Deep Neural Networks (DNN)

Dr.Varodom Toochinda

Dept. of Mechanical Engineering

Kasetsart University

Outline

- Logistic regression
- Neural network with one hidden layer
- Nonlinear activation functions
- DNN model
- Computational graph
- Multi-class classification
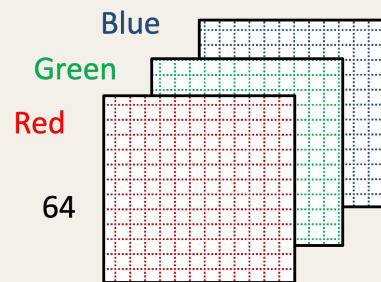
Logistic regression
(*การถดถอยลอจิสติก*)

$$X = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots \\ X^{(1)} & X^{(2)} & \dots & X^{(m)} \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

$$Y = \begin{bmatrix} y^{(1)} & y^{(2)} & \dots & y^{(m)} \end{bmatrix}$$

binary classification

Outcome

$$Y = \begin{cases} 1 : \text{cat} \\ 0 : \text{no cat} \end{cases}$$

Input vector

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \\ x_n \end{bmatrix} \begin{matrix} \left.\vphantom{x_1}\right\}\text{Red} \\ \left.\vphantom{x}\right\}\text{Green} \\ \left.\vphantom{x_n}\right\}\text{Blue} \end{matrix} \Bigg\} n_x$$

Blue
Green
Red

$n = n_x = 64 \times 64 \times 3 = 12{,}288$ elements
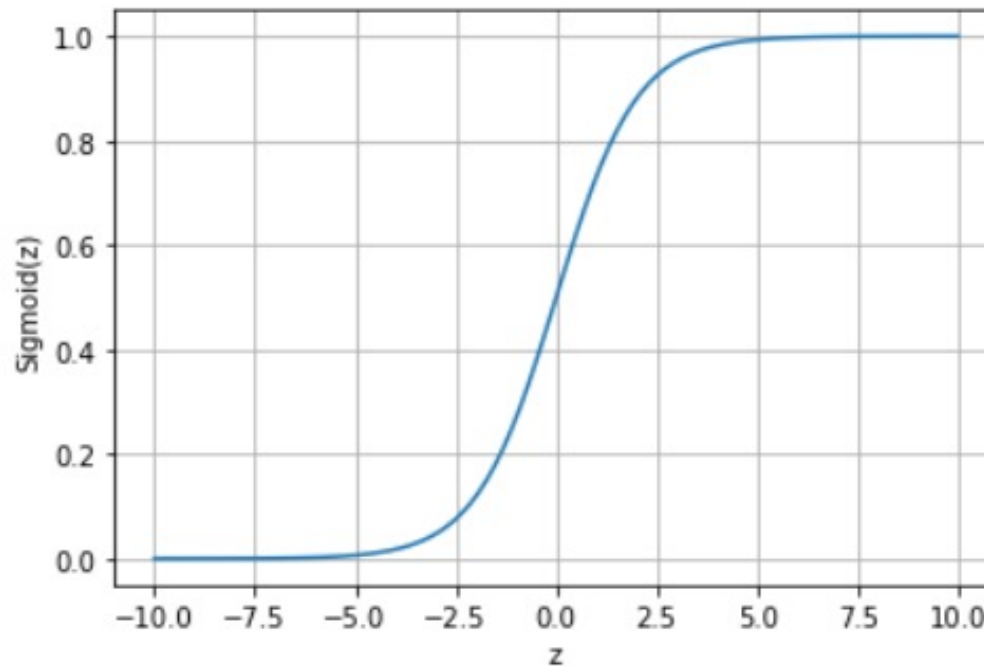
Sigmoid activation function

in range [0,1]

