# Linear Algebra

## Vector Properties and Interactions

Michael Ruddy

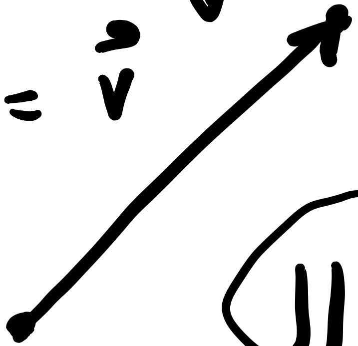
# Overview

- Vector Norm
- Vector Dot Product

# Vector Norm


$$d(p_1, p_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

# Vector Norm




"one-norm"

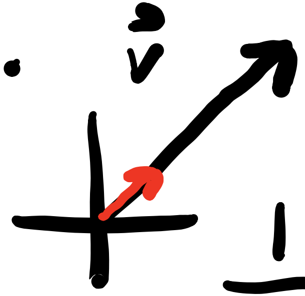
$$\|\vec{v}\|_1 = |v_1| + |v_2|$$

$$\|\vec{v}\| = \sqrt{(v_1 - 0)^2 + (v_2 - 0)^2}$$

$$= \sqrt{v_1^2 + v_2^2}$$

$\|\vec{v}\|_2$ "two-norm" or "norm"

**Unit Vector**: Any vector with norm equal to 1.




$$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2}$$

$$\frac{1}{\|\vec{v}\|} \cdot \vec{v} = \begin{pmatrix} \frac{v_1}{\|v\|} \\ \frac{v_2}{\|v\|} \end{pmatrix}$$

$$\left\| \frac{\vec{v}}{\|\vec{v}\|} \right\| = \sqrt{\frac{(v_1)^2}{\|\vec{v}\|^2} + \frac{(v_2)^2}{\|\vec{v}\|^2}}$$

$$= \|\vec{v}\| / \|\vec{v}\| = 1$$

# Example

$$\vec{v} = \begin{pmatrix} 1 \\ 2 \\ 0 \\ -1 \end{pmatrix} \qquad \frac{\vec{v}}{\|\vec{v}\|} = \begin{pmatrix} 1/\sqrt{6} \\ 2/\sqrt{6} \\ 0 \\ -1/\sqrt{6} \end{pmatrix}$$

$$\|\vec{v}\| = \sqrt{1^2 + 2^2 + 0^2 + (-1)^2}$$

$$= \sqrt{1+4+0+1} = \sqrt{6}$$

# Vector Norm

# Dot Product

$$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{pmatrix}$$

$N \times 1$

$$|\vec{v}| = \sqrt{v_1^2 + v_2^2 + \dots + v_N^2} = \sqrt{\sum_{i=1}^{N} v_i^2}$$

Transpose

$$\begin{pmatrix} v_1 & v_2 & \cdots & v_N \end{pmatrix} = \vec{v}^T$$

$1 \times N$

$$\sqrt{\vec{v}^T \cdot \vec{v}} = \sqrt{v_1^2 + v_2^2 + \dots + v_N^2}$$

## Dot Product

$$\vec{v}^T \cdot \vec{v} = \vec{v} \cdot \vec{v} = v_1^2 + v_2^2 + \cdots + v_N^2$$

$$\vec{v} \cdot \vec{w} = v_1 \cdot w_1 + v_2 \cdot w_2 + \cdots + v_N \cdot w_N$$

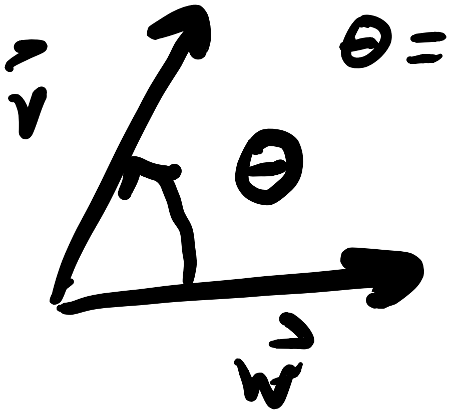
$$(\vec{v}^T \cdot \vec{w}) = \sum_{i=1}^{N} v_i \cdot w_i$$

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 0 \end{pmatrix} = 1 \cdot (-1) + 2(0) = -1 + 0 = -1$$

# Dot Product

$$\sqrt{\vec{v} \cdot \vec{v}} = \|\vec{v}\|$$

$$\vec{v} \cdot \vec{w}$$




$\theta = 0$ iff

$\vec{v} \perp \vec{w}$

orthogonal

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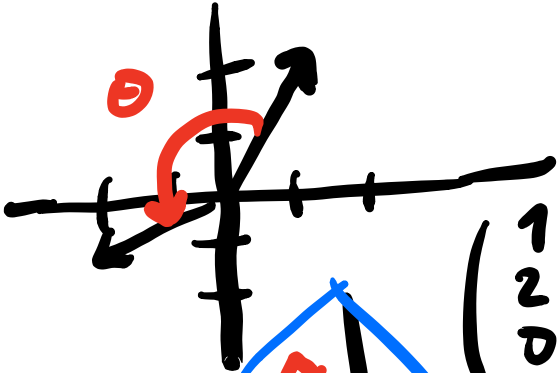
Two vectors have dot product $= 0$ if and only if they are orthogonal, i.e. the angle between vectors is $90°$.

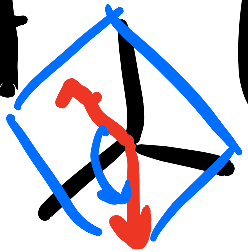
# Dot Product

$$\begin{pmatrix}1\\2\end{pmatrix} \qquad \begin{pmatrix}-2\\-1\end{pmatrix}$$
$$\vec{v} \qquad\qquad \vec{w}$$

$$\vec{v}\cdot\vec{w} = 1(-2) + 2(-1)$$
$$= -2 - 2 = -4$$
$$\neq 0$$



$$\begin{pmatrix}1\\2\\0\end{pmatrix}\cdot\begin{pmatrix}-2\\1\\3\end{pmatrix} = 1(-2) + 2(1) + 0(3)$$
$$= -2 + 2 + 0 = 0$$



$$\begin{pmatrix}1\\2\\3\\4\end{pmatrix}\begin{pmatrix}0\\1\\0\\1\end{pmatrix}$$

$$\vec{v} \cdot \vec{w} = \|\vec{v}\| \cdot \|\vec{w}\| \cos(\theta)$$


$$\cos(90^\circ) = 0$$