RECALL: OPERATIONS ON INTEGERS

1. Addition of Integers

Case I: positive + positive

You just add them right away.

Ex.

1. 12 + 8 = 20
2. 100 + 200 = 300
3. 33 + 44 = 77
4. 17 + 23 = 40
5. 50 + 70 = 120

Case II: negative + negative

You just add them right away, then add a negative sign on the sum you got.

Ex.

1. -5 + (-7) = -12
2. -10 + (-3) = -13
3. -20 + (-30) = -50
4. -12 + (-18) = -30
5. -40 + (-60) = -100

Case III: positive + negative

When you add a positive number and a negative number, you should:

1. Disregard the signs and subtract the two numbers.
2. Once you have the difference, you will take the sign of the number with the higher magnitude.
3. Apply that sign to the answer you obtained.

For instance, consider this example:

6 + (-3)

Here, you disregard the signs, so you have 6 - 3 = 3. Since 6 has a higher magnitude than 3 and it is positive, the answer will be positive:

6 + (-3) = 3

Ex.

1. 5 + (-3) = 2
2. 8 + (-6) = 2
3. 10 + (-8) = 2
4. 15 + (-12) = 3
5. 20 + (-15) = 5

Case IV: negative + positive

This is just the same as Case III.

1. Disregard the signs and subtract the two numbers.
2. Once you have the difference, you will take the sign of the number with the higher magnitude.
3. Apply that sign to the answer you obtained.

Ex.

1. -5 + 3 = -2
2. -8 + 6 = -2
3. -10 + 8 = -2
4. -15 + 12 = -3
5. -20 + 15 = -5

Here is a better summary:

addition	positive	negative
positive	(add) positive	(subtract) sign of higher magnitude
negative	(subtract) sign of higher magnitude	(add) negative

2. Multiplication of Integers

Case I: (positive)(positive)

You just multiply them right away.

REMEMBER: positive times positive = positive

Ex.

1. 3 * 4 = 12
2. 5 * 2 = 10
3. 6 * 7 = 42
4. 8 * 9 = 72
5. 10 * 3 = 30

Case II: (negative)(negative)

You just multiply them right away , then change the sign to positive.

REMEMBER: negative times negative = positive

Ex.

1. (-3) * (-4) = 12
2. (-5) * (-2) = 10
3. (-6) * (-7) = 42
4. (-8) * (-9) = 72
5. (-10) * (-3) = 30

Case III: (positive)(negative)

If the two numbers you're multiplying have different signs, you just multiply them, then add a negative sign to the answer.

REMEMBER: positive times negative = negative

Ex.

1. 2 * (-5) = -10
2. 7 * (-3) = -21
3. 4 * (-8) = -32
4. 9 * (-6) = -54
5. 12 * (-2) = -24

Case IV: (negative)(positive)

Just like Case III, if the two numbers you're multiplying have different signs, you just multiply them, then add a negative sign to the answer.

REMEMBER: negative times positive = negative

Ex.

1. (-2) * 5 = -10

2. (-7) * 3 = -21
3. (-4) * 8 = -32
4. (-9) * 6 = -54
5. (-12) * 2 = -24

Here is a better summary:

multiplication	positive	negative
positive	(multiply) positive	(multiply) negative
negative	(multiply) negative	(multiply) positive

LESSON LOOK BACK: SEQUENCES AND SERIES

Sequence vs. Series:

Sequence:

A sequence is an ordered list of numbers following a specific pattern or rule. Each number in the sequence is referred to as a term.

Infinite Arithmetic Sequence: 2, 4, 6, 8, 10, ...

Finite Arithmetic Sequence: [4, 8, 12, 16, 24]

Infinite Geometric Sequence: 3, 6, 12, 24, 48, ...

Finite Geometric Sequence: [2, 4, 8, 16, 32]

Series:

A series is the sum of the terms of a sequence. It is the addition of all the terms in a sequence.

Arithmetic series: 2 + 4 + 6 + 8 + 10

Infinite Geometric Series: 3 + 6 + 12 + 24 + 48, ... (Non-converging)

Infinite Geometric Series: 4 + 1 + ¼ + $1/_{16}$ ... (Converging)

Finite Geometric Series: 3 + 9 + 27 + 81 + 243

Arithmetic Sequence:

is a sequence of numbers in which the difference of any two successive members is a constant. This difference is referred to as the common difference, denoted by d.

Ex.

In the arithmetic sequence: [2, 5, 8, 11, 14, ...], we know that the common difference, d, equals 3 because 5-2 = 3, 8-5 = 3, 11-8 = 3 and so on. We always get 3 whenever we subtract two consecutive terms on the sequence.

