Linear Algebra

Matrix Arithmetic

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Matrix times a Vector

$$\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} * \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & -2 & -1 \\ 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 2(1) + 1(-1) + -2(0) \\ 2(0) + 1(-2) + -2(-1) \\ 2(1) + 1(0) + -2(1) \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

3x3

- The resulting column vector is given by the dot product of each row w/ the vector.
- The # of $\underbrace{\text{columns}}_{3}$ must match vector dim.

Matrix times a Vector

2-dim

$$\underbrace{\begin{bmatrix} 2 & 0 & 2 \\ -5 & 1 & 2 \end{bmatrix}}_{A \checkmark \; 2\times3} \underbrace{\begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}}_{3\times1} = \begin{bmatrix} 0 \\ -13 \end{bmatrix}$$

- I can multiply any 3D vector by a matrix of size $M \times 3$. The result will be a M-D vector.

$v^T A \to \begin{bmatrix} 2 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 1 & 1 \end{bmatrix} \to$ 2D vector.

Matrix times a Vector

$$\underset{A}{\begin{bmatrix} 1 & -1 & 0 \\ 0 & -2 & -1 \\ 1 & 0 & 1 \end{bmatrix}} \underset{\vec{x}}{\begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}} = \begin{bmatrix} 2(1) + 1(-1) - 2(0) \\ 2(0) + 1(-2) - 2(-1) \\ 2(1) + 1(0) - 2(1) \end{bmatrix}$$

$$= 2 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + 1 \begin{bmatrix} -1 \\ -2 \\ 0 \end{bmatrix} + (-2) \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$A\vec{x}$ is equivalent to

- computing a lin. comb. of the cols of A
- Mapping a 3D vector to an 3D vector

Matrix times a Matrix

$$\underset{A\ \ 2\times 2}{\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}} * \underset{2\times 3}{\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix}} = \underset{2\times 3}{\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \end{bmatrix}}$$

$\xrightarrow{x}$

In general $A \cdot B$

$\dim(A) = M_1 \times N_1$ $\qquad$ $\underline{N_1 = M_2}$

$\dim(B) = M_2 \times N_2$ $\qquad$ $\underline{M_1 \times N_2}$

$M_1 \times N_1 \quad M_2 \times N_2$

Matrix times a Matrix

Commutative mult: $xy = yx$

$2(3) = 3(2)$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$2 \times 3 \quad 2 \times 2$

$3 \neq 2$

$AB \neq BA$

$$\overset{A}{\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}} \overset{B}{\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}} = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$$

$$\overset{A}{\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}} \overset{B}{\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}} = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

!!

Matrix Addition

$$\begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} 5 & 10 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+5 & -1+10 \\ -1+0 & 0-1 \end{bmatrix} = \begin{bmatrix} 6 & 9 \\ -1 & -1 \end{bmatrix}$$

Matrix Arithmetic

Intuition: $A\vec{x}$ as a linear map

Want

- $(A+B)\vec{x} = A\vec{x} + B\vec{x}$
- $(AB)\vec{x} = A(B\vec{x})$