

Linear Algebra

[KOMS120301] - 2023/2024

14.1 - Intuition behind matrix transformation

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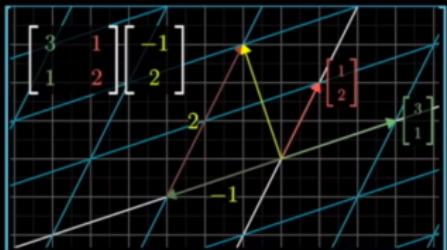
Week 14 (December 2023)

Learning objectives

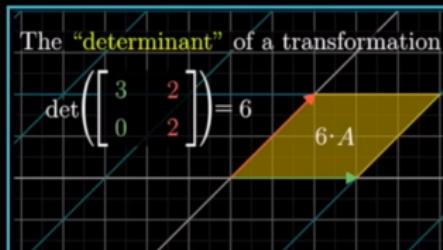
- Recap what we learned in the previous weeks;
- Get an intuitive understanding of the concept;
- Relate it to the concept of linear transformation.

What we have learned

Linear transformations



Determinants



Linear systems

$$\begin{aligned} 2x + 5y + 3z &= -3 \\ 4x + 0y + 8z &= 0 \\ 1x + 3y + 0z &= 2 \end{aligned} \rightarrow \begin{bmatrix} 2 & 5 & 3 \\ 4 & 0 & 8 \\ 1 & 3 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}$$

Change of basis

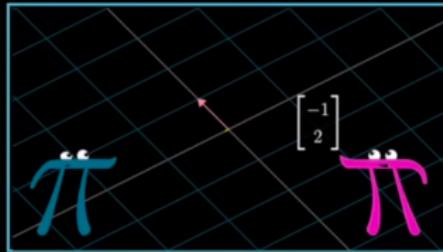
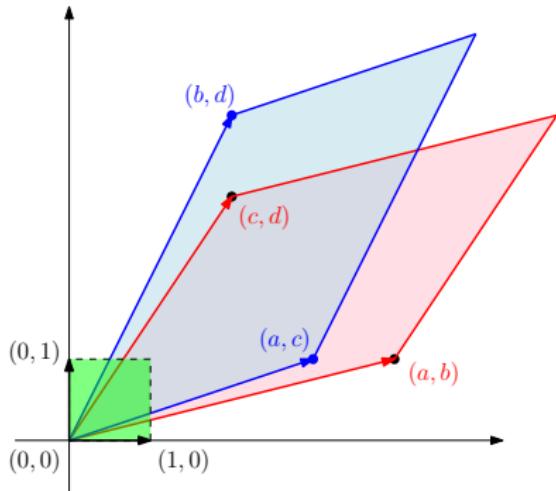


Figure: Prerequisites (source: Youtube of 3Blue1Brown)

Geometric interpretation of determinant (from Week 5)



Matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ can be viewed as an “arrangement” of:

- row vectors:
 $\begin{bmatrix} a & b \end{bmatrix}$ and $\begin{bmatrix} c & d \end{bmatrix}$
- or, column vectors:
 $\begin{bmatrix} a \\ c \end{bmatrix}$ and $\begin{bmatrix} b \\ d \end{bmatrix}$

The matrix defines the so-called *linear transformation* of the unit square (in green) formed by the *basis vectors* $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, with respect to:

- the **row vectors**, shown by the **red** parallelogram; or
- the **column vectors**, shown by the **blue** parallelogram

Both parallelograms have the **same area**. Prove it!

Vectors that “stay in their position” after transformation

Transformation of basis vectors (1)

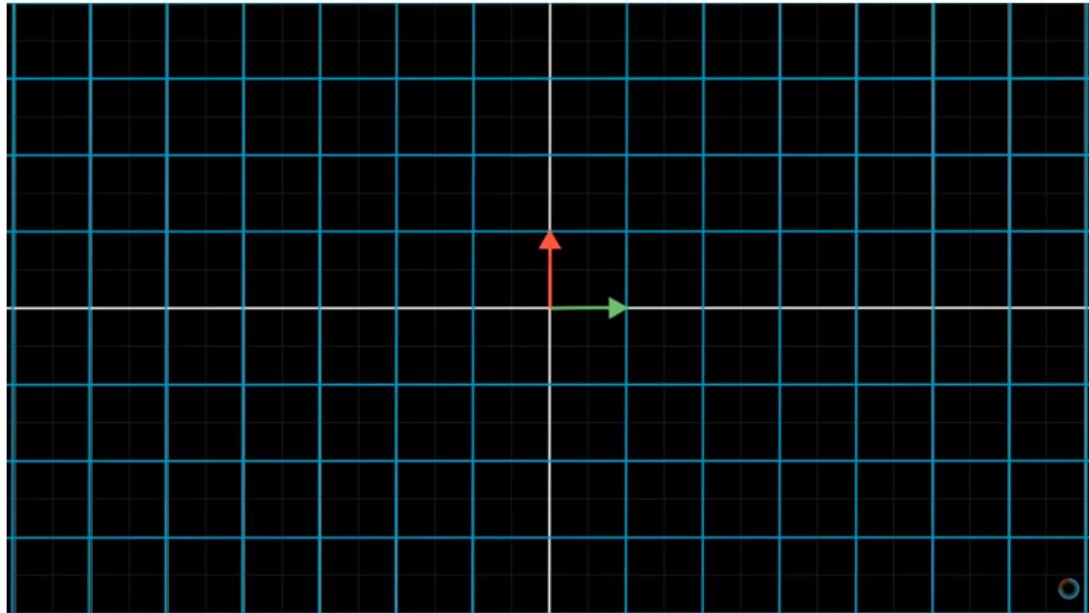


Figure: Two basis vectors in standard system (source: *Youtube of 3Blue1Brown*)

Transformation of basis vectors (2)

