Hierarchical Modeling

CS 432 Interactive Computer Graphics

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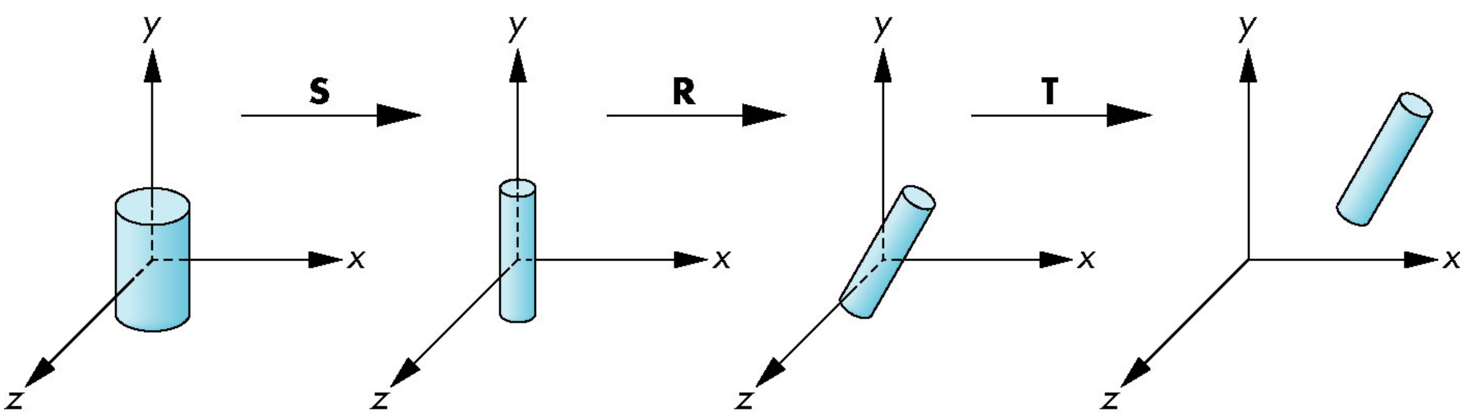
Objectives

- Examine the limitations of linear modeling
  - Symbols and instances
- Introduce hierarchical models
  - Articulated models
  - Robots
- Introduce Tree and DAG models

Instance Transformation

- Start with a prototype object (a *symbol*)
- Each appearance of the object in the model is an *instance*
  - Must scale, orient, position
  - Defines instance transformation



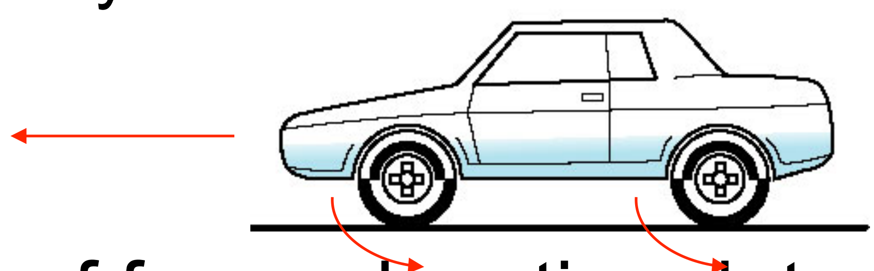
Symbol-Instance Table

Can store a model by assigning a number to each symbol and storing the parameters for the instance transformation

Symbol	Scale	Rotate	Translate
1	s_x, s_y, s_z	$\theta_x, \theta_y, \theta_z$	d_x, d_y, d_z
2			
3			
1			
1			
.			
.			

Relationships in Car Model

- Symbol-instance table does not show relationships between parts of model
- Consider model of car
  - Chassis + 4 identical wheels
  - Two symbols



- Rate of forward motion determined by rotational speed of wheels

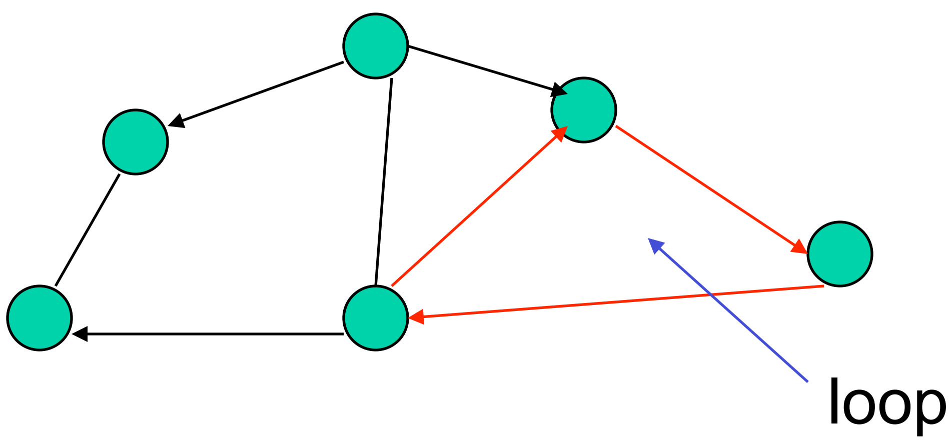
Structure Through Function Calls

```
car(speed)
{
     chassis()
     wheel(right_front);
     wheel(left_front);
     wheel(right_rear);
     wheel(left_rear);
}
```

- Fails to show relationships well
- Look at problem using a graph

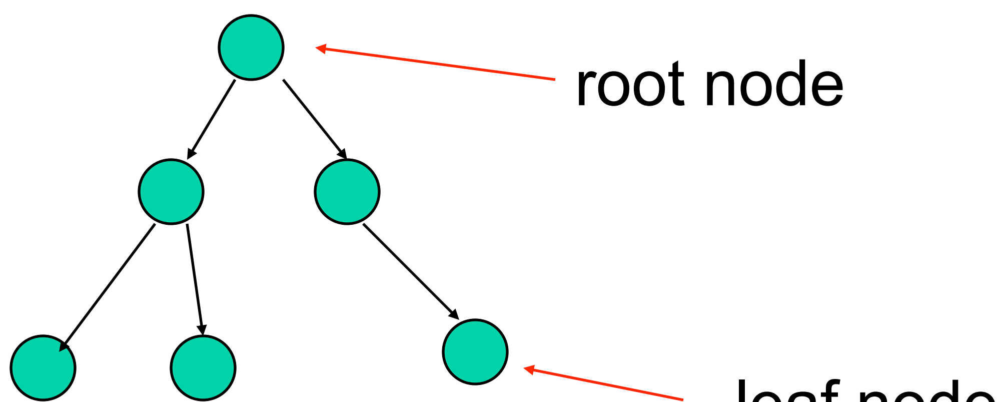
Graphs

- Set of *nodes* and *edges (links)*
- Edge connects a pair of nodes
  - Directed or undirected
- *Cycle*: directed path that is a loop

