Women In Data Datathon: Conjunction Risk Analysis

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1 Background

1.1 Introduction

From 2020 to 2024, satellites operating in Earth orbit grew from 3,371 to 11,539. In this year alone, more than 1,200 satellites were launched into orbit from January to April. SpaceX led with 573 Starlink satellites during the Q1 of 2025.

Our space environment is becoming increasingly crowded as the number of satellites and large constellations like Starlink continues to grow. In addition to these new launches, inactive satellites in Low Earth Orbit (LEO) can remain in orbit for years to centuries.

Each additional satellite increases conjunction frequency and thus creates more chances for collision. When two satellites collide, they can produce thousands of pieces of debris and trigger cascading collision events.

Sources:

- Satellite Industry Association Releases the 28th Annual State of the Satellite Industry Report
- Orbital debris and the market for satellites
- Modeling Orbital Decay of Low-Earth Orbit Satellites due to Atmospheric Drag
- NASA Spacecraft Conjunction Assessment and Collision Avoidance Best Practices Handbook
- Satellite orbital conjunction reports assessing threatening encounters in space (SOCRATES)

1.2 Key Terms and Concepts

1.2.1 What is a shell?

A shell is band of altitudes where satellites are placed. It is not as single orbit but a "layer" above Earth where satellites can exist with different inclinations and longitudes.

Risks of different shells

- Shells below 500 km are less crowded, but satellites decay faster due to atmospheric drag.
- Shells between 500–600 km very popular because they balance longer lifetime with lower launch cost. However, conjunctions risks are higher since the space is more crowded. Lifetime could be years to decades, e.g. a dead satellite at 550 km might remain in orbit for 10-25 years before atmospheric reentry.

- There is less drag in shells above 800 km, so satellites can stay for decades to centuries. This is bad for long-term sustainability since debris also lingers forever.
- Objects in the geostationary orbit shell (~36,000 km) remain essentially forever as there is meaningful drag at all. Satellites must be moved to a "graveyard orbit" when retired.

Lifetimes

Consider a typical satellite-sized object that is about 100–1,000 kg with moderate drag area.

Below are its orbital lifetime estimates by altitude:

• 300-400 km: ~0-2 years before atmospheric reentry

• 500-600 km: $\sim 5-30 \text{ years}$

• 700-800 km: $\sim 80-400 \text{ years}$

• 900-1,000 km: \sim 500-1,500 years

• 1,200 km and above: 2,000+ years

1.2.2 Space Object Types

Object Type	Description	Importance in Conjunction Risk Analysis
Payload	Operational or defunct satellites	Valuable, often maneuverable, critical to protect
Rocket body	Spent propulsion units to deploy satellites into orbit, i.e. launch vehicle stages	Large, non-maneuverable, collision threat
Debris	Fragments from explosions, collisions, breakups	Numerous and unpredictable
Unknown	Identified but unclassified objects	Complicates modeling with uncertainty, could be a hazard or payload

Why are rocket bodies catalogued differently than standard debris?

• From Space Track Documentation:

They can have mechanisms or fuel on board that can affect the orbital behavior of the rocket body even after long periods of time. Rocket bodies are also constructed to endure high temperatures and stresses associated with launch, so they have a greater probability of surviving reentry and require closer attention than most debris.

1.2.3 Conjunction vs. Collision

Conjunction: A close approach between two objects in space, defined by a threshold distance.

Collision: An event where we objects hit each other.

1.3 Historical Context

2009 - The First Satellite Collision

The collision of Iridium 33 and Cosmos 2251 produced more than 1800 pieces of debris that were larger than 10 cm. Some of which will remain in orbit through 2100.

1.4 Goal: Forecast Conjunction Risk

Phase 1: Setup

- Define scope
- Import libraries and dataset
- Data wrangling
- Configure conjunction analysis
- Build propagators

Phase 2: Execution

- Run coarse propagation
- Deduplicate coarse candidates
- Refine locally

Phase 3: Analysis and Reporting

- Risk report
- Summary and recommendations

2 Setup

2.1 Scope

Objective

As satellites are being launched at an accelerating rate each year, we want to know: Is it getting too crowded in popular shells? Can we track or predict the risk of conjunctions?

Our reported findings are intended to support policymakers in making evidence-based decisions regarding space safety and environmental regulations.

Dataset

We will work with a combined dataset from:

- 1. United States Space Force (USSF)'s 18th Space Defense Squadron Element Sets
- 2. NASA Goddard Space Flight Center's CDDIS (Crustal Dynamics Data Information System) Ephemeris Archive

Deliverable

Risk report and recommendations.

Model Choice

The SGP4 is a standard orbital propagation model used to predict to position and velocity of satellites over time.

- Strengths: fast, efficient, standardized
- Limitations: not the most precise, best for short-term predictions

Methodology

1) What time horizon are we forecasting?

Screen for close approaches over the next 24 hours. This is long enough to see interesting traffic but short enough to run fast on a laptop with thousands of objects.

2) How fine is our sampling of time?

For the first "coarse" pass, propagate every 5 minutes. For any potential close approach we find, we'll re-check just those two satellites around that time at 30-second steps to refine the minimum distance. This will keep runtime manageable and still finds the real minimum distance accurately.

3) What counts as a "conjunction"?

Flag pairs that ever get within 10 km (and we'll also count the stricter 5 km subset). We'll use an initial search radius of 20 km during the coarse pass to make sure we don't miss events that dip below 10 km between 5-minute samples.

4) Which objects are we analyzing?

The densest shell by object count. Exclude anything that's not currently orbiting (isOrbiting == False) so we don't propagate dead/decayed entries.

5) What coordinate system/units are we using?

SGP4 returns positions in the TEME/ECI frame, in kilometers. We'll compute distances directly in that frame with plain Euclidean distance.

6) What should we expect from TLE/SGP4?

TLE+SGP4 is screening-level only (good for finding candidates, not for computing formal probability of collision). Accuracy drops as you move far from the TLE's epochDate, so we'll start the forecast at the latest epoch among your selected objects to reduce bias.

7) What do we need from the dataframe?

