Procedural Methods

CS 432 Interactive Computer Graphics
Prof. David E. Breen
Department of Computer Science

Shader Applications

- Moving vertices
  - Wave motion
  - Morphing
  - Particle systems
  - Newtonian dynamics
  - Fractals
- Lighting
  - More realistic models
  - Cartoon shaders

Wave Motion Vertex Shader

```
uniform float time;
uniform float xs, zs, // frequencies
uniform float h; // height scale
uniform mat4 ModelView, Projection;
in vec4 vPosition;

void main() {
  vec4 t =vPosition;
  t.y = vPosition.y
     + h*sin(time + xs*vPosition.x)
     + h*sin(time + zs*vPosition.z);
  gl_Position = Projection*ModelView*t;
}
```


Modeling

- Geometric
  - Meshes
  - Hierarchical
  - Curves and Surfaces
- Procedural
  - Particle Systems
  - Fractal

Particle Systems

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Introduction

- Most important of procedural methods
- Used to model
  - Natural phenomena
    - Clouds
    - Terrain
    - Plants
  - Crowd Scenes
  - Real physical processes

Particle Systems – A Technique for Modeling a Class of Fuzzy Objects

William T. Reeves, Lucasfilm
ACM Transactions on Graphics, 1983

Presented in CS536 by Walt Mankowski
19 October 2006

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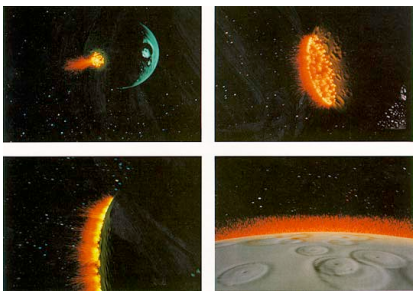
Genesis Project Demo



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Prior to Khan

State of the art in computer animation:

- Static geometric objects
- Primitive surface elements (e.g. polygons)
- Deterministic
- Simple affine transformations

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Prior to Khan

State of the art in computer animation:

- Smooth surfaces
- Well-defined
- Shiny surfaces

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What about "fuzzy" objects?

- Clouds, smoke, water, fire
- Irregular, ill-defined, complex
- Dynamic
- Fluid

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Representation of "fuzzy" objects

- Cloud of primitive particles that define its volume
- Particles are born, move, change form, and die over time
- Stochastic processes are basis for all dynamics

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Advantages of particles

- Simpler than polygons
- Easier and faster to compute
- Easier to motion-blur
- Model is procedural and random
- Less human modeling required
- Particle systems are "alive"
- More natural way to represent dynamic motion than polygons

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High level view

- System is collection of many of tiny particles (25,000–750,00 in sample frames)
- Over time, particles are added to the system, move and change, and then are removed from the system

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For each frame...

- New particles are "born" into the system
- Each new particle is assigned its own individual, random attributes
- Any particles that are too old "die" and are removed from the system
- Remaining particles move and are transformed according to their attributes
- Image is rendered to frame buffer

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Particle generation

- For entire frame
  - $NParts_f = MeanParts_f + Rand() * VarParts_f$
- For smaller screen area
  - $NParts_f = (MeanParts_{saf} + Rand() * VarParts_{saf}) * ScreenArea$
- Uniformly distributed over a range
- Dynamically change density →Vary mean over all frames
  - $MeanParts_f = InitMeanParts + DeltaMeanParts * (f - f_0)$

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Particle attributes

For each new particle, must determine:

- initial position
- initial velocity
- initial size
- initial color
- initial transparency
- shape
- lifetime

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Initial position

- sphere
- circle
- rectangle
- etc.

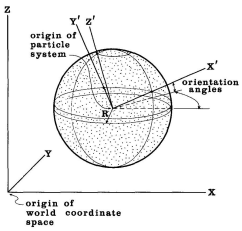


Fig. 1. Typical particle system with spherical generation shape.

Initial settings

- Initial speed is uniformly distributed:
  - InitialSpeed = MeanSpeed + Rand() * VarSpeed
- Color, size, transparency set the same way

Particle dynamics

- To move a particle, just add its velocity vector to its position.
- Can simulate gravity by using an acceleration factor.
- Color, transparency, size controlled by similar rate-of-change parameters.
- Everything is **completely deterministic**.

