July 2021

Open in Colab

01211433 Homework # 3

Requirement : Python 3 with Python control systems library

Execute the commands below to install Python control systems library in Colab

In [ ]:
```
!pip install slycot
```

In [ ]:
```
!pip install control
```

Import the required libraries

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import control as ctl
```

In [2]:
```
# compute bounds
S_bnd = 20*np.log10(0.05)
print("Low frequency bound on S = " + str(S_bnd) + " dB")
print("Low frequency bound on L = " + str(-S_bnd) + " dB")
T_bnd = 20*np.log10(0.15)
print("High frequency bound on T and L = "+str(T_bnd)+" dB")
```

```
Low frequency bound on S = -26.020599913279625 dB
Low frequency bound on L = 26.020599913279625 dB
High frequency bound on T and L = -16.478174818886377 dB
```

Problem: let us design a controller for our same old robot joint driven by DC motor developed since the first module

$$P(s) = \frac{1}{7s^2 + 0.05s} \tag{1}$$

(1)

with the following design specs

1. steady state error is eliminated
2. low frequency disturbance is attenuated at least 0.05 below 0.5 rad/s
3. high frequency measurement noise is attenuated 0.15 above 80 rad/s
4. closed-loop stable, with phase margin at least 50 degrees

From the above discussion, this can be translated to stability and performance bounds

1. $L(s)$ has an integrator. Note that $P(s)$ already has one
2. $|S(j\omega)| \leq -26\ dB \rightarrow |L(j\omega)| \geq 26\ dB$ below 0.5 rad/s
3. $|T(j\omega)| \leq -16\ dB \rightarrow |L(j\omega)| \leq -16\ dB$ above 80 rad/s
4. $L(j\omega)$ has at least 50 degrees phase margin

To aid this design problem, we write a function lshape() in the cell below.

In [3]:
```
def lshape(C,P, lf, lfb, hf, hfb, pm ):
    assert lf > 0   # avoid bad values
    assert lfb > 0
    assert hf > lf
    assert hfb < 0
    assert 0 < pm < 90
    L = C*P # form loop transfer function
    # create a suitable range of frequency from lf, hf
    lf_log10 = np.log10(lf)
    w_start = np.floor(lf_log10)-1
    hf_log10 = np.log10(hf)
    w_end = np.ceil(hf_log10)+1
    w = np.logspace(w_start,w_end, 1000)

    # frequency response of L
    Lmag, Lph, om = ctl.freqresp(L, w)
    Lmag_db = np.squeeze(20*np.log10(Lmag))
    Lph_deg = np.squeeze(np.degrees(Lph))

    # create bound vectors
    lf_mask = np.where(om<lf, lfb, 0)
    hf_mask = np.where(om<hf, 0, hfb)
    lf_bnd = lf_mask*np.ones(om.shape)
    hf_bnd = hf_mask*np.ones(om.shape)

    # check whether violation occurs
    lf_idxv = np.where(om>lf)
    lf_idx = lf_idxv[0][0]   # find index of low-freq region
    hf_idxv = np.where(om<hf)
    hf_idx = hf_idxv[0][-1]  # find index of high-freq region
    lfmag = Lmag_db[:lf_idx]
    hfmag = Lmag_db[hf_idx:]
    if min(lfmag)<lfb:
        lf_legend = "LF bound (violated)"
    else:
        lf_legend = "LF bound"
    if max(hfmag)>hfb:
        hf_legend = "HF bound (violated)"
    else:
        hf_legend = "HF bound"

    # desired phase margin vectors
    pmvec = (pm-180)*np.ones(om.shape)

    # compute gain/phase margins
    g_margin, ph_margin, wgm, wpm = ctl.margin(L)
    ph_at_crossover = (ph_margin-180)
    # Loopshaping plot
    fig, (ax1, ax2) = plt.subplots(2, figsize=(8,8))
    fig.suptitle('$L(j\omega)$ v.s. bounds')
    ax1.semilogx(om, Lmag_db,'k-', om, lf_bnd, 'm-.', om, hf_bnd,'b-.')
    ax1.legend(["$|L(j\omega)|$",lf_legend,hf_legend],loc="lower left")
    ax1.grid(True)
    ax1.set_ylabel('magnitude (dB)')

    ax2.semilogx(om, Lph_deg,'k-',om, pmvec,'b-', wpm, ph_at_crossover,'r*')
    if ph_margin > pm:
        pmtext = "phase margin = " + str(round(ph_margin)) + " degrees"
    else:
        pmtext = "phase margin = " + str(round(ph_margin)) + " degrees (violated)"
```

