Announcements

- Lab 00 + Welcome Form: Released! Due Tue 1/27
  - On the fence about **Regular** vs. **Bridge** vs. **Video** discussion?
    - Choose **Regular/Bridge to be assigned a time** that works for you. Video students (the default) cannot attend in-person.
    - Switch section times/formats until Add/Drop deadline (**Week 4**)
  - **61C Scholars** Pilot Program (self-identify on welcome survey)
    - Scholars-specific regular discussion + other activities/socials

- **CONCURRENT ENROLLMENT**
  - Everyone can get in, but official roster enrollment will take longer
    - Do NOT email us at this time!
  - If by **Monday evening**, you cannot complete lab00 because you cannot sign up for an instructional account, email cs61c@berkeley.edu

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CS61C

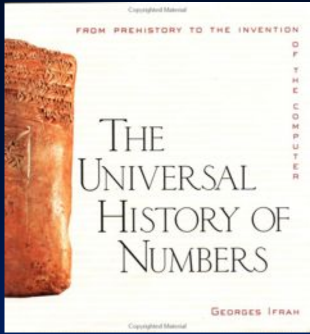
Great Ideas in Computer Architecture

(a.k.a. Machine Structures)

Assistant Teaching Professor
Lisa Yan

Number Representation

Great book ⇒
The Universal History of Numbers
by Georges Ifrah



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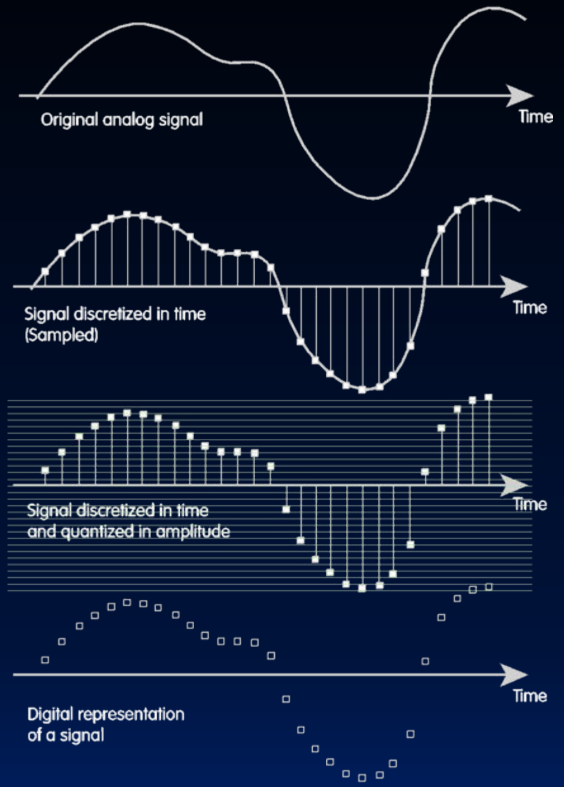
Data input: Analog → Digital

Real world is analog!
To import analog information, we must do two things:

- Sample
  - E.g., for a CD, every 44,100ths of a second, we ask a music signal how loud it is.
- Quantize
  - For every one of these samples, we figure out where, on a 16-bit (65,536 tick-mark) “yardstick”, it lies.



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Digital data not necessarily born Analog…

hof.povray.org

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Discuss

1. How many "things" can be represented by 4 bits?

A. 4
B. 8
C. 16
D. 64
E. Something else

2. What does this particular 4-bit pattern represent?

`1011`

3. How many bits do you need to represent π (pi)?

A. 1
B. 9 (π=3.14, so 0.011 "." 001100)
C. 64 (Macs are 64-bit machines)
D. Every bit the machine has
E. ∞

BIG IDEA: Bits can represent anything!!

With N bits, you can represent at most 2^N things.

4 bits,
each 0 or 1

____ ____ ____ ____

1011

The meaning of a bitstring depends on the program context!

Today: Many ways to use N bits to represent numbers

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Pop quiz??

1. [no pollEV, just discuss]
How many "things" can be represented by 4 bits?

A. 4
B. 8
C. 16
D. 64
E. Something else

2. [no pollEV, just discuss]
What does this particular 4-bit pattern represent?

```
1011
```

3. How many bits do you need to represent π (pi)?

A. 1
B. 9 (π=3.14, so 0.011"." 001100)
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D. Every bit the machine has
E. ∞

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BIG IDEA: Bits can represent anything!!

- Logical values? 1 bit
  - One possible convention: 0 → False, 1 → True
- Characters?
  - A, ..., Z: 26 letters → 5 bits (26 ≤ 32)
  - ASCII: upper/lower case + punctuation → 7 bits → round to 1 byte
  - Unicode (www.unicode.com): standard code to cover all the world's languages ⇒ 8, 16, 32 bits
- Colors?
  - HTML color codes: 24 bits (3 bytes)
- Locations / addresses? Commands?
  - IPv4 (32 bit), IPv6 (64 bit), etc.

