26.09.22

Neumerical Method

Solution of Algebric & transcendental

- Bracketting method:
  - ↳ Bisection method
  - ↳ False method

→ Open method

Bisection method:

Find the root of the following function using Bisection method. $f(x) = 3x - \cos x - 1$

Soln: $a = 0$ $f(a) = -2$

$b = 1$ $f(b) = 1.46$

$\therefore f(a) * f(b) < 0$

a	b	f(a)	f(b)	$c = a+b/2$	f(c)
0	1	-2	1.44	0.5	-0.38
0.5	1	-0.38	1.44	0.75	0.52
0.5	0.75	-0.38	0.52	0.625	0.06
0.5	0.675	-0.38	0.06	0.56	-0.17
.56	.625	-0.17	0.06	0.59	-0.05
.59	.625	-0.05	0.06	0.61	+0.01
.59	.61	-0.05	0.01	0.61	0.01

$\therefore$ root of $f(x) = 0.61$

False position method:

$$c = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

Find the root of the following function using false position method.

$$f(x) = 3x - \cos x - 1$$

$a = 0 \quad f(a) = -2$

$b = 1 \quad f(b) = 1.46$

a	b	f(a)	f(b)	c	f(c)
0	1	−2	1.46	.58	−0.10
.58	1	−0.10	1.46	.61	0.01

(i) $f(x) = 3x - \cos x - 1$? Bisection methode

(ii) # $f(x) = x^3 - 4x - 9$ using Bisection & false position.

$f(x) = x^3 - 4x - 9$

$f(0) = -9$

$f(1) = -12$

$f(2) = -9, \ a = 2$

$f(3) = 6, \ b = 3$

a	b	f(a)	f(b)	$c = \frac{a+b}{2}$	f(c)
2	3	−9	6	2.5	−3.38
2.5	3	−3.38	6	2.8	1.75
2.5	2.8	−3.38	1.75	2.65	−0.99
2.65	2.8	−0.99	1.75	2.73	0.43
2.65	2.73	−0.99	0.43	2.69	−0.29
2.69	2.73	−0.29	0.43	2.71	0.06
2.69	2.71	−0.29	0.06	2.7	−0.117
2.7	2.71	−0.117	0.06	2.71	0.062

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$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

Newton Raphson method:

Use Newton Raphson method to find the real root near 2 of the eq^n $x^4 - 11x + 8 = 0$ accurate to five decimal values.

Soln: $f(x) = x^4 - 11x + 8$

$f'(x) = 4x^3 - 11$,

let, $x_1 = 2$ – যে কোনো একটা হিসেবে হবে,

$f(x_1) = 2^4 - 11 \times 2 + 8$

$= 2$

$f'(x_1) = 4 \times 2^3 - 11 = 21$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2 - \frac{2}{21}$$

$$= 1.\underline{90476}$$

five decimal এ দেবে

Similarly,

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 1.90476 - \frac{(1.90476)^4 - 11(1.90478) +}{4 \times (1.90476)^3 - 11}$$

$$= 1.89209$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.89188$$

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$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = 1.89188$$

$\therefore$ The root the eq^n is: 1.89188

Secant method

formula; $x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$

Use the ~~Seant~~ Secant method to estimate the root of the eq^n: $x^2 - 4x - 10 = 0$ with the initial estimates of $x_1 = 4$ & $x_2 = 2$.

initial value

Sol^n: $f(x) = x^2 - 4x - 10$

& $x_1 = 4$ & $x_2 = 2$

Iteration-1: $f(x_1) = -10$, $f(x_2) = -14$

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

$$= 2 - \frac{-14(2-4)}{-14-(-10)} = 9$$

Second Iteration:

$x_1 = x_2 = 2$

$x_2 = x_3 = 9$

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

$$= 9 - \frac{f(9)(9-2)}{f(9)-f(2)}$$

$\therefore x_3 = 4$

Third Iteration:

$x_1 = x_2 = 9$

$x_2 = x_3 = 4$

$$\therefore x_3 = 4 - \frac{-10(4-9)}{-10-35} = 5.1111$$

Fourth iteration:

$x_1 = x_2 = 4$

$x_2 = x_3 = 5.1111$

$x_3 = 5.9563$

Fifth iteration:

$x_1 = x_2 = 5.1111$

$x_2 = x_3 = 5.9563$

$x_3 = \cancel{7}\ 5.7225$

Sixth iteration: $x_1 = x_2 = 5.9563$

$x_2 = x_3 = 5.7255$

$x_3 = 5.6182$

The value can be further refine by continueing the process, if necessary

Soln of Linear Algebra:

Gauss Elemination:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$2x_1 + x_2 + 4x_3 = 12$
$4x_1 + 11x_2 - x_3 = 33$ ——— (i)
$8x_1 - 3x_2 + 2x_3 = 20$

Soln: The system can be written as $AX = B$

where, $A = \begin{bmatrix} 2 & 1 & 4 \\ 4 & 11 & -1 \\ 8 & -3 & 2 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $B = \begin{bmatrix} 12 \\ 33 \\ 20 \end{bmatrix}$

The augmented matrix of (i)

$$[A:B] = \left[\begin{array}{ccc:c} 2 & 1 & 4 & 12 \\ 4 & 11 & -1 & 33 \\ 8 & -3 & 2 & 20 \end{array}\right]$$

$R_2' \rightarrow R_2 - 2R_1$
$R_3' \rightarrow R_3 - 4R_1$

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$$[A:B] = \begin{bmatrix} 2 & 1 & 4 & : & 12 \\ 0 & 9 & -9 & : & 9 \\ 0 & -7 & -14 & : & 28 \end{bmatrix}$$

$R_3' \rightarrow 9R_3 + 7R_2$

$$[A:B] = \begin{bmatrix} 2 & 1 & 4 & : & 12 \\ 0 & 9 & -9 & : & 9 \\ 0 & 0 & -189 & : & -189 \end{bmatrix}$$

$2x_1 + x_2 + 4x_3 = 12$

$9x_2 - 9x_3 = 9$

$-189x_3 = -189$

$x_3 = 1$

$9x_2 = 18$

$\therefore x_2 = 2$

$2x_1 + 2 + 4 \times 1 = 12$

$2x_1 = 6$

$\therefore x_1 = 3$ Ans

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Gauss Jordan Elimination Method:

$$\left.\begin{aligned} 2x - 3y + 10z &= 3 \\ -x + 4y + 2z &= 20 \\ 5x + 2y + z &= -12 \end{aligned}\right\} \text{---(i)}$$

The system can be written as,

$$AX = B \quad \text{---(ii)}$$

where,

$$A = \begin{bmatrix} 2 & -3 & 10 \\ -1 & 4 & 2 \\ 5 & 2 & 1 \end{bmatrix};\ X = \begin{bmatrix} x_1\ x \\ x_2\ y \\ x_3\ z \end{bmatrix};\ B = \begin{bmatrix} 3 \\ 10 \\ -12 \end{bmatrix}$$

Now, the augmented matrix of the given system

$$[A:B] = \begin{bmatrix} 2 & -3 & 10 & : & 3 \\ -1 & 4 & 2 & : & 10 \\ 5 & 2 & 1 & : & -12 \end{bmatrix}$$

$R_2' = \frac{2}{11} R_2$

$1 \quad 3/2 \quad 5 : 3/2$

$6 \quad 1 \quad \frac{14}{11}$

$3/2 + 4$

$= \frac{3+}{2}$

$R_1' = \frac{R_1}{2}$

$$[A:B] = \begin{bmatrix} 1 & 3/2 & 5 & : & 3/2 \\ -1 & 4 & 2 & : & 10 \\ 5 & 2 & 1 & : & -12 \end{bmatrix}$$

