Computer Viewing

CS 432 Interactive Computer Graphics

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Objectives

- Introduce the mathematics of projection
- Introduce OpenGL viewing functions
- Look at alternate viewing APIs

Computer Viewing

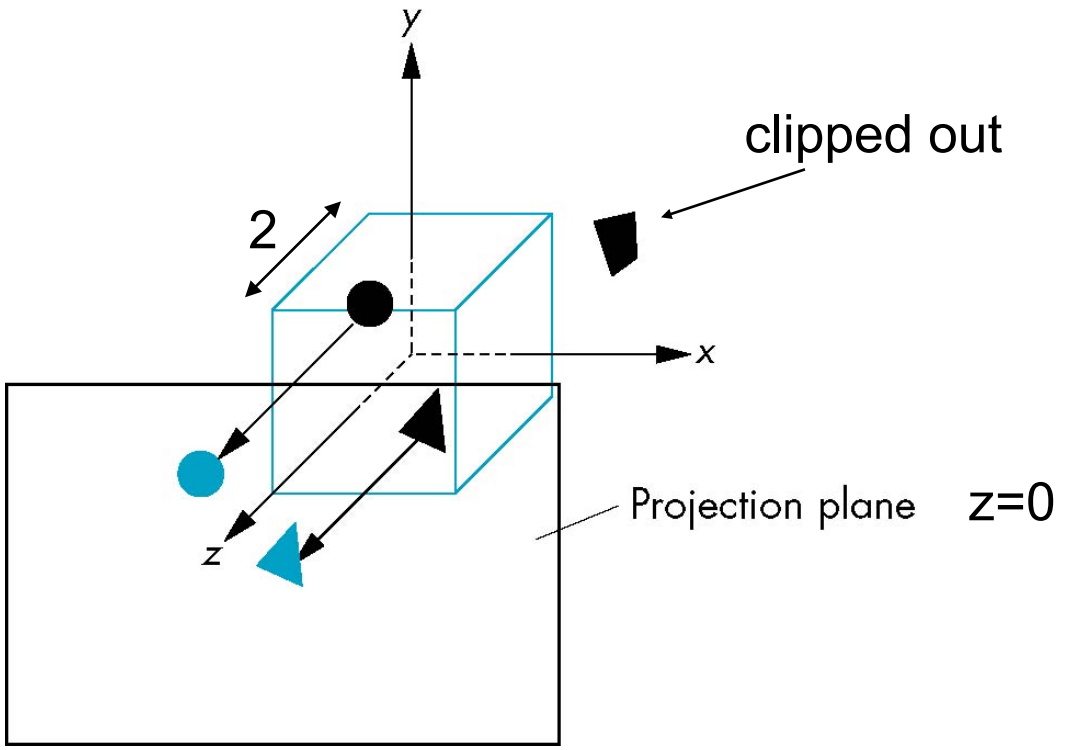
- There are three aspects of the viewing process, all of which should be / are implemented in the pipeline,
  - Positioning the camera
    - Setting the model-view matrix
  - Selecting a lens
    - Setting the projection matrix
  - Clipping
    - Setting the view volume

The OpenGL Camera

- In OpenGL, initially the object and camera frames are the same
  - Default model-view matrix is an identity
- The camera is located at origin and points in the negative z direction
- OpenGL also specifies a default view volume that is a cube with sides of length 2 centered at the origin
  - Default projection matrix is an identity

Default Projection

Default projection is orthographic



Moving the Camera Frame

- If we want to visualize object with both positive and negative z values we can either
  - Move the camera in the positive z direction
    - Translate the camera frame
  - Move the objects in the negative z direction
    - Translate the world frame
- Both of these views are equivalent and are determined by the model-view matrix
  - Want a translation (`Translate(0.0,0.0,-d);`)

