26.09.22

Neumerical Method

Solution of Algebric & transcendental

- Bracketting method:
  - ↳ Bisection method
  - ↳ False method
- → Open method

Bisection method:

Find the root of the following function using Bisection method. $f(x) = 3x - \cos x - 1$

Soln: $a = 0$ $f(a) = -2$

$b = 1$ $f(b) = 1.46$

$\therefore f(a) * f(b) < 0$

a	b	f(a)	f(b)	$c = a+b/2$	f(c)
0	1	-2	1.44	0.5	-0.38
0.5	1	-0.38	1.44	0.75	0.52
0.5	0.75	-0.38	0.52	0.625	0.06
0.5	0.675	-0.38	0.06	0.56	-0.17
.56	.625	-0.17	0.06	0.59	-0.05
.59	.625	-0.05	0.06	0.61	+0.01
.59	.61	-0.05	0.01	0.61	0.01

$\therefore$ root of $f(x) = 0.61$

False position method:

$$c = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

find the root of the following function using false position method.

$$f(x) = 3x - \cos x - 1$$

$a = 0 \quad f(a) = -2$
$b = 1 \quad f(b) = 1.46$

a	b	f(a)	f(b)	c	f(c)
0	1	-2	1.46	.58	-0.10
.58	1	-0.10	1.46	.61	0.01

(i) $f(x) = 3x - \cos x - 1$? Bisection methode
(ii) # $f(x) = x^3 - 4x - 9$ using Bisection & false position.

$f(x) = x^3 - 4x - 9$
$f(0) = -9$
$f(1) = -12$
$f(2) = -9$, $a = 2$
$f(3) = 6$, $b = 3$

a	b	f(a)	f(b)	$c = \frac{a+b}{2}$	f(c)
2	3	-9	6	2.5	-3.38
2.5	3	-3.38	6	2.8	1.75
2.5	2.8	-3.38	1.75	2.65	-0.99
2.65	2.8	-0.99	1.75	2.73	0.43
2.65	2.73	-0.99	0.43	2.69	-0.29
2.69	2.73	-0.29	0.43	2.71	0.06
2.69	2.71	-0.29	0.06	2.7	-0.117
2.7	2.71	-0.117	0.06	2.71	0.062

17.10.22

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

Newton Raphson method:

Use Newton Raphson method to find the real root near 2 of the eq^n $x^4 - 11x + 8 = 0$ accurate to five decimal values.

Soln: $f(x) = x^4 - 11x + 8$

$f'(x) = 4x^3 - 11$,

let, $x_1 = 2$ - যে কোনো একটা হিসাবেই হবে,

$f(x_1) = 2^4 - 11 \times 2 + 8$

$= 2$

$f'(x_1) = 4 \times 2^3 - 11 = 21$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2 - \frac{2}{21}$$

$$= 1.\underbrace{90476}_{\text{five decimal-এ গুনবে}}$$

Similarly,

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 1.90476 - \frac{(1.90476)^4 - 11(1.90478) +}{4 \times (1.90476)^3 - 11}$$

$$= 1.89209$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.89188$$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = 1.89188$$

$\therefore$ The root the eq^n is: 1.89188

Secant method

formula: $$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Use the ~~secant~~ secant method to estimate the root of the eq^n: $x^2 - 4x - 10 = 0$ with the initial estimates of $x_1 = 4$ & $x_2 = 2$. initial value

Sol^n: $f(x) = x^2 - 4x - 10$

& $x_1 = 4$ & $x_2 = 2$

Iteration-1: $f(x_1) = -10$, $f(x_2) = -14$

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

$$= 2 - \frac{-14(2-4)}{-14-(-10)} = 9$$

Second Iteration: $x_1 = x_2 = 2$

$x_2 = x_3 = 9$

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

$$= 9 - \frac{f(9)(9-2)}{f(9)-f(2)}$$

$\therefore x_3 = 4$

Third Iteration:

$$x_1 = x_2 = 9$$
$$x_2 = x_3 = 4$$
$$\therefore x_3 = 4 - \frac{-10(4-9)}{-10-35} = 5.1111$$

Fourth iteration:

$$x_1 = x_2 = 4$$
$$x_2 = x_3 = 5.1111$$
$$x_3 = 5.9563$$

Fifth iteration:

$$x_1 = x_2 = 5.1111$$
$$x_2 = x_3 = 5.9563$$
$$x_3 = \cancel{7}\ 5.7225$$

Sixth iteration: $x_1 = x_2 = 5.9563$

$$x_2 = x_3 = 5.7255$$
$$x_3 = 5.6182$$

The value can be further refine by continueing the procese, if necessary

Soln of Linear Algebra:

