Assignment 6

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1. **Problem: APT (50 points).** Consider an economy with $N_A + N_B$ Risky assets corresponding to two families of (A and B) stocks, and one risk-free asset with return $R_0 = 0.04$. The return of the risky assets satisfy the factor model:

$$R_{Ai} - R_0 = \alpha_A + \beta_A(F + \mu_F) + \sigma_A \epsilon_{Ai} \quad \forall i = 1, \ldots, N_A$$
$$R_{Bi} - R_0 = \alpha_B + \beta_B(F + \mu_F) + \sigma_B \epsilon_{Bj} \quad \forall j = 1, \ldots, N_B$$

where the $F, \epsilon_{A,i}, \epsilon_{B,j}$ are all independently and identically distributed random variables with zero mean and unit variance. Suppose that $\mu_F = 0.08$, $\beta_A = \beta_B = 0.5$ and $\sigma_A = \sigma_B = 0.3$. In addition $\alpha_A = 0$, but $\alpha_B = 0.03$.

We define the excess return of an asset by $R_i^e = R_i - R_0$. Note that an excess return is a zero-cost trading strategy return since it is the return to a \$1 investment in asset i funded by borrowing \$1 at the risk-free rate.

   (a) Assume first that there is only 1 stock with positive alpha, that is $N_B = 1$. Compute the systematic risk, the idiosyncratic risk, the total risk (or standard deviation), and the 'Sharpe ratio' (since the strategy is zero cost we define its Sharpe Ratio as the ratio of its average return to its volatility) of a \$1 position in the positive alpha stock $R_{B,1}$ financed by borrowing at the risk-free rate.

   (b) Determine the factor bet portfolios w that invests in all the zero-alpha stocks $i = 1, \ldots, N_A$, that has a factor exposure (a beta) of 1 to the factor F and smallest idiosyncratic risk. The return on the factor bet portfolio is then simply $R_F = \sum_{i=1}^{N_A} w_i R_{Ai}$.

(c) Construct a zero cost portfolio that invests \$1 in the positive alpha asset $R_{B,1}$ and hedges the factor risk by shorting the proper amount of the factor bet portfolio.

Plot the systematic risk, the idiosyncratic risk, the total risk (or standard deviation), the expected return and the 'Sharpe ratio' of that portfolio as a function of $N_A = 1, 2, 3, \ldots$

(d) How does that portfolio compare to the Maximum Sharpe ratio portfolio that you could build from optimally investing in the $N_A + 1$ assets? Can you construct a strategy investing in these $N_A + 1$ assets that has zero cost and infinite sharpe ratio in the limit as $N \to \infty$?

(e) Suppose now that $N_B = N_A = N$. Can you construct a strategy investing in these $2N$ assets that has zero cost and infinite sharpe ratio in the limit as $N_A \to \infty$? Plot the systematic risk, the idiosyncratic risk, the total risk (or standard deviation), the expected return and the 'Sharpe ratio' of that portfolio as a function of $N = 1, 2, 3, \ldots$

Explain why your results imply that N_B cannot be very large.

2. **Problem: Understanding Warren Buffett's performance (50 points).** Warren Buffett is widely considered as one of the most successful investor of the last 50 years. He is the chairman, CEO and largest shareholder of Berkshire Hathaway (BRK). His outstanding performance is illustrated by Figures 1 and 2. Figure 1 shows the annualized Information Ratio (IR) of BRK vs. all actively managed equity funds on the CRSP mutual fund database with at least 30 years of return history.[1] Similarly, Figure 2 shows the annualized IR of BRK vs. all common stock on the CRSP database with at least 30 years of return history. In both cases, he is clearly an outlier. In this exercise you will examine the extent to which Buffett's performance is due to exposure to systematic risk factors.

(a) Download from CRSP the historical time series of the BRK (permno=17778) monthly stock returns from 1976 to 2019.

