CS 432/637
Interactive Computer Graphics

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More Texture Mapping

Slide Credits

- Jonathan Cohen - Johns Hopkins University
- John Hart - University of Illinois
- Leonard McMillan - UNC, Chapel Hill
- Dani Lischinski - Hebrew University
- Craig Schroeder - Drexel University

Procedural Textures and Shading

Procedural Texture/Shading

- Define texture color/value with a procedure
- Allows for a wide variety of (programmable) surface materials and embellishments
- Many independent variables
  - Position, normal, curvature, geodesic distance, time

Potential Advantages of Procedural Textures

- Compact representation
- No fixed resolution
- No fixed area
- Parameterized - generates class of related textures

Disadvantages of Procedural Textures

Difficult to build and debug

Surprising results

Slow evaluation

Procedural Texture Conventions

Avoid conditionals

- Convert to mathematical functions when possible
- Makes anti-aliasing easier

Parameterize rather than building in constants

- Assign reasonable defaults which may be overridden

Simple Building Blocks

Mix (lerp)

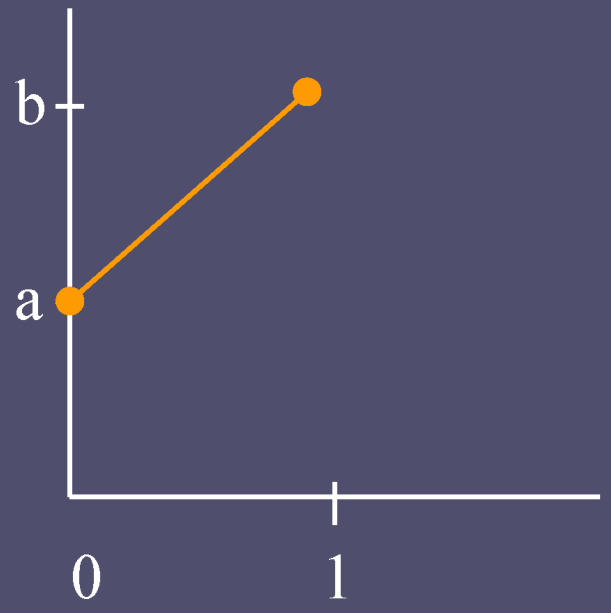
Step, smoothstep, pulse

Min, max, clamp, abs

Sin, cos

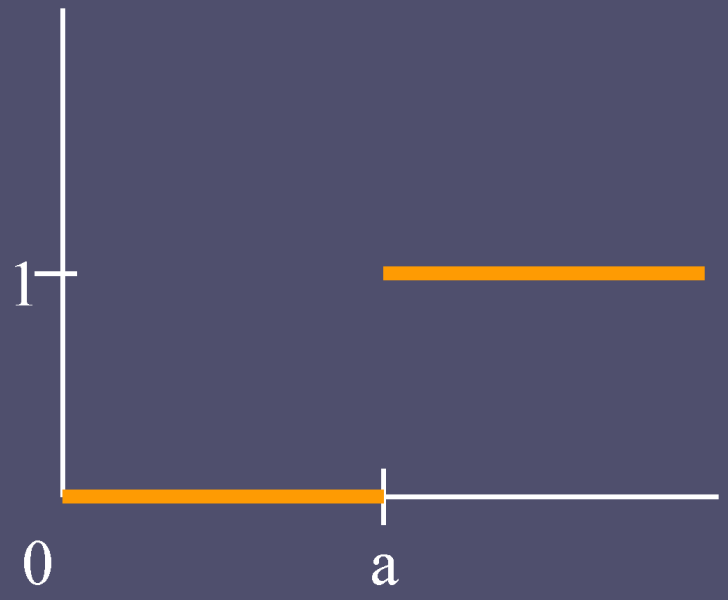
Mod, floor, ceil

Mix



mix(a,b,x)

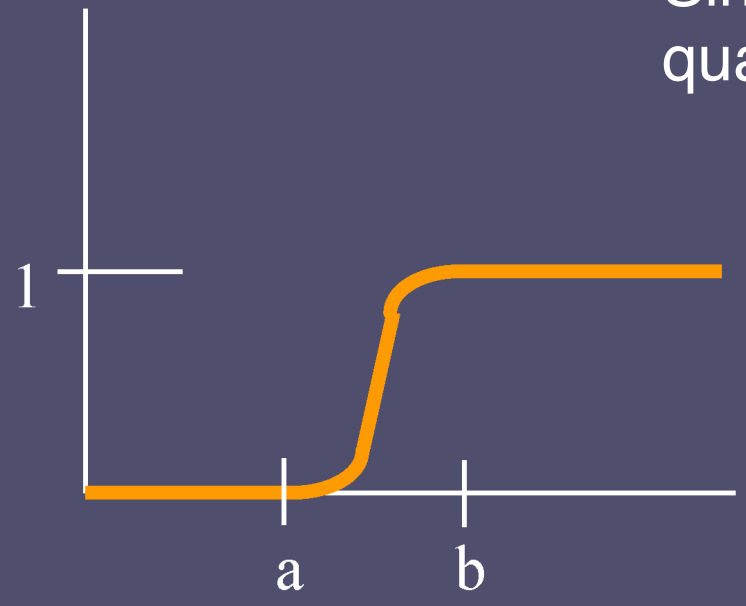
Step



step(a, x)

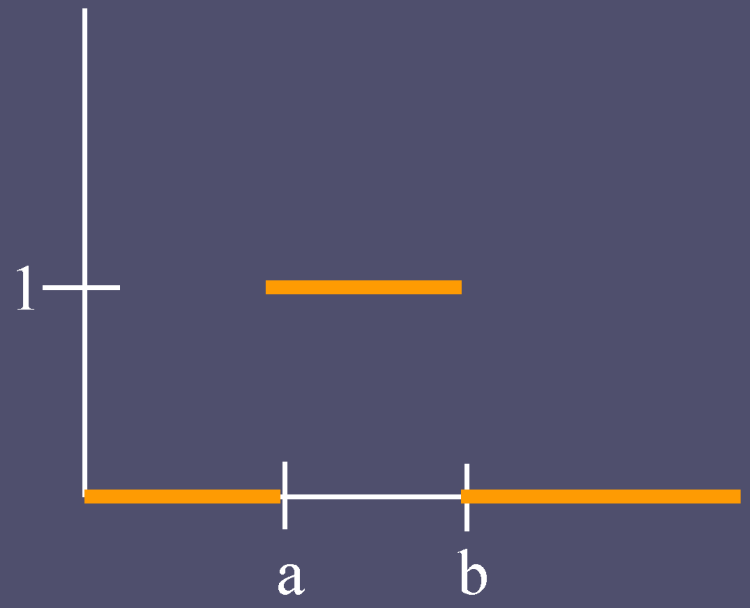
Smoothstep

Sin() or piece-wise quadratics work well



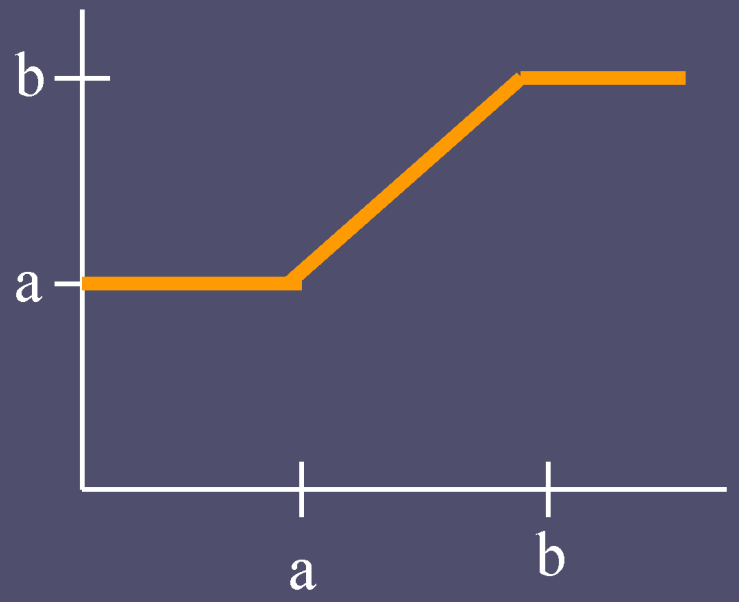
smoothstep(a,b,x)

Pulse



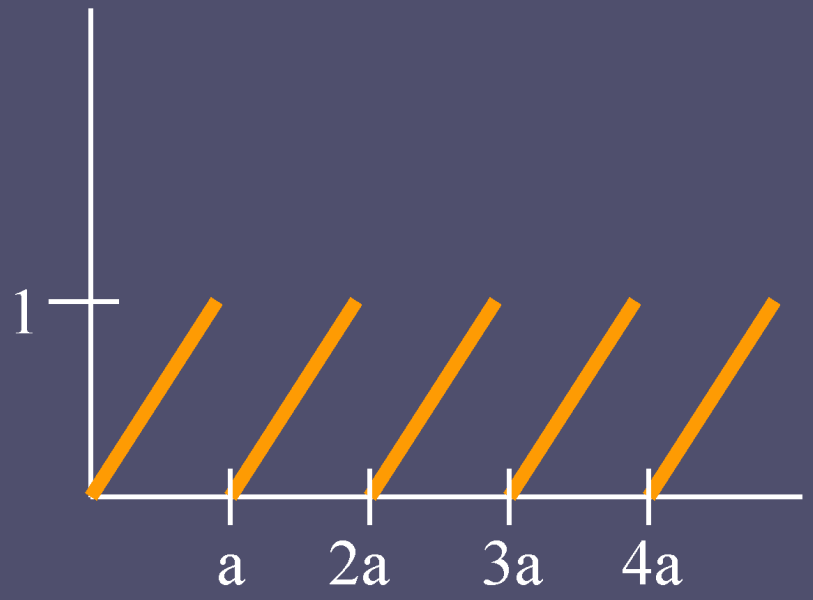
$$pulse(a,b,x) = step(a,x) - step(b,x)$$

Clamp



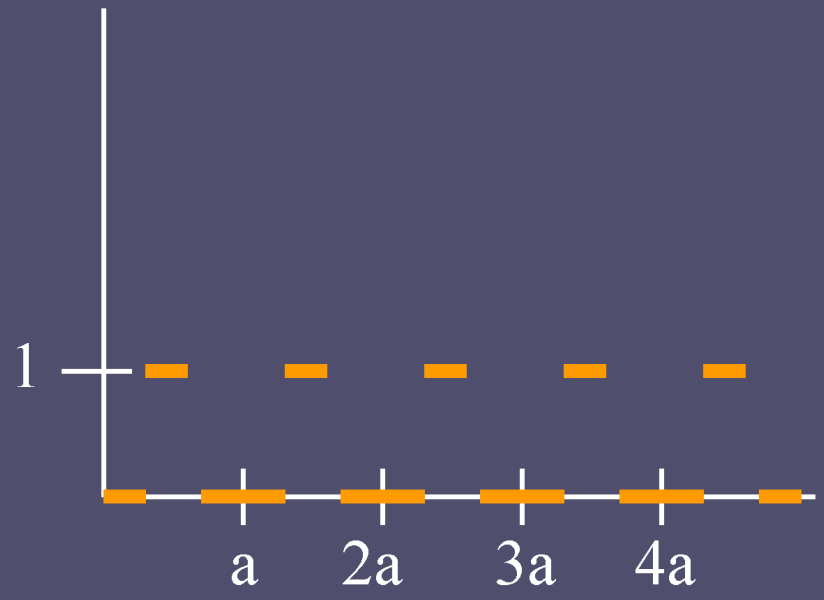
$$\text{clamp}(x,a,b) = \min(\max(x,a), b)$$

Mod



mod(x,a) / a

Periodic Pulse



pulse(0.4, 0.6, mod(x,a)/a)

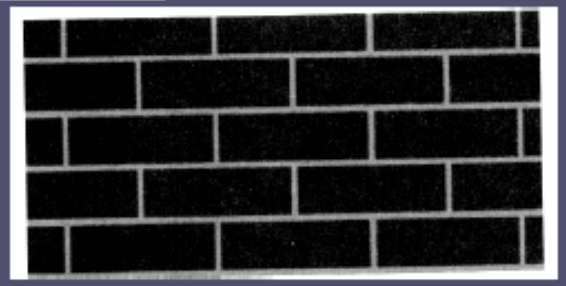
Example 1 - brick

Brick is primarily a 2D pulse

Input parameters may include:

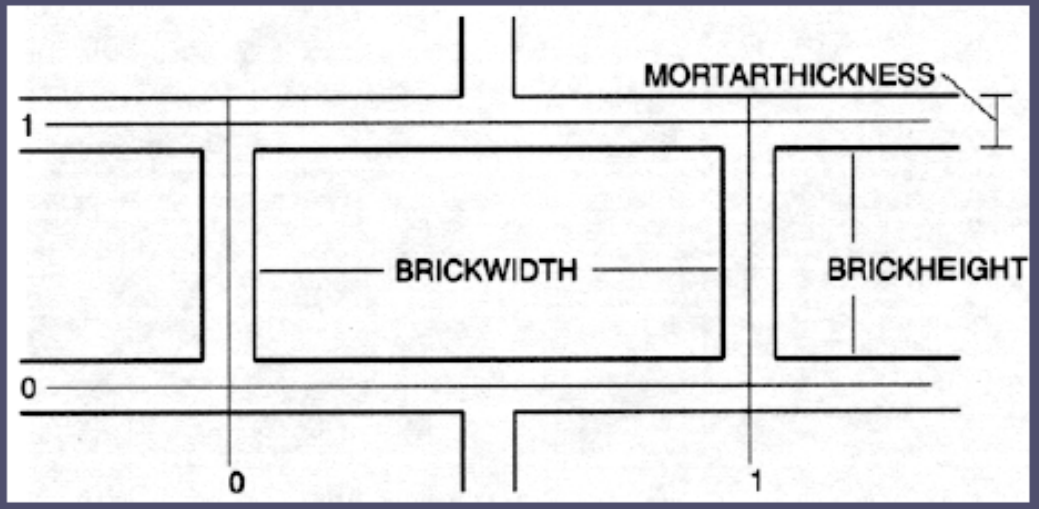
- color of brick and mortar
- size of brick
- thickness of mortar
- mortar bump size
- frequency of brick color variation
- etc.