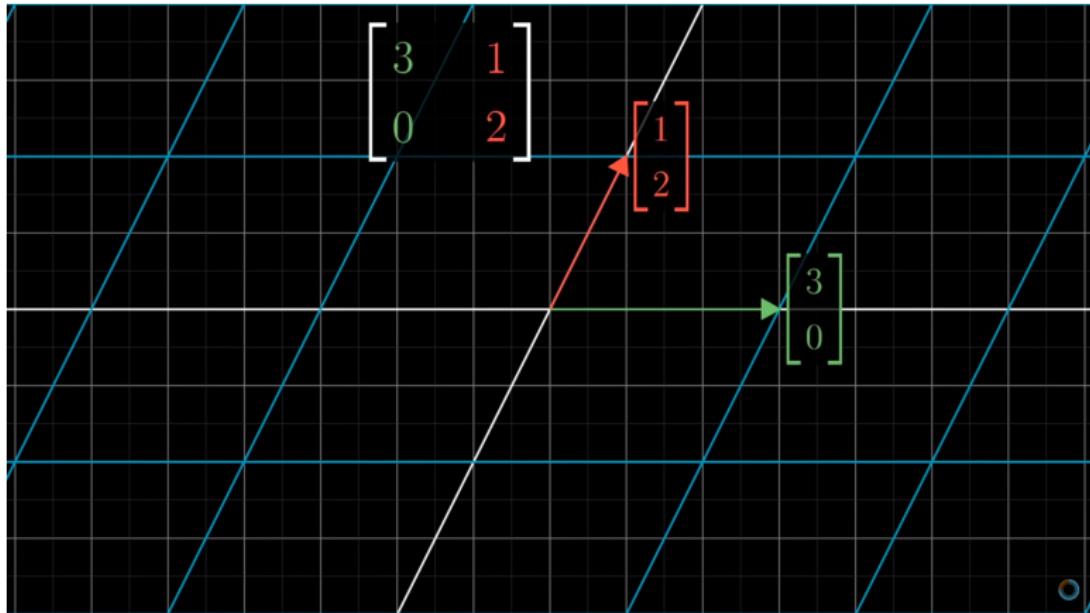


Figure: Result of transformation of the basis vectors remain in its “position” (source: *Youtube of 3Blue1Brown*)

Transformation of basis vectors (3)

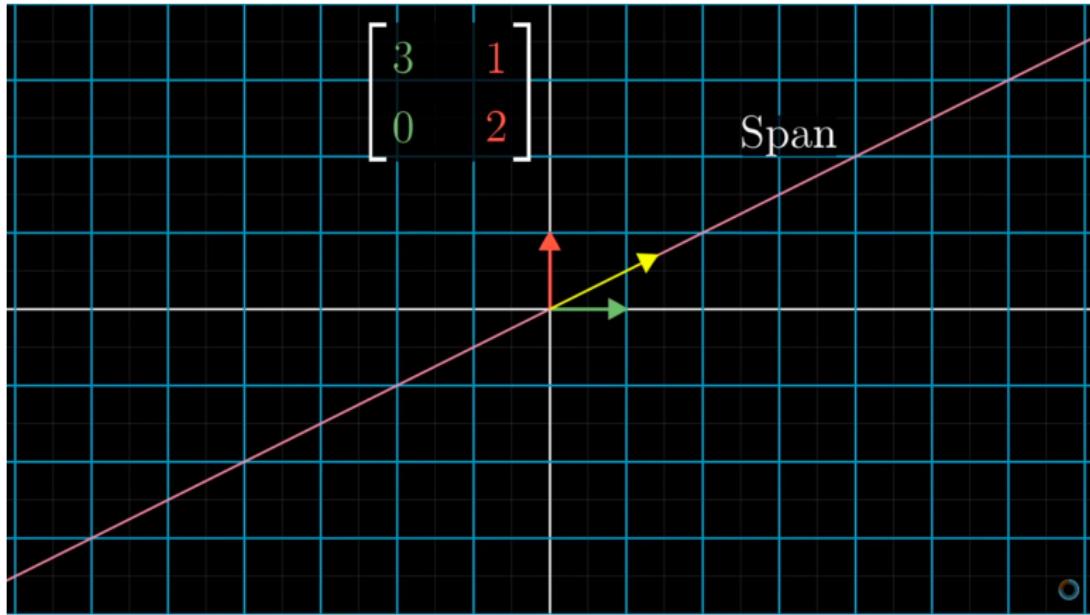


Figure: A (yellow) vector and its span (*source: Youtube of 3Blue1Brown*)

Transformation of basis vectors (4)

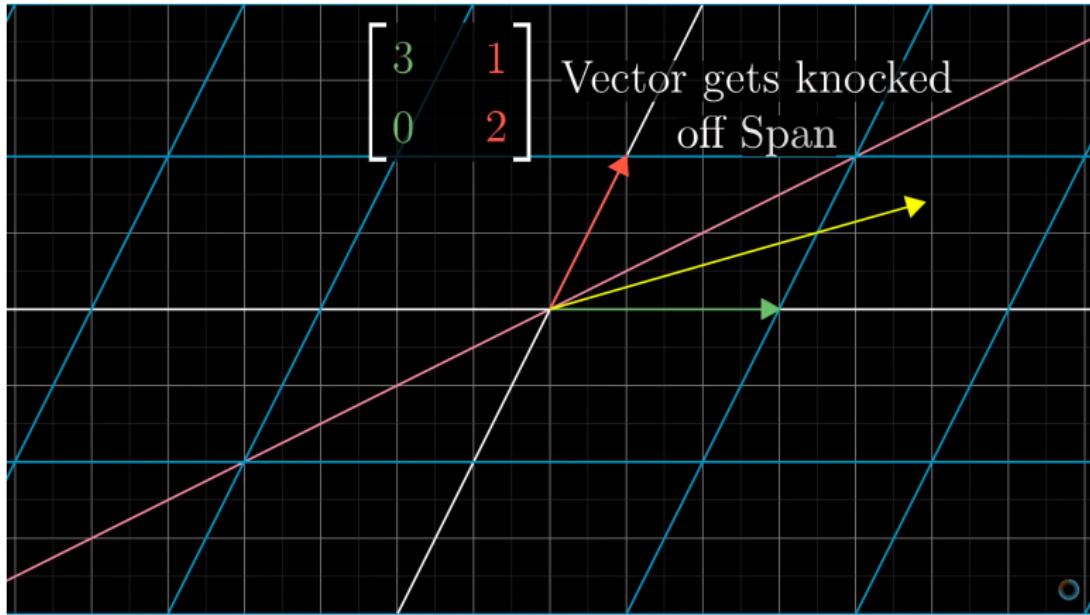


Figure: The yellow vector does not stay in its position (source: *Youtube of 3Blue1Brown*)

Transformation of basis vectors (5)

