Computer Vision [H02K5a]

Final Project: Model Reconstruction

If we build an Active Shape Model (ASM), we align a training set of landmarks, and analyze the variations in shape using a Principal Component Analysis (PCA). From this set, we can extract the mean shape of the training set. The eigenvectors represent the axes along which this mean model can vary. Thus, given the mean shape of an aligned training set, along with the extracted eigenvectors and eigenvalues, all possible shapes within this class of shapes can generated by linearly combining the mean shape with the weighted eigenvectors. The eigenvectors are scaled with an amount equal to the eigenvalues multiplied with their corresponding coefficients.

Given a set T of *n* points, we would like to find the best approximation of our model M of *n* points to this target point set T. Note that finding a reconstruction (or approximation) can best be done when model and target are **already aligned** in some way…). In [1], a method is presented for finding an optimal approximation using a regularization parameter η (see original paper). This parameter controls the trade-off between obtaining a statistically correct shape (high values will enforce shapes close to the mean shape) and a least-squares minimization of the fit (lower values for η will allow the model to deform more to obtain a better fit). A summary of the method is given below.

<u>Note:</u> the landmark matrices are converted to landmark vectors. This means that if you are working with 50 2-dimensional landmarks ([50x2]), a [100x1] column vector will be obtained.

x'	= approximation of our model M to target T
x	= our model M (mean shape of aligned training set)
Q	= projection matrix (eigenvectors scaled with corresponding eigenvalues)
c	= projection coefficients (these are what we are looking for)

Our goal is stated by the following objective:

$$\mathbf{x' = x + Qc}$$

meaning that our reconstruction is a linear combination of the mean shape and the projection matrix scaled by the correct coefficients. Instead of looking for the reconstruction directly, we can also look for the difference between the model and the target shape. We can thus restate our problem by taking the difference between our target and model:

$$\mathbf{y = T - x}$$

where T is our target model. The problem then simplifies to finding a reconstruction of the differences:

$$\mathbf{y' = Qc}$$

We are thus trying to solve the problem by finding an approximation to the difference between target and model, instead of finding an approximation to the target directly. This problem can be solved in many different ways (linear system solver, SVD, …).

[1] A statistical method for robust 3D surface reconstruction from sparse data

$$c = \left[(Q^T Q)^{-1} Q^T y \right]_{k \times 1}$$