Sky tesselation

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I. ICOSAHEDRON

The vertices of an icosahedron centered at the origin and with circumradius 1 are:

$$V_1 = (0, 0, 1) \tag{1a}$$
$$V_2 = (0, 0, -1) \tag{1b}$$
$$V_3 = \left(\frac{2}{\sqrt{5}}, 0, \frac{1}{\sqrt{5}}\right) \tag{1c}$$
$$V_4 = \left(-\frac{2}{\sqrt{5}}, 0, -\frac{1}{\sqrt{5}}\right) \tag{1d}$$
$$V_5 = \left(-\frac{5+\sqrt{5}}{10}, \sqrt{\frac{5-\sqrt{5}}{10}}, \frac{1}{\sqrt{5}}\right) \tag{1e}$$
$$V_6 = \left(-\frac{5+\sqrt{5}}{10}, -\sqrt{\frac{5-\sqrt{5}}{10}}, \frac{1}{\sqrt{5}}\right) \tag{1f}$$
$$V_7 = \left(\frac{5+\sqrt{5}}{10}, \sqrt{\frac{5-\sqrt{5}}{10}}, -\frac{1}{\sqrt{5}}\right) \tag{1g}$$
$$V_8 = \left(\frac{5+\sqrt{5}}{10}, -\sqrt{\frac{5-\sqrt{5}}{10}}, -\frac{1}{\sqrt{5}}\right) \tag{1h}$$
$$V_9 = \left(\frac{5-\sqrt{5}}{10}, \sqrt{\frac{5+\sqrt{5}}{10}}, \frac{1}{\sqrt{5}}\right) \tag{1i}$$
$$V_{10} = \left(\frac{5-\sqrt{5}}{10}, -\sqrt{\frac{5+\sqrt{5}}{10}}, \frac{1}{\sqrt{5}}\right) \tag{1j}$$
$$V_{11} = \left(-\frac{5-\sqrt{5}}{10}, \sqrt{\frac{5+\sqrt{5}}{10}}, -\frac{1}{\sqrt{5}}\right) \tag{1k}$$
$$V_{12} = \left(-\frac{5-\sqrt{5}}{10}, -\sqrt{\frac{5+\sqrt{5}}{10}}, -\frac{1}{\sqrt{5}}\right) \tag{1l}$$

II. ORTHOGRAPHIC PROJECTION

If we have a unit sphere centered on the origin O and 2 points $P_{1,2} = (x_{1,2}, y_{1,2}, z_{1,2})$ on it, find the Cartesian equation of the great circle which passes through both of these points.

III. CONSTRUCTING FINER TESSELATIONS

We have a unit sphere centered on the origin O and 2 points $P_{1,2} = (x_{1,2}, y_{1,2}, z_{1,2})$ on it. Take a point M which lies on the line P_1P_2 and is inside the sphere, such that

$$\frac{P_1M}{MP_2} = \frac{a}{b}, a, b \in \mathbb{R}. \tag{2}$$

Find the coordinates of the point M in terms of the coordinates of P_1 and P_2, a, and b.

If the ray $\vec{OM}$ intersects the sphere at point Q, find the coordinates of Q.

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