Creating a Digital Lensmeter for Unknown Prescription Lenses

BME 5020

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Abstract

Myopia is corrected using a concave lens while hyperopia uses convex lens system for correction. The lens systems used, optically modifies the magnification of the object seen at a far distance. The concave lenses converge the light rays and reduce the perceived size of the original object whereas the convex lenses diverge the rays, increasing the perceived size of the original object. This occurs with reference to standard vision. A lensmeter can be used to determine the prescription strength (optical power) of unknown lenses using the properties of concave or convex lens. The setup used, utilizes a computer, smartphone (camera), pair of glasses, and a credit card aligned at a fixed distance. The camera is interfaced wirelessly and is controlled using Matlab. The program captures two images: with and without lenses, and equally crops the portion of the credit card seen in the image. The program then converts the captured red-green-blue (RGB) images into binary images where a white area of the credit card is generated. Two binary images are taken: with and without the corrective lenses. The number of pixels covered by each white area is compared to create a ratio between the two binary images. This ratio of magnification and its standard prescription value associated with the glasses is added to a calibration database. A fitting curve and equation are generated for the magnification vs. prescription power. This equation allows the determination of the prescription power of an unknown lens. The use of any smartphone with a fixed distance and the calibration equation, allows an end user to simply determine the unknown prescription of any pair of glasses utilizing a computer.

Introduction

According to the World Health Organization, around 1.3 billion people around the world suffer from some form of visual impairment (Holden, Fricke and Wilson). The main causes of visual impairment stem from refractive errors, cataracts, and glaucoma. However, these usually stem from congenital factors, but increasing research has demonstrated that environmental factors affect visual acuity as well. In order to correctly assess visual impairment, biological and clinical factors must be distinguished (Huang and Moore). Congenital defects of the eyes are the most important way of diagnosing problems in vision. Figure 1 below depicts the different types of eyes a person can be born with.

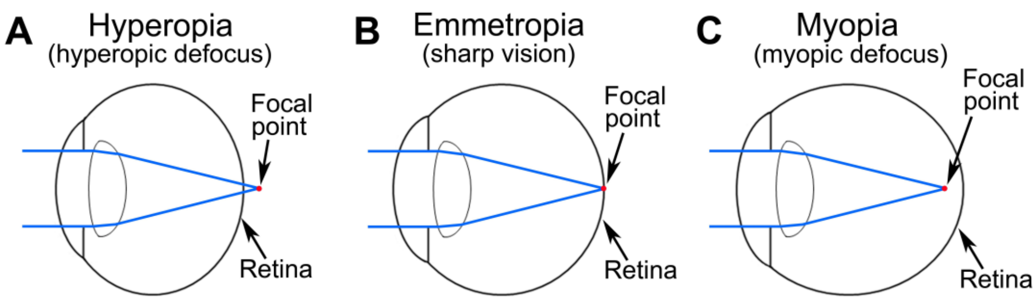

Figure 1. Congenital Eye Forms
Different forms of visual impairments that a patient can be born with. This is directly related to the shape of the eye itself. (1a) Hyperopic defocus – slightly shorter eye. (1b) Normal vision – normal size eye (1c) Myopic defocus – slightly oblong eye.

Emmetropic eyes are normal eyes that do not require any vision correction. This is also the normal shape and size of the eye and vision is viewed as sharp. In contrast, myopic and hyperopic eyes require some form of visual correction. Myopic eyes have a tendency for the focus to be in front of the retina thus, causing blurry vision. Here, the eye is also much more oblong in shape causing such a change. Lastly, hyperopic eyes are shorter in size and focus behind the retina causing blurry vision in objects seen very close to the eyes. These congenital defects can in fact be corrected but if left untreated may result in some loss of vision (Simpson, Wojciechowski and Oexle).

These biological defects lead to uncorrected refractive errors that hamper a person's ability to correctly and sharply see. In turn, this can seriously reduce performance in school, employability, productivity, and generally impair quality of life. However, these refractive errors can successfully be corrected with the use of glasses. Glasses are the most cost-effective innovation that can result in normal vision (Holden, Fricke and Wilson). As uncorrected refractive errors are the main reason of low vision and the second leading cause of blindness, glasses have provided a way to view the world normally with refracting light correctly into the back of the eye. However, most people who wear glasses, barely know their prescriptions. In order to solve such an issue, this paper will attempt to demonstrate an easy and cost-effective way to find the prescription strength of a pair of glasses with the use of a prototype lensmeter using MatLab and everyday items.

Background Information

The eye is a structure that is designed to detect visual stimuli. It is able to form an image on the retina, which detects light and converts these stimuli into action potentials that are sent to the brain. Light enters the eye by passing through the cornea, the clear portion at the front of the eye. Then light is refracted as it passes through the lens in the eye and through the vitreous chamber. As it moves through the jelly-like fluid in the vitreous chamber, excess light is absorbed by the choroid (Foster and Jiang). Finally, the light is focused on the back of the eye on the retina. It contains many electromagnetic receptor cells called photoreceptors that are responsible for detecting light. They are in the form of rods and cones. The rod and cone cells synapse with the nerve cells and travel from the eye toward the occipital lobe of the brain where complex image analysis occurs, and images are visualized.