The dateframe contains classical mean elements we can feed into SGP4: inclination, raan, argOfPerigee, meanAnomaly, eccentricity, meanMotion, bStar, plus epochDate. Check these fields exist and have minimal missing data for the chosen shell.

2.2 Import Libraries and Dataset

```
[]: import pandas as pd
import numpy as np

from pathlib import Path
from datetime import datetime, timedelta, timezone
from dataclasses import dataclass
from sgp4.api import Satrec, SGP4_ERRORS, jday, WGS72
from scipy.spatial import cKDTree

FINAL_DIR = Path.cwd().parents[1] / "data" / "02_final"
```

print(FINAL_DIR)

c:\Users\ash\Desktop\wid-datathon\data\02_final

```
[42]: df = pd.read_csv(FINAL_DIR / "satellite_data_clean.csv")

# global setting to show all columns
pd.set_option('display.max_columns', None)

df.info()
```

<class 'pandas.core.frame.DataFrame'>
RangeIndex: 29140 entries, 0 to 29139
Data columns (total 57 columns):

Data	ta columns (total 57 columns):		
#	Column	Non-Null Count	Dtype
0	argOfPerigee	29140 non-null	float64
1	bStar	29132 non-null	float64
2	${\tt createdAt}$	29140 non-null	object
3	eccentricity	29140 non-null	float64
4	${\tt semiMajorAxis}$	29109 non-null	float64
5	satNo	29140 non-null	int64
6	revNo	29109 non-null	float64
7	raan	29140 non-null	float64
8	period_els	29109 non-null	float64
9	${\tt meanMotionDot}$	29118 non-null	float64
10	${\tt meanMotionDDot}$	29118 non-null	float64
11	meanMotion	29140 non-null	float64
12	meanAnomaly	29140 non-null	float64
13	inclination_els	29140 non-null	float64
14	idOnOrbit	29140 non-null	int64
15	epoch	29140 non-null	object
16	epochDate	29140 non-null	object
17	intldes	29140 non-null	object
18	noradCatId	29140 non-null	int64
19	objectType	29140 non-null	object
20	satName	29140 non-null	object
21	country	29140 non-null	object
22	launch	29140 non-null	object
23	site	29140 non-null	object
24	decay	1198 non-null	object
25	inclination_sat	29134 non-null	float64
26	rcsValue	29140 non-null	int64
27	rcsSize	28728 non-null	object
28	file	29140 non-null	int64
29	launchYear	29140 non-null	int64
30	launchNum	29140 non-null	int64
31	launchPiece	29133 non-null	object

```
32
    objectName
                               29140 non-null
                                               object
 33
    objectId
                               29140 non-null
                                               object
 34
    objectNumber
                               29140 non-null
                                               int64
 35
    perigee_alt_km
                               29109 non-null
                                               float64
                               29109 non-null
     apogee alt km
                                               float64
 36
 37
     apogee_mismatch
                               29140 non-null
                                               bool
 38
    perigee mismatch
                               29140 non-null
                                               bool
    orbitClass
 39
                               29140 non-null
                                               object
    launchDecade
                               29140 non-null
                                               object
    inclinationBand
 41
                               29140 non-null
                                               object
 42
    eccClass
                               29140 non-null
                                               object
 43
    ageInYears
                               29140 non-null
                                               float64
                                               float64
     shell_idx_100km
 44
                               29109 non-null
 45
     shell_100km
                               29140 non-null
                                               object
 46
    shell_center_km
                               29109 non-null
                                               float64
 47
    isDecayed
                               29140 non-null
                                               bool
 48
    isOrbiting
                               29140 non-null
                                               bool
 49
    isStarlink
                               29140 non-null
                                               bool
 50
    isOneweb
                               29140 non-null
                                               bool
 51
    isIridium
                               29140 non-null
                                               bool
 52
    isConstellation
                               29140 non-null
                                               bool
    dwelling alt km
                               29109 non-null
 53
                                               float64
    dwelling_alt_km_weighted 29109 non-null float64
 55
    dwelling_shell_idx
                               29109 non-null
                                               float64
 56 dwelling_shell_100km
                               29140 non-null
                                               object
dtypes: bool(8), float64(21), int64(8), object(20)
memory usage: 11.1+ MB
```

