Econometrics Week7

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Final Assignement. Developed in Python and code hidden with jupyterutils. Code version can be submitted as well.

Each portion is separated into "PART X" and the reponses are in bold below the hidden or unhidden code portions.

In [3]:
```
import numpy as np
import pandas as pd
from matplotlib.pyplot import *
import statsmodels.api as sma
import statsmodels.stats as sms
import statsmodels.tsa.stattools as smts
import seaborn as sns
import sklearn as skl
%matplotlib inline

import warnings
warnings.filterwarnings('ignore')
```

In [4]:
```
from dfast.jupyterutils.togglecode import hideCode
hideCode()
```

Out[4]: Toggle Code.

Bring in the data and check it out. Then fully shift (lag) the data for future use and print to confirm.

In [5]:
```
path = 'C:\\Users\\SchillW\\Documents\\Econ_Coursera\\Wk7\\'
df = pd.read_excel(path+'data.xls', index_col=0)
df.head(3)
```

Out[5]:

	GDP	GDPIMPR	LOGGDP	GrowthRate	li1	li2	T
Date							
1950Q1	94.300	NaN	4.546481	NaN	0	0	0
1950Q2	95.200	1.0	4.555980	0.009499	0	0	1
1950Q3	97.663	1.0	4.581523	0.025543	3	1	2

In [6]:
```
## FULLY SHIFT AND LAG THE DATA SET HERE
full = pd.concat([df, df.shift(1), df.shift(2)], axis=1, join='outer')
cols = []
for lg in range(0,3):
    for i in df.columns:
        if lg==0:
            cols.append(i)
        else:
            cols.append(i+'(-'+str(lg)+')')

full.columns = cols
```

Fully lagged dataframe of variables.

In [7]:
```
print full.head(3)
```

```
           GDP  GDPIMPR    LOGGDP  GrowthRate  li1  li2  T  GDP(-1)  \
Date
1950Q1  94.300      NaN  4.546481         NaN    0    0  0      NaN
1950Q2  95.200      1.0  4.555980    0.009499    0    0  1     94.3
1950Q3  97.663      1.0  4.581523    0.025543    3    1  2     95.2

        GDPIMPR(-1)  LOGGDP(-1)  ...    li1(-1)  li2(-1)  T(-1)  GDP(-2)  \
Date                             ...
1950Q1          NaN         NaN  ...        NaN      NaN    NaN      NaN
1950Q2          NaN    4.546481  ...        0.0      0.0    0.0      NaN
1950Q3          1.0    4.555980  ...        0.0      0.0    1.0     94.3

        GDPIMPR(-2)  LOGGDP(-2)  GrowthRate(-2)  li1(-2)  li2(-2)  T(-2)
Date
1950Q1          NaN         NaN             NaN      NaN      NaN    NaN
1950Q2          NaN         NaN             NaN      NaN      NaN    NaN
1950Q3          NaN    4.546481             NaN      0.0      0.0    0.0

[3 rows x 21 columns]
```

PART A - Logliklihood Ratio Test

In [30]:
```
try:
    del x, y
except:
    pass

start = '1951Q1'
end = '2010Q4'

x = full[['li1(-1)','li2(-1)']]
xA = x[(x.index>=start)&(x.index<=end)]

y = full[['GDPIMPR']]
yA = y[(y.index>=start)&(y.index<=end)]

modA = sma.Logit(endog=yA, exog=sma.add_constant(xA))
fitA = modA.fit()
print fitA.summary2()
```

```
Optimization terminated successfully.
         Current function value: 0.559076
         Iterations 5
                         Results: Logit
=================================================================
Model:              Logit            Pseudo R-squared: 0.122
Dependent Variable: GDPIMPR          AIC:              274.3565
Date:               2017-03-08 12:05 BIC:              284.7984
No. Observations:   240              Log-Likelihood:   -134.18
Df Model:           2                LL-Null:          -152.76
Df Residuals:       237              LLR p-value:      8.4833e-09
Converged:          1.0000           Scale:            1.0000
No. Iterations:     5.0000
-------------------------------------------------------------------
           Coef.    Std.Err.      z      P>|z|     [0.025    0.975]
-------------------------------------------------------------------
const      0.7288     0.1536    4.7454   0.0000    0.4278    1.0298
li1(-1)   -0.3719     0.0727   -5.1176   0.0000   -0.5143   -0.2294
li2(-1)   -0.1203     0.0377   -3.1936   0.0014   -0.1941   -0.0465
=================================================================
```

Build the model from the data and match with the output on the test. Set up each specific model fit.