$$\hat{y} = \sigma(w^T x + b)$$

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

```
z = np.linspace(-10, 10, 100)
y = 1/(1 + np.exp(-z))

plt.plot(z,y)
plt.xlabel("z")
plt.ylabel("Sigmoid(z)")
plt.grid()
plt.show()
```
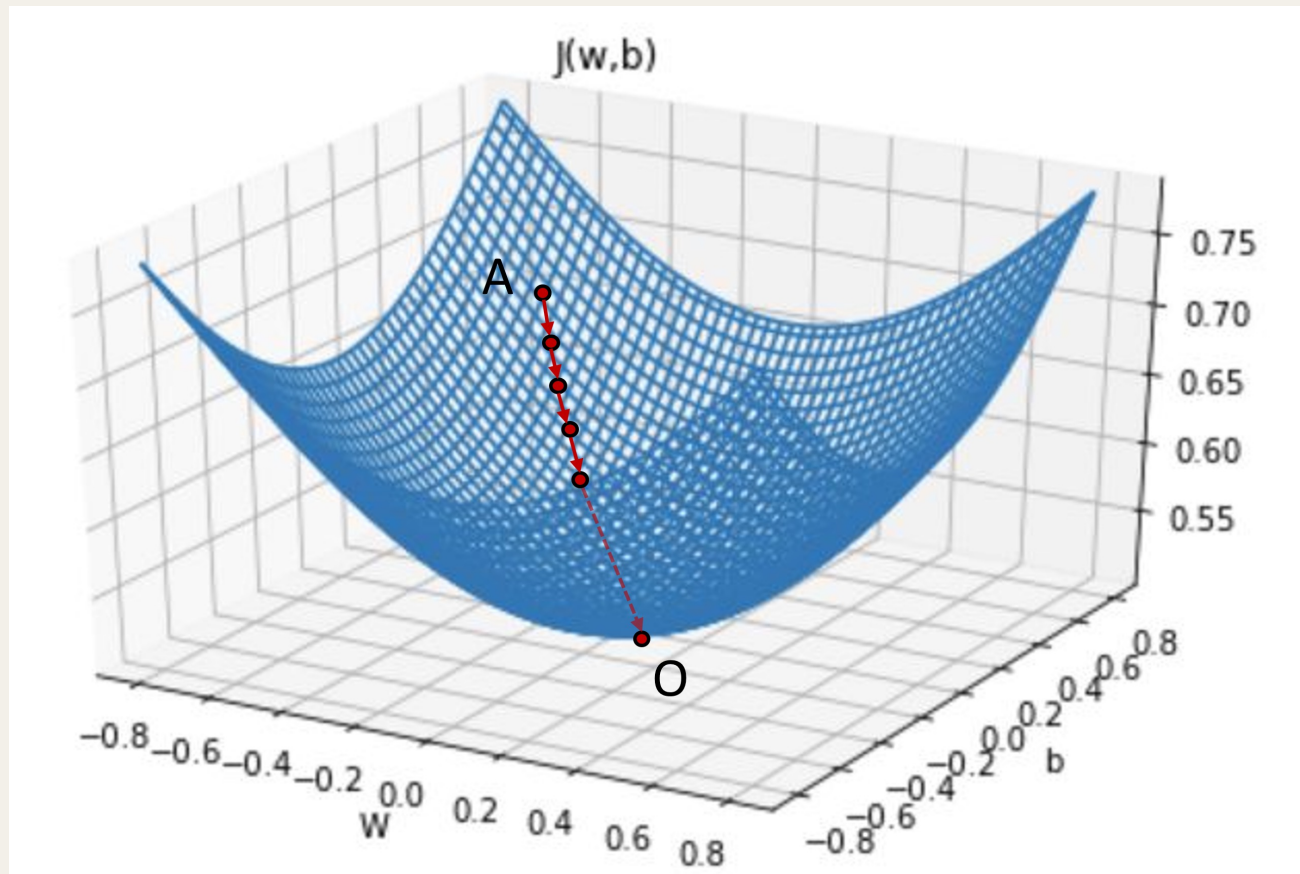
Loss function