Gauss Elemination:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$2x_1 + x_2 + 4x_3 = 12$
$4x_1 + 11x_2 - x_3 = 33$ ——— (i)
$8x_1 - 3x_2 + 2x_3 = 20$

Soln:

The system can be written as $AX = B$

where, $A = \begin{bmatrix} 2 & 1 & 4 \\ 4 & 11 & -1 \\ 8 & -3 & 2 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $B = \begin{bmatrix} 12 \\ 33 \\ 20 \end{bmatrix}$

The augmented matrix of (i)

$$[A:B] = \left[\begin{array}{ccc:c} 2 & 1 & 4 & 12 \\ 4 & 11 & -1 & 33 \\ 8 & -3 & 2 & 20 \end{array}\right]$$

$R_2' \rightarrow R_2 - 2R_1$
$R_3' \rightarrow R_3 - 4R_1$

$$[A:B] = \begin{bmatrix} 2 & 1 & 4 & : & 12 \\ 0 & 9 & -9 & : & 9 \\ 0 & -7 & -14 & : & 28 \end{bmatrix}$$

$R_3' \to 9R_3 + 7R_2$

$$[A:B] = \begin{bmatrix} 2 & 1 & 4 & : & 12 \\ 0 & 9 & -9 & : & 9 \\ 0 & 0 & -189 & : & -189 \end{bmatrix}$$

$2x_1 + x_2 + 4x_3 = 12$

$9x_2 - 9x_3 = 9$

$-189x_3 = -189$

$x_3 = 1$

$9x_2 = 18$

$\therefore x_2 = 2$

$2x_1 + 2 + 4 \times 1 = 12$

$2x_1 = 6$

$\therefore x_1 = 3$ An

23.10.22

Gauss Jordan Elimination Method:

$$\left.\begin{array}{l} 2x-3y+10z=3 \\ -x+4y+2z=20 \\ 5x+2y+z=-12 \end{array}\right\} \text{---} (i)$$

The system can be written as,

$$AX=B \text{ --- } (ii)$$

where,

$$A=\begin{bmatrix} 2 & -3 & 10 \\ -1 & 4 & 2 \\ 5 & 2 & 1 \end{bmatrix};\ X=\begin{bmatrix} x \\ y \\ z \end{bmatrix};\ B=\begin{bmatrix} 3 \\ 10 \\ -12 \end{bmatrix}$$

Now, the augmented matrix of the given system

$$[A:B]=\begin{bmatrix} 2 & -3 & 10 & : & 3 \\ -1 & 4 & 2 & : & 10 \\ 5 & 2 & 1 & : & -12 \end{bmatrix}$$

$R_1'=\frac{R_1}{2}$

$$[A:B]=\begin{bmatrix} 1 & 3/2 & 5 & : & 3/2 \\ -1 & 4 & 2 & : & 10 \\ 5 & 2 & 1 & : & -12 \end{bmatrix}$$

$\frac{3}{2}+4$

$=\frac{3+}{2}$

$R_2'=R_2+R_1$

$R_3'=R_3-5R_1$

$-12-\frac{15}{2}$

$=\frac{-24-15}{2}$

$$\begin{bmatrix} 1 & 3/2 & 5 & 3/2 \\ 0 & \frac{11}{2} & 7 & 23/2 \\ 0 & -11/2 & -24 & : -39/2 \end{bmatrix}$$

$R_2'=\frac{2}{11}R_2$

$1 \quad 3/2 \quad 5 : 3/2$

$0 \quad 1 \quad \frac{14}{11}$

Now, the augmented matrix of the given system,

$$[A:B] = \left[\begin{array}{ccc:c} 2 & -3 & 10 & 3 \\ -1 & 4 & 2 & 20 \\ 5 & 2 & 1 & -12 \end{array}\right]$$

$$r_1' = \frac{r_1}{2}$$

$$[A:B] = \left[\begin{array}{ccc:c} 1 & -3/2 & 5 & 3/2 \\ -1 & 4 & 2 & 20 \\ 5 & 2 & 1 & -12 \end{array}\right]$$