Brick



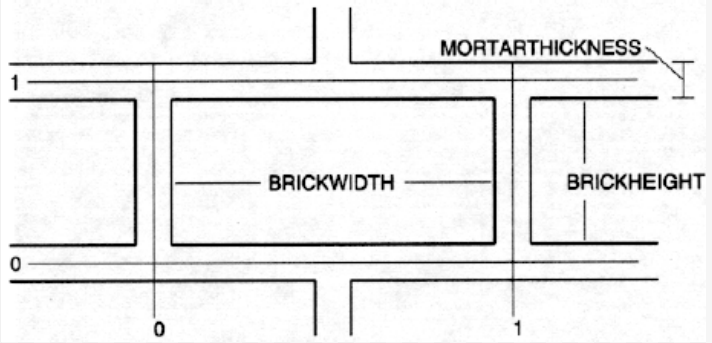
(pulse)

(pulse)



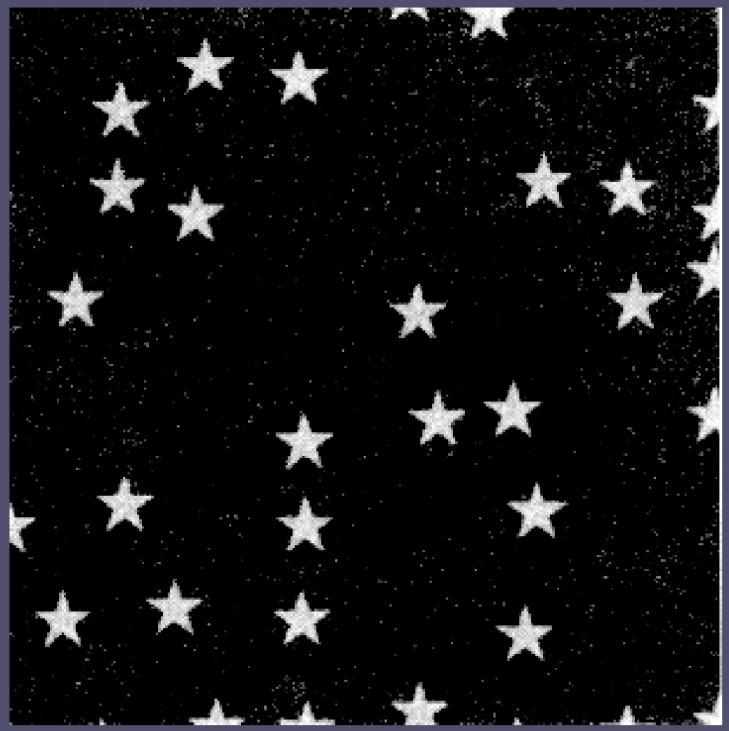
from Ebert, ed., *Texturing and Modeling: a Procedural Approach, 1994, pages 37-38.*

Brick texture code




```
uniform vec3  BrickColor, MortarColor;
uniform vec2  BrickSize,  BrickPct;
varying vec2  MCposition;
void main(void) {
  vec3  color;   vec2  position, useBrick;
  position = MCposition / BrickSize;
  if (fract(position.y * 0.5) > 0.5)    position.x += 0.5;
  position = fract(position);
  useBrick = step(position, BrickPct);
  color  = mix(MortarColor, BrickColor, useBrick.x * useBrick.y);
}
```

Example - Star Wallpaper



Example - Star Wallpaper

Divide 2D texture space into uniform grid

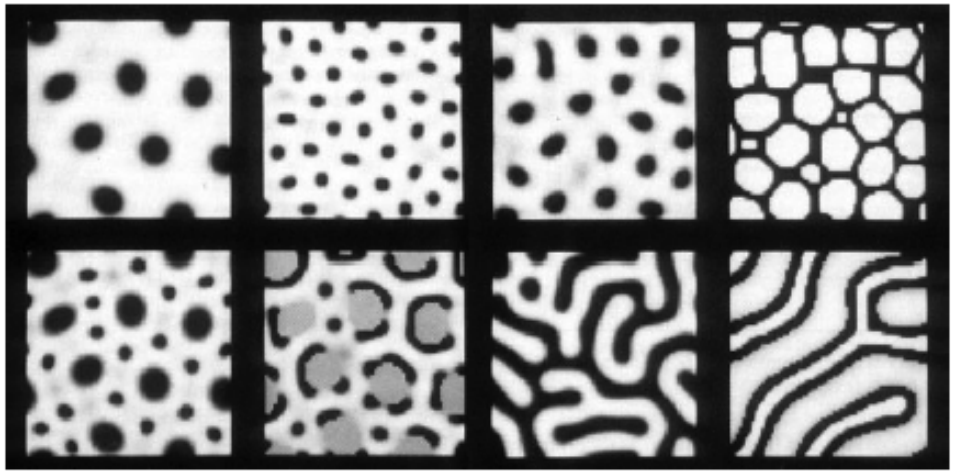
Decide whether or not to place a star in each cell

Perturb position of star within each cell

To render a point on surface, check nearby cells for stars which may cover point

Reaction-Diffusion Textures
Turk 91; Witkin & Kass 91

- Reaction-diffusion is a mathematical model for generation of natural patterns, arising due to local non-linear interactions of excitation and inhibition.
- First proposed by Alan Turing in 1952



Main Idea

- Certain cell properties (such as generation of pigments) are determined in the embryo based on the local concentration of one or more chemicals, called morphogens.
- Morphogen concentrations are determined by two concurrent processes:
  - Diffusion - spreading of morphogens through the tissue
  - Reaction - chemical reactions that create or destroy morphogens, based on their concentration in each cell
- Natural patterns can be generated by simulating the R-D processes.

$$U + 2V \rightarrow 3V$$
$$V \rightarrow P$$

Mathematical Model

- A system of non-linear partial differential equations.
- Let C(x,y) denote the concentration of a morphogen in a (2D) system. The governing equation is:

$$\dot{C} = \frac{\partial C}{\partial t} = a^2 \nabla^2 C - bC + R$$

- where a is the diffusion coefficient, b is the dissipation coefficient, R is the rate of change due to reaction, and $\nabla^2 C = \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}$ is the Laplacian of C.

Finite Differences Apprx

- On a 2D integer grid with spacing h, the Laplacian is approximated as:

$$\nabla^2 C \approx \frac{C_{i+1,j} + C_{i-1,j} + C_{i,j+1} + C_{i,j-1} - 4C_{i,j}}{h^2}$$

- In other words, the Laplacian operator is a convolution with:

$$L = \frac{1}{h^2}\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Finite Differences (cont'd)

The original equation $\dot{C} = \frac{\partial C}{\partial t} = a^2 \nabla^2 C - bC + R$

can be rewritten as $\dot{C} = M * C + R$

where $$M = \frac{1}{h^2}\begin{bmatrix} 0 & a^2 & 0 \\ a^2 & -4a^2 - h^2 b & a^2 \\ 0 & a^2 & 0 \end{bmatrix}$$

Euler's Method

- We are really after C, rather than its derivative. Therefore, we must integrate the equation:

$$C_t = C_0 + \int_0^t \dot{C}\, dt$$

- The integration is performed using small discrete time steps:

$$C_{t+\Delta t} = C_t + \Delta t\left(M * C_t + R_t\right)$$

Anisotropic Diffusion

- More interesting pattern can be created if the diffusion rates are different in different directions. Instead of modeling diffusion with the Laplacian operator

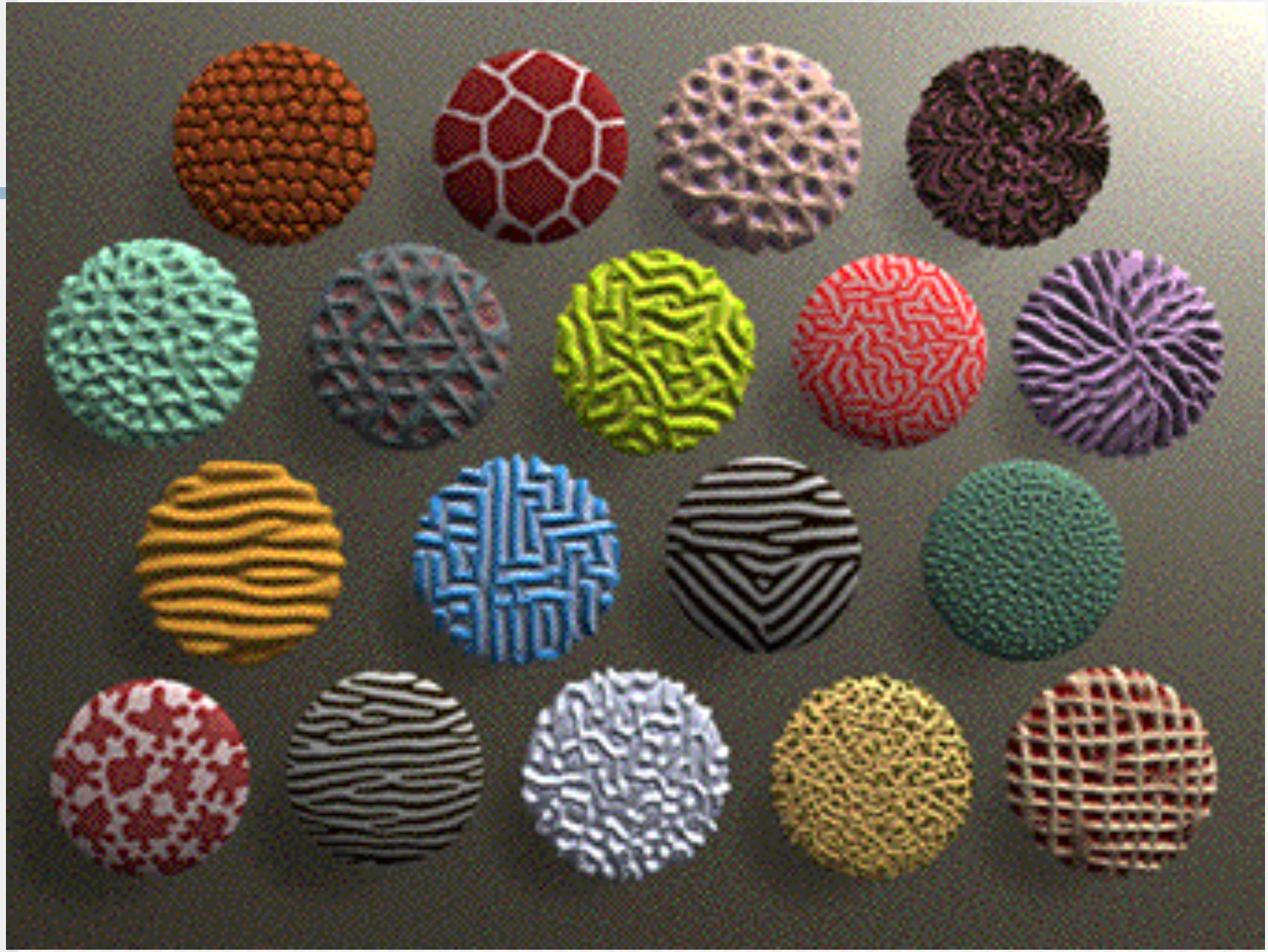
$$a^2 \nabla^2 C = a^2 \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right)$$

- use different rates along X and Y directions:

$$a_1^2 \frac{\partial^2 C}{\partial x^2} + a_2^2 \frac{\partial^2 C}{\partial y^2}$$

Space-Varying Diffusion

- Control diffusion pattern spatially by specifying a "diffusion map" - a map that gives the diffusion coefficients at each position on the surface.

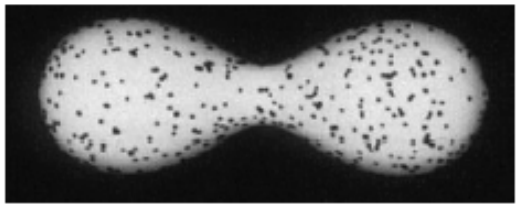


Reaction-Diffusion on Surfaces

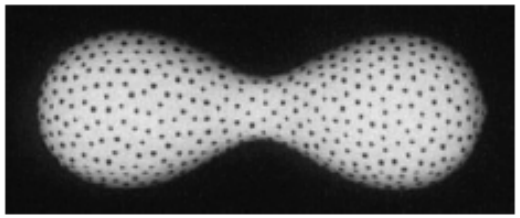
- To avoid the difficulties associated with mapping 2D textures onto objects in 3D, we'd like to perform the RD simulation directly on the surface of an object.
  - Randomly distribute points over the object's surface
  - Create a mesh of even-sized cells
  - Simulate R-D on this mesh

Relaxation of Random Points

- Given a polyhedral model, randomly place points (for uniform distribution, the probability of a point being placed in a polygon is proportional to its area).



- Relaxation: move each point according to repulsion forces applied to it by other nearby points (iterate several times).