Formula for arithmetic sequence:

$$a_n = a_1 + (n - 1)\,d$$

a_n = the nth term

a_1 = the first term

n = the term number

d = the common difference

meaning, we can get the term a_n, by adding (n-1) common differences to the first term.

In the example sequence [2, 5, 8, 11, 14, ...], let's try to get the 100th term using the formula.

nth term	a_1	a_2	a_3	...	a_{100}
value	2	5	8	...	?

$a_n = a_1 + (n - 1)\,d$

$a_{100} = 2 + (100 - 1)(3)$

$a_{100} = 2 + (99)(3)$

$a_{100} = 2 + 297$

$\underline{a_{100} = 299}$

The 100th term on the sequence is 299.

The arithmetic mean:

The arithmetic mean of any two terms in an arithmetic sequence is simply the average of those terms. It can be represented by this formula:

$$Arithmetic\ Mean = \frac{a+b}{2}$$

For example, if you have an arithmetic sequence [2, 4, 6, 8, 10], and you want to find the arithmetic mean of the second and fourth terms (4 and 8), you would add them and then divide by 2:

$$\frac{4+8}{2}$$

$$\frac{12}{2}$$

$$= 6$$

So, the arithmetic mean of the second and fourth terms (4 and 8) in the sequence is 6.

The common difference:

The common difference in an arithmetic sequence is the fixed number that is added or subtracted to get from one term to the next. It is denoted by the symbol, d.

Ex.

in the sequence 2, 5, 8, 11, 14, ... the common difference is 3 because you add 3 to each term to get the next one.

similarly, in the sequence 10, 7, 4, 1, -2, ... the common difference is -3 because you subtract 3 from each term to get the next one.

The common difference helps us identify the pattern and relationship between consecutive terms in an arithmetic sequence.

$$d = \frac{a_x - a_y}{x - y}$$

Geometric Sequence:

is a sequence of numbers where each term is found by multiplying the previous term by a fixed, non-zero number. This fixed number is known as the common ratio, denoted by r.

Ex.

consider the sequence: 2, 6, 18, 54, 162, ...

In this sequence, the common ratio between each term is 3, because each term is obtained by multiplying the preceding term by 3.

Formula for geometric sequence:

$$a_n = a_1 * (r)^{(n-1)}$$

a_n = the nth term

a_1 = the first term

r = the common ratio

n = the term number

meaning, we can get the term a_n, by multiplying r raised to (n-1) common ratios to the first term.

In the example sequence [2, 6, 18, 54, 162, ...], let's try to get the 12th term using the formula.

nth term	a_1	a_2	a_3	...	a_{12}
value	2	6	18	...	?

$a_n = a_1 * (r)^{(n-1)}$

$a_{12} = 2 * (3)^{(12-1)}$

$a_{12} = 2 * (3)^{(12-1)}$

$a_{12} = 2 * (3)^{11}$

$a_{12} = 2 * 177\ 147$

$a_{100} = 354\ 294$

The geometric mean:

The geometric mean of two numbers is the square root of their product.

$$Geometric\ Mean = \sqrt{a * b}$$

Consider this sequence, with a common ratio of 4, [2, 8, 32, 128, 512, ...] and you want to find the geometric mean of the second and fourth terms (8 and 128), you need to get the square root of their product.

$$\sqrt{8 * 128}$$

$$\sqrt{1024}$$

$$\underline{= 32}$$

So, the geometric mean of the second and fourth terms (8 and 128) in the sequence is 32.

The common ratio:

The common ratio in a geometric sequence is the constant value by which each term is multiplied to obtain the next term in the sequence. It is denoted by the symbol, r.

In the sequence: [2, 8, 32, 128, 512, ...], the common ratio is 4 because each term is obtained by multiplying the previous term by 4. So, 8 (the second term) is obtained by multiplying 2 (the first term) by 4, 32 (the third term) is obtained by multiplying 8 by 4, and so on.

$$r = \sqrt[x-y]{\frac{a_x}{a_y}}$$

Arithmetic Series (Arithmetic Sum):

The arithmetic sum refers to the total sum of the terms in an arithmetic series. It is calculated by using the formula for the sum of an arithmetic series.

$$Formula\ 1: \quad S_n = \frac{n\,(2a_1 + (n-1)d])}{2}$$

$$Formula\ 2: \quad S_n = \frac{n\,(a_1 + a_n)}{2}$$

S_n = sum of the series

a_1 = the first term

a_n = the last number

n = total number of terms

d = the common difference

Ex.

Consider the arithmetic sequence: [3, 6, 9, 12, 15] where the first term = 3, the common difference = 3, and the number of terms = 5.

