#### 9.2 - Algoritma pada Graf (part 2)

[KOMS120403]

Desain dan Analisis Algoritma (2023/2024)

Dewi Sintiari

Prodi S1 Ilmu Komputer Universitas Pendidikan Ganesha

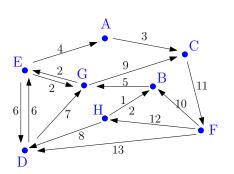
Week 11 (April 2024)

#### Daftar isi

- Shortest path problem (masalah jarak terpendek)
  - ► Algoritma Dijkstra

# Bagian 1. Shortest Path

**Masalah:** diberikan graf berarah berbobot sisi. Temukan jalur terpendek dari s ke t.



8

Pohon jalur terpendek (shortest-path tree) yang berakar pada simpul v adalah pohon rentang (spanning tree) T dari graf G, sedemikian sehingga jarak jalur dari akar v ke simpul lain mana pun u di T adalah jalur terpendek jarak dari v ke u di G.

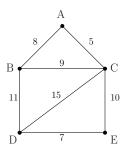


Figure: Graf G

Pohon jalur terpendek (shortest-path tree) yang berakar pada simpul v adalah pohon rentang (spanning tree) T dari graf G, sedemikian sehingga jarak jalur dari akar v ke simpul lain mana pun u di T adalah jalur terpendek jarak dari v ke u di G.

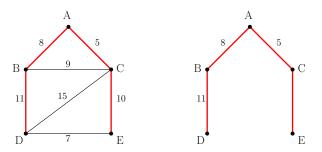


Figure: Yang berwarna merah adalah pohon jalur terpendek dari G berakar pada A

Pohon jalur terpendek (shortest-path tree) yang berakar pada simpul v adalah pohon rentang (spanning tree) T dari graf G, sedemikian sehingga jarak jalur dari akar v ke simpul lain mana pun u di T adalah jalur terpendek jarak dari v ke u di G.

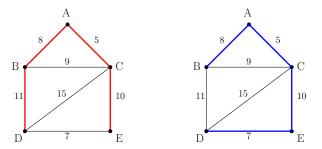


Figure: Yang berwarna merah merupakan pohon jalur terpendek, dan yang biru merupakan MST dari G

#### Varian dari shortest path

#### Berdasarkan titik

- Sumber tunggal (single source): dari satu simpul s ke setiap simpul lainnya
- Source-sink: dari simpul sumbery s ke suatu simpul akhir t
- Semua pasangan: antara semua pasangan simpul

#### Batasan pada bobot sisi?

- Bobot yang tak-negatif
- Bobot yang merupakan jarak Euclid
- Bobot sebarang

#### Sirkuit?

- Tanpa sirkuit berarah
- Tanpa "sirkuit negatif"



#### Penerapan dari shortest path

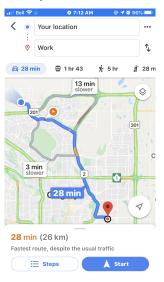


Figure: Mencari rute terpendek pada Google maps

#### Penerapan dari shortest path



Figure: Sistem navigasi pada mobil

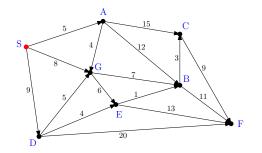
# Bagian 2. Algoritma Dijkstra



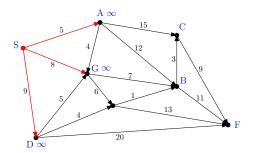
Figure: Edsger W. Dijkstra

#### Single-source shortest path:

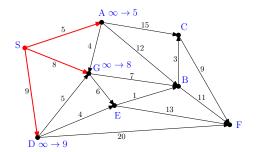
**Tujuan:** Temukan jalur terpendek dari sumber s ke setiap simpul lainnya.