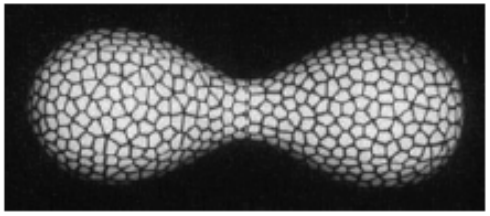


Relaxation

- Loop k times:
  - For each point P on surface
    - Determine nearby points to P
    - Map these points onto plane of P's polygon
    - Compute and store repulsive forces on P
  - For each point P on surface
    - Compute the new position of P based on repulsive forces

Mesh Construction

- Compute Voronoi regions, based on the points computed in the previous stage:

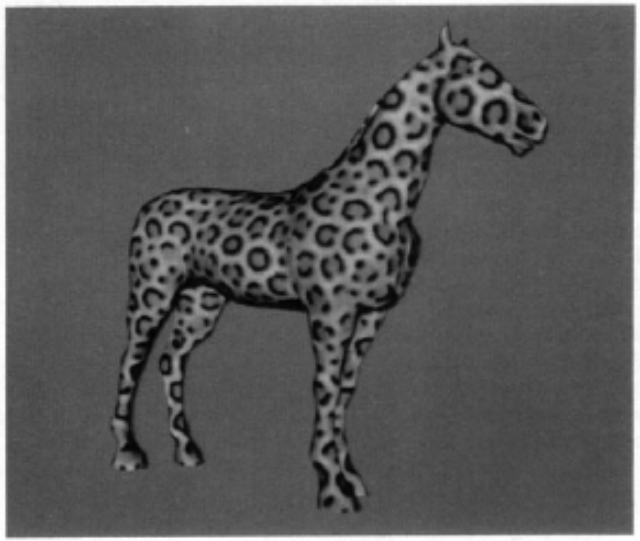


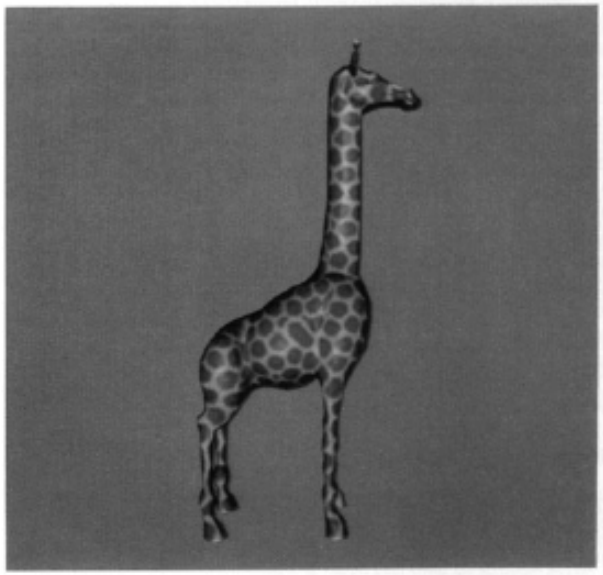
- The Voronoi region of a point P contains all points that are closest to P
- For each point we compute it's Voronoi neighbors and the lengths of the edges between adjacent Voronoi regions.

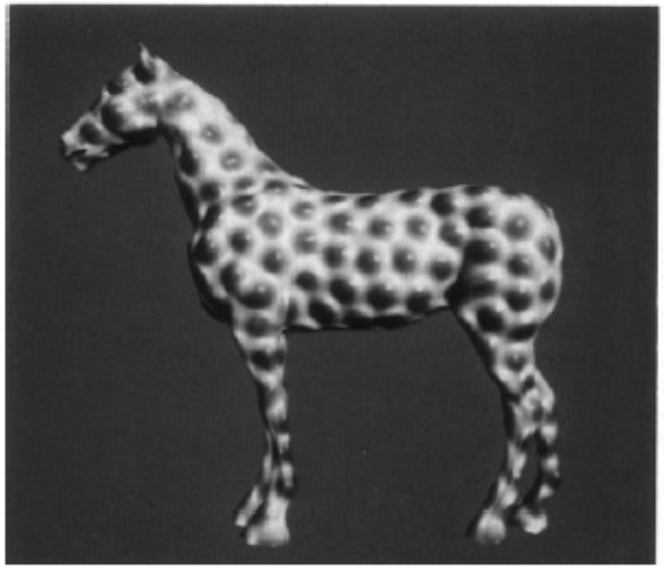
Reaction-Diffusion on a Mesh

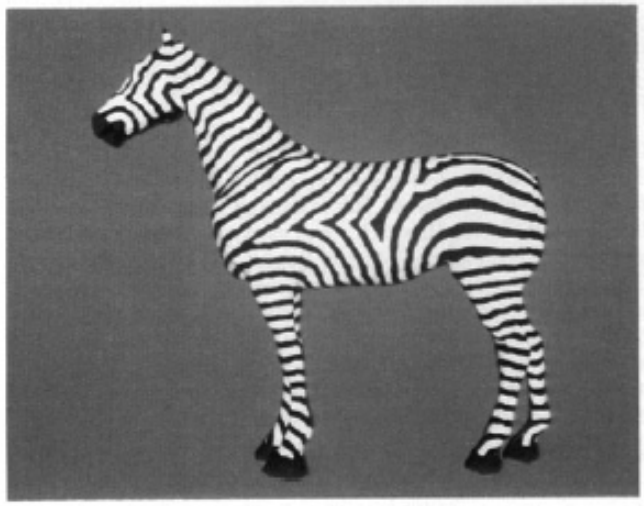
- The edge lengths become the diffusion coefficients (normalized to sum to 1)
  - Results in isotropic diffusion.
  - How can anisotropic diffusion be accomodated?

- The Laplacian of a cell is computed taking into account all of the cells neighbors. Each neighbor's contribution is weighted by the diffusion coefficient corresponding to the edge between the two cells.







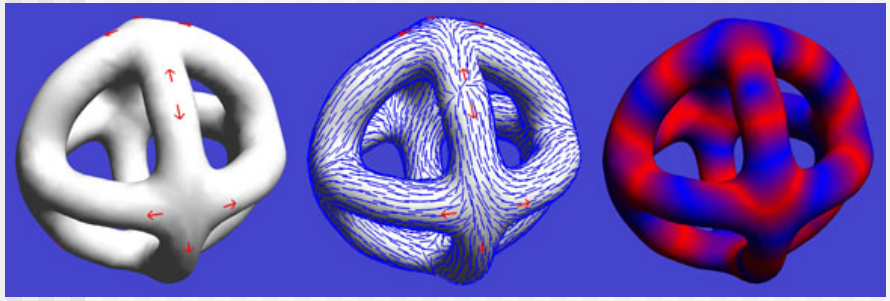


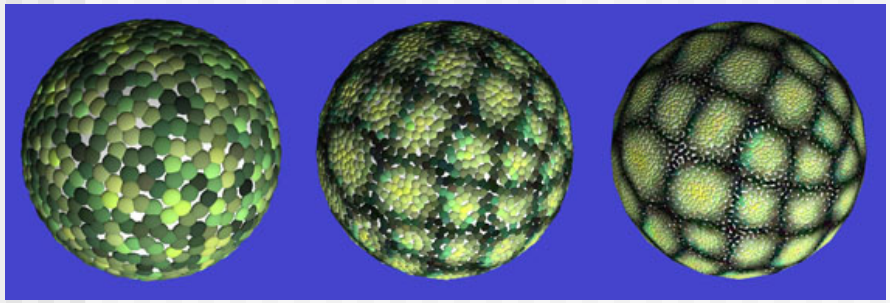
Texture Synthesis

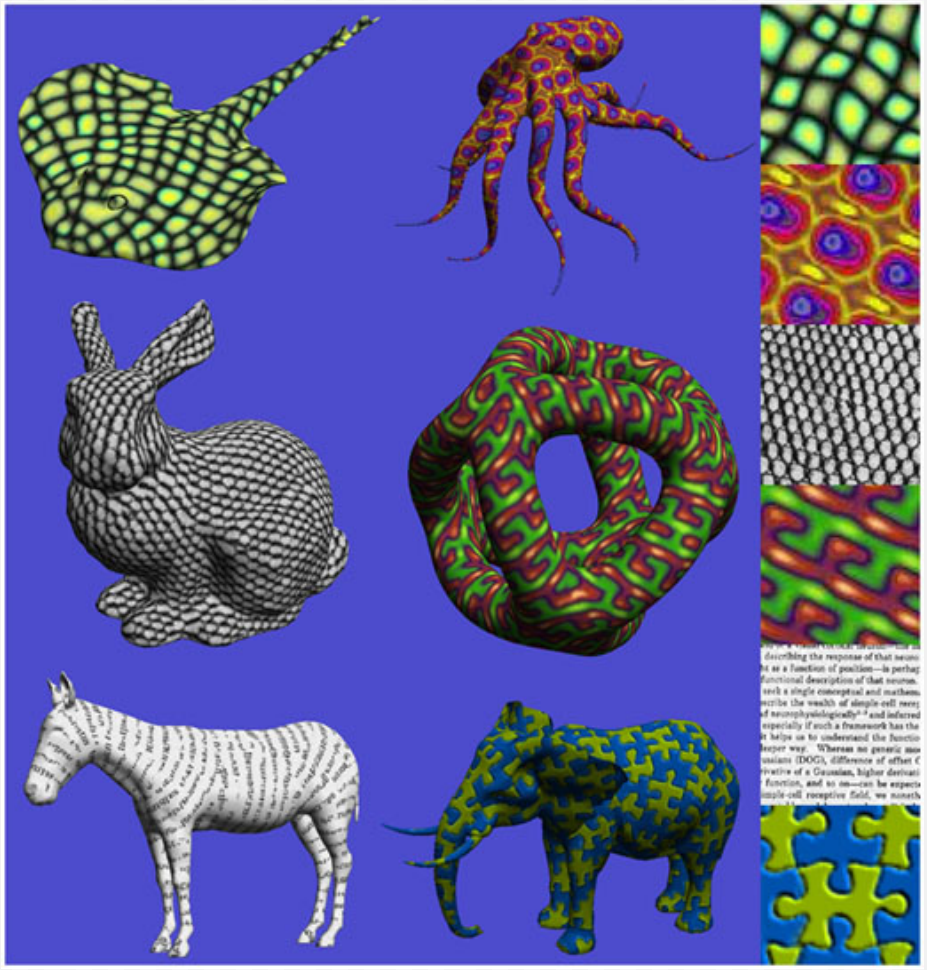
"Texture Synthesis on Surfaces", Greg Turk, Proc. SIGGRAPH 2001, pp. 347-354

- Spread small texture patch over an arbitrary multiresolution mesh
- Color vertices from coarse to fine, while following a user-defined vector field
- Determine color by examining the color of neighboring points and finding best match to a similar pixel neighborhood in texture sample

Texture Synthesis







Procedural Noise

Noise Functions

- Break up regularity
- Add realism
- Enable modeling of irregular phenomena and structures

White Noise

Sequence of random numbers

Uniformly distributed

Totally uncorrelated

- no correlation between successive values

Not desirable for texture generation

- Too sensitive to sampling problems
- Arbitrarily high frequency content

Ideal Noise for Texture Generation

Repeatable pseudorandom function of inputs

Known range [-1, 1]

Band-limited (maximum freq. about 1)

No obvious periodicities

Stationary and isotropic

- statistical properties invariant under translation and rotation

Lattice Noise

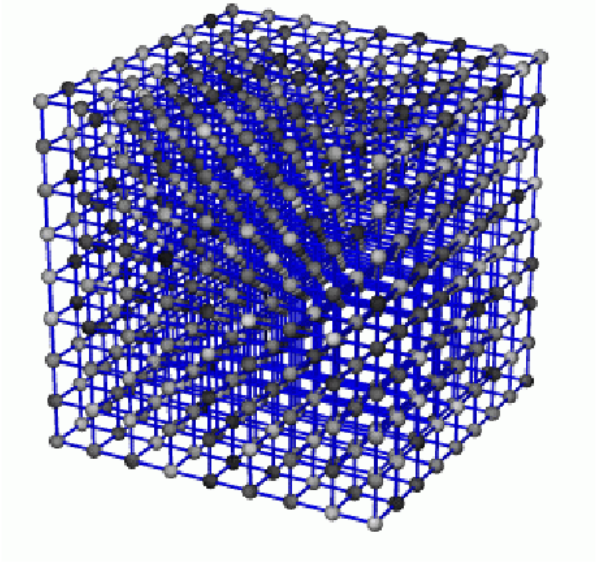
Low pass filtered version of white noise

- Random values associated with integer positions in noise space
- Intermediate values generated by some form of interpolation
- Frequency content limited by spacing of lattice

Noise and Turbulence

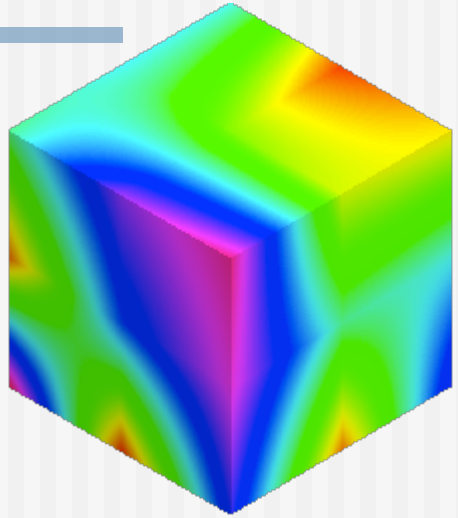
When we say we want to create an "interesting" texture, we usually dont care exactly what it looks like -- were only concerned with the overall appearance. We want to add random variations to our texture, but in a controlled way. Noise and turbulence are very useful tools for doing just that.