Refractive Errors

However, during this process, too much or too little curvature of the cornea of the lens can result in some form of visual defect. Too much curvature causes the light that enters to be bent too much and in turn is focused in front of the retina (Huang and Moore). This results in myopia, or nearsightedness. A person suffering from myopia has clear vision when looking at objects up close but struggles to view objects at a distance. The images that are viewed at a distance are blurred and not in focus. As shown in Figure 2, when light enters the eye in a person with myopia, the lens has an oblong shape distorting light in front of the retina. This does not allow a clear image to be sent through the optic nerve. This results in blurred vision that is seen. This can be corrected with the use of a concave (diverging) lens, which causes light rays to diverge slightly before they reach the cornea correcting vision (Roth, Coulouvrat and Hajak). In contrast, hyperopia, or farsightedness,

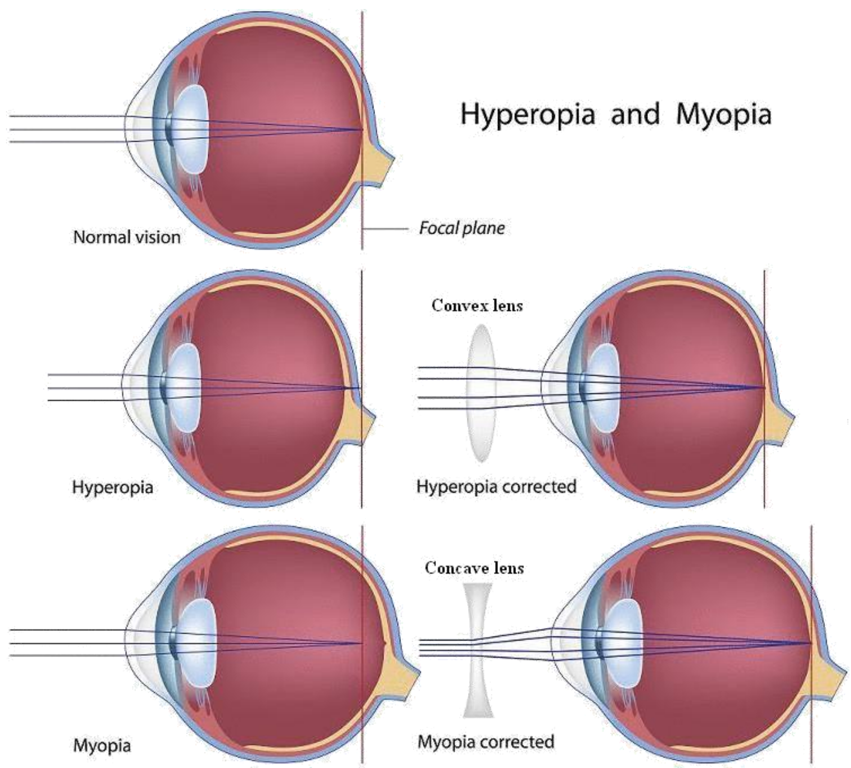

Figure 2. Hyperopia vs. Myopia
(a) Normal vision - parallel light enters the eyes and is refracted on the retina. This signal is sent down through the optic nerve and normal vision is seen. (b) Hyperopia – light enters the eyes and is refracted incorrectly behind the retina. This causes blurred images at close distances. (c) Myopia – light enters the eyes and is refracted incorrectly in front of the retina. This causes blurry vision for images seen at a distance.

results from the light focusing behind the retina. As Figure 2 shows, light enters the eye and the lens is slightly smaller than in normal vision. This accounts for not enough refraction and in turn the light focuses behind the retina. The optic nerve cannot relay the image clearly and images at a distance are unchanged, but up-close images are blurry. Hyperopia can be corrected with the use of convex (converging) lenses (Wu, Huang and Yu). These convex lenses cause the light rays to converge before reaching the cornea and allow for the focus of light back onto the retina.

Optic Laws

When light travels through any homogeneous medium, it travels in a straight line. This parallel light is known as rectilinear propagation. The behavior of light as it passes through any medium can be evaluated through the theory of geometrical optics. Geometrical optics explains reflection as well as refraction when applicable to lenses. When looking at the scope of this paper, refractive errors are the main cause of impaired vision. The rest of this study will elaborate on refraction, which is the bending of light as it passes from one medium to anther while changing speeds (Holden, Fricke and Wilson).

Since light can travel from either side of a lens, there are generally two focal points, with one on each side. The focal length can be measured in either direction. However, for thin spherical lenses, the focal lengths are equal so there is only one focal length that is assumed. Figure 3 shows the difference between converging and diverging lenses. Figure 3 illustrates that

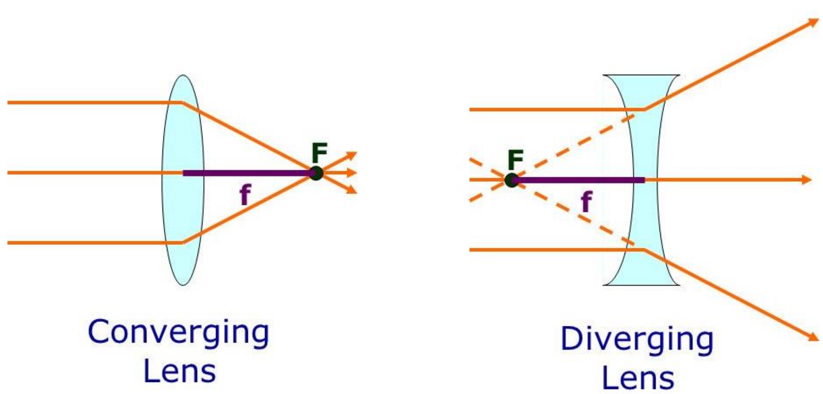

Figure 3. Converging vs Diverging Lenses
(a) Converging lenses always have the focal point at the side that light travels towards. (b) Diverging lenses have the focal point on the side that light starts at.

a converging lens will always be thicker at the center thus forcing light to "converge" to a single focal point. In contrast, diverging lenses are always thinner at the center, resulting in light to "diverge" and the focal point will form in front of the lens itself (Jones and Luensmann). Converging lenses (reading glasses) are used to aid those who are farsighted, while diverging lenses (standard glasses) are used to aid people who are nearsighted.

In addition, there are several important lengths that are associated with lenses illustrated in Figure 4. The focal length (f) is the distance between the focal point (F) and the lens itself. The distance between the object and the lens is called the object distance (u) and the distance between the image and the lens is the image distance (v) (Huang and Moore).