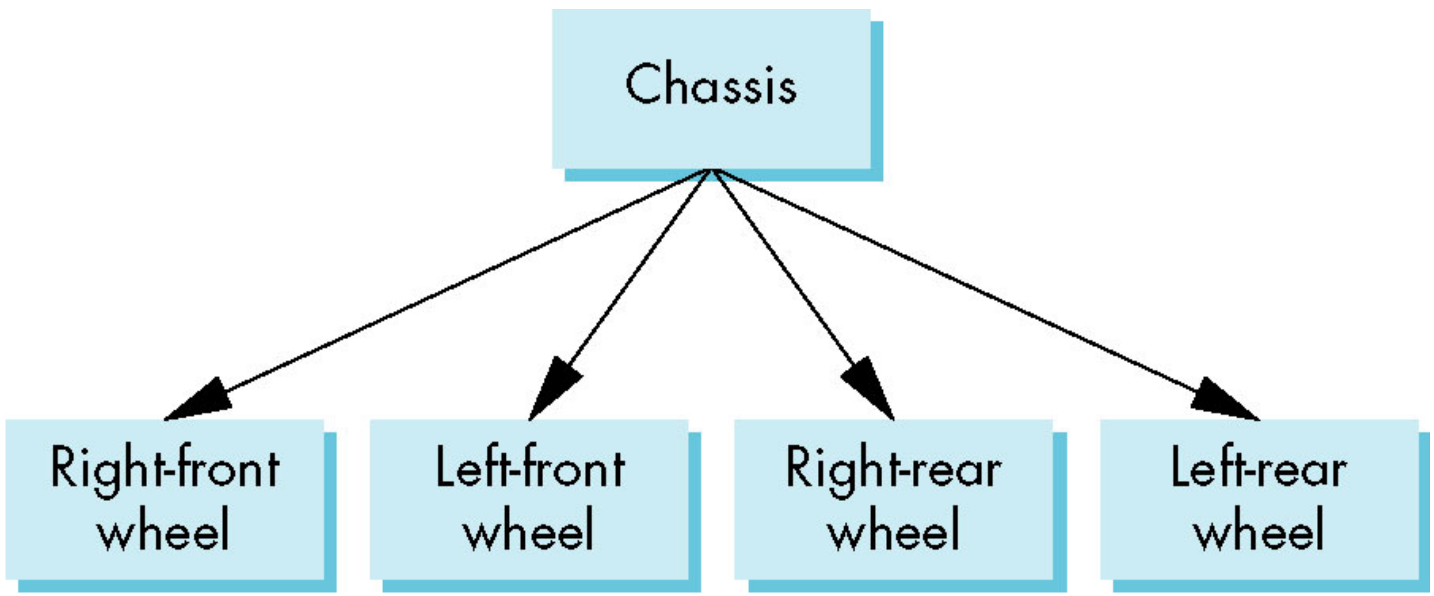


Tree

- Graph in which each node (except the root) has exactly one parent node
  - May have multiple children
  - Leaf or terminal node: no children

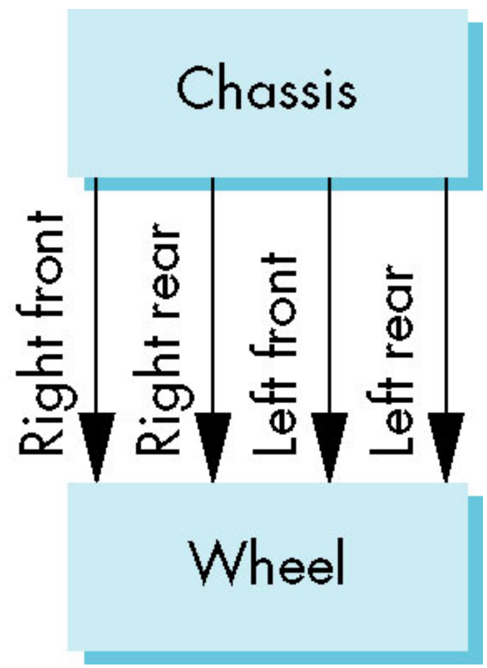


Tree Model of Car



DAG Model

- If we use the fact that all the wheels are identical, we get a *directed acyclic graph*
  - Not much different than dealing with a tree

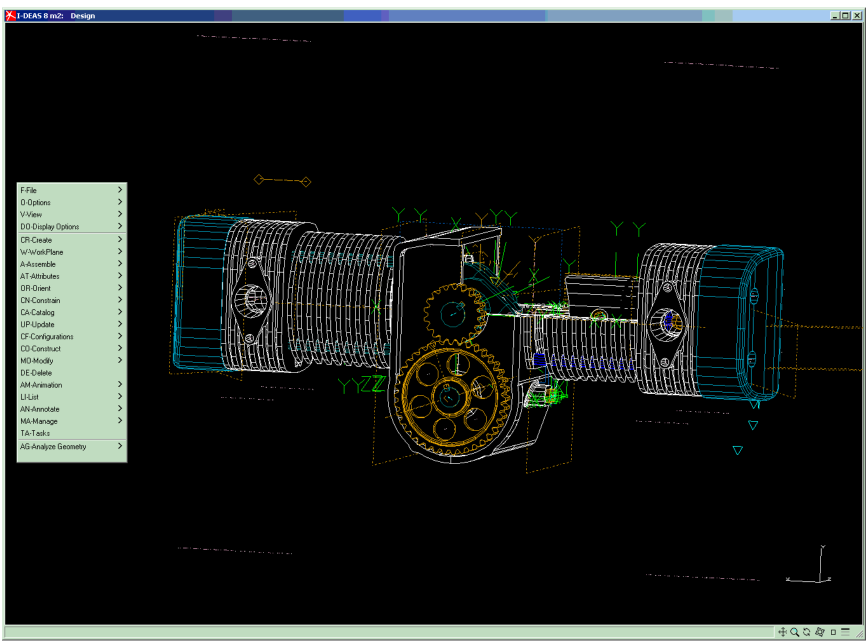


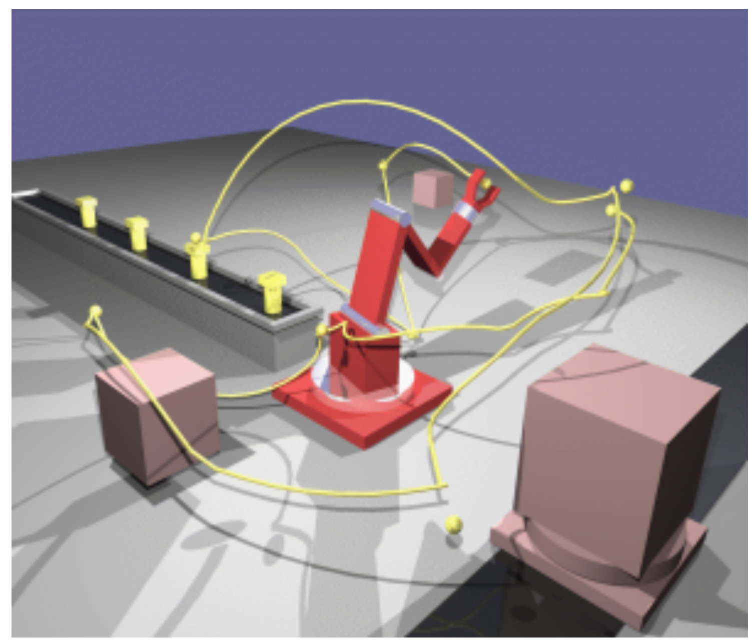
Modeling with Trees

- Must decide what information to place in nodes and what to put in edges
- Nodes
  - What to draw
  - Pointers to children
- Edges
  - May have information on incremental changes to transformation matrices (can also store in nodes)

Transformations to Change Coordinate Systems

- Issue: the world has many different relative frames of reference
- How do we transform among them?
- Example: CAD Assemblies & Animation Models





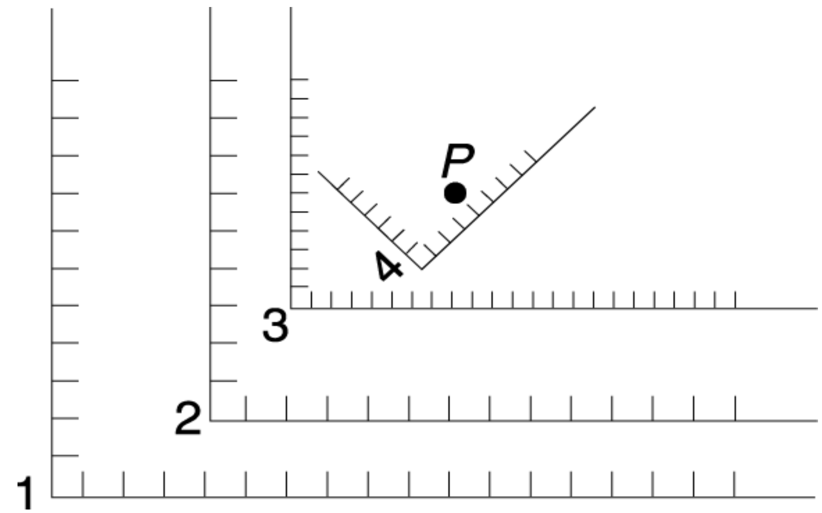
Transformations to Change Coordinate Systems