$$r_2' = r_2 + r_1$$
$$r_3' = r_3 - 5r_1$$

$$[A:B] = \left[\begin{array}{ccc:c} 1 & -3/2 & 5 & 3/2 \\ 0 & 5/2 & 7 & 43/2 \\ 0 & 19/2 & -24 & -39/2 \end{array}\right]$$

$$r_2' = \frac{2r_2}{5}$$

$$[A:B] = \left[\begin{array}{ccc:c} 1 & -3/2 & 5 & 3/2 \\ 0 & 1 & 14/5 & 43/5 \\ 0 & 19/2 & -24 & -39/2 \end{array}\right]$$

$$\frac{3}{2} + \frac{3}{2} \times \frac{43}{5} \qquad \frac{3}{2} + \frac{129}{10} = \frac{15+129}{10} = \frac{144}{10} = \frac{72}{5}$$

$$r_1' = r_1 + 3/2\, r_2$$
$$r_3' = r_3 - \frac{19}{2} r_2$$

$$[A:B] = \left[\begin{array}{ccc:c} 1 & 0 & \frac{46}{5} & \frac{72}{5} \\ 0 & 1 & 14/5 & 43/5 \\ 0 & 0 & -\frac{253}{5} & -\frac{506}{5} \end{array}\right]$$

$$r_3' = -\frac{5}{253} r_3$$

$$[A:B] = \left[\begin{array}{ccc|c} 1 & 0 & 46/5 & 72/5 \\ 0 & 1 & 19/5 & 43/5 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\therefore [A:B] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

The reduced system is:

$$x + 0 + 0 = -4$$
$$0 + y + 0 = 3$$
$$0 + 0 + z = 2$$

we get, $x = -4$, $y = 3$ $z = 2$ Ans

30.10.12

Cholesky's ~~traicir~~ triangularisation method or Crout's triangularisation method or method of factorisation or Cholesky's factorisation method:

Solve the system of linear equations using Cholesky's factorization method.

$$2x - 6y + 8z = 24$$
$$5x + 4y - 3z = 2$$
$$3x + y + 2z = 16$$

Soln:

The ~~ado~~ above system can be written

as, $AX = B$

where, $A = \begin{bmatrix} 2 & -6 & 8 \\ 5 & 4 & -3 \\ 3 & 1 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $B = \begin{bmatrix} 24 \\ 12 \\ 16 \end{bmatrix}$

let, $A = LU$ — decomposition follows as

where, $L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$ and $U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \times \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} 2 & -6 & 8 \\ 5 & 4 & -3 \\ 3 & 1 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} \\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33} \end{bmatrix} = \begin{bmatrix} 2 & -6 & 8 \\ 5 & 4 & -3 \\ 3 & 1 & 2 \end{bmatrix}$$

$\Rightarrow$

$u_{11} = 2$	$l_{21} = 5/2$	$u_{23} = -23$
$u_{12} = -6$	$u_{22} = 19$	$l_{32} = 10/19$
$u_{13} = 8$	$l_{31} = 3/2$	$u_{33} = 40/19$

P.T.O.

$$U = \begin{bmatrix} 2 & -6 & 8 \\ 0 & 19 & -23 \\ 0 & 0 & \frac{40}{19} \end{bmatrix} \text{ and } L = \begin{bmatrix} 1 & 0 & 0 \\ 5/2 & 1 & 0 \\ 3/2 & 10/19 & 1 \end{bmatrix}$$

$\therefore AX = B$

$\Rightarrow LUX = B$

let, $UX = Y$, where, $Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

$\therefore LY = B$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 5/2 & 1 & 0 \\ 3/2 & \frac{10}{19} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 2 \\ 16 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} y_1 \\ 5/2\, y_1 + y_2 \\ 3/2\, y_1 + \frac{10}{19} y_2 + y_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 2 \\ 16 \end{bmatrix}$$

$y_1 = 24$

$y_2 = -58$

$y_3 = \frac{200}{19}$

$$\therefore Y = \begin{bmatrix} 24 \\ -58 \\ \frac{200}{19} \end{bmatrix}$$

$UX = Y$

$$\Rightarrow \begin{bmatrix} 2 & -6 & 8 \\ 0 & 19 & -23 \\ 0 & 0 & \frac{40}{19} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 24 \\ -58 \\ \frac{200}{19} \end{bmatrix}$$

$$\Rightarrow 2x - 6y + 8z = 24$$

$$\Rightarrow 19y - 23z = -58$$

$$\frac{40}{19} z = \frac{200}{19}$$

Solving these $z = 5$, $y = 3$, $x = 1$.