California Gold
0xFDB515

Red (FD)	Green (B5)	Blue (15)

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Agenda

Binary, Decimal, Hex

- Binary, Decimal, Hex
- Integer Representations
- Sign-Magnitude, Ones' Complement
- Two's Complement
- Bias Encoding

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Number vs Numeral

Numeral

A symbol or name that stands for a number

e.g., *4* , *four* , *quatro* , *IV* , *IIII* , ...

...and **Digits** are symbols that make numerals

Above the abstraction line

Abstraction Line

Below the abstraction line

Number

The "idea" in our minds...there is only ONE of these

e.g., *the concept of "4"*

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Decimal: Base 10 (Ten) #s

Digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Example:

3271 = $\mathbf{3271}_{10}$ = $(\mathbf{3}\text{x}10^3) + (\mathbf{2}\text{x}10^2) + (\mathbf{7}\text{x}10^1) + (\mathbf{1}\text{x}10^0)$

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Base 2 (Two) #s, Binary

Digits: 0, 1 (**bi**nary digi**ts** → bits)

Example: Binary number "**1101**"

Convert to decimal:

$$\mathbf{0b1101} = \mathbf{1101}_2 = (\mathbf{1}\mathrm{x}2^3) + (\mathbf{1}\mathrm{x}2^2) + (\mathbf{0}\mathrm{x}2^1) + (\mathbf{1}\mathrm{x}2^0)$$
$$= 8 + 4 + 0 + 1$$
$$= 13$$

Common binary shorthand:
`0b1101`

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Discuss #2

1. How to represent 165 in binary?

2. Is this binary value human-readable?
   What if it appeared multiple times in a program?

0b11110100101

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Consider

- How to represent 165 in binary?

1	0	1	0	0	1	0	1
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1

~~165~~ ~~37~~ ~~5~~ ~~1~~ 0

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Base 16 (Sixteen) #s, Hexadecimal

Digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

10, 11, 12, 13, 14, 15

Example: Hexadecimal number "**A5**"

Convert to decimal:

0xA5 $= \mathbf{A5}_{16} = (\mathbf{10} \times 16^1) + (\mathbf{5} \times 16^0)$

$= 160 + 5$

$= 165$

"Hex" for short.
Common hex shorthand: `0xA5`

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Hexadecimal is a Useful Shorthand for Binary (1/2)

Dec	Hex	Bin
00	0	0000
01	1	0001
02	2	0010
03	3	0011
04	4	0100
05	5	0101
06	6	0110
07	7	0111
08	8	1000
09	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

Memorize this table.

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Hexadecimal is a Useful Shorthand for Binary (2/2)

Dec	Hex	Bin
00	0	0000
01	1	0001
02	2	0010
03	3	0011
04	4	0100
05	5	0101
06	6	0110
07	7	0111
08	8	1000
09	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

The number 165

- In binary ("prefix 0b")

$$0b\underbrace{1010}_{A}\underbrace{0101}_{5}$$

$$0xA5$$

- In hexadecimal (prefix "0x")

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Back to Question 2 (1/2)

1. How to represent 165 in binary? `0b10100101`

   `= 0xA5`

2. Is this binary value human-readable?
   What if it appeared multiple times in a program?

`0b11110100101`

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Zero pad leftmost group of 4 bits as needed

1. How to represent 165 in binary? `0b10100101`

   `= 0xA5`

2. Is this binary value human-readable?
   What if it appeared multiple times in a program?

$$\underbrace{\texttt{0011}}_{\texttt{3}}\quad\underbrace{\texttt{1010}}_{\texttt{A}}\quad\underbrace{\texttt{0101}}_{\texttt{5}}$$