- 4 coordinate systems
  1 point *P*

$$M_{1\leftarrow 2} = T(4,2)$$

$$M_{2\leftarrow 3} = T(2,3) \bullet S(0.5,0.5)$$

$$M_{3\leftarrow 4} = T(6.7,1.8) \bullet R(45^\circ)$$



$$M_{i\leftarrow k} = M_{i\leftarrow j} \cdot M_{j\leftarrow k}$$

1994 Foley/VanDam/Finer/Huges/Phillips ICG

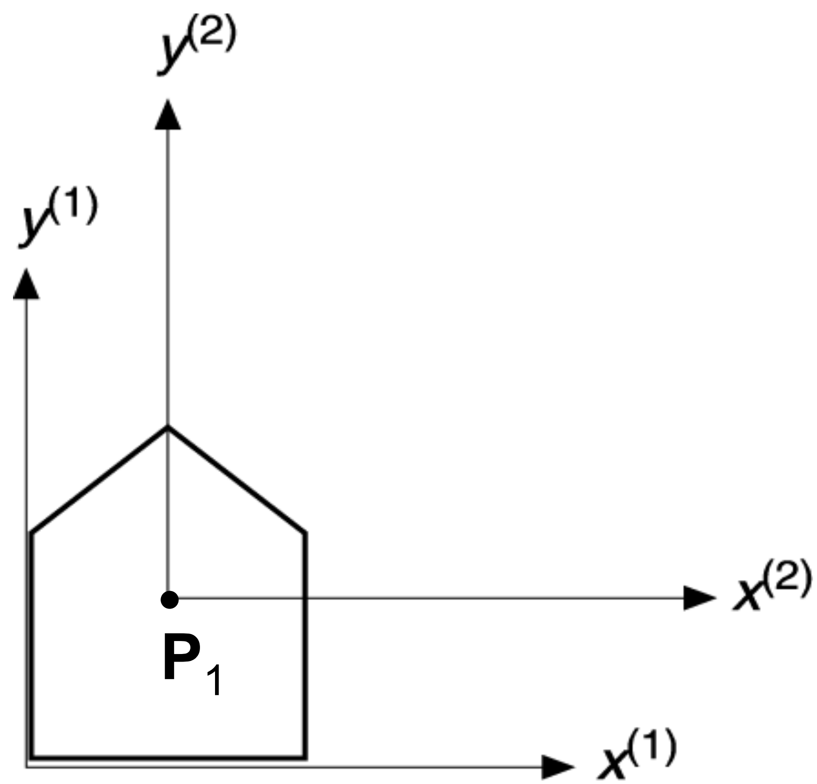
Coordinate System Example (1)

- Translate the House to the origin

$$M_{1\leftarrow 2} = T(x_1, y_1)$$

$$M_{2\leftarrow 1} = \left(M_{1\leftarrow 2}\right)^{-1}$$

$$= T(-x_1, -y_1)$$

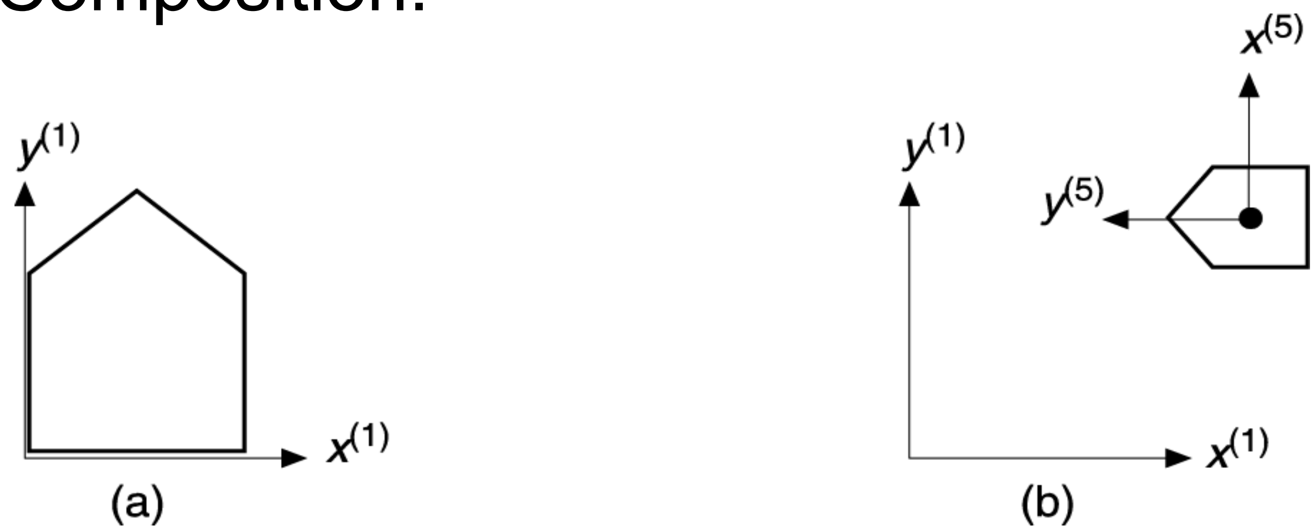


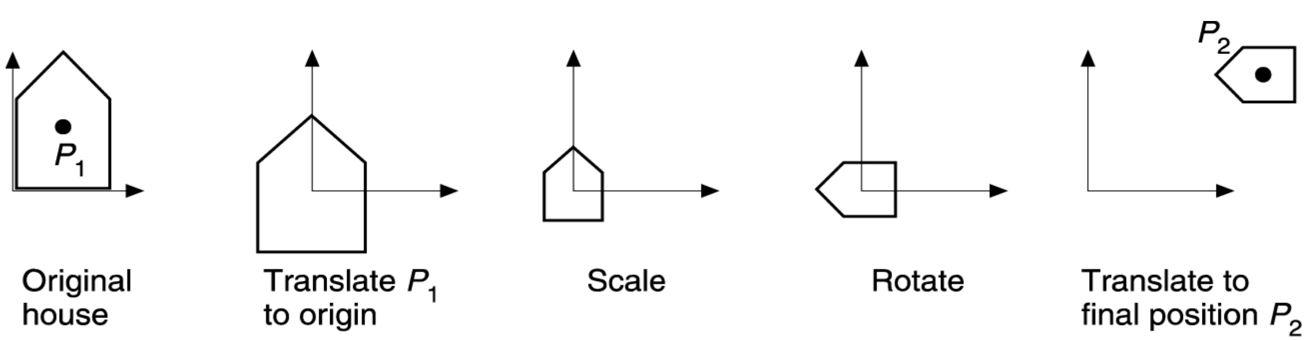
The matrix M_{ij} that maps points from coordinate system j to i is the inverse of the matrix M_{ji} that maps points from coordinate system j to coordinate system i.

Coordinate System Example (2)

- Transformation Composition:

$$M_{5\leftarrow 1} = M_{5\leftarrow 4} \bullet M_{4\leftarrow 3} \bullet M_{3\leftarrow 2} \bullet M_{2\leftarrow 1}$$





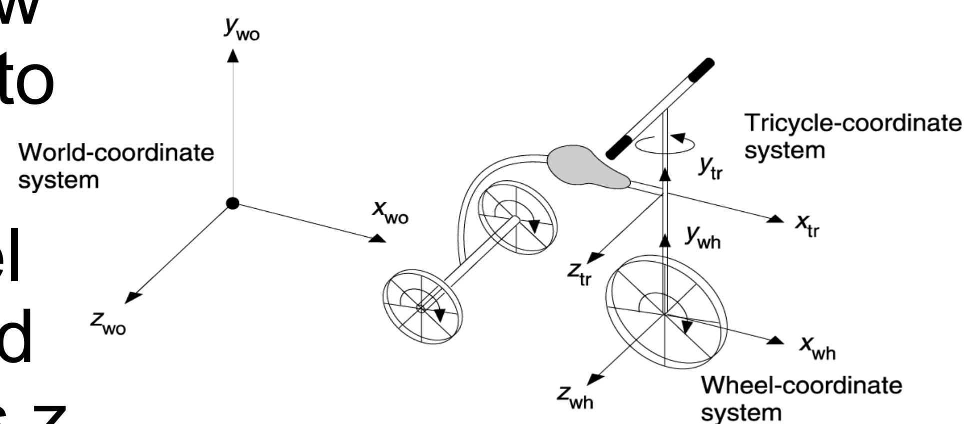
World Coordinates and Local Coordinates

- To move the tricycle, we need to know how all of its parts relate to the WCS
- Example: front wheel rotates on the ground wrt the front wheel's z axis:

$$P^{(wo)} = T(\alpha r, 0, 0) \cdot R_z(\alpha) \cdot P^{(wh)}$$

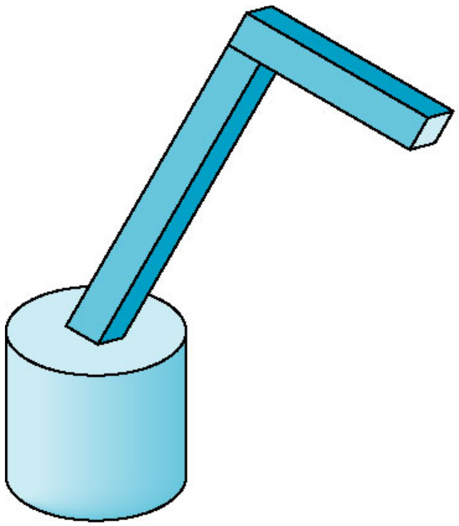
  Coordinates of *P* in wheel coordinate system:

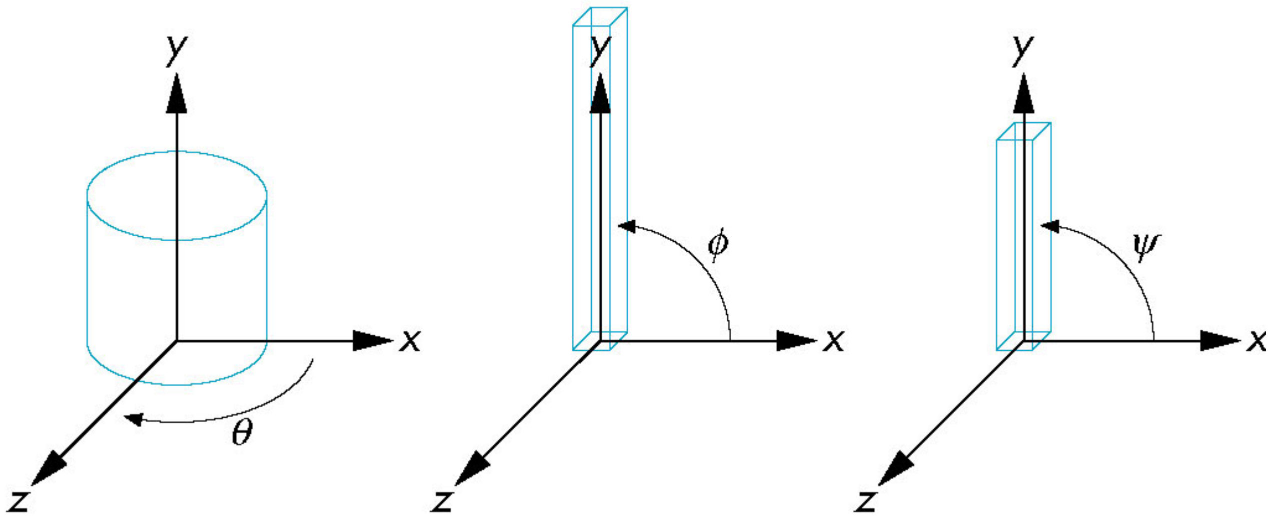
$$P'^{(wh)} = R_z(\alpha) \cdot P^{(wh)}$$



1994 Foley/VanDam/Finer/Huges/Phillips ICG

Robot Arm



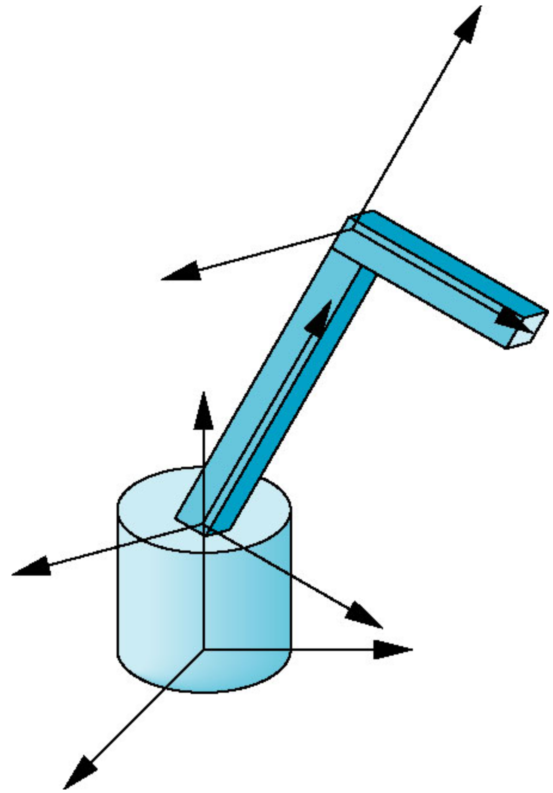


robot arm

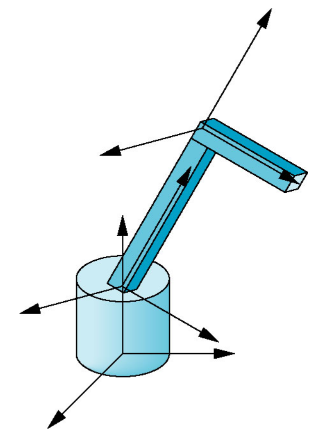
parts in their own coodinate systems

Articulated Models

- Robot arm is an example of an *articulated model*
  - Parts connected at joints
  - Can specify state of model by giving all joint angles



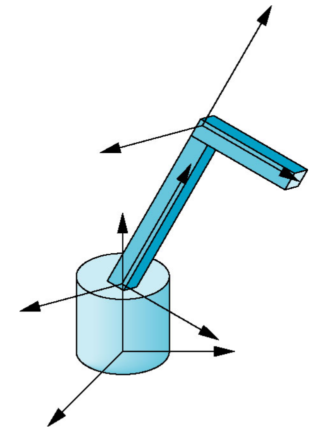
Relationships in Robot Arm



- Base rotates independently
  - Single angle determines position
- Lower arm attached to base
  - Its position depends on rotation of base
  - Must also translate relative to base and rotate about connecting joint
- Upper arm attached to lower arm
  - Its position depends on both base and lower arm
  - Must translate relative to lower arm and rotate about joint connecting to lower arm

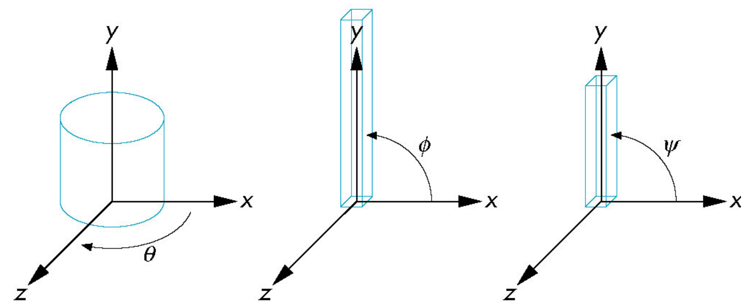
Required Matrices

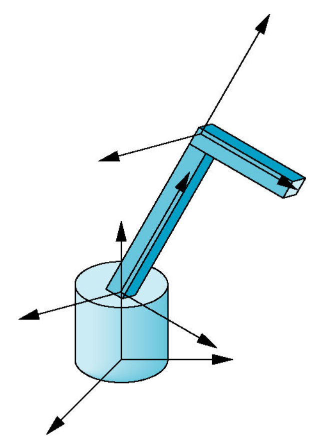
- Rotation of base: $\mathbf{R}_b$
  - Apply $\mathbf{M} = \mathbf{R}_b$ to base
- Translate lower arm <u>relative</u> to base: $\mathbf{T}_{lu}$
- Rotate lower arm around joint: $\mathbf{R}_{lu}$
  - Apply $\mathbf{M} = \mathbf{R}_b \mathbf{T}_{lu} \mathbf{R}_{lu}$ to lower arm
- Translate upper arm <u>relative</u> to upper arm: $\mathbf{T}_{uu}$
- Rotate upper arm around joint: $\mathbf{R}_{uu}$
  - Apply $\mathbf{M} = \mathbf{R}_b \mathbf{T}_{lu} \mathbf{R}_{lu} \mathbf{T}_{uu} \mathbf{R}_{uu}$ to upper arm



OpenGL Code for Robot

```
mat4 ctm;  // current transformation matrix
robot_arm()
{
    ctm = RotateY(theta);
    base();
    ctm *= Translate(0.0, h1, 0.0);
    ctm *= RotateZ(phi);
    lower_arm();
    ctm *= Translate(0.0, h2, 0.0);
    ctm *= RotateZ(psi);
    upper_arm();
}
```



