$$P(w_j|x) = \frac{p(x|w_j)P(w_j)}{p(x)} \qquad (1)$$

$$P(\text{error}) = \int_{-\infty}^{\infty} P(\text{error}, x)\,dx = \int_{-\infty}^{\infty} P(\text{error}|x)\,p(x)\,dx \qquad (5)$$

$$P(\text{error}|x) = \min[P(w_1|x), P(w_2|x)] \qquad (7)$$

$$\min[a,b] \leq a^{\beta} b^{1-\beta}, \quad a,b \geq 0, \ 0 \leq \beta \leq 1 \qquad (72)$$

$$\begin{aligned}
P(\text{error}) &= \int_{-\infty}^{\infty} P(\text{error}|x)\,p(x)\,dx \qquad (73) \\
&= \int \min[P(w_1|x), P(w_2|x)]\,p(x)\,dx \\
&= \int \min[p(x|w_1)P(w_1), p(x|w_2)P(w_2)]\,dx \\
&\leq \int p^{\beta}(x|w_1)\,P^{\beta}(w_1) \cdot p^{1-\beta}(x|w_2)\,P^{1-\beta}(w_2)\,dx \\
&= P^{\beta}(w_1)P^{1-\beta}(w_2) \int p^{\beta}(x|w_1)\,p^{1-\beta}(x|w_2)\,dx, \quad 0 \leq \beta \leq 1 \\
&= P^{\beta}(w_1)P^{1-\beta}(w_2)\, e^{-k(\beta)}
\end{aligned}$$