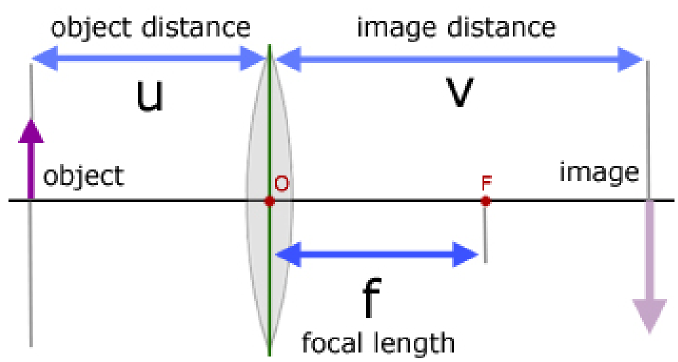

Figure 4. Relationship between Object Distance, Image Distance, and Focal Length
1 divided by the object distance (u) plus 1 divided by the image distance (v) equals 1 divided by the focal length.

This simple relationship can also be evaluated through the thin lens equation. In Equation

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Equation 1. Thin Lens Equation

1: one divided by f is equal to one divided by u plus one divided by v, where f is focal length, u is object distance, and v is image distance. Here, focal length is the distance between the retina and the lens. This makes it somewhat difficult to use this equation when dealing with optics if this distance is unknown. In addition, this equation can only be utilized when the thickness of a lens is negligible. If the thickness is so small that it will not make any difference in refraction, then this equation can be used. For most cases, this equation can be used without any error in the focal length reading (Foster and Jiang). However, when the thickness of a lens cannot be neglected, another equation must be utilized to consider the distance. For these types of lenses, the focal length is related to the curvature of the lens surface and the index of refraction by the lens. An equation that can take into account this relationship is the Lensmaker's equation:

$$\frac{1}{f} = (n-1)(\frac{1}{r_1} - \frac{1}{r_2})$$

Equation 2. Lensmaker's Equation

where n is the index of refraction of the lens material, r_1 is radius of the first curvature of the lens surface, and r_2 is the second curvature of the lens surface. To demonstrate this relationship, it is much easier to visualize this in Figure 5. Light first hits the front of the lens (left side) and

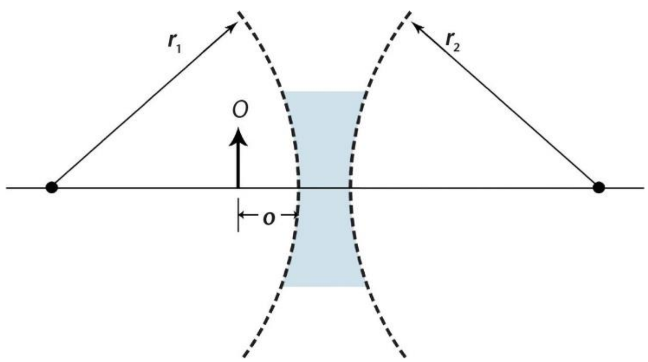

Figure 5. Lensmaker's Diagram
Pathway of light is refracted due to the r_1 and then passes through the lens, and then again due to r_2 before entering the eye.

refracts based on the curvature of the concavity of the lens. Then the light travels through the lens and refracts again due to the second curvature of the convexity of the lens before leaving the lens and entering the eye (Huang and Moore). This is all based on the index of refraction of the lens material.

Since the eye is a very complex refractive instrument that uses real lenses itself, the cornea acts like the primary source of refractive power. It is at this point that the use of either converging or diverging lenses can be used to distort refraction to correct vision if need be. Light is then passed through the adaptive lens that can change the focal length before reaching the vitreous humor. It is then diffused through the layers of retinal tissue before reaching the rods and cones. At this point, the image focuses and is minimized significantly, but is still relatively blurry. The nervous system then processes the image and the remaining errors are removed to display a sharp view of the world.

Diopters

Optometrists describe a lens in terms of its power (P). This is measured in diopters. A diopter is a magnifying power of a lens or a lens system. If can be quantified by Equation 3:

$$P = \frac{1}{f}$$

Equation 3. Power Equation

where f is the focal length in meters. Power has the same sign as focal length but can be positive for converging lenses and negative for diverging lenses. People who are nearsighted require diverging lenses, while people who are farsighted require converging lenses. Physicians usually prescribe glasses in intervals of 0.25 diopters (D) even though the human eye can detect 0.15 D intervals (Roth, Coulouvrat and Hajak). The reason that this is done, is because the 0.15 D interval is not easily rounded. When visiting an optometrist and requesting any prescription for a lens, they will give the reading in terms of OD and OS. OD is the right eye prescription reading while OS is the left eye. An example of a basic prescription would be OD: -4.25. The way one should read this is 4.25 diopters of nearsightedness in the right eye.

Prescriptions

Ophthalmologists use an instrument called a lensmeter to determine the correct prescription of a pair of glasses. This prescription can be somewhat difficult to understand and read. Equation 4 shows the basic formula of a prescription:

$$Prescription: S \quad \pm C \ \times Axis$$

Equation 4. Prescription Readings

The S stands for the "spherical" portion of the prescription and this depicts the degree of nearsightedness or farsightedness. The C stands for the "cylinder" portion or the astigmatism value. An astigmatism is the result of the cornea or the lens of the eye being shaped like a football rather than a sphere (Huang and Moore). This reading can be negative or positive depending on if a patient is myopic or hyperopic. The negative values display the astigmatism of nearsightedness while the positive values display the astigmatism of farsightedness. The C value is followed by an Axis reading. This reading is the orientation of the astigmatism (Foster and Jiang). It is essentially an angle reading of the cylinder astigmatism. It explains the difference in curvature occurring in the cornea. At the axis value, an astigmatism is corrected. To read a lens prescription, all three data readings must be present, or a reading may be incorrect. If a patient has a reading of -2.00 -1.50x180, this means that the patient has 2 diopters of nearsightedness with 1.5 diopters of astigmatism on an axis of 180 degrees (correction horizontally). In contrast, if there is no other readings after the spherical value, it can be assumed that the lens to correct either nearsightedness or farsightedness without an astigmatism.