(b) Compute the annualized mean and standard deviation of BRK excess returns as well as the annualized Sharpe ratio. Contrast with that of the three Fama-French

[1]The IR is defined as the intercept in a regression of monthly excess returns divided by the standard deviation of the residuals. The explanatory variable in the regression is the monthly excess returns of the CRSP value-weighted market portfolio.

factor portfolio returns ($R^e_m = R_m - r_f$, SMB, HML) and the momentum (MOM) as well as the risk-free rate which you may download from Ken French's data web site:

$https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html$

(c) Run the following regressions:

$$R^e_t = \alpha + \beta_1 R^e_{mt} + \varepsilon_t \tag{1}$$

$$R^e_t = \alpha + \beta_1 R^e_{mt} + \beta_2 SMB_t + \beta_3 HML_t + \varepsilon_t \tag{2}$$

$$R^e_t = \alpha + \beta_1 R^e_{mt} + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 MOM_t + \varepsilon_t \tag{3}$$

$$R^e_t = \alpha + \beta_1 R^e_{mt} + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 MOM_t + \beta_5 RMW_t + \beta_5 CMA_t + \varepsilon_t \tag{4}$$

The first regression controls for market exposure.

The second regression controls for standard factors that capture the effects of size and value (Fama and French (1993)). The third regression controls for momentum (Carhart (1997), Jegadeesh and Titman (1993)). The momentum factor (MOM) is a strategy of buying recent "winner" stocks and shorting recent "loser" stocks. The fourth regression also controls for two additional factors identified by Fama-French in their recent paper "A 5-factor asset pricing model" (posted on moodle). They are called Robust-minus-Weak and Conservative-minus-Aggressive.

For each regression report the estimated α and βs along with the associated t-statistics. Also report the R^2 in the regression and the information ratio of BRK.

(d) Based on your reading of the FF 5-factor paper, explain how RMW and CMA are constructed and why these factor portfolios may capture sources of priced return.

(e) Interpret what the β-estimates say about Warren Buffett's investment strategy.[2]

(f) Does exposure to common risk factors explain Warren Buffett's performance (what happens to the α and information ratio)?

(g) If you can choose your optimal mean-variance efficient portfolio that combines all 6 factor portfolios ($R^e_M, SMB, UMD, MOM, RMW, CMA$), the risk-free rate ($R_f$ proxied by the average short-term T-Bill rate over the same period) and BRK

[2]See also `http://www.forbes.com/sites/phildemuth/2013/06/27/the-mysterious-factor-p-charlie-munger-robert-novy-marx-and-the-profitability-factor`.

what would be the optimal portfolio weight vector if you target an expected return of 10% for your portfolio? What would be that portfolio's volatility?

(h) Rerun all the regressions for data until 1995 and compare with the full-sample results. How does Buffett's performance compare in the first half of the sample and in the full sample (or second half)? What do you think can explain the difference in performance between the two sub-periods?