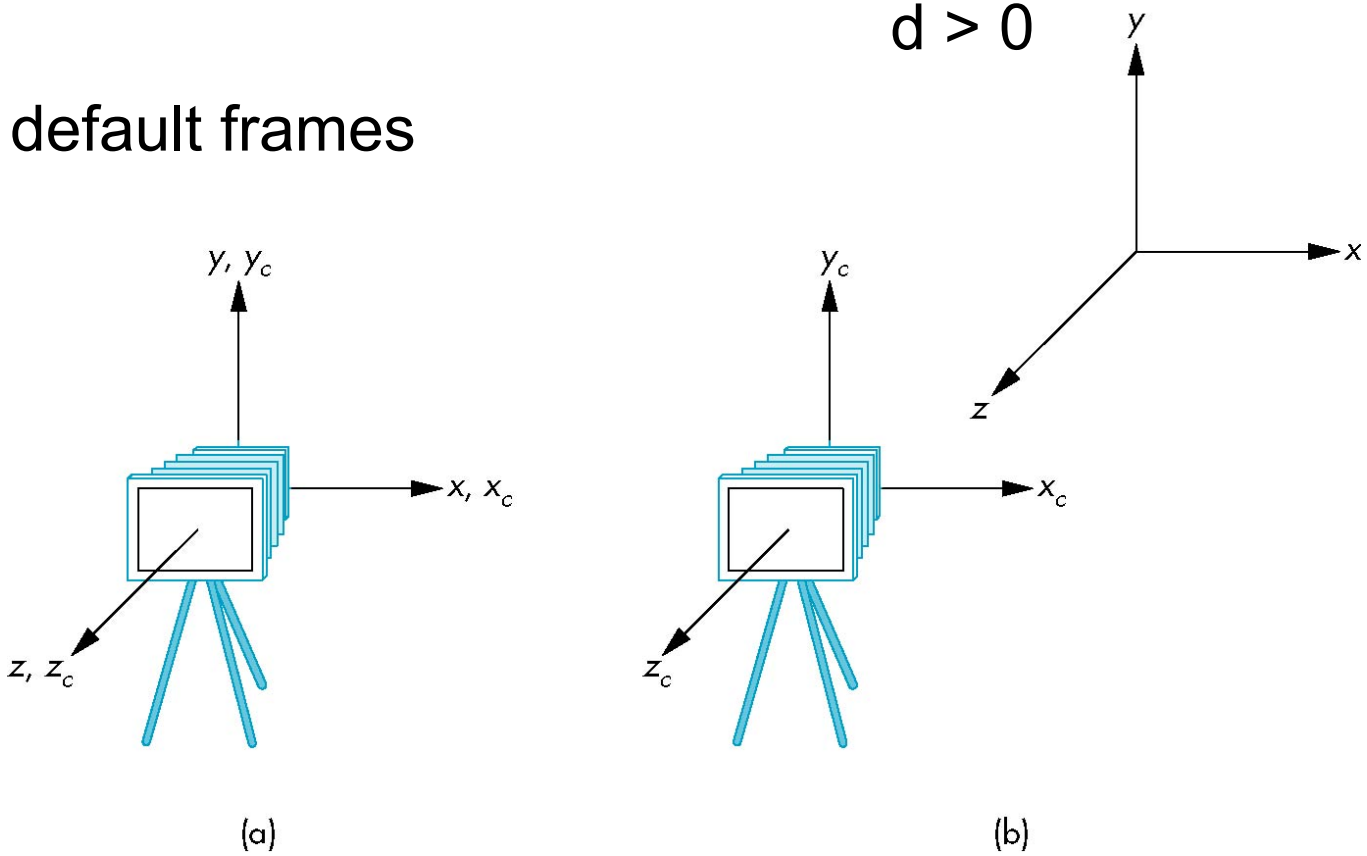
`-d > 0`

Moving Camera back from Origin

frames after translation by –d

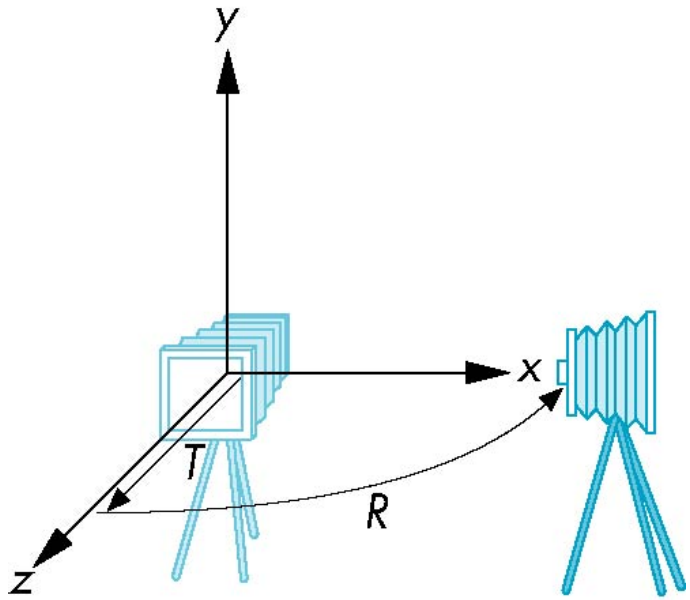
d > 0

default frames



Moving the Camera

- We can move the camera to any desired position by a sequence of rotations and translations
- Example: side view
  - Rotate the camera
  - Move it away from origin
  - Model-view matrix C = TR

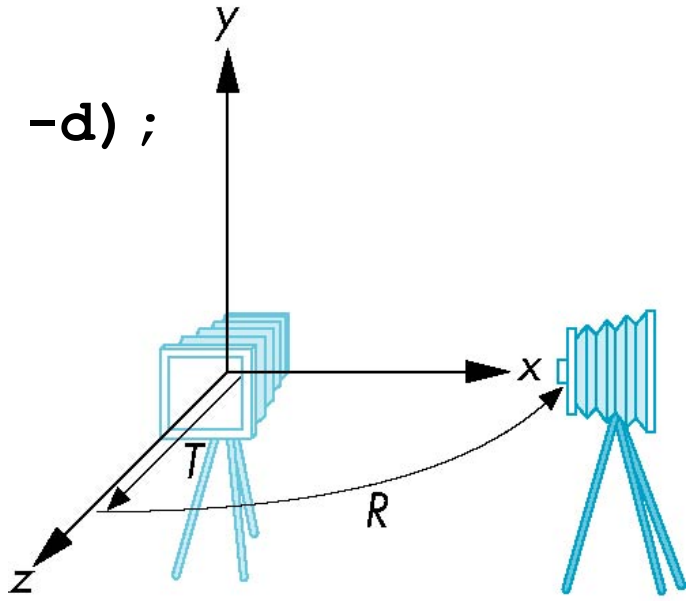


OpenGL code

- Remember that last transformation specified is first to be applied

```
// Using mat.h

mat4 t = Translate(0.0, 0.0, -d);
mat4 ry = RotateY(-90.0);
mat4 m = t*ry;
```



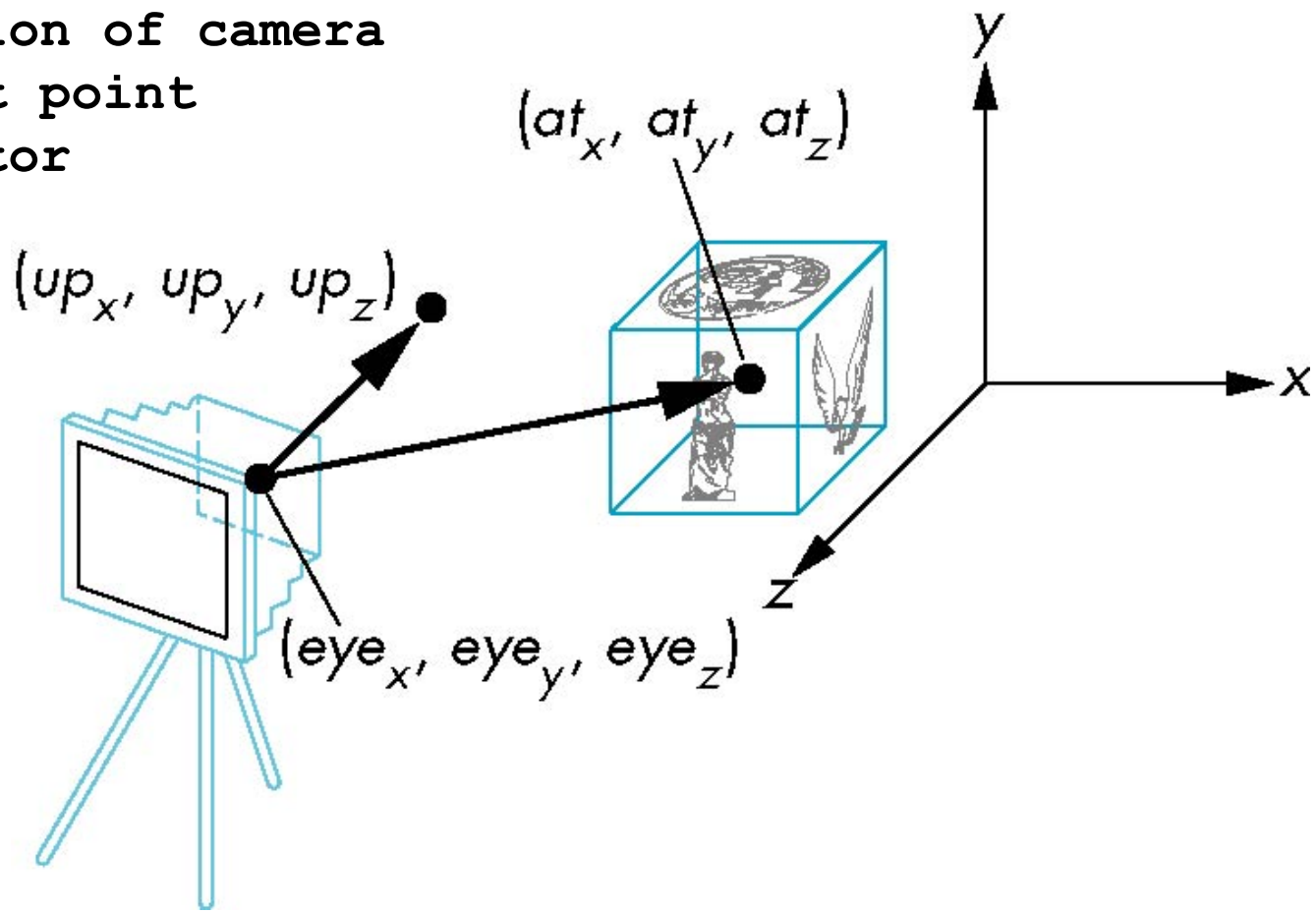
The LookAt Function

- The GLU library contained the function gluLookAt to form the required modelview matrix through a simple interface
- Note the need for setting an up direction
  - Should not be parallel to look-at direction
- Replaced by LookAt() in mat.h
  - Can concatenate with modeling transformations
- Example: isometric view of cube aligned with axes

```
mat4 mv = LookAt(vec4 eye, vec4 at, vec4 up);
```

LookAt Function

```
LookAt(eye, at, up)
 eye - location of camera
 at - look-at point
 up - up vector
```



Other Viewing APIs

- The LookAt function is only one possible API for positioning the camera
- Others include
  - View reference point, view plane normal, view up (PHIGS, GKS-3D)
  - Yaw, pitch, roll
  - Elevation, azimuth, twist
  - Direction angles

Projections and Normalization

- The default projection in the eye (camera) frame is orthographic
- For points within the default view volume

$$x_p = x$$
$$y_p = y$$
$$z_p = 0$$

- Most graphics systems use *view normalization*
  - All other views are converted to the default view by transformations that determine the projection matrix
  - Allows use of the same pipeline for all views

