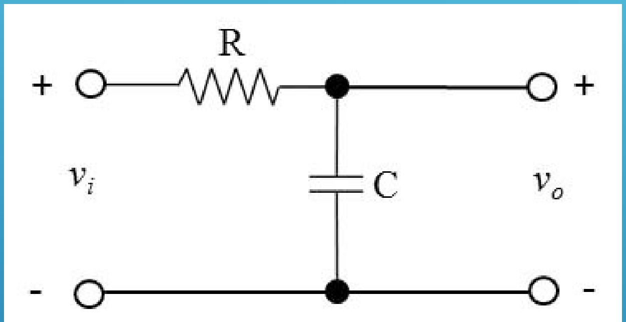
$$P = P_1 + P_2$$

Feedback
(negative)

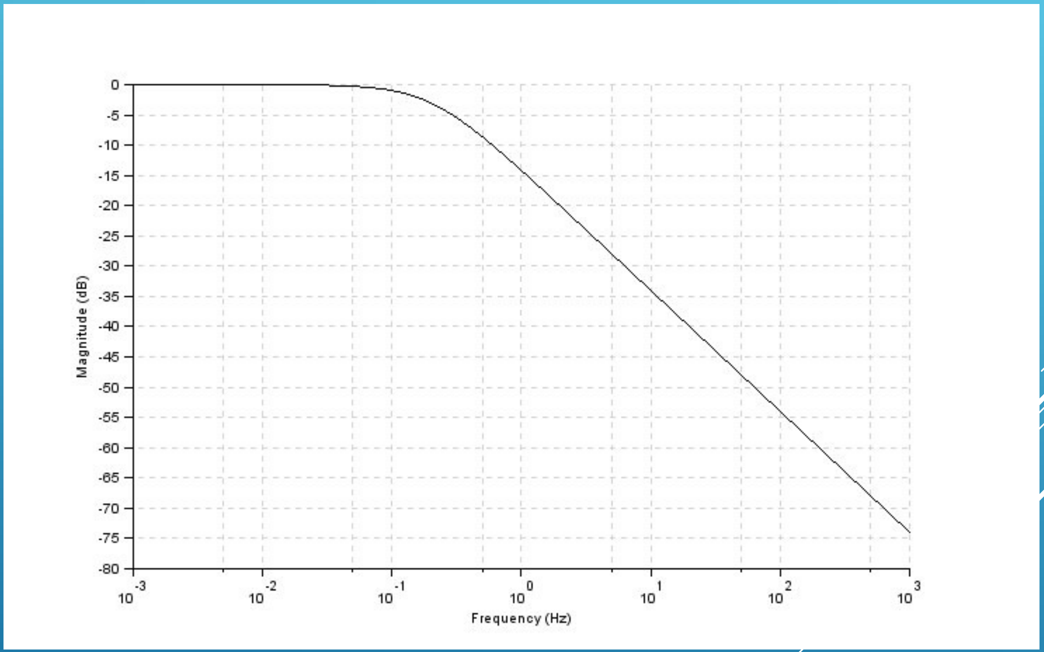