H.W) Solve the following eqns by Cholesky's factorisation method:

$$2x + y + 4z = 12$$

$$8x - 3y + 2z = 20$$

$$4x + 11y - z = 33$$

31.10.22

Cramer's rule

$$2x + 4y - z = -5$$
$$-4x + 3y + 5z = 14$$
$$6x - 3y - 2z = 5$$

$$D = \begin{vmatrix} 2 & 4 & -1 \\ -4 & 3 & 5 \\ 6 & -3 & -2 \end{vmatrix}$$

$$= 2(-6+15) - 4(8-30) - 1(12-18)$$
$$= 112$$

$$Dx = \begin{vmatrix} -5 & 4 & -1 \\ 14 & 3 & 5 \\ 5 & -3 & -2 \end{vmatrix}$$

$$= -5(-6+15) - 4(-28-25) - 1(-42-15)$$
$$= 224$$

$$Dy = \begin{vmatrix} 2 & -5 & -1 \\ -4 & 14 & 5 \\ 6 & 5 & -2 \end{vmatrix} \quad ; \quad Dz = \begin{vmatrix} 2 & 4 & -5 \\ -4 & 3 & 14 \\ 6 & -3 & 5 \end{vmatrix}$$

$$2(15+42) - 4(-20-84) - 5(12-18)$$
$$= 560$$

$$= 2(-28-25) + 5(8-30) - 1(-20-84)$$
$$= -106 - 110 + 104 = -\cancel{320}\,112$$

$$\therefore x = \frac{D_x}{D} = \frac{224}{112} = 2$$

$$y = \frac{D_y}{D} = \frac{-3\cancel{2}0}{1\cancel{1}2} = \frac{-112}{112} = -1$$

$$z = \frac{D_z}{D} = \frac{560}{112} = 5$$

Numerical Integration

Trapezoidal rule: (Any value of n)

$$\int_{a=x_0}^{b(x_n)} f(x)\,dx = \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+\cdots+y_{n-1})\right]$$

$$\boxed{h = \frac{b-a}{n}}$$

Simpson 1/3 rule: (n should be multiple of 2)

$$\int_{a=x_0}^{b=x_n} f(x)\,dx = \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+\cdots+y_{n-1}) + 2(y_2+y_4+\cdots+y_{n-2})\right]$$

$$\boxed{h = \frac{b-a}{n}}$$

Simpson 3/8 rule: (n should be multiple of 3)

$$\int_{a=x_0}^{b=x_n} f(x)\,dx = \frac{3}{8}h\left[(y_0+y_n)+3(y_1+y_2+y_4+y_5+\cdots+y_{n-4}+y_{n-2}+y_{n-1})+2(y_3+y_6+\cdots+y_{n-3})\right]$$

weddle's rule: (n should be multiple of 6)

$$\int_{a=x_0}^{b=x_n} f(x)\,dx = \frac{3h}{10}\Big[(y_0+5y_1)+(y_2+6y_3)+(y_4+5y_5) + y_6+(y_6+5y_7)+(y_8+6y_9)+(y_{10}+5y_{11}) + y_{12}+(y_{12}+5y_{13})+(y_{14}+6y_{15})+(y_{16}+5y_{17}) + y_{18}+\cdots\Big]$$

Problem:

$\int_0^{12} \frac{dx}{1+x^2}$; trapezodia value $n=6$.

$f(x)=\frac{1}{1+x^2}$

$a=0,\ b=12,\ n=6$

$h=\frac{b-a}{n}=\frac{12-0}{6}=2$

CamScanner

#P

x	0	2	4	6	8	10	12
$y = f(x)$	1	$1/5$	$1/17$	$1/37$	$1/65$	$\frac{1}{101}$	$1/145$
y	1.0000 (y_0)	0.2000 (y_1)	0.0588 (y_2)	0.0270 (y_3)	0.0154 (y_4)	0.0099 (y_5)	0.0069 (y_6)

$$\int_0^{12} \frac{dx}{1+x^2}$$

$$\left[\tan^{-1} x\right]_0^{12}$$

$$= \tan^{-1}(12) - \tan^{-1}(1)$$

$$= 1.48766$$

$$\frac{h}{2}\left[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5)\right]$$

$$= \frac{2}{2}\left[(1.0000 + 0.0069) + 2(0.20 + 0.0588 + 0.0270 + 0.0154 + 0.0099)\right]$$

$$= (1.0069 + 0.6222) = 1.6291.$$

The exact value is,

$$\int_0^{12} \frac{1}{1+x^2}\, dx = \left[\tan^{-1} x\right]_0^{12} = 1.48766.$$