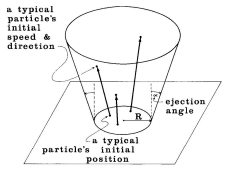


Fig. 3. Form of an explosion-like particle system.

Particle extinction

- Remove particles that exceed lifetime
- Can also remove particles that stray outside a window

Particle rendering

Two simplifying assumptions:

- Particles don't interact with other objects
  - They're composited in later
  - Added "glow" on planet from the fire with an additional light source
- Each particle is a point light source
  - No hidden surfaces
  - Each particle only contributes to the color of the pixel it covers

Particle hierarchy

- Maintain a tree of particle systems.
- Allows more global control over system.
- In Genesis Demo:
  - Top level was point of impact
  - Particles it generates are themselves other particle systems.

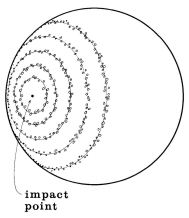


Fig. 2. Distribution of particle systems on the planet's surface.

Motion blur

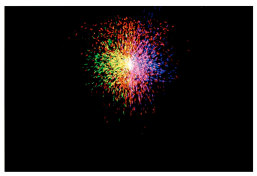
- Computer animation up to then produced one static image at an instant in time
- This can lead to aliasing and strobing
- They motion-blurred the particles:
  - Calculate position at start of frame and half way through
  - Draw an antialiased line between points
- Stills look blurry but video looks better

Go to videos

-Making of the Genesis Sequence from Star Trek II

- Particle Dreams
  - Karl Sims

Other applications



Fireworks

Grass

Future directions (circa 1982)

- Clouds
  - difficult to model
  - can throw shadows on themselves
  - very large number of particles
- Water
- More sophisticated motion-blur

Newtonian Particle

- Particle system is a set of particles
- Each particle is an ideal point mass
- Six degrees of freedom
  - Position
  - Velocity
- Each particle obeys Newtons' law

$$f = ma$$

Particle Equations

$\mathbf{p}_i = (x_i, y_i z_i)$

$\mathbf{v}_i = d\mathbf{p}_i / dt = \mathbf{p}_i' = (dx_i / dt, dy_i / dt , z_i / dt)$

$m \mathbf{v}_i' = \mathbf{f}_i$

Hard part is defining force vector

Force Vector

- Independent Particles
  - Gravity
  - Wind forces
  - O(n) calulation
- Coupled Particles O(n)
  - Meshes
  - Spring-Mass Systems
- Coupled Particles O(n^2)
  - Attractive and repulsive forces

Solution of Particle Systems

```
float time, delta state[6n], force[3n];
state = initial_state();
for(time = t0; time<final_time;
                          time+=delta) {
  force = force_function(state, time);
  state = ode(force, state, time,
                                delta);
  render(state, time)
}
```


Simple Forces

- Consider force on particle i

  $\mathbf{f}_i = \mathbf{f}_i(\mathbf{p}_i, \mathbf{v}_i)$
- Gravity $\mathbf{f}_i = \mathbf{g}$

  $\mathbf{g}_i = (0, -g, 0)$
- Wind forces
- Drag
  - $\mathbf{f}(-\mathbf{v}_i)$

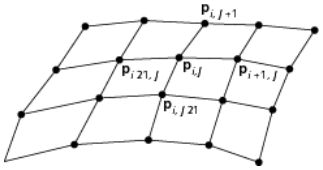
$\mathbf{p}_i(t_0), \mathbf{v}_i(t_0)$

Particle System

```
uniform vec3 init_vel;
uniform float g, m, t;
uniform mat4 Projection, ModelView;
in vPosition;
void main(){
  vec3 object_pos;
  object_pos.x = vPosition.x + vel.x*t;
  object_pos.y = vPosition.y + vel.y*t
                          - g/(2.0*m)*t*t;
  object_pos.z = vPosition.z + vel.z*t;
  gl_Position = Projection*
            ModelView*vec4(object_pos,1);
}
```


Meshes

- Connect each particle to its closest neighbors
  - O(n) force calculation
- Use spring-mass system



