Linear Algebra

What is a Matrix?

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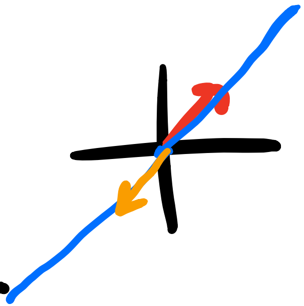
Is a given set of vectors linearly independent?

- Is there a subset of these vectors with the same span?
- What is the dimension of the span of these vectors?

Vectors and Geometry

The span of a set of vectors always forms a linear space.

- The dimension of this linear space is = to the size of a basis for this set.



$\text{Span}(v_1) \rightarrow$ line

$\text{Span}(\{v_1, v_2\})$ $\rightarrow$ line lin. dep.

$\rightarrow$ plane lin. indep.

The Matrix : Collection of vectors

$$\underset{v_1}{\begin{bmatrix}1\\1\\0\end{bmatrix}}, \underset{v_2}{\begin{bmatrix}-1\\1\\0\end{bmatrix}}, \underset{v_3}{\begin{bmatrix}1\\2\\1\end{bmatrix}} \in \mathbb{R}^3 \Rightarrow \underset{3\times 3}{\begin{bmatrix}1 & -1 & 1\\1 & 1 & 2\\0 & 0 & 1\end{bmatrix}}$$

Column Space of A: The span of the columns of A, $colsp(A)$ $(= span\{v_1, v_2, v_3\})$

Row Space of A: The span of the rows of A.

The dimension of A is $M \times N$
M - # of rows N - # of cols.

The Matrix

RREF - Reduced Row Echelon Form

→ Manipulating the rows of the Matrix A
(preserve the Row Space of A)

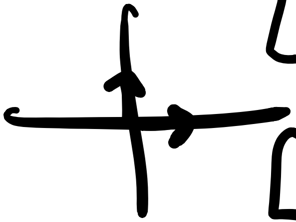
↓

Span of vectors

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \text{ swap}$$

$2 \cdot$ ↑ scale

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



$$\begin{bmatrix} 2 & 41 \\ 6 & -1 \end{bmatrix}$$

The Matrix: Linear Map from one linear space to another.

$A\vec{x} \longrightarrow \vec{y}$

$\vec{y}$ $\mathbb{R}^3$

$\mathbb{R}^2$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ -x_2 - 2x_3 \end{bmatrix}$$

$r \times c$

2×3

3-dim input

2-dim output

The Matrix