convex function

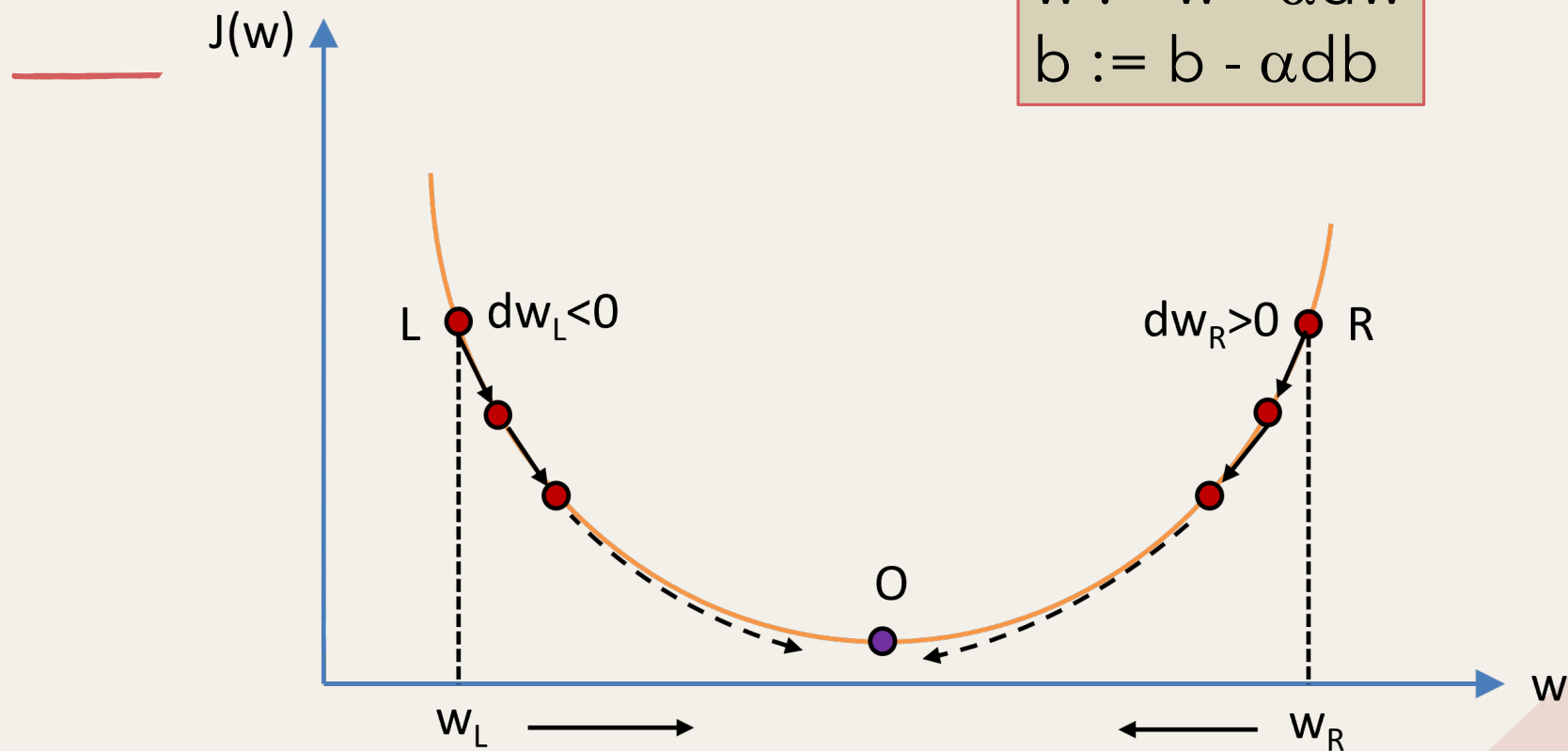$$\mathcal{L}(\hat{y}, y) = -(y\ log\hat{y} + (1 - y)log(1 - \hat{y}))$$

$$J(w, b) = \frac{1}{m}\sum_{i=1}^{m}\mathcal{L}(\hat{y}^{(i)}, y^{(i)}) = -\frac{1}{m}\sum_{i=1}^{m}[y^{(i)}log\hat{y}^{(i)} + (1 - y^{(i)})log(1 - \hat{y}^{(i)})]$$

sometimes called "cost funciton"