In [9]:
```
fitA_Cli1 =  sma.Logit(endog=yA, exog=sma.add_constant(xA[['li1(-1)']])).fit()
fitA_Cli2 =  sma.Logit(endog=yA, exog=sma.add_constant(xA[['li2(-1)']])).fit()
```

```
Optimization terminated successfully.
         Current function value: 0.582277
         Iterations 5
Optimization terminated successfully.
         Current function value: 0.623002
         Iterations 5
```

In [10]:
```
loglikA = np.reshape(np.array([fitA.llnull, fitA_Cli1.llf, fitA_Cli2.llf,
fitA.llf]), (4,1))

loglik = np.reshape(np.array([-152.763, -139.747, -149.521, -134.178]), (4,1))
lldf = pd.DataFrame(loglik, index=['Const','Const+li1','Const+li2','All'], col
umns=['Loglikelihood'])
```

In [11]:
```
print "The loglikelihood estimations are below. \n "
print "Loglikelihood:\n",  loglikA
# print "Loglikelihood:\n", "\n", loglik, "\n \n LL Calc\n", loglikA
```

```
The loglikelihood estimations are below.

Loglikelihood:
[[-152.76340039]
 [-139.74657981]
 [-149.52046643]
 [-134.17823347]]
```

In [14]:
```
from scipy.stats import chisqprob
def likelihood_ratio(llmin, llmax):
    return(2.0*(llmax-llmin))

LR = np.zeros((len(loglikA),len(loglikA)))
dof = LR.copy()
for i in range(4):
    for j in range(4):
        LR[i,j] = likelihood_ratio(loglikA[i],loglikA[j])
        if i==j:
            dof[i,j] = 0
        elif np.abs(j-i)<3:
            dof[i,j] = 1
        else:
            dof[i,j] = 2

# p = np.around(chisqprob(LR, dof), 3) # L2 has 1 DoF more than L1
p = np.around(chisqprob(LR, dof), 5)
pF = np.reshape(p[0,1:], (1,3))

print "P-Values for Loglikelihood Comparisons of Model \n"
pd.DataFrame(pF, columns = ['Const+li1','Const+li2','All Params'],
                   index = ['vs Null'])
```

```
P-Values for Loglikelihood Comparisons of Model
```

Out[14]:

	Const+li1	Const+li2	All Params
vs Null	0.0	0.01087	0.0

(a):

The matrix above reflects that the model is valid as all of the p-values are significant to thw 1% level with the Constant and li2 model being significant to the 5% level.

Additionally, this can be confirmed by examing the llr_pvalue features of the statsmodels models as seen below:

In [16]:
```
lrsma = np.array([fitA_Cli1.llr_pvalue, fitA_Cli2.llr_pvalue,
fitA.llr_pvalue],ndmin=2)
lrsm = pd.DataFrame(lrsma.T, columns=['Likelihood Ratio P-Values'])
lrsm.index = ['Const+li1','Const+li2','Full Model']
np.around(lrsm, 7)
```

Out[16]:

	Likelihood Ratio P-Values
Const+li1	3.000000e-07
Const+li2	1.087350e-02
Full Model	0.000000e+00

PART B

The cacluation for Mcfadden's is : Mcfaddens R^2 = 1 - LL_Model/LL_Null.