Prob 2: Ans: 1.43495

$\int_0^{12} \frac{dx}{1+x^2}$ Simpson's 3/8 rule, $n=6$

The Simpso's $\frac{3}{8}$ rule is

$$I = \frac{3\cancel{8}2}{8}\left[(y_0+y_6)+3(y_1+y_2+y_4+y_5) + 2(y_3)\right]$$

~~a=0, b=12,~~

x	0	2	4	6	8	10	12
$y=f(x)$	1	1/5	1/17	1/37	1/65	$\frac{1}{101}$	$\frac{1}{145}$
y	1.000	0.2000	0.0588	0.02370	0.0154	0.0099	.0069

$a=0$
$b=12$
$n=6$
$h=\frac{12-0}{6}$
$=2$

$$= \frac{3\cancel{8}\times 2}{8}\left[(1+0.0069)+3(.2+0.0588+0.0154+0.0099) + 2(0.02370)\right]$$

$$= \frac{6}{8}\left[1.9066\right]$$

$$= 1.42995$$

$\int_2^6 \log_{10} x\,dx$ trapezoidal rule $n=8$

$a=2$ $b=6$, $n=8$ — $n+1 = 8+1 = 9$ → 9 point

$$h = \frac{b-a}{n} = \frac{6-2}{8} = 0.5$$

2.0 2.5 3.0 . 3.5 4.0 4.5 5.0 5.5. 6.0

H.W

i) $\int_0^{12} e^x dx$; $n=12$

ii) $\int_2^6 \log_{10} x dx$; $n=6$

all method.

x	2	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0
$f(x)$	0.30103	0.39794	0.47712	0.54407	0.60206	0.65321	0.69897	0.74036	0.77815

the trapzoidal rule is.

$$I = \frac{h}{2}\left[(y_0 + y_8) + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7)\right]$$

$$= \frac{0.5}{2}\left[(0.30103 + 0.77815) + 2(0.39794 + 0.47712 + 0.54407 + 0.60206 + 0.65321 + 0.69897 + 0.74036)\right]$$

$$= 2.32666$$

$y(0)=0$ $y(x_0)=y_0$ $y_2=y(x_2)$ $x\,y_0=y(x_0)$

13.11.22

Runge-Kutta method:

* using forth order Range Kutta method to estimate $y(0.4)$ when $y'(x)=x^2+y^2$ & $y(0)=0$ [consider $h=0.2$]

$y'(x)=\frac{dy}{dx}$

$f(x,y)=\frac{dy}{dx}=y'(x)$

$y(0)=0$

Soln: Given, $y'(x)=x^2+y^2$ & $y(0)=0$ & $h=0.2$

$\therefore f(x,y)=x^2+y^2,\ x_0=0,\ y_0=0$

By, 4th order Runge Kutta method for the first approximation, we have

$$k_1=hf(x_0,y_0)$$
$$=h(x_0^2+y_0^2)$$
$$=0.2(0+0)$$
$$=0$$

$$k_2=hf\left(x_0+\frac{h}{2},y_0+\frac{k_1}{2}\right)$$
$$=h\left\{\left(x_0+h/2\right)^2+\left(y_0+\frac{k_1}{2}\right)^2\right\}$$
$$=0.602$$

$$K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$$

$$= h\left\{\left(x_0 + \frac{h}{2}\right)^2 + \left(y_0 + \frac{K_2}{2}\right)^2\right\}$$

$$= 0.002$$

$$\cancel{K_4 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_3}{2}\right)}$$

$$K_4 = hf(x_0 + h, y_0 + K_3)$$

$$= h\{(x_0 + h)^2 + (y_0 + K_3)^2\}$$

$$= 0.008$$

$$\Delta y_1 = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

$$= 0.002667$$

$$y_1 = y_0 + \Delta y_1$$

$$= 0 + 0.002667$$

$$= 0.002667$$

$y_1 = y(x_1)$

$x_1 = x_0 + h$

$= 0 + 0.2$

$= 0.2$

y (0.4) বের করবার

$$y(0.2) = 0.002667$$

For the second approximation,

also we have, $x_1 = 0.2$ & $y_1 = 0.002667$

$$K_1 = hf(x_1, y_1) = h(x_1^2 + y_1^2) = 0.008$$

$$K_2 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{K_1}{2}\right)$$

$$= 0.2\left\{\left(x_1 + h/2\right)^2 + \left(y_1 + \frac{K_1}{2}\right)^2\right\}$$