Spring Forces

- Assume each particle has unit mass and is connected to its neighbor(s) by a spring
- Hooke's law: force proportional to distance ($d = ||\mathbf{p} - \mathbf{q}||$) between the points

Hooke's Law

- Let s be the distance when there is no force

$$\mathbf{f} = -k_s(|\mathbf{d}| - s)\,\mathbf{d}/|\mathbf{d}|$$

k_s is the spring constant

$\mathbf{d}/|\mathbf{d}|$ is a unit vector pointed from $\mathbf{p}$ to $\mathbf{q}$

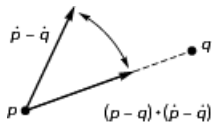
- Each interior point in mesh has four forces applied to it

Spring Damping

- A pure spring-mass will oscillate forever
- Must add a damping term

$$\mathbf{f} = -(k_s(|\mathbf{d}| - s) + k_d\,\dot{\mathbf{d}}\cdot\mathbf{d}/|\mathbf{d}|)\mathbf{d}/|\mathbf{d}|$$

- Must project velocity



Repulsion

- Inverse square law

$$\mathbf{f} = k_r\mathbf{d}/|\mathbf{d}|^2$$

- General case requires $O(n^2)$ calculation
- In most problems, the drop off is such that not many particles contribute to the forces on any given particle
- Sorting problem: is it O(n log n)?

Boxes

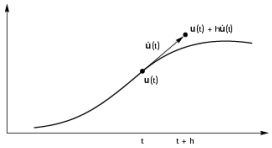
- Spatial subdivision technique
- Divide space into boxes
- Particle can only interact with particles in its box or the neighboring boxes
- Must update which box a particle belongs to after each time step

Linked Lists

- Each particle maintains a linked list of its neighbors
- Update data structure at each time step
- Must amortize cost of building the data structures initially

Solution of ODEs

- Particle system has 6n ordinary differential equations
- Write set as $d\mathbf{u}/dt = g(\mathbf{u},t)$
- Solve by approximations using Taylor's Thm



Euler's Method

$\mathbf{u}(t + h) \approx \mathbf{u}(t) + h\, d\mathbf{u}/dt = \mathbf{u}(t) + h\mathbf{g}(\mathbf{u}, t)$

Per step error is $O(h^2)$

Require one force evaluation per time step

Problem is numerical instability

Depends on step size

Improved Euler

$\mathbf{u}(t + h) \approx \mathbf{u}(t) + h/2(\mathbf{g}(\mathbf{u}, t) + \mathbf{g}(\mathbf{u}, t+h))$

Per step error is $O(h^3)$

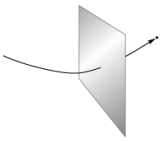
Also allows for larger step sizes

But requires two function evaluations per step

Also known as Runge-Kutta method of order 2

Contraints

- Easy in computer graphics to ignore physical reality
- Surfaces are virtual
- Must detect collisions separately if we want exact solution
- Can approximate with repulsive forces



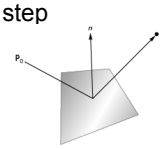
Collisions

Once we detect a collision, we can calculate new path

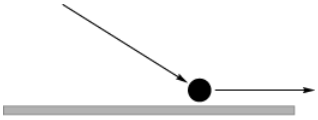
Use coefficient of resititution

Reflect vertical component

May have to use partial time step



Contact Interactions



- Sliding contact
- Normal force that prevents particle from penetrating surface
- Friction force in the tangential direction

Flocks, Herds, and Schools: A Distributed Behavioral Model

Craig W. Reynolds

SIGGRAPH 1987

Presented by Duc Nguyen

November 15, 2007

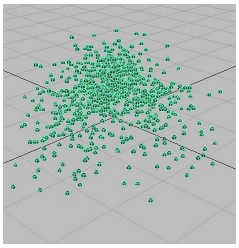
Simulated Flocks of "Boids"

- Simulate motion of flocks of birds or other animals following individual behaviors
- Extends particle systems
- More realistic (and easier!) than scripting the paths of the individual birds

Prior work

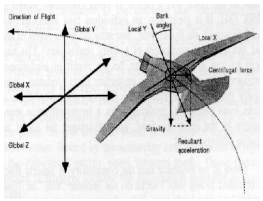
- *Eurythmy* (1985)
  - Flocking simulations used force-fields around each bird and around each object.
  - The animator sets the initial positions, headings, and velocities.
- Other behavioral control work by Karl Sims was based around groups of single objects, not flocks.