$R_2' = R_2 + R_1$

$R_3' = R_3 - 5R_1$

$-12 - \frac{15}{2}$

$= \frac{-24 - 15}{2}$

$$\begin{bmatrix} 1 & 3/2 & 5 & : & 3/2 \\ 0 & \frac{11}{2} & 7 & : & 23/2 \\ 0 & -11/2 & -24 & : & -39/2 \end{bmatrix}$$

Now, the augmented matrix of the given system,

$$[A:B] = \begin{bmatrix} 2 & -3 & 10 & : & 3 \\ -1 & 4 & 2 & : & 20 \\ 5 & 2 & 1 & : & -12 \end{bmatrix}$$

$$r_1' = \frac{r_1}{2}$$

$$[A:B] = \begin{bmatrix} 1 & -3/2 & 5 & : & 3/2 \\ -1 & 4 & 2 & : & 20 \\ 5 & 2 & 1 & : & -12 \end{bmatrix}$$

$$r_2' = r_2 + r_1$$
$$r_3' = r_3 - 5r_1$$

$$[A:B] = \begin{bmatrix} 1 & -3/2 & 5 & : & 3/2 \\ 0 & 5/2 & 7 & : & 43/2 \\ 0 & 19/2 & -24 & : & -39/2 \end{bmatrix}$$

$$r_2' = \frac{2r_2}{5}$$

$$[A:B] = \begin{bmatrix} 1 & -3/2 & 5 & : & 3/2 \\ 0 & 1 & 14/5 & : & 43/5 \\ 0 & 19/2 & -24 & : & -39/2 \end{bmatrix}$$

$$\frac{3}{2} + \frac{3}{2} \times \frac{43}{5} \qquad \frac{3}{2} + \frac{129}{10} = \frac{15+129}{10} = \frac{144}{10} = \frac{72}{5}$$

$$r_1' = r_1 + 3/2\, r_2$$
$$r_3' = r_3 - \frac{19}{2} r_2$$

$$[A:B] = \begin{bmatrix} 1 & 0 & \frac{46}{5} & : & \frac{72}{5} \\ 0 & 1 & 14/5 & : & 43/5 \\ 0 & 0 & \frac{-253}{5} & : & \frac{-506}{5} \end{bmatrix}$$

$$r_3' = -\frac{5}{253} r_3$$

$$[A:B] = \begin{bmatrix} 1 & 0 & 46/5 & : & 72/5 \\ 0 & 1 & 14/5 & : & 43/5 \\ 0 & 0 & 1 & : & 2 \end{bmatrix}$$

$$\therefore [A:B] = \begin{bmatrix} 1 & 0 & 0 & : & -4 \\ 0 & 1 & 0 & : & 3 \\ 0 & 0 & 1 & : & 2 \end{bmatrix}$$

The reduced system is:

$$x + 0 + 0 = -4$$
$$0 + y + 0 = 3$$
$$0 + 0 + z = 2$$

we get, $x = -4$, $y = 3$ $z = 2$ Ans

30.10.12

Cholesky's ~~tracir~~ triangularisation method or Crout's triangularisation method or method of factorisation or Cholesky's factorisation method:

Solve the system of linear equations using Cholesky's factorization method.

$$2x - 6y + 8z = 24$$
$$5x + 4y - 3z = 2$$
$$3x + y + 2z = 16$$

Soln:

The ~~ado~~ above system can be written as, $AX = B$

where, $A = \begin{bmatrix} 2 & -6 & 8 \\ 5 & 4 & -3 \\ 3 & 1 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $B = \begin{bmatrix} 24 \\ 12 \\ 16 \end{bmatrix}$

let, $A = LU$ – decomposition follows as

where, $L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$ and $U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \times \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} 2 & -6 & 8 \\ 5 & 4 & -3 \\ 3 & 1 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} \\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33} \end{bmatrix} = \begin{bmatrix} 2 & -6 & 8 \\ 5 & 4 & -3 \\ 3 & 1 & 2 \end{bmatrix}$$

$u_{11} = 2$	$l_{21} = 5/2$	$u_{23} = -23$
$u_{12} = -6$	$u_{22} = 19$	$l_{32} = 10/19$
$u_{13} = 8$	$l_{31} = 3/2$	$u_{33} = 40/19$

