Discrete-Continuum Correspondence

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1.1 The Fundamental Conjecture

For the theory to be fully successful, we must in some way be able to reconstruct continuum spacetime as a limiting case of a causal set. In reference [1] Rafael Sorkin, a key proponent of the causal set methodology, discusses the theory's *'Hauptvermutung'* - its fundamental conjecture. It states that if manifolds M_1 and M_2 both approximate the same causal set, then $M_1 \approx M_2$ (where '$\approx$' signifies approximate isometry - a mapping between manifolds which preserves distance between points). However this statement needs tightening - what does it mean for a manifold to 'approximate a causal set'?. The central conjecture says that causal set C is well-approximated by a continuum (M, g) if there's an embedding $f : C \hookrightarrow M$ which:

1. Respects order: $x \preceq y$ iff$f(x)$ is in the causal past of $f(y)$
2. The number of elements embedded in a region of M is approximately equal to the volume of that region in fundamental units

An embedding which satisfies these constraints is a 'faithful' embedding. Further work is required to tighten these statements in order to form a robust 'central conjecture' [2].

1.2 Random Sprinkling

A further conjecture of causal set theory is that C is faithfully embeddable in M if it 'could have arisen by a sprinkling process'. A 'sprinkling' process is a Poisson process whereby spacetime elements are 'embedded' in partitioned regions of spacetime proportional to the region's volume. It has been hypothesised - but not proved - that a Poisson process is the *only* way to do this. This is a non-physical 'game' that can be played to artificially create causal sets that are guaranteed to have a continuum spacetime as an approximation to them. It is important to remember that the causal set is the physical 'thing' that exists - this concept of a continuous manifold M and a sprinkling process are not.

In summary, the causal set is the physical substance of the theory. Faithful embedding is a process which distinguishes causal sets which can be well-approximated by a continuum spacetime and those that can't, whilst Poisson sprinkling is an artificial process which allows us to manufacture causal sets with a continuum approximation to use for further research.

References

[1] R. D. Sorkin. Causal sets: Discrete gravity (notes for the valdivia summer school). *Lectures on Quantum Gravity, Proceedings of the Validivia Summer School, Valdivia, Chile, January 2002*, 2005.

[2] L. Bombelli. Statistical lorentzian geometry and the closeness of lorentzian manifolds. *Journal of Mathematical Physics*, 41(10):6944, 2000.