```
    ax2.text(wpm,ph_at_crossover,pmtext)

    dpmtext = "Desired PM (" + str(pm) + " degrees)"
    ax2.set_xlabel('frequency (rad/s)')
    ax2.set_ylabel('phase (deg)')
    ax2.legend(["$\measuredangle L(j\omega)$",dpmtext],loc="lower left")
    ax2.grid(True)

    # plot magnitude of S and T v.s bounds
    S = 1/(1+L)
    T = L/(1+L)

    # frequency responses of S and T
    Smag, Sph, om = ctl.freqresp(S, w)
    Tmag, Tph, om = ctl.freqresp(T, w)

    Smag_db = np.squeeze(20*np.log10(Smag))
    Tmag_db = np.squeeze(20*np.log10(Tmag))

    # check whether violation occurs
    lf_idxv = np.where(om>lf)
    lf_idx = lf_idxv[0][0]   # find index of low-freq region
    hf_idxv = np.where(om<hf)
    hf_idx = hf_idxv[0][-1]  # find index of high-freq region
    lfSmag = Smag_db[:lf_idx]
    hfTmag = Tmag_db[hf_idx:]
    if max(lfSmag)>-lfb:
        lf_legend = "LF bound (violated)"
    else:
        lf_legend = "LF bound"
    if max(hfTmag)>hfb:
        hf_legend = "HF bound (violated)"
    else:
        hf_legend = "HF bound"

    # create data vector for stability bound in mid freq region

    om_mid = om[lf_idx:hf_idx]
    pm_r = np.radians(pm)
    x = np.sin(0.5*(np.pi - pm_r))/(np.sin(pm_r))
    x = 20*np.log10(x)
    bnds_mid = x*np.ones(om_mid.shape)

    ST_peak = max(max(Smag_db),max(Tmag_db))
    if ST_peak > x:
        mf_legend = "Stability bound (violated)"
    else:
        mf_legend = "Stability bound"