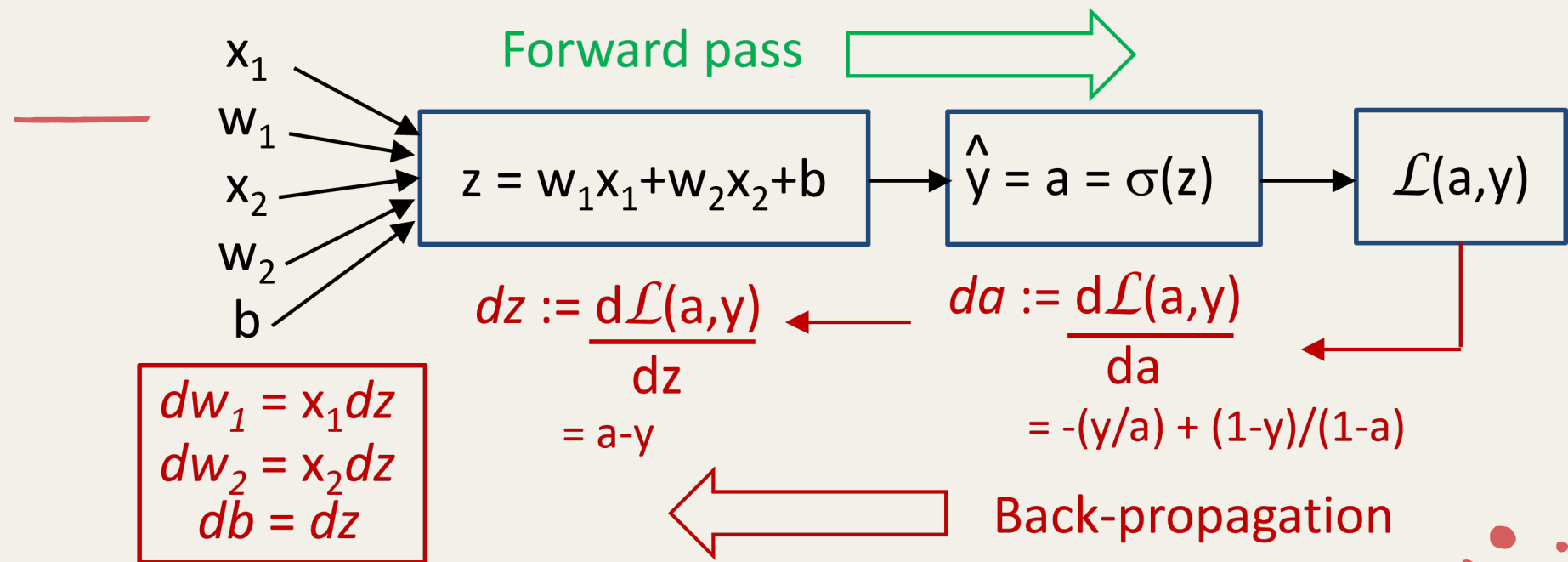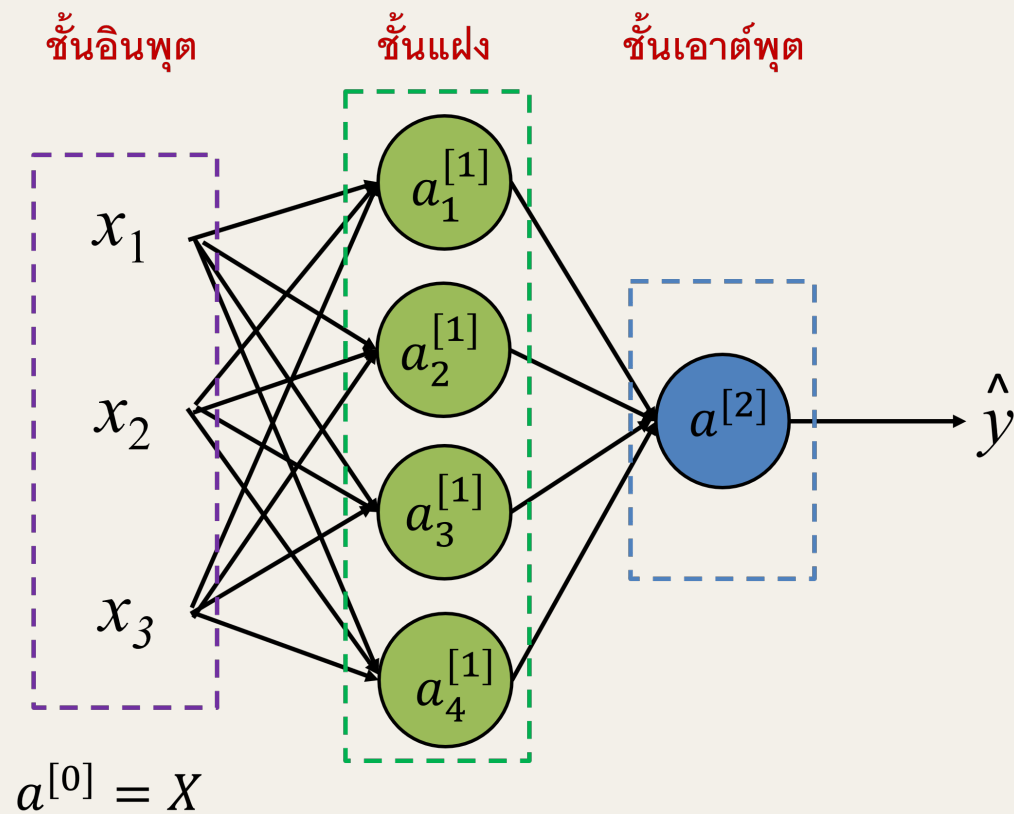# Gradient descent