2.3 Data Wrangling

```
[43]:
                               index n_missing
      24
                               decay
                                           27942
      27
                             rcsSize
                                             412
                     perigee_alt_km
      35
                                              31
      54
          dwelling_alt_km_weighted
                                              31
      53
                    dwelling_alt_km
                                              31
      46
                    shell_center_km
                                              31
                    shell_idx_100km
      44
                                              31
      36
                      apogee_alt_km
                                              31
      55
                 dwelling_shell_idx
                                              31
```

```
4
                     semiMajorAxis
                                            31
      8
                                            31
                        period_els
      6
                             revNo
                                            31
                    meanMotionDDot
                                            22
      10
      9
                     meanMotionDot
                                            22
      1
                             bStar
                                             8
      31
                       launchPiece
                                             7
      25
                   inclination_sat
                                             6
[44]: # parse epochDate to timezone-aware UTC timestamps
      df = df.copy()
      df["epochDate"] = pd.to_datetime(df["epochDate"], utc=True, errors="coerce")
      # replace missing bStar with 0.0 (common practice for SGP4 init if unknown)
      if "bStar" in df.columns:
          df["bStar"] = df["bStar"].fillna(0.0)
      # create a reliable inclination in degrees
      df["inclination_deg"] = df["inclination_els"].where(
          ~df["inclination_els"].isna(),
          df["inclination_sat"]
      )
      # coerce all numeric inputs we'll send to SGP4 to numeric dtype
      for c in_
       →["eccentricity", "meanAnomaly", "raan", "argOfPerigee", "meanMotion", "inclination_deg", "bStar"]
          df[c] = pd.to_numeric(df[c], errors="coerce")
[45]: shell_counts = (
          df.loc[df["isOrbiting"] == True]
            .groupby("shell_100km", dropna=False)
            .size()
            .sort_values(ascending=False)
      )
      print("Objects by 100-km shell (orbiting only):")
      print(shell_counts.head(10))
     Objects by 100-km shell (orbiting only):
     shell_100km
      500- 599 km
                        6253
      400- 499 km
                        4624
      700- 799 km
                        3286
      800-899 km
                        2285
      600- 699 km
                        2201
      300- 399 km
                        1364
      900- 999 km
                        1167
```

```
1400-1499 km
                       1048
     35700-35799 km
                        726
     1000-1099 km
                        661
     dtype: int64
[46]: SHELL_TO_USE = shell_counts.idxmax() # densest shell
      SHELL_COUNT = int(shell_counts.max())
      print(f"\nChosen shell: {SHELL_TO_USE}, {SHELL_COUNT} objects")
      # filter dataframe to shell and still-orbiting objects
      df shell = df[
          (df["shell_100km"] == SHELL_TO_USE) &
          (df["isOrbiting"] == True) &
          (df["orbitClass"] == "LEO")
      ].copy()
      # valid eccentricity range for SGP4: [0,1)
      ecc mask = df shell["eccentricity"].between(0.0, 1.0, inclusive="left")
      df_shell_clean = df_shell[ecc_mask].copy()
      print(f" After setting eccentricity range: {len(df_shell_clean)} rows remain.")
      # mean motion must be positive (revs/day)
      mm mask = df shell clean["meanMotion"].astype(float) > 0.0
      df_shell_clean = df_shell_clean[mm_mask].copy()
      print(f" After meanMotion > 0: {len(df shell clean)} rows remain.")
      # epochDate must be valid (not NaT)
      df_shell_clean = df_shell_clean[df_shell_clean["epochDate"].notna()].copy()
      print(f" After valid epoch: {len(df_shell_clean)} rows remain.")
      df_shell = df_shell_clean.copy()
     Chosen shell: 500-599 km, 6253 objects
       After setting eccentricity range: 6093 rows remain.
       After meanMotion > 0: 6093 rows remain.
       After valid epoch: 6093 rows remain.
[47]: df_shell["objectType"].value_counts(dropna=False)
[47]: objectType
     Payload
                     4967
     Debris
                      845
     Unknown
                      150
      Rocket body
                      131
     Name: count, dtype: int64
```

2.4 Configure Conjunction Analysis

```
[48]: from dataclasses import dataclass
      from typing import Optional
      @dataclass
      class Config:
          # what we're analyzing
                                      # e.g."500-600 km"
          shell_name: str = "mixed"
          n_objects: int = 0
                                        # filled after df_shell is built
          # constant
          earth_radius_km = 6378.137  # Earth's mean equatorial radius (WGS-84)
          # time window
          forecast_hours: int = 24
              # how far into the future to forecast
              # start with 24h for speed; you can extend later
          explicit_start_utc: Optional[datetime] = None # set to fix start; else_
       ⇔latest TLE epoch
          # sampling step sizes
          coarse_step_minutes: int = 5  # coarse propagation step for full set
          refine_step_seconds: int = 30  # refinement step for candidate pairs
          # thresholds
          search_radius_km: float = 20.0 # coarse neighbor query radius
          report_thresh_km: float = 10.0 # main reporting threshold
          report_strict_km: float = 5.0 # stricter subset for "very close"
       \hookrightarrowapproaches
          # derived (computed after df_shell is known)
          t_start: Optional[datetime] = None
          t_end: Optional[datetime] = None
      cfg = Config()
      def initialize_config(cfg: Config, df_shell: pd.DataFrame) -> Config:
          if df_shell is None or df_shell.empty:
              raise ValueError("df_shell cannot be empty")
          # shell name from column
          if "shell 100km" not in df shell.columns:
              raise ValueError("expected 'shell_100km' in df_shell for labeling the ⊔
       ⇔working shell")
          shell_values = df_shell["shell_100km"].dropna().unique()
```

```
if len(shell_values) == 1:
                     cfg.shell_name = str(shell_values[0])
                     print(f"Single shell detected: {cfg.shell_name}")
          else: # edge case: mixed labels
                     cfg.shell_name = "mixed"
                     print("WARNING: Multiple shell labels found in df_shell:")
                     for val in shell_values:
                                print(f" - {val}")
                     print("Proceeding with shell_name='mixed'")
          cfg.n_objects = int(len(df_shell))
          if cfg.explicit_start_utc is not None:
                     t_start = cfg.explicit_start_utc
          else:
                     t_start = df_shell["epochDate"].max() if "epochDate" in df_shell.
   ⇔columns else None
                     # if epoch parsing failed earlier and this is NaT, fall back to 'now'
   \hookrightarrow in \ UTC.
                     if t_start is None or pd.isna(t_start):
                                t_start = datetime.now(timezone.utc)
          cfg.t_start = t_start
          cfg.t_end = t_start + timedelta(hours=cfg.forecast_hours)
          return cfg
def print_config(cfg: Config) -> None:
          print("\nConjunction summary:")
          rows = {
                     "Shell label":
                                                                                              cfg.shell_name,
                     "Object count":
                                                                                               cfg.n_objects,
                     "Time window":
                                                                                             f"{cfg.t_start} to {cfg.t_end}",
                      "Coarse step":
                                                                                            f"{cfg.coarse_step_minutes} min",
                     "Refine step": f"{cfg.refine_step_seconds} sec",
"Coarse search radius": f"{cfg.search_radius_km} km",
"Risk thresholds": f"<{cfg.report thresh km} km. <{c
                     "Risk thresholds":
                                                                                              f"<{cfg.report_thresh_km} km, <{cfg.</pre>
   →report_strict_km \right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right
                     "Frame & units":
                                                                                             "SGP4 TEME/ECI; distances in km (Euclidean).
   \hookrightarrowII
          }
          for field, val in rows.items():
                     print(f"{field}: {val}")
```

```
[49]: cfg = initialize_config(cfg, df_shell)
      print_config(cfg)
     Single shell detected: 500-599 km
     Conjunction summary:
     Shell label: 500- 599 km
     Object count: 6093
     Time window: 2025-08-03 00:00:00+00:00 to 2025-08-04 00:00:00+00:00
     Coarse step: 5 min
     Refine step: 30 sec
     Coarse search radius: 20.0 km
     Risk thresholds: <10.0 km, <5.0 km
     Frame & units: SGP4 TEME/ECI; distances in km (Euclidean).
[50]: # EPOCH FRESHNESS FILTER
      # qoal: keep satellites whose TLE epoch is "close" to the common start time
       \hookrightarrow t_start.
      # avoids SGP4 numerical/pathology issues when propagating far from an object's
       →own epoch
      # compute age of each epoch relative to t_start (days; positive means epoch
       \hookrightarrow BEFORE \ t\_start)
      df shell = df shell.copy()
      df_shell["epoch_age_days"] = (cfg.t_start - df_shell["epochDate"]).dt.
       ototal_seconds() / 86400.0
      # pick a freshness window, like ±3 days for LEO screening
      max_age_days = 1.0
      fresh_mask = df_shell["epochDate"].between(
          cfg.t_start - pd.Timedelta(days=max_age_days),
          cfg.t_start + pd.Timedelta(days=max_age_days)
      )
      df_shell_fresh = df_shell[fresh_mask].copy()
      print("Epoch freshness filter:")
      print(f" Window: |epoch - t_start| <= {max_age_days:.1f} days")</pre>
      print(f" Kept:
                         {len(df_shell_fresh)}")
      print(f" Dropped: {len(df_shell) - len(df_shell_fresh)}")
      # summary of how far the kept epochs are from t_start
      print("\nAge (days) among kept rows:")
      print(df_shell_fresh['epoch_age_days'].describe().to_string())
```

```
# if you dropped too many rows and want to relax the window - bump max_age_days_\ to 5-7.

# if you still drop a lot - consider redefining t_start (e.g., median/quantile_\ of epochs).
```

