ECONOMETRICS · WK3

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1.

ⓐ We have $y = X\beta + \varepsilon$ and model 1 has p_1 regressors and model 0 has $p_0 < p_1$ regressors

we know $AIC = \log(s^2) + \frac{2k}{n}$ and $\frac{s_0^2}{s_1^2} < e^{\frac{2}{n}(p_1 - p_0)}$

suppose $p_0 < p_1$ such that $p_0 = p_1 - 1$ at least

$$\frac{s_0^2}{s_1^2} < e^{\frac{2}{n}(p_1 - (p_1 - 1))} = \frac{s_0^2}{s_1^2} < e^{\frac{2}{n}} \rightarrow \log\left(\frac{s_0^2}{s_1^2}\right) < \log\left(e^{\frac{2}{n}}\right)$$

$$\Rightarrow \log(s_0^2) - \log(s_1^2) < \frac{2}{n} \Rightarrow n\left[\log(s_0^2) - \log(s_1^2)\right] < 2$$

• replacing $\log(s_*^2)$ with AIC_*, we have

$n\left[\left(AIC_0 - \frac{2k}{n}\right) - \left(AIC_1 - \frac{2k}{n}\right)\right] < 2 = n\left[AIC_0 - AIC_1\right] < 2$ which is only TRUE if $AIC_1 > AIC_0$, thus the smallest AIC is preferred. ∴

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ⓑ Assuming n is very large we know that $\lim_{n\to\infty} e^x \approx 1 + x$,

we then have that $\frac{s_0^2}{s_1^2} < e^{2/n(p_1 - p_0)} \approx \frac{s_0^2}{s_1^2} < 1 + \frac{2}{n}(p_1 - p_0)$

multiplying by s_1^2 on both sides yields:

$$s_0^2 < s_1^2 + \frac{2}{n}(p_1 - p_0)s_1^2 \Rightarrow s_0^2 - s_1^2 < \frac{2}{n}(p_1 - p_0)s_1^2$$

$$\Rightarrow \frac{s_0^2 - s_1^2}{s_1^2} < \frac{2}{n}(p_1 - p_0) \quad ∴$$

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ⓒ Assuming e_R is the residual of the restricted model and e_U is the residual of the full model and that "n" is still large,

we know that s_* in $\frac{s_0^2 - s_1^2}{s_1^2}$ is the standard error of the regressions from the AIC conditions and s is defined as $s = \sqrt{\frac{1}{n-1}\sum_{t=1}^{n} e_t^2} = \sqrt{\frac{1}{n-1}(e'e_1)}$ in matrix notation. As $n \to \infty$, $\lim_{n\to\infty} \frac{1}{n-1} \Rightarrow 1$

and $s \to \sqrt{e'e}$, thus for large n we have ⟶

(c) CONTINUED.

$$\rightarrow \frac{s_0^2 - s_1^2}{s_1^2} < \frac{2}{n}(p_1 - p_0) \sim \frac{\sqrt{e_0'e_0}^2 - \sqrt{e_1'e_1}^2}{\sqrt{e_1'e_1}^2} < \frac{2}{n}(p_1 - p_0)$$

where $e_0 = e_R$ and $e_1 = e_u$ then,

$$\frac{e_R'e_R - e_u'e_u}{e_u'e_u} < \frac{2}{n}(p_1 - p_0) \approx \frac{s_0^2 - s_1^2}{s_1^2} < \frac{2}{n}(p_1 - p_0) \quad \therefore$$

(d) We wish to show that with n still large, the findings in (c) are approximately equivalent to an F test with critical value $= 2$.

we have, $\frac{e_R'e_R - e_u'e_u}{e_u'e_u} < \frac{2}{n}(p_1 - p_0)$

we know that $p_0 < p_1$ where p is the number of variables in each particular model. This degree of freedom is equivalent to the factor g.

Let $p_1 - p_0 = g$, then $\left[\frac{e_R'e_R - e_u'e_u}{e_u'e_u}\right]\frac{n}{g} < 2$

$$\Rightarrow \frac{[e_R'e_R - e_u'e_u]/g}{[e_u'e_u]/n} < 2$$

we have assumed that n is large. let us also assume n is so large that $n >> k$ then $(n-k) \approx n$

the F test is $\frac{(e_R'e_R - e_u'e_u)/g}{(e_u'e_u)/(n-k)}$ where if $n >> k$

$$\Rightarrow \frac{(e_R'e_R - e_u'e_u)/g}{(e_u'e_u)/n} = \frac{(e_R'e_R - e_u'e_u)/g}{e_u'e_u/n} < 2$$

and our critical value is 2! $\therefore$