    plt.figure(figsize=(8,4))
    plt.semilogx(om,Smag_db,'k-',om,Tmag_db,'g-',om,-lf_bnd,'m-.',om,hf_bnd,'b-.',om_mi
    plt.xlabel('frequency (rad/s)')
    plt.ylabel('magnitude (dB)')
    plt.legend(["$|S(j\omega)|$","$|T(j\omega)|$",lf_legend,hf_legend, mf_legend])
    plt.grid(True)
    plt.title('$|S(j\omega)|$ and $|T(j\omega)|$ v.s. bounds')
    plt.show()
```

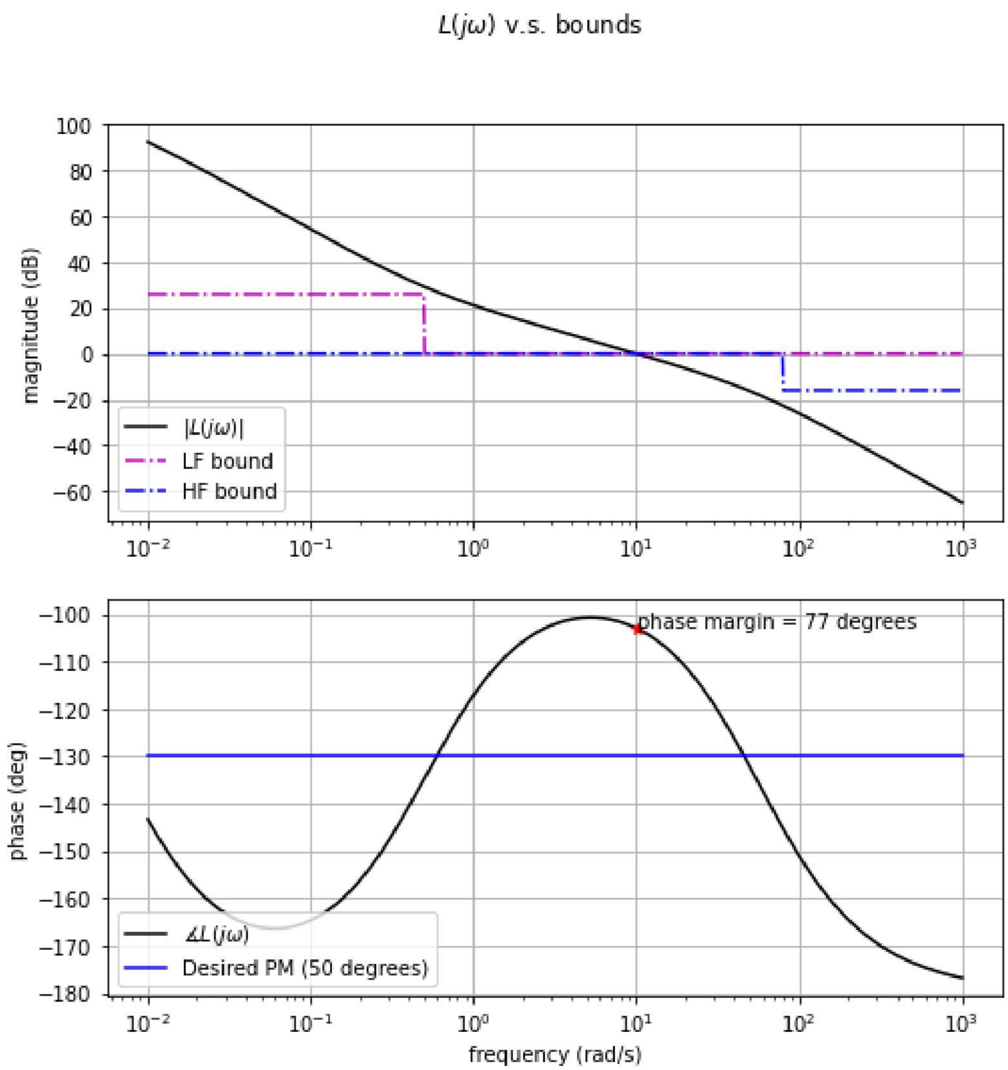
where the arguments are as follows:

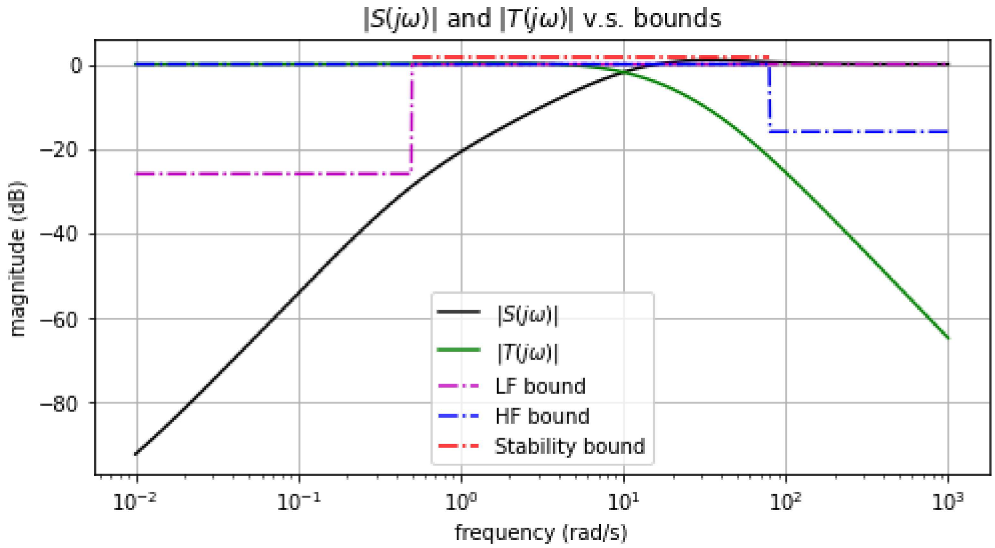
- **C :** Controller
- **P :** Plant
- **lf :** Define low frequency region from 0 - lf (rad/s)
- **lfb :** Low frequency bound for $L(s)$