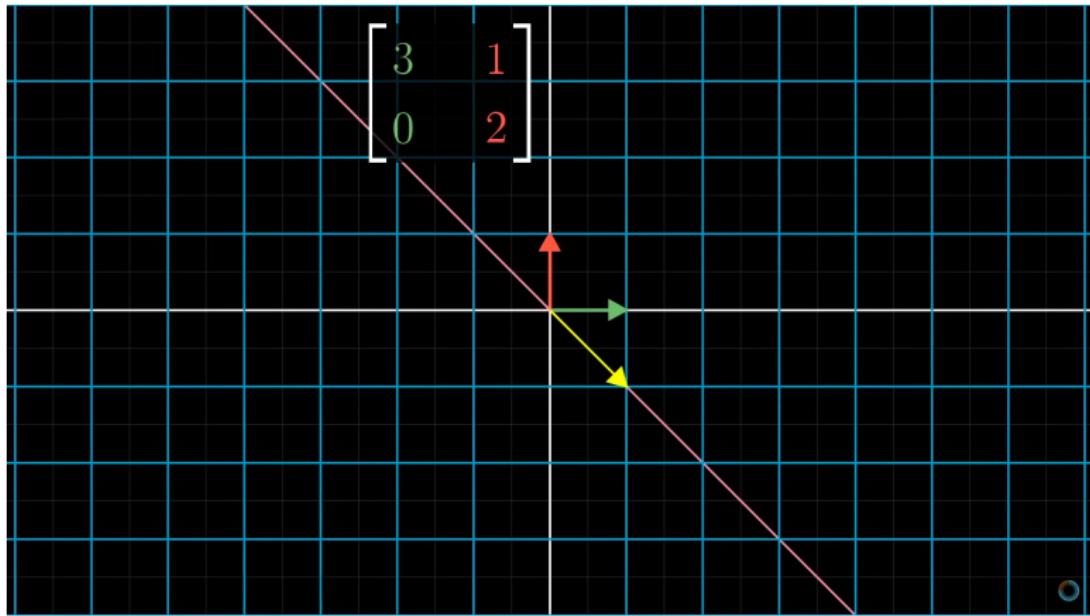


Figure: Another yellow vector (source: *Youtube of 3Blue1Brown*)

Transformation of basis vectors (6)

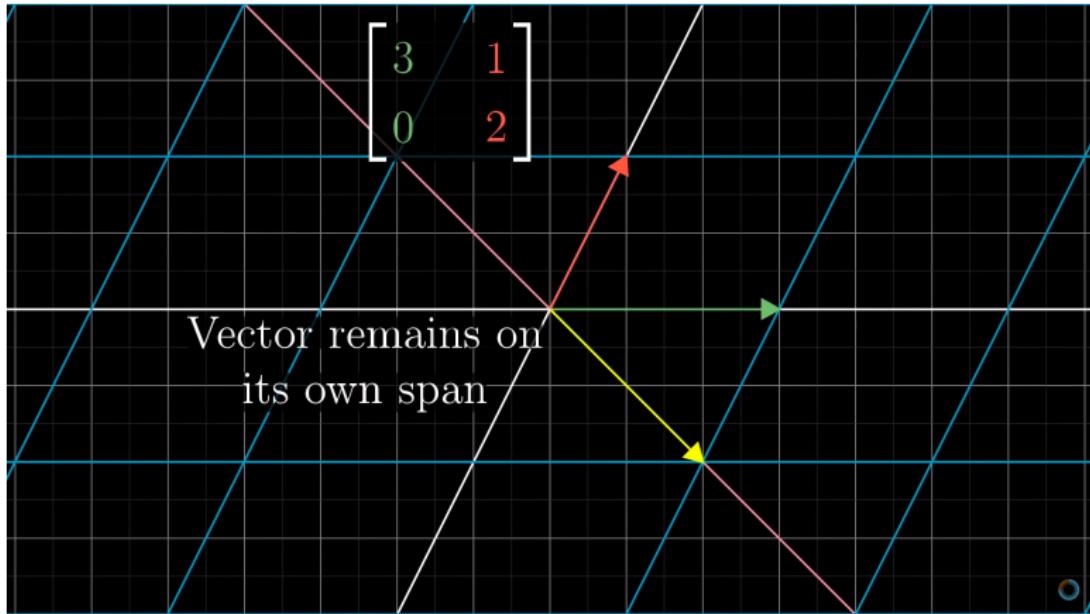


Figure: The vector remains in its position after transformation (source: [Youtube of 3Blue1Brown](#))

Transformation of basis vectors (7)

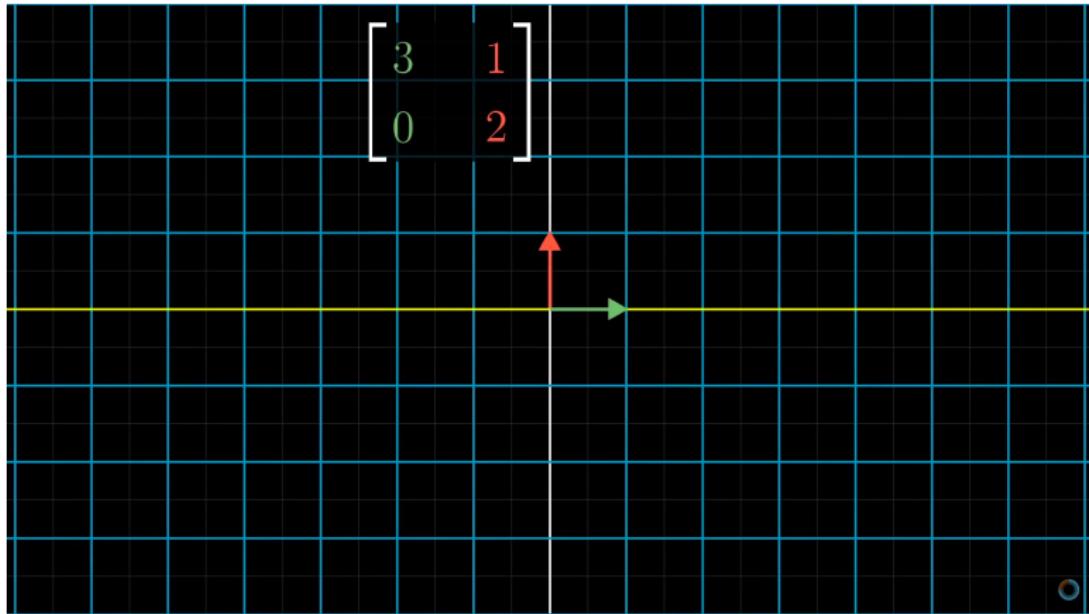


Figure: What happens to the green basis vector and its span? (source: [Youtube of 3Blue1Brown](#))

Transformation of basis vectors (8)

