Friendly Practice with Polynomials

Don't worry if these problems seem challenging at first - we'll take them one step at a time! Remember, polynomials are just combinations of terms with different powers, and you've got this. Take your time and check your work as you go, there are plenty of opportunities.

1 Practice Problems

1. Factor this expression completely:

$$x^2 + 7x + 12$$

2. Use the FOIL method to multiply, then sketch the resulting polynomial:

$$(2x + 3)(x - 1)$$

Be sure to identify key points including zeros and y-intercept.

3. Factor and graph:

$$6x^2 + 18x - 24$$

Show the vertex and x-intercepts on your sketch.

4. Multiply and graph:

$$(\frac{1}{2}x + 2)(\frac{1}{3}x - 1)$$

Identify and label all intercepts and any key points.

5. Find all roots of and sketch:

$$x^2 - 4x + 4$$

Your sketch should clearly show the multiplicity of the root(s).

6. Solve, identify the type of roots, and graph:

$$x^2 + \frac{1}{4}x - 2$$

7. Graph the polynomial:

$$f(x) = x^2 - 2x - 3$$

Identify the y-intercept and x-intercepts.

8. Graph and analyze:
$$g(x) = (x+1)^2(x-2)$$
Identify all zeros and their multiplicities.

9. Factor completely and analyze graphically, where a is a constant:
$$ax^2 + ax - 2a$$

10. For what values of b will the polynomial have a repeated root:
$$x^2 + 2bx + b^2$$
Sketch for several values of b.

11. Factor completely and analyze graphically, where k is a constant:
$$k^2x^2 - 9$$

2 Key Graphing Tips

1. General Strategy:
   - First identify key points (zeros, y-intercept)
   - Determine shape based on degree and leading coefficient
   - Consider how parameters affect the basic shape
   - Sketch lightly first, then refine
2. Important Features:
   - Multiple roots (multiplicity > 1) mean the graph touches but doesn't cross
   - Odd-degree polynomials always cross their horizontal asymptotes
   - Even-degree polynomials stay above or below their horizontal asymptotes
3. When working with parameters:
   - Consider how different values affect:
     - Direction (opens up/down)
     - Location of zeros
     - Stretching/compression
     - Position of vertex
   - Try specific values to understand behavior
   - Pay attention to special cases (when parameter = 0)
4. To find y-intercept:
   - Let $x = 0$
   - The y-intercept is always c in standard form
5. To find x-intercepts:
   - Let $y = 0$
   - Factor or use quadratic formula
   - Check multiplicity of each root
6. For transformations:
   - $f(x) + k$ shifts up k units
   - $f(x - h)$ shifts right h units
   - $-f(x)$ reflects over x-axis
   - $f(-x)$ reflects over y-axis
   - $af(x)$ stretches vertically by factor of $|a|$

- $f(bx)$ compresses horizontally by factor of $|b|$

7. For higher degree polynomials:

   - Number of turning points $\leq$ degree - 1
   - Number of x-intercepts $\leq$ degree
   - Always crosses x-axis at roots of odd multiplicity
   - Touches x-axis at roots of even multiplicity

Remember: It's perfectly normal if some of these problems take time to solve. The key is understanding each step and building confidence gradually. Great job working through these challenging concepts!

3 Essential Formulas and Quick Reference

3.1 Quadratic Formulas

1. Quadratic Formula:

   - For $ax^2 + bx + c = 0$
   - $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
   - Use when factoring isn't obvious

2. Vertex Formula for $ax^2 + bx + c$:

   - x-coordinate: $x = -\frac{b}{2a}$
   - y-coordinate: plug x back into original equation
   - Or use: $y = -\frac{b^2}{4a} + c$

3. Discriminant $(b^2 - 4ac)$ tells you about roots:

   - If > 0: two real roots
   - If $= 0$: one real root (multiplicity 2)
   - If < 0: two complex conjugate roots

4. Difference of Squares:
$$a^2 - b^2 = (a + b)(a - b)$$

3.2 Key Relationships

1. For quadratic $ax^2 + bx + c$:
    - Sum of roots $= -\frac{b}{a}$
    - Product of roots $= \frac{c}{a}$
    - Distance between roots $= \frac{\sqrt{b^2-4ac}}{|a|}$
2. Vieta's Formulas (if roots are r and s):
    - $r + s = -\frac{b}{a}$
    - $r \times s = \frac{c}{a}$

3.3 Polynomial End Behavior

1. Even degree with positive leading coefficient:
    - Both ends point up ($\cup$)
2. Even degree with negative leading coefficient:
    - Both ends point down ($\cap$)
3. Odd degree with positive leading coefficient:
    - Left end down, right end up (/)
4. Odd degree with negative leading coefficient:
    - Left end up, right end down (\)

Remember: These formulas are tools to help you understand the underlying concepts. Don't just memorize them - try to understand why they work and how they relate to each other. When solving problems, think about which formula would be most helpful and why. You've got this!