P.T.O.

$$U = \begin{bmatrix} 2 & -6 & 8 \\ 0 & 19 & -23 \\ 0 & 0 & \frac{40}{19} \end{bmatrix} \text{ and } L = \begin{bmatrix} 1 & 0 & 0 \\ 5/2 & 1 & 0 \\ 3/2 & 10/19 & 1 \end{bmatrix}$$

$\therefore AX = B$

$\Rightarrow LUX = B$

let, $UX = Y$, where, $Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

$\therefore LY = B$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 5/2 & 1 & 0 \\ 3/2 & \frac{10}{19} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 2 \\ 16 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} y_1 \\ 5/2\, y_1 + y_2 \\ 3/2\, y_1 + \frac{10}{19} y_2 + y_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 2 \\ 16 \end{bmatrix}$$

$y_1 = 24$

$y_2 = -58$

$y_3 = \frac{200}{19}$

$$\therefore Y = \begin{bmatrix} 24 \\ -58 \\ \frac{200}{19} \end{bmatrix}$$

$UX = Y$

$$\Rightarrow \begin{bmatrix} 2 & -6 & 8 \\ 0 & 19 & -23 \\ 0 & 0 & \frac{40}{19} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 24 \\ -58 \\ \frac{200}{19} \end{bmatrix}$$

$\Rightarrow 2x - 6y + 8z = 24$

$\Rightarrow 19y - 23z = -58$

$\frac{40}{19} z = \frac{200}{19}$

Solving these $z = 5$, $y = 3$, $x = 1$.

H.W) Solve the following eqns by Cholesky's factorisation method:

$2x + y + 4z = 12$

$8x - 3y + 2z = 20$

$4x + 11y - z = 33$

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Cramer's rule

$$2x + 4y - z = -5$$
$$-4x + 3y + 5z = 14$$
$$6x - 3y - 2z = 5$$

$$D = \begin{vmatrix} 2 & 4 & -1 \\ -4 & 3 & 5 \\ 6 & -3 & -2 \end{vmatrix}$$

$$= 2(-6+15) - 4(8-30) - 1(12-18)$$
$$= 112$$

$$D_x = \begin{vmatrix} -5 & 4 & -1 \\ 14 & 3 & 5 \\ 5 & -3 & -2 \end{vmatrix}$$

$$= -5(-6+15) - 4(-28-25) - 1(-42-15)$$
$$= 224$$

$$D_y = \begin{vmatrix} 2 & -5 & -1 \\ -4 & 14 & 5 \\ 6 & 5 & -2 \end{vmatrix} \quad ; \quad D_z = \begin{vmatrix} 2 & 4 & -5 \\ -4 & 3 & 14 \\ 6 & -3 & 5 \end{vmatrix}$$

$$2(15+42) - 4(-20-84) - 5(12-18)$$
$$= 560$$

$$= 2(-28-25) + 5(8-30) - 1(-20-84)$$
$$= -106 - 110 + 104 = -\cancel{320}\ 112$$

$$\therefore x = \frac{D_x}{D} = \frac{224}{112} = 2$$

$$y = \frac{D_y}{D} = \frac{-3\cancel{2}0}{1\cancel{1}2} = \frac{-112}{112} = -1$$

$$z = \frac{D_z}{D} = \frac{560}{112} = 5$$

Numerical Integration

Trapezoidal rule: (Any value of n)

$$\int_{a=x_0}^{b(x_n)} f(x)\,dx = \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+\cdots+y_{n-1})\right]$$

$$\boxed{h = \frac{b-a}{n}}$$

Simpson 1/3 rule: (n should be multiple of 2)

$$\int_{a=x_0}^{b=x_n} f(x)\,dx = \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+\cdots+y_{n-1}) + 2(y_2+y_4+\cdots+y_{n-2})\right]$$

$$\boxed{h = \frac{b-a}{n}}$$