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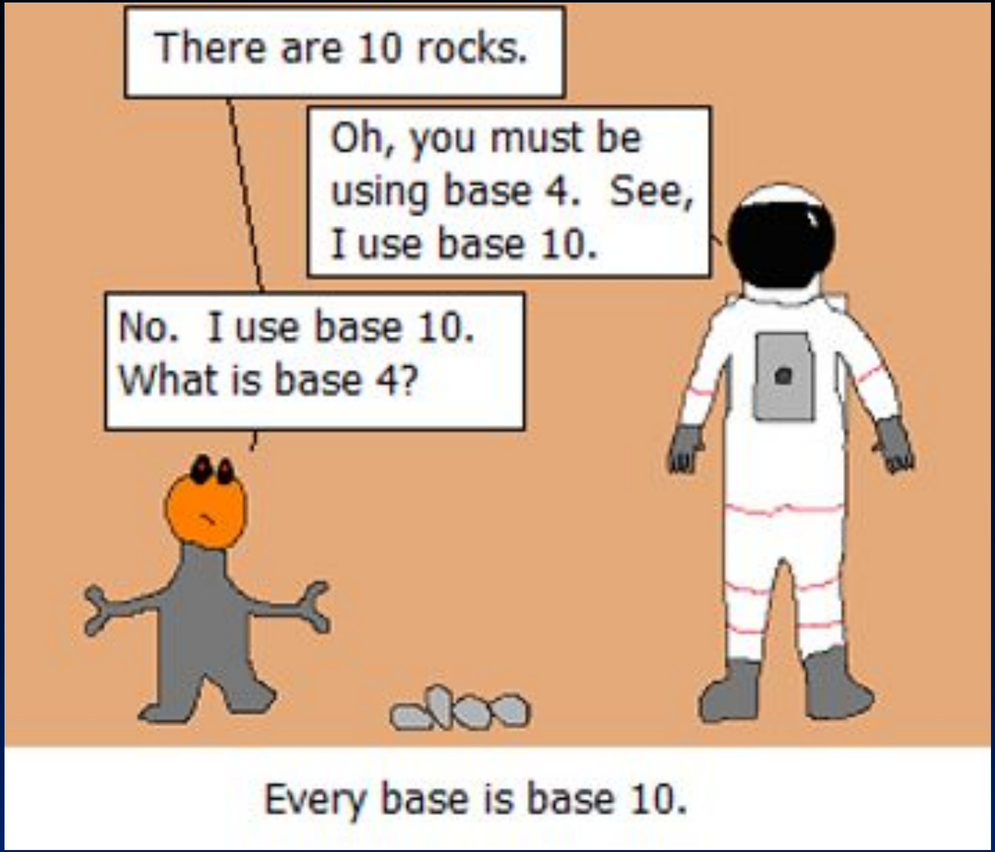
Which base do we use? (1/2)



How many rocks?

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Which base do we use? (2/2)

- **Decimal:** great for humans, especially when doing arithmetic
- **Hex:** if human looking at long strings of binary numbers, it's much easier to convert to hex and see 4 bits/symbol
  - Terrible for arithmetic on paper
- **Binary:** what computers use
  - **To a computer, numbers are always binary**

- Different reps of the same number:
  - 32_{ten} == 32_{10} == `0x20` == 100000_2 == `0b100000`

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Agenda

Integer Representations

- Binary, Decimal, Hex
- Integer Representations
- Sign-Magnitude, Ones' Complement
- Two's Complement
- Bias Encoding

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How do we pick a representation for <u>integers</u>? (1/2)

- Want a representation that supports common integer operations:
    - Add them
    - Subtract them
    - Multiply them
    - Divide them
    - Compare them (<, =, ≠, ≤, etc.)

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N-Bit Unsigned Integer Representation

- Most computers use the **unsigned** integer representation from mathematics (that we just discussed.)
    - C's `uint8_t`, `uint16_t`, etc.: [0, 2^N-1]
- Example: 10 + 7 = 17
    - 10, 7 can be represented with 4 bits:
    - Addition, subtraction just as you would in decimal!!
    - So simple to **add** in binary that we can build circuits to do it!
        - **This design decision would make hardware simple!**
    - ...wait...

```
 11    carry bits
 1010
+ 0111
 10001
```


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How do we pick a representation for integers?(2/2)

- While strictly speaking, binary numerals have an infinite number of digits, **hardware has limits**.
  - Must also specify the number of bits for an integer representation.
- An N-bit representation can only represent 2^N things. When doing arithmetic, numbers might "wrap around."

Car odometer

With a 4-bit unsigned integer representation, 10 + 7 = 1.

$$
\begin{array}{r@{\quad}l}
11 & \text{carry bits} \\
1010 & \\
+\ \underline{0111} & \\
\text{dropped}\ 1\,0001 &
\end{array}
$$

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Number line vs. number "wheel"

Integer overflow: The arithmetic result is outside the representable range.

Number wheels visualize integer overflow.

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Agenda

Sign-Magnitude, Ones' Complement

- Binary, Decimal, Hex
- Integer Representations
- Sign-Magnitude, Ones' Complement
- Two's Complement
- Bias Encoding

Let's move to discussing **signed** integer representations, which can represent positive and negative #s.

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N-Bit Signed Integer Representations

For the proposed N-bit signed integer representations, consider:

1. How to represent 5? Is it easy to convert to -5?

2. Is it easy to identify positive vs. negative from a bit pattern?

3. Is addition easy to implement? Example: 5 + -5 = 0

4. How to represent zero?

5. [at home] What is the positive integer range? Negative integer range?