- **hf :** Define high frequency region from hf - ∞ (rad/s)
- **hfb :** High frequency bound for $L(s)$
- **pm :** Phase margin (degrees)

In [4]:
```
# create the plant
s = ctl.tf('s')
P = 1/(7*s**2 + 0.05*s)
```

In [5]:
```
lf = 0.5 # low frequency here
lfb = 26 # your answer
hf = 80 # high frequency here
hfb = -16 # your answer
pm = 50
```

In [6]:
```
C = 4000*(s+0.5)/(s+56) # your controller
lshape(C,P, lf, lfb, hf, hfb,pm )
```

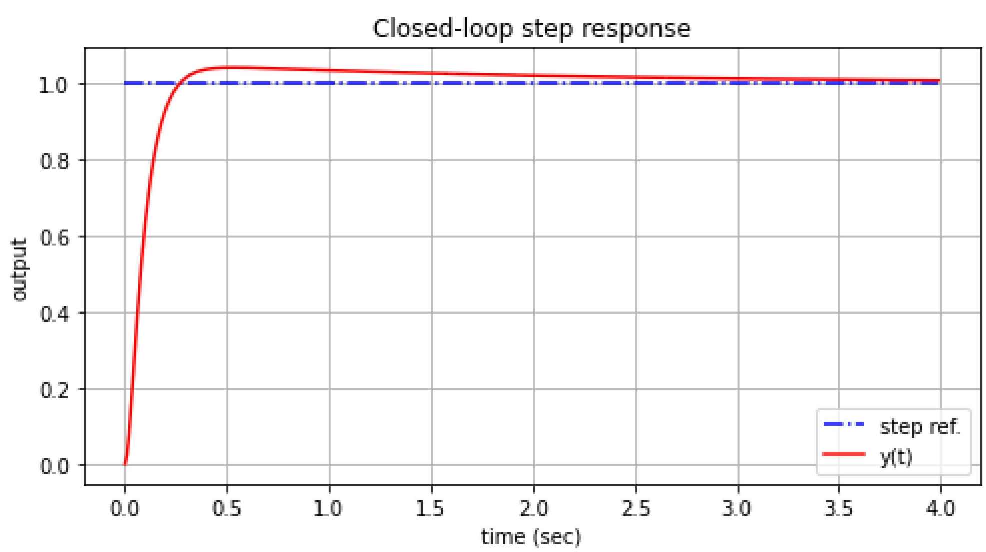




Iterate the above cell until you achieve a controller that meets the specs.

Then plot the closed-loop step response (adjust tvec if necessary)

In [7]:
```
L = C*P
T = L/(1+L)
