WHITE NOISE and COLORED NOISE*

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Consider a collodial particle in a fluid. The corresponding langevin equation is as below:

$$m\ddot{x} = -\gamma\dot{x} + \eta(t) \tag{1}$$

I guess you are familiar with the notations here. We will focus on the noise term $\eta(t)$ which has a normal distribution of fashion $\mathcal{N}(0, \sigma^2 \cdot t)$. You will notice that this distribution is not stationary, because the variance changes as time progresses. But once you fix the time and look at different realizations of this "signal" you will get a stationary normal distributed pdf.

The autocorrelation function of $\eta(t)$ is written as:

$$< \eta(t)\eta(t+\tau) >= k\delta(\tau) \tag{2}$$

where 'k' is some constant. So now where is "whiteness" coming from? You will easily recognize that the fourier transform of this "time-domain" autocorrelation function i.e the scaled delta function will give you a flat "frequency-domain" power spectral density(PSD) function. The flat PSD says that each infinitesimal band contains equal power content contributed by each frequency component. Something analogous to white color. But keep in mind that there is no ideal white noise in our real world. So in a colored noise you will see a PSD something like in FIG. 3, which has different power contributions from different frequencies.

$$Autocorrelation \xrightarrow{\mathscr{F}} Powerspectraldensity \tag{3}$$

$$\delta(t) \xrightarrow{\mathscr{F}} Constant \tag{4}$$

One more point which I need to emphasis is that the ideal white noise is a mathematical construct. And practically all noises you generate(whether using a computer or not) will have some kind of correlation insted of delta correlation and so practically you end up generating colored noise.

*This is a humble attempt from an engineer's side to shed light on the concepts of white noise and colored noise. If you find any mistake please do report to me at devanand.t.1986@gmail.com. :)

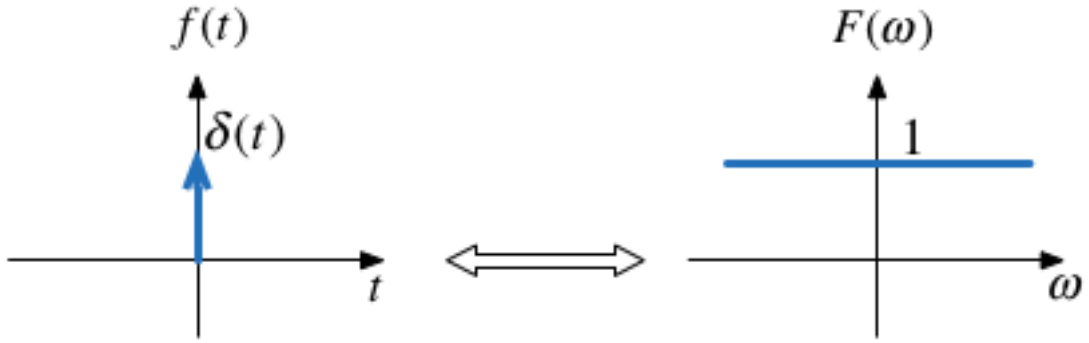

Figure 1: Fourier duals: delta function and flat function(white noise PSD). Ref:https://www.quora.com/The-Law-Of-Interconnection-Are-all-things-in-nature-and-science-somehow-related

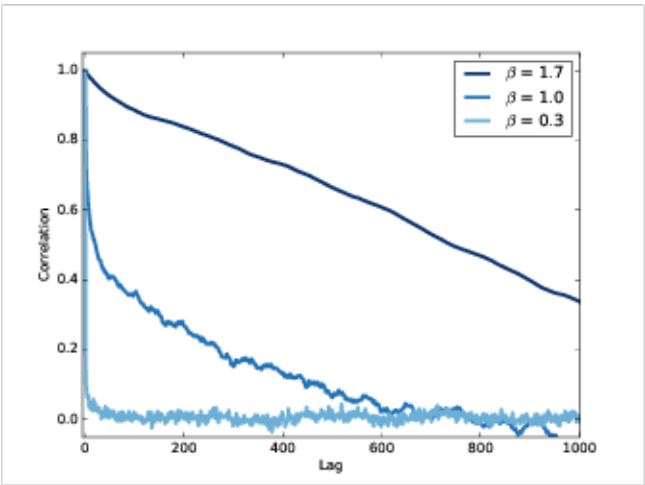

Figure 2: A sample colored noise autocorrelation. Notice the heavy tails compared to zero tail of delta function (This is Pink Noise). Ref:http://greenteapress.com/thinkdsp/html/thinkdsp029.png

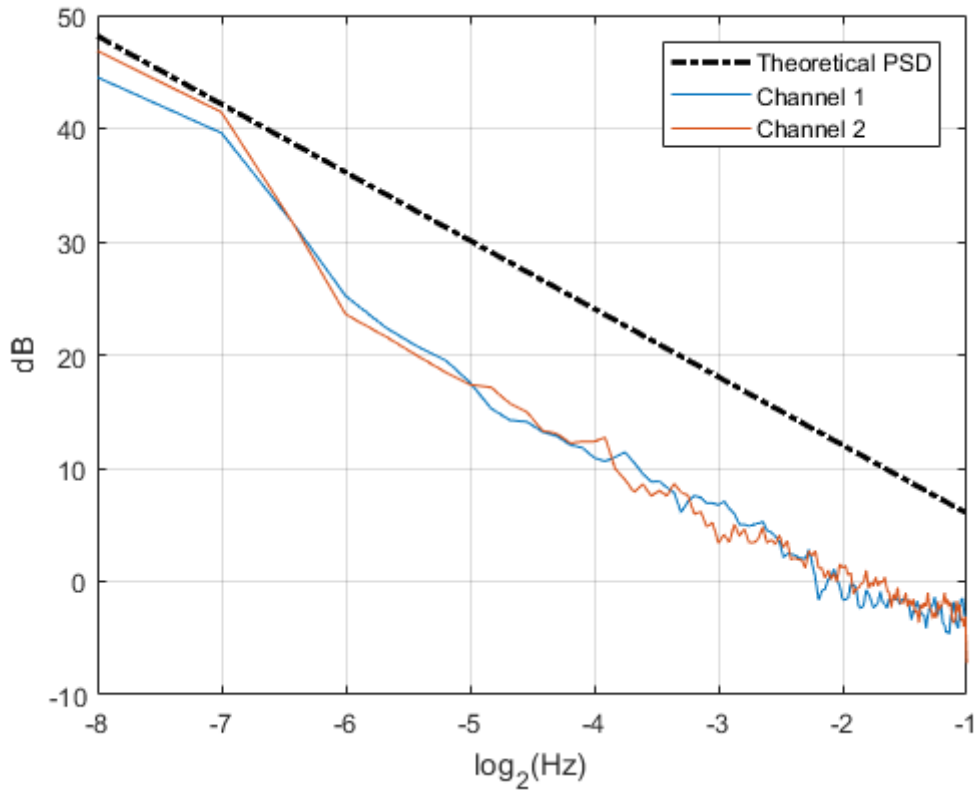

Figure 3: A sample colored noise PSD (This is Pink Noise I guess). Ref:`https://www.mathworks.com/help/examples/dsp/win64/TwoChannelBrownianNoiseExample_02.png`