Ain't no free lunch: We still have N bits.
Goal: half positive, half negative, and zero

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Two "strawman" designs for N-bit signed integer rep (1/2)

- Sign-Magnitude
  - Leftmost bit is **sign bit**
  - Remaining N-1 bits is magnitude, i.e., numerical value

$+5_{ten}$ = `0b 0000 0101`

-5_{ten} = `0b 1000 0101`

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Two "strawman" designs for N-bit signed integer rep. (2/2)

- One's Complement
  - (reserve leftmost bit for sign)
  - If positive, unsigned rep
  - If negative, **flip** bits of positive rep

$+5_{ten}$ = `0b 0000 0101`

-5_{ten} = `0b 1111 1010`

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Discuss #3

- Sign-Magnitude
  - Leftmost bit is **sign bit**
  - Remaining N-1 bits is magnitude, i.e., numerical value
- One's Complement
  - (reserve leftmost bit for sign)
  - If positive, unsigned rep
  - If negative, **flip** bits of positive rep

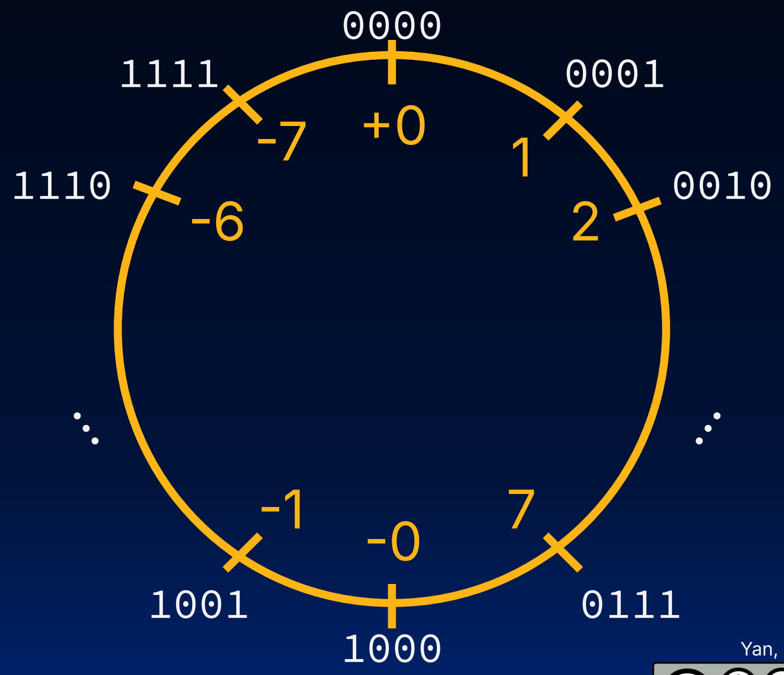
1. How to represent 5? -5? [done]
2. How to represent zero?
3. Is it easy to identify positive vs. negative?
4. Is addition easy to implement? Example: 5 + -5 = 0

Sign-Magnitude: Rarely Used

$+5_{ten}$ = 0b 0000 0101

-5_{ten} = 0b 1000 0101

- Sign-magnitude is **rarely used**, due to many shortcomings:
  - Incrementing "binary odometer" increases then decreases values
  - Arithmetic circuit complicated: depends on signs same/different
  - **Two zeros** (how to compare??)
- Reasonable for signal processing, not for general purpose computers



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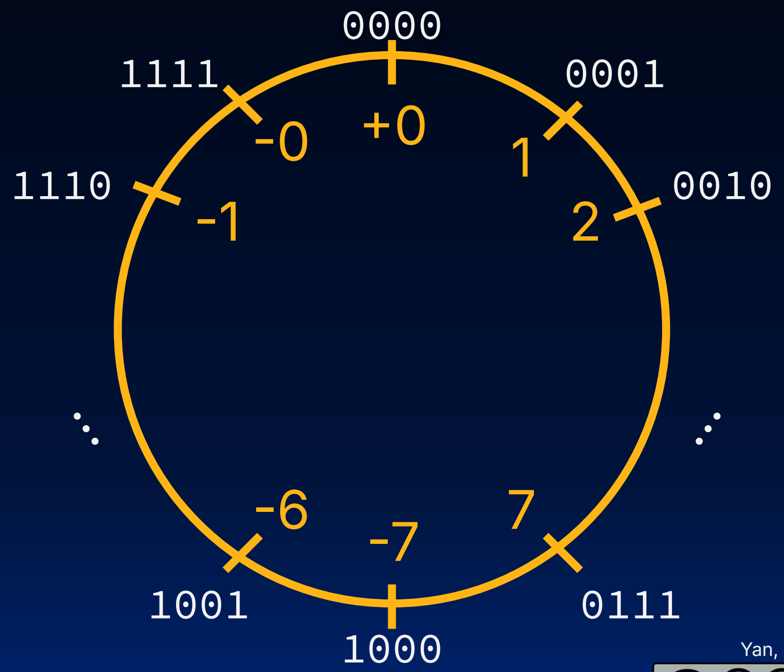
One's Complement: Old but OK