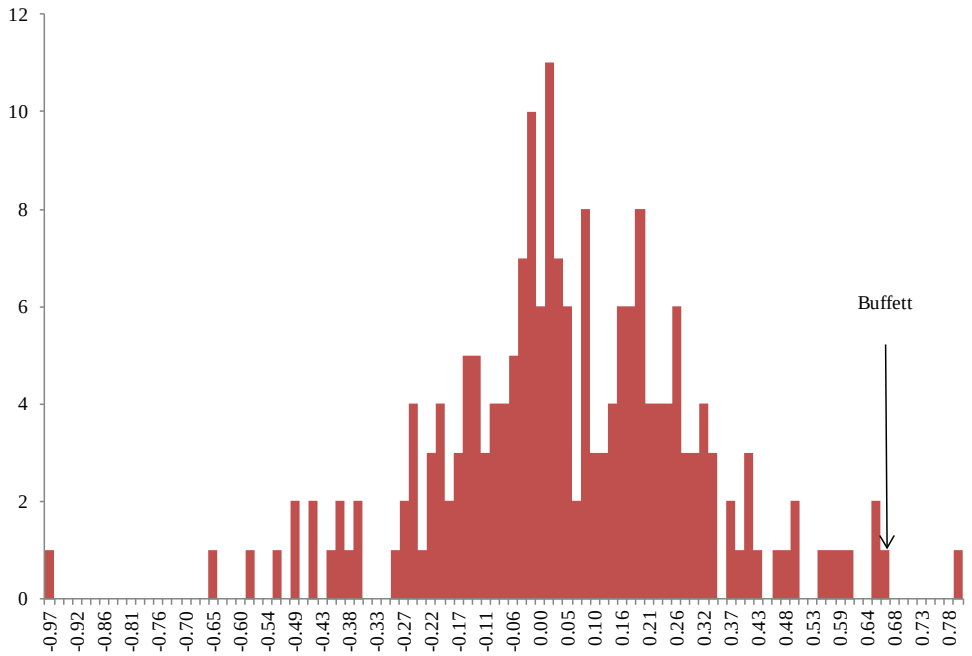

Figure 1: Berkshire vs. mutual funds

This figure shows the distribution of annualized Information Ratios of all actively managed equity funds on the CRSP mutual fund database with at least 30 years of return history. Information ratio is defined as the intercept in a regression of monthly excess returns divided by the standard deviation of the residuals. The explanatory variable in the regression is the monthly excess returns of the CRSP value-weighted market portfolio. The vertical line shows the Information ratio of Berkshire Hathaway.

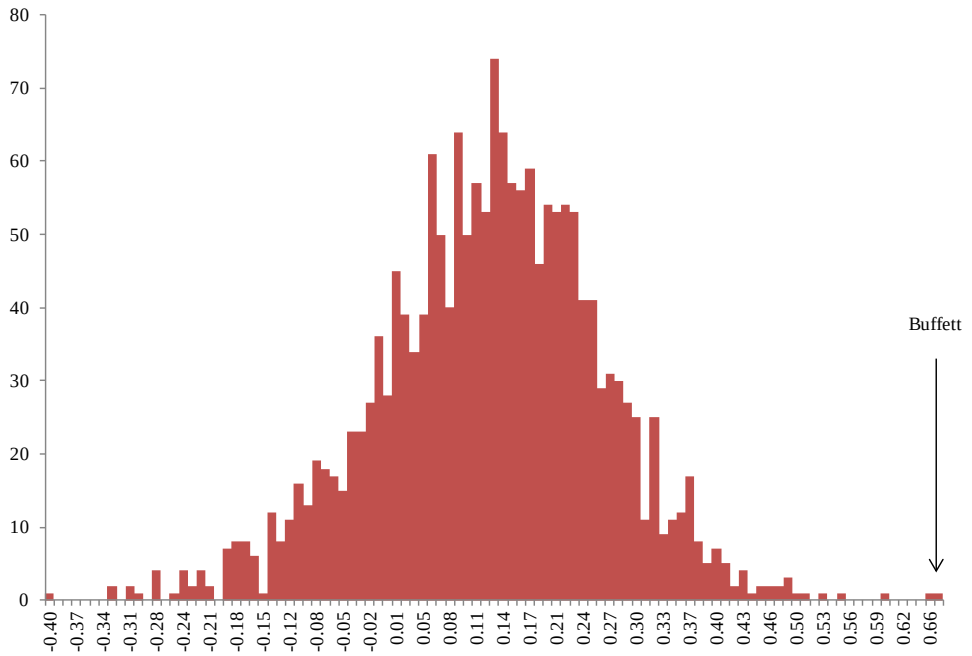

Figure 2: Berkshire vs. common stocks

This figure shows the distribution of annualized Information Ratios of all common stock on the CRSP database with at least 30 years of return history. Information ratio is defined as the intercept in a regression of monthly excess returns divided by the standard deviation of the residuals. The explanatory variable in the regression is the monthly excess returns of the CRSP value-weighted market portfolio. The vertical line shows the Information ratio of Berkshire Hathaway.

Figure 3: Berkshire