Gradient descent in 2D

$$w := w - \alpha dw$$
$$b := b - \alpha db$$

$J(w)$

L $dw_L < 0$

$dw_R > 0$ R

O

w_L w_R

w

Computational graph of logistic regression

Forward pass →

x_1
w_1
x_2
w_2
b

$$z = w_1x_1 + w_2x_2 + b$$

→ $$\hat{y} = a = \sigma(z)$$

→ $$\mathcal{L}(a,y)$$

$$dz := \frac{d\mathcal{L}(a,y)}{dz}$$
$$= a - y$$

$$da := \frac{d\mathcal{L}(a,y)}{da}$$
$$= -(y/a) + (1-y)/(1-a)$$

$$dw_1 = x_1 dz$$
$$dw_2 = x_2 dz$$
$$db = dz$$

← Back-propagation

see details in logistic_regression.ipynb

Neural network with one hidden layer

ชั้นอินพุต

ชั้นแฝง

ชั้นเอาต์พุต

x_1

x_2

x_3

$a_1^{[1]}$

$a_2^{[1]}$

$a_3^{[1]}$

$a_4^{[1]}$

$a^{[2]}$

$\hat{y}$

$a^{[0]} = X$

Neural network with one hidden layer

$z_1^{[1]} = w_1^{[1]T} x + b_1^{[1]}$

$a_1^{[1]} = g^{[1]}(z_1^{[1]})$

x_1

x_2

x_3

$a_1^{[1]}$

$a_2^{[1]}$

$a_3^{[1]}$

$a_4^{[1]}$

$a^{[2]}$

$\hat{y}$
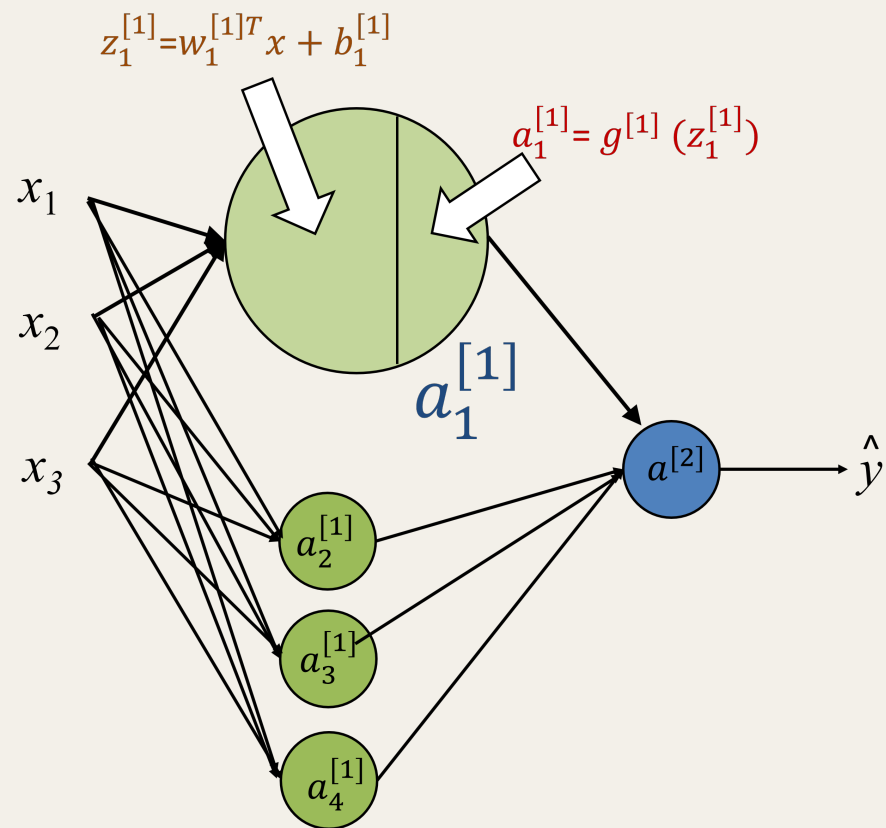
Neural network with one hidden layer (vectorized)

$$\hat{Y} = \begin{bmatrix} \hat{y}^{(1)} & \hat{y}^{(2)} & \dots & \hat{y}^{(m)} \end{bmatrix}$$

$$Z^{[l]} = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots \\ z^{[l](1)} & z^{[l](2)} & \dots & z^{[l](m)} \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

$$A^{[1]} = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots \\ a^{[l](1)} & a^{[l](2)} & \dots & a^{[l](m)} \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