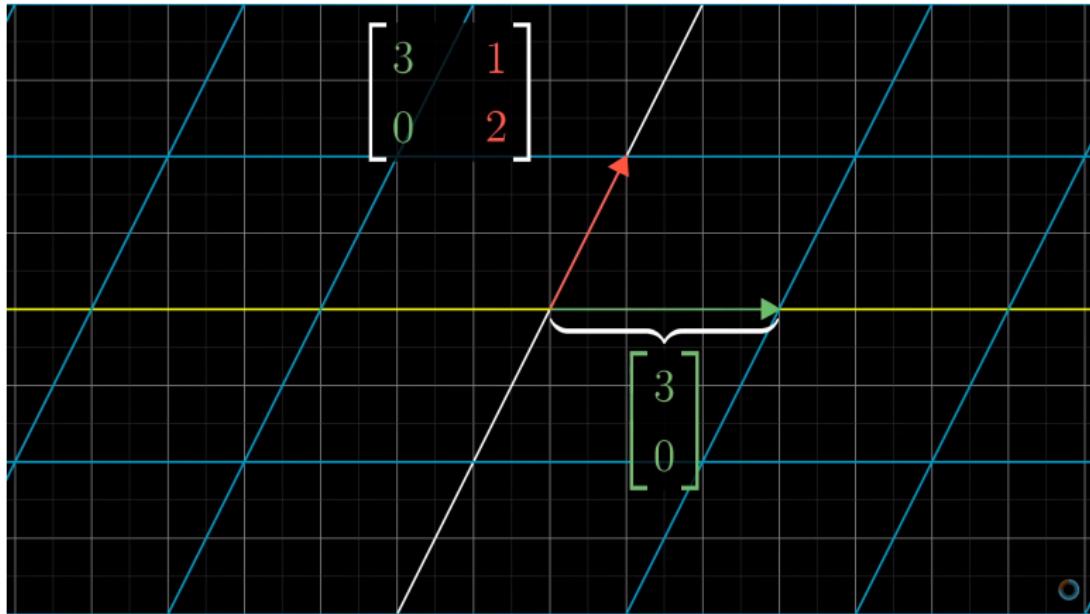


Figure: The green vector remains in its position, and multiplies by 3
(source: *Youtube of 3Blue1Brown*)

Transformation of basis vectors (9)

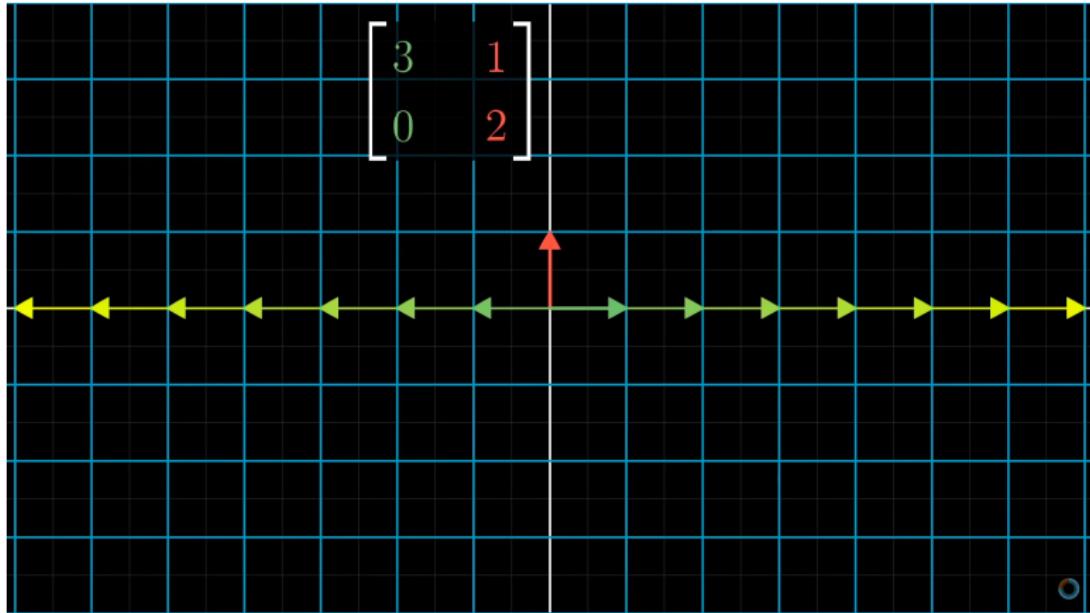


Figure: This happens to all vectors with the same (reverse) direction as the green vector (*source: Youtube of 3Blue1Brown*)

Transformation of basis vectors (10)

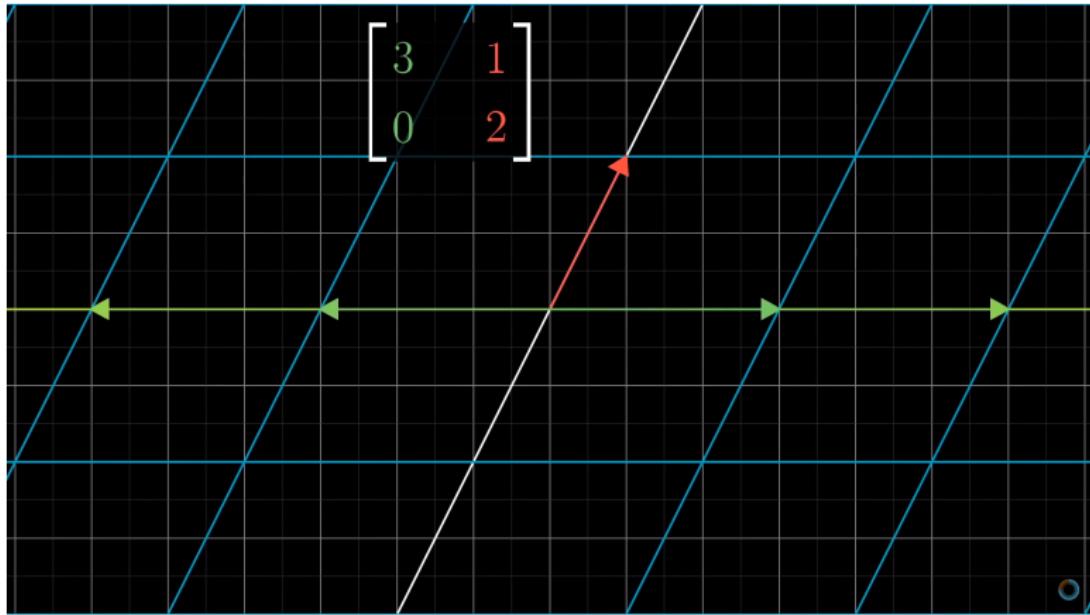


Figure: They are all stretched to 3 times the original vector (source: *Youtube of 3Blue1Brown*)

Transformation of basis vectors (11)