$+5_{ten}$ = 0b 0000 0101

-5_{ten} = 0b 1111 1010

- Advantages:
  - Leftmost bit ("**most significant bit**") is still effectively sign bit
  - Incrementing binary odometer consistent on the # line
- Some disadvantages still persist:
  - Still two zeros
  - Arithmetic still somewhat complicated
- While used for a while on some computer products
  - It's not currently used in current hardware



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Agenda

Two's Complement

- Binary, Decimal, Hex
- Integer Representations
- Sign-Magnitude, Ones' Complement
- Two's Complement
- Bias Encoding

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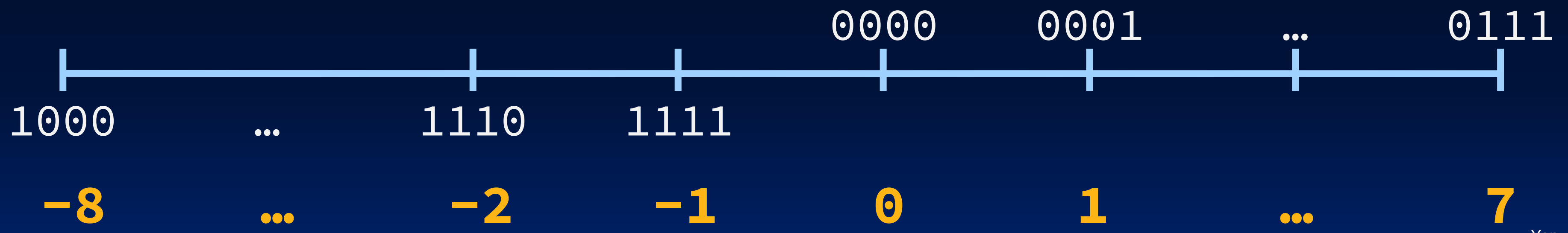
Two's Complement: The Motivation

- Ones' complement:

- The problem: "Overlap" creates two 0s.
- The solution: **Shift the negative mappings left by one.**



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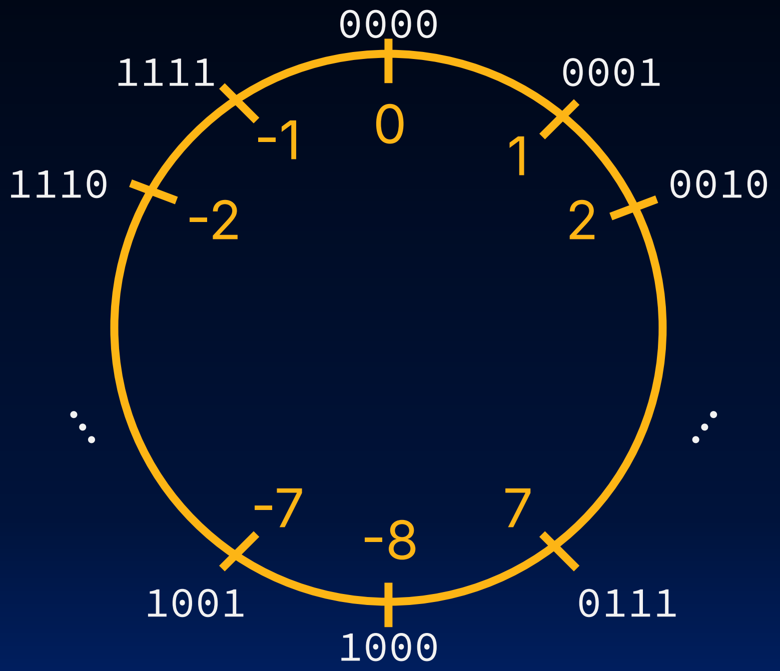
Two's Complement: Why?

$+5_{ten}$ = `0b 0000 0101`

-5_{ten} = `0b 1111 1011`

- Advantages:
  - Leftmost bit ("**most significant bit**") is still effectively sign bit
  - One zero
  - **Simple hardware for addition**
  - Incrementing binary odometer consistent on the # line
- Two's complement is the representation used for all C23 signed integers.
  - `int8_t`, `int16_t`, etc.