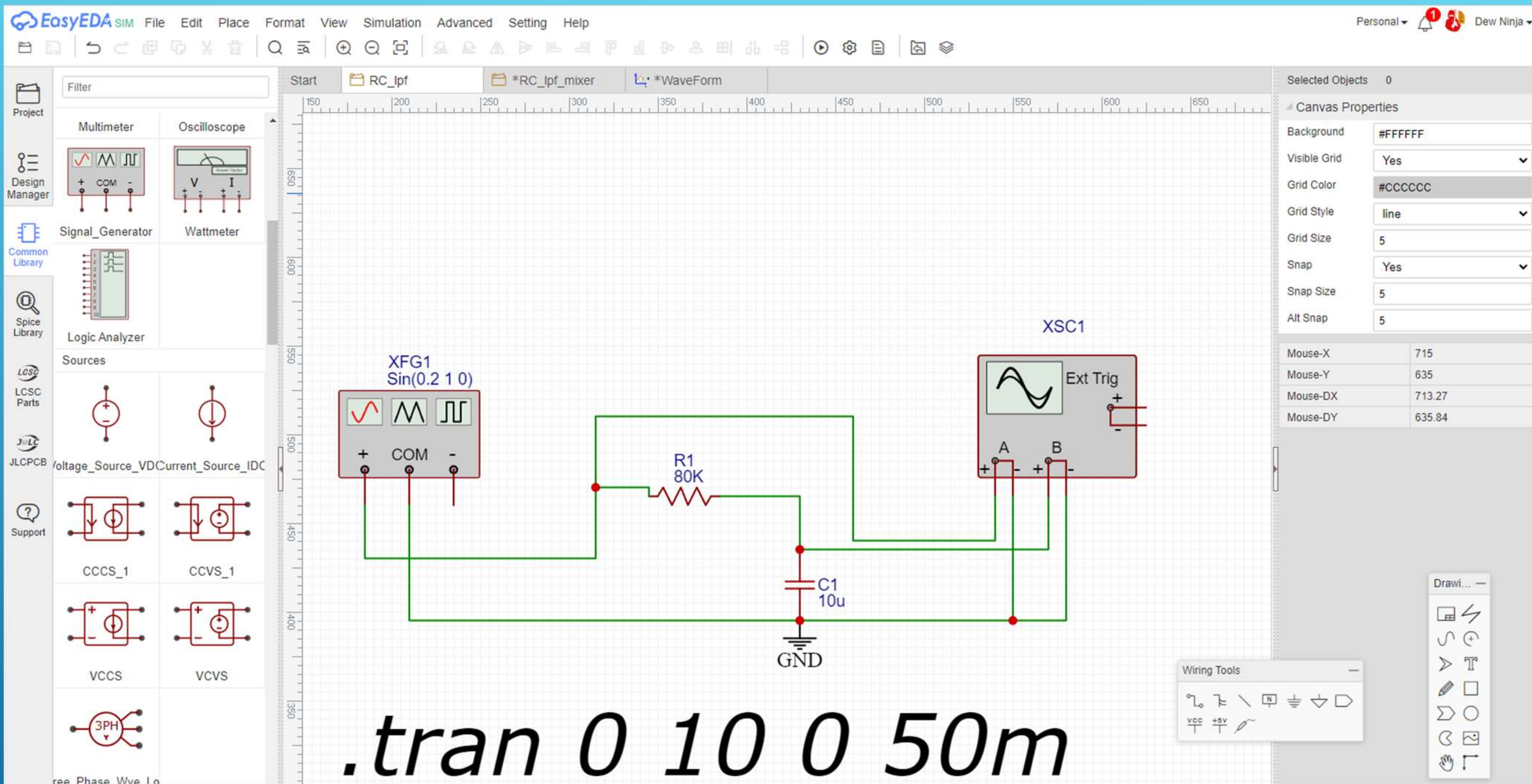
$$P = \frac{P_1}{1 + P_1 P_2}$$

EX. PASSIVE LOW-PASS FILTER (LPF)