Epoch freshness filter:

```
Window: |epoch - t_start| <= 1.0 days
```

Kept: 5874
Dropped: 219

Age (days) among kept rows:

count	5874.000000
mean	0.148621
std	0.355745
min	0.000000
25%	0.000000
50%	0.000000
75%	0.000000
max	1.000000

2.5 Build Propagators

sgp4 is a standard model for predicting satellite positions from TLE data (two-line elements).

Our dataframe doesn't store raw tle strings, but it does have the equivalent parameters. We'll feed those into sgp4.api.Satrec objects using the function sgp4init.

Units and conversions: - sgp4init expects angles in radians, not degrees - meanMotion must be converted from revolutions/day to radians/minute - epochDate must be expressed as a Julian date split into (jd, fraction)

Variable names (inclo, nodeo, argpo, mo, no_kozai, etc.) below were chosen to match the sgp4 C/fortran heritage. The python wrapper sgp4.api preserves those names for consistency.

sgp4init() will return a satellite record object, called a satrec, that knows how to compute that satellite's position at any time. satellites is a list where each element is dictionary containing a satellite's metadata and how to propagate it.

```
[51]: from sgp4.api import Satrec, SGP4_ERRORS, jday, WGS72

satellites = []  # list of dictionaries with satNo, name, and Satrec object
errors = []  # tracks any rows we fail to convert

EPOCHO = datetime(1949, 12, 31, 0, 0, 0, tzinfo=timezone.utc)

for idx, row in df_shell.iterrows(): # loop through each row in df_shell
    try:
        epoch_dt = row["epochDate"].to_pydatetime() # extract epoch as datetime

# days since 1949-12-31 (as float)
```

```
epoch_days = (epoch_dt - EPOCHO).total_seconds() / 86400.0
        # convert to radians
       inclo = np.deg2rad(row["inclination_deg"])
       nodeo = np.deg2rad(row["raan"])
       argpo = np.deg2rad(row["argOfPerigee"])
              = np.deg2rad(row["meanAnomaly"])
        # convert revs/day to rad/min
       no_kozai = float(row["meanMotion"]) * 2.0 * np.pi / (24.0 * 60.0)
        # orbital scalers
       ecco = float(row["eccentricity"]) # must be in [0,1)
       bstar = float(row.get("bStar", 0.0)) # ok if 0.0
        # initialize satellite record
       satrec = Satrec()
       satrec.sgp4init(
           WGS72,
                                 # Earth gravity model (standard for SGP4)
                                  # 'i' = initialize
            'i',
           int(row["satNo"]),  # satellite ID
                                  # Julian date
           epoch_days,
                                 # drag term
           bstar,
                                 # ndot, nddot (not used here; 0.0 okay)
           0.0, 0.0,
                                 # eccentricity
           ecco,
                                 # argument of perigee [rad]
           argpo,
                                 # inclination [rad]
           inclo,
                                 # mean anomaly [rad]
           mo,
           no_kozai,
                                 # mean motion [rad/min]
                                 # RAAN [rad]
           nodeo
       )
       satellites.append({ # add satellite to list
            "satNo": row["satNo"],
            "satName": row.get("satName", ""),
            "objectType": row.get("objectType", ""),
           "satrec": satrec
       })
   except Exception as e:
        errors.append((idx, str(e)))
print(f"Built {len(satellites)} propagators successfully.")
if errors:
   print(f"Failed on {len(errors)} rows. Example error:")
   print(errors[0])
```

Built 6093 propagators successfully.

```
[52]: # TEST PROPAGATION
      def norm3(vec):
          return float(np.sqrt(vec[0]**2 + vec[1]**2 + vec[2]**2))
          # returns Euclidean norm of a 3-vector
      # pick the first satellite we built
      if not satellites:
          raise RuntimeError("No satellites in `satellites`, build propagators first")
      test_sat = satellites[0] # change index if desired
      # convert t_start to Julian date parts for SGP4
      jd, fr = jday(
          cfg.t_start.year,
          cfg.t_start.month,
          cfg.t_start.day,
          cfg.t_start.hour,
          cfg.t_start.minute,
          cfg.t_start.second + cfg.t_start.microsecond * 1e-6
      # propagate and inspect results
      err, r, v = test_sat["satrec"].sgp4(jd, fr)
      print(f"Testing '{test_sat.get('satName','')}' (satNo. {test_sat['satNo']}) atu
      →t start={cfg.t start}\n")
      if err != 0:
          print(f" SGP4 ERROR: {SGP4_ERRORS[err]}")
      else:
          # r, v are TEME/ECI position (km) and velocity (km/s)
          r_mag = norm3(r) # distance from Earth's center (km)
          v mag = norm3(v) # speed (km/s)
          alt_km = r_mag - cfg.earth_radius_km
          print(" r (km):", [round(x, 3) for x in r])
          print(" v (km/s):", [round(x, 5) for x in v])
          print(f" |r| = {r_mag:,.2f} km (altitude {alt_km:,.2f} km above mean_
       ⇔equator)")
          print(f'' | v| = \{v_mag:, .3f\} \ km/s''\}
          # LEO range check
          if 6500 <= r_mag <= 7500 and 6.5 <= v_mag <= 8.5:</pre>
              print("\nCheck: Values look reasonable for LEO ")
          else:
              print("\nCheck: Values are unusual for LEO, recheck inputs ")
```