Homogeneous Coordinate Representation

default orthographic projection

$$x_p = x$$
$$y_p = y$$
$$z_p = 0$$
$$w_p = 1$$

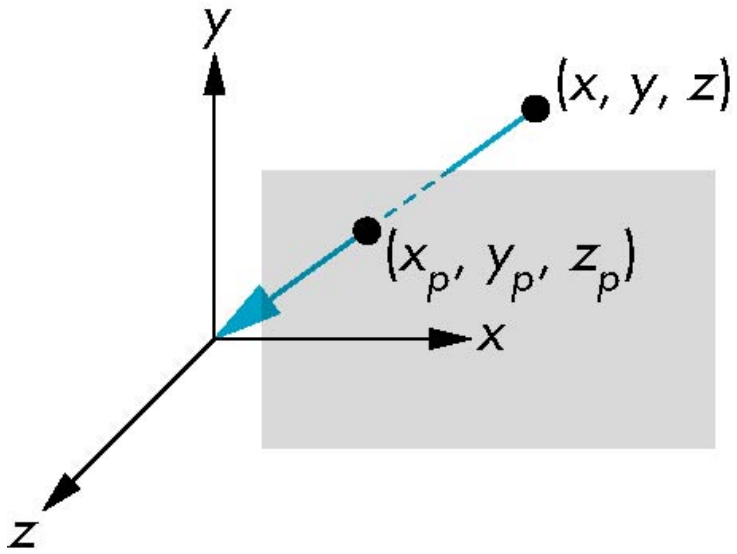
$$\mathbf{p}_p = \mathbf{M}\mathbf{p}$$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In practice, we can let $\mathbf{M} = \mathbf{I}$ and set the z term to zero later

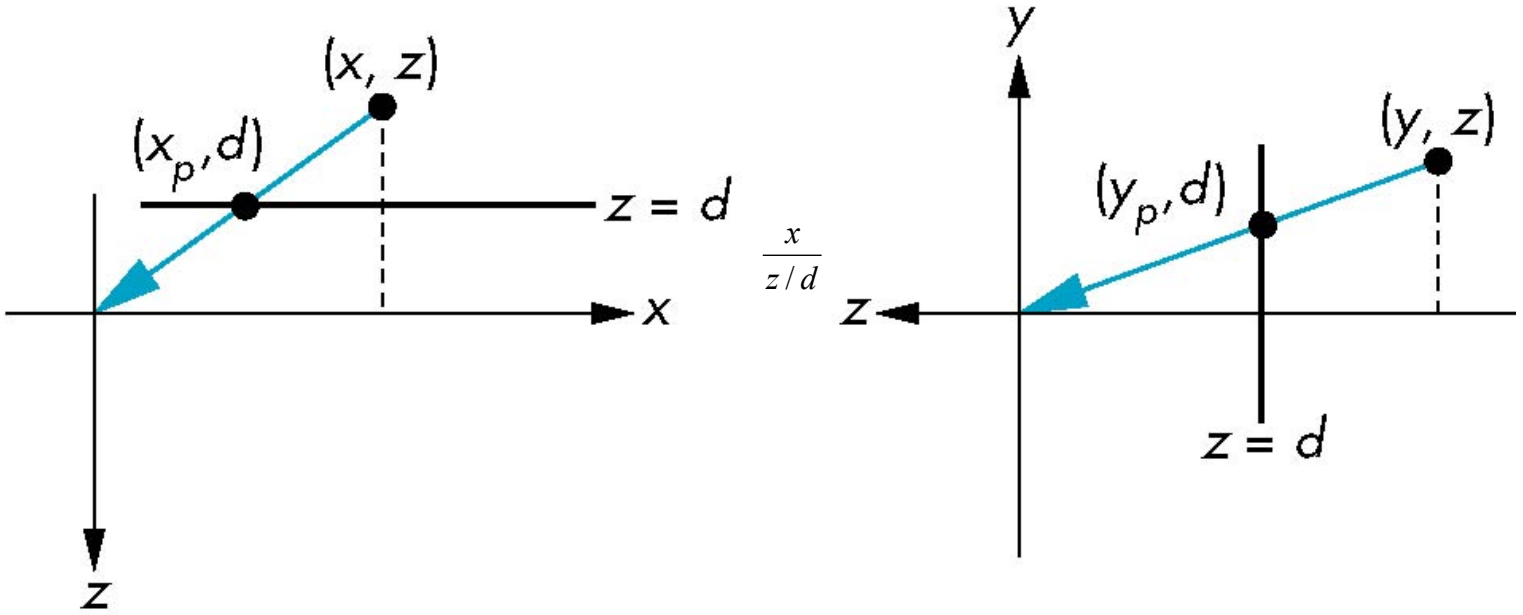
Simple Perspective

- Center of projection at the origin
- Projection plane $z = d, d < 0$



Perspective Equations

Consider top and side views



$$x_p = \frac{x}{z/d} \qquad y_p = \frac{y}{z/d} \qquad z_p = d$$

Homogeneous Coordinate Form

consider $\mathbf{q} = \mathbf{M}\mathbf{p}$ where $\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$

$$\mathbf{q} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad \Rightarrow \quad \mathbf{p} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$

Perspective Division

- However $w \neq 1$, so we must divide by w to return from homogeneous coordinates
- This *perspective division* yields

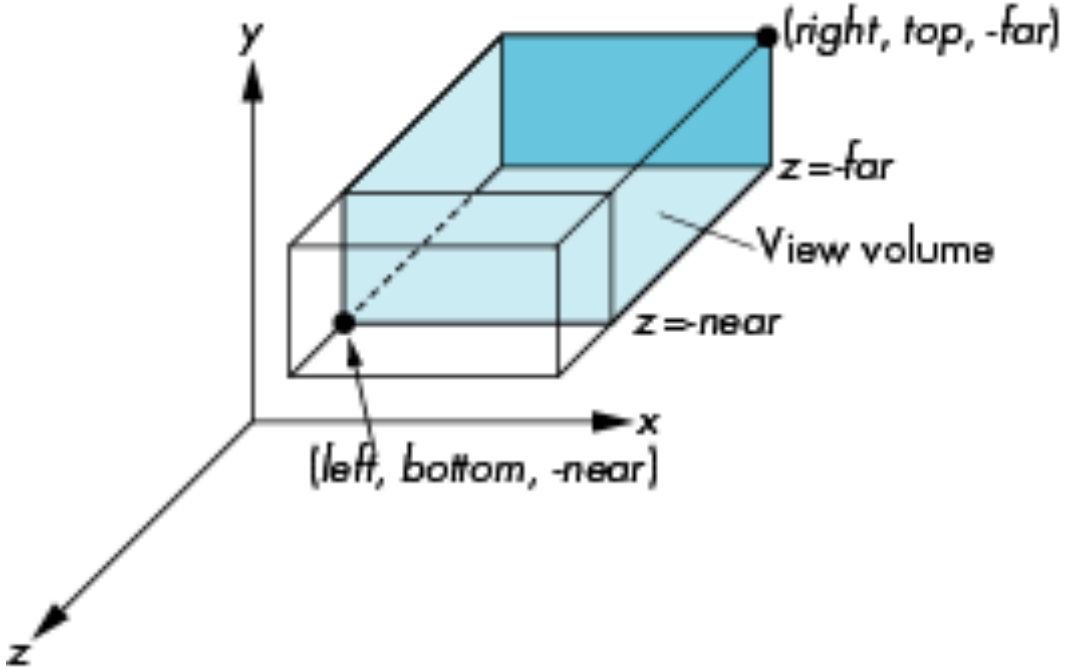
$$x_p = \frac{x}{z/d} \qquad y_p = \frac{y}{z/d} \qquad z_p = d$$

the desired perspective equations

- We will consider the corresponding clipping volume with mat.h functions that are equivalent to deprecated OpenGL functions