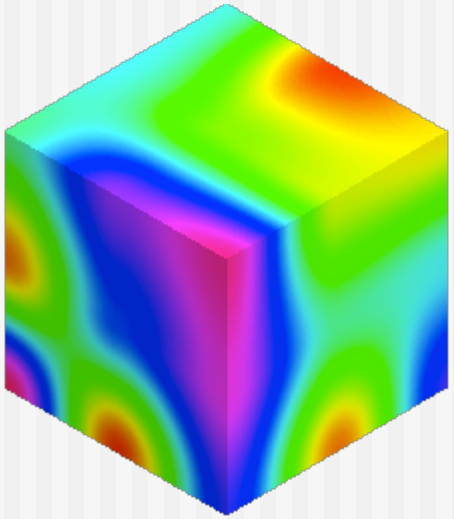
A *noise function* is a continuous functions that varies throughout space at a uniform frequency. To create a simple noise function, consider a 3D lattice, with a random value assigned to each triple of integer coordinates:



Making Noise

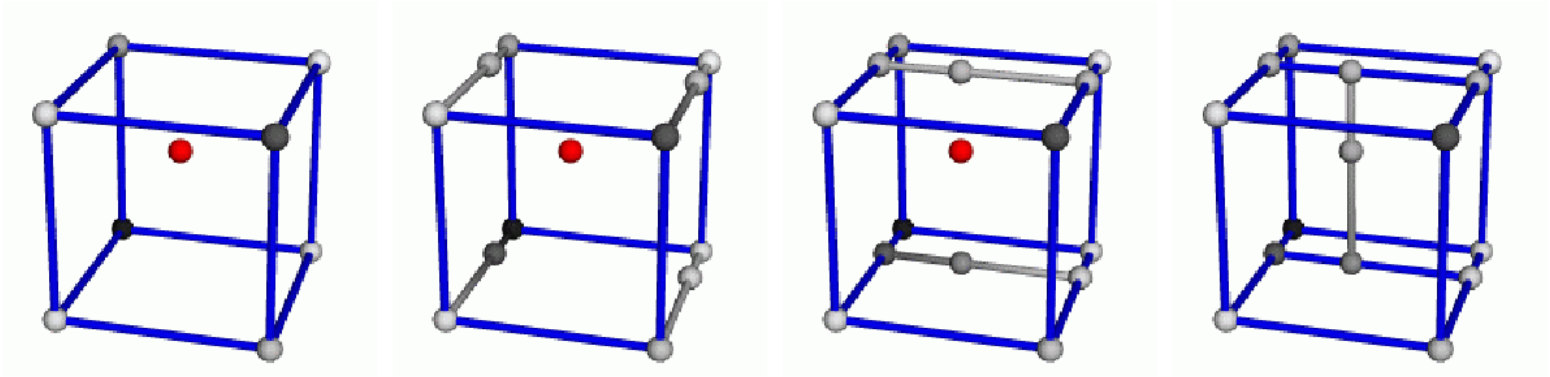
- Good:
  - Create 3-D array of random values
  - Trilinearly interpolate
- Better
  - Create 3-D array of random 3-vectors
  - Hermite interpolate





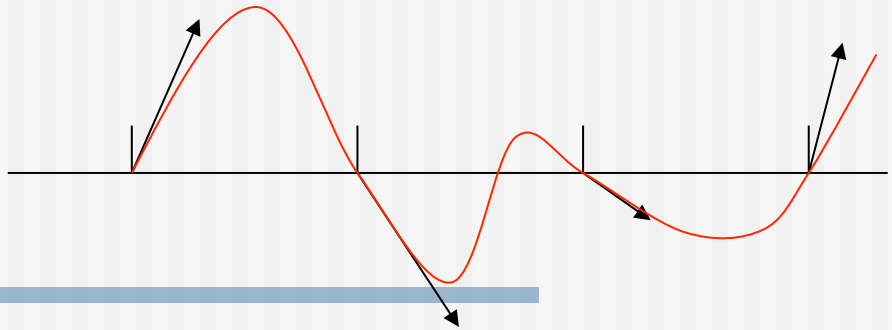
Interpolating Noise

To calculate the noise value of any point in space, we first determine which cube of the lattice the point is in. Next, we interpolate the desired value using the 8 corners of the cube:



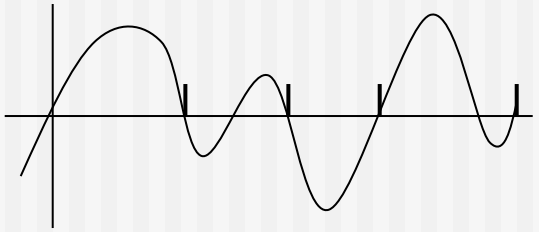
Trilinear interpolation is illustrated above. Higher-order interpolation can also be used.

Hermite Interpolation

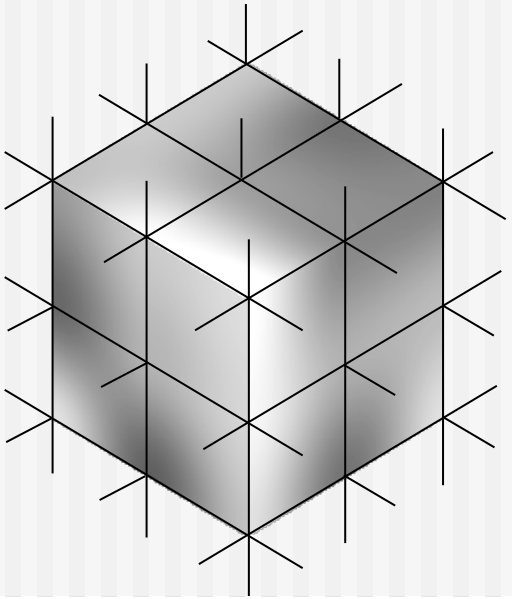


- Some cubic $h(t) = at^3 + bt^2 + ct + d$ s.t.
  - $h(0) = 0 \; (d = 0)$
  - $h(1) = 0 \; (a + b + c = 0)$
  - $h'(0) = r_0 \qquad (c = r_0)$
  - $h'(1) = r_1 \qquad (3a + 2b + r_0 = r_1)$
- Answer:
  - $h(t) = (r_0 + r_1)\, t^3 - (2r_0 + r_1)\, t^2 + r_0 t$

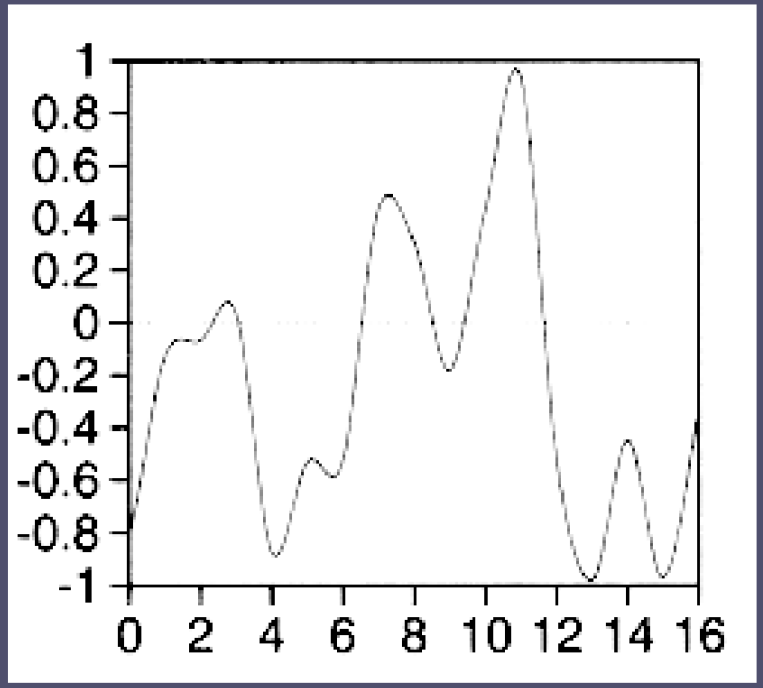
Noise Functions

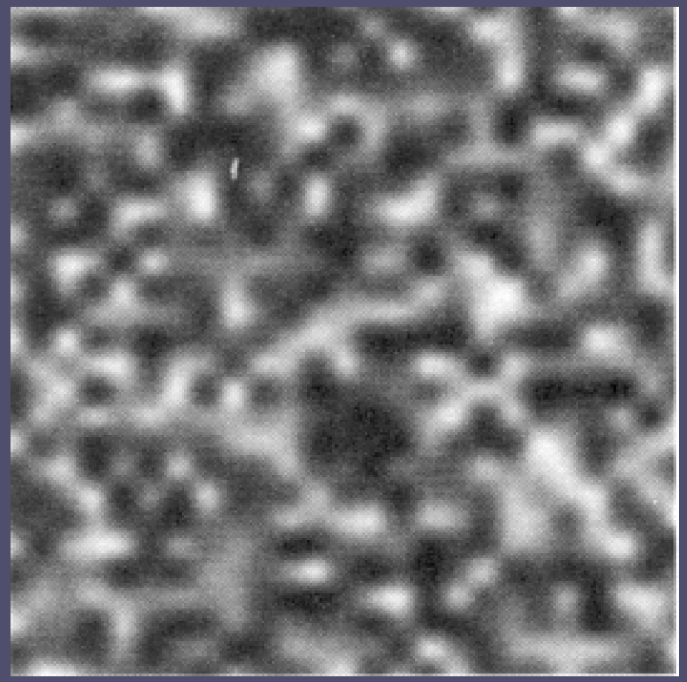


- Add “noise” to make textures interesting
- Perlin noise function $N(x,y,z)$
  - Smooth
  - Correlated
  - Bandlimited
- $N(x,y,z)$ returns a single random number in [-1,1]
- Gradient noise (like a random sine wave)
  - $N(x,y,z)=0$ for int x,y,z
  - Values not at lattice points are interpolated, using gradients as spline coefficients
- Value noise (another random sine wave)
  - $N(x,y,z)$=random for int x,y,z

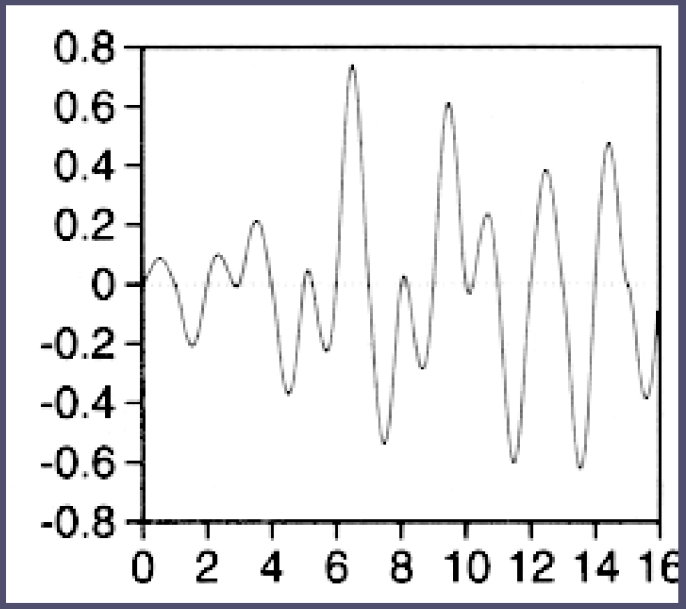


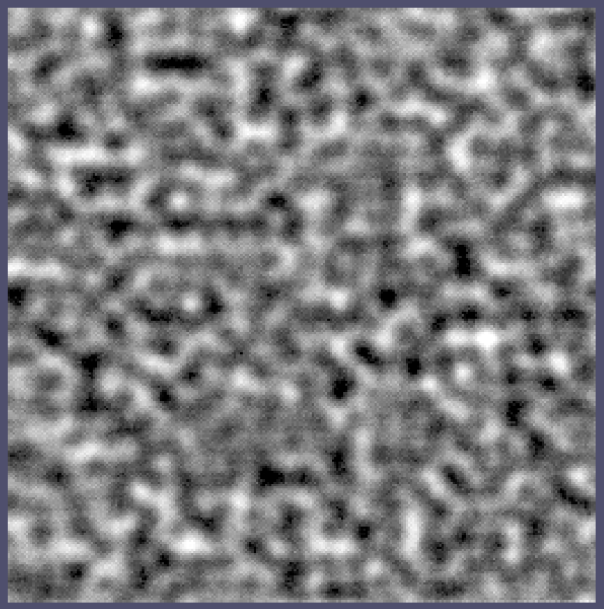
1D and 2D Value Noise





1D and 2D Gradient Noise





Value vs. Gradient Noise

Both noises have limited frequencies

Value noise slightly simpler to compute

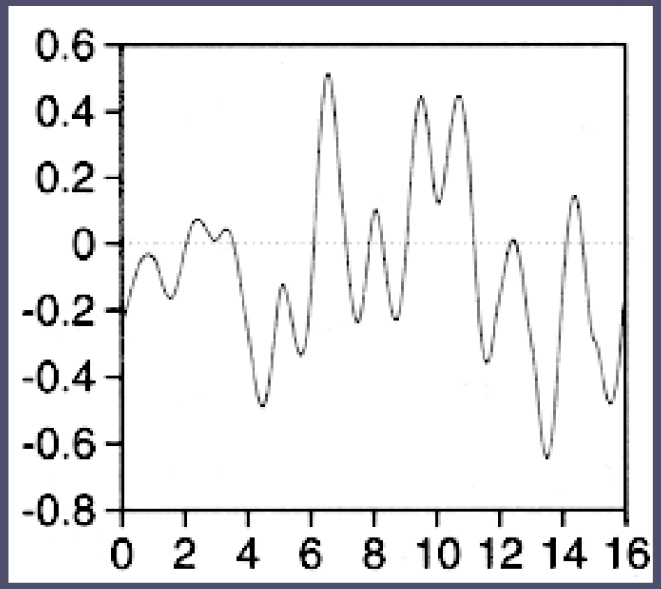
Gradient noise has most of the energy in the higher frequencies

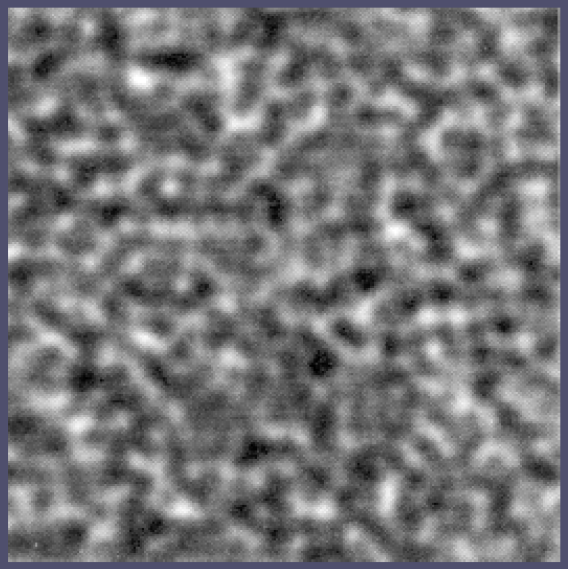
- forced zero crossings

Gradient noise has regularity because of zero crossings

Value Gradient Noise

Weighted sum of value and gradient noises





Example - Perturbed Texture

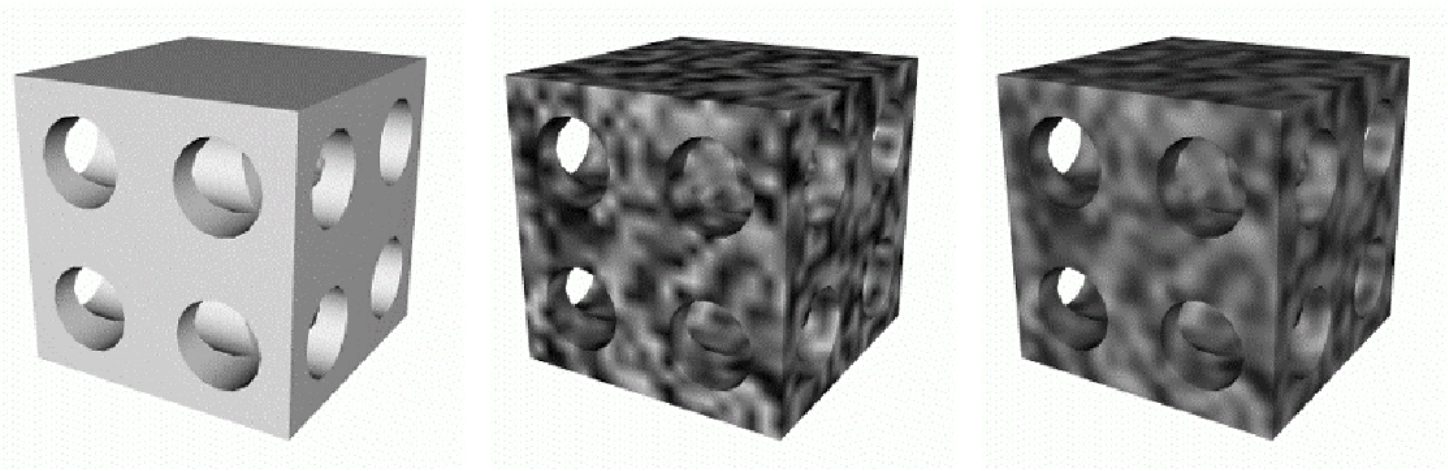
Use noise function to apply perturbation to texture coordinates