$$= 0.018$$

$$K_3 = hf\left(x_1 + h/2, y_1 + \frac{K_2}{2}\right)$$

$$= 0.2\left\{\left(x_1 + h/2\right)^2 + \left(y_1 + \frac{K_2}{2}\right)^2\right\}$$

$$= 0.018$$

$$K_4 = hf\left(x_1 + h, y_1 + K_3\right)$$

$$= 0.2\left\{\left(x_1 + h\right)^2 + \left(y_1 + K_3\right)^2\right\}$$

$$= 0.032$$

$$\Delta y_2 = \frac{1}{6}\left(K_1 + 2K_2 + 2K_3 + K_4\right)$$

$$= 0.018667$$

$$y_2 = y_1 + \Delta y_2$$

$$= 0.002667 + 0.018667$$

$$= 0.0213 \text{ (correct to four decimals)}$$

$y(0.4) = 0.0213$ (correct of four decimals)

$y_2 = y(x_2)$

$x_2 = x_1 + h$

$= 0.2 + 0.2$

$= 0.4$

Ans!

21.11.22

curve fitting

Straight line	Parabola or quadratic curve $y=a+bx+cx^2$	exponential curve
$y=a+bx$	$\Sigma y = na + b\Sigma x + c\Sigma x^2$	$y = ab^x$
$\Sigma y = na + b\Sigma x$	$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$	$\Rightarrow \log y = \log ab^x$
$\Sigma xy = a\Sigma x + b\Sigma x^2$	$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$	$= \log a + \log b^x$
		$\underbrace{\log y}_{Y} = \underbrace{\log a}_{A} + x\underbrace{\log b}_{B}$
		$\Sigma Y = nA + B\Sigma x$
		$\Sigma xY = A\Sigma x + B\Sigma x^2$

Fit a straight line to the following data regarding x as the independent variable:

x:	0	1	2	3	4
y:	1	1.8	3.3	4.5	6.3

Soln: let, the straight line to be fitted to the given data be, $y=a+bx$. Then the normal eqns are: $\Sigma y = na + b\Sigma x$

and $\Sigma xy = a\Sigma x + b\Sigma x^2$

P.T.O.

x	y	xy	x^2
0	1	0	0
1	1.8	1.8	1
2	3.3	6.6	4
3	4.5	13.5	9
4	6.3	25.2	16
$\Sigma x=10$	$\Sigma y=16.9$	$\Sigma xy=47.1$	$\Sigma x^2=30$

Here m on n is number of item

Here, $m=5$

Substituting these values in the normal eqns, we have, $16.9 = 5a + 10b$

$47.1 = 10a + 30b.$

Solving the above eqns, we have

$a = 0.72$ & $b = 1.33.$

Hence, the fitted line is: $y = 0.72 + 1.33x$

Problem-02:

Fit a second degree parabola to the following data:

x	0	1	2	3	4
y	1	5	10	22	38

P.T.O.

Soln: let, the parabola to be fitted to the given data be $y = a + bx + cx^2$, then the normal ~~eqn~~ eqns are:

$$\Sigma y = ma + b\Sigma x + c\Sigma x^2$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$$

$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$$

x	y	x^2	x^3	x^4	xy	x^2y
0	1	0	0	0	0	0
1	5	1	1	1	5	5
2	10	4	8	16	20	40
3	22	9	27	81	66	198
4	38	16	64	256	152	608
$\Sigma x = 10$	$\Sigma y = 76$	$\Sigma x^2 = 30$	$\Sigma x^3 = 100$	$\Sigma x^4 = 354$	$\Sigma xy = 243$	$\Sigma x^2 y = 851$

Here, $m = 5$.

Subsituting these values in the normal eqns, we get,

$$76 = 5a + 10b + 30c$$

$$243 = 10a + 30b + 100c$$

$$851 = 30a + 100b + 354c$$

Solving these eqns, we get, $a = 1.43$, $b = 0.24$, $C = 2.21$.