Particle Systems



- Particle System is a collection of large number of particles each with their own behaviors
  - At the time of the paper, they were used to model fire, smoke, clouds and ocean waves.
- A Boid is a generalization of a particle
  - Each boid follows a flight model

Boid Model

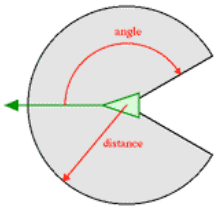


- Local coordinate system
- Direction of flight
- Bank angle
- Affected by
  - Gravity
  - Centrifugal force

Boid Simulation

- Create a boid model that supports flight
- Add individual behaviors that interact with each other:
  - Separation
  - Alignment
  - Cohesion
  - Obstacle Avoidance
- Boids must be able to arbitrate conflicting behaviors as well

Boid Neighborhood

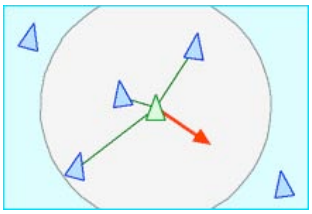


Only sense boids

- within a certain distance
- that are in "front" of you

Boid Behavior: Separation

- Keep a certain distance from neighboring boids



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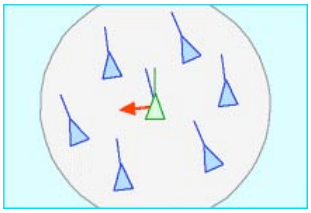
Boid Behavior: Alignment

- Steer toward the general heading of neighboring boids



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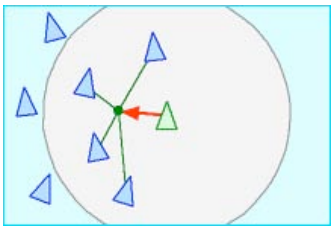
Boid Behavior: Cohesion

- Move toward the average position of neighboring boids



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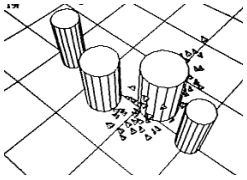
Avoiding Obstacles

- Independently model shape for rendering and collision avoidance
- Boids model uses a *steer-to-avoid* as opposed to force-field approach



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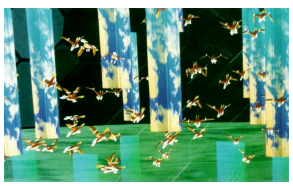
Putting it all together

- Simulated flocking behavior that mimics real life flocks, schools and herds
- Combined with a low priority goal seeking results in a scripted path of the flock



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- Go to videos

  -Boids demo

  -Stanley & Stella in Breaking the Ice

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Summary

- Boids is a model of group motion of flocks based on actions of individual boids
- Boid simulation is an example of emergent behavior
  - Simple local rules lead to complex global behavior
- This paper (from 1987!) has spawned the field of collective behavior modeling/ analysis in physics, biology, artificial life and computer science

Fractals

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Overview

- Fractal representations
- Algorithms for drawing fractals
- Mandelbrot Set
- Brownian Motion

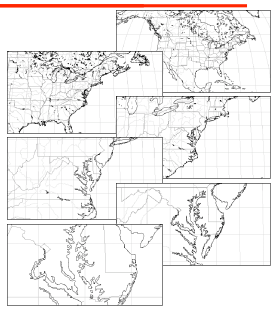
Introduction to Fractals

- Term Fractal coined by Benoit Mandelbrot
- Properties of fractal:
  - Self-similarity (small portion looks like the whole object)
  - Have fractional dimensions
  - Non-differentiable
  - Infinite length

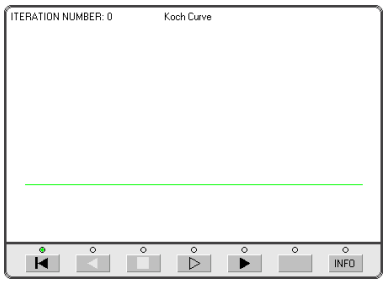
Construction of Koch Curve

Length of the Fractals

- Lewis Richardson (1961) measured length of coastlines
- Total length increase as measurement stick reduced
- Does this mean coastlines are infinite in length?