tvec = np.arange(0,4.0,0.01)
r = np.ones(tvec.shape)
tout, y = ctl.step_response(T,tvec)
plt.figure(figsize=(8,4))
plt.plot(tout,r,'b-.',tout,y,'r-')
plt.grid('True')
plt.xlabel('time (sec)')
plt.ylabel('output')
plt.legend(['step ref.','y(t)'])
plt.title('Closed-loop step response')
plt.show()
```



To make sure that this design meets all the specification, the disturbance and noise responses in the time-domain need to be evaluated. Create a function to plot output response of arbitrary function.

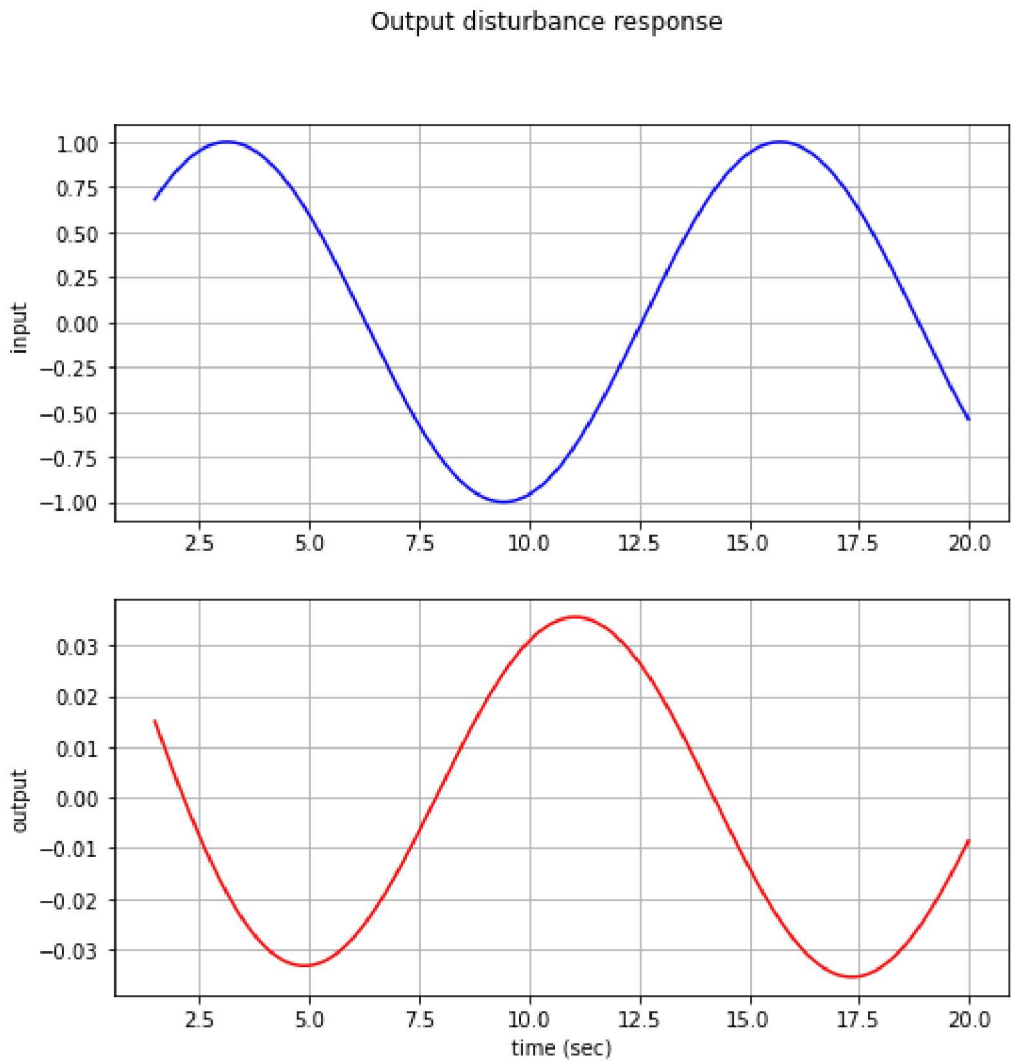
In [8]:
```
def plot_response(sys,u,t,title):
    tout, y = ctl.forced_response(sys,t, u)
```