In [17]:
```
x = full[['li1(-1)','li2(-1)','li1(-2)','li2(-2)']]
xB = x[(x.index>=start)&(x.index<=end)]

y = full[['GDPIMPR']]
yB = y[(y.index>=start)&(y.index<=end)]

modB = sma.Logit(endog=yB, exog=sma.add_constant(xB))
fitB = modB.fit()

fitB_Cl11l21 =  sma.Logit(endog=yB, exog=sma.add_constant(xB[['li1(-1)','li2(-
1)']])).fit()
fitB_Cl11l22 =  sma.Logit(endog=yB, exog=sma.add_constant(xB[['li1(-1)','li2(-
2)']])).fit()
fitB_Cl12l21 =  sma.Logit(endog=yB, exog=sma.add_constant(xB[['li1(-2)','li2(-
1)']])).fit()
fitB_Cl12l22 =  sma.Logit(endog=yB, exog=sma.add_constant(xB[['li1(-2)','li2(-
2)']])).fit()

## Doing it this way left us with different results than expected.
```

```
Optimization terminated successfully.
         Current function value: 0.540830
         Iterations 6
Optimization terminated successfully.
         Current function value: 0.559076
         Iterations 5
Optimization terminated successfully.
         Current function value: 0.558860
         Iterations 5
Optimization terminated successfully.
         Current function value: 0.543106
         Iterations 6
Optimization terminated successfully.
         Current function value: 0.543588
         Iterations 6
```

In [18]:
```
LL_null = -152.763
LL_cl11l21 = -134.178
LL_cl11l22 = -134.126
LL_cl12l21 = -130.346
LL_cl12l22 = -130.461

LLB = pd.DataFrame(np.reshape(np.array([-152.763, -134.178, -134.126,
-130.346, -130.461]),(1,5)))
LLB.columns = ['LL_null','LL_cl11l21','LL_cl11l22','LL_cl12l21','LL_cl12l22']


MR2 = pd.DataFrame(np.reshape(np.array([1.0 -
LLB['LL_cl11l21']/LLB['LL_null'],
                                        1.0 -
LLB['LL_cl11l22']/LLB['LL_null'],
                                       1.0 - LLB['LL_cl12l21']/LLB['LL_null'],
                                       1.0 -
LLB['LL_cl12l22']/LLB['LL_null']]),(1,4)))
MR2.columns = ['LL_cl11l21','LL_cl11l22','LL_cl12l21','LL_cl12l22']

print "Log Likelihoods : \n", LLB
print "\n McFaddens R Squared : \n", MR2
```

```
Log Likelihoods :
   LL_null  LL_cl11l21  LL_cl11l22  LL_cl12l21  LL_cl12l22
0 -152.763    -134.178    -134.126    -130.346    -130.461

 McFaddens R Squared :
   LL_cl11l21  LL_cl11l22  LL_cl12l21  LL_cl12l22
0    0.121659    0.121999    0.146744    0.145991
```

(b):

Based on the calculated McFaddens R^2 above, the most optimal model would be to go with the $Constant + li1(-2) + li2(-1)$ model. But not by very much.

PART C

Probability of economic growth over evaluation sample.

In [31]:
```
## The model was created above:
## It should be noted that they do not accurately specify the time constraints
 for these models!!!!!
print fitB_Cl12l21.summary2()
```

```
                          Results: Logit
=================================================================
Model:              Logit            Pseudo R-squared: 0.147
Dependent Variable: GDPIMPR          AIC:              266.6909
Date:               2017-03-08 12:05 BIC:              277.1328
No. Observations:   240              Log-Likelihood:   -130.35
Df Model:           2                LL-Null:          -152.76
Df Residuals:       237              LLR p-value:      1.8366e-10
Converged:          1.0000           Scale:            1.0000
No. Iterations:     6.0000
-------------------------------------------------------------------
           Coef.    Std.Err.      z      P>|z|     [0.025   0.975]
-------------------------------------------------------------------
const      0.7457     0.1573    4.7397   0.0000    0.4373   1.0540
li1(-2)   -0.4287     0.0763   -5.6175   0.0000   -0.5783  -0.2791
li2(-1)   -0.1312     0.0386   -3.3994   0.0007   -0.2068  -0.0556
=================================================================
```

