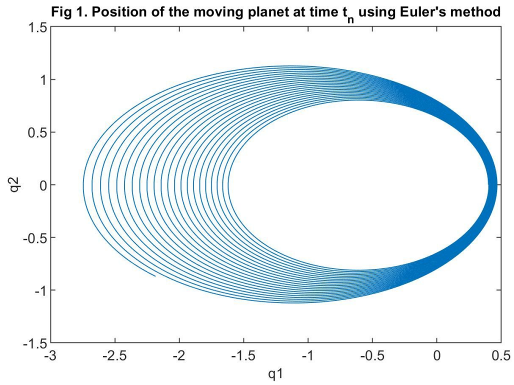
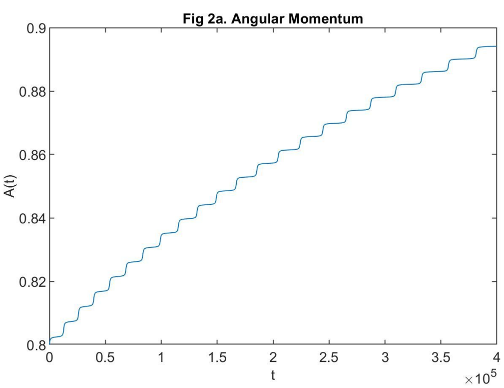
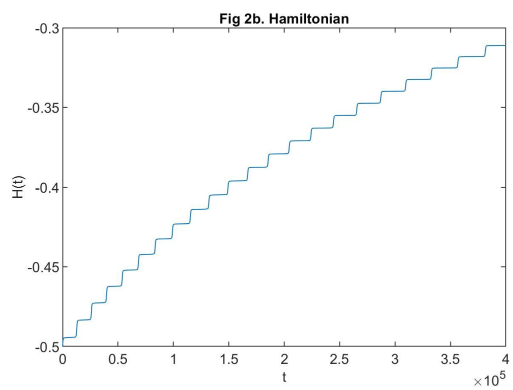
Submitted by Deniz Evrendilek (301340591) for MACM 316 on the 26th of July 2019

Computing Assignment 8: Numerical Solution of Kepler's Problem (the Two-Body Problem)



This study reports the two numerical solutions of Kepler's problem by Euler's (EM) versus symplectic Euler (SE) methods. By using the following parameters of time $0 \le t \le T = 200$, stepsize $h = 0.0005$, $N = 400{,}000$, and the initial values of $q_1(0) = 1 - e$, $q_2(0) = 0$, $p_1(0) = 0$, and $p_2(0) = \sqrt{\frac{1+e}{1-e}}$ where $e = 0.6$, the EM solution is plotted in the q_1-q_2 plane (**Fig. 1**). EM only uses conditions at the current time (t) to compute condition at a later time (t+1). The EM solution is not a method that accounts for the law of conservation of energy $H(t)$, and angular momentum $A(t)$. The velocity is not updated first in order to compute the position with the new velocity. Due to this physical nonsense, EM shows a qualitatively wrong behavior (a distorted ellipse), that is, the EM solution spirals outwards. The size of the error in $A(t)$ (**Fig. 2a**) and $H(t)$ (**Fig. 2b**) increases with time linearly for EM.





The EM solution was improved on by SE considering both the current state and the state at a later time to update the state; namely, updating the velocity with the force at the new position (**Fig. 3a**). SE ensures the preservation of $A(t)$ at around 0.8, and the near-conservation of $H(t)$ between -0.49932 and -0.50068 in planetary motion since its symplecticity means its area-preserving property in space. The size of the error in $A(t)$ (**Fig. 3b**) and $H(t)$ (**Fig. 3c**) remains bounded and small for SE. SE keeps the curves of the numerical and exact solutions closer to one another. Both EM and SE exhibit a precession effect. SE would be my choice to compute planetary orbits owing to its area-preserving property, conservation of $A(t)$ and $H(t)$ and better precision and accuracy.