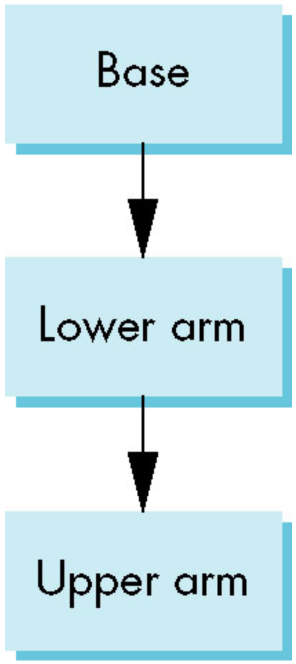


OpenGL Code for Robot

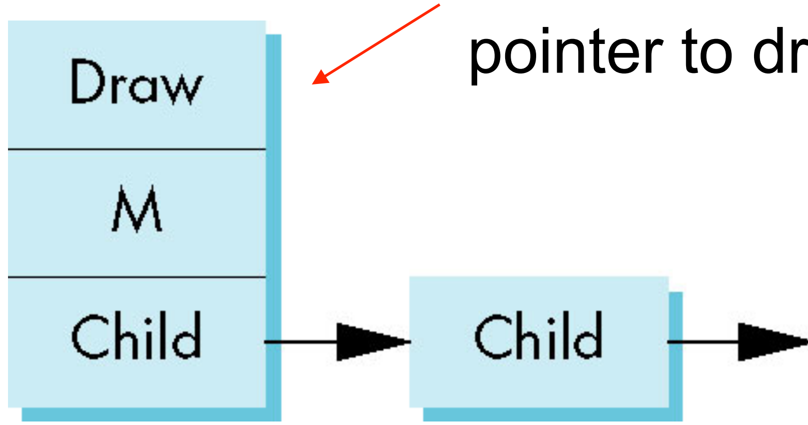
- At each level of hierarchy, calculate **`ctm`** matrix in application.
- Send matrix to shaders
- Draw geometry for one level of hierarchy
- Apply **`ctm`** matrix in shader

Tree Model of Robot

- Note code shows relationships between parts of model
  - Can change "look" of parts easily without altering relationships
- Simple example of tree model
- Want a general node structure for nodes



Possible Node Structure



Code for drawing part or pointer to drawing function

linked list of pointers to children

matrix relating node to parent

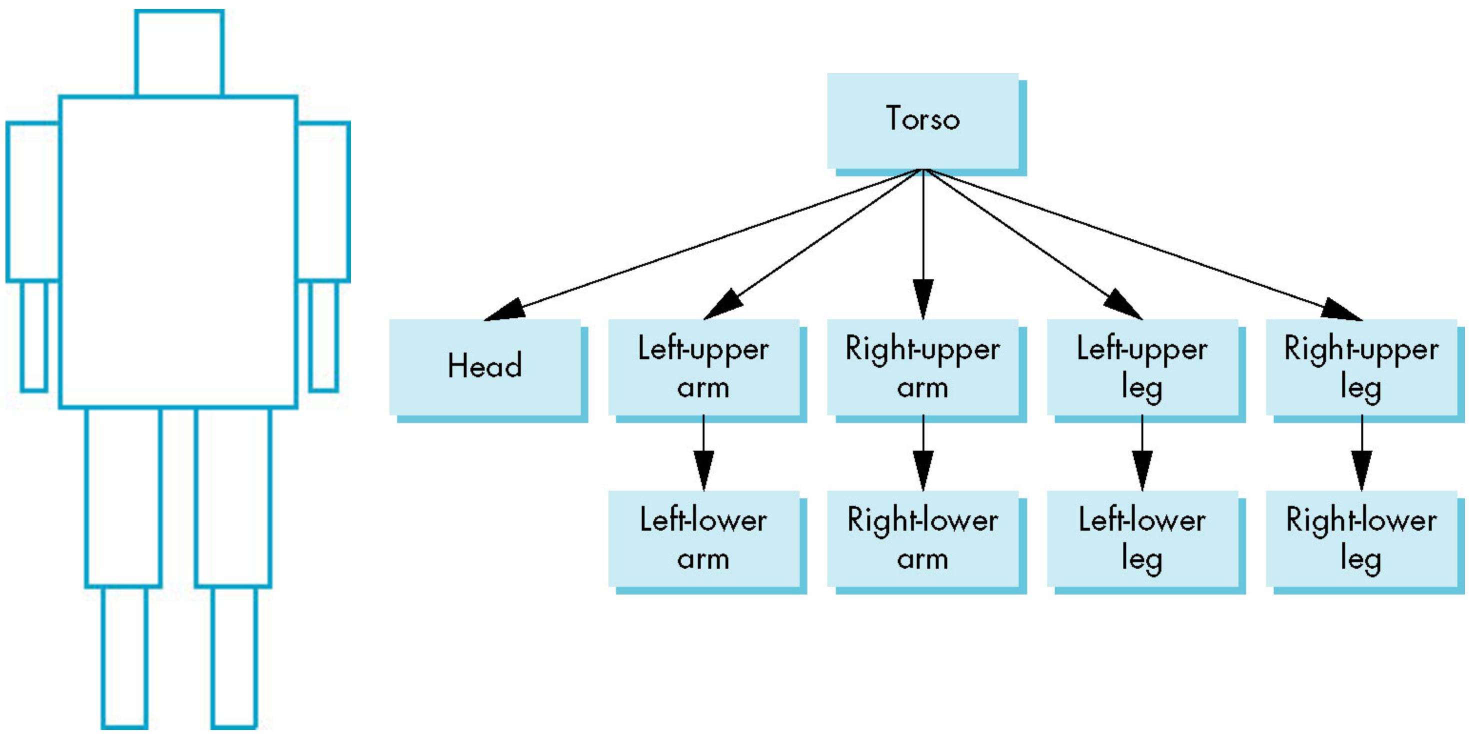
Generalizations

- Need to deal with multiple children
  - How do we represent a more general tree?
  - How do we traverse such a data structure?
- Animation
  - How to use dynamically?
  - Can we create and delete nodes during execution?

Objectives

- Build a tree-structured model of a humanoid figure
- Examine various traversal strategies
- Build a generalized tree-model structure that is independent of the particular model