forward pass

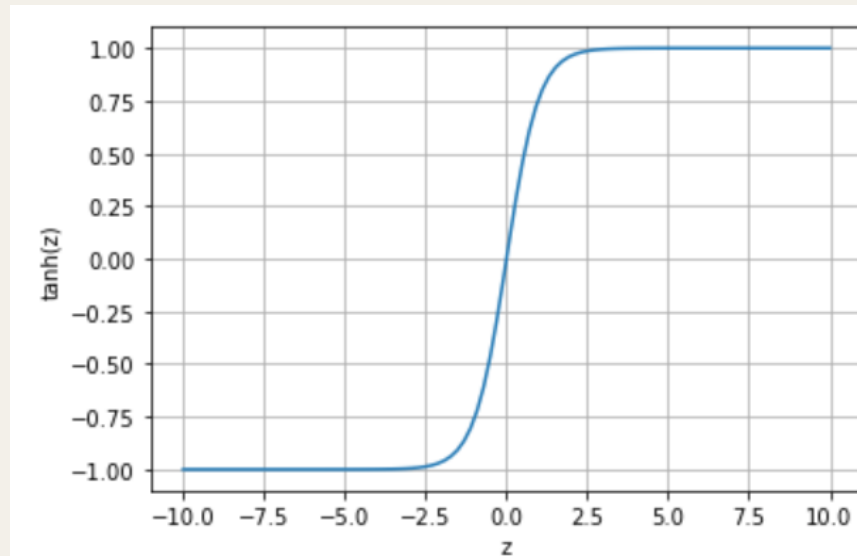$$Z^{[1]} = W^{[1]}X + b^{[1]}$$

$$A^{[1]} = g^{[1]}(Z^{[1]})$$

$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$

$$\hat{Y} = A^{[2]} = g^{[2]}(Z^{[2]})$$

tanh() activation function


```
z = np.linspace(-10, 10, 100)
y = np.tanh(z)
plt.plot(z,y)
plt.xlabel("z")
plt.ylabel("tanh(z)")
plt.grid()
plt.show()
```
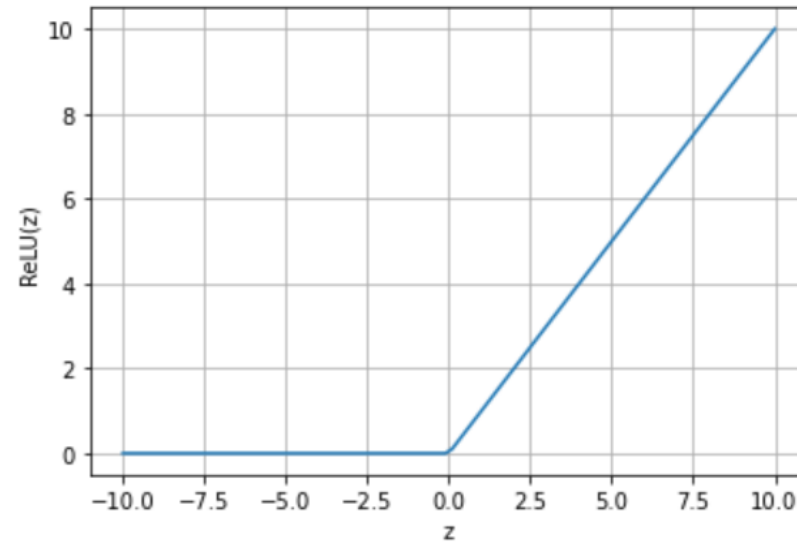
$$tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

ReLU activation function

```
def ReLU(z):
    return z*(z>0)

z = np.linspace(-10, 10, 100)
y = ReLU(z)
plt.plot(z,y)
plt.xlabel("z")
plt.ylabel("ReLU(z)")
plt.grid()
plt.show()
```
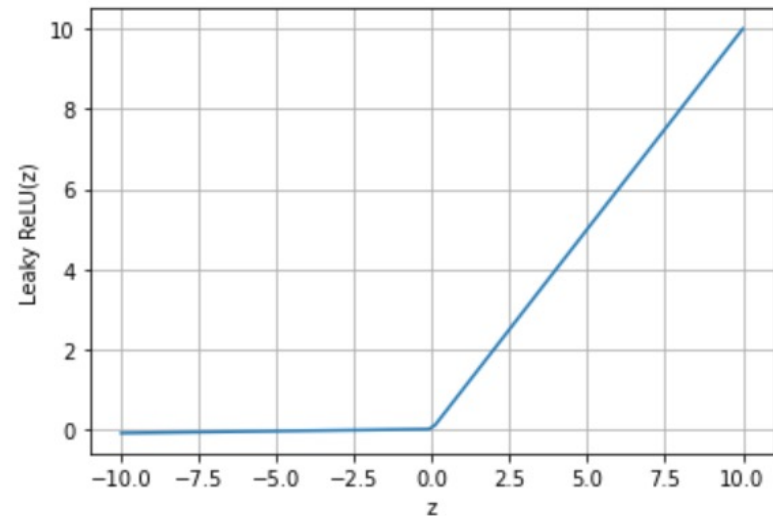