OpenGL Orthogonal Viewing

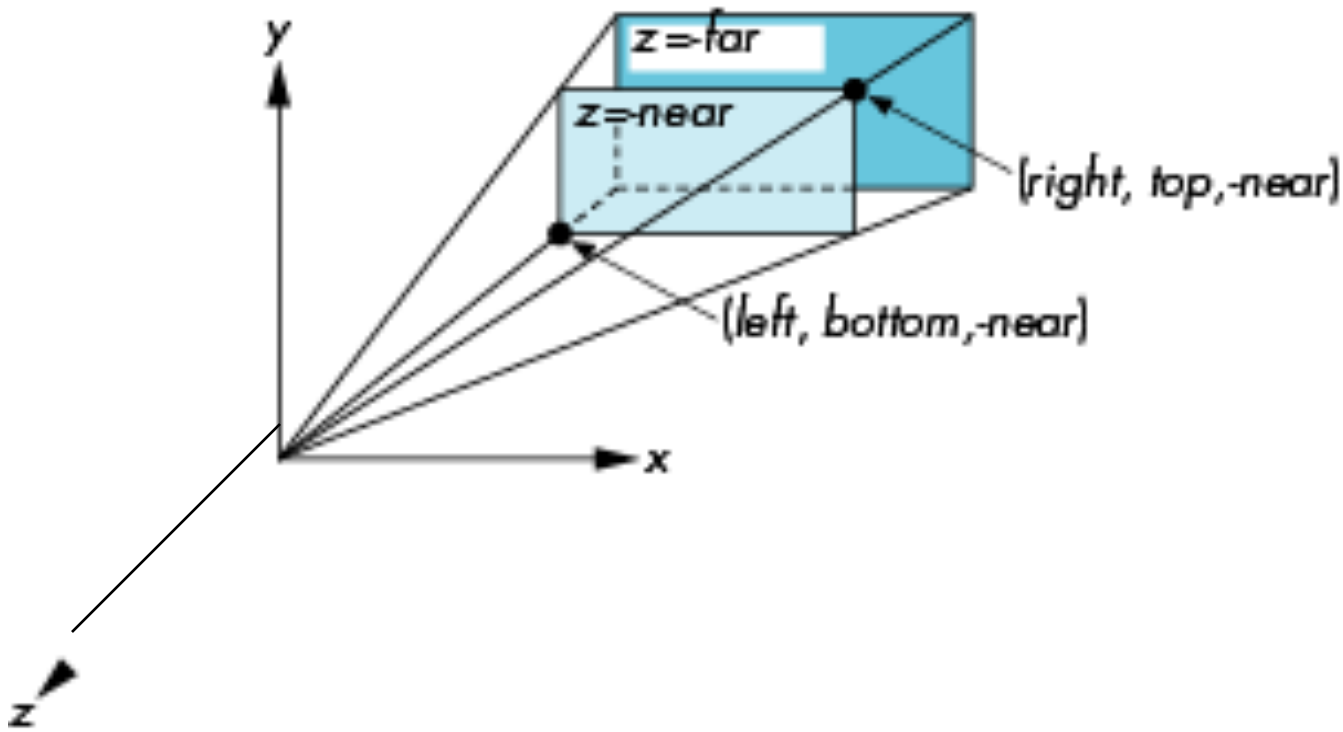
`Ortho(left,right,bottom,top,near,far)`



`near` and `far` measured from camera

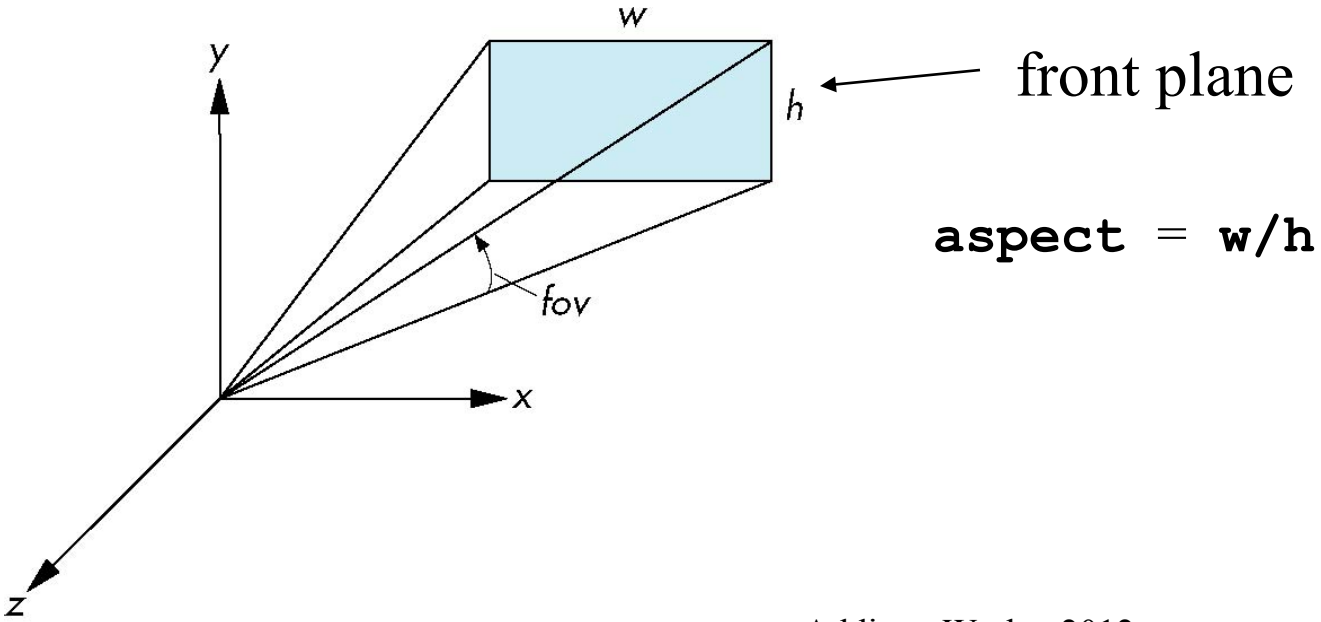
OpenGL Perspective

```
Frustum(left,right,bottom,top,near,far)
```



Using Field of View

- With **`Frustum`** it is often difficult to get the desired view
- **`Perspective(fovy, aspect, near, far)`** often provides a better interface



