Linear Algebra

Matrix Determinant

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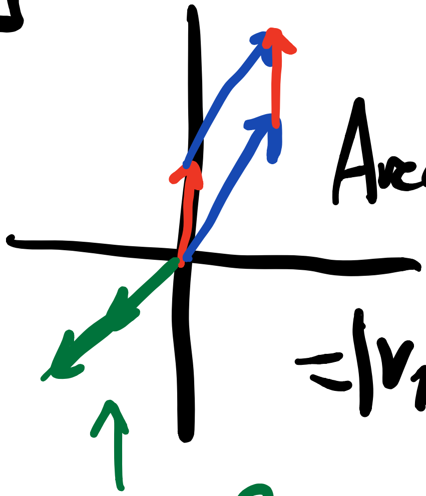
2x2 Example

$\vec{v} = \begin{bmatrix}1\\2\end{bmatrix}$ $\vec{w} = \begin{bmatrix}0\\1\end{bmatrix}$ $\vec{v} = \begin{bmatrix}v_1\\v_2\end{bmatrix}$ $\vec{w} = \begin{bmatrix}w_1\\w_2\end{bmatrix}$

$$\begin{bmatrix}a & b\\ c & d\end{bmatrix}$$

$$\boxed{\begin{array}{c}\underline{ad - bc}\\ \underline{\text{determinant}}\end{array}}$$



Area(Parallelogram) $= |v_1 w_2 - v_2 w_1|$

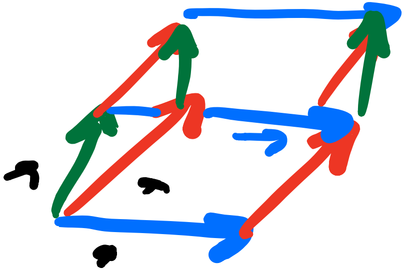
Area = 0

$\begin{bmatrix}-1\\-1\end{bmatrix}$ $\begin{bmatrix}-2\\-2\end{bmatrix}$ $(-1(-2)) - (-1(-2)) = 0$

Determinant

Definition For any square $N \times N$ matrix there is a $f^{\underline{n}}$ on the entries of the matrix returning a scalar value which we call the determinant.

- Invertible iff non-zero det.



volume $= |\, 3\times3 \text{ det}\,|$

$\det = 0$

3x3 Determinant

$$\begin{bmatrix} 1 & 3 & 7 \\ 0 & 2 & 0 \\ -2 & 0 & -1 \end{bmatrix}$$

$$1(2(-1) - 0(0))$$
$$-3(0(-1) - 0(-2))$$
$$+7(0(0) - -2(2))$$

$$1(-2) - 3(0) + 7(4)$$
$$= -2 + 28 = 26$$

Laplace Expansion

Definition The (i,j)-minor of a matrix is the determinant of M_{ij} which is the matrix by deleting the i^{th} row & j^{th} column.

$$\begin{bmatrix} 1 & 3 & 7 \\ 0 & 2 & 0 \\ -2 & 0 & -1 \end{bmatrix} \quad \det(M_{1,2}) = 0(-1) - 0(-2) = 0$$

$$\begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ -2 & -1 \end{bmatrix} \quad \det(M_{2,3}) = 1(0) - (-2)3 = 6$$

Laplace Expansion

For any fixed row i, $M_{13} = \begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$

$$\underline{\det(A)} = \sum_{j=1}^{N} (-1)^{i+j} a_{ij} \det(M_{ij})$$

$i=1$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$a_{11} \det(M_{11}) + (-1) a_{12} \det(M_{12}) + (-1)^{1+3} a_{13} \det(M_{13})$$

$M_{12} = \begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix}$ $M_{11} = \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix}$

Inverse through Determinants

Definition) Cofactor Matrix is a matrix of minors

- C where $c_{ij} = (-1)^{i+j} \det(M_{ij})$
- $\mathrm{Adj}(A) = C^T$
- $A^{-1} = \frac{1}{\det(A)} \mathrm{Adj}(A)$