Linear Algebra

What is a Vector?

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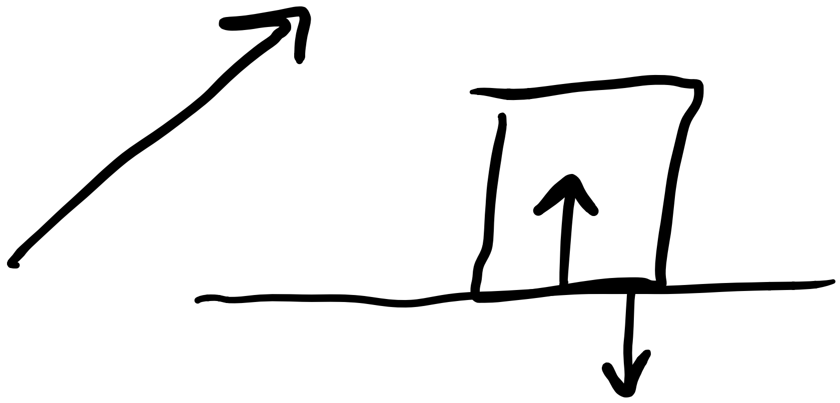
Overview

- Intuition behind Vectors
- Vector Arithmetic
- Geometric Interpretation

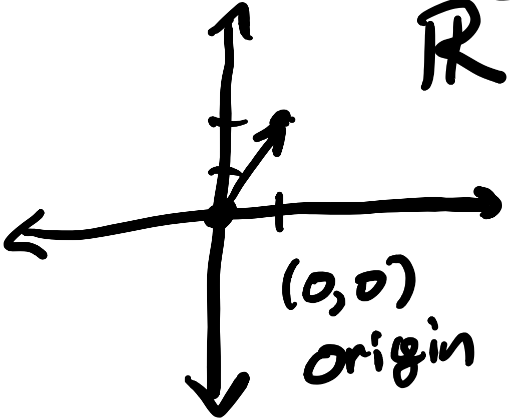
Vectors



Points vs. Vectors

$$v = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \qquad (1, 2)$$

$$\mathbb{R}^2 = \{ (x, y) : x, y \in \mathbb{R} \}$$



- floating point
- continuous value

Vector Space

$$\mathbb{R}^N = \{ (x_1, x_2, \ldots, x_N) \mid x_i \in \mathbb{R} \}$$

$$\vec{0} = (0, 0, 0, \ldots, 0)$$

"origin"

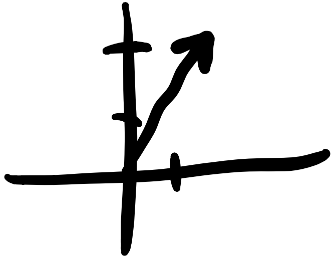
"zero vector"

Vectors

$\begin{pmatrix} x \\ y \end{pmatrix}$ $\qquad$ $(x \ \ y)$

$$\vec{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}_{2\times 1} \qquad \vec{v} = \begin{pmatrix} 1 & 2 \end{pmatrix}_{1\times 2}$$

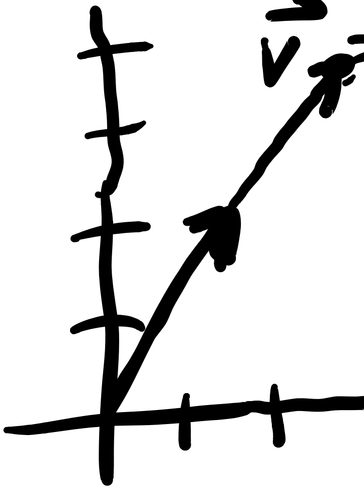
column vector $\qquad$ row vector



r x c

rows $\quad$ columns

Scalar Multiplication



$$\vec{v} = \begin{pmatrix}1\\2\end{pmatrix}$$

$$\underline{2}\cdot\vec{v} = \begin{pmatrix}2\cdot 1\\2\cdot 2\end{pmatrix} = \begin{pmatrix}2\\4\end{pmatrix}$$

scalar

$$c\cdot\vec{v} = c\begin{pmatrix}v_1\\v_2\\\vdots\\v_N\end{pmatrix} = \begin{pmatrix}c\cdot v_1\\c\cdot v_2\\\vdots\\c\cdot v_N\end{pmatrix}$$

Scalar

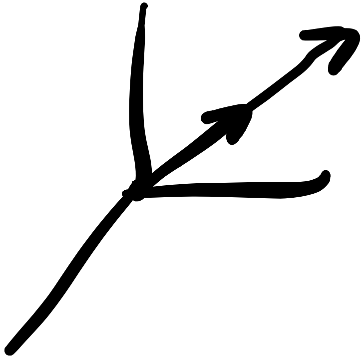
vector

Scalar Multiplication

$$-2\begin{pmatrix}1\\2\end{pmatrix}=\begin{pmatrix}-2\\-4\end{pmatrix}$$

$$2\begin{pmatrix}0\\0\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$$

$$\boxed{4\begin{pmatrix}1\\2\\3\\4\end{pmatrix}=\begin{pmatrix}4\\8\\12\\16\end{pmatrix}}$$



Vector Addition

$$\vec{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \vec{w} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\vec{v} + \vec{w} = \begin{pmatrix} 1 + 0 \\ 2 + 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{pmatrix} \quad \vec{w} = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{pmatrix} \quad \vec{v} + \vec{w} = \begin{pmatrix} v_1 + w_1 \\ v_2 + w_2 \\ \vdots \\ v_N + w_N \end{pmatrix}$$

Vector Addition

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1+0 \\ 2+0 \\ 3+0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

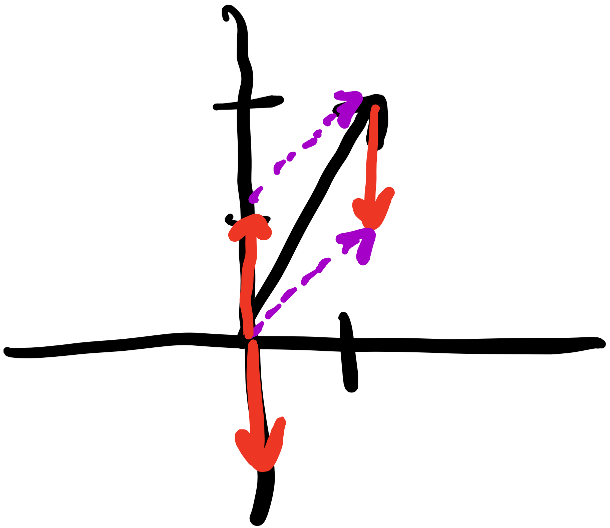
$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 3 \\ 5 \end{pmatrix}$$

Vector Addition

Vector Subtraction

$$\underbrace{\begin{pmatrix} 1 \\ 2 \end{pmatrix}}_{\vec{v}} - \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{\vec{w}} = \underbrace{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}_{\vec{v}-\vec{w}}$$



$$\vec{v}-\vec{w} + \vec{w} = \vec{v}$$

All Together Now

$$3\begin{pmatrix}1\\2\\10\end{pmatrix} - 2\begin{pmatrix}0\\1\\100\end{pmatrix}$$

$$= \begin{pmatrix}3\\6\\30\end{pmatrix} - \begin{pmatrix}0\\2\\200\end{pmatrix} = \begin{pmatrix}3\\4\\-170\end{pmatrix}$$