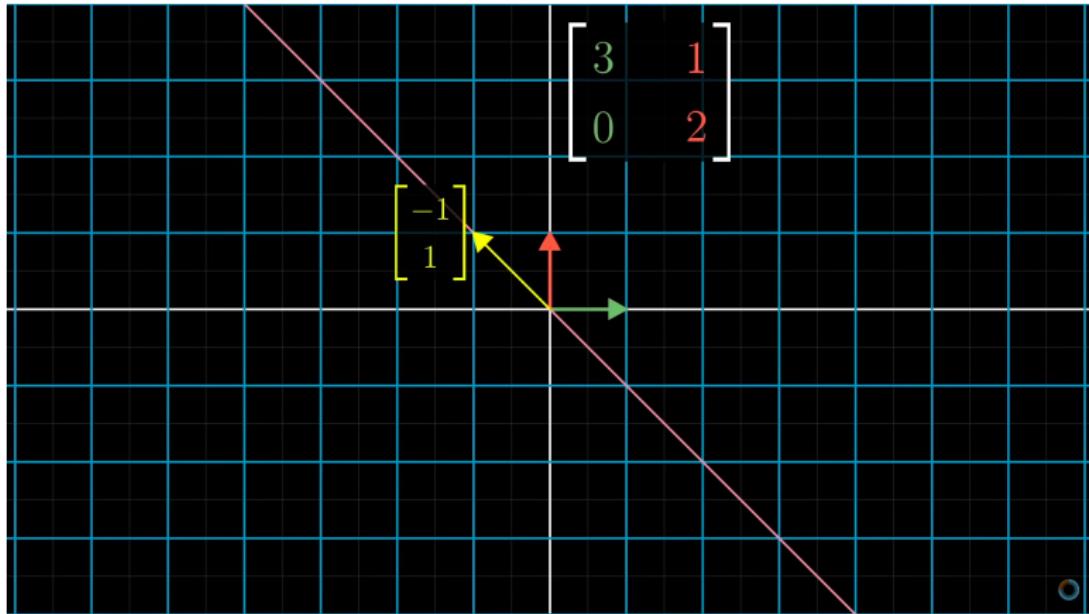


Figure: Another vector with similar property (source: *Youtube of 3Blue1Brown*)

Transformation of basis vectors (12)

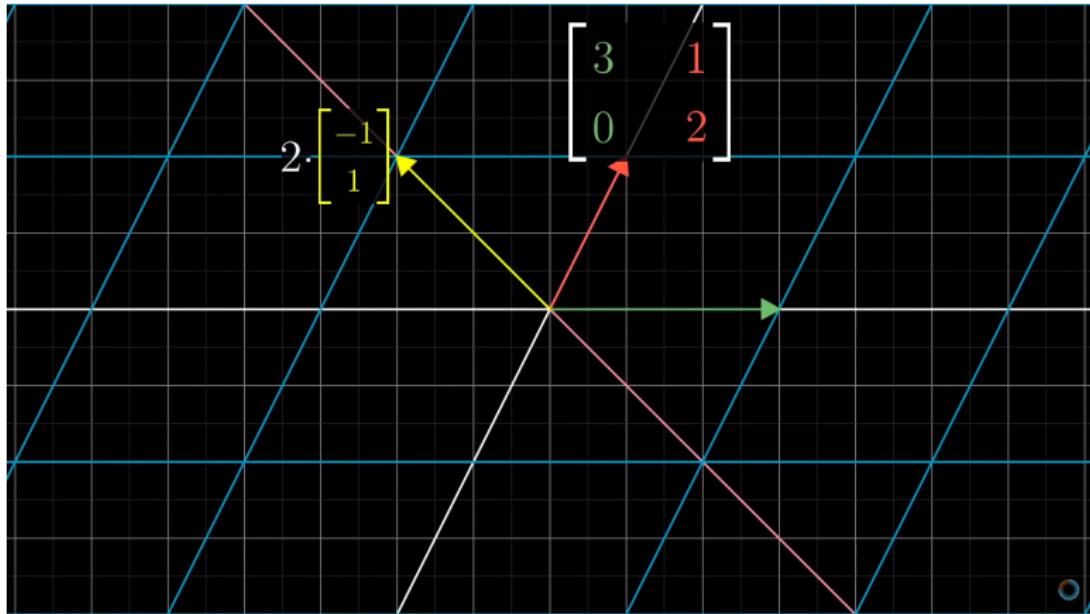


Figure: This vector remains in its position after transformation (source: [Youtube of 3Blue1Brown](#))

Transformation of basis vectors (13)

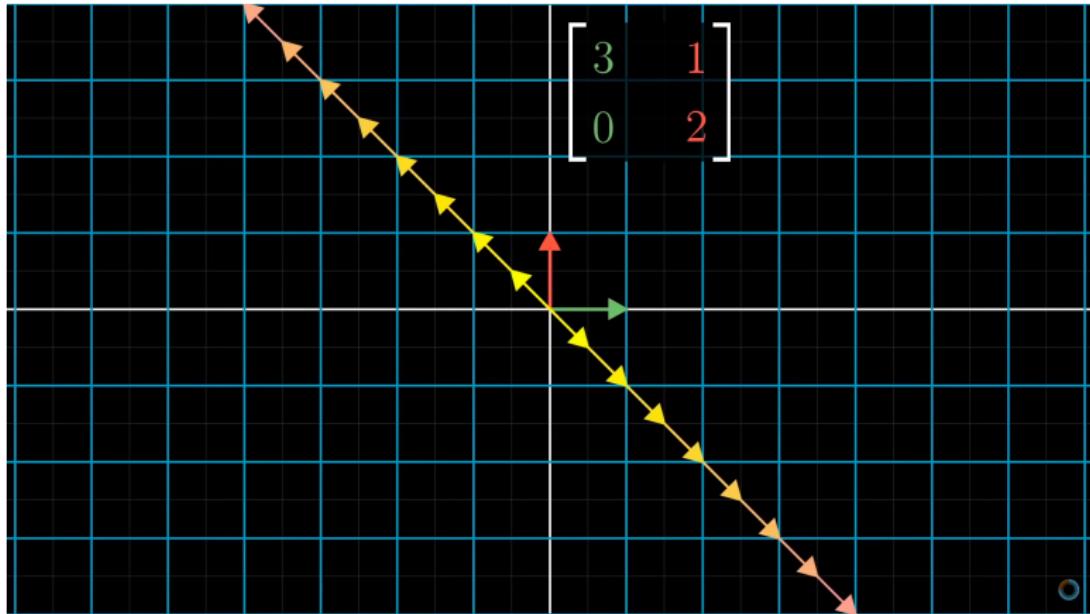


Figure: The property holds for all vectors in the span of its vector
(source: *Youtube of 3Blue1Brown*)

Eigenvectors (1)