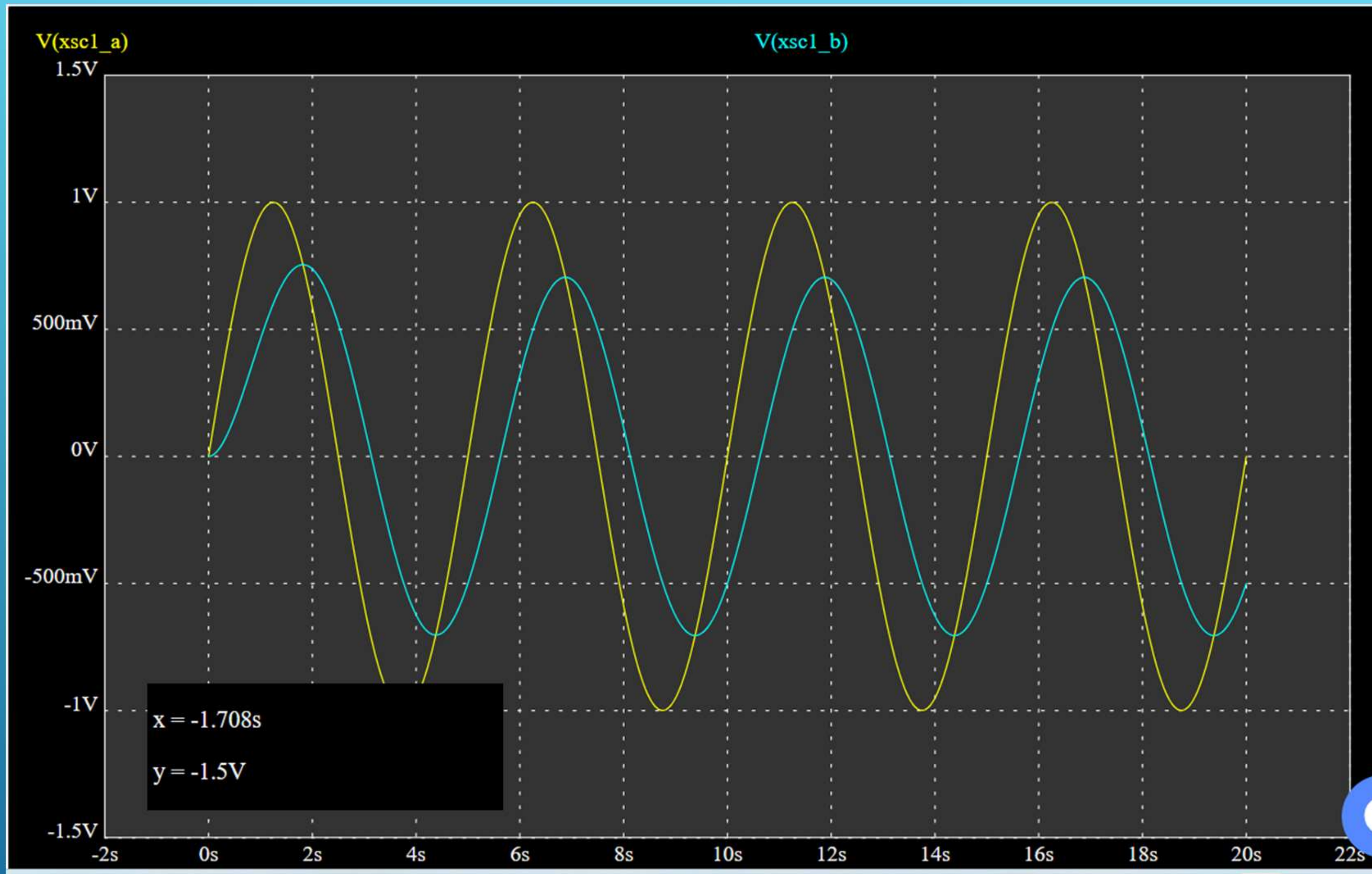


$$\omega_o = 2\pi f = \frac{1}{RC}$$

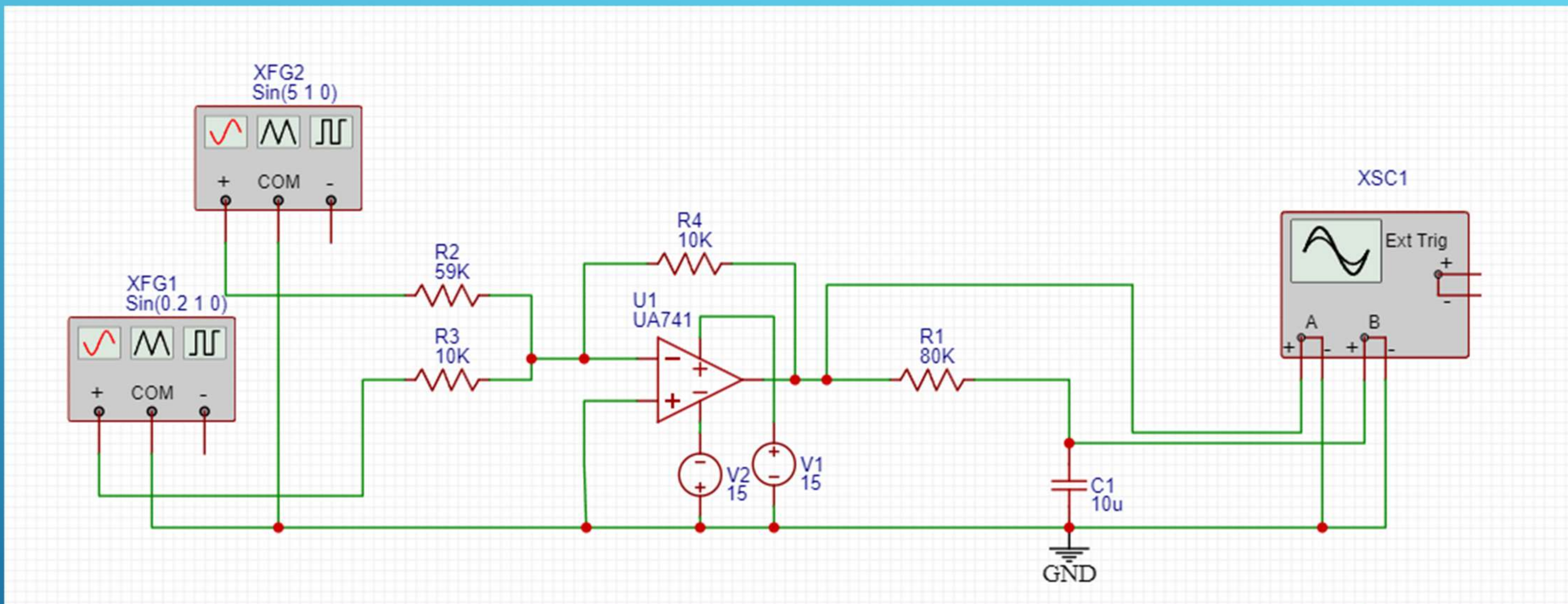




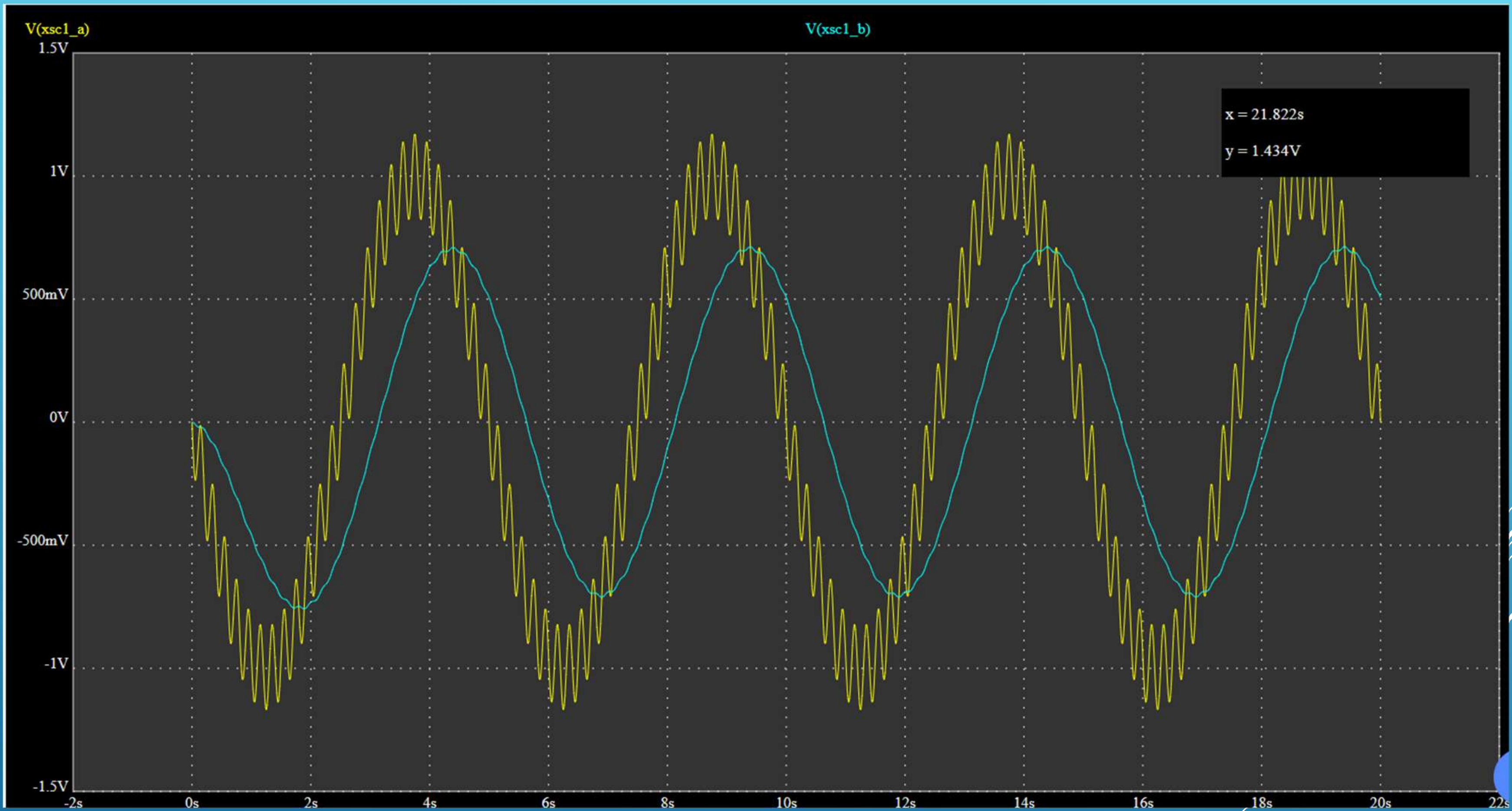
LPF SIMULATION ON EASYEDA



INPUT-OUTPUT FROM OSCILLOSCOPE

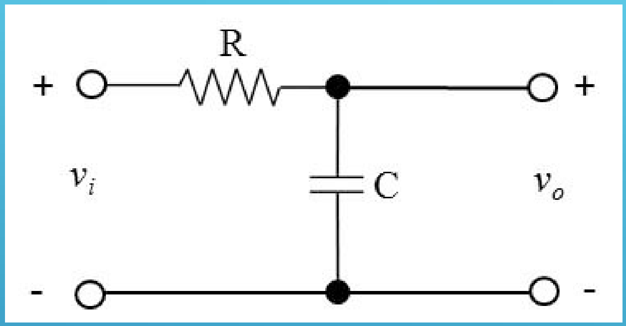


SIGNAL + NOISE SIMULATION



INPUT – OUTPUT COMPARISION

EXERCISE:



- Design an LPF filter with cutoff frequency 5 Hz
- Plot frequency response
- Apply base frequency (1 Hz) plus noise (20 Hz) to the LPF. Compare input and output signals

OTHER NOISE FILTERING METHODS

- Moving average
- Exponentially-weighted moving average

see averaging.ipynb