BioE 135/235 Homework 4

Due Date: Friday, 4/16/2021, 5:00pm PST to Gradescope
Late work will be deducted 20% for each day it is late.

1. Glyolytic Oscillator and the Trapping Region

In the fundamental biochemical process referred to as glycolysis, living cells obtain energy by breaking down sugar. In intact yeast cells as well as in yeast or muscle extracts, glycolysis has been observed to proceed in an oscillatory fashion, with the concentration of various intermediates waxing and waning with a period of several minutes (Chance et. al. 1973, Goldbeter 1980). A simple model of these oscillations has been proposed by Sel'kov (1968). In non-dimensionalized form, its equations are:

$$\frac{d}{dt}x = -x + ay + x^2y$$
$$\frac{d}{dt}y = b - ay - x^2y$$

where x and y are the concentrations of ADP (adenosine diphosphate) and F6P (fructose-6-phosphate), and $a, b > 0$ are kinetic parameters.

(a) What are the nullclines of this system? Plot them for $a = 0.08$ and $b = 0.6$.

(b) Plot the corresponding vector field for the system on the same plot as you used for part (a). *Hint: you may find the quiver function, and this demonstration, very useful, if you plan to use Python for this.*

(c) A region of the state space is said to be a *trapping region* if a state that starts within or enters the trapping region, never leaves the trapping region upon entry. Mathematically, it turns out that an equivalent definition of a trapping region is to say that all vectors along the boundary of the trapping region, don't point outwards from the trapping region. In other words, all vectors along the boundary of the trapping region either point inwards, or point parallel to the boundary.

Sketch a potential trapping region for the system on the plot from part (b). What is the biological implication of a trapping region in this context?

(d) What does the existence of a trapping region in a system suggest about the stability of that system? Will any biological systems have no possible trapping region?

(e) Solve for the fixed point(s) of this system. Under what conditions is(are) the fixed point(s) stable?

2. Nullclines and Vector Fields

For each of the following systems, graph the nullclines. Also sketch the vector field in each segment of the phase plane as partitioned by the nullclines. What are the fixed points of each system?

(a) System 1:

$$\frac{d}{dt}x = x(-x - 3y + 150)$$
$$\frac{d}{dt}y = y(-2x - y + 100)$$

(b) System 2:

$$\frac{d}{dt}x = x(10 - x - y)$$
$$\frac{d}{dt}y = y(30 - 2x - y)$$

(c) System 3:

$$\frac{d}{dt}x = 2x(1 - \frac{1}{2}x) - xy$$
$$\frac{d}{dt}y = y(\frac{9}{4} - y^2) - x^2y$$

3. Final Project Beginnings

Complete the first draft of your final project—that is, complete the abstract; background & significance; and specific aims sections of your project. For specific feedback, please don't hesitate to reach out to Nick at nick.nolan@berkeley.edu!