$$f(z) = max(0, z)$$

leaky ReLU activation function


```
def LeakyReLU(z):
    return np.where(z>0, z, z*0.01)

z = np.linspace(-10, 10, 100)
y = LeakyReLU(z)
plt.plot(z,y)
plt.xlabel("z")
plt.ylabel("Leaky ReLU(z)")
plt.grid()
plt.show()
```

$$f(z) = max(\alpha z, z)$$

2.3.4 อนุพันธ์ของฟังก์ชันกระตุ้น

เริ่มจากฟังก์ชันซิกมอยด์ $\sigma()$ (2.4) สามารถแสดงโดยแคลคูลัสได้อนุพันธ์ดังนี้

$$g'(z) = \frac{1}{1+e^{-z}}\left(1 - \frac{1}{1+e^{-z}}\right) = g(z)(1 - g(z)) = a(1 - a)$$

โดยด้านขวาสุดของ (2.36) มาจากการใช้สัญกรณ์ $a = g(z)$

กรณีฟังก์ชัน $tanh()$ (2.33) ได้อนุพันธ์เท่ากับ

$$g'(z) = 1 - (tanh(z))^2 = 1 - g^2(z) = 1 - a^2$$

สำหรับฟังก์ชัน ReLU (2.34)

$$g'(z) = \begin{cases} 0 & \text{if} \quad z < 0 \\ 1 & \text{if} \quad z > 0 \end{cases}$$

และสำหรับ Leaky ReLU (2.35)