Methods and Setup

The objective of this paper is to demonstrate a prototype lensmeter utilizing MatLab and every day items to find the prescription strength readings of an unknown pair of glasses. In order to have a successful setup, a few fixed parameters had to be set. The distance between the lens (glasses), camera (phone), and card (image of interest) were all fixed and kept constant throughout the trials. In addition, a statistical database was created using the ratios of all the images with respect to a converging or a diverging lens to one without one. After the database was created, a fitting function for the graph of the ratios of magnification vs known prescriptions

(D) were made. The use of a best fit equation was utilized to determine the power of unknown lenses (in D). Then the last step was to linearize the best fit like to make sure that this line was in fact the one that should be used. Figure 6 displays a top view of how the setup looked like in terms of the distances between the credit card (image of interest), lens (glasses), and the camera (smartphone).

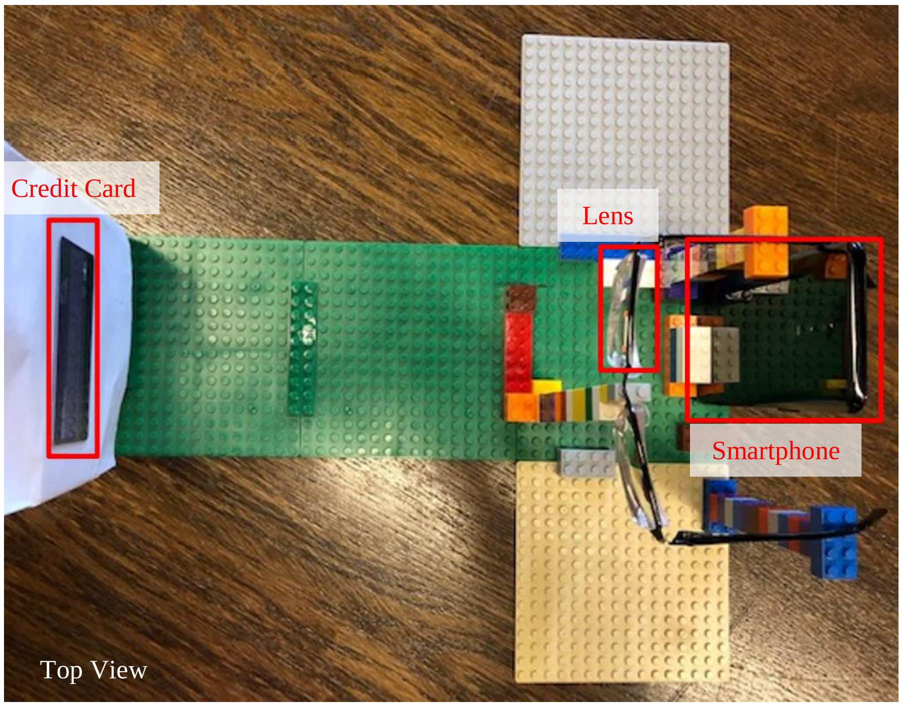

Figure 6. Setup Overview
The credit card, lens, and smartphone have a fixed placement for every single trial to create the database.

GUI

Before a GUI or anything on MatLab was initiated, a flow chart of image processing as well as computing the prescriptions were created in order to direct the overall flow of the GUI. The overall structure of the image processing flowchart was to attempt to store values that were read into a database. This process was started off by taking any image and attempting to convert it into a grayscale image. Then the area was computed by transforming this image into binary thresholds. The data was processes and attempted to be stored in a database that was created based on prescription power (D) and ratios of the images taken. Additionally, a second flowchart was required to compute prescriptions for both farsightedness and nearsightedness. The farsighted data was linearly fit as this gave the best fit line very easily. However, in contrast, the nearsighted data, could be done in multiple ways. An option was created to implement a linear fit as well as a non-linear fit. Figure 7 depicts the way that both processes were started and what were the outcomes of such:

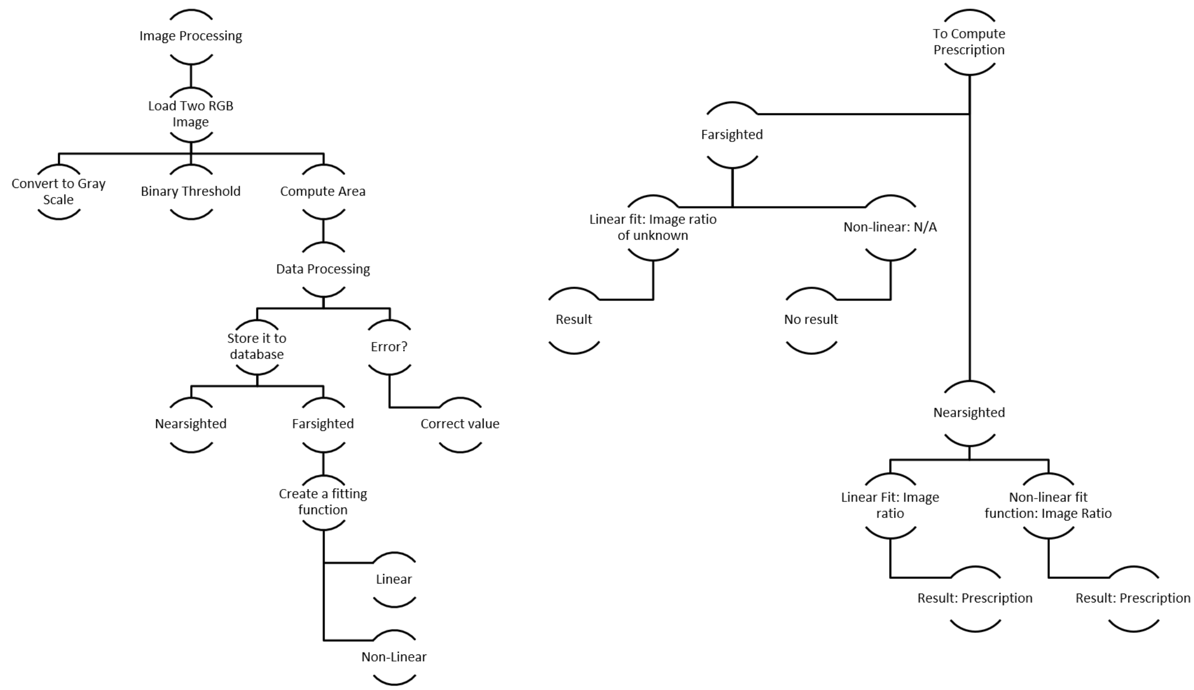

Figure 7. Flowchart Depicting the GUI Pathway
Image processing and compute prescription pathways that was made to create the GUI that is the basis of this paper to find unknown prescriptions.