```
    truncated_idx = 150  # get rid of transient
    tout = tout[truncated_idx:]
    u = u[truncated_idx:]
    y = y[truncated_idx:]
    fig, (ax1, ax2) = plt.subplots(2, figsize=(8,8))
    fig.suptitle(title)
    ax1.plot(tout,u,'b-')
    ax1.grid(True)
    ax1.set_ylabel('input')

    ax2.plot(tout,y,'r-')
    ax2.grid(True)
    ax2.set_ylabel('output')
    ax2.set_xlabel('time (sec)')

    plt.show()
```

From the design specifications, the required disturbance attenuation is at least 0.05 for frequency below 0.5 rad/s. Since the attenuatiton is the least at $\omega = 0.5$ rad/s, we use this frequency as our test point.

Recall that the closed-loop transfer function for the output disturbance response is $S(s)$. The plot from this cell must confirm that the controller meets this attenuation performance.

In [9]:
```
L = C*P
S = 1/(1+L)
t = np.arange(0,20,0.01)  # adjust if needed
w =0.5  # rad/s
u = np.sin(w*t)
plot_response(S,u, t, 'Output disturbance response')
```

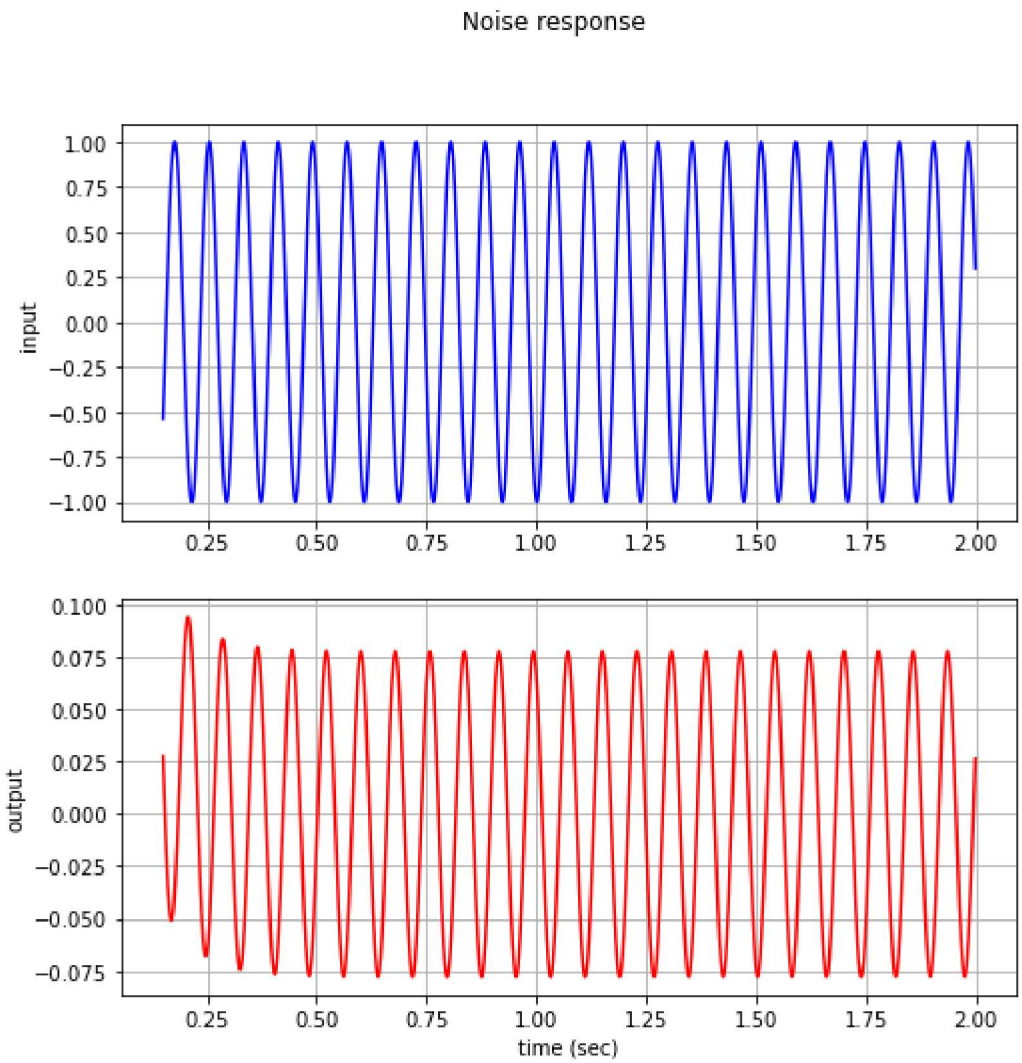


Note : The output magnitude must swing within 0.05

Use the same plot function on the complementary sensitivity $T(s)$ to verify that, with a noise input $u(t) = \sin(\omega t)$ where $\omega = 80$ rad/s, the output should swing within ± 0.15 unit.

In [10]:

```
t = np.arange(0,2,0.001) # adjust if needed
w = 80  # rad/s
u = np.sin(w*t)
T = L/(1+L)
plot_response(T,u, t, 'Noise response')
```



In [ ]:

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dew.ninja