4 Step-by-Step Solutions

1. $x^2 + 7x + 12$
   - Look for factors of 12 that add to 7
   - 3 and 4 work: $3 + 4 = 7$, $3 \times 4 = 12$
   - Therefore: $x^2 + 7x + 12 = (x + 3)(x + 4)$
2. $(2x + 3)(x - 1)$
   - F: $(2x)(x) = 2x^2$
   - O: $(2x)(-1) = -2x$
   - I: $(3)(x) = 3x$
   - L: $(3)(-1) = -3$
   - Combine like terms: $2x^2 - 2x + 3x - 3$
   - Final answer: $2x^2 + x - 3$

   Graphing steps:
   - Find y-intercept: $f(0) = -3$
   - Find zeros by factoring: $2x^2 + x - 3 = (2x - 2)(x + 1.5)$
   - $x = 1$ and $x = -1.5$
   - Vertex: $x = -0.25$ (using $-b/2a$)
   - Vertex point: $(-0.25, -3.125)$
   - Parabola opens upward ($a > 0$)
   - Sketch shows a U-shaped curve passing through $(-1.5, 0)$, $(0, -3)$, and $(1, 0)$
3. $6x^2 + 13x + 6$

   (a) First, factor out the GCD of all terms:

   $$6x^2 + 18x - 24 = 6(x^2 + 3x - 4)$$

   (b) Factor the quadratic inside the parentheses:

   $$6(x^2 + 3x - 4) = 6(x + 4)(x - 1)$$

   (c) Find x-intercepts by setting the equation equal to 0:

   $$6(x + 4)(x - 1) = 0$$

   $$x + 4 = 0 \text{ or } x - 1 = 0$$
   $$x = -4 \text{ or } x = 1$$

(d) Find the vertex using $x = -\frac{b}{2a}$ when we consider our quadratic in standard form: $ax^2 + bx + c$

$$x = -\frac{18}{2(6)} = -\frac{18}{12} = -\frac{3}{2}$$

(e) Find the y-coordinate of the vertex by plugging $x = -\frac{3}{2}$ into the original equation:

$$6(-\frac{3}{2})^2 + 18(-\frac{3}{2}) - 24$$

$$6(\frac{9}{4}) - 27 - 24$$

$$\frac{54}{4} - 27 - 24$$

$$13.5 - 27 - 24$$

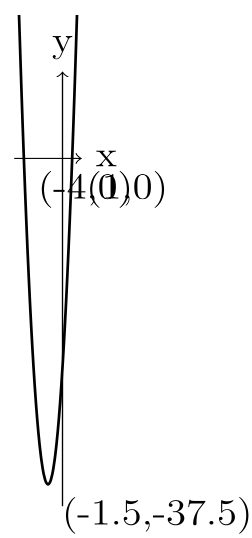
$$-37.5$$

Therefore, the vertex is

$$(-\frac{3}{2}, -37.5)$$

(f) Key points for graphing:

- x-intercepts: (-4, 0) and (1, 0)
- vertex: $(-\frac{3}{2}, -37.5)$
- Opens upward since coefficient of x^2 is positive (6 ¿ 0)



This parabola opens upward with two x-intercepts and a vertex below the x-axis.