To get the sum of the sequence, we use the formula:

$$S_n = \frac{n\,(a_1 + a_n)}{2}$$

$$S5 = \frac{5\,(3 + 15)}{2}$$

$$S5 = \frac{5\,(18)}{2}$$

$$S5 = \frac{90}{2}$$

$$\underline{S5 = 45}$$

So, the sum of the arithmetic series 3, 6, 9, 12, 15 is 45.

Geometric Series (Geometric Sum):

A geometric series is the sum of the terms of a geometric sequence.

It is calculated by using the formula:

$$Finite\ Series: \quad S_n = \frac{a_1\,(1 - r^n)}{1 - r}$$

$$Infite\ Series: \quad S_n = \frac{a_1}{1 - r}$$

S_n = sum of the series

a_1 = the first term

n = total number of terms

r = the common ratio

Ex.

Suppose we have the series: [2 + 6 + 18 + 54 + 162], where the first term is 2, the common ratio is 3, and the number of terms is 5.

Using the formula for the sum of a finite geometric series:

$$Finite\ Series: \quad S_n = \frac{a_1\,(1 - r^n)}{1 - r}$$

$$S_n = \frac{2\,(1 - 3^5)}{1 - 3}$$

$$S_n = \frac{2\,(1 - 243)}{1 - 3}$$

$$S_n = \frac{2\,(-242)}{-2}$$

$$S_n = \frac{-484}{-2}$$

$$\underline{S_n = 242}$$

So, the sum of the finite geometric series 2 + 6 + 18 + 54 + 162 is 242.

Ex 2.

Suppose we have the series:

$$2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$$

where the first term is 2 and the common ratio is $\frac{1}{2}$.

Using the formula for the sum of an infinite geometric series:

$$Infite\ Series: \quad S_n = \frac{a_1}{1-r}$$

$$S_n = \frac{2}{1-\frac{1}{2}}$$

$$S_n = \frac{2}{\frac{1}{2}}$$

$$\underline{S_n = 4}$$

So, the sum of the infinite geometric series is 4.

Modeling Arithmetic Series Using Sigma Notation.

Sigma notation is a better way of writing a series. All arithmetic series can be modeled using sigma notation given you have, the last term and the explicit formula for that sequence.

Ex. [7 + 9 + 11 + ... + 403 + 405]

First, get the total number of terms in the series.

$a_n = a_1 + (n - 1)\, d$

$405 = 7 + (n - 1) * 2$

$405 = 7 + 2n - 2$

$405 - 7 + 2 = 2n$

$405 - 7 + 2 = 2n$

$400 = 2n$

$n = 200$

Second, get the explicit form of the series.

$a_n = a_1 + (n - 1)\, d$

$a_n = 7 + (n - 1)\, 2$

$a_n = 7 + 2n - 2$

$a_n = 7 + 2n - 2$

$a_n = 2n + 5$, meaning, we will get all of the succeeding terms by following this formula.

Third, plug equations to the sigma notation.

$$\sum_{n=1}^{200} 2n + 5$$

This

Is the same as this

[7 + 9 + 11 + ... + 403 + 405], but in a more organized form.

Modeling Geometric Series Using Sigma Notation.

All geometric series can be modeled using sigma notation given you have, the last term and the explicit formula for that sequence.

Ex. [3 – 1 + 1/3 – 1/9 + 1/27]

First, get the total number of terms in the series, which we know right away.

$n = 5$

Second, get the explicit form of the series.

$a_n = a_1 * (r)^{(n-1)}$

$a_n = 3 * (-1/3)^{(n-1)}$, we know that -1/3 is the common ratio by inspection.

$a_n = 3 * (-1/3)^{(n-1)}$, meaning, we will get all of the succeeding terms by following this formula.

Third, plug equations to the sigma notation.

$$\sum_{n=1}^{5} 3\left((-\frac{1}{3})^{n-1}\right)$$

This

Is the same as this

[3 – 1 + 1/3 – 1/9 + 1/27], but in a more organized form.

Applications of Arithmetic Sequence.

Here are some of the real world applications of Arithmetic Sequences.

1. Finances and Economics:

- Simple interest
- Annuity

2. Physics and Engineering:

- Uniform motion
- Uniform Acceleration
- Position of a moving object with constant speed.

3. Computer Science and Programming:

- Time it takes to finish a loop.

4. Construction and Architecture:

- Evenly spaced constructions (columns or beams).

Applications of Arithmetic Sequence.

Here are some of the real world applications of Geometric Sequences.

1. Finance and Economics:

- Compound Interest

2. Population Studies:

- Population Growth
- Radioactive Decay