Testing 'FENGYUN 1C DEB' (satNo. 35230) at t_start=2025-08-03 00:00:00+00:00

```
r (km): [-4223.135, -905.095, -5454.942]
v (km/s): [5.25127, 2.94944, -4.5768]
|r| = 6,957.76 km (altitude 579.63 km above mean equator)
|v| = 7.565 km/s
```

Check: Values look reasonable for LEO

3 Execution Phase

3.1 Coarse Propagation

Build a time grid from $t_start \rightarrow t_end every coarse_step_minutes$.

For each time: - Propagate each satellite via satrec.sgp4() to get position r = (x,y,z) in TEME/ECI (km) - Build a KD-tree on the 3D positions - Query for all pairs within a search radius - Store those pairs as candidates with their coarse distance and timestamp

```
[53]: from scipy.spatial import cKDTree  # fast KD-tree (compiled)

# returns Euclidean norm |/a-b|| if b is provided

def norm3(a, b=None):
    if b is None:
        return float(np.sqrt((a*a).sum()))

d = a - b
    return float(np.sqrt((d*d).sum()))

times_coarse = pd.date_range( # build shared time grid
        start = cfg.t_start,
        end = cfg.t_end,
        freq = f"{cfg.coarse_step_minutes}min",
        inclusive = "both"  # include t_end
).to_pydatetime().tolist()

print(f"Coarse grid has {len(times_coarse)} timestamps "
        f"\n({cfg.t_start} to {cfg.t_end}, step={cfg.coarse_step_minutes} min)")
```

Coarse grid has 289 timestamps $(2025-08-03\ 00:00:00+00:00\ to\ 2025-08-04\ 00:00:00+00:00\ ,\ step=5\ min)$

```
[54]: # convenience arrays for satellite metadata
n = len(satellites)
satNos = np.array([int(s["satNo"]) for s in satellites], dtype=np.int64)
satNames = np.array([s.get("satName","") for s in satellites], dtype=object)

candidates = [] # store in a list of dicts

# simple progress printing every k steps
print_every = max(1, len(times_coarse)//12) # ~12 status lines over the run
```

3.2 Neighbor Search (KD-tree)

Why use a KD-tree?

It avoids $O(N^2)$ all-pairs checks. For ~6,000 satellites, all-pairs would be ~18M distance checks per timestep. KD-tree gives you only the nearby ones.

```
[55]: for t_idx, t in enumerate(times_coarse):
          # convert this timestamp to Julian date parts
          jd, fr = jday(t.year, t.month, t.day,
                        t.hour, t.minute, t.second + t.microsecond * 1e-6)
          # pre-allocate arrays for positions; mark invalid slots with NaN
          R = np.full((n, 3), np.nan, dtype=float)
          valid_mask = np.zeros(n, dtype=bool)
          for i, s in enumerate(satellites): # propagate all satellites to time t
              err, r, v = s["satrec"].sgp4(jd, fr)
              if err == 0:
                  R[i, :] = r \# km
                  valid_mask[i] = True
              else:
                  if t idx == 0 and i < 3:
                      print("sgp4 error:", SGP4_ERRORS.get(err, err), "for satNo", __

¬s["satNo"])
          # keep only valid positions for the KD-tree
          if not valid_mask.any():
              continue # unlikely if epochs were filtered
          Rv = R[valid mask] # positions of valid satellites
          idx_valid = np.where(valid_mask)[0] # lookup table, tells you which_
       →original satellite each row of Rv came from
          tree = cKDTree(Rv) # build KD-tree and query all pairs within coarse search
       \rightarrow radius
          pair_set = tree.query_pairs(r=cfg.search_radius_km) # returns indices (i,__
       \hookrightarrow j) in the *compressed* array Rv
          if not pair_set: # skip if zero candidates at this time
              if t_idx % print_every == 0:
                  print(f"t={t.isoformat()} candidates=0")
              continue
          # for each candidate pair, compute the coarse distance and collect metadata
          for (ia, ib) in pair_set:
              gi = idx_valid[ia] # global index into satellites list
              gj = idx_valid[ib]
```

```
ra = Rv[ia]
        rb = Rv[ib]
        d_km = norm3(ra, rb)
        # altitudes (quick context, not used as a filter here)
        altA = norm3(ra) - cfg.earth_radius_km
        altB = norm3(rb) - cfg.earth_radius_km
        candidates.append({
            "t coarse": t,
                                      # coarse timestamp
           "idxA": gi, "idxB": gj,
                                       # indices into `satellites` list
            "satNoA": int(satNos[gi]),
            "satNoB": int(satNos[gj]),
            "d_coarse_km": d_km,
            "altA_km": altA,
            "altB_km": altB,
        })
    # PROGRESS LOG OPTIONS
    # --- VERBOSE: print every timestep
    ⇔total={len(candidates)}")
    # --- QUIET: only print when candidates exist
    # if pair_set:
         print(f"HIT t=\{t:\%Y-\%m-\%d \%H:\%M\} +\{len(pair_set)\}
 ⇔total={len(candidates)}")
    # --- LIGHT: periodic heartbeat (default)
    if t_idx % print_every == 0:
        print(f"t={t:%Y-%m-%d %H:%M} cand={len(pair_set)} _
 ⇔total={len(candidates)}")
t=2025-08-03 00:00 cand=18896 total=18896
t=2025-08-03 02:00 cand=21424 total=462047
t=2025-08-03 04:00 cand=15567 total=886074
```

```
t=2025-08-03 02:00 cand=21424 total=462047
t=2025-08-03 04:00 cand=15567 total=886074
t=2025-08-03 06:00 cand=20181 total=1310431
t=2025-08-03 08:00 cand=15430 total=1725958
t=2025-08-03 10:00 cand=19796 total=2145682
t=2025-08-03 12:00 cand=15349 total=2556526
t=2025-08-03 14:00 cand=19428 total=2977171
t=2025-08-03 16:00 cand=19428 total=3384110
t=2025-08-03 18:00 cand=19067 total=3797569
t=2025-08-03 20:00 cand=15288 total=4200928
t=2025-08-03 22:00 cand=18671 total=4612588
```