Using the above flowchart, a GUI was coded that could acquire an image with the use of a smartphone and give the prescription strength of a pair of glasses. The way that this was implemented was to split the GUI into three different parts. The first part was the image processing portion of the GUI. Here is where two images were acquired: a background image (constant) and an image with glasses. The RGB images were converted to binary and the area were computed with and without glasses. The ratio between these as then calculated and found.

In the data processing portion of the GUI, the ratio was acquired from the image processing portion and set with a known prescription of a pair of glasses. This allows the user to start creating a database of the glasses that will be later used to create a calibration curve for all the data. Here is where the prescriptions were split between farsighted and nearsighted. Lastly, the ultimate portion of the GUI was the output portion. This allows the user to attain a calibration equation that was predetermined and used to estimate an unknown prescription of a lens. Then it also gave a reading of the prescriptions once a button was pressed to the nearest 0.25 D. Figure 8 shows how this GUI was formatted.

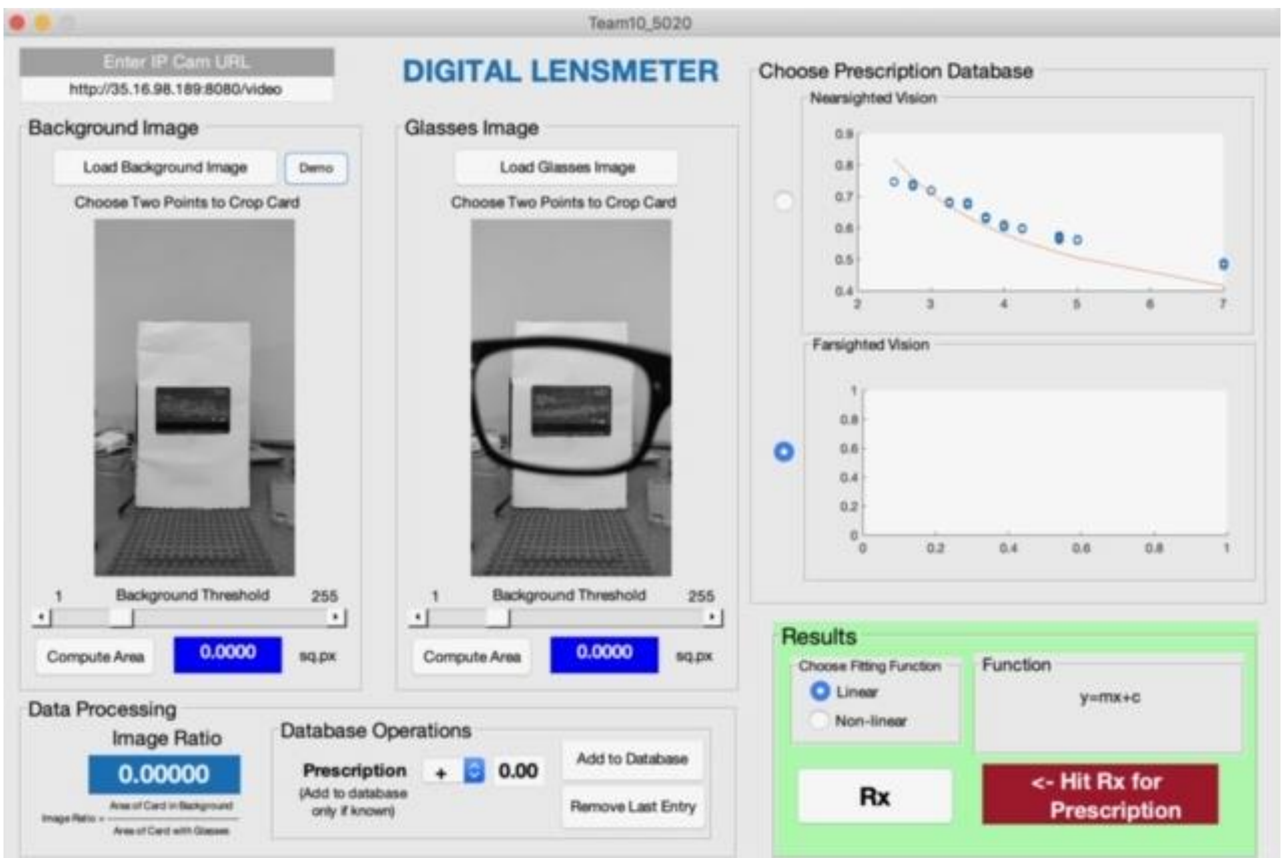

Figure 8. Digital Lensmeter GUI
GUI created in MatLab to create a database that can be used to find any prescription of an unknown lens.

Data Analysis

After creating a statistical database, for both farsighted and nearsighted lenses, an equation had to be made for both in order to display the best fit for all of the data points. This had to be done in order to get a predicted prescription value for any unknown pair of glasses. However, when trying to analyze the data, the very first model that was chosen was a linear fit. If this worked with a high R^2 value (close to 1), then no other models were attempted because this was an accurate interpretation of the statistical database. However, if the R^2 value was not up to par, then another one was chosen to try to represent the database in a better way.

