Transformations on Lines and Quadratics

Joshua Aaron

November 2024

Don't worry if these problems seem challenging at first - we'll take them one step at a time! Remember, transformations are ways of representing how we move objects in space (or on a page). This is more about getting comfortable with functions and the algebra associated with them than some complicated set of equations... you've got this, take your time!

Translation and Reflection and Scaling of a Linear Equation

1. Original equation: $y = 2x + 3$
   - Translate the graph 4 units to the right and 2 units down, then reflect across the x-axis
   - Find the new equation
2. Original equation: $y = -3x + 1$
   - Translate the graph 2 units to the left and 4 units up, then scale horizontally by a factor of $\frac{1}{2}$ and vertically by a factor of 3, and finally reflect across the y-axis
   - Find the new equation

Combined Transformations of a Quadratic Equation

3. Original equation: $y = (x - 1)^2 + 2$
   - Scale the graph horizontally by a factor of $\frac{1}{3}$ and vertically by a factor of 2, then reflect across the y-axis
   - Find the new equation
4. Original equation: $y = 2(x + 1)^2 - 4$
   - Translate the graph 3 units to the right and 1 unit down, then reflect across the x-axis, and finally scale vertically by a factor of $\frac{1}{2}$
   - Find the new equation

Comparing Quadratic Functions with Transformations

5.
   - Original equation: $y = x^2 + 2x + 1 \rightarrow$ Transformed equation: $y = (x-3)^2 + 5$
   - Determine the translation applied to the original function to get the transformed function, if you also reflect the transformed function, how does the translation change, if at all?

6.
   - Original equation: $y = x^2 - 4x + 4 \rightarrow y + 3 = \frac{1}{2}x^2 - 2x + 2$
   - Determine the transformations applied to the original function and reflect the transformed function over an axis of your choice, how does the equation change? Can you recenter the function so that the vertex is on the origin?

7.
   - Original equation: $y = (x-2)^2 + 1 \rightarrow y = -(x+2)^2 - 1$
   - Determine the transformation applied to the original function and scale the transformed function in a direction of your choice, after translating its vertex to the point $(4, 4)$ how does the equation change?

8.
   - Original equation: $y = 4x^2 - x + 3 \rightarrow y = 2(4x+5)(x-3) + 2$
   - Determine all the transformations applied to the original function

Abstract Problems:

9. Describe (in words) how the graph of $y = f(x)$ would change if:
   - The function $f(x)$ was replaced with $-f(x)$
   - The function $f(x)$ was replaced with $f(-x) + 3$
   - The function $f(x)$ was replaced with $2f(2x)$

10. Consider the equation of a circle: $x^2 + y^2 = r^2$. See if you can show how changing this equation changes the circle. It might help to use Desmos or another graphing program for this one! If you can, provide the general equation for each type of transformation we've discussed, starting with a given circle equation and moving to another one after the transformation (as above).

11. Explain how you would determine if two quadratic functions represent the same parabola, but with different transformations applied. What steps would you take to identify the specific transformations?

12. Consider the equation of an ellipse: $(\frac{x}{a})^2 + (\frac{y}{b})^2 = 1$. See if you can show how changing this equation changes the ellipse. It might help to use Desmos or another graphing program for this one! If you can, provide the general equation for each type of transformation we've discussed starting with a given ellipse equation and moving to another one after the transformation (as above).