Projection Matrices

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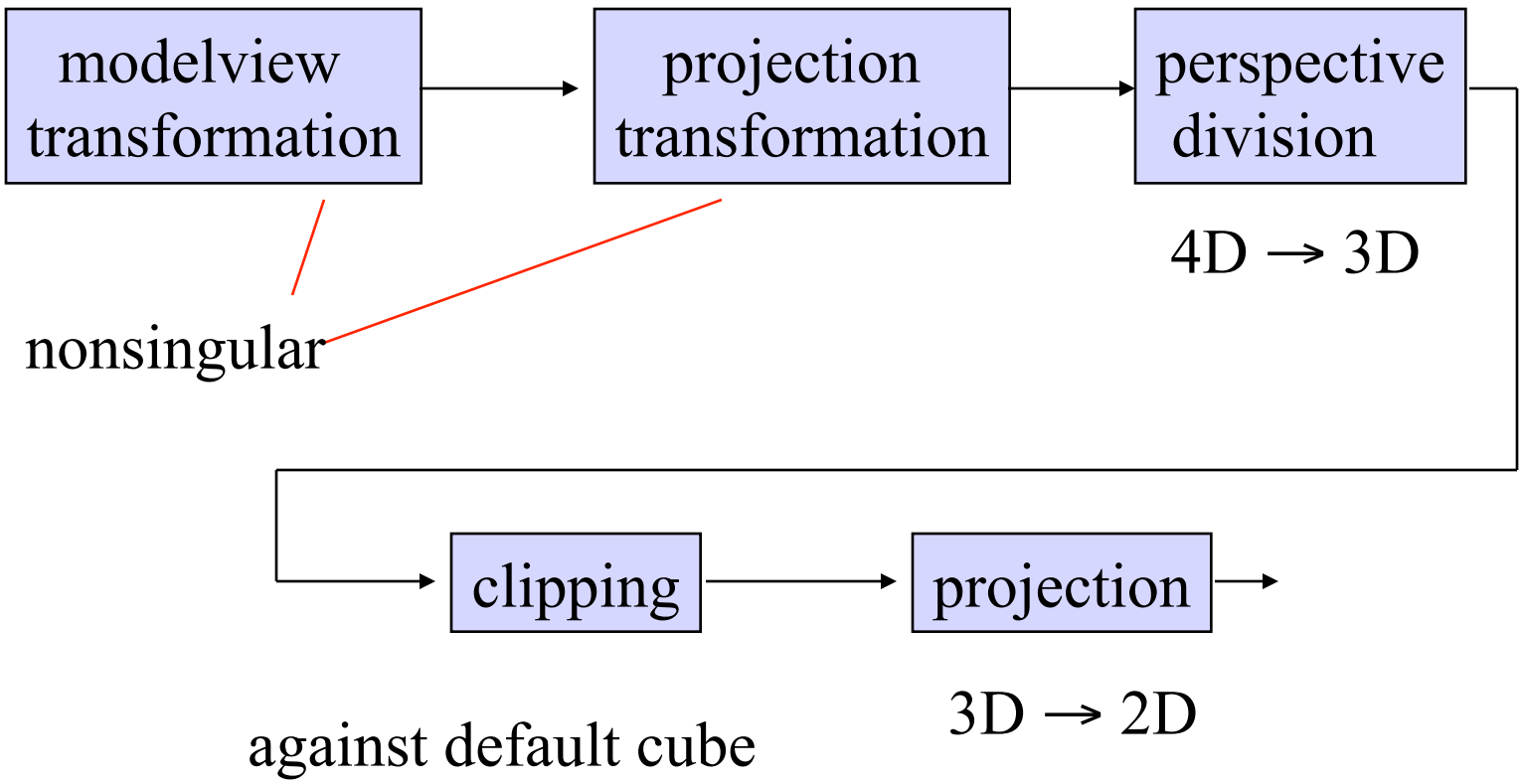
Objectives

- Derive the projection matrices used for standard OpenGL projections
- Introduce oblique projections
- Introduce projection normalization

Normalization

- Rather than derive a different projection matrix for each type of projection, we can convert all projections to orthogonal projections with the default view volume
- This strategy allows us to use standard transformations in the pipeline and makes for efficient clipping

Pipeline View



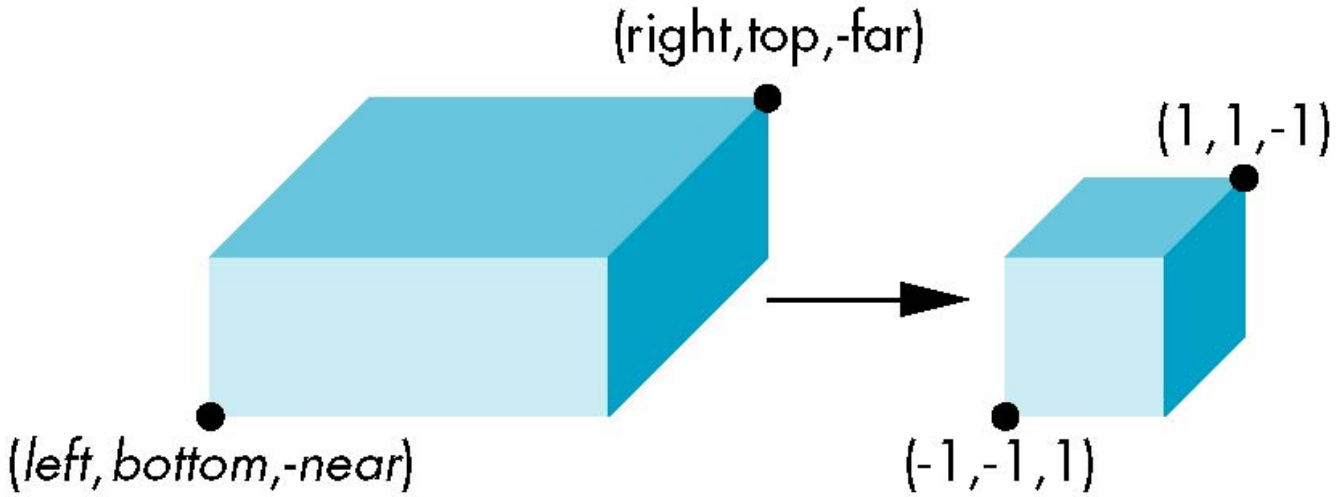
Notes

- We stay in four-dimensional homogeneous coordinates through both the modelview and projection transformations
  - Both these transformations are nonsingular
  - Default to identity matrices (orthogonal view)
- Normalization lets us clip against simple cube regardless of type of projection
- Delay final projection until end
  - Important for hidden-surface removal to retain depth information as long as possible

Orthographic Normalization

`Ortho(left,right,bottom,top,near,far)`

normalization ⇒ find transformation to convert specified clipping volume to default



left < right bottom < top near < far

Orthographic Matrix

- Two steps
  - Move center to origin

    T(-(left+right)/2, -(bottom+top)/2,(near+far)/2))
  - Scale to have sides of length 2

    S(2/(left-right),2/(top-bottom),2/(near-far))

$$\mathbf{P} = \mathbf{ST} = \begin{bmatrix} \frac{2}{right-left} & 0 & 0 & -\frac{right+left}{right-left} \\ 0 & \frac{2}{top-bottom} & 0 & -\frac{top+bottom}{top-bottom} \\ 0 & 0 & \frac{2}{near-far} & \frac{far+near}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Final Projection

- Set $z = 0$
- Equivalent to the homogeneous coordinate transformation

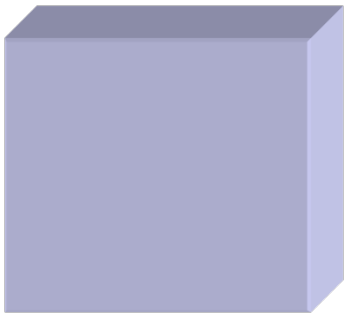
$$\mathbf{M}_{\text{orth}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Hence, general orthographic projection in 4D is