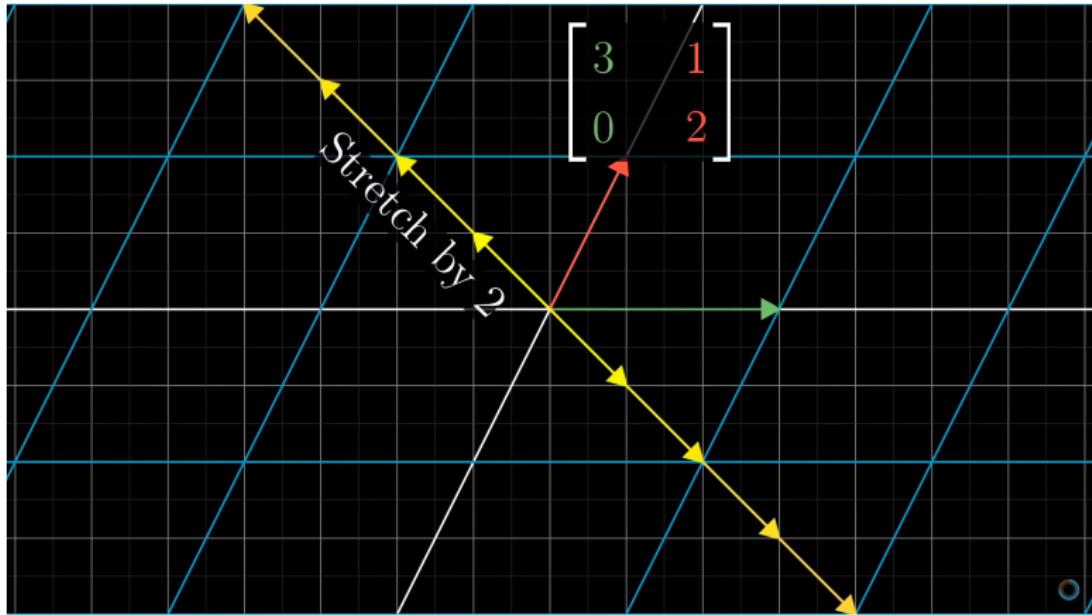


Figure: The yellow vector is stretched by 2 (source: *Youtube of 3Blue1Brown*)

Eigenvectors (2)

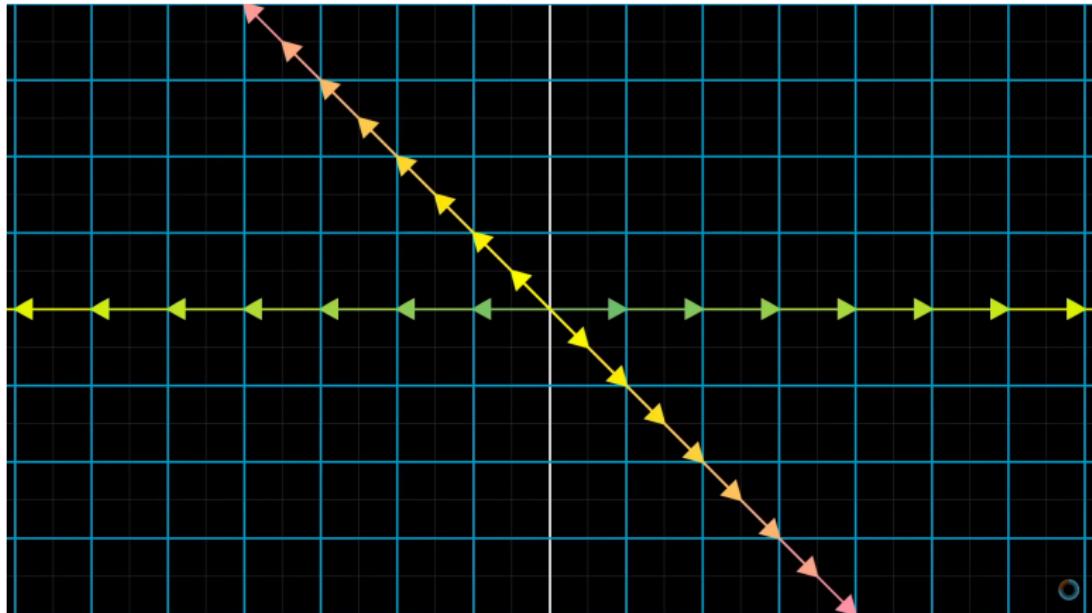


Figure: The green vector is stretched by 3 (source: *Youtube of 3Blue1Brown*)

Eigenvectors (3)

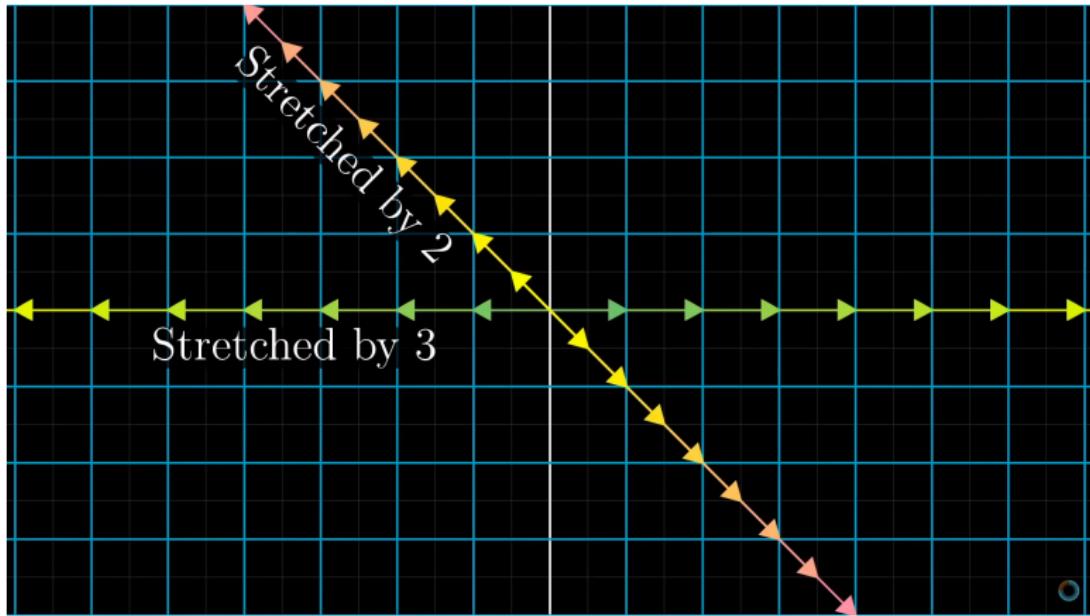


Figure: Source: *Youtube of 3Blue1Brown*

Eigenvectors (4)