Hence, the filled parabola is:

$$y = 1.43 + 0.24x + 2.21x^2$$

H.W)

1. Show that the line of fit to the following data is given by $y = -0.5x + 8$

x:	6	7	7	8	8	8	9	9	10
y:	5	5	4	5	4	3	4	3	3

Soln! Let, the straight line to be fitted to the given data be, $y = a + bx$. Then the normal eqn are: $\Sigma y = ma + b\Sigma x$

$\& \ \Sigma xy = a\Sigma x + b\Sigma x^2$

x	y	xy	x^2
6	5	30	36
7	5	35	49
7	4	28	49
8	5	40	64
8	4	32	64
8	3	24	64
9	4	36	81
9	3	27	81
10	3	30	100
$\Sigma x = 72$	$\Sigma y = 36$	$\Sigma xy = 282$	$\Sigma x^2 = 588$

Here, $m = 9$

Substituting these values in the normal eqn,

We have,

$$36 = 9a + 72b \quad \text{—(i)}$$

$$282 = 72a + 588b \quad \text{—(ii)}$$

Solving these eqn, we have $(i \times 8) - (ii)$

$$\begin{array}{rcl} 288 &=& \cancel{72a} + 576b \\ 282 &=& \cancel{72a} + 588b \\ (-) & & (-) \quad (-) \\ \hline 6 &=& -12b \end{array}$$

$$\therefore b = -\frac{1}{2} = -0.5$$

(i) $\Rightarrow$

$$36 = 9a + 72(-0.5)$$

$$\Rightarrow 36 + 36 = 9a$$

$$\therefore a = 8$$

Hence, the fitted line is, $y = 8 - 0.5x$

Ans.

$x = a + by$ अर्थ y independ 2.21

20. Fit a second degree parabola taking x as independent variable.

x:	1	2	3	4	5	6	7	8	9
y:	2	6	7	8	10	11	11	10	9

Soln:
Let, the parabola to be fitted to the given data be, $y = a + bx + cx^2$, then the normal eqn are;

$$\Sigma y = ma + b\Sigma x + c\Sigma x^2$$
$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$$
$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$$

x	y	x^2	x^3	x^4	xy	x^2y
1	2	1	1	1	2	2
2	6	4	8	16	12	24
3	7	9	27	81	21	63
4	8	16	64	256	32	128
5	10	25	125	625	50	256
6	11	36	216	1296	66	396
7	11	49	343	2401	77	539
8	10	64	512	4096	80	640
9	9	81	729	6561	81	729
$\Sigma x = 45$	$\Sigma y = 74$	$\Sigma x^2 = 285$	$\Sigma x^3 = 2025$	$\Sigma x^4 = 15333$	$\Sigma xy = 421$	$\Sigma x^2 y = 2771$

Here, $m = 9$

Subsituting these value in the ~~no~~ norm. eq^n, we get,

$$74 = 9a + 45b + 285c$$
$$421 = a45 + 285b + 2025c$$
$$2771 = 285a + 2025b + 15333c$$

Solving these eq^n, we get,

(i) ~~×95 − (ii)×19 − (iii)×3~~

$$7030 = 85\cancel{5}\cancel{a} + 4275 + 27075c$$
$$7999 = 8\cancel{55}a +$$

(ii) ~~×~~ − (i)×5

$$421 = 4\cancel{5}a + 285b + 2025c$$
$$370 = \cancel{45a} + 225b + 1425c$$

$$51 = 60b + 600c$$

$a = -0.93$, $b = 3.52$, $c = -0.27$

Hence, the filled parabola is:

$$y = -0.93 + 3.52x - 0.27x^2$$

Ans

28.11.22

Fit an exponential curve of the form $y = ab^x$ to the following data:

x:	1	2	3	4	5	6	7	8
y:	1.0	1.2	~~1.8~~ 1.8	2.5	3.6	4.7	6.6	9.1

$y = ab^x$

$\Rightarrow \log y = \log ab^x$

$\Rightarrow \underbrace{\log y}_{Y} = \underbrace{\log a}_{A} + x\underbrace{\log b}_{B}$

$y = ab^x$ takes the form, $Y = A + Bx$

where, $Y = \log y$, $A = \log a$, $B = \log b$

The normal eq^n are: $\sum Y = nA + B\sum x$

$\sum xY = A\sum x + B\sum x^2$

x	y	$Y = \log y$	xY	x^2
1	1.0	0.0000	0.0000	1
2	1.2	0.0792	0.1584	4
3	1.8	0.2553	0.7659	9
4	2.5	0.3979	1.5916	16
5	3.6	0.5563	2.7815	25
6	4.7	0.6721	4.0326	36
7	6.6	0.8195	5.7365	49
8	9.1	0.9590	7.6720	64
$\sum x = 36$	$\sum y = 30.5$	$\sum Y = 3.7393$	$\sum xy = 22.7385$	$\sum x^2 = 204$