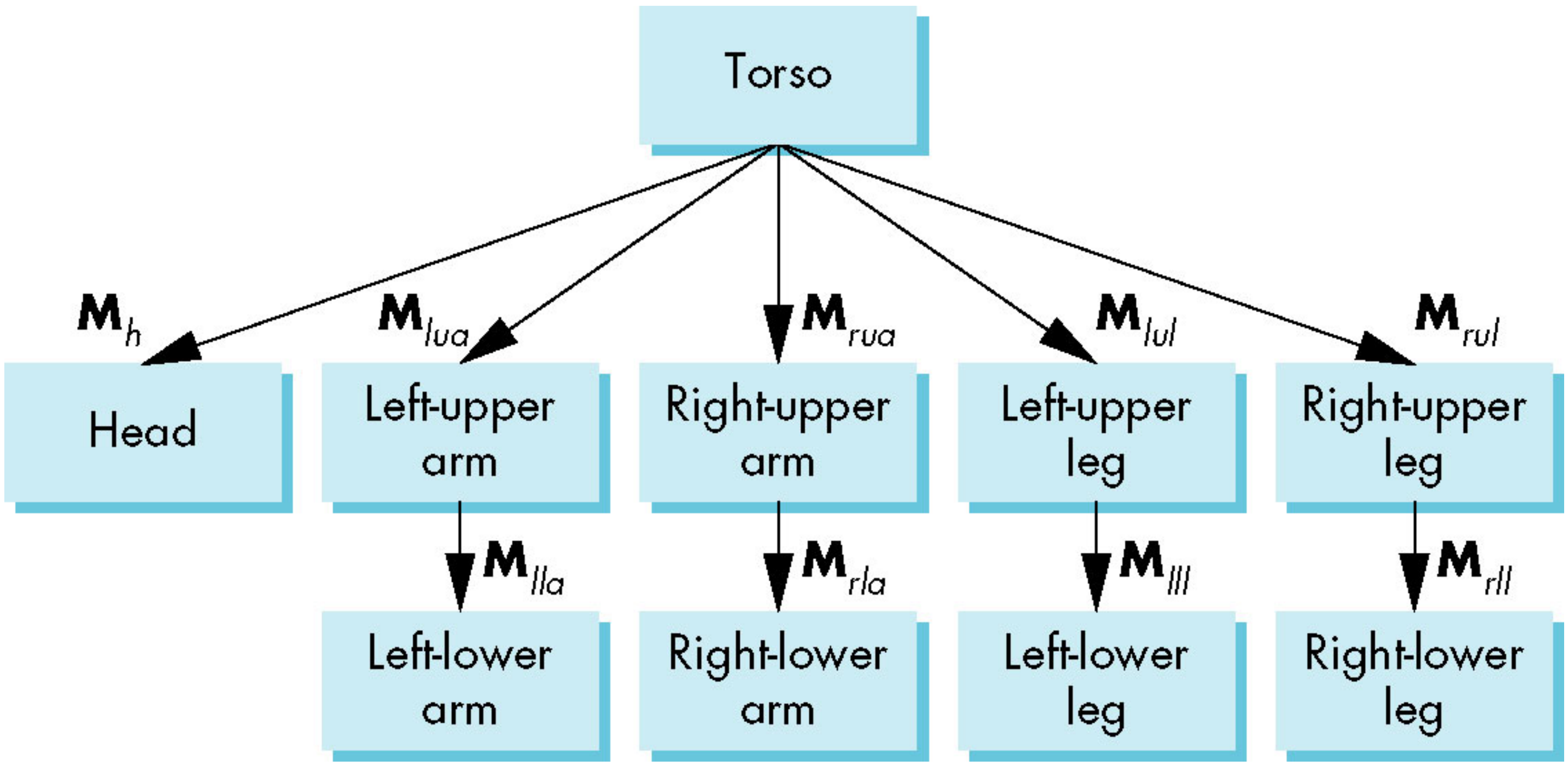
Humanoid Figure



Building the Model

- Can build a simple implementation using quadrics: ellipsoids and cylinders
- Access parts through functions
  - `torso()`
  - `left_upper_arm()`
- Matrices describe position of node with respect to its parent
  - $\mathbf{M}_{lla}$ positions left lower arm with respect to left upper arm

Tree with Matrices



Display and Traversal

- The position of the figure is determined by 11 joint angles (two for the head and one for each other part)
- Display of the tree requires a *graph traversal*
  - Visit each node once
  - Display function at each node that describes the part associated with the node, applying the correct transformation matrix for position and orientation

Transformation Matrices

- There are 10 relevant matrices
  - $\mathbf{M}$ positions and orients entire figure through the torso which is the root node
  - $\mathbf{M}_h$ positions head with respect to torso
  - $\mathbf{M}_{lua}$, $\mathbf{M}_{rua}$, $\mathbf{M}_{lul}$, $\mathbf{M}_{rul}$ position arms and legs with respect to torso
  - $\mathbf{M}_{lla}$, $\mathbf{M}_{rla}$, $\mathbf{M}_{lll}$, $\mathbf{M}_{rll}$ position lower parts of limbs with respect to corresponding upper limbs

Stack-based Traversal

- Set model-view matrix to $\mathbf{M}$ and draw torso
- Set model-view matrix to $\mathbf{MM}_{\mathrm{h}}$ and draw head
- For left-upper arm need $\mathbf{MM}_{\mathrm{lua}}$ and so on
- Rather than recomputing $\mathbf{MM}_{\mathrm{lua}}$ from scratch or using an inverse matrix, we can use the matrix stack to store $\mathbf{M}$ and other matrices as we traverse the tree

Traversal Code

```
figure() {
    PushMatrix()
    torso();
    Rotate (…);
    head();
    PopMatrix();
    PushMatrix();
    Translate(…);
    Rotate(…);
    left_upper_arm();
    PopMatrix();
    PushMatrix();
```

save present currents xform matrix

update ctm for head

recover original ctm

save it again

update ctm for left upper arm

recover and save original ctm again

rest of code

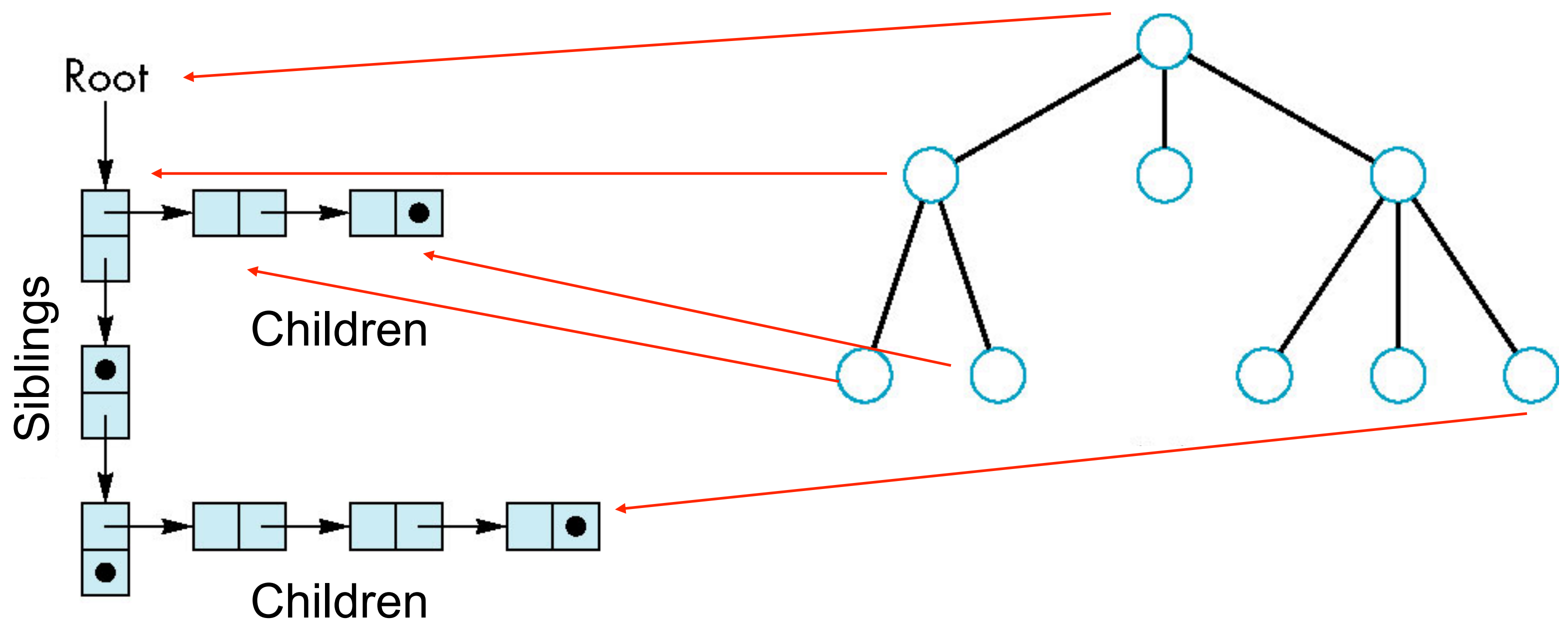
Analysis

- The code describes a particular tree and a particular traversal strategy
  - Can we develop a more general approach?
- Note that the sample code does not include state changes, such as changes to colors
  - May also want to use a `PushAttrib` and `PopAttrib` to protect against unexpected state changes affecting later parts of the code

General Tree Data Structure

- Need a data structure to represent tree and an algorithm to traverse the tree
- We will use a *left-child right sibling* structure
  - Uses linked lists
  - Each node in data structure is two pointers
  - Left: linked list of children
  - Right: next node (i.e. siblings)

Left-Child Right-Sibling Tree



Tree node Structure

- At each node we need to store
  - Pointer to sibling
  - Pointer to child
  - Pointer to a function that draws the object represented by the node
  - Homogeneous coordinate matrix to multiply on the right of the current model-view matrix
    - Represents changes going from parent to node
    - In OpenGL this matrix is a 1D array storing matrix by columns

C Definition of treenode

```
typedef struct treenode
{
    mat4 m;
    void (*f)();
    struct treenode *sibling;
    struct treenode *child;
} treenode;
```

torso and head nodes

```
treenode torso_node, head_node, lua_node, … ;

torso_node.m = RotateY(theta[0]);
torso_node.f = torso;
torso_node.sibling = NULL;
torso_node.child =  &head_node;

 head_node.m = translate(0.0, TORSO_HEIGHT
  +0.5*HEAD_HEIGHT, 0.0)*RotateX(theta[1])
  *RotateY(theta[2]);
head_node.f = head;
head_node.sibling = &lua_node;
head_node.child = NULL;
```