Look up image texture (or generate procedural texture) using modified coordinates

Example - Perturbed Texture

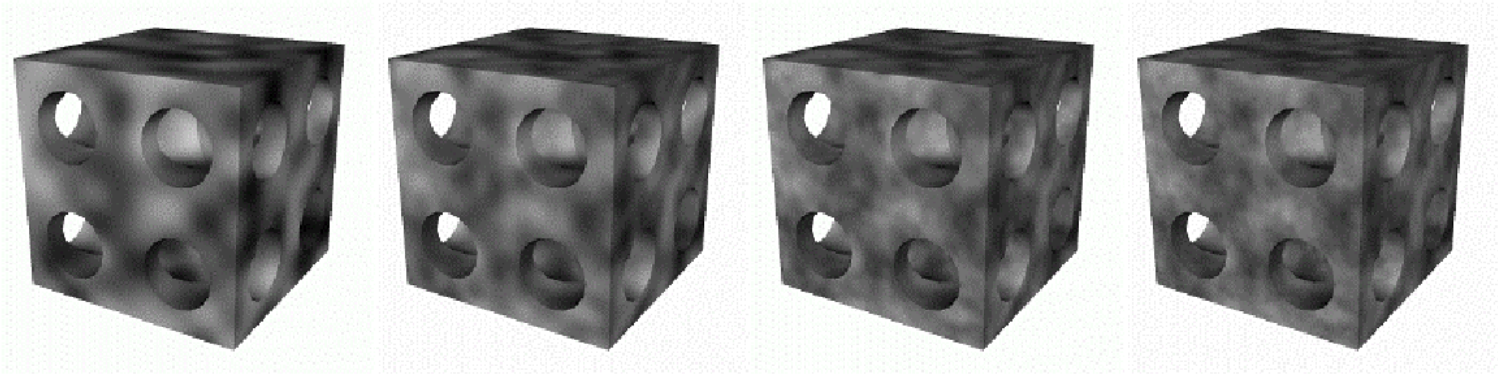
Evaluating Noise

Since noise is a 3D function, we can evaluate it at any point we want. We dont have to worry about about mapping the noise to the object, we just use (x, y, z) at each point as our 3D texture coordinates! It is as if we are carving our object out of a big block of noise.



Turbulence

Noise is a good start, but it looks pretty ugly all by itself. We can use noise to make a more interesting function called turbulence. A simple turbulence function can be computed by summing many different frequencies of noise functions:



One Frequency *Two Frequencies* *Three Frequencies* *Four Frequencies*

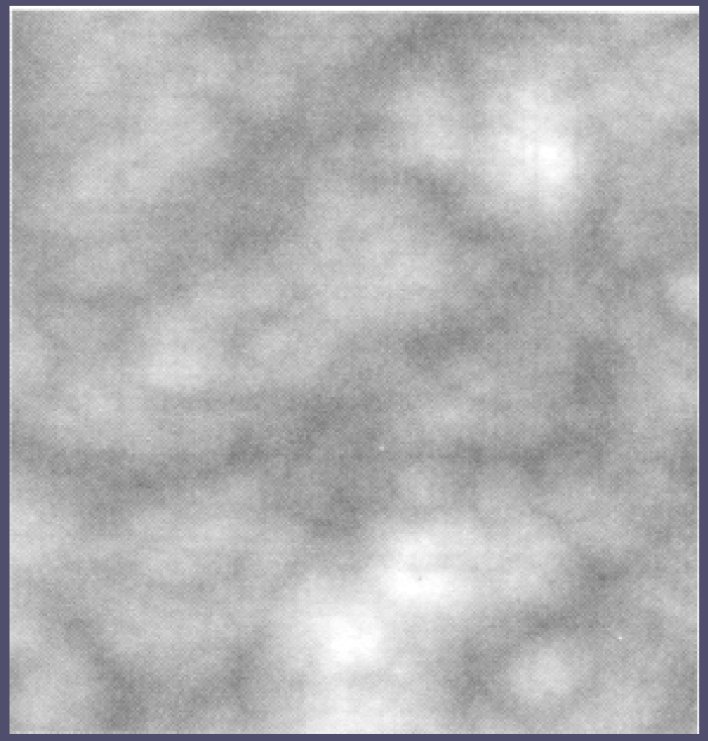
Now were getting somewhere. But even turbulence is rarely used all by itself. We can use turbulence to build even more fancy 3D textures...

Turbulence

```
float turbulence(point Q)
{
   float value = 0;
   for (f= MINFREQ; f < MAXFREQ; f *= 2)
        value += abs(noise(Q*f))/f;
   return value;
}
```

(in practice, don't use a round number like 2)

Turbulence



3D Procedural Texture Maps

Texture Mapping

- Maps image onto surface
- Depends on a surface parameterization (s,t)
  - Difficult for surfaces with many features
  - May include distortion
  - Not necessarily 1:1

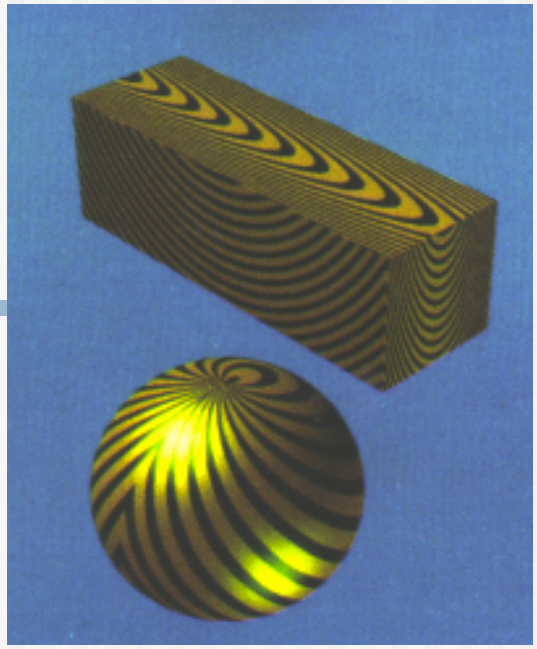
Kettle, by Mike Miller

Solid Texturing

- Uses 3-D texture coordinates (s,t,r)
- Can let $s = x$, $t = y$ and $r = z$
- No need to parameterize surface
- No worries about distortion
- Objects appear sculpted out of solid substance

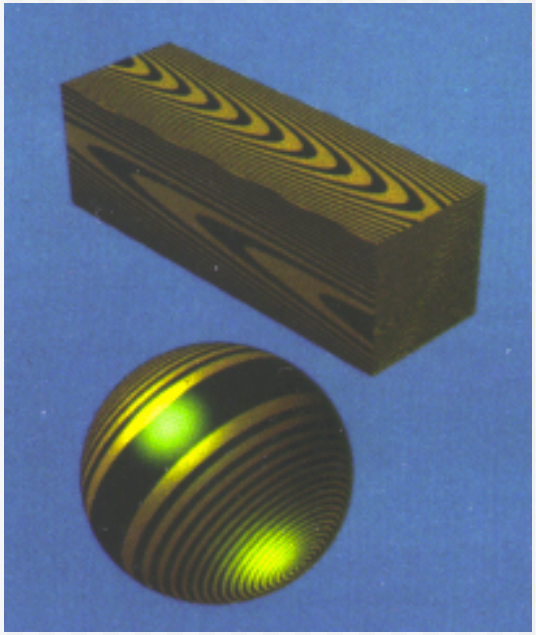
Surface Texture

features don't line up



Solid Texture

features do line up

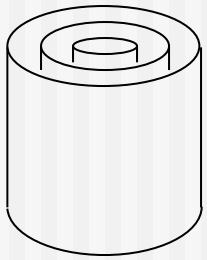


Darwyn Peachey, 1985

Solid Texture Problems

- How can we deform an object without making it swim through texture?
- How can we efficiently store a procedural texture?

Procedural Texturing



$f(s,t,r) = s^2 + t^2$

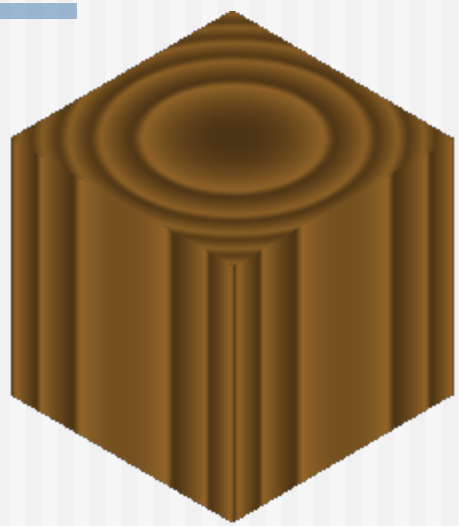
Woodmap(f)

- Texture map is a function
- Write a procedure to perform the function
  - input: texture coordinates - s,t,r
  - output: color, opacity, shading
- Example: Wood
  - Classification of texture space into cylindrical shells

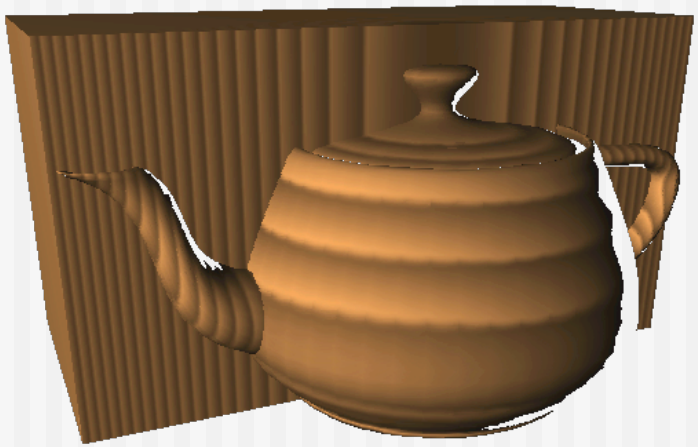
$$f(s,t,r) = s^2 + t^2$$

  - Outer rings closer together, which simulates the growth rate of real trees
  - Wood colored color table
    - Woodmap(0) = brown "earlywood"
    - Woodmap(1) = tan "latewood"

$$\text{Wood}(s,t,r) = \text{Woodmap}(f(s,t,r) \bmod 1)$$



Wood(s,t,r)



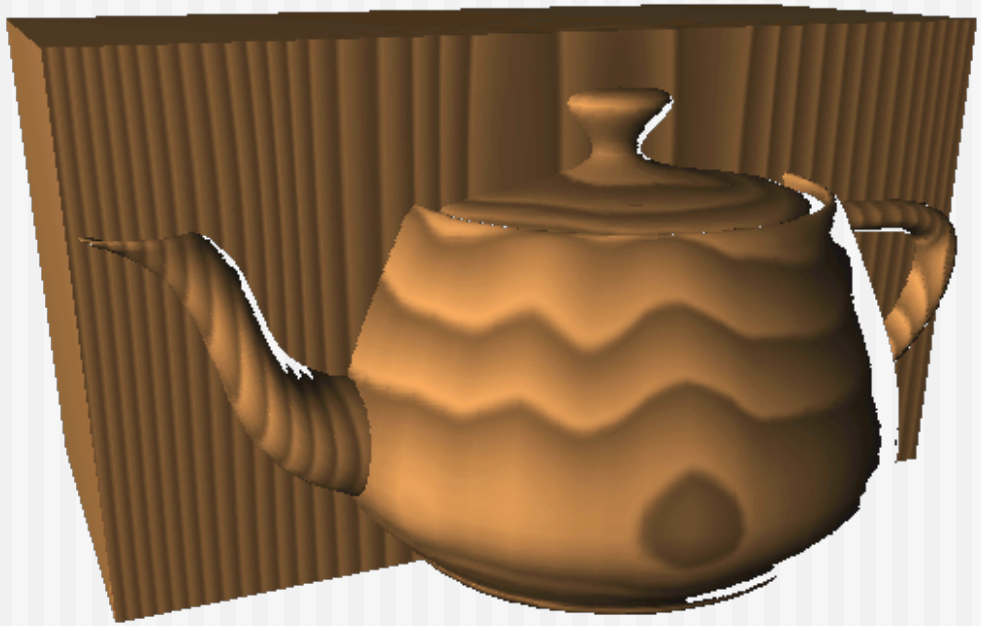
Using Noise

- Add noise to cylinders to warp wood
  - Wood($s^2 + t^2 + N(s,t,r)$)
- Controls
  - Amplitude: power of noise effect