In [20]:
```
x = full[['li1(-2)','li2(-1)']]
xC = x[x.index>end]

predC = fitB_Cl12l21.predict(sma.add_constant(xC))
predCTable = fitB_Cl12l21.pred_table(threshold=0.5)
rtot = np.sum(predCTable,axis=1)
tot = np.sum(rtot)

predCPRT = pd.DataFrame(predCTable/tot)
predCPRT.columns = ['yhat=0','yhat=1']
predCPRT.index = ['y=0','y=1']

predSum = pd.DataFrame(rtot)
predSum.columns=['Sum']
predSum.index = ['y=0','y=1']
prt = pd.concat([predCPRT, predSum], axis=1, join='outer')


print "Prediction Realization Table: \n"
print prt, "\n"
print "Sums :"
print np.sum(prt,axis=0), "\n"
print "Hit Rate :"
print prt.iloc[0,0] + prt.iloc[1,1]
```

```
Prediction Realization Table: 

       yhat=0  yhat=1    Sum
y=0  0.133333  0.2000   80.0
y=1  0.104167  0.5625  160.0

Sums :
yhat=0      0.2375
yhat=1      0.7625
Sum       240.0000
dtype: float64

Hit Rate :
0.695833333333
```

(c): THE PREDICTION REALIZATION TABLE AND HIT RATE ARE PRINTED ABOVE.

The hit rate is 69.58 and the probability of economic growth is approximately 76.25 in total.

The hit rate represents the accuracy of the model. With 1 being that the model is increasing we are likely to continue to see economic growth.

PART D

ADF Test for Log GDP. The ADF test is formulated and in Python you can print out the model results as seen below.

In [32]:
```
x = full[['LOGGDP']]
xD = x[(x.index>=start)&(x.index<=end)]

loggdp_ADF = sma.tsa.stattools.adfuller(xD['LOGGDP'], maxlag=1,
                                        autolag=None, regression='ct',
                                        regresults=True) #c for constant only,
 no trend
print 'ADF Test \n'
print 'Statistic :', loggdp_ADF[0], ',  P-value :', loggdp_ADF[1],'\n'
print 'Confidence Levels :', loggdp_ADF[2], '\n'
print 'Model \n'
print loggdp_ADF[3].resols.summary2(xname=['LOG GDP lag1','Diff LOG GDP
lag1','Constant','Trend'])
### CONFIRMED THE ORDER OF CONST TREND
```

```
ADF Test

Statistic : -2.51821089277 ,  P-value : 0.318918118592

Confidence Levels : {'5%': -3.4290999471622556, '1%': -3.9973200578432064, '1
0%': -3.1379848180498104}

Model

                  Results: Ordinary least squares
===================================================================
Model:              OLS              Adj. R-squared:     0.393
Dependent Variable: y                AIC:                -1888.3395
Date:               2017-03-08 12:05 BIC:                -1874.4504
No. Observations:   238              Log-Likelihood:     948.17
Df Model:           3                F-statistic:        52.20
Df Residuals:       234              Prob (F-statistic): 7.23e-26
R-squared:          0.401            Scale:              2.0630e-05
-------------------------------------------------------------------
                     Coef.  Std.Err.    t    P>|t|   [0.025  0.975]
-------------------------------------------------------------------
LOG GDP lag1        -0.0204   0.0081 -2.5182 0.0125 -0.0364 -0.0044
Diff LOG GDP lag1    0.6325   0.0509 12.4338 0.0000  0.5323  0.7328
Constant             0.0956   0.0375  2.5512 0.0114  0.0218  0.1695
Trend                0.0001   0.0000  2.4979 0.0132  0.0000  0.0001
-------------------------------------------------------------------
Omnibus:              23.009       Durbin-Watson:          2.012
Prob(Omnibus):        0.000        Jarque-Bera (JB):       36.479
Skew:                 0.578        Prob(JB):               0.000
Kurtosis:             4.530        Condition No.:          23965
===================================================================
* The condition number is large (2e+04). This might indicate
strong multicollinearity or other numerical problems.
```

(d):

The coefficient value for LOG GDP at 1 Lag is -0.0204 and the subsequent p-value is 0.0125 which is significant to the 5% level. I do not think this is a necessary statistic to point out but it was in previous tests.

