Linear Algebra

Matrix Inverse

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Identity Matrix

1 - mult. identity

$1 \cdot x = x = x \cdot 1$

$I_N \vec{v} = \vec{v}$

$A: N \times N$

$I_N A = A$

$A I_N = A$

Defn) The N-dim identity matrix is given by the square matrix of size $N \times N$ w/ ones on the diagonal & zero elsewhere

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3 \times 3} \qquad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2 \times 2}$$

Matrix Inverse

Def'n) The <u>Inverse</u> of a Square $N \times N$ Matrix A is the matrix A^{-1} such that

$$AA^{-1} = A^{-1}A = I_N$$

$$\boxed{(A^{-1})^{-1} = A}$$

NOTE: Not every matrix has an inverse.

Def'n) A matrix is <u>invertible</u> if there exists an inverse A^{-1}.

Left/Right Inverse

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 0 \end{bmatrix}_{2\times 3} = A$$

NOTE: A non-square matrix cannot have both a left & right inverse.

- I_2 on the left: $I_2 A = A$
- I_3 on the right: $A I_3 = A$

$B_{N\times M}$

For an $M\times N$ matrix A
- The matrix B is a <u>left inverse</u> if $BA = I_M$
- The matrix B is a <u>right inverse</u> if $AB = I_N$

Finding an Inverse

<u>Methods</u>

- Reduced Row Echelon Form (RREF)
- The Determinant

$ad-bc \neq 0$

2x2

$AA^{-1} = I_2$

$A^{-1}A = I_2$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad A^{-1} = \left(\frac{1}{\underbrace{ad-bc}_{ad-bc\neq 0}}\right)\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Why is the inverse so important?

- Inverse Matrix is an inverse function.
  (Identity Matrix is a identity $f^{\underline{n}}$)
- A square matrix is invertible if and only if it columns (rows) are linearly independent (form a basis) ⟵

TFAE: The Following Are Equivalent: - Full Rank

- Invertible - Non-Zero Determinant