$$\mathbf{P} = \mathbf{M}_{\text{orth}}\mathbf{ST}$$

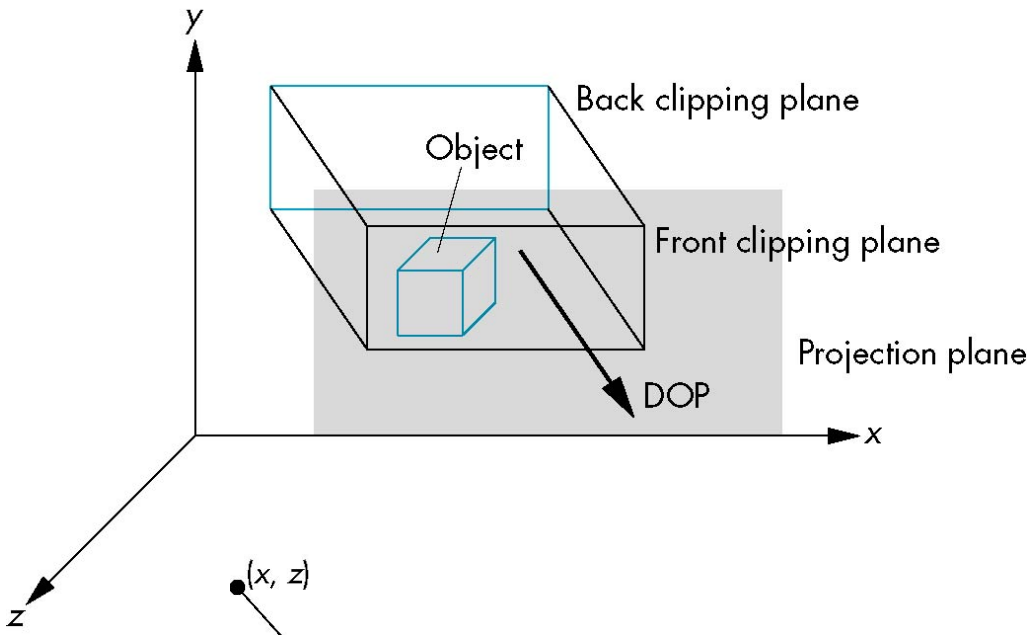
Oblique Projections

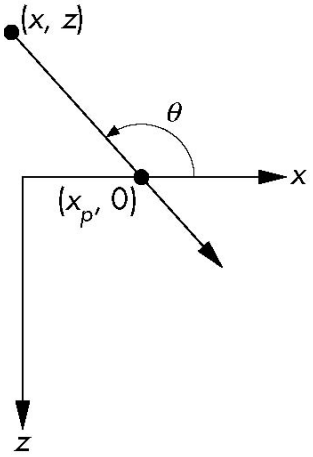
- The OpenGL projection functions cannot produce general parallel projections such as



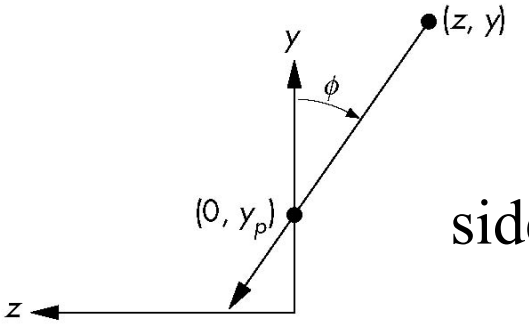
- However if we look at the example of the cube it appears that the cube has been sheared
- Oblique Projection = Shear + Orthographic Projection

General Shear





top view



side view

Shear Matrix

xy shear (*z* values unchanged)

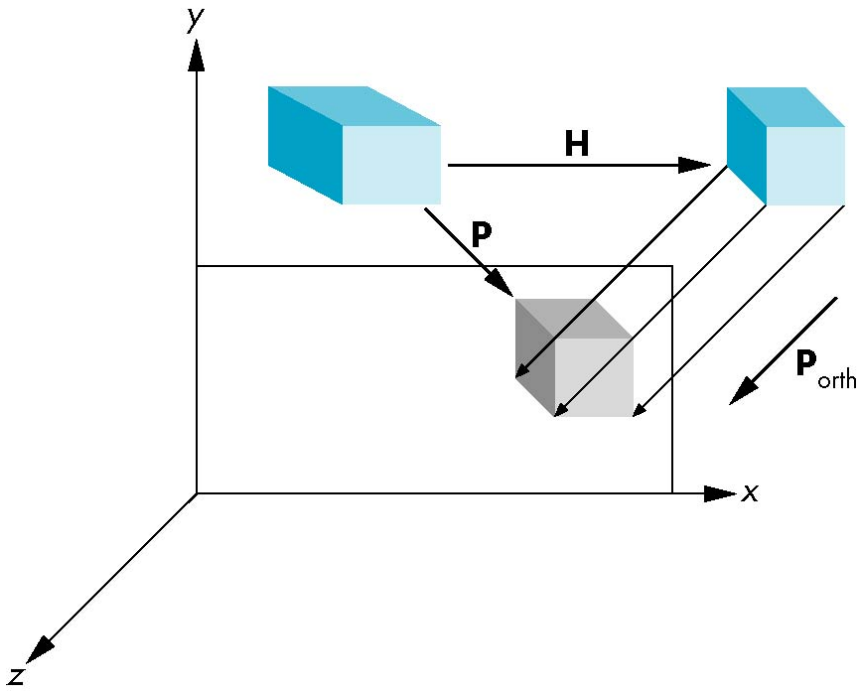
$$\mathbf{H}(\theta,\phi) = \begin{bmatrix} 1 & 0 & -\cot\theta & 0 \\ 0 & 1 & -\cot\varphi & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Projection matrix

$$\mathbf{P} = \mathbf{M}_{\text{orth}}\ \mathbf{H}(\theta,\phi)$$

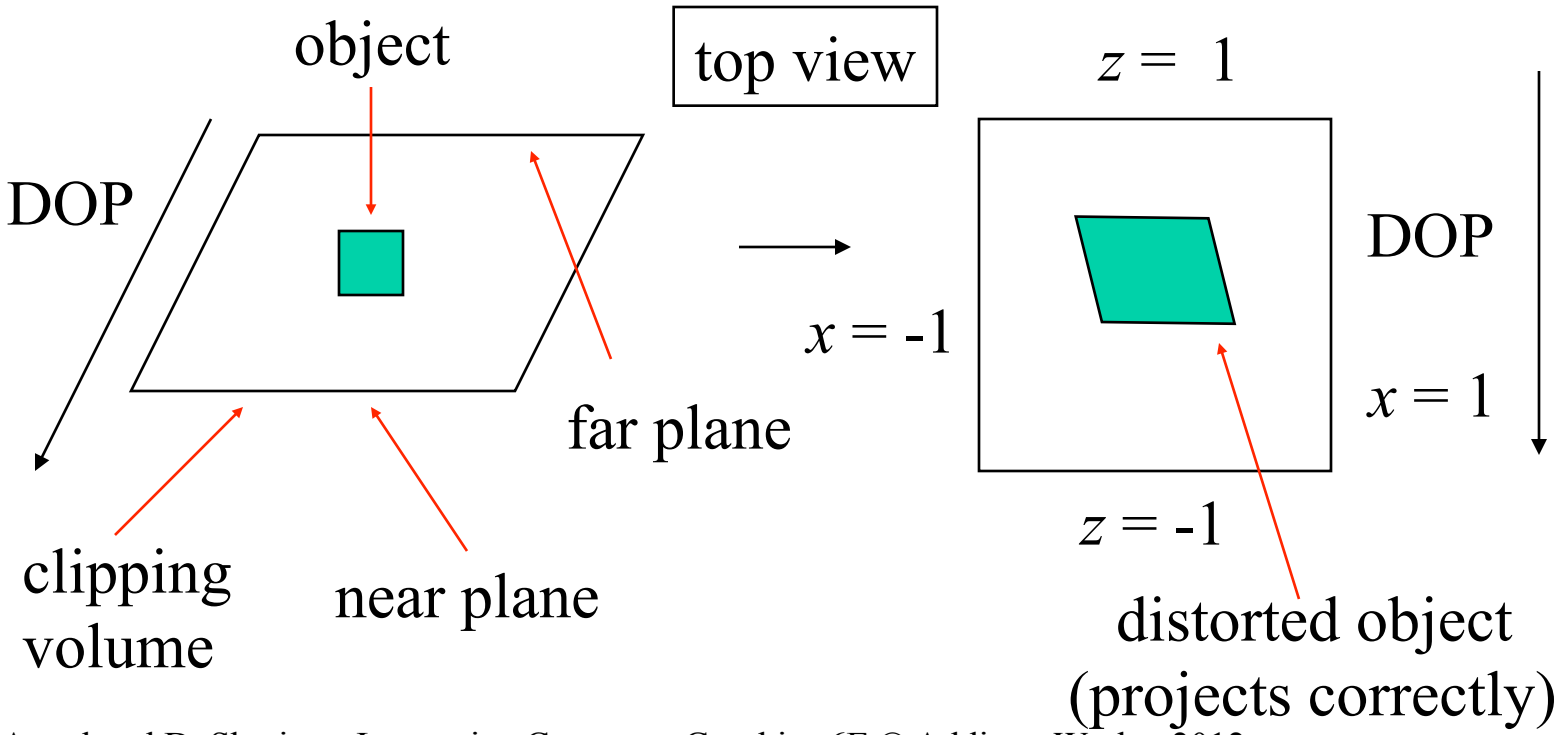
General case: $\mathbf{P} = \mathbf{M}_{\text{orth}}\ \mathbf{STH}(\theta,\phi)$