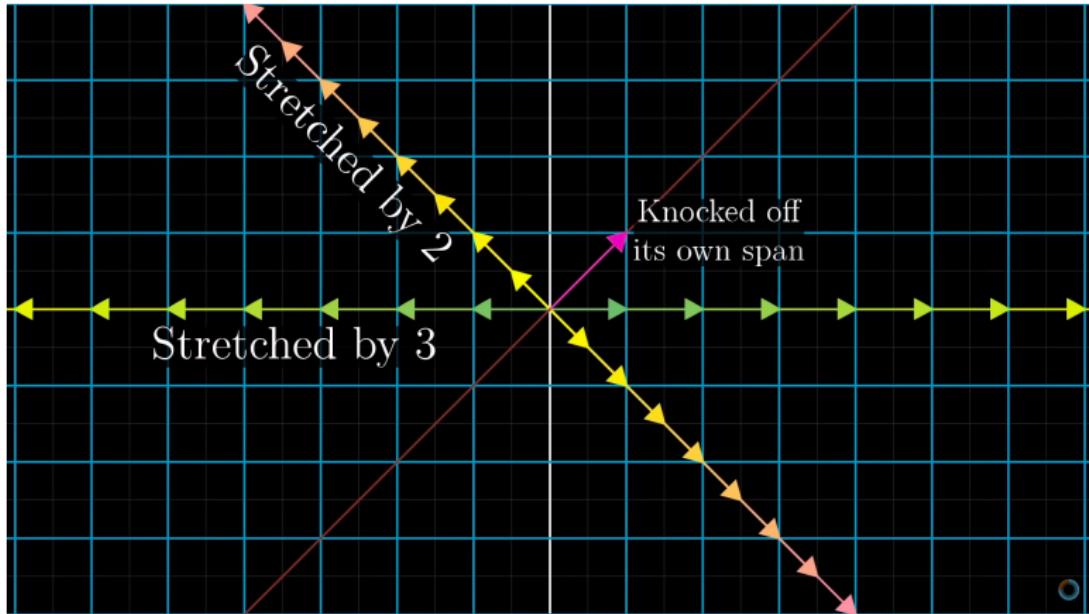


Figure: Other vectors do not stay in their span
Source: Youtube of 3Blue1Brown

Eigenvectors (5)

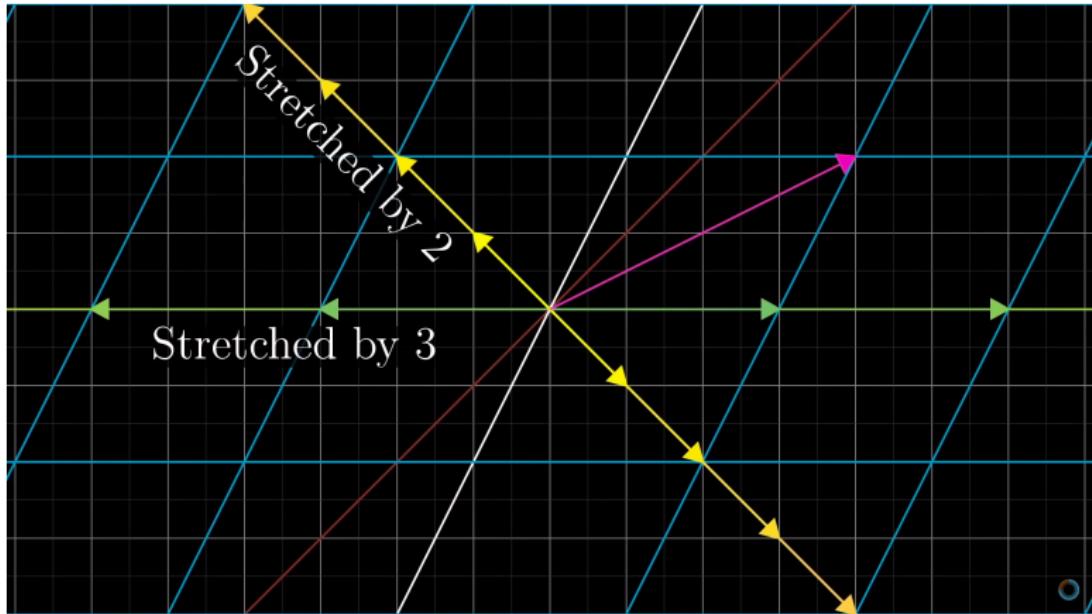


Figure: The transformation keeps the two vectors (yellow and green) in their position (*source: Youtube of 3Blue1Brown*)

Eigenvectors (6)

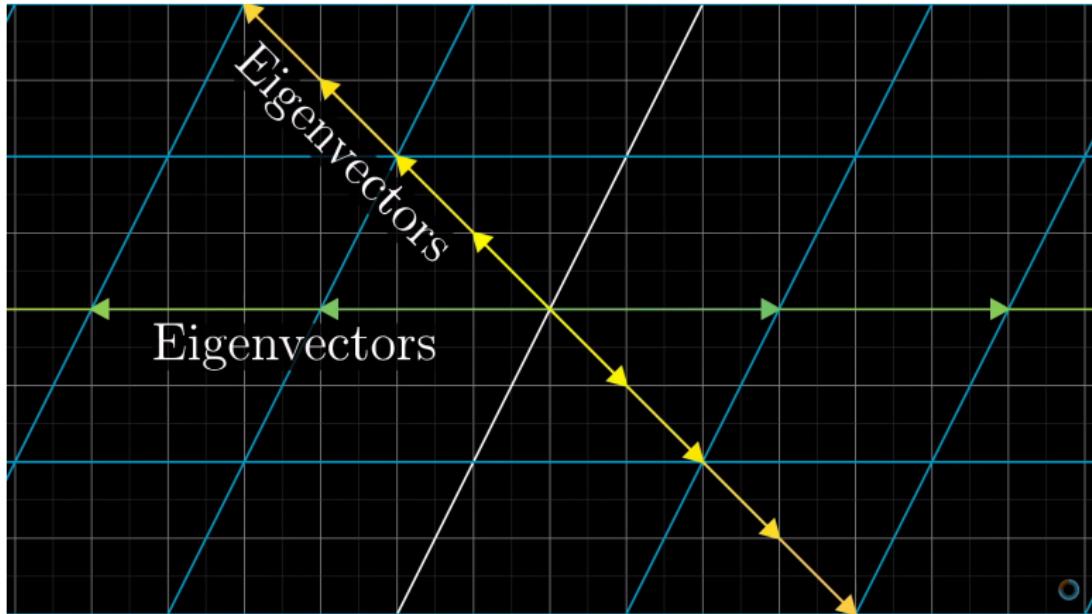


Figure: The transformation keeps the two vectors in their position
(source: *Youtube of 3Blue1Brown*)

Eigenvectors (7)