Notes

- The position of figure is determined by 11 joint angles stored in **`theta[11]`**
- Animate by changing the angles and redisplaying
- We form the required matrices using **`Rotate`** and **`Translate`**
  - Because the matrix is formed using the model-view matrix, we may want to first push original model-view matrix on matrix stack

Preorder Traversal

```
void traverse(treenode* root)
{
   if(root==NULL) return;
   mvstack.push(ctm);
   ctm = ctm*root->m;
   root->f();
   if(root->child!=NULL) traverse(root->child);
   ctm = mvstack.pop();
   if(root->sibling!=NULL)
                              traverse(root->sibling);
}
```

Traversal Code & Matrices

- `figure()` called with CTM set
- M_{fig} defines figure's place in world

```
figure() {
    PushMatrix()
    torso();
    Rotate (…);
    head();
    PopMatrix();
    PushMatrix();
    Translate(…);
    Rotate(…);
    left_upper_arm();
```

Stack	CTM
	M_{fig}

Stack	CTM
M_{fig}	M_{fig}

Stack	CTM
M_{fig}	$M_{fig}M_h$

Stack	CTM
	M_{fig}

Stack	CTM
M_{fig}	M_{fig}

Stack	CTM
M_{fig}	$M_{fig}M_{lua}$

Traversal Code & Matrices

```
PushMatrix()
Translate(…);
Rotate(…);
left_lower_arm();
PopMatrix();
PopMatrix();
PushMatrix()
Translate(…);
Rotate(…);
right_upper_arm();
    …
    …
```

Stack	CTM
$M_{fig}M_{lua}$	$M_{fig}M_{lua}$
M_{fig}	

Stack	CTM
$M_{fig}M_{lua}$	$M_{fig}M_{lua}M_{lla}$
M_{fig}	

Stack	CTM
M_{fig}	$M_{fig}M_{lua}$

Stack	CTM
	M_{fig}

Stack	CTM
M_{fig}	M_{fig}

Stack	CTM
M_{fig}	$M_{fig}M_{rua}$

Notes

- We must save current transformation matrix before multiplying it by node matrix
  - Updated matrix applies to children of node but not to siblings which contain their own matrices
- The traversal program applies to any left-child right-sibling tree
  - The particular tree is encoded in the definition of the individual nodes
- The order of traversal matters because of possible state changes in the functions

Dynamic Trees

- If we use pointers, the structure can be dynamic

```
typedef treenode *tree_ptr;
tree_ptr torso_ptr;
torso_ptr = malloc(sizeof(treenode));
```

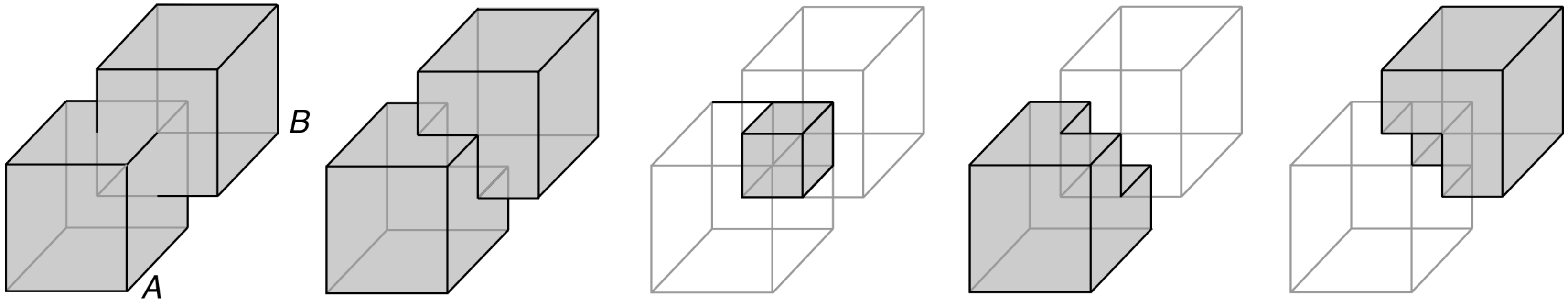
- Definition of nodes and traversal are essentially the same as before but we can add and delete nodes during execution

Solids and Solid Modeling

- Solid modeling introduces a mathematical theory of solid shape
  - Domain of objects
  - Set of operations on the domain of objects
  - Representation that is
    - Unambiguous
    - Accurate
    - Unique
    - Compact
    - Efficient

Solid Objects and Operations

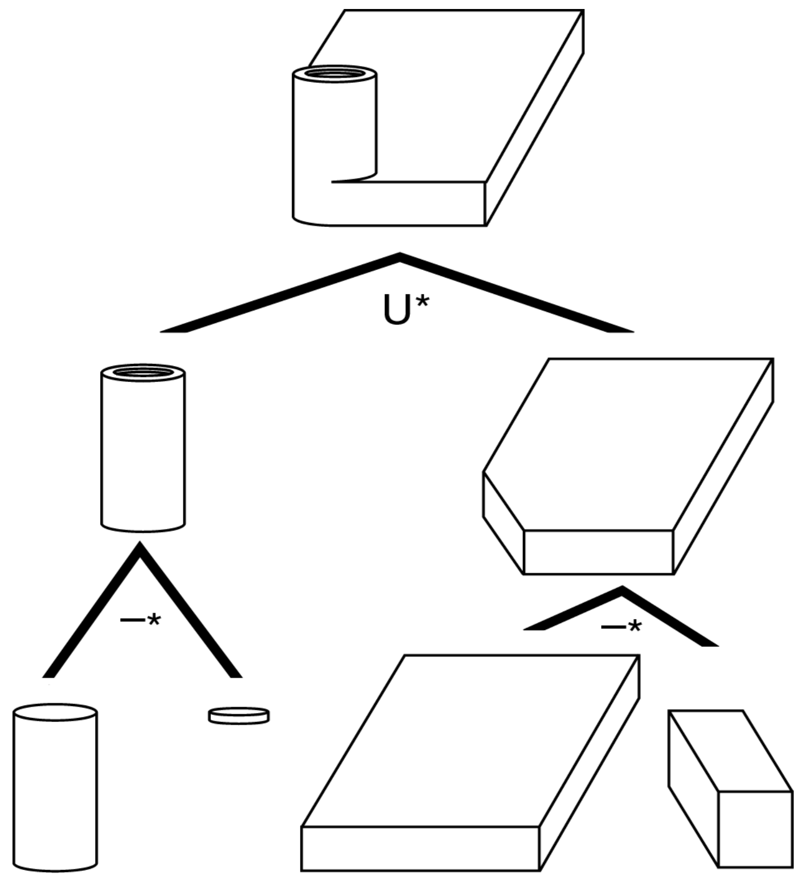
- Solids are point sets
  - Boundary and interior
- Point sets can be operated on with boolean algebra (union, intersect, etc)



Foley/VanDam, 1990/1994

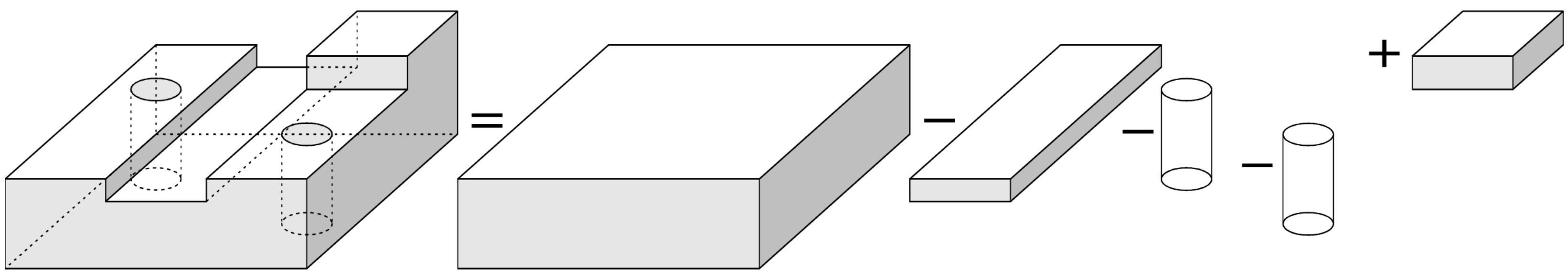
Constructive Solid Geometry (CSG)

- A tree structure combining primitives via regularized boolean operations
- Primitives can be solids or *half spaces*