Equivalency



Effect on Clipping

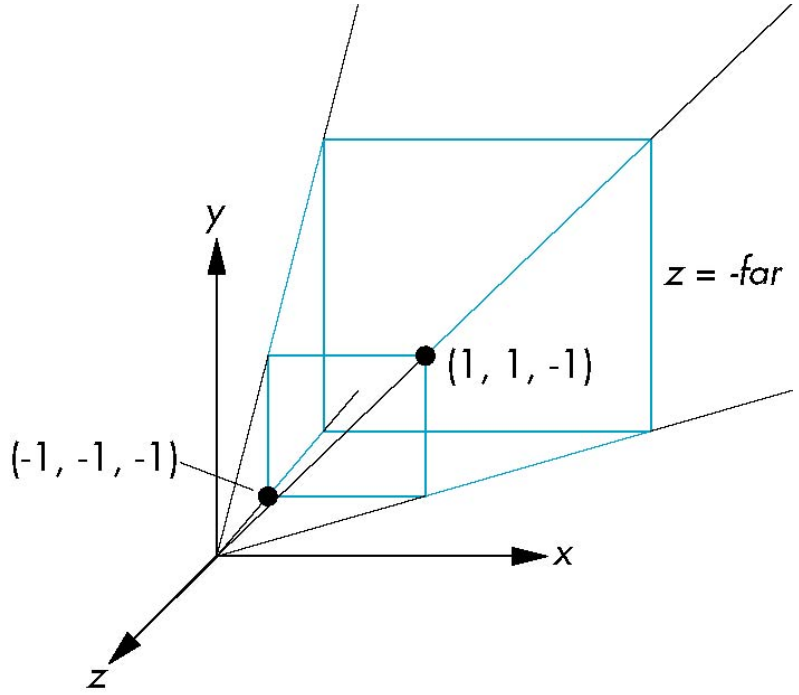
- The projection matrix $\mathbf{P} = \mathbf{STH}$ transforms the original clipping volume to the default clipping volume



Simple Perspective

Consider a simple perspective with the COP at the origin, the near clipping plane at $z = -1$, and a 90 degree field of view determined by the planes

$x = \pm z, y = \pm z$



Perspective Matrices

Simple projection matrix in homogeneous coordinates

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Note that this matrix is independent of the far clipping plane

Generalization

$$\mathbf{N} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & \beta \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

after perspective division, the point $(x, y, z, 1)$ goes to

$$x'' = x/z$$
$$y'' = y/z$$
$$Z'' = -(\alpha+\beta/z)$$

which projects orthogonally to the desired point regardless of α and β

Picking α and β

If we pick

$$Z'' = -(\alpha + \beta/z)$$

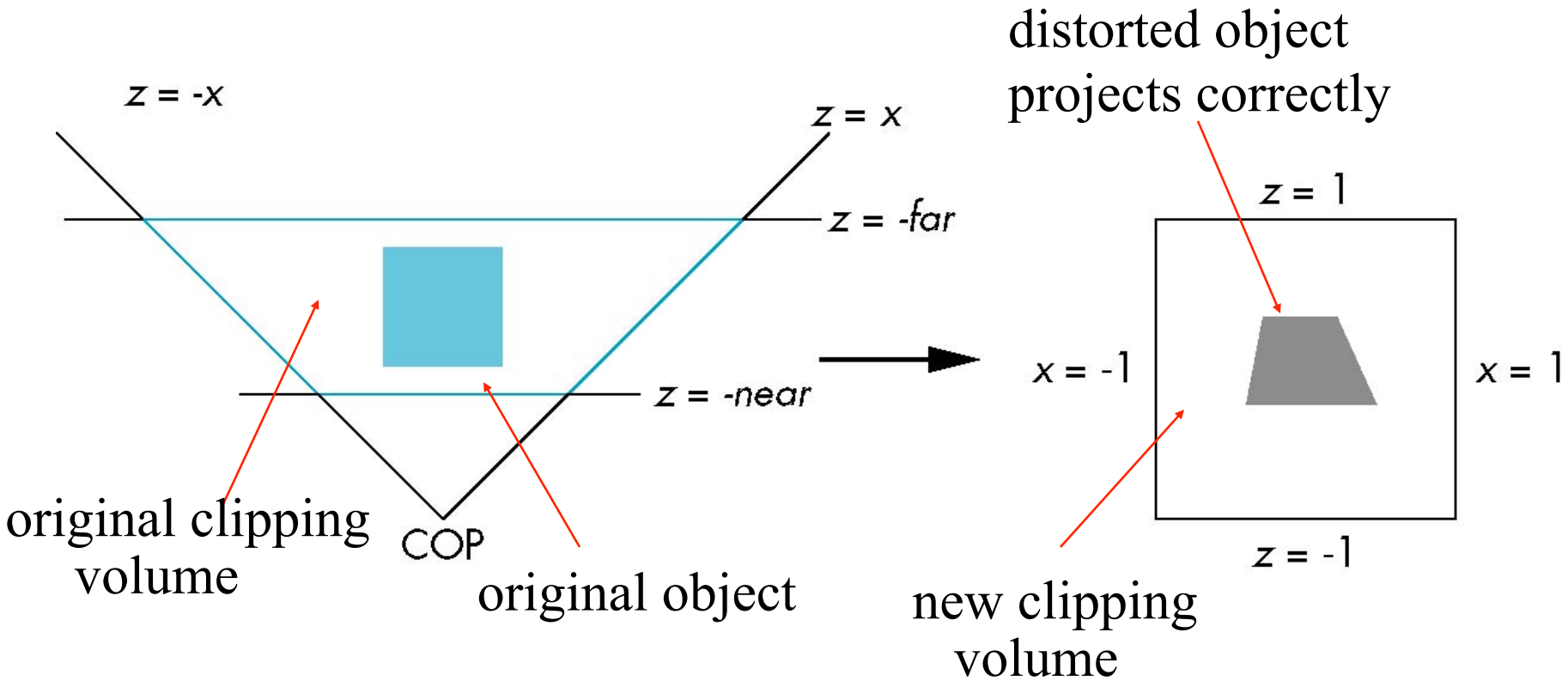
$$\alpha = \frac{\text{near} + \text{far}}{\text{far} - \text{near}}$$

$$\beta = \frac{2\text{near} * \text{far}}{\text{near} - \text{far}}$$

the near plane is mapped to $z = -1$
the far plane is mapped to $z = 1$
and the sides are mapped to $x = \pm 1, y = \pm 1$