Koch Curve

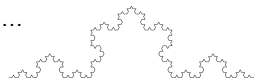


Properties of Koch Curve

- Has no tangent at any point
- Length of the curve at iteration n is $(4/3)^n$

Step	Num of segments	Length of segment	Total length
0	1	1	1
1	4	0.3333333	1.3333333
2	16	0.1111111	1.7777778
100	$1.04858*10^{60}$	$1.94033*10^{-48}$	$3.11798*10^{12}$

- Koch curve is 1.26186 … dimensional object



Fractal Dimensions

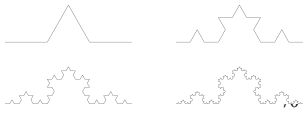
- 1D line of unit length divided into n segments looks like the whole line scaled by $1/n$
- 2D square of unit area divided into n segments looks like the whole square scaled by $1/n^{1/2}$
  - i.e. 4 segments, each one 1/2 the size of the original
- Divide an object into n segments where each segment is scaled by $1/s$
- $s = n^{1/d}$ $d = \log(n)/\log(s)$ $2 = \log(4)/\log(2)$
- The square is divided into 4 parts and each part is 1/2 the size of the original
- $d = \log(4)/\log(2) = 2$

Fractal Dimensions

- $s = n^{1/d}$ $d = \log(n)/\log(s)$
- Koch curve divides line into 4 parts, each part is 1/3 the size of the original
- Therefore the dimension of the Koch curve is:

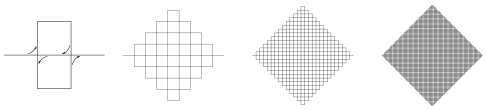
$$4^{\frac{1}{d}} = 3, \quad n^{1/d} = s$$

$$d = \frac{\log(4)}{\log(3)} = \frac{\log(n)}{\log(s)} = 1.26186 \ \ldots$$



Space Filling Fractals

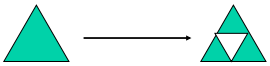
- Peano curve (1890)
- Fills a 2D region
- Fractal dimension - 2
- $s = n^{1/d}$ 9 segments 1/3 original size
  $d = \log(9)/\log(3) = 2$



Construction of Peano Curve

Sierpinski Gasket

Rule based:



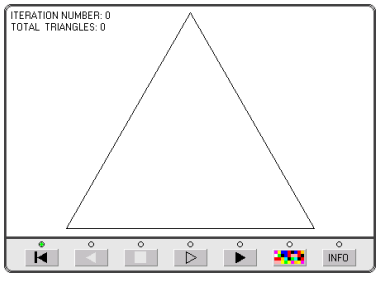
Repeat n times. As n →∞

Area→0

Perimeter →∞

Not a normal geometric object

Sierpinski Gasket



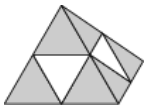
http://www.arcytech.org/java/fractals/

Examples

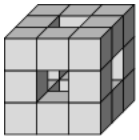
- Koch Curve
  - Scale by 3 each time
  - Create 4 new objects
  - d = ln 4 / ln 3 = 1.26186
- Sierpinski gasket
  - Scale by
  - Create 3 new objects
  - d = ln 3 / ln 4 = 1.58496

Volumetric Examples

d = ln 4/ ln 2 = 2



D = ln 20 / ln 3 = 2.72683



Naturally Occurring Fractals

- Fractals often occur in nature
- All natural fractals are *grown*
- We can model natural objects with fractals

Brain

Lungs

Kidney

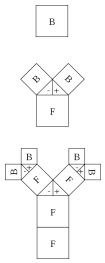
Production Systems

- In 1968 A. Lindenmayer developed a formalism describing plant growth
- Called L-system
- L-system consists of:
  - Seed cell (*axiom*)
  - Production rules