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Two's Complement: Procedural Algorithm

- Two's Complement
  - If positive, unsigned rep
  - If negative:
    - Start from positive rep
    - **flip** bits
    - **add one**

5_{ten} `0101` → `1010` → `1011` -5_{ten}

- Negative → positive is same procedure: flip and add one.
  - Makes hardware easy!

-5_{ten} `1011` → `0100` → `0101` 5_{ten}

In notes: Read the singular mathematical formula for both positive and negative

On your own: Prove math formula is equivalent to this slide's algorithm

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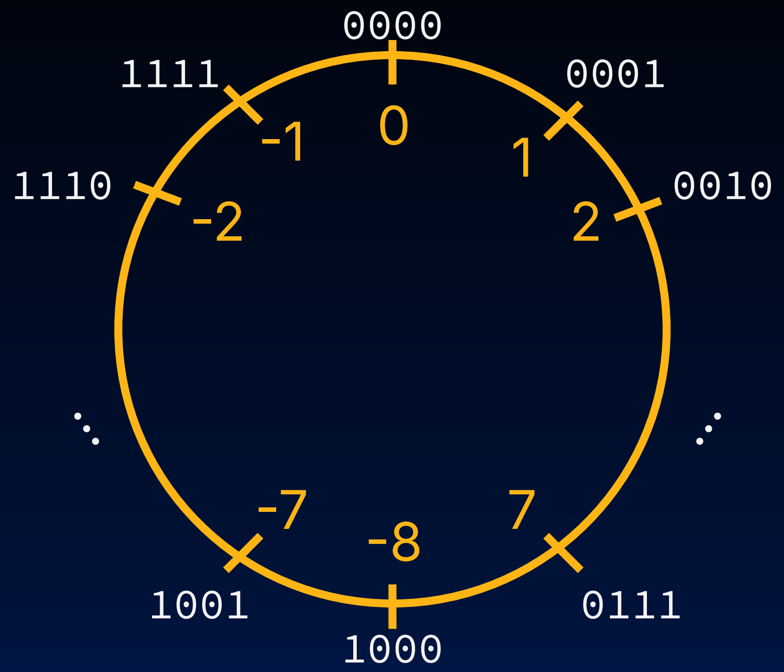
Two's Complement: Addition

Decimal	5
	+ −5
	0
	111 carry bits
Two's complement	0101
	+ 1011
	dropped 10000



Integer overflow in two's complement is a **feature**!
It means we can use the same HW to add numbers of different signs.

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Agenda

Bias Encoding

[at home] Used for certain applications

- Binary, Decimal, Hex
- Integer Representations
- Sign-Magnitude, Ones' Complement
- Two's Complement
- Bias Encoding

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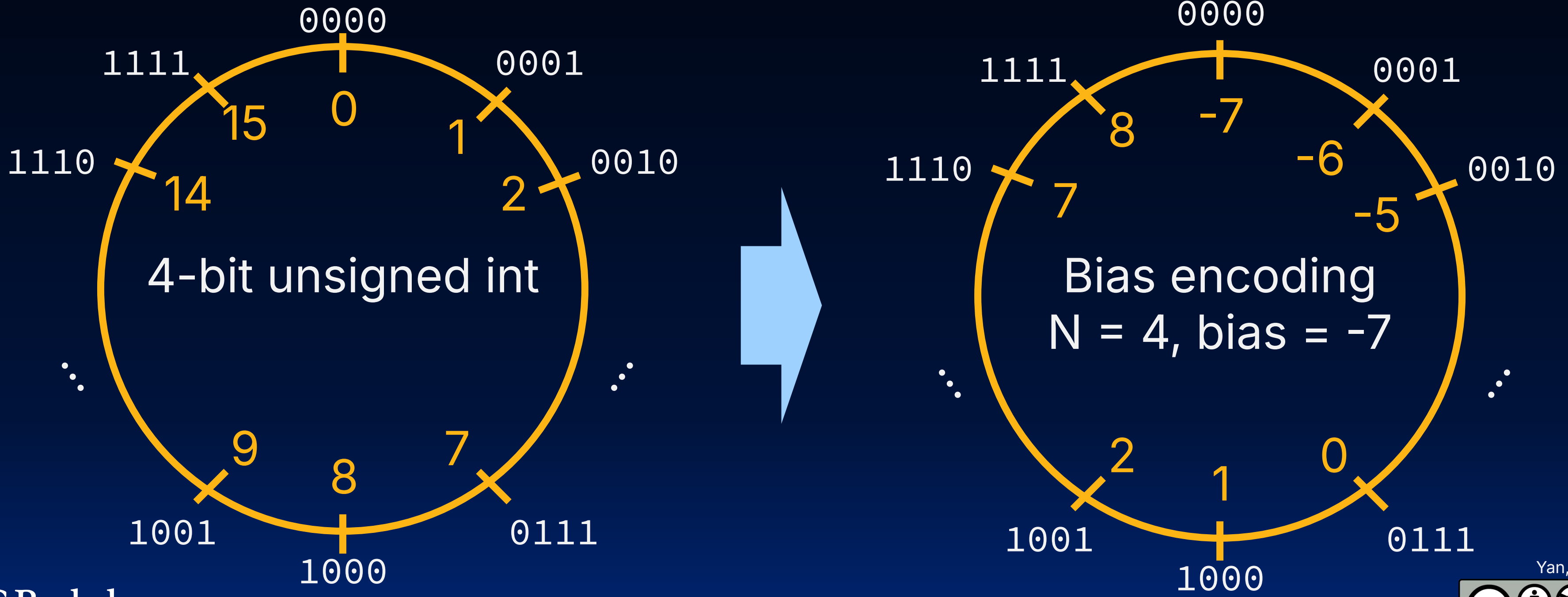
Bias Encoding

