# FIN405-PS9

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May 2021

## 1 Closed-end fund discount

1)

The value of the fund at time $t$ is the value at time $t-1$ compounded by its return of time $t$ minus a fraction of the value at time $t$ before paying dividends and costs. The dynamics of the NAV of the fund is thus

$$\begin{aligned} V_t &= V_{t-1}(1+R_t) - (f+\delta)V_{t-1}(1+R_t) \\ &= V_{t-1}(1+R_t)(1-(f+\delta)). \end{aligned} \tag{1}$$

We can write this as a function of the initial value $V_0$ by recursively substituting the preceding values $V_{t-i}$

$$\begin{aligned} V_t &= V_{t-1}(1+R_t)(1-(f+\delta)) \\ &= V_{t-2}(1+R_{t-1})(1-(f+\delta))(1+R_t)(1-(f+\delta)) \\ &= V_{t-2}(1-(f+\delta))^2 \prod_{i=0}^{1}(1+R_{t-i}) \\ &\vdots \\ &= V_0(1-(f+\delta))^t \prod_{i=0}^{t-1}(1+R_{t-i}) \end{aligned} \tag{2}$$

2)

Let $D_t = V_{t-1}(1+R_t)\delta$ and $F_t = V_{t-1}(1+R_t)f$ be the the dividends and the fees payed at time $t$ to the investors and the manager respectively. The present value of a sequence of cash flows being the sum of the discounted expected cash flows, we compute the present value of the dividends at time $t$ as

$$
\begin{aligned}
PV_D(t) &= \sum_{i=0}^{\infty} E[V_{t-1+i}(1+R_{t+i})\delta]\frac{1}{(1+k)^i} \\
&= \delta(1+k)\sum_{i=0}^{\infty} E[V_{t-1+i}]\frac{1}{(1+k)^i} \\
&= \delta(1+k)V_0\sum_{i=0}^{\infty}(1+k)^{t-1+i}(1-(f+\delta))^{t-1+i}\frac{1}{(1+k)^i} \\
&= \delta V_0\sum_{i=0}^{\infty}(1+k)^{t+i}(1-(f+\delta))^{t-1+i}\frac{1}{(1+k)^i} \\
&= \delta V_0(1+k)^t\sum_{i=0}^{\infty}(1-(f+\delta))^{t-1+i} \\
&= \delta V_0(1+k)^t(1-(f+\delta))^{t-1}\sum_{i=0}^{\infty}(1-(f+\delta))^i \\
&= \frac{\delta V_0(1+k)^t(1-(f+\delta))^{t-1}}{1-(1-(f+\delta))} \\
&= \frac{\delta V_0(1+k)^t(1-(f+\delta))^{t-1}}{f+\delta}. \qquad (3)
\end{aligned}
$$

Since $PV_F(t)$ only replaces $\delta$ in $PV_D(t)$ by a time-invariant constant $f$, we naturally get

$$
PV_F(t) = \frac{fV_0(1+k)^t(1-(f+\delta))^{t-1}}{f+\delta}. \qquad (4)
$$

This shows that if CAPM holds, the idiosyncratic risk of the fund ($Var(\epsilon_t)$) does not affect values to the investors nor the manager. This is because if CAPM holds, the fund's performance is only driven by its systematic risk exposure, and consequently, so are the investors' dividends and the manager's fees.

3)

The discount at any time $t$ is given by the relative difference between the NAV at time $t$ and the market (trading) price of the fund at time $t$, in our case $d_t = (V_t - PV_D(t))/V_t$. Since we want the average discount implied by our formula, we take the expectation of this discount which is given by

$$
E[d_t] = 1 - \frac{PV_D(t)}{E[V_t]}, \qquad (5)
$$

since $PV_D(t)$ is deterministic. Since CAPM holds, we have that

$$
E[V_t] = V_0(1-(f+\delta))^t(1+k)^t. \qquad (6)
$$

Therefore,

$$E[V_t] = 1 - \frac{\delta}{(1-(f+\delta))(f+\delta)}, \tag{7}$$

which with the numbers given yields

$$E[V_t] = 1 - \frac{0.027}{(1-(0.0044+0.027))(0.0044+0.027)} = 11.23\% \tag{8}$$

The discount implied by our model is indeed lower than the observed average discount over 26 years, which sparks the closed-end fund discount puzzle discussed below.

4)

Management fees can indeed be a credible explanation for the additional discount investors impose on a closed-end fund. The explanation could be that investors perceive the fees earned by the manager as excessive and not justifiable especially when the closed-end fund produces no alpha and only takes systematic exposure, which can easily be reproduced by any investor, without special skill or talent needed.

## 2 Reversal and momentum strategies

*see PS9-Code.ipynb*