Candidate preview:

```
t_coarse
                          satNoA satNoB d_coarse_km
                                                         altA_km
                                                                    altB_km
2025-08-03 00:00:00+00:00
                            53215
                                   53213
                                             0.012744 566.036699 566.040957
2025-08-03 00:00:00+00:00
                           48367
                                   48374
                                             0.014597 547.983245 547.995372
                           58190
                                             0.019235 558.789126 558.804504
2025-08-03 00:00:00+00:00
                                   56119
2025-08-03 00:00:00+00:00
                           55420
                                   55271
                                             0.026606 574.568089 574.590435
2025-08-03 00:00:00+00:00
                           52858
                                             0.028376 540.663827 540.640100
                                   52856
2025-08-03 00:00:00+00:00
                           56918
                                   57997
                                             0.032693 558.750160 558.735401
2025-08-03 00:00:00+00:00
                           55435
                                   55432
                                             0.034817 574.578019 574.572876
2025-08-03 00:00:00+00:00
                            58195
                                             0.040379 558.828136 558.830876
                                   56134
2025-08-03 00:00:00+00:00
                                             0.043731 558.911380 558.916606
                            56699
                                   57106
2025-08-03 00:00:00+00:00
                                             0.055167 520.602541 520.578733
                            64532
                                   64531
```

Total coarse candidates collected: 5011577

3.3 Deduplication

De-duplicate coarse candidates so we don't refine the same pair many times. We'll keep the shortest coarse distance per unique pair.

```
[57]: # check: ensure we have coarse candidates
if 'candidates_df' not in locals() or candidates_df.empty:
    raise RuntimeError("candidates_df is missing or empty.")

coarse = candidates_df.copy()

# create an order-independent pair key
pair_key = np.where(coarse["satNoA"] < coarse["satNoB"],</pre>
```

```
coarse["satNoA"].astype(str) + "-" + coarse["satNoB"].
 ⇒astype(str),
                    coarse["satNoB"].astype(str) + "-" + coarse["satNoA"].
 →astype(str))
coarse["pair_id"] = pair_key
# keep the closest coarse occurrence per pair
# (to get multiple per pair across time, group by day/hour buckets later)
coarse_best = (
    coarse.sort_values(["pair_id", "d_coarse_km"])
          .groupby("pair_id", as_index=False)
          .first()
)
print(f"Unique pairs to refine: {len(coarse_best)} (from {len(coarse)} coarse_u
 ⇔rows)")
print(coarse_best[["t_coarse","satNoA","satNoB","d_coarse_km"]].head(10).
 ⇔to_string(index=False))
```

```
Unique pairs to refine: 41670 (from 5011577 coarse rows)
                 t_coarse satNoA satNoB d_coarse_km
2025-08-03 02:20:00+00:00
                            10095
                                             17.895829
                                    43678
2025-08-03 07:40:00+00:00
                            42831
                                    10096
                                             18.119522
2025-08-03 00:40:00+00:00
                            65055
                                    10188
                                             14.506981
2025-08-03 10:15:00+00:00
                            10761
                                    40290
                                             12.681518
2025-08-03 04:00:00+00:00
                            30879
                                    10974
                                             16.870528
2025-08-03 01:30:00+00:00
                            34316
                                    10974
                                             7.241115
2025-08-03 00:00:00+00:00
                            52377
                                    11114
                                             16.188481
2025-08-03 00:25:00+00:00
                            11267
                                    39416
                                             17.834428
2025-08-03 10:00:00+00:00
                            11267
                                    43490
                                             11.359098
2025-08-03 00:45:00+00:00
                            11267
                                    44886
                                              5.931519
```

3.4 Local Refinement

Find true TCA and minimum distance! Steps:

- For each unique pair, build a refinement time window centered on its coarse timestamp (e.g., ± 10 minutes, step_seconds).
- Propagate only those two satellites across the window.
- Compute distance at each substep; pick the minimum \rightarrow that's the TCA and d min.
- Grab relative speed at TCA (from SGP4 velocities).
- Save a tidy row for each refined event.

```
[58]: def refine_pair(satA, satB, t_center, half_window_minutes=10, step_seconds=30):

# build the fine time grid centered at t_center
```

```
t_start = t_center - timedelta(minutes=half_window_minutes)
         = t_center + timedelta(minutes=half_window_minutes)
  # se seconds-based frequency (capital 'S')
  times = pd.date_range(start=t_start, end=t_end,
                         freq=f"{step_seconds}S", inclusive="both").
→to_pydatetime()
   # pre-allocate arrays for distances and relative speed
  nT = len(times)
  dists = np.full(nT, np.nan, dtype=float)
  vrels = np.full(nT, np.nan, dtype=float)
  # loop over sub-steps; propagate both sats and compute separation and rel_{\sqcup}
\hookrightarrowspeed
  for k, tk in enumerate(times):
       jd, fr = jday(tk.year, tk.month, tk.day,
                     tk.hour, tk.minute, tk.second + tk.microsecond * 1e-6)
       errA, rA, vA = satA["satrec"].sgp4(jd, fr)
       errB, rB, vB = satB["satrec"].sgp4(jd, fr)
       if errA != 0 or errB != 0:
           # skip this sub-step if either failed (can happen near stale epochs)
           continue
       # distance between positions (km)
       dists[k] = norm3(np.array(rA) - np.array(rB))
       # relative speed magnitude (km/s)
      rel_v = np.array(vA) - np.array(vB)
      vrels[k] = norm3(rel_v)
   # choose minimum valid distance
  if np.all(np.isnan(dists)):
      return {
           "satNoA": satA["satNo"], "satNoB": satB["satNo"],
           "satNameA": satA.get("satName",""), "satNameB": satB.

¬get("satName",""),
           "t_TCA": None, "d_min_km": np.nan, "v_rel_km_s": np.nan,
           "t_center": t_center, "n_steps": nT,
           "status": "refine_failed_all_nan"
      }
  kmin = np.nanargmin(dists)
  return {
       "satNoA": satA["satNo"], "satNoB": satB["satNo"],
```

```
"satNameA": satA.get("satName",""), "satNameB": satB.get("satName",""),
              "t TCA": times[kmin],
              "d_min_km": float(dists[kmin]),
              "v rel km s": float(vrels[kmin]) if not np.isnan(vrels[kmin]) else np.
       ⇔nan,
              "t center": t center, # coarse time around which we refine
              "n_steps": nT,
              "status": "ok"
          }
[59]: refined_rows = []
      half_window_minutes = 10
                                 # ±10 minutes around the coarse time
      step_seconds = cfg.refine_step_seconds
      for _, row in coarse_best.iterrows():
          iA = int(row["idxA"])
                                 # indices into satellites
          iB = int(row["idxB"])
          t_center = pd.to_datetime(row["t_coarse"], utc=True).to_pydatetime()
          satA = satellites[iA]
          satB = satellites[iB]
          result = refine_pair(
              satA, satB, t_center,
              half_window_minutes=half_window_minutes,
              step_seconds=step_seconds
          )
          refined_rows.append(result)
      refined_df = pd.DataFrame(refined_rows)
      print(f"Refinement complete. Rows: {len(refined_df)}")
      print(refined_df.head(10).to_string(index=False))
     C:\Users\ash\AppData\Local\Temp\ipykernel_22416\2767047286.py:8: FutureWarning:
     'S' is deprecated and will be removed in a future version, please use 's'
     instead.
       times = pd.date_range(start=t_start, end=t_end,
     Refinement complete. Rows: 41670
      satNoA satNoB
                            \mathtt{satNameA}
                                        satNameB
                                                                      t_TCA d_min_km
                                 t_center n_steps status
     v rel km s
                          COSMOS 921 DIWATA 2B 2025-08-03 02:20:00+00:00 17.895829
       10095 43678
     3.029118 2025-08-03 02:20:00+00:00
                                              41
       42831
               10096
                      FLYING LAPTOP SL-14 R/B 2025-08-03 07:40:00+00:00 18.119522
     11.653229 2025-08-03 07:40:00+00:00
```

65055

10188

41

PRSC-S1 DELTA 1 DEB 2025-08-03 00:40:00+00:00 14.506981

```
1.997945 2025-08-03 00:40:00+00:00
                                        41
                                                ok
                                  CZ-2C DEB 2025-08-03 10:15:00+00:00 12.681518
  10761
          40290
                   DELTA 1 DEB
14.566158 2025-08-03 10:15:00+00:00
                                          41
                                                 ok
         10974 FENGYUN 1C DEB
                                  SL-14 R/B 2025-08-03 04:00:00+00:00 16.870528
15.185883 2025-08-03 04:00:00+00:00
                                          41
  34316
          10974 COSMOS 2251 DEB
                                  SL-14 R/B 2025-08-03 01:30:00+00:00 7.241115
1.227373 2025-08-03 01:30:00+00:00
                                         41
  52377
          11114
                  STARLINK-3805
                                   SL-8 DEB 2025-08-03 00:00:00+00:00 16.188481
2.736324 2025-08-03 00:00:00+00:00
                                         41
                                                ok
                      SL-14 R/B APRIZESAT 7 2025-08-03 00:25:00+00:00 17.834428
  11267
          39416
13.970884 2025-08-03 00:25:00+00:00
                                          41
                                                 ok
                      SL-14 R/B
                                  CZ-2D DEB 2025-08-03 10:00:00+00:00 11.359098
  11267
         43490
7.671557 2025-08-03 10:00:00+00:00
                                         41
                                                ok
                      SL-14 R/B
                                   OBJECT H 2025-08-03 00:45:00+00:00 5.931519
  11267
         44886
2.051345 2025-08-03 00:45:00+00:00
                                         41
                                                ok
```

3.4.1 status column tags: ok or fail

Not every coarse candidate can be successfully refined. Common reasons:

- bad/missing TLE parameters for one of the objects
- numerical failure in the propagator
- epochs too far out of date

```
[60]: # drop rows where refinement failed completely
events_df = refined_df[refined_df["status"] == "ok"].copy()
events_df.info()
```

<class 'pandas.core.frame.DataFrame'>
RangeIndex: 41670 entries, 0 to 41669
Data columns (total 10 columns):

```
Non-Null Count Dtype
    Column
    ----
                -----
 0
    \mathtt{satNoA}
                41670 non-null int64
 1
    satNoB
                41670 non-null int64
 2
    \mathtt{satNameA}
              41670 non-null object
 3
    \mathtt{satNameB}
                41670 non-null object
 4
    t_TCA
                41670 non-null datetime64[ns, UTC]
 5
                41670 non-null float64
    d_min_km
 6
    v_rel_km_s 41670 non-null float64
    t_center
                41670 non-null datetime64[ns, UTC]
 7
 8
    n_steps
                41670 non-null int64
    status
                41670 non-null object
dtypes: datetime64[ns, UTC](2), float64(2), int64(3), object(3)
memory usage: 3.2+ MB
```

```
[61]: total = len(refined_df) # total coarse candidates refined status_counts = refined_df["status"].value_counts()
```

```
success = status_counts.get("ok", 0)
fail = total - success
success_rate = 100.0 * success / total if total > 0 else 0.0

print(f"Refinement results:")
print(f" Total coarse candidates: {total}")
print(f" Successful refinements: {success} ({success_rate:.1f}%)")
print(f" Failed refinements: {fail}")
```

Refinement results:

Total coarse candidates: 41670

Successful refinements: 41670 (100.0%)