    $a\ N(s,\ t,\ r)$
  - Frequency: coarse vs. fine detail

    $N(f_s\ s,\ f_t\ t,\ f_r\ r)$
  - Phase: location of noise peaks

    $N(s + \phi_s,\ t + \phi_t,\ r + \phi_r)$

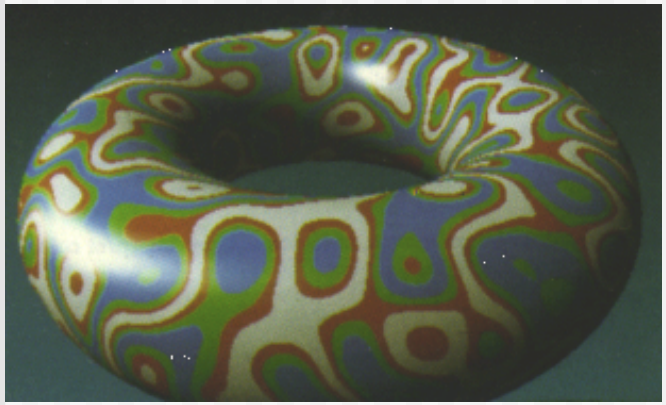


An Image Synthesizer

K. Perlin, Proc. SIGGRAPH '85, July 1985, p. 287

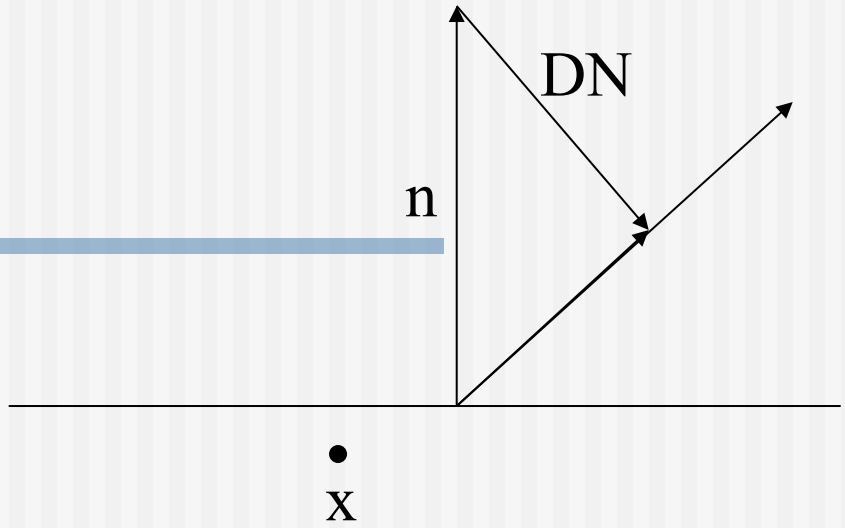
- Created an interpreted, high level language for describing 2D/3D textures
  - Easy to program
  - No compilation necessary
  - Define a set of intrinsic procedural primitives
    - More efficient, reusable, flexible, rich set of texturing tools
  - High level operations (arithmetic, comparisons)
  - Include looping and branching
  - High level, intrinsic types like vectors
- New primitives
  - Noise, Dnoise, Turbulence, Composition

Colormap Donuts



- Spotted donut
  - Gray(N(40*x,40*y,40*z))
  - Gray() - ramp colormap
  - Single 40Hz frequency
- Bozo donut
  - Bozo(N(4*x,4*y,4*z))
  - Bozo() - banded colormap
  - Cubic interpolation means contours are smooth

Bump Mapped Donuts



n += DNoise(x,y,z); normalize(n);

- DNoise(*s*,*t*,*r*) = ∇Noise(*s*,*t*,*r*)
- Bumpy donut
  - Same procedural texture as spotted donut
  - Noise replaced with Dnoise and bump mapped

Composite Donuts

- Stucco donut
  - Noise(x,y,z)*DNoise(x,y,z)
  - Noisy direction
  - noisy amplitude

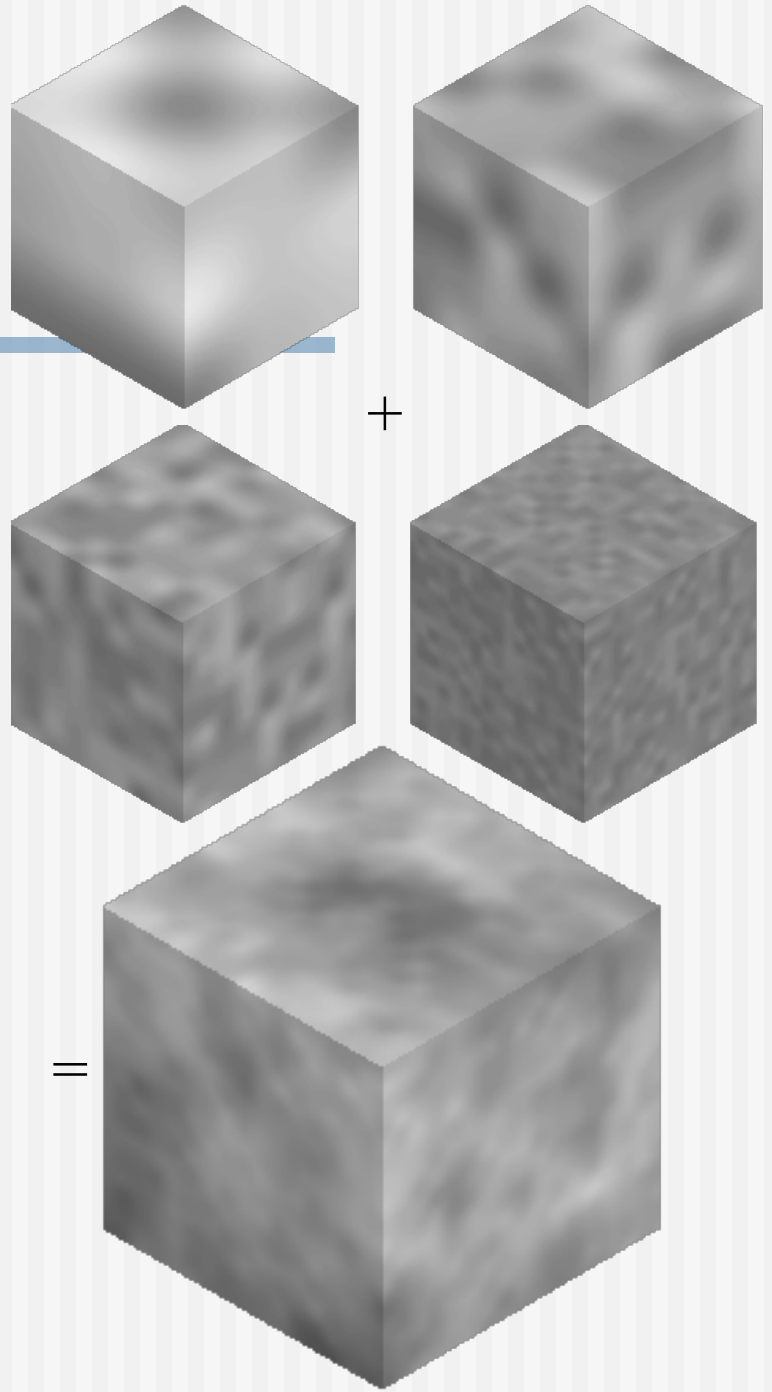
- Fleshy donut
  - Same texture
  - Different colormap

Fractional Brownian Textures

- $1/f^{\beta}$ distribution
- Roughness parameter β
  - Ranges from 1 to 3
  - $\beta = 3$ - smooth, not flat, still random
  - $\beta = 1$ - rough, not space filling, but thick
- Construct using spectral synthesis

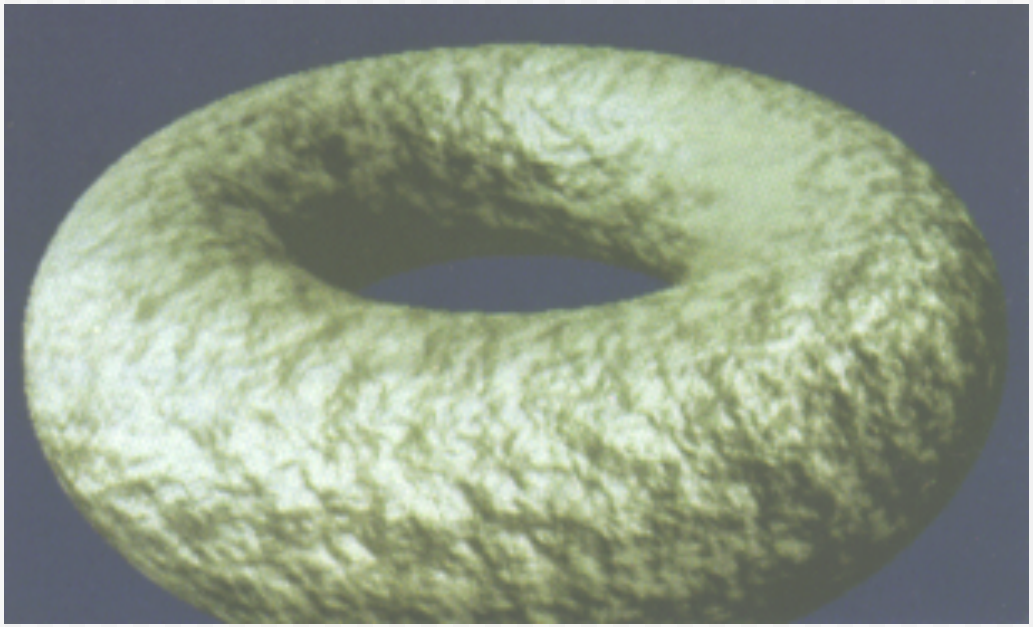
$$f(\mathbf{s}) = \sum_{i=1}^{4} 2^{-i\beta} n(2^i \mathbf{s})$$

  - Add several octaves of noise function
  - Scale amplitude appropriately



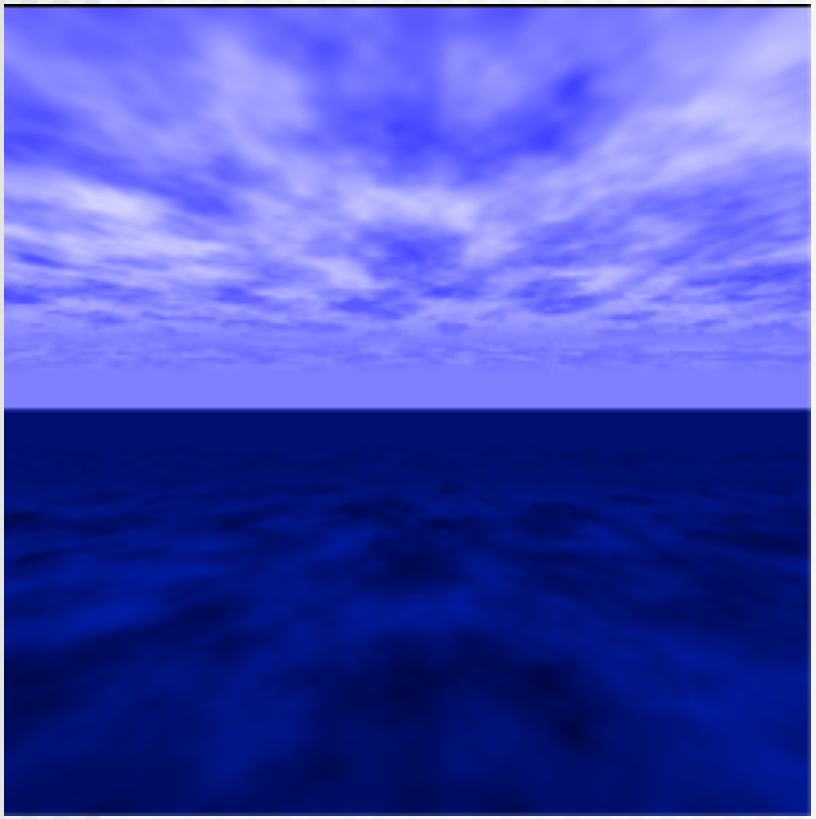
Fractal Bump-Mapped Donut

```
fbm(beta) {
    val = 0; vec = (0,0,0);
    for (i = 0; i < octaves; i++) {
        val += Noise(2^i *x, 2^i *y, 2^i *z)/pow(2,i*beta);
        vec += DNoise(2^i *x, 2^i *y, 2^i *z)/pow(2,i*beta);
    }
    return vec or val;
}
```



Clouds
Water

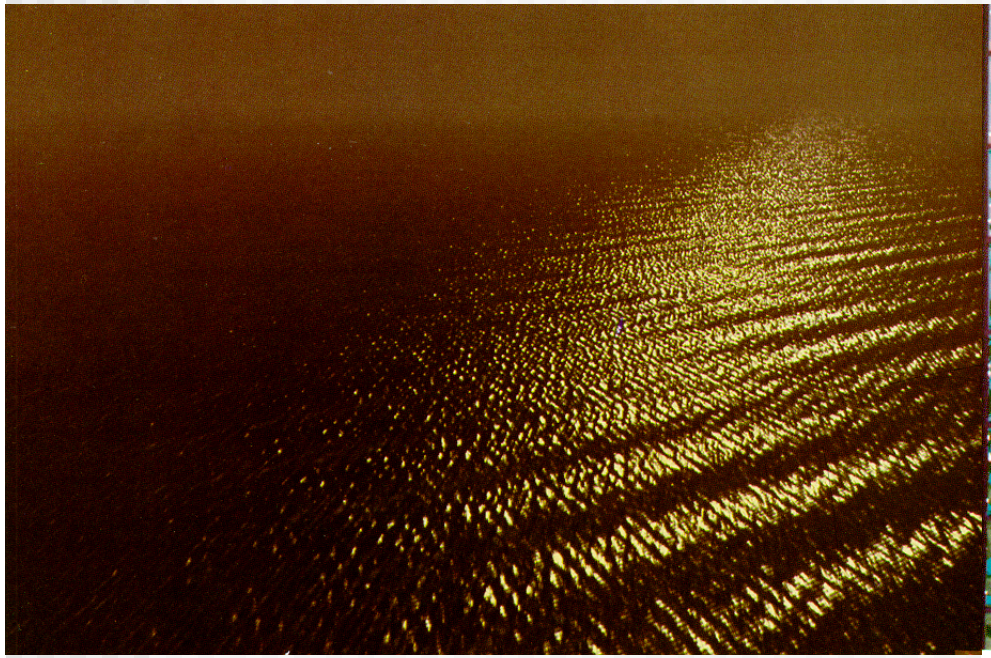
$$f(\mathbf{s}) = \sum_{i=1}^{4} 2^{-i} n(2^i \mathbf{s})$$

