Exponent Rules – Discovery Worksheet

Maths for IT – Autumn 2025

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For each section: after practicing, write your own “discovered rule” in words. Then invent 5 wacky problems of your own

1 What’s a Base? What’s an Exponent?

When we write a^3, the *base* is a and the *exponent* is 3. It tells us how many times to multiply the base by itself. For example, $a^3 = a \cdot a \cdot a$.

Let’s try:

1. How might we write b^2 as repeated multiplication?

2. How might we write c^4 as repeated multiplication?

3. How might we write d^5 as repeated multiplication?

4. What would e^{12} look like if we wrote it all the long way?

5. How might we write f^m when the exponent is a variable?

6. How might we write g^{p+1}?

7. How might we write $\left(\frac{h}{2}\right)^3$?

8. What about $\left(\frac{1}{j}\right)^4$?

Exploration:

1. How does 2^3 compare to 3^2? Which is larger?

2. What if the base is not a whole number? For example, what happens with $\left(\frac{1}{2}\right)^2$ or $(0.1)^3$?

3. If the base is a fraction less than 1, like $\left(\frac{1}{2}\right)^n$, what happens as n gets larger?

4. How does the expansion of a^m look? And what happens if we multiply $a^m \cdot a^n$ — what does that suggest?

Now that you've seen how exponents can be written, you might wonder what unusual ones could look like. Try making up 5 of your own problems. You could explore repeated multiplication, large exponents, fractions or decimals as bases, or even variables as exponents. Which of your examples feels most surprising?

2 Multiplying Same Bases

1. $a^2 \cdot a^3$

2. $b^4 \cdot b^1$

3. $c^7 \cdot c^2$

4. $d^{1205} \cdot d^{321}$

5. $e^m \cdot e^2$

6. $f^3 \cdot f^n$

7. $g^2 \cdot g^4 \cdot g^5$

8. $h^1 \cdot h^2 \cdot h^3 \cdot h^4$

9. $i^m \cdot i^n \cdot i^p$

10. $j^{100} \cdot j^{200} \cdot j^{300}$

Exploration:

1. What different pairs of exponents could multiply to give b^{12}? Can you think of several?

2. What different sets of three exponents could multiply to give c^{12}?

3. How might we try out examples like $d^2 \cdot d^5$ or $d^7 \cdot d^4$? What do we notice?

4. What happens if the base is a fraction or a negative number, like $\left(\frac{1}{2}\right)^m \cdot \left(\frac{1}{2}\right)^n$ or $(-3)^m \cdot (-3)^n$?

5. How many different ways can you make a problem that simplifies to e^{20}?

As you create your own problems, you might explore: multiple terms, very large exponents, or variables in the exponents. Can you come up with 5 unusual multiplications that still simplify neatly?

3 Dividing Same Bases

1. $\dfrac{r^5}{r^2}$

2. $\dfrac{m^9}{m^4}$

3. $\dfrac{n^{100}}{n^{99}}$

4. $\dfrac{p^{5000}}{p^{4997}}$

5. $\dfrac{q^m}{q^2}$

6. $\dfrac{t^7}{t^n}$

7. $\dfrac{s^8}{s^2\, s^2}$

8. $\dfrac{\frac{u^{20}}{u^5}}{u^2}$

9. $\dfrac{v^m}{v^n\, v^p}$

10. $\dfrac{w^{200}}{w^{100}\, w^{50}}$

Exploration:

1. What kind of division leaves us with f^3? Can you find several?

2. What kind of division would leave us with g^0? How many ways could you write it?

3. If the base is a fraction like $\frac{1}{2}$, what happens when you divide $\left(\frac{1}{2}\right)^m$ by $\left(\frac{1}{2}\right)^n$?

4. If the exponents are variables, how does something like $\dfrac{h^m}{h^n}$ simplify? What if $m < n$?

5. What messy-looking division could end up being really simple, maybe even just 1?

Try inventing 5 division problems of your own. Could one of them look complicated but simplify to 1? Could another include three terms, or mix fractions and variables?