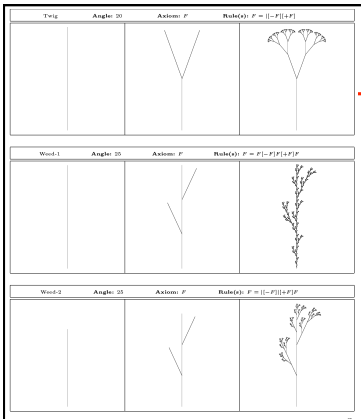
Axiom: B

Rules: $B=>F[-B][+B]$

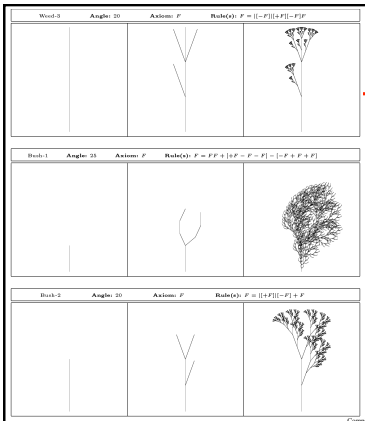
$F=>FF$

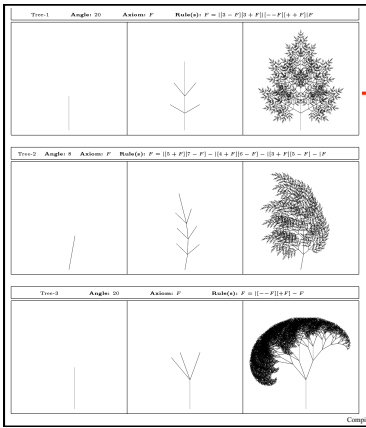


Plantlike Fractals 1



Plantlike Fractals 2





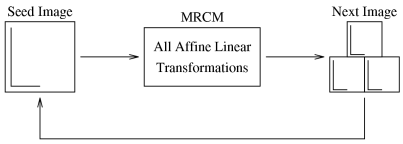
Plantlike Fractals 3

Affine Transformation Fractals

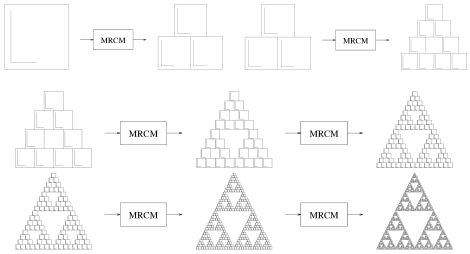
- Fractal often described to contain miniature versions of itself.
- We can use affine transformations to describe where these miniatures should be placed
  - Translation
  - Rotation
  - Scaling
- Self-affine fractals may have different scaling factors in different dimensions

Multiple Reduction Copy Machine Algorithm

- Can be simulated with real copy machine
- Copy machine takes a seed image
- Creates several transformed copies of it as an output.
- Procedure is recursive

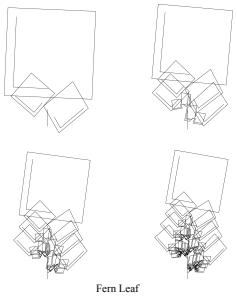


MRCM Examples



Problems with MRCM

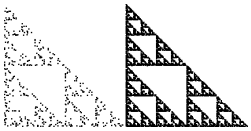
- To get good image we must compute MRCM to large depth
- Computing time grows exponentially with depth
- Problem amplified when the reduction in size during one iteration is small



Fern Leaf

Iterated Functional System

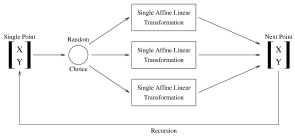
- Observations:
  - Fractal consist entirely of points
  - Randomly and recursively apply affine transformation $L_i(p)$ to point p
  - Converges to a fractal



500 points 5000 points

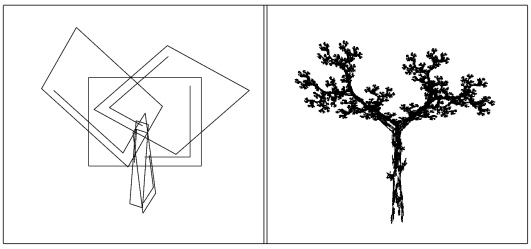
IFS Algorithm

- Pick random point of the seed image
- Randomly pick one of the affine transformations
- Transform the point
- Continue recursively