<u>Farsighted Lenses</u>

After creating the database, a linear best fit line was tested, and Figure 9 depicts how it matched up with the data:

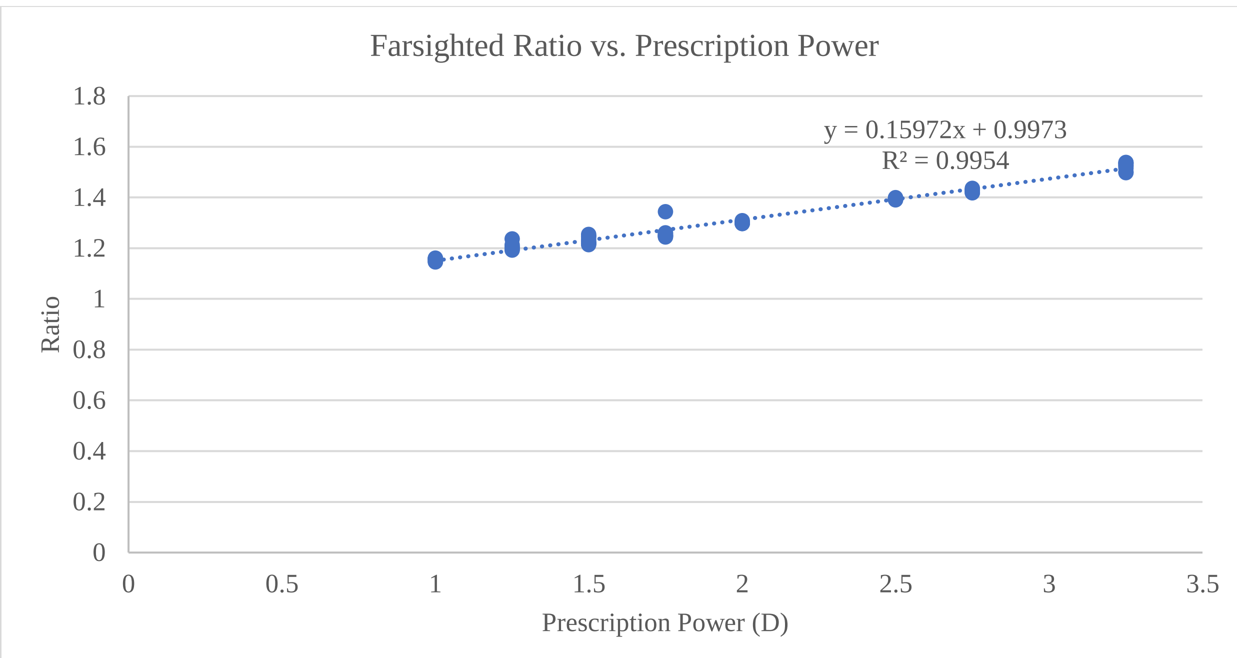

Figure 9. Farsighted Linear Fit
A best fit line of y=0.15972x+0.9973 was found with a R^2 value of 0.9954.

The best fit line suggest that this is clearly a linear representation. As the prescription power of the farsighted glasses increases, so does the ratio. Since the R^2 value is 0.9954, this explains that the data is in fact linear since the value is extremely close to 1. With this model it is quite easy to say that any prescriptions that are used for farsighted vision would be determined with this equation since it matches up with the database so well. Since the R^2 value is very high, there is no need for another best fit line estimate. However, a limitation that was observed was that since prescription reading glasses range from +3.5 to +5, and there was no cost-effective way to obtain these glasses, this range remains untested. However, for the time being, it is safe to predict that they will in turn follow this trend.

<u>Nearsighted Lenses</u>

However, after creating a database for the nearsighted glasses, some discrepancy was determined as can be seen in Figure 10:

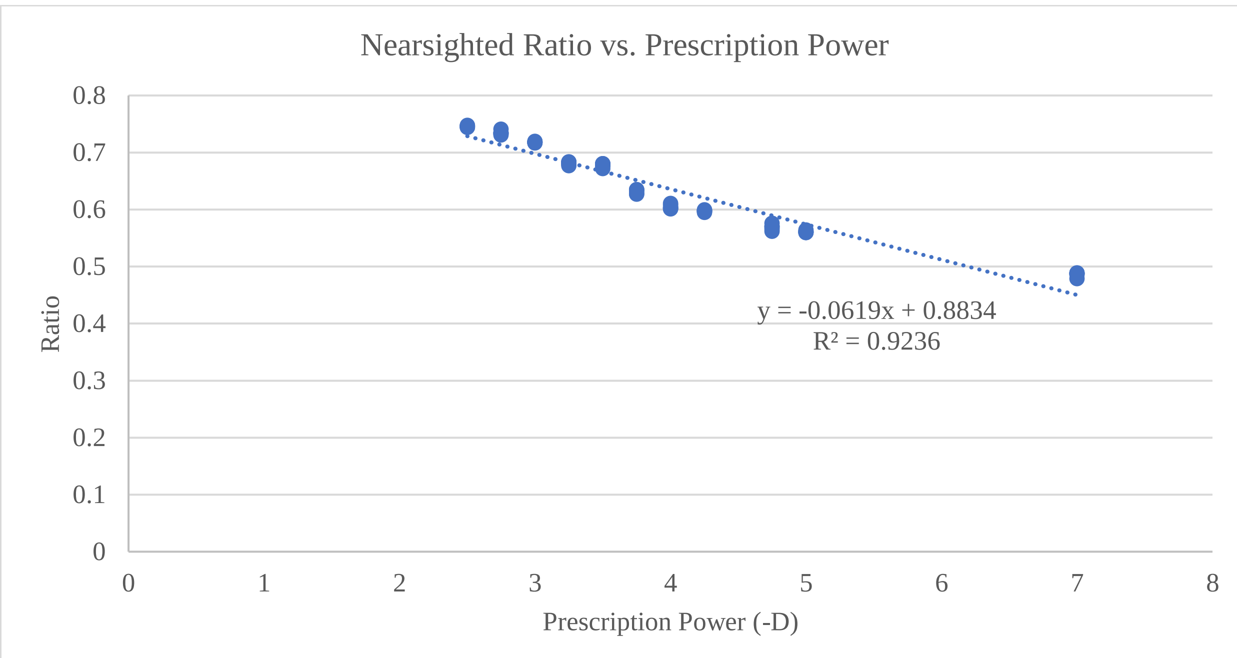

Figure 10. Nearsighted Linear Fit
A best fit line of y=-0.0619x+0.8834 was found with a R^2 value of 0.9236.