4 Zero Exponents

1. $\dfrac{x^4}{x^4}$

2. $\dfrac{y^{10}}{y^{10}}$

3. $\dfrac{z^1}{z^1}$

4. $\dfrac{a^{999}}{a^{999}}$

5. $\dfrac{b^m}{b^m}$

6. $\dfrac{c^n}{c^n}$

7. $\dfrac{d^5 \cdot d^2}{d^3 \cdot d^4}$

8. $\dfrac{\frac{e^6}{e^2}}{e^4}$

9. $\dfrac{f^3 \cdot f^7}{f^5 \cdot f^5}$

10. $\left(\dfrac{g^2}{g}\right) \cdot \left(\dfrac{g^3}{g^4}\right)$

11. $\left(\dfrac{h^5}{h^5}\right) \cdot \left(\dfrac{i^3}{i^3}\right)$

12. $\dfrac{\frac{j^7}{j^7}}{\frac{k^2}{k^2}}$

13. $\dfrac{r^3m^3}{r^3m^3}$

14. $\dfrac{p^4q^2}{p^4q^2}$

Exploration:

1. What happens if we try $\dfrac{(-2)^5}{(-2)^5}$?

2. How does $\dfrac{\left(\frac{1}{3}\right)^7}{\left(\frac{1}{3}\right)^7}$ simplify?

3. What about $\dfrac{(0.5)^{12}}{(0.5)^{12}}$?

4. Does the same idea work if the exponent is a variable, like $\dfrac{a^m}{a^m}$?

5. Can we make a division that looks messy — like $\dfrac{b^5c^3}{b^5c^3}$ — but still simplifies to 1?

Can you invent 5 of your own examples where things cancel all the way down to 1? Try using fractions, decimals, or negative bases.

5 Negative Exponents

1. $\dfrac{g^3}{g^5}$

2. $\dfrac{h^2}{h^7}$

3. $\dfrac{i^{20}}{i^{25}}$

4. $\dfrac{j^{500}}{j^{501}}$

5. $\dfrac{k^m}{k^{m+1}}$

6. $\dfrac{r^p}{r^{p+3}}$

7. $\dfrac{m^2}{m^5\,m^2}$

8. $\dfrac{\frac{n^{10}}{n^4}}{n^{10}}$

9. $\dfrac{w^3}{w^4}$

10. $\dfrac{p^2}{p^{m+2}}$

11. $\dfrac{q^5r^3}{q^7r^3}$

12. $\left(\frac{s^4}{s^6}\right) \cdot \left(\frac{t^2}{t^2}\right)$

13. $\frac{u^2v^3}{u^5v^3}$

14. $\frac{\frac{w^{12}}{w^{20}}}{\frac{x^5}{x^5}}$

Exploration:

1. How might $\frac{2^3}{2^5}$ relate to fractions?

2. What about $\frac{(-3)^2}{(-3)^5}$?

3. How does $\frac{\left(\frac{1}{2}\right)^4}{\left(\frac{1}{2}\right)^7}$ simplify?

4. What does $\frac{p^m}{p^{m+2}}$ look like?

5. How does $\frac{q^{10}}{q^{12}}$ compare to $\frac{q^{12}}{q^{10}}$?

Based on your experiments, how would you describe the rule for negative exponents in your own words? Try writing down 5 examples that illustrate your rule, including ones with negative and fractional bases.

6 Powers of Powers

1. $(y^2)^3$

2. $(z^5)^2$

3. $(a^{10})^4$

4. $(b^{100})^{20}$

5. $(c^m)^2$

6. $(d^3)^n$

7. $(e^2 \cdot e^3)^2$

8. $(f^m \cdot f^n)^p$

9. $(g^{10})^{10} \cdot (g^2)^2$

10. $\big((h^2)^3\big)^4$

11. $\big((i^2 j^3)^2\big)^2$

12. $(k^m r^n)^2$

13. $\big((m^4)^2 (m^3)^2\big)^2$

14. $(n^{p+1})^q$

Exploration:

1. How does $(r^2)^3$ expand if you write it all out?

2. How does $(s^3)^4$ compare to $(s^4)^3$?

3. What happens with $(t^m)^n$ compared to $(t^n)^m$?

4. What if the base is negative, like $((-2)^2)^3$?

5. What if the base is a fraction, like $\left(\left(\frac{1}{2}\right)^3\right)^4$?

6. How do problems like $(u^2 \cdot u^5)^2$ simplify?

7. What about very large nested ones like $(((v^2)^3)^4)$?

See if you can invent 5 of your own power-of-a-power problems. Try mixing in negative and fractional bases, or making the exponents themselves variables.