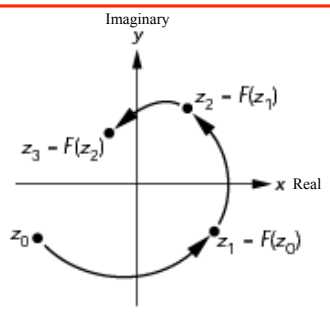


Fern Leaf

IFS Generated Tree



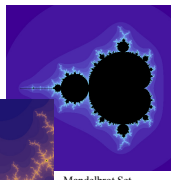
Iteration in the Complex Plane



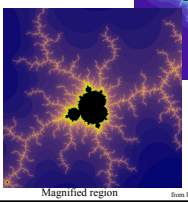
Mandelbrot Set

- $x_n = x_{n-1}^2 + c$ for some complex number c
  - For some c, $x \to 0$ as $n \to \infty$ $\in$ Mandelbrot Set
  - For some c, $x \to \infty$ as $n \to \infty$
  - For others *neither*

For each c in the complex plane
$x_0 = 0$
for ($n=1$ to n_{max}) {
$x_n = x_n^2 + c$
if ($|x_n| > 2$) break
}
If ($n < n_{max}$) color c = white
else color c = black



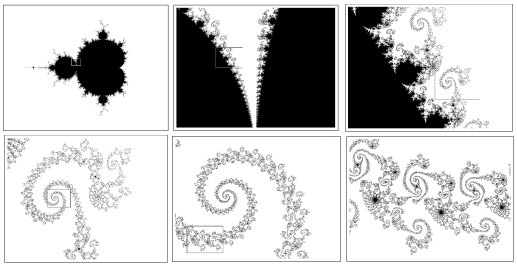
Mandelbrot Set



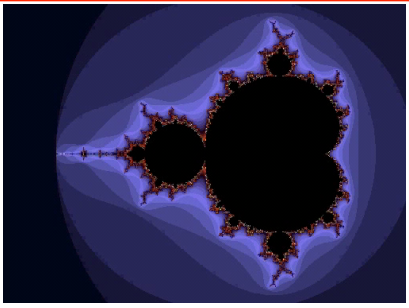
Magnified region

from http://www.astro.su.se/~alexis/fractals/

Mandelbrot Set

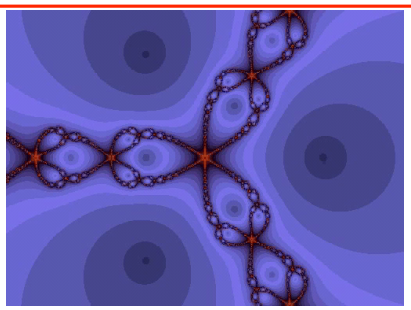


Mandelbrot Set



Newton Set

$f(x) = x^3 - 1 \qquad x_{n+1} = x_n - f(x) / f'(x)$



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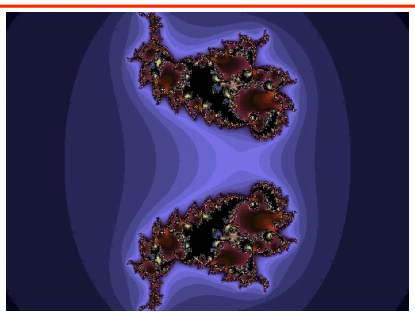
Phoenix Set

$z_{n+1} = z_n^2 + \mathrm{Re}(c) + \mathrm{Im}(c) * z_{n-1}$



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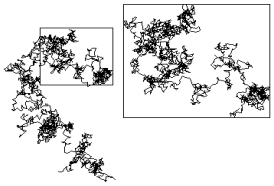
Brownian Motion

- The dance of pollen grains in a water drop (Robert Brown in 1827)
- Explained by Albert Einstein in 1905
- Self-similar motion
- Infinite length
- Used to simulate rivers mountains and other random natural phenomena



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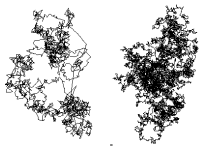
Fractional Brownian Motion

- Fractional BM (fBM) is a generalization of BM to include memory
  - Integral on progress of random walk
- fBM characterized by it's power spectrum
  - BM has $1/f^2$ power spectrum
  - fBM had $1/f^{\beta}$ power spectrum with $1.0 \le \beta \le 3.0$
- Think of β as controlling terrain roughness