Hence the new clipping volume is the default clipping volume

Normalization Transformation



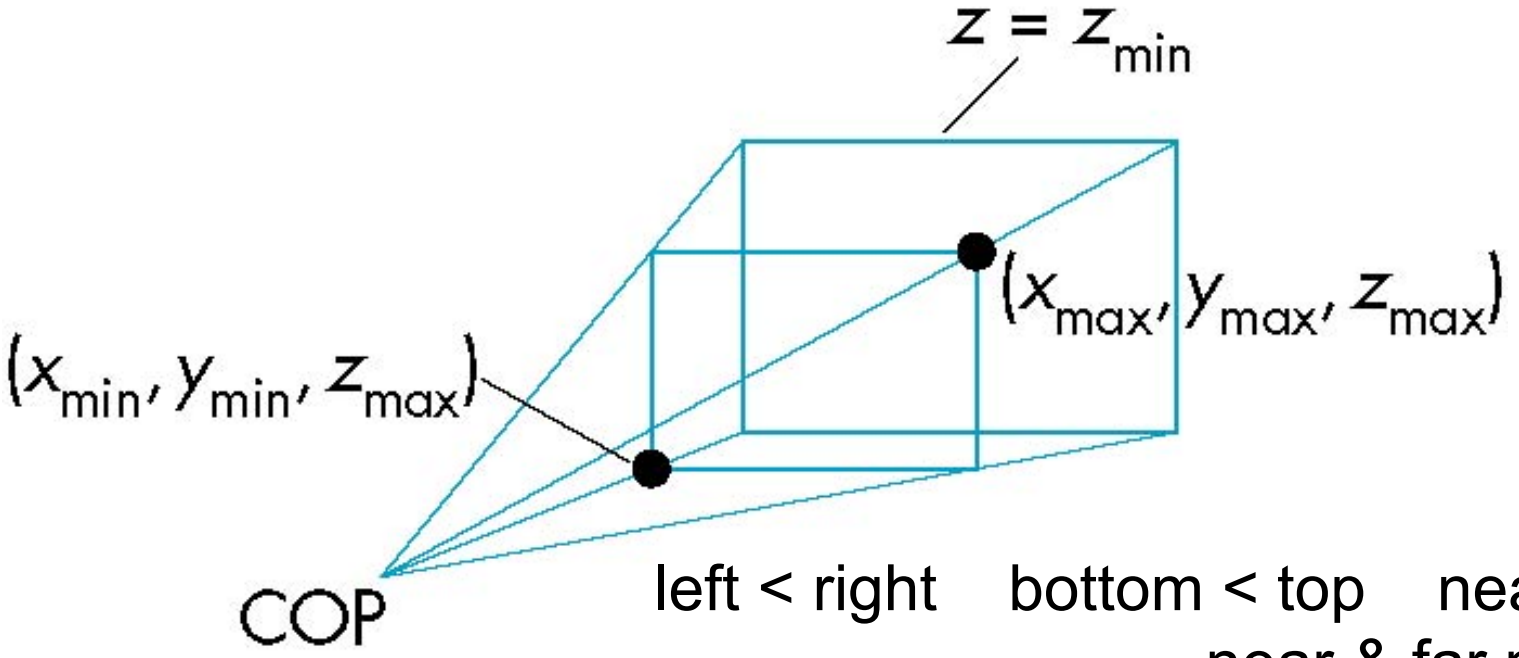
Normalization and Hidden-Surface Removal

- Although our selection of the form of the perspective matrices may appear somewhat arbitrary, it was chosen so that if $z_1 > z_2$ in the original clipping volume then the for the transformed points $z_1' > z_2'$
- Thus hidden surface removal works if we first apply the normalization transformation
- However, the formula $z'' = -(\alpha+\beta/z)$ implies that the distances are distorted by the normalization which can cause numerical problems especially if the near distance is small

OpenGL Perspective

- **`Frustum`** allows for an unsymmetric viewing frustum (although **`Perspective`** does not)

`Frustum(left,right,bottom,top,near,far)`



left < right bottom < top near < far

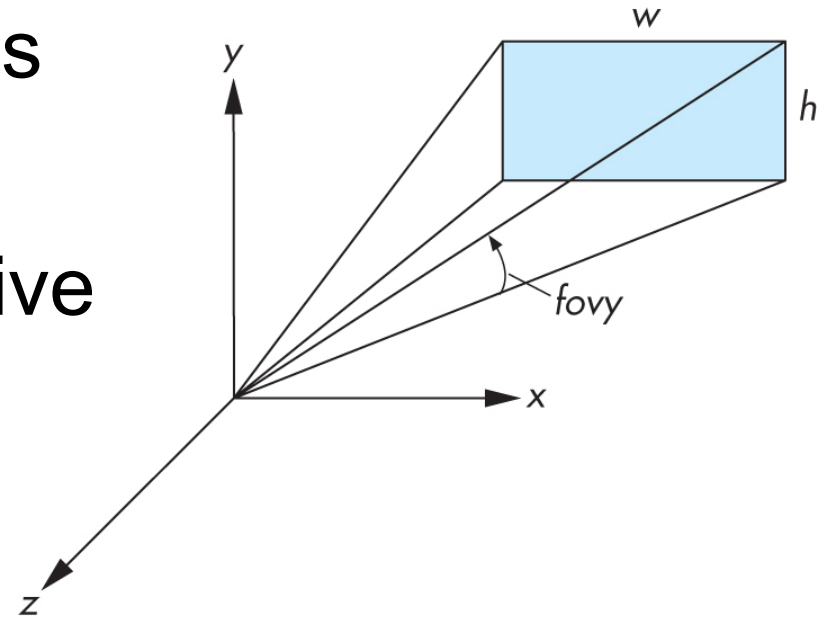
near & far positive

OpenGL Perspective

- **`Perspective`** provides less flexible, but more intuitive perspective viewing

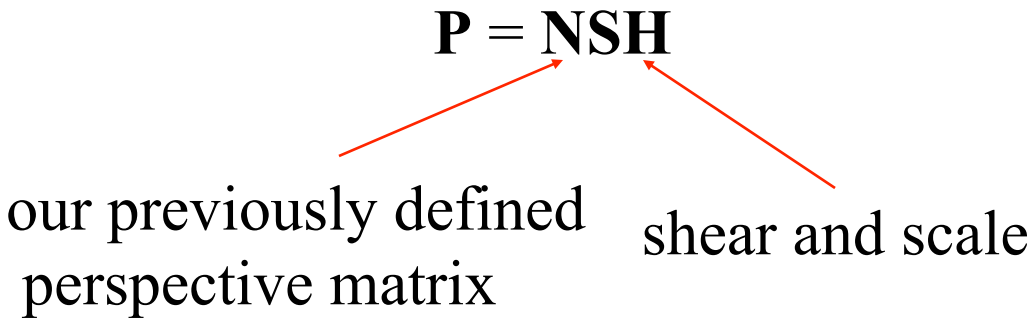
```
Perspective(fov, aspect, near, far);
```

- Field of view in angles
- aspect: w/h
- near < far, both positive



OpenGL Perspective Matrix

- The normalization in `Frustum` requires an initial shear to form a right viewing pyramid, followed by a scaling to get the normalized perspective volume. Finally, the perspective matrix results in needing only a final orthographic transformation



Why do we do it this way?

- Normalization allows for a single pipeline for both perspective and orthogonal viewing
- We stay in four dimensional homogeneous coordinates as long as possible to retain three-dimensional information needed for hidden-surface removal and shading
- We simplify clipping

Viewing OpenGL Code

- It's not too bad, thanks to Ed Angel
- In application

```
model_view =  LookAt(eye, at, up);
projection = Ortho(left, right, bottom, top,
                                   near, far);
```

or

```
projection = Perspective( fov, aspect, near,
                                        far);
```

- In vertex shader

```
gl_Position = projection*model_view*vPosition;
```

OpenGL code

- Remember that matrices are column major order in GLSL, so …

Transpose your matrices when sending them to the shaders!

```
glUniformMatrix4fv(matrix_loc, 1, GL_TRUE,
                                   model_view);
```