7 Roots and Fractional Exponents

1. $\left(r^{4}\right)^{\frac{1}{2}}$

2. $\left(s^{9}\right)^{\frac{1}{2}}$

3. $t^{\frac{3}{2}}$

4. $\left(u^{16}\right)^{\frac{1}{2}}$

5. $(v^{m})^{\frac{1}{2}}$

6. $\left(w^{2n}\right)^{\frac{1}{2}}$

7. $\left(x^{9}\right)^{\frac{1}{3}}$

8. $\left(y^{27}\right)^{\frac{1}{3}}$

9. $\left(z^{12}\right)^{\frac{1}{4}}$

10. $\left(a^{100}\right)^{\frac{1}{10}}$

11. $\left(b^{6}c^{6}\right)^{\frac{1}{2}}$

12. $\left(d^{12}e^{4}\right)^{\frac{1}{2}}$

13. $\left(f^8 g^{12}\right)^{\frac{1}{4}}$

14. $\left((h^2)^6\right)^{\frac{1}{2}}$

Exploration:

1. What does the denominator of a fractional exponent seem to mean? How about the numerator?

2. How does $\left(\frac{1}{2}\right)^{\frac{1}{2}}$ behave compared to $(2)^{\frac{1}{2}}$?

3. What happens if the base is negative and the exponent is fractional? Can we always make sense of it?

4. How do roots connect to repeated multiplication? For example, what does it mean that $\sqrt[3]{8} = 2$?

5. Can you create an example with a variable base and a fractional exponent that simplifies neatly?

Invent 5 fractional exponent problems of your own. Can one use a fraction as the base? Can another have both numerator and denominator in the exponent greater than 1?

8 Stretching Fractional Exponents

1. $f^{\frac{1}{3}}$

2. $g^{\frac{2}{3}}$

3. $\left(h^6\right)^{\frac{1}{3}}$

4. $i^{\frac{120}{30}}$

5. $j^{m/n}$

6. $(k^n)^{\frac{1}{m}}$

7. $l^{\frac{5}{4}}$

8. $\left(m^{12}\right)^{\frac{1}{6}}$

9. $\left(n^{15}\right)^{\frac{3}{5}}$

10. $\left(p^{60}\right)^{\frac{1}{12}}$

11. $\left(q^3r^6\right)^{\frac{1}{3}}$

12. $\left(s^{10}t^{20}\right)^{\frac{1}{5}}$

13. $\left(u^{24}v^{12}\right)^{\frac{2}{3}}$

14. $(w^{m+n})^{\frac{1}{2}}$

Exploration:

1. How do the numerator and denominator of the fractional exponent each affect the outcome?

2. What is another way to write $f^{\frac{2}{3}}$?

3. How does $(x^{12})^{\frac{1}{6}}$ connect to roots?

4. What happens with $(y^{15})^{\frac{3}{5}}$ if we expand it step by step?

5. Can you find two different paths to simplify the same expression, like $(z^{12})^{\frac{1}{6}}$?

Invent 5 problems using fractional exponents. Can one involve three variables? Can another mix large exponents with fractions?

9 Mixed Simplification Challenge

Simplify each expression as far as possible.

1. $(a^2b^3)(a^4b)$

2. $\dfrac{c^5d^2}{c^2d^5}$

3. $(e^3f^2)^2$

4. $\dfrac{g^6h^3}{g^2h^6}$

5. $\left(i^4j^2\right)^{\frac{1}{2}}$

6. $\left(k^3r^5\right)^{\frac{1}{3}}$

7. $(m^2n^3)(m^4n^{-2})$

8. $\dfrac{p^5q^2}{p^2q^5}$

9. $\left(r^2s^3\right)^0$

10. $\dfrac{t^5}{t^5} \cdot u^0$

11. $(v^{-2}w^3)(v^4w^{-1})$

12. $\dfrac{(x^2y^3)^2}{x^3y}$

Exploration:

1. Which of these simplify to 1? Which stay as powers? Which turn into fractions?

2. Can you create a problem that looks messy but simplifies to just a single letter?

3. Can you write one with three variables that simplifies neatly?

4. How might you use both negative and fractional exponents together in one problem?

5. Which of your invented problems looks the scariest but has the simplest answer?

Invent 5 "scary" mixed problems of your own that combine many different rules.