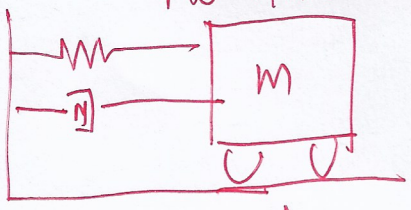
# Lyapunov example

nonlinear mass-spring-damper




$$m\ddot{x} + b\dot{x}|\dot{x}| + k_0 x + k_1 x^3 = 0 \quad (1)$$

Stable or not → no general solution

Choose Lyapunov candidate as total energy (kinetic + potential)

$$V(x) = \frac{1}{2}m\dot{x}^2 + \int_0^x (k_0\tilde{x} + k_1\tilde{x}^3)\, d\tilde{x}$$

$$= \frac{1}{2}m\dot{x}^2 + \frac{1}{2}k_0 x^2 + \frac{1}{4}k_1 x^4 \quad (2)$$

Find $\dot{V}(x)$ along trajectory

$$\dot{V}(x) = m\dot{x}\ddot{x} + (k_0 x + k_1 x^3)\dot{x}$$

$$= m\dot{x}\left(-\frac{b\dot{x}|\dot{x}| - k_0 x - k_1 x^3}{m}\right) + (k_0 x + k_1 x^3)\dot{x}$$

$$= \dot{x}(-b\dot{x}|\dot{x}|)$$

$$= -b\dot{x}^2|\dot{x}| < 0$$

energy being dissipated → stable

* From J.J Slotine and W. Li, Applied Nonlinear Control, pp 57-58