Sky tesselation

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I. ICOSAHEDRON

The vertices of an icosahedron centered at the origin and with circumradius 1 are:

$$V_1 = (0,0,1) \tag{1a}$$
$$V_2 = (0,0,-1) \tag{1b}$$
$$V_3 = \left(\frac{2}{\sqrt{5}}, 0, \frac{1}{\sqrt{5}}\right) \tag{1c}$$
$$V_4 = \left(-\frac{2}{\sqrt{5}}, 0, -\frac{1}{\sqrt{5}}\right) \tag{1d}$$
$$V_5 = \left(-\frac{5+\sqrt{5}}{10}, \sqrt{\frac{5-\sqrt{5}}{10}}, \frac{1}{\sqrt{5}}\right) \tag{1e}$$
$$V_6 = \left(-\frac{5+\sqrt{5}}{10}, -\sqrt{\frac{5-\sqrt{5}}{10}}, \frac{1}{\sqrt{5}}\right) \tag{1f}$$
$$V_7 = \left(\frac{5+\sqrt{5}}{10}, \sqrt{\frac{5-\sqrt{5}}{10}}, -\frac{1}{\sqrt{5}}\right) \tag{1g}$$
$$V_8 = \left(\frac{5+\sqrt{5}}{10}, -\sqrt{\frac{5-\sqrt{5}}{10}}, -\frac{1}{\sqrt{5}}\right) \tag{1h}$$
$$V_9 = \left(\frac{5-\sqrt{5}}{10}, \sqrt{\frac{5+\sqrt{5}}{10}}, \frac{1}{\sqrt{5}}\right) \tag{1i}$$
$$V_{10} = \left(\frac{5-\sqrt{5}}{10}, -\sqrt{\frac{5+\sqrt{5}}{10}}, \frac{1}{\sqrt{5}}\right) \tag{1j}$$
$$V_{11} = \left(-\frac{5-\sqrt{5}}{10}, \sqrt{\frac{5+\sqrt{5}}{10}}, -\frac{1}{\sqrt{5}}\right) \tag{1k}$$
$$V_{12} = \left(-\frac{5-\sqrt{5}}{10}, -\sqrt{\frac{5+\sqrt{5}}{10}}, -\frac{1}{\sqrt{5}}\right) \tag{1l}$$

II. TASKS

- If we have a unit sphere centered on the origin O and 2 points $P_{1,2} = (x_{1,2}, y_{1,2}, z_{1,2})$ on it, find the Cartesian equation of the great circle which passes through both of these points.
- We have a unit sphere centered on the origin O and 2 points $P_{1,2} = (x_{1,2}, y_{1,2}, z_{1,2})$ on it. Take a point M which lies on the line P_1P_2 and is inside the sphere, such that

$$\frac{P_1M}{MP_2} = \frac{a}{b}, a, b \in \mathbb{R}. \tag{2}$$

  Find the coordinates of the point M in terms of the coordinates of P_1 and P_2, a, and b.

  If the ray $\vec{OM}$ intersects the sphere at point Q, find the coordinates of Q.
- Take an equilateral triangle ABC, divide it in smaller equilateral triangles by dividing its sides into $n = 2, 3, 4, ...$ smaller segments.
- Take our icosahedron from before. For $n = 2$, on each face, construct smaller n^2 smaller equilateral triangles. Find the projections of the vertices triangles onto the unit sphere. Record the vertices and faces of the new polyhedron in a file. Repeat for all $n <= 50$.
- Find the (approximate) area of a face of the new polyhedron as a function of n.
- Find the (approximate) area of a spherical triangle formed by the projection of a face of the polyhedron onto the unit sphere. For what $n = \tilde{n}$ is this area less than $1\,\mathrm{deg}^2$?
- Find the area factor as a function of n:

$$F_A(n) = \frac{A_n}{4\pi}$$

- Find the volume factor as a function of n:

$$F_V(n) = \frac{V_n}{4\pi/3}$$

- Implement in a computer program the conversion from Cartesian coordinates (x, y, z) to spherical polar coordinates (r, θ, φ).
- Implement in a computer program the conversion from spherical polar coordinates (θ, φ) to Mollweide coordinates (X, Y) (here we take $r = 1$ for the unit sphere).

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