The best fit line suggests that there is still some error in this fit. Since the R^2 value is 0.9236, it shows that this is not the most accurate fit for the data. If any unknown prescription pair of glasses were tested with this model, they would give a reading that would be off. The reason being is that a linear model does not exactly fit this type of data very well. A possible reason that this can be is because there is much more variance in the database. When making the database, some pairs of glasses had astigmatisms that skewed some of the points within the database. This will be further evaluated in the errors and limitation section as to why this occurred. Since a linear fit did not seem to match up with the database very accurately, it was deemed that another fit should be attempted to try to find a better fitting line to get more accurate results when reading unknown prescriptions.

Since the database has some parabolic trends, a polynomial fit of the second order was attempted to match the data base. Figure 11 depicts how this best fit line looked like:

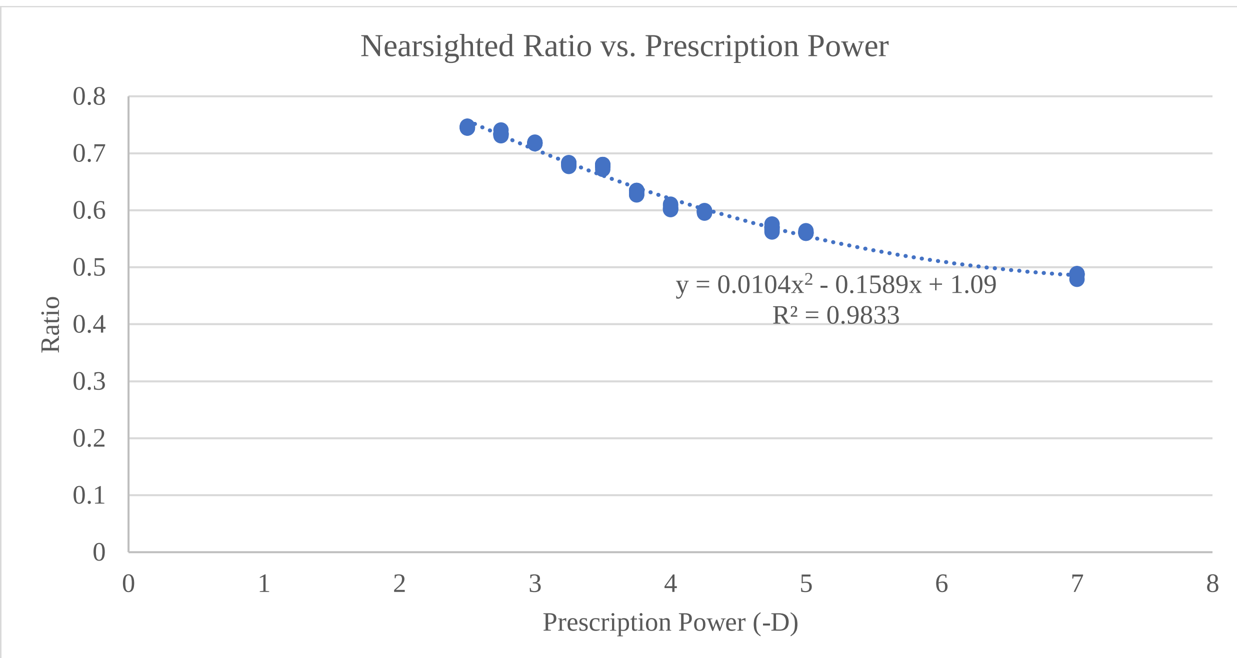

Figure 11. Nearsighted Polynomial (2nd Order) Fit
A best fit line of $y=-0.0104x^2-0.1589x+1.09$ was found with a R^2 value of 0.9833.

This was a much more accurate fit as the R^2 value was 0.9833. This does in fact match up very well with the data. However, there is a major flaw in this type of polynomial fit. After a certain point, there has to be an increase in the fit. What this means is that after a certain prescription value, the readings on this best fit line will tend to increase and this could cause some form of issues when predicting prescriptions of unknown glasses. However, -7 is an extremely high-power prescription so it may be deemed that there are no more possible prescriptions that will be much higher. Nevertheless, it cannot be assumed that this is the case. For the database, this is almost a perfect best fit and will most likely provide data points that are accurate for unknown prescription lenses. In order to really check if this fit is reliable, the data must be linearized.

Linearization

In order to linearize the data, the non linear functions must be placed back into a linear model. This will be done using the linearization formula itself.

Linearization Formula Method

The following best fit polynomial equation will be used to determine the linearization:

$$f(x) = 0.0104x^2 - 0.1589x + 1.09 \text{ at } x_0 = 4.25$$

Equation 5. Second Order Polynomial Fit

$$L(x) = f(x_0) + f'(x_0)(x - x_0)$$

$$f(4.25) = \frac{24101}{40000} \approx 0.602525$$

$$f'(x) = \frac{13x}{625} - \frac{1589}{10000}$$

$$f'(4.25) = \frac{13(4.25)}{625} - \frac{1589}{10000} \approx -0.0705$$

$$L(x) = \frac{24101}{40000} - \frac{141}{2000}(x - 4.25)$$

$$y = -0.0705 + 9.022$$

A graph of the linearized line is available on the next page.