4. $(\frac{1}{2}x + 2)(\frac{1}{3}x - 1)$

- Multiply: $\frac{1}{6}x^2 + \frac{1}{6}x - 2$

Graphing steps:

- y-intercept: $f(0) = -2$
- Find zeros by factoring: $\frac{1}{6}x^2 + \frac{1}{6}x - 2 = \frac{1}{6}(x^2 + x - 12)$
- $x = -4$ and $x = 3$
- Vertex: $x = -0.5$
- Vertex point: $(-0.5, -2.125)$
- Parabola opens upward $(a > 0)$
- Sketch shows wide U-shaped curve (due to small leading coefficient)

5. $x^2 - 4x + 4 = (x - 2)^2$

   Graphing steps:

   - This is a perfect square trinomial
   - Only one x-intercept at $x = 2$ (multiplicity 2)
   - y-intercept: $f(0) = 4$
   - Vertex at $(2, 0)$
   - Parabola opens upward
   - Graph touches but doesn't cross x-axis at $x = 2$
   - Sketch shows parabola that bounces off x-axis at $(2, 0)$

6. $x^2 + \frac{1}{4}x - 2$

   - Use quadratic formula: $\frac{-\frac{1}{4} \pm \sqrt{(\frac{1}{4})^2 + 4(1)(2)}}{2}$
   - Simplify under radical: $\frac{-\frac{1}{4} \pm \sqrt{\frac{129}{16}}}{2}$
   - Two real, irrational roots: approximately -1.62 and 1.37

   Graphing steps:

   - y-intercept: $f(0) = -2$
   - x-intercepts at $x \approx -1.62$ and $x \approx 1.37$
   - Leading coefficient is 1, so parabola opens upward
   - Vertex: $x = -\frac{1}{8}$ (using $-b/2a$)
   - Vertex point: $(-\frac{1}{8}, -2.016)$
   - Sketch shows standard upward parabola through $(0, -2)$

7. $f(x) = x^2 - 2x - 3$

   - Factor: $(x + 1)(x - 3)$

Graphing steps:

- y-intercept: $f(0) = -3$
- x-intercepts: $x = -1$ and $x = 3$
- Leading coefficient is 1, so parabola opens upward
- Vertex at $x = 1$ (halfway between roots)
- Vertex point: $(1, -4)$
- Sketch shows symmetric parabola through points $(-1, 0)$, $(0, -3)$, and $(3, 0)$

8. $g(x) = (x + 1)^2(x - 2)$

   Graphing steps:

   - Factor shows roots at $x = -1$ (double root) and $x = 2$ (simple root)
   - y-intercept: $g(0) = (1)^2(-2) = -2$
   - Leading coefficient is 1, cubic opens upward
   - At $x = -1$: graph touches but doesn't cross (multiplicity 2)
   - At $x = 2$: graph crosses x-axis
   - Sketch shows cubic curve that:
     - Comes down from upper left
     - Touches x-axis at $x = -1$ without crossing
     - Continues down to a minimum
     - Crosses at $x = 2$
     - Continues up to upper right

9. $ax^2 + ax - 2a$

   - Factor out a: $a(x^2 + x - 2)$
   - Factor inside: $a(x + 2)(x - 1)$

   Graphing notes:

   - For $a > 0$:
     - Opens upward
     - x-intercepts at $x = -2$ and $x = 1$
     - y-intercept at $(0, -2a)$
   - For $a < 0$:
     - Opens downward
     - Same x-intercepts
     - y-intercept at $(0, -2a)$
   - For $a = 0$:

  - Horizontal line at $y = 0$

10. $x^2 + 2bx + b^2$

    - Rewrite as $(x+b)^2 = x^2 + 2bx + b^2$

    Graphing notes:

    - For any value of b, graph is a parabola that:
      - Opens upward
      - Has vertex at $(-b, 0)$
      - Touches but doesn't cross x-axis at $x = -b$
      - y-intercept at $(0, b^2)$
    - As $|b|$ increases:
      - Vertex moves further from origin
      - Parabola becomes wider at y-axis
    - As b approaches 0:
      - Vertex approaches origin
      - Graph approaches $x = 0$

11. $k^2x^2 - 9$

    - Factor as: $(kx+3)(kx-3)$, $k \neq 0$

    Graphing notes:

    - For $k > 0$:
      - Opens upward
      - x-intercepts at $x = \pm\frac{3}{k}$
      - y-intercept at $(0, -9)$
      - Symmetric about y-axis
    - For $k < 0$:
      - Opens upward (k^2 is positive)
      - Same x-intercepts
      - Same y-intercept
    - As $|k|$ increases:
      - Parabola gets narrower
      - x-intercepts move toward origin
    - As $|k|$ decreases:
      - Parabola gets wider
      - x-intercepts move away from origin
    - For $k = 0$:
      - Not a quadratic; becomes horizontal line $y = -9$