Based on the output listed above using the statsmodels ADF test, the test statistic is -2.518 and its corresponding p-value is 0.3189. It is very far right of even the 10% confidence level and would thus imply that we cannot reject H0 and the variable is NOT stationary.

PART E

Modeling the Growth Rate at 1 Lag with varying lags of the First and Second Leading Indicators. Only the R Squares from the models are presented.

In [23]:
```
x1 = full[['GrowthRate(-1)','li1(-1)','li2(-1)']]
x11 = x1[(x1.index>=start)&(x1.index<=end)]

x2 = full[['GrowthRate(-1)','li1(-2)','li2(-1)']]
x12 = x2[(x2.index>=start)&(x2.index<=end)]

x3 = full[['GrowthRate(-1)','li1(-1)','li2(-2)']]
x21 = x3[(x3.index>=start)&(x3.index<=end)]

x4 = full[['GrowthRate(-1)','li1(-2)','li2(-2)']]
x22 = x4[(x4.index>=start)&(x4.index<=end)]

y1 = full[['GrowthRate']]
y11 = y1[(y1.index>=start)&(y1.index<=end)]
y12 = y11.copy()
y21 = y11.copy()
y22 = y11.copy()

modE11 = sma.OLS(endog=y11, exog=sma.add_constant(x11)).fit()
modE12 = sma.OLS(endog=y12, exog=sma.add_constant(x12)).fit()
modE21 = sma.OLS(endog=y21, exog=sma.add_constant(x21)).fit()
modE22 = sma.OLS(endog=y22, exog=sma.add_constant(x22)).fit()

r2 = pd.DataFrame([modE11.rsquared, modE12.rsquared, modE21.rsquared, modE22.r
squared])
r2.index = ['mod11','mod12','mod21','mod22']
r2.columns = ['R Squares']
r2
```

Out[23]:

	R Squares
mod11	0.507975
mod12	0.477193
mod21	0.507665
mod22	0.477130

In [24]:
```
modE11Coefs = pd.DataFrame(modE11.params,
                           columns=['Coefficients of Model k1=k2=1'])
modE11Coefs
```

Out[24]:

	Coefficients of Model k1=k2=1
const	0.001737
GrowthRate(-1)	0.461579
li1(-1)	-0.001023
li2(-1)	-0.000149

(e):

The results above show the best R2 is indeed 0.5079 for the model where $k1 = k2 = 1$ (Mod11) barely beating out Mod21. The coefficients are then listed below that where the Growth Rate lagged variable has a coefficient of 0.461579. Such a large value in comparison to the other coefficients might indicate that we have a mispecification in our model.

PART F

Breusch Godfrey Test to examine the lag structure and see if the model was misspecified.

In [25]:
```
bgall = sms.diagnostic.acorr_breush_godfrey(modE11, nlags=1, store=True)
bg = bgall[0:4]
modE11_BG = pd.DataFrame(np.reshape(np.array(bg),(1,4)),
                         columns=['LM Stat','LM p-val','Fvalue','Fval:p-val'],
                        index=['BG Test for Model k1=k2=1'])
modE11_BG
```

Out[25]:

	LM Stat	LM p-val	Fvalue	Fval:p-val
BG Test for Model k1=k2=1	0.230366	0.631253	0.225783	0.63511

(f):

The Breusch Godfrey test for the model where $k1 = k2 = 1$ at 1 Lag shows a high p-value for the test statistic of 0.230366. This test has H0 that there is no serial correlation and with a low p-value we reject the Null. Thus with a p-value of 0.631253 we cannot reject the null and there is no serial correlation.

PART G

Forecast for the Growth Rate.