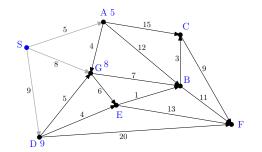
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S	0	-
A	5.0	$S \to A$
В	14.0	$\mathrm{E} \to \mathrm{B}$
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G	8.0	$S \to G$



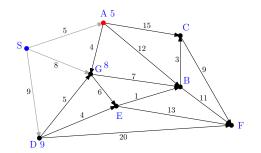
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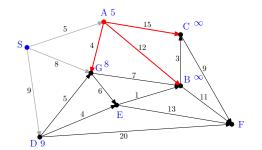
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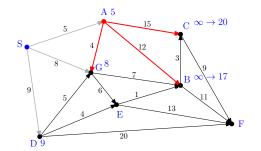
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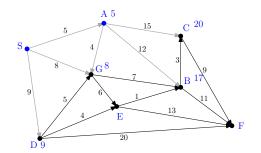
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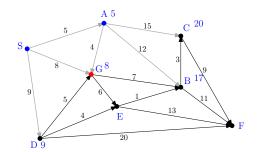
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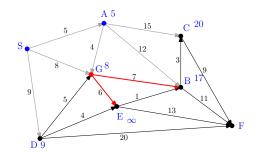
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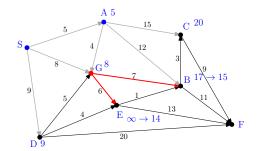
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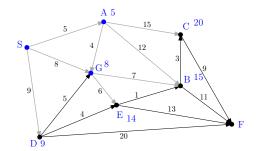
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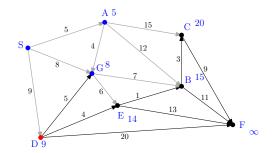
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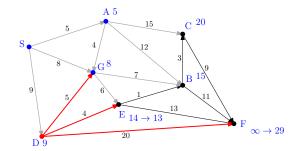
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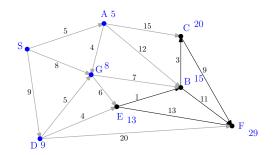
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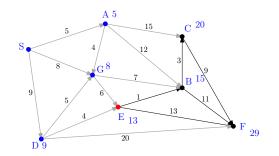
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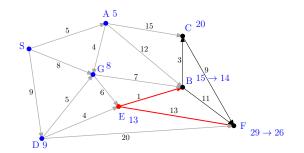
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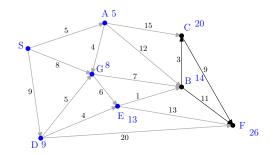
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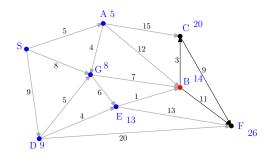
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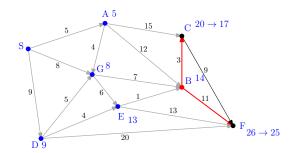
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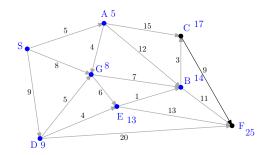
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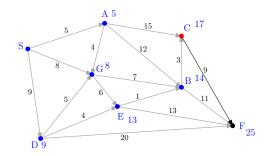
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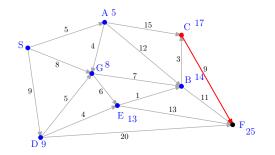
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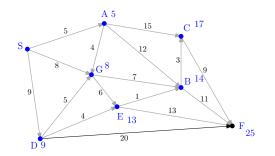
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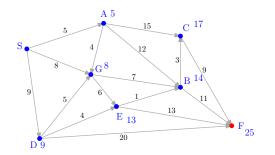
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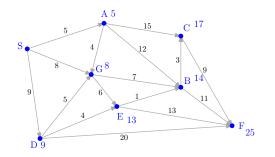
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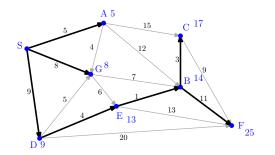
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