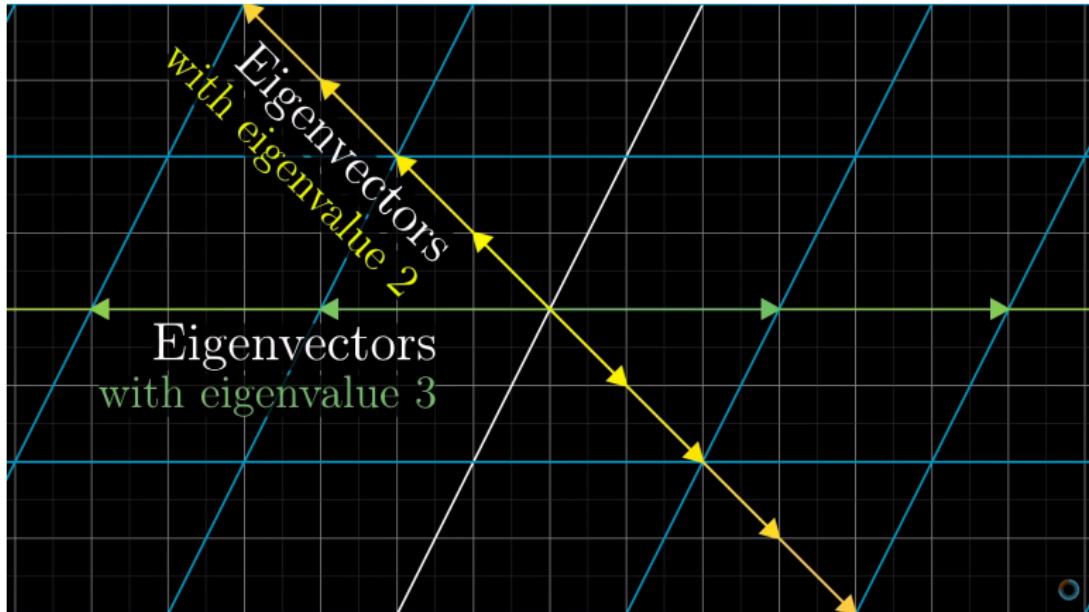


Figure: They are called **eigenvectors** (The transformation keeps the two vectors in their position *source: Youtube of 3Blue1Brown*)

Eigenvectors (8)

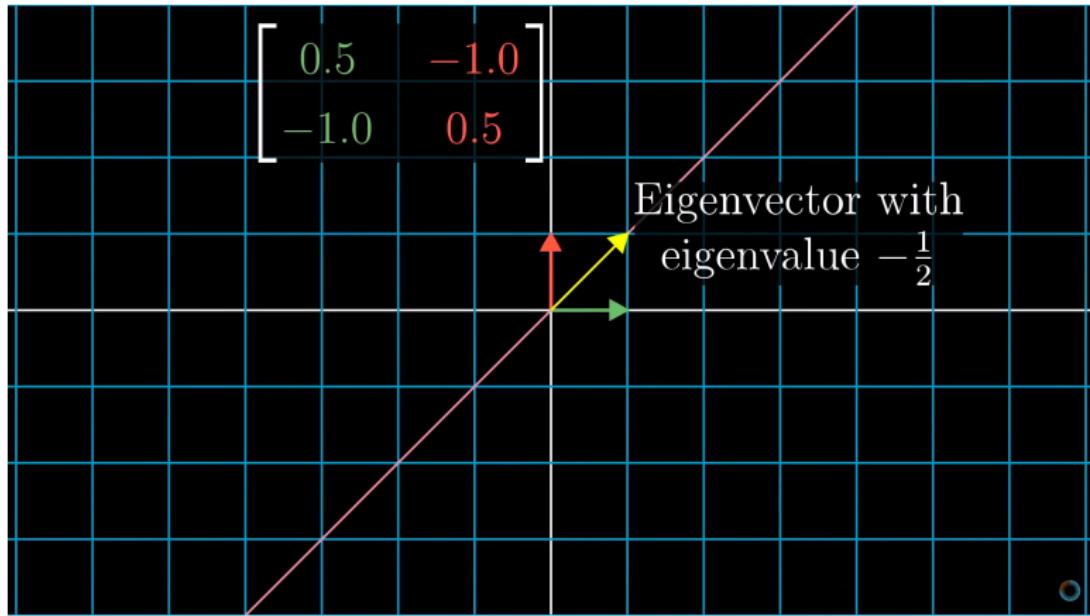


Figure: Prerequisites (source: *Youtube of 3Blue1Brown*)

Eigenvectors (9)

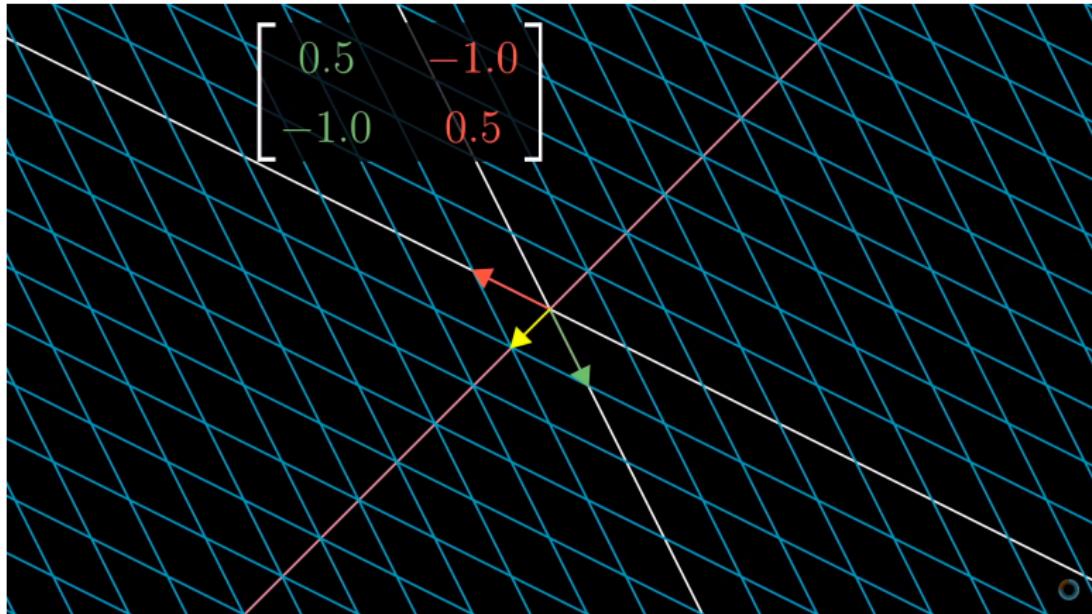


Figure: Prerequisites (source: *Youtube of 3Blue1Brown*)