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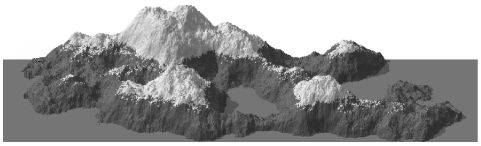
Voss & Saupe in "The Science of Fractal images"

XMOUNTAINS



- Landscape generated by the `XMOUNTAINS` program by Steven Booth
- Realistic, self-similar image

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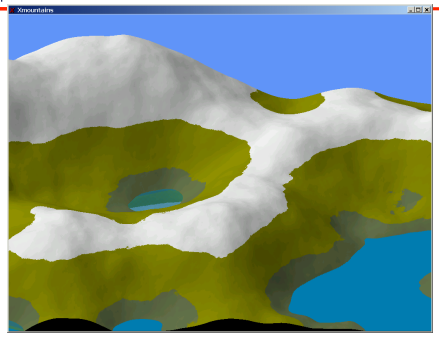
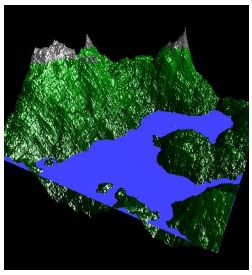
Compiled from Gary W. Flake "The Computational Beauty of Nature"

XMOUNTAINS : Changing Fractal Dimension



Random Midpoint Displacement

- fBM computations are time consuming
- RMD are faster to compute but less realistic
- Basic idea:
  - Start with 2 (2D) 4 (3D) random points.
  - Compute the midpoint for each interval
  - Displace the midpoint at random but based on the difference between end points



RMD Algorithm

- Generate points a and b at random

$y_{mid} = (1/2)(y(a)+y(b))+r$

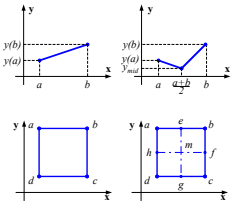
$r = s\cdot(|b-a|)\cdot r_g()$

s - is a surface roughness parameter

$r_g()$- returns a Gaussian random value (mean = 0, variance = 1)

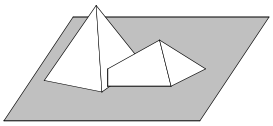
In 3D version:

$y_m = (1/4)(y(a)+y(b)+y(c)+y(d))+r$

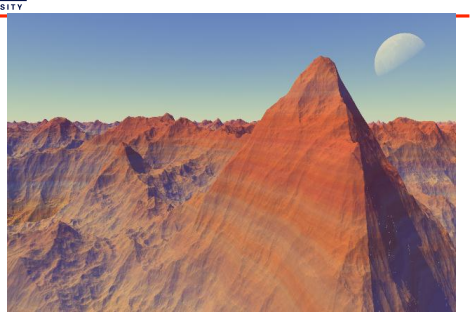


Controlling the Topography

- To model real environments we want to control the placement of peaks and valleys
- We can do it by setting up *Control Surfaces*.
- Calculate random elevations based on:
  - Elevation of control surface
  - Average elevation of the midpoint

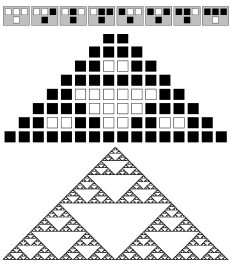


Fractal Mountains
by Ken Musgrave

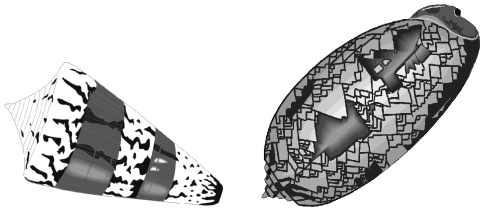


Cellular Automata

- Invented by John von Neumann in 1940s
- Thoroughly studied by Wolfram
- Mechanism to study reproduction
- Dynamic system
  - Discrete in space
  - Discrete in time



Cellular Automata in Nature



- Process which creates seashells has been linked to a one-dimensional CA

Conus Textile Seashell

Source: wikipedia.org

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