- **Bias encoding**: "Shift" the numbers to represent a target range of integers



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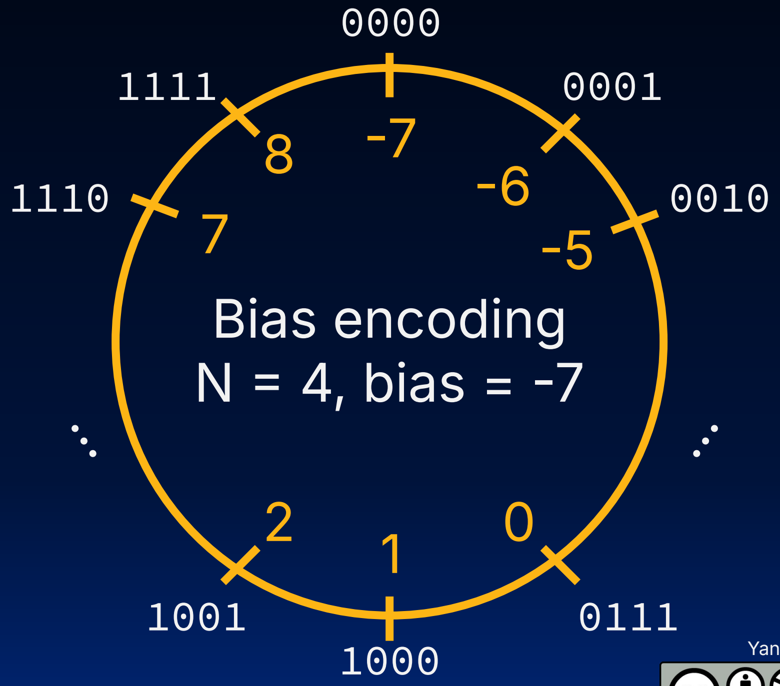
Bias Encoding

- **Bias encoding**: “Shift” the numbers to represent a target range of integers

- Number = (unsigned rep) + (bias)
  - Define a “bias”
  - To interpret stored binary: Read the data as an unsigned number, then add the bias
  - To store a data value: Subtract the bias, then store the resulting number as an unsigned number



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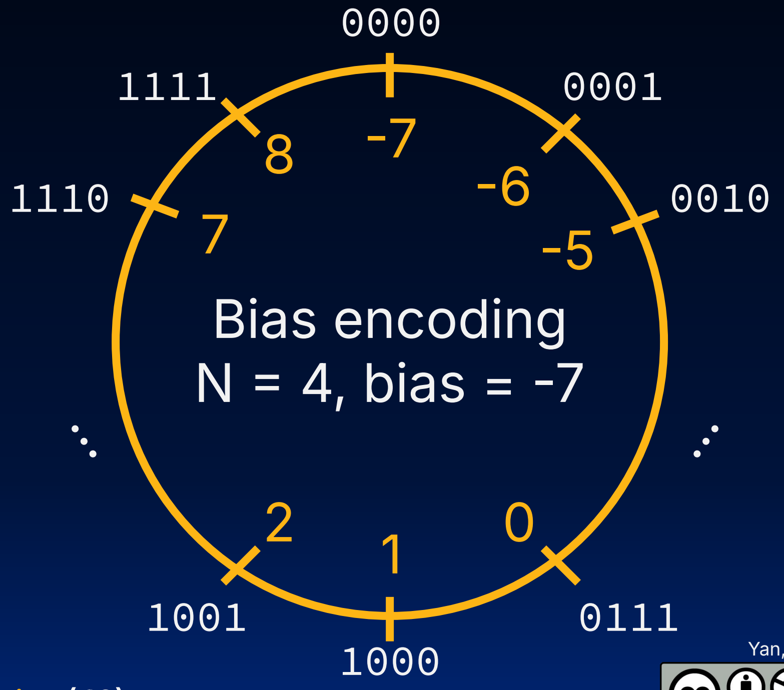
When is Bias Encoding Used?