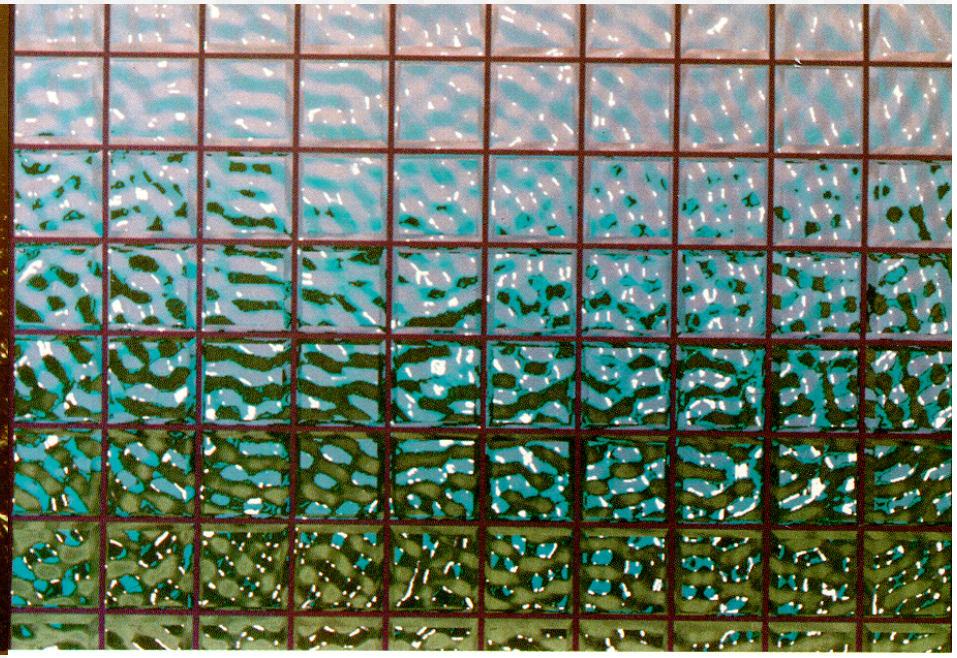


Gunther Berkus via Mojoworld

Water

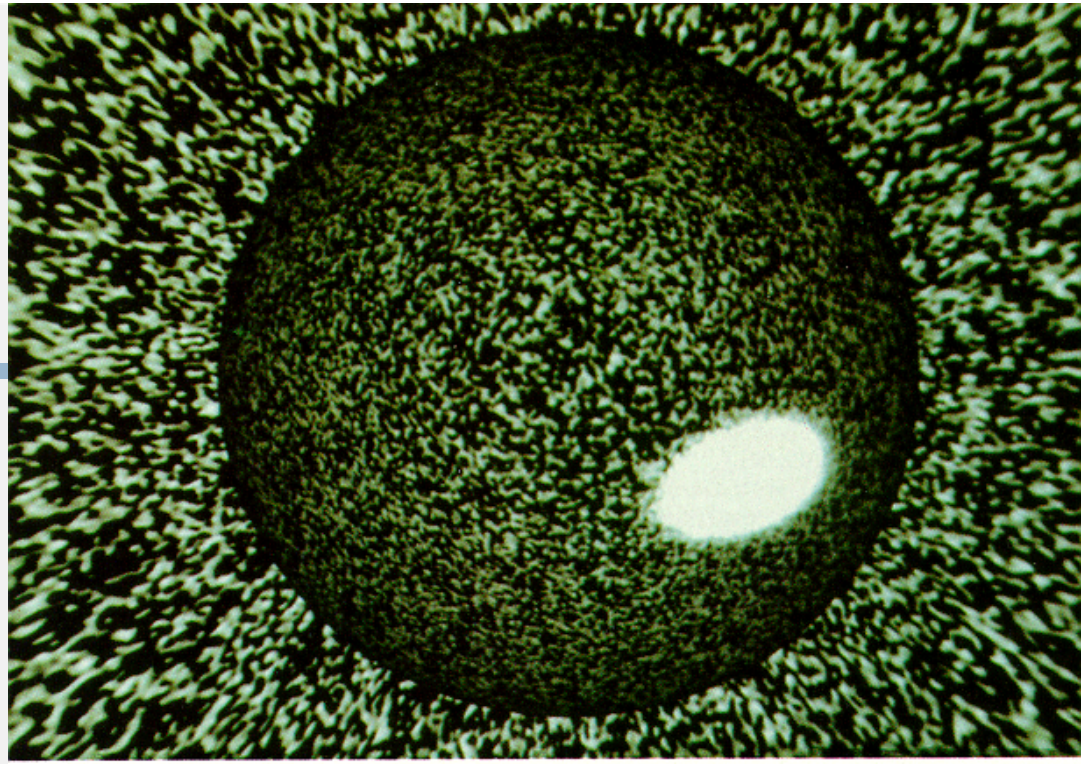
- Created by making a wave with Dnoise
- Create multiple waves to simulate water

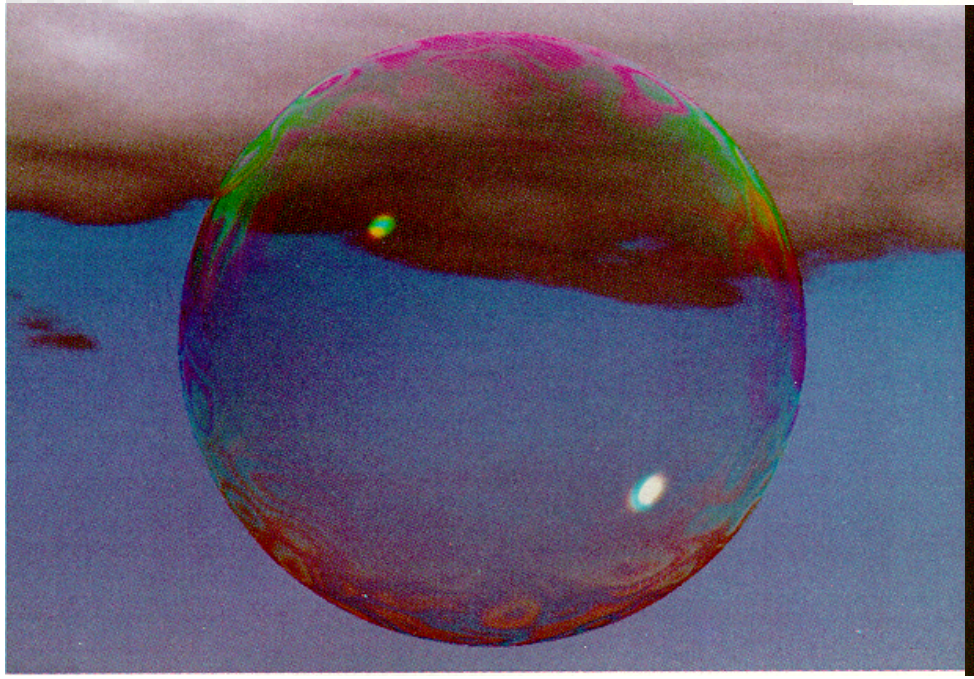


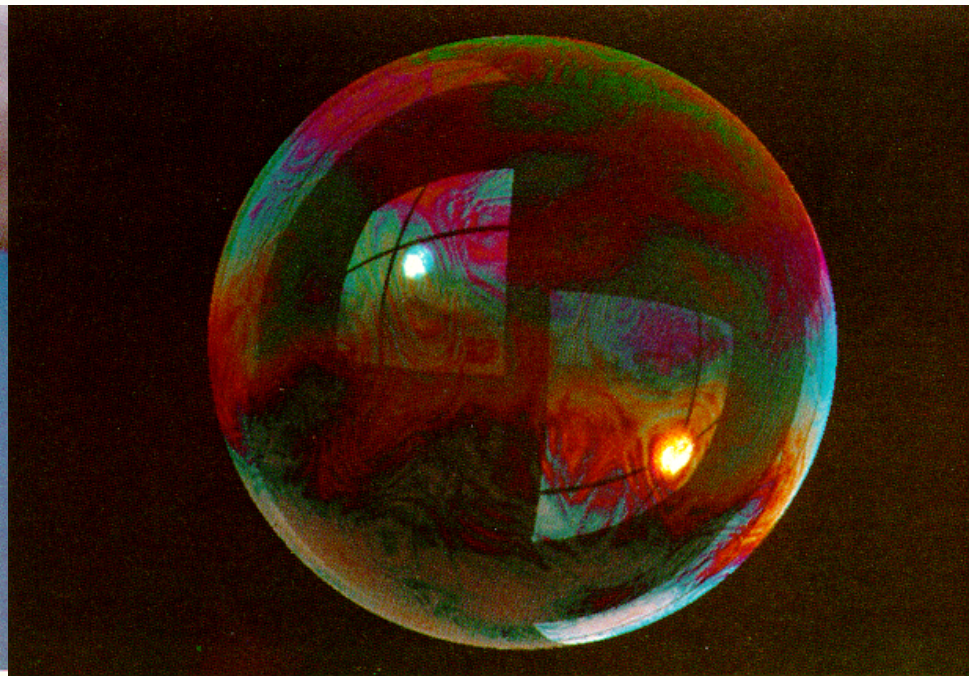


Clouds & Bubbles

Created using turbulence, reflection, and refraction



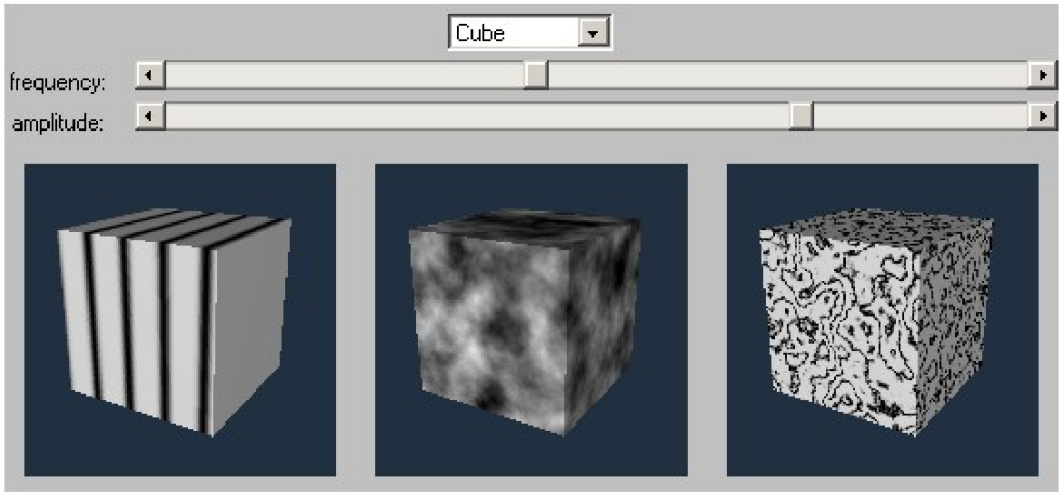




Marble Example

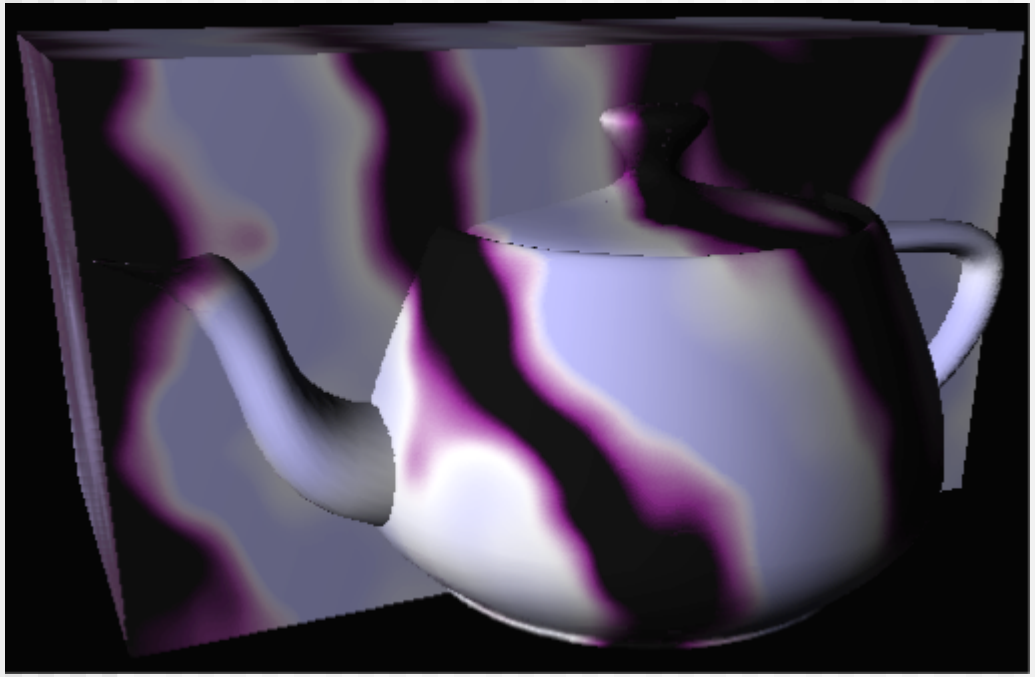
We can use turbulence to generate beautiful 3D marble textures, such as the marble vase created by Ken Perlin. The idea is simple. We fill space with black and white stripes, using a sine wave function. Then we use turbulence at each point to distort those planes. By varying the frequency of the sin function, you get a few thick veins, or many thin veins. Varying the amplitude of the turbulence function controls how distorted the veins will be.

