# APPLICATIONS OF GEOMETRIC SEQUENCES AND SERIES

## Compound Interest

Before we can talk about compound interest as geometric series, let's first talk about simple interest as an arithmetic sequence.

Simple Interest:

Simple interest is calculated only on the initial principal amount. It remains constant throughout the entire period, and it does not take into account any interest that has been previously earned or added to the principal amount.

$I = PRT$

$A = P + I_t$

I = Simple Interest

P = Principal Amount

R = Rate

T = Time

A = Total Amount

$I_t$ = Total Interest Over Time

Ex.

Yasmin invest $1 200 for 5 years earning a fixed 5% interest per annum simple interest. What is the value of her investment and the end of 5 years?

We can solve this in two ways:

1 . By using simple interest formula

$I = PRT$

$I = (1200)(0.05)(5)$

$I = 300$

Since we have 5 years' worth of interest, we have 300*5 = 1500

$A = P + I_t$

$A = 1200 + 1500$

$A = \$\ 2\ 700$

2. By thinking that it is an arithmetic sequence.

We first need to know what is the interest since it is the common difference. We know that it is 300. So, the sequence

| term | value |
|---|---|
| year 0 | 1 200 |
| year 1 | 1 200 + 300 |
| year 2 | 1200 + 300 + 300 |
| ... | ... |

Using the formula for the arithmetic sequence, we can solve for the fifth term.

$a_n = a_1 + (n - 1)d$

$a_5 = 1500 + (5 - 1)(300)$

$a_5 = \$\ 2\ 700$

Compound Interest:

Compound interest means you earn interest not just on the money you put in (the principal) but also on the interest that money has already earned. So, every time interest is calculated, it's like you're adding the interest to your original money, and then the next time you calculate interest, it's as if you're earning interest on that bigger amount. This makes your money grow faster compared to simple interest where you only earn interest on the original amount.

$$A = P(1 + r/n)^{n*t}$$

$$I = A - P$$

A = Total Amount

P = Principal Amount

r = Rate

n = compounding period

t = Time

I = Interest Accumulated

Ex.

Luke puts $1000 in an account that pays interest of 9%. What will be the value of the account after 10 years?

We can solve this in two ways:

1 . By using compound interest formula

$$A = P(1 + r/n)^{n*t}$$

$$A = 1000(1 + 0.09/1)^{1*10}$$

$$A = \$2\,367.36$$

2. By thinking of it like a geometric sequence.

We first need to know what is the rate since it determines our common ratio. We know that it is 9%. So, the sequence

| term | value |
|---|---|
| year 0 | 1000 |
| year 1 | $1000 + (1 + 0.9)^1$ |
| year 2 | $1000 + (1 + 0.9)^1$ |
| ... | ... |

By following this trend, we get

$A_{10} = 1000 + (1 + 0.9)^{10}$

$A_{10} = \$2\ 367.36$

**Population Growth:**

Population growth in simple terms refers to the increase in the number of people living in a particular area over a specific period. It occurs when the number of births and the number of people moving into the area (immigration) are more than the number of deaths and the number of people leaving the area (emigration). This can lead to a larger and more densely populated community, city, or country. Understanding population growth is essential for planning resources, infrastructure, and services to meet the needs of the growing population.

Ex.

In 2010, a certain island contains 200 dogs. If the population increases by 6% each year, how many dogs are expected to be on the island by 2025?

Subtract 2010 from 2015 to get the total number of years, 15.

Now, use the formula for population growth, which is the same as compound interest with n = 1.

$A = P\ (1 + r/n)^{n*t}$

$A = 200\ (1 + 0.06/1)^{1*15}$

$A = 200\ (1.06)^{15}$

A ~~ 480 dogs.