- Number = (unsigned rep) + (bias)
- Useful for, say, AC signals with a DC offset
  - e.g., for 8V and 10V; just store 2V range

Bias could be anything we want!

- In practice, with N bits, reasonable choice is bias = $-(2^{N-1} - 1)$
  - E.g., 4 bits bias = $-(2^3-1) = -(8-1) = -7$
- Used for floating point (more later)



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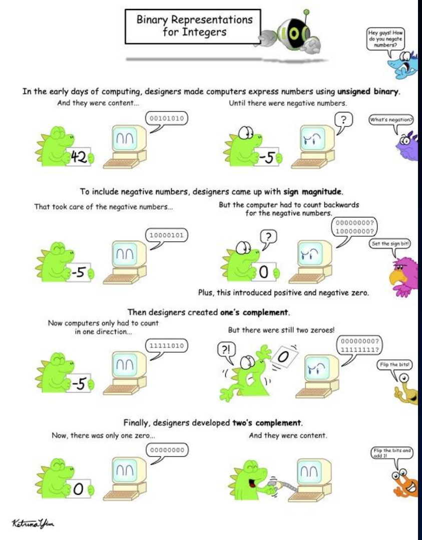
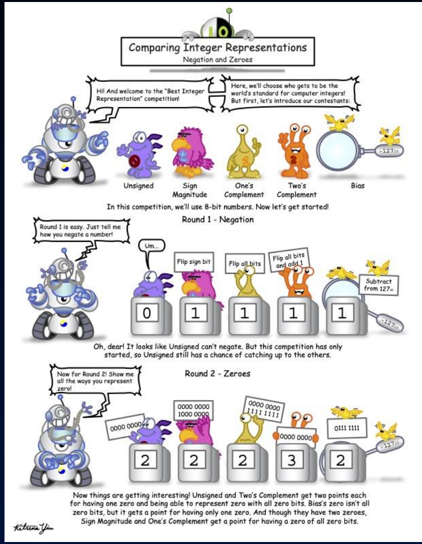
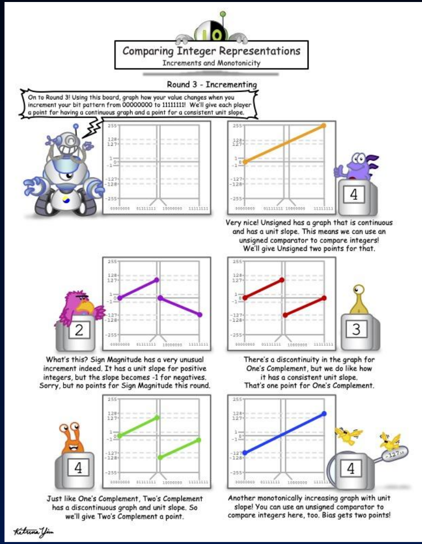
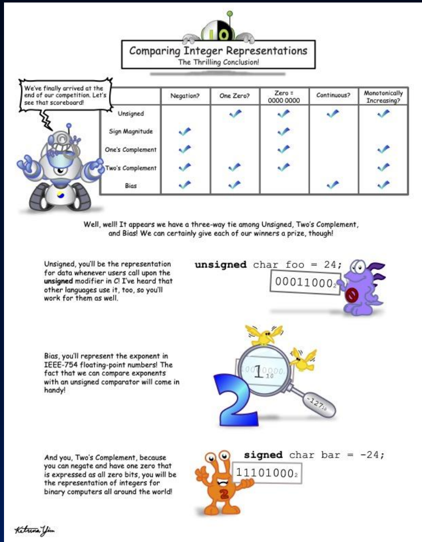
Amazing Illustrations by Ketrina (Yim) Thompson









See course notes for reference links!

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And in summary...

- We represent "things" in computers as particular bit patterns:
  - With N bits, you can represent at most 2^N things.
- Today, we discussed five different encodings for integers:
  - Unsigned integers
  - Signed integers:
    - Sign-Magnitude
    - Ones' Complement
    - Two's Complement
    - Bias Encoding
- Computer architects make design decisions to make HW simple
  - Unsigned and Two's complement are C standard. Learn them!!
- Integer overflow: The result of an arithmetic operation is outside the representable range of integers.
  - Numbers have infinite digits, but computers have finite precision. This can lead to arithmetic errors. More later!

For you to consider:
How could we represent -12.75?

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