Dependable Systems and Networks

HW #4

Due Nov. 07, 2022

1. (10 points) How many check bits are needed if the Hamming single error correcting code is used to detect single bit errors in a 512-bit data word?

Sol:

Check bits K 要滿足 $512 + K \leq 2^K - 1 \Rightarrow K \geq 10$

故 check bits 至少要 = 10

2. (10 points) Develop an SEC code for a 16-bit data word. Generate the code for the data word *0101000000111001*. Show that the code will correctly identify an error in data bit 7.

Bit Position	Position Number	Check Bits	Data Bits
21	10101		D16
20	10100		D15
19	10011		D14
18	10010		D13
17	10001		D12
16	10000	C16	
15	01111		D11
14	01110		D10
13	01101		D9
12	01100		D8
11	01011		D7
10	01010		D6
9	01001		D5
8	01000	C8	
7	00111		D4
6	00110		D3
5	00101		D2
4	00100	C4	
3	00011		D1
2	00010	C2	
1	00001	C1	

Sol:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
	C1	C2	D1	C4	D2	D3	D4	C8	D5	D6	D7	D8	D9	D10	D11	C16	D12	D13	D14	D15	D16
data	1	0	1	0	0	0	1	0	1	1	0	0	0	0	0	0	0	1	0	1	0
C1	*		*		*		*		*		*		*		*		*		*		*
C2		*	*			*	*			*	*			*	*			*	*		
C4				*	*	*	*					*	*	*	*					*	*
C8								*	*	*	*	*	*	*	*						
C16																*	*	*	*	*	*
error data	1	0	1	0	0	0	1	0	1	1	1	0	0	0	0	0	0	1	0	1	0
	0	1						1													

C16=0, C8=0, C4=0, C2=0, C1=1

因此，code word = 010100000001101000101

如果 data bit 7 發生錯誤，則變為 C16=0, C8=1, C4=0, C2=1,C1=0

$00001 \oplus 01010 = 01011$，表示在 1+2+8 個 bit 發現錯誤，也就是 D7

3. (10 points) **Checksum Code:**

1) Devise an example to illustrate the problem with the Single-Precision Checksum (SPC) code, that is, the information thus the ability to detect errors can be lost in the ignored overflow.

2) Describe two solutions to solve the problem.

4. (10 points) ***Non-separable*** **Cyclic Code:**

1) Design a combinatorial circuit that is capable of encoding four-bit information words into a *non-separable* cyclic code using the generator polynomial $G(X)=1+X+X^2+X^5$.

2) Show the resulting code words for the data words $(d_3d_2d_1d_0) = (1011)$.

5. (10 points) ***Separable*** **Cyclic Code:**

Show the *separable* cyclic code words for the data word $(d_3d_2d_1d_0) = (1011)$ using the generator polynomial $G(X)=1+X+X^2+X^5$.

Sol:

6. (10 points) Show that the Hamming distance of an M-of-N code is 2.

Sol:

N bits 中有 M 個 bits 為 1

因此$m + 1$ or $m - 1$ bits 為 1 時都可以被 detected

故Hamming distance $d = e + 1 = 1 + 1 = 2$

7. (10 points) Compare two parity codes for data words consisting of 64 data bits: (1) a (72, 8) Hamming code and (2) a single parity bit per byte. Both codes require eight check bits. Indicate the error correction and detection capabilities, the expected overhead, and list the types of multiple errors that are detectable by these two codes.

$Capability$ ：1. $correct\ a\ single\ bit\ error$

2. $detect\ 2\ bit\ errors$

Overhead ：由於電路較複雜和 time delay，(1) 預計會高於 (2)。

8. (10 points) A communication channel has a probability of 10^{-3} that a bit transmitted on it is erroneous. The data rate is 12000 bps. Data packets contain 240 information bits, a 32-bit CRC for error detection, and 0, 8, or 16 bits for error correction coding (ECC). Assume that if eight ECC bits are added, all single-bit errors can be corrected, and if sixteen ECC bits are added all double-bit errors can be corrected.

(a) Find the throughput in information bits per second of a scheme consisting of error detection with retransmission of bad packets (i.e., no error correction).

(b) Find the throughput if eight ECC check bits are used, so that single-bit errors can be corrected. Uncorrectable packets must be retransmitted.

(c) Finally find the throughput if sixteen ECC check bits are appended, so that two-bit errors can be corrected. As in (b), uncorrectable packets must be retransmitted. Would you recommend increasing the number of ECC check bits from 8 to 16?

Throughput in bits per second = P (a packet has no errors) ∗ data rate of the code ∗ data rate

9. (10 points) Derive all code words for the separable 5-bit cyclic code based on the generating polynomial X +1 and compare the resulting code words to those for the non-separable code.

Data word	non-separable code word	separable code word
0000	00000	00000
0001	00011	00011
0010	00110	00101
0011	00101	00110
0100	01100	01001
0101	01111	01010
0110	01010	01100
0111	01001	01111
1000	11000	10001
1001	11011	10010
1010	11110	10100
1011	11101	10111
1100	10100	11000
1101	10111	11011
1110	10010	11101
1111	10001	11110

10. (10 points) Given that $X^7 - 1 = (X + 1)g_1(X)g_2(X)$, where $g_1(X) = X^3 + X + 1$
   (a) Calculate $g_2(X)$.
   (b) Identify all the (7, k) cyclic codes that can be generated based on the factors of $X^7 - 1$. How many different such cyclic codes exist?
   (c) Show all the code words generated by $g_1(X)$ and their corresponding data words.

Sol:

$(a)\ g_2(x) = x^3 + x^2 + 1.$

(b)

Generator	Cyclic code
X+1	(7,6) code
$g_1(x)$	(7,4) code
$g_2(x)$	(7,4) code
$(X + 1) * g_1(x)$	(7,3) code
$(X + 1) * g_2(x)$	(7,3) code
$g_1(x)g_2(x)$	(7,1) code

(c)

Data word	non-separable code word	separable code word
0000	0000000	0000000
0001	0001011	0001011
0010	0010110	0010110
0011	0011101	0011101
0100	0101100	0100111
0101	0100111	0101100
0110	0111010	0110001
0111	0110001	0111010
1000	1011000	1000101
1001	1010011	1001110
1010	1001110	1010011
1011	1000101	1011000
1100	1110100	1100010
1101	1111111	1101001
1110	1100010	1110100
1111	1101001	1111111