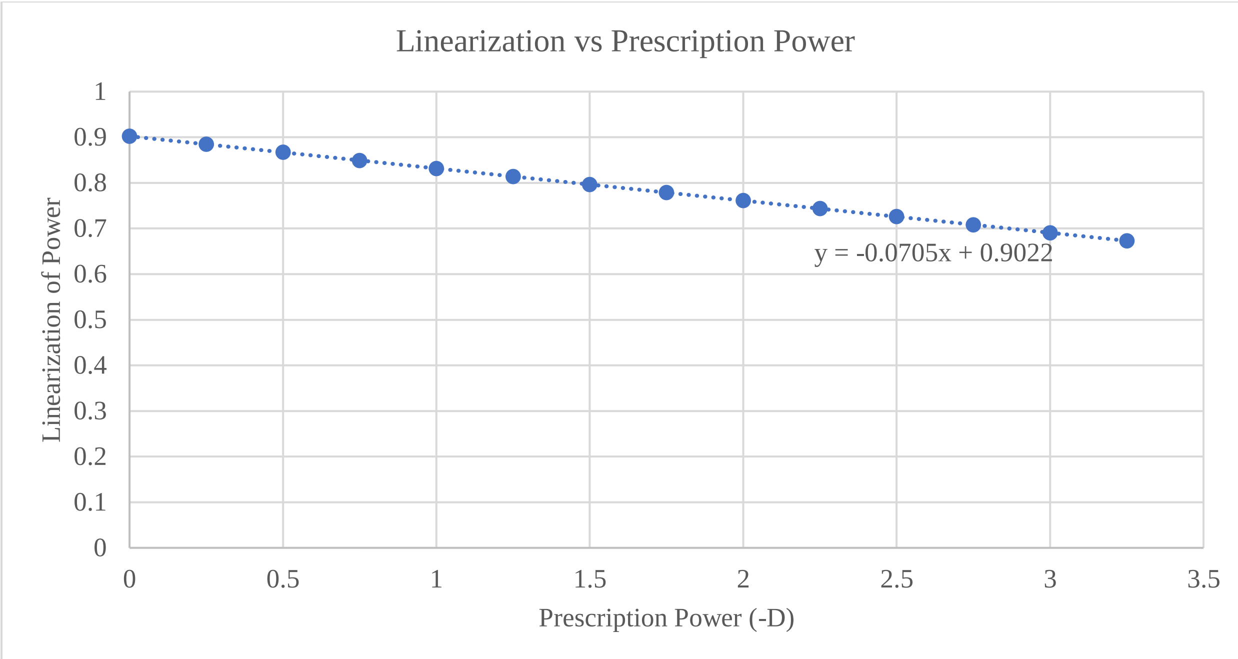

Figure 12. Linearization of Second Order Polynomial Fit
The linearization was found to be y=-0.075x+0.9022.

The linearization of this data proves to be an accurate depiction of prescription power vs. the ratio.

Errors and Limitations

Through the course of this study, there have been quite a few errors and limitations that will be addressed in the following section. The most blatant limitation was the fact that there when selecting for a range of glasses to create the database, only a certain number were available. For the hyperopic database, over the counter glasses were tested only. This only includes strengths of +1.00 D to +3.25 D. However, prescription strengths can up to as high as +5. The issue with getting such high power prescriptions would be primarily the cost. It is almost impossible to buy prescriptions of that high power for a cheap cost. In addition, a similar issue is prevalent with the myopic database. Only prescription strengths of -2.50 D to -7.00 D were tested. This also creates so issues because it is almost unknown now the lower side of the prescriptions would have created the ratio.

Additionally, another limitation pertaining to prescriptions was evident that was alluded to earlier in the paper. Only spherical powers can be evaluated through this program. What this means is that when going back to Equation 4, prescription is: $S \pm C \times Axis$. The C (cylindrical) and Axis (degrees) values refer to astigmatisms. With the database that was created for this project, it was found that glasses that were used to correct any form of astigmatisms skewed results in either way in terms of the size of the ratios seen. However, there was a slight trend that was seen that when a pair of glasses has astigmatism, the ratio values are in fact smaller. This cannot be stated for certain, since more testing needs to be done on the C and Axis values to know which one has a larger effect on the ratio values.

Another limitation is the fact that the program setup was only able to work with Android phones. The application that was used to send a captured image could only be sent to the program through an IP Cam application that was only available on Android devices. However, in the coding, there was a work around where a manual entry for images with and without glasses can be utilized to calculate the ratios. In addition, another limitation is the fact that the program cannot read anisometropic glasses. What this means is that glasses that have both transitioning nearsighted and farsighted areas cannot be used. The program cannot detect the image between the two and no real ratio will be created. Lastly, a possible error could be in the image analysis itself when taking the surface area. There was an evident shadow that was taken into account for on some of the trials for the data collection. This was removed immediately by shining a light on the area to illuminate the area. A shadow would effectively add area to the ratio and make them larger than they should be. However, this was accounted for with the instituted error of ±0.25.

Conclusion

Myopia and hyperopia are prevalent in many parts of the world and affect around 15% of the world's population (Wu, Huang and Yu). Since 2004, refractive errors have become ever so increasing and now it has gradually reached the point where more and more people suffer from it every day. With the increasing number of people suffering from impaired vision, glasses around the world have become a common and cost-effective way to fix refractive errors. Since many people do not know their exact prescription, this GUI and setup provided in this paper illustrates a very simple way to find one's prescription within roughly 0.25 D intervals. The end user basically has to click 4 buttons and the unknown prescription will not be a mystery. Throughout the course of this project, it has become evident that astigmatisms have provided to be a huge hinderance on the progression of the readings that this GUI and setup can display. As a result, a static error of ±0.25 D was instituted to compensate for such an error. Nevertheless, further trials are required as well as more prescriptions to add the effects of cylindrical and axial values that are in most prescriptions to explore the effects of astigmatism. With the help of this GUI, one can find out their unknown prescriptions, with simple household items through a created statistical database.

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