$$\text{Marble} = \sin(f * (x + A*\text{Turb}(x,y,z)))$$



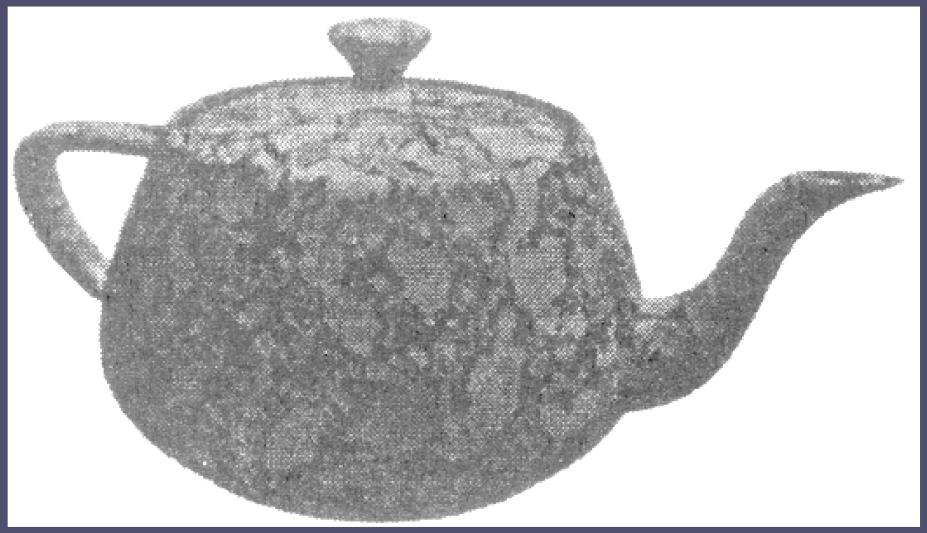
Marble

$$f(s,t,r) = r + \sum_{i=1}^{4} 2^{-i} n(2^i s, 2^i t, 2^i r))$$



Ken Perlin, 1985

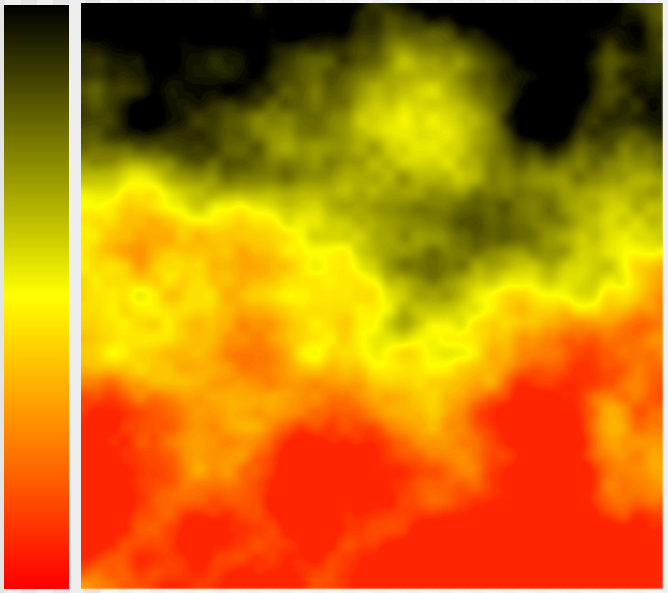
Example - Blue Marble

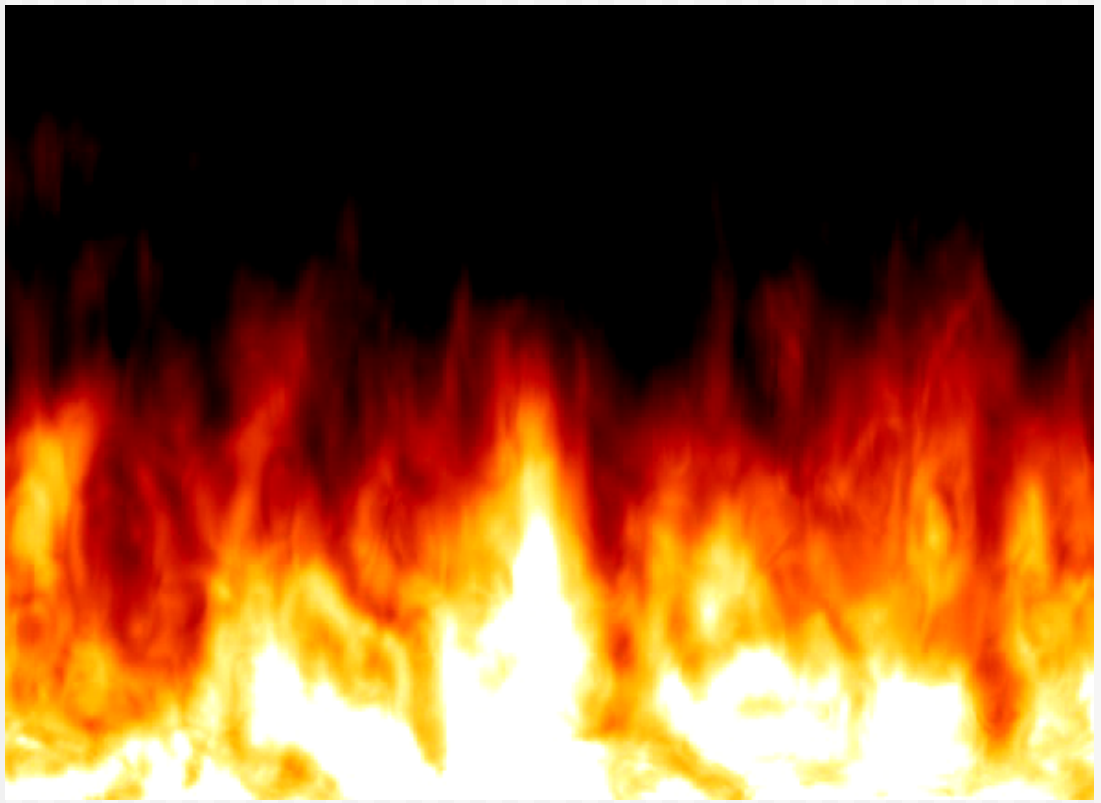


Marble vase (right) from Foley, van Dam, Feiner, and Hughes. *Computer Graphics: Principles and Practice*.

Fire

See “Texture and Modeling: A Procedural Approach,” Ebert et al., Morgan Kaufmann , 2003



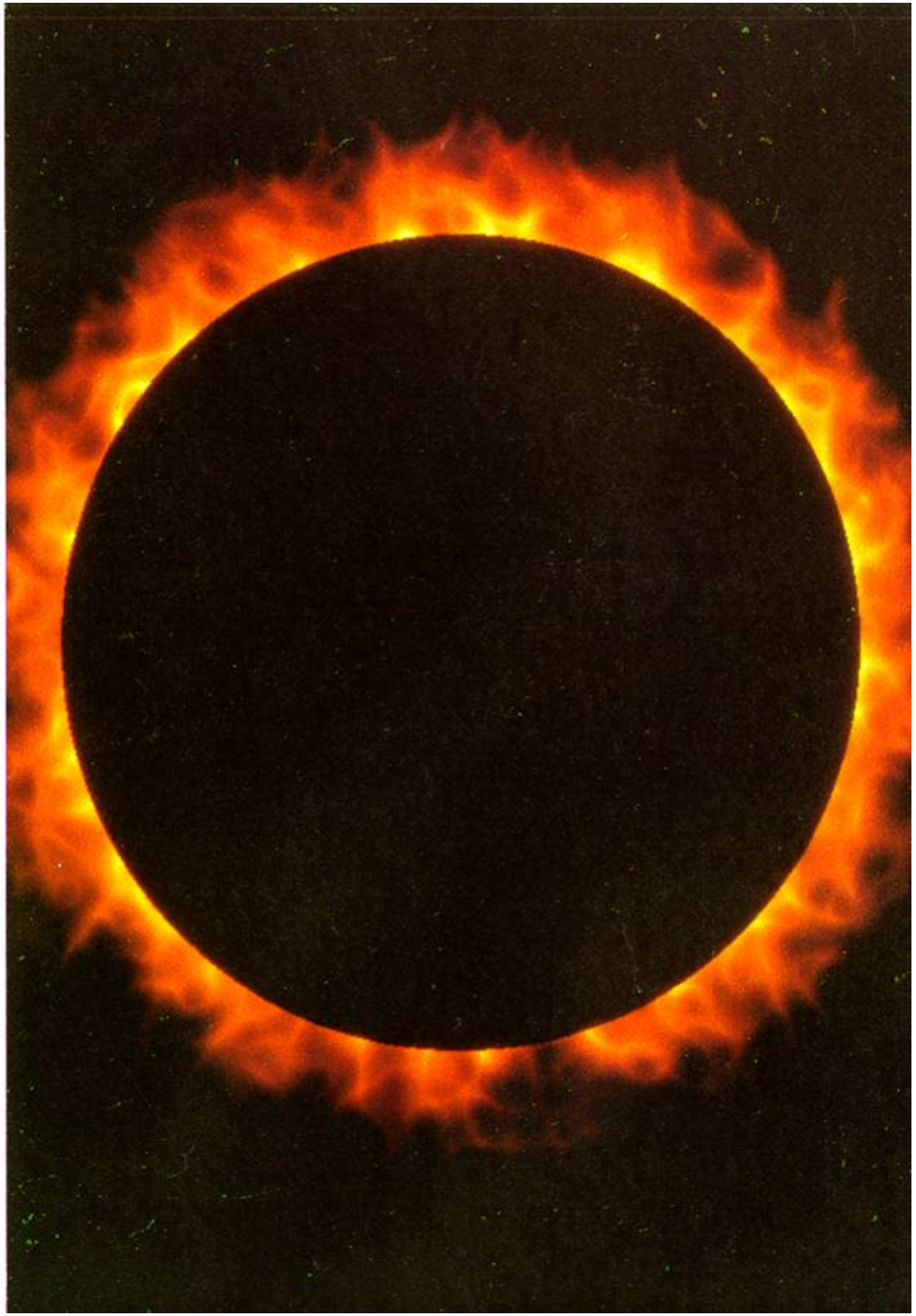


Ken Musgrave

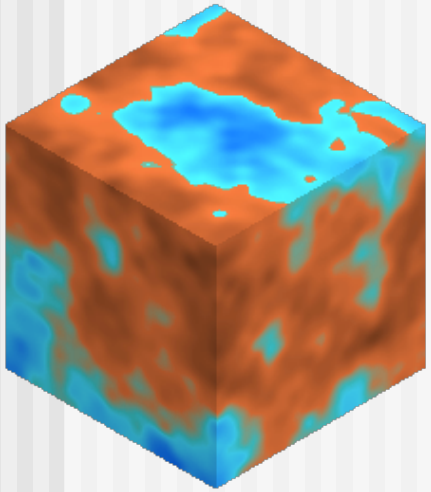
Fire (Corona)

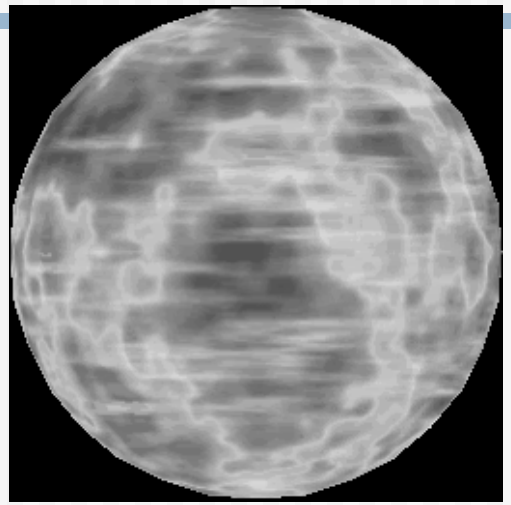
- Apply turbulence when flow exists
- Fire in a solar corona is flowing away from the center

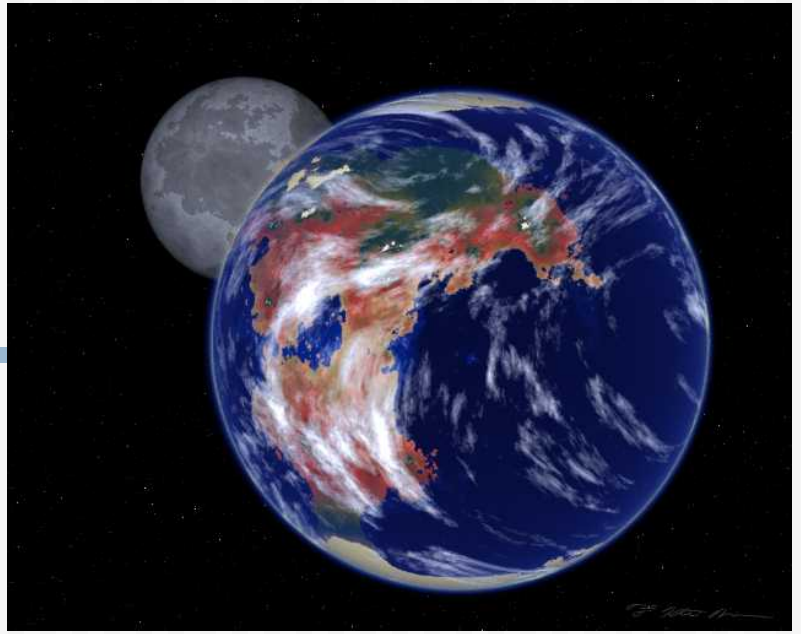
```
Function corona(v)
    radius = norm(v)
    dr = turbulence(v)
    return color_of_corona(radius + dr)
```

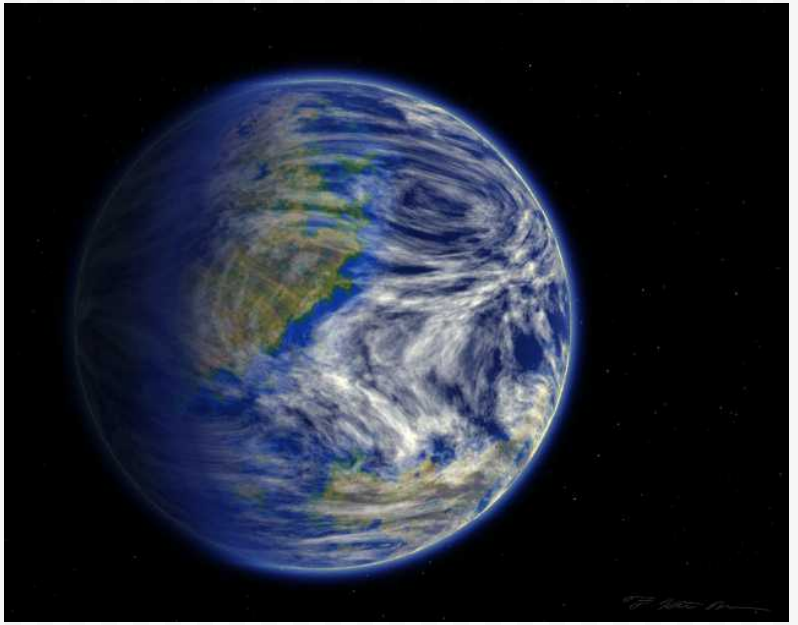


Planets









Ken Musgrave