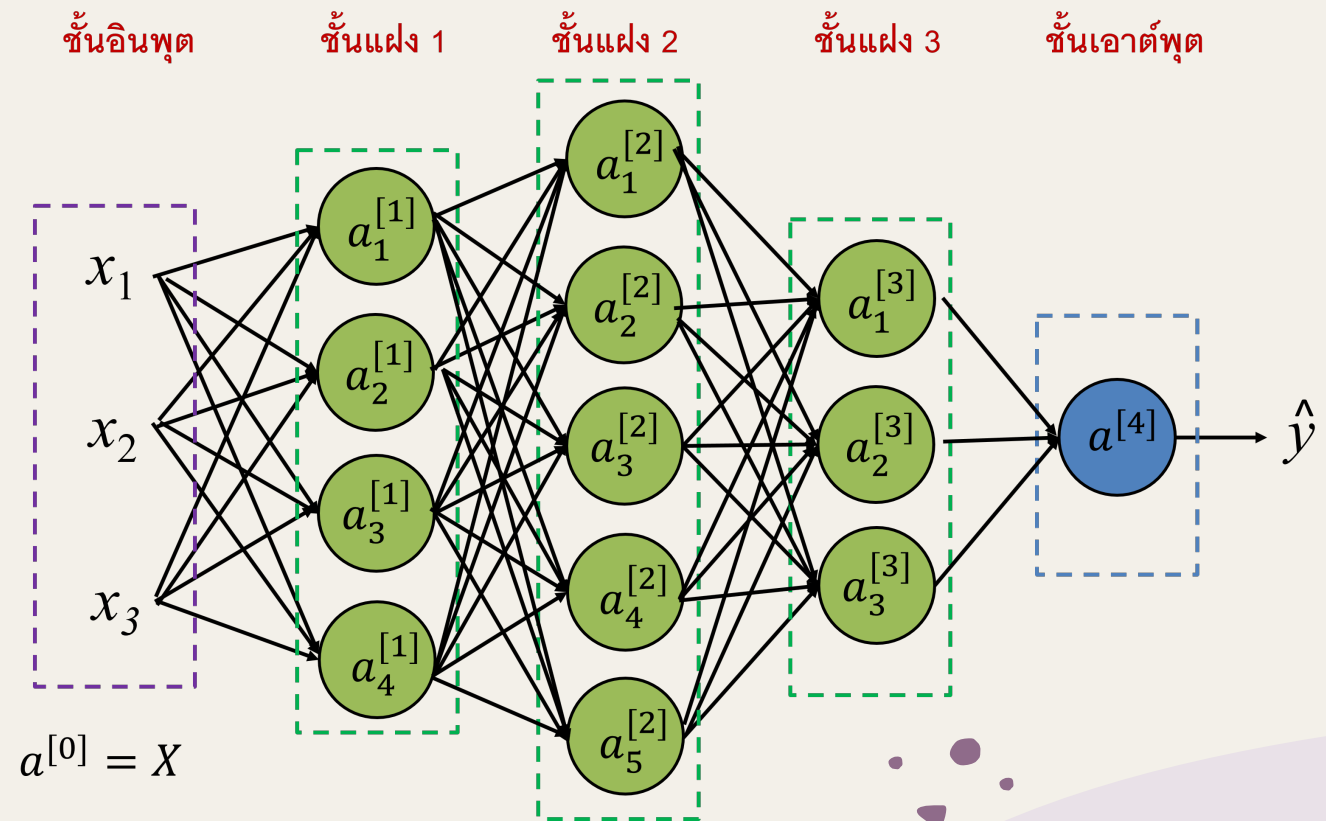$$g'(z) = \begin{cases} \alpha & \text{if} \quad z < 0 \\ 1 & \text{if} \quad z > 0 \end{cases}$$
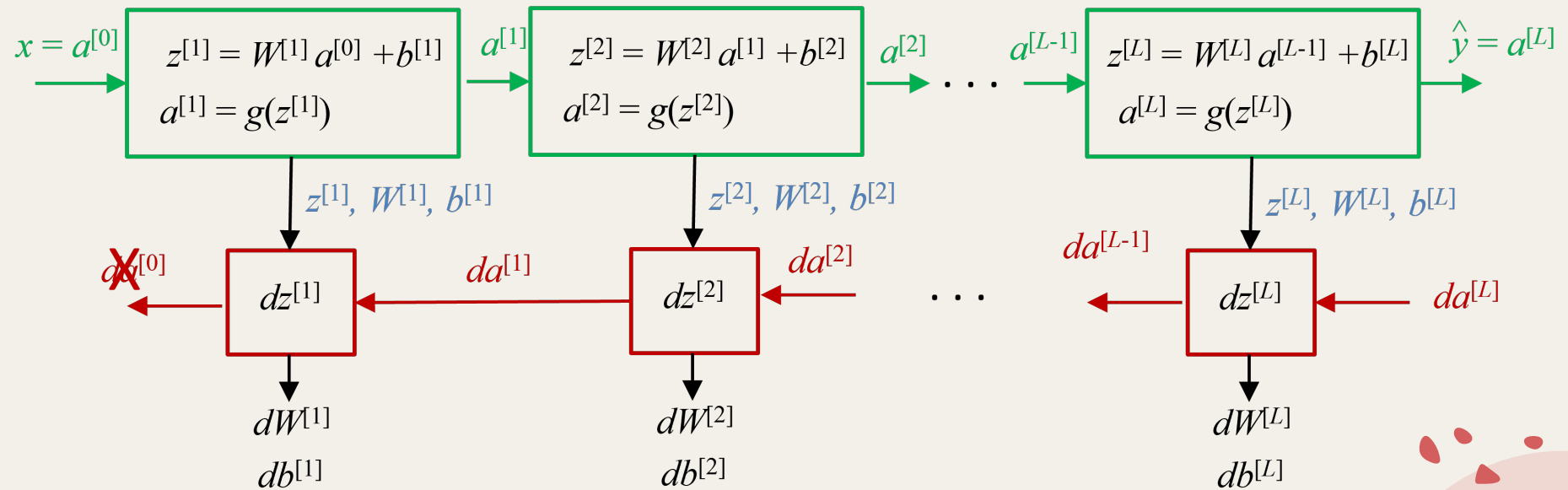
ตัวอย่าง *2.3*

see single_layer_dnn.ipynb

DNN model

also called
MLP (Multi-Layer
Perceptron)

ชั้นอินพุต   ชั้นแฝง 1   ชั้นแฝง 2   ชั้นแฝง 3   ชั้นเอาต์พุต

$a^{[0]} = X$

Computational graph of L-layer DNN

see multi_layer_dnn.ipynb

Supplementary notebooks

- model_tf.ipynb
- multiclass.ipynb

Exercises

- Problem 2-4