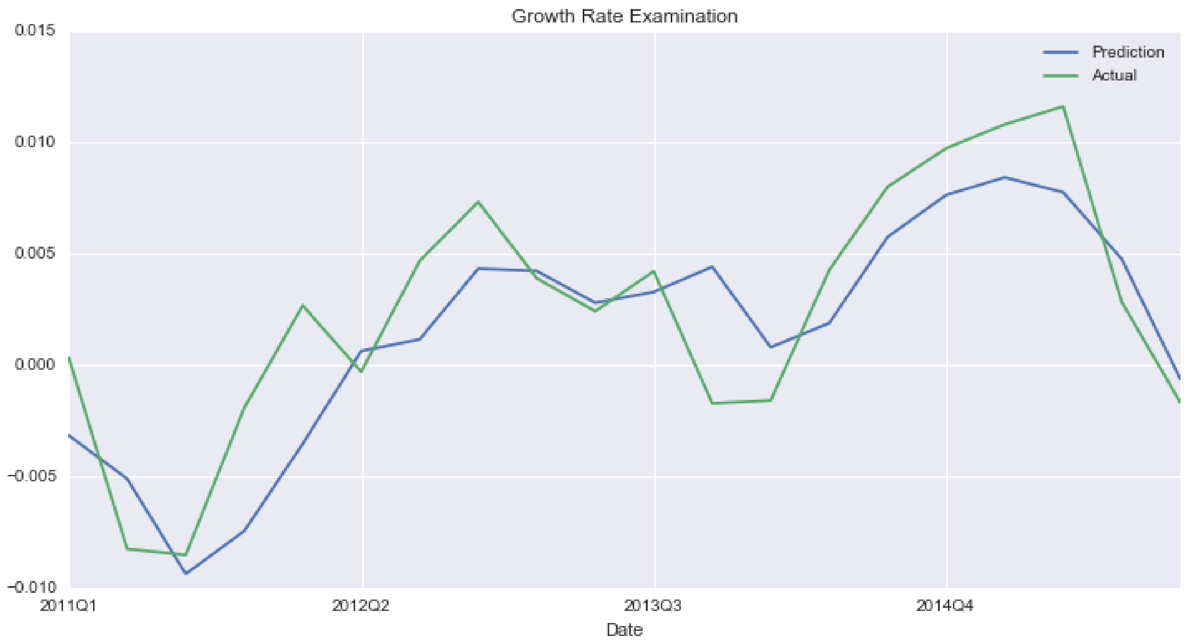
In [26]:
```
x = full[['GrowthRate(-1)','li1(-1)','li2(-1)']]
xG = x[x.index>end]
xGR = pd.DataFrame(full[full.index>end]['GrowthRate'])

predG = modE11.predict(sma.add_constant(xG))

predGdf = pd.DataFrame(predG, index=xG.index)
outDf = pd.concat([predGdf, xGR], axis=1, join='outer')
outDf.columns = ['Prediction','Actual']
```

In [28]:
```
fig2, ax=subplots(1,1,figsize=(12,6))
outDf.plot(ax=ax, title='Growth Rate Examination')
```

Out[28]: <matplotlib.axes._subplots.AxesSubplot at 0xb599710>



In [29]:
```
maeG = np.sum(np.absolute(predGdf.values-xGR.values))
rmseG = np.sqrt(((predGdf.values-xGR.values) ** 2).mean())
print "FIT G  :  RMSE  \  MAE  = ", rmseG, "  \  ", maeG, "\n"
```

```
FIT G  :  RMSE  \  MAE  =  0.00315576342885   \   0.0528065895385
```

(g):

The above is the time series representation of the prediction and actuals over the 'evaluation' period. The RMSE is 0.003156 and the MAE is 0.0528.

The prediction is significantly smoother than the actuals which is to be expected and based on appearance and error the prediction model fits the actuals very well.

In [72]:
```
from dfast.jupyterutils.togglecode import hideCode
hideCode()
```

Out[72]: Toggle Code.

In [ ]:
```
```