Linear Algebra

[KOMS120301] - 2023/2024

4.3 - Applications of Linear System in CS

(the content of this slide is adapted from the lecture's slide of Rinaldi Munir, ITB)

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Exploratory activities

Exercise

Solve each of the following systems:

$$\begin{aligned} x_1 + x_2 - 2x_3 + 4x_4 &= 5 \\ 2x_1 + 2x_2 - 3x_3 + x_4 &= 3 \\ 3x_1 + 3x_2 - 4x_3 - 2x_4 &= 1 \end{aligned}$$

(a)

$$\begin{aligned} x_1 + x_2 - 2x_3 + 3x_4 &= 4 \\ 2x_1 + 3x_2 + 3x_3 - x_4 &= 3 \\ 5x_1 + 7x_2 + 4x_3 + x_4 &= 5 \end{aligned}$$

(b)

$$\begin{aligned} x + 2y + z &= 3 \\ 2x + 5y - z &= -4 \\ 3x - 2y - z &= 5 \end{aligned}$$

(c)

(a) Reduce its augmented matrix M to echelon form and then to row canonical form as follows:

$$M = \begin{bmatrix} 1 & 1 & -2 & 4 & 5 \\ 2 & 2 & -3 & 1 & 3 \\ 3 & 3 & -4 & -2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & -2 & 4 & 5 \\ 0 & 0 & 1 & -7 & -7 \\ 0 & 0 & 2 & -14 & -14 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & -10 & -9 \\ 0 & 0 & 1 & -7 & -7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Rewrite the row canonical form in terms of a system of linear equations to obtain the free variable form of the solution. That is,

$$\begin{aligned} x_1 + x_2 \qquad - 10x_4 &= -9 \\ x_3 - 7x_4 &= -7 \end{aligned} \qquad \text{or} \qquad \begin{aligned} x_1 &= -9 - x_2 + 10x_4 \\ x_3 &= -7 + 7x_4 \end{aligned}$$

(The zero row is omitted in the solution.) Observe that x_1 and x_3 are the pivot variables, and x_2 and x_4 are the free variables.

(b) First reduce its augmented matrix M to echelon form as follows:

$$M = \begin{bmatrix} 1 & 1 & -2 & 3 & 4 \\ 2 & 3 & 3 & -1 & 3 \\ 5 & 7 & 4 & 1 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & -2 & 3 & 4 \\ 0 & 1 & 7 & -7 & -5 \\ 0 & 2 & 14 & -14 & -15 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & -2 & 3 & 4 \\ 0 & 1 & 7 & -7 & -5 \\ 0 & 0 & 0 & 0 & -5 \end{bmatrix}$$

There is no need to continue to find the row canonical form of M, because the echelon form already tells us that the system has no solution. Specifically, the third row of the echelon matrix corresponds to the degenerate equation

$$0x_1 + 0x_2 + 0x_3 + 0x_4 = -5$$

which has no solution. Thus, the system has no solution.

(c) Reduce its augmented matrix M to echelon form and then to row canonical form as follows:

$$M = \begin{bmatrix} 1 & 2 & 1 & 3 \\ 2 & 5 & -1 & -4 \\ 3 & -2 & -1 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 1 & -3 & -10 \\ 0 & -8 & -4 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 1 & -3 & -10 \\ 0 & 0 & -28 & -84 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 1 & -3 & -10 \\ 0 & 0 & 1 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

Thus, the system has the unique solution $x = 2$, $y = -1$, $z = 3$, or, equivalently, the vector $u = (2, -1, 3)$. We note that the echelon form of M already indicated that the solution was unique, because it corresponded to a triangular system.

Page 22-23 Howard Anton Book (*to find, check pages 36-37*)

Guidelines:

1. Divide yourselves into 8 groups
2. Do Q1 - Q2 (3 questions @)
3. Do Q3 - Q4 (1 question @)
4. Do Q5 - Q8 (1 questions @)
5. Do Q9 - Q12 (1 questions @)
6. Do Q13 - Q14 (1 questions @)
7. Do Q15 - Q22 (1 questions @)