## Linear Algebra

Reduced Row Echelon Form

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RREF; Gauss-Jarden Elimination Defn) A matrix is in RREF provided: 1) The 1st non-zero entry of each row is one.
2) All values above/below a pivot ar zero.
3) The rows are arranged by their pivots
i.e. The top row has the left most pivot, all rows at Zeros one at the bottom. Every matrix has a unique RREF!

(iff co if and and if) a square matrix is invertible iff its RREF is the idulity matrix - The RREF of a matrix has the same rou space. (column) - The dimension of the row space of a matrix is the #ofpinals in its RREF.

**RREF** 

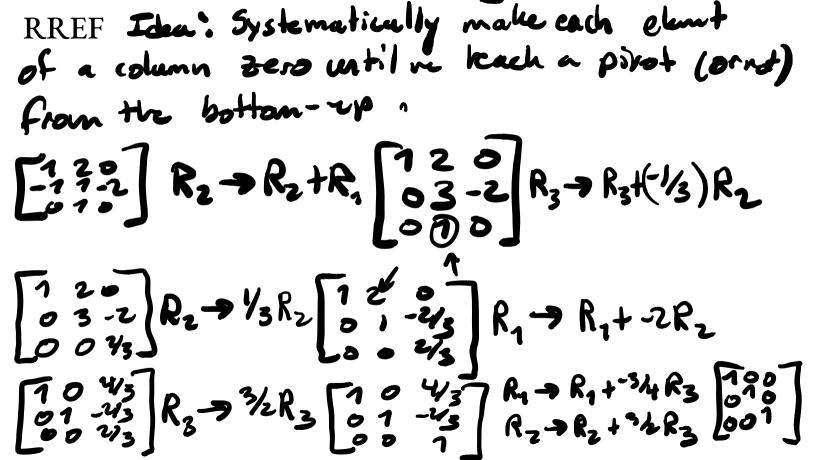
Elementary Row Operations: Manipulate the rows of A Wo changing the row space. 1) Swap any two rows (non-zero)
2) Multiply any row by a scalar
(non-zero)
3) Add a multiple of one row to wother.

All Here operations preserve the row space

Elementary Row Operations 120
$$\begin{bmatrix}
120 \\
-1-1-2 \\
010
\end{bmatrix}$$

$$+2(-1-12)$$
1) Row Swap;  $R_1 \leftrightarrow R_2 \rightarrow \begin{bmatrix} -1-1-2 \\ 010 \end{bmatrix}$ 
2) Scalar Multiple;  $R_1 \rightarrow 2R_1 \rightarrow \begin{bmatrix} 240 \\ 010 \end{bmatrix}$ 
3) Ald arow:  $R_1 \rightarrow R_1 + 2R_2$ 

$$- \sum_{i=1}^{-1} -7-2i \\
07-2i \rightarrow 7-2i$$



RREF
$$\begin{bmatrix}
1 & 2 & 0 \\
3 & -1 & 2 \\
-2 & 3 & -2
\end{bmatrix}
R_3 \rightarrow R_3 + 2R_1
\begin{bmatrix}
1 & 2 & 0 \\
3 & -1 & 2 \\
0 & 7 & -2
\end{bmatrix}
R_2 \rightarrow R_2 - 3R_1$$

$$\begin{bmatrix}
1 & -\frac{2}{3} & 0 \\
8 & -\frac{2}{3} & 2
\end{bmatrix}
R_3 \rightarrow R_3 + R_2
\begin{bmatrix}
1 & 2 & 0 \\
0 & -\frac{2}{3} & 2
\end{bmatrix}
R_2 \rightarrow R_2$$

$$\begin{bmatrix}
1 & -\frac{2}{3} & 0 \\
8 & -\frac{2}{3} & 2
\end{bmatrix}
R_2 \rightarrow R_2$$

$$\begin{bmatrix}
1 & -\frac{2}{3} & 0 \\
0 & -\frac{2}{3} & 2
\end{bmatrix}
R_2 \rightarrow R_2$$

$$\begin{bmatrix}
1 & -\frac{2}{3} & 0 \\
0 & -\frac{2}{3} & 2
\end{bmatrix}
R_2 \rightarrow R_2$$

$$\begin{bmatrix}
1 & -\frac{2}{3} & 0 \\
0 & -\frac{2}{3} & 2
\end{bmatrix}
R_2 \rightarrow R_2$$

$$\begin{bmatrix}
1 & -\frac{2}{3} & 0 \\
0 & -\frac{2}{3} & 2
\end{bmatrix}
R_2 \rightarrow R_2$$

$$\begin{bmatrix}
1 & -\frac{2}{3} & 0 \\
0 & -\frac{2}{3} & 2
\end{bmatrix}
R_1 \rightarrow R_1 - 2R_2
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & -\frac{2}{3} & 2
\end{bmatrix}$$

$$\begin{bmatrix}
1 & -\frac{2}{3} & 0 \\
0 & 0 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
1 & -\frac{2}{3} & 0 \\
0 & 0 & 0
\end{bmatrix}$$

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\end{bmatrix}$$

$$\begin{bmatrix}
1 & -\frac{2}{3} & 0 \\
0 & 0 & 0
\end{bmatrix}$$

Lord 1007 Augmentet Matrix Lord 0010 (12 R R) 1 (AAR) Matrix Inverse

Matrix Inverse 720 100 01-2 110 01-2 110 002 1-1-11 104 -1-207 100 102 001 001 001 001 001 -4-1/2 12