$3.7393 = 8A + 36B$

$22.7385 = 36A + 204B$

Solving these,

$A = 1.8336, \quad B = 0.1406$

$A = \log a \qquad B = \log b$

$a = \log^{-1}(A) \qquad b = \log^{-1}(B)$

$\therefore a = 0.68234 \qquad b = 1.3828$

The required exponential curve to be fitted,

$y = (0.68234)(1.3828)^x$

H.W)

1. fit a curve $y = ax^b$ to the following data:

x:	1	2	3	4	5	6
y:	2.98	4.26	5.21	6.10	6.80	7.50

7.12.22

Picard's method:

$$\frac{dy}{dx} = f(x, y)$$

formula: $y_n = y_0 + \int_{x_0}^{x} f(x, y_{n-1})\, dx \quad n = 1, 2, 3$

with the initial condition $y(x_0) = y_0$

\# Use Picards method to approximate y when $x = 0.2$, give that $y = 1$, when $x = 0$ & $\frac{dy}{dx} = x - y$

$y(x_0) = y_0 = 1$

Soln: 1st approximate,

$$y_1 = y_0 + \int_{x_0}^{x} f(x, y_0)\, dx$$

$$= 1 + \int_0^x (x - 1)\, dx$$

$$= 1 + \frac{x^2}{2} - x \qquad \text{At } x = 0.2,\ y_1 = 0.82$$

2nd approximate:

$$y_2 = y_0 + \int_{x_0}^{x} f(x, y_1)\,dx$$

$$= 1 + \int_0^x (x-1-\frac{x^2}{2}+x)\,dx$$

$$= 1 + x^2 - x - \frac{x^3}{6}$$

At, $x = 0.2$, $y_2 = 0.838667$.

3rd approximation:

$$y_3 = y_0 + \int_{x_0}^{x} f(x, y_2)\,dx$$

$$= 1 + \int_0^x (2x-1-x^2+\frac{x^3}{6})\,dx$$

$$= 1 + x^2 - x - \frac{x^3}{3} + \frac{x^4}{24}$$

At, $x = 0.2$, $y_3 = 0.8374$.

4th approximation:

$$y_4 = y_0 + \int_{x_0}^{x} f(x, y_3)\,dx$$

$$= 1 + \int_0^x (2x-1-x^2+\frac{x^3}{3}+\frac{x^4}{24})\,dx$$

$$= 1 + x^2 - x - \frac{x^3}{3} + \frac{x^4}{12} - \frac{x^5}{120}$$

At, $x = 0.2$, $y_4 = 0.83746$

Ans

nx^{n-1}

$3x$

Use picards method to approximate y when $x=0.1$ given that $y=1$ when $x=0$ & $\frac{dy}{dx}=3x+y^2$.

$y(x)=1$

$y_0(x)=1$

Soln: $y_1 = y_0+\int_{x_0}^{x} f(x,y_0)dx$

$= 1+\int_0^x (3x+y^2)\,dx$

$= 1+[\cancel{3.\frac{x^2}{2}}+3+y^2]\,dx$

$= 1+\int_{x_0}^{x}(3x+1)\,dx$

$= 1+3.\frac{x^2}{2}+x$

At, $x=0.1$, $y_1 = 1.115$

2nd approximate:

$y_2 = y_0+\int_{x_0}^{x} f(x,y_1)dx$

$= 1+\int_0^x \left\{3x+\left(1+x+\frac{3x^2}{2}\right)^2\right\}dx$

$= 1+\int_0^x \left(3x+1+x^2+\frac{9x^4}{4}+2x+3x^3+3x^2\right)dx$

$= 1+\int_0^x \left(1+5x+4x^2+3x^3+\frac{9x^4}{4}\right)dx$

CamScanner

$$y_2 = 1 + x + \frac{5x^2}{2} + \frac{4x^3}{3} + \frac{3x^4}{4} + \frac{9x^5}{20}$$

At, $x = 0.1$, $y_2 = 1.126$

Since next integration is difficult so considering y_2 as final approximate soln. of given ordinary differential ~~equent~~ equation.