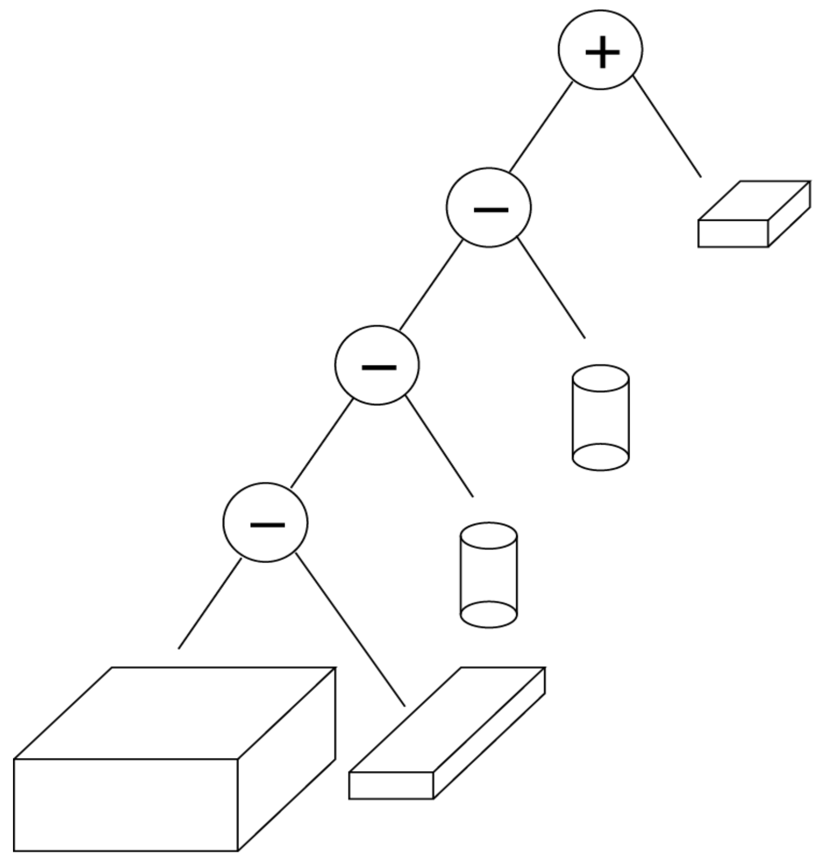
A Sequence of Boolean Operations

- Boolean operations
- Rigid transformations



Pics/Math courtesy of Dave Mount @ UMD-CP

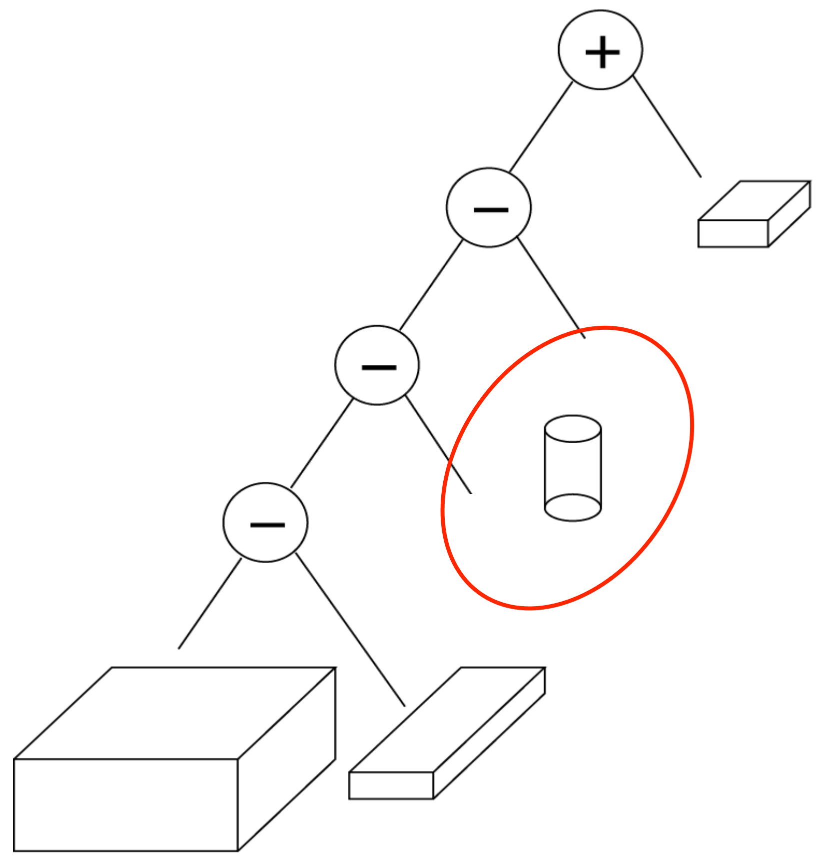
The Induced CSG Tree



Pics/Math courtesy of Dave Mount @ UMD-CP

The Induced CSG Tree

- Can also be represented as a directed acyclic graph (DAG)



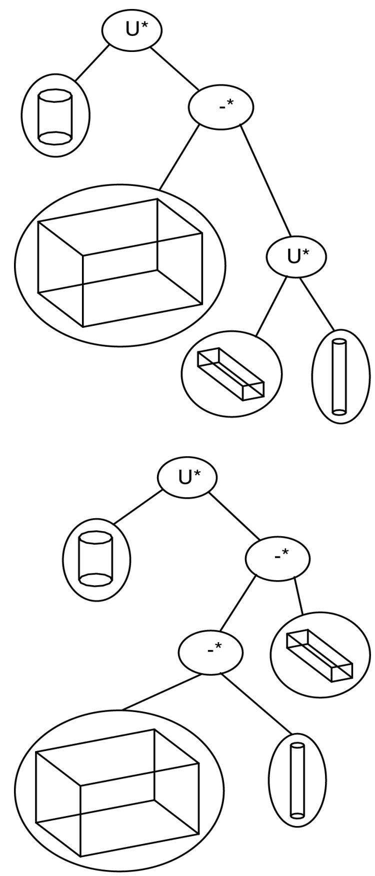
Pics/Math courtesy of Dave Mount @ UMD-CP

Issues with Constructive Solid Geometry

- Non-uniqueness
- Choice of primitives
- How to handle more complex modeling?
  - Sculpted surfaces? Deformable objects?

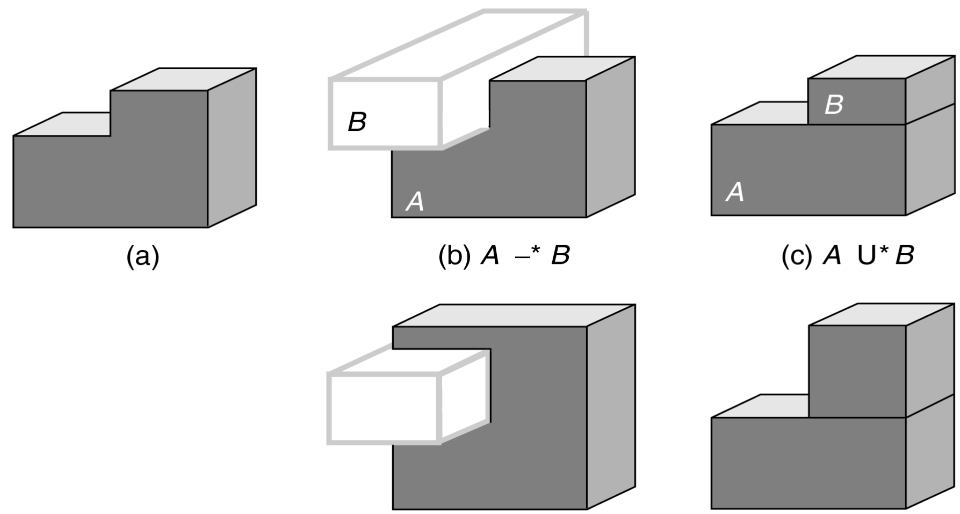
Issues with Constructive Solid Geometry

- Non-Uniqueness
  - There is more than one way to model the same artifact
  - Hard to tell if A and B are identical



Issues with CSG

- Minor changes in primitive objects greatly affect outcomes
- Shift up top solid face



(a) (b) *A* –* *B* (c) *A* U* *B*

(d) (e)

Foley/VanDam, 1990/1994

Uses of Constructive Solid Geometry

- Found (basically) in every CAD system
- Elegant, conceptually and algorithmically appealing
- Good for
  - Rendering, ray tracing, simulation
  - BRL CAD