Assignment 5

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1. **Problem: Mean-variance portfolio choice and leverage constraints (40 points).** Consider an economy with $N = 3$ risky assets R_1, R_2, R_3 and one risk-free asset R_0. The expected return vector is $\mu = [0.08; 0.14; 0.16]$ and standard deviation $\sigma = [0.10; 0.30; 0.35]$. The pair-wise correlation between any two returns is 0.2. There is a risk-free rate $R_0 = 0.05$. We want to solve the problem of a mean-variance investor who faces leverage constraints and cannot borrow more than 30% of his wealth. The investor seeks the portfolio R_P such that $\max E[R_P] - \frac{a}{2}V[R_p]$ subject to $w'\mathbf{1} \leq m$ where $m = 1.3$ and w is the vector of weights invested in the risky assets.

   You can make all calculations and plot the graph in Python (or in other programs). Do not forget about the proofs, which should be explicitly written (by hand, LaTex, or markdown in Python)

   - Determine the tangency portfolio w_t, its mean, variance and Sharpe ratio.
   - Determine the zero beta portfolio w_z (which has zero correlation with the tangency portfolio), its mean, variance and Sharpe ratio.
   - Prove that the that the investor will optimally choose to invest in a comination of a risky-asset-only mean-variance efficient portfolio and the risk-free rate. Prove further tha this implies that we can restrict his optimal portfolio choice to portfolios with returns of the form $R_P = R_0 + x_t(R_t - R_0) + x_z(R_z - R_0)$. Setup the Lagrangian of the agent's problem, derive the first-order condition, and compute the optimal portfolio in terms of x_t, x_z the holdings of tangency and zero-beta portfolio. Then given the portfolio in terms of the underlying securities w_0, w_1, w_2, w_3.

- Prove that there exists a risk-aversion level a^* so that if $a > a^*$ then the agent is unconstrained and does not hold the zero-beta portfolio. Instead, if $a < a^*$ then the agent will also invest in the zero-beta portfolio.
- Plot the Sharpe ratio on the optimal portfolio as a function of the risk-aversion level. What happens to the Sharpe ratio of the optimal portfolio as a falls below a^*? Interpret the finding.

2. **Problem: Betting against Beta and information ratio (30 points).**

   Suppose you estimate the following regression $R_i - R_0 = \alpha_i + \beta_i(R_M - R_0) + \epsilon_i$ for $i = H, L$ two portfolios of High and low beta firms respectively. You find the following $\alpha_L = 2.36\%, \alpha_H = -4.36\%,\ \beta_L = 0.55,\ \beta_H = 1.78$. The risk-free rate is 0.5%. In addition, $\sigma_L = \sigma_H = 12\%$ and the residuals are uncorrelated.

   - Construct a zero cost portfolio, using H and L, with a positive alpha and zero beta. What is the alpha of that portfolio? what is its information ratio?
   - Given $\sigma_M = 14\%,\ \mu_M - R_0 = 7.5\%$ what is the optimal position in the BAB portfolio that you should take? What is the optimal position in the high-beta portfolio and in the low beta portfolio?
   - What is the Sharpe ratio of your optimal portfolio? How does it compare to that of the market?

3. **Problem: APT (30 point)**

   Consider the following model of returns for stock returns R_i with $i = 1, \ldots, N$:

$$R_i = \alpha_i + \sum_{k=1}^{K} B_{ik} F_k + \epsilon_i$$

   where the factor exposure coefficient B_{ik} are known constants, the F_k $\forall k = 1, \ldots, K$ are returns of specific stock portfolios which are uncorrelated with each other and have a normal distribution $F_k \sim N(m_k, \sigma_k^2)$ and ϵ_i are independent normal random variables with $\epsilon_i \sim N(0, \sigma^2)$ $\forall i$. In addition assume there is a risk-free rate R_0.

   - According to the APT what should be expected stock return $E[R_i]$ for stocks $i = 1, \ldots, N$? What restriction does it imply for α_i?

- Show that if the market portfolio is spanned by the factors in the sense that there exists some weights w_k such that $R_M = \sum_{k=1}^{K} w_k F_k$, then the CAPM holds if and only if the APT holds. Find how w_k is related to factor risk-premia and volatility.
- Now suppose that

$$R_i = R_0 + \sum_{k=1}^{K} B_{ik}\lambda_k + \epsilon_i$$

where the $\lambda_k > 0$ are constants and we assume that the stock exposures are all bounded away from zero $B_{ik} \geq b > 0 \ \forall i, k$. So unlike in the previous question the factors F_k are not random. So stock returns are only affected by the ϵ_i shocks that are iid normal random variables. Is this model consistent with the APT? If not, find an asymptotic arbitrage portfolio that as the number of stocks N grows arbitrarily large will have zero risk and strictly positive profits.