Failed refinements: 0

4 Analysis and Reporting

4.1 Totals

```
[62]: # apply thresholds
      events_df["below_10km"] = events_df["d_min_km"] < cfg.report_thresh_km
      events_df["below_5km"] = events_df["d_min_km"] < cfg.report_strict_km</pre>
      events_10 = events_df[events_df["below_10km"]].copy()
      events_5 = events_df[events_df["below_5km"]].copy()
      # sort by TCA then by distance
      events_10.sort_values(["t_TCA", "d_min_km"], inplace=True)
      events_5.sort_values(["t_TCA", "d_min_km"], inplace=True)
[125]: print(f"Events under {cfg.report_thresh_km} km: {len(events_10)}")
      print(f"Events under {cfg.report_strict_km} km: {len(events_5)}\n")
      print(f"Conjunctions by shortest distance (Top 10):")
      preview_cols = ["satNoA", "satNoB", "d_min_km", "v_rel_km_s"]
      print(events_5[preview_cols].head(10).to_string(index=False))
      Events under 10.0 km: 20307
      Events under 5.0 km: 12959
      Conjunctions by shortest distance (Top 10):
       satNoA satNoB d_min_km v_rel_km_s
        52092 52310 0.045807
                                 0.000165
        63096 63080 0.143227
                                  0.000305
               64543 0.191631 0.000303
        64544
        62712
               62706 0.350875 0.000761
        64092
               64090 0.383235 0.000776
        56349
               56352 0.391792 0.000352
```

```
58211
               58218 0.519249
                                  0.000640
       64155
               64166 0.600434
                                  0.001913
       51152
               51754 0.621888
                                  0.000807
[82]: # count by day/hour (for multi-day runs)
      events_10["date"] = events_10["t_TCA"].dt.floor("D")
      daily_10 = events_10.groupby("date").size()
      events_5["date"] = events_5["t_TCA"].dt.floor("D")
      daily 5 = events 5.groupby("date").size()
      daily_table = pd.DataFrame({
          "<5 km": daily_5,
          "<10 km": daily_10
      })
      daily_table.index = daily_table.index.strftime("%Y-%m-%d")
      daily_table
```

0.000531

[82]: <5 km <10 km date 2025-08-02 115 173 2025-08-03 12842 20132 2025-08-04 2 2

51758

51756 0.482989

4.2 Which object types dominate conjunction risk?

- by pair count
- by weighted risk

4.2.1 By pair count

<class 'pandas.core.frame.DataFrame'>
RangeIndex: 41670 entries, 0 to 41669
Data columns (total 13 columns):

```
Column
      #
                      Non-Null Count Dtype
          _____
                      _____
      0
          satNoA
                      41670 non-null int64
      1
          satNoB
                      41670 non-null int64
      2
          \mathtt{satNameA}
                      41670 non-null object
      3
          \mathtt{satNameB}
                      41670 non-null object
      4
         t TCA
                      41670 non-null datetime64[ns, UTC]
      5
          d_min_km
                      41670 non-null float64
      6
          v_rel_km_s 41670 non-null float64
      7
          t_center
                      41670 non-null datetime64[ns, UTC]
      8
          n_steps
                      41670 non-null int64
          status
                      41670 non-null object
      10 below_10km 41670 non-null
                                      bool
                      41670 non-null
      11 below_5km
                                      bool
      12 pair_id
                      41670 non-null object
     dtypes: bool(2), datetime64[ns, UTC](2), float64(2), int64(3), object(4)
     memory usage: 3.6+ MB
[66]: # map satNo -> objectType
      id_to_type = df_shell.set_index("satNo")["objectType"]
      # annotate conjunction pairs with object types
      df_conj = df_events.assign(
          type1 = df_events["satNoA"].map(id_to_type),
          type2 = df_events["satNoB"].map(id_to_type)
      ).copy()
      # categorize the pair (order independent)
      def pair_category(row):
          t1, t2 = sorted([row["type1"], row["type2"]])
          return f"{t1}-{t2}"
      df_conj["pair_type"] = df_conj.apply(pair_category, axis=1)
      df_conj.info()
     <class 'pandas.core.frame.DataFrame'>
     RangeIndex: 41670 entries, 0 to 41669
     Data columns (total 16 columns):
          Column
                      Non-Null Count Dtype
      0
          \mathtt{satNoA}
                      41670 non-null int64
      1
          \mathtt{satNoB}
                      41670 non-null int64
      2
          satNameA
                      41670 non-null object
          satNameB
                      41670 non-null object
      4
         t_TCA
                      41670 non-null datetime64[ns, UTC]
      5
          d_min_km
                      41670 non-null float64
      6
          v_rel_km_s 41670 non-null float64
          t_center
                      41670 non-null datetime64[ns, UTC]
```

```
8
           n_steps
                        41670 non-null
                                        int64
       9
           status
                        41670 non-null
                                        object
       10 below_10km 41670 non-null
                                        bool
       11 below_5km
                        41670 non-null
                                        bool
       12
           pair id
                        41670 non-null
                                        object
       13
           type1
                        41670 non-null
                                        object
                        41670 non-null
       14 type2
                                        object
       15 pair_type
                       41670 non-null
                                        object
      dtypes: bool(2), datetime64[ns, UTC](2), float64(2), int64(3), object(7)
      memory usage: 4.5+ MB
[115]: # aggregate
       pair summary = (
           df_conj.groupby("pair_type")
                  .agg(
                      n_events=("d_min_km", "size"),
                      median_miss_km=("d_min_km", "median"),
                      min_miss_km=("d_min_km", "min"),
                      n_events_below_1km=("d_min_km", lambda x: (x < 1).sum())</pre>
                  )
                  .sort_values("n_events", ascending=False)
                  .rename_axis("pair_type").reset_index()
       pair_summary
[115]:
                        pair_type n_events
                                              median_miss_km min_miss_km \
                  Payload-Payload
       0
                                       38713
                                                   10.145515
                                                                  0.006495
       1
                  Payload-Unknown
                                                   10.452237
                                         958
                                                                  0.110107
       2
                   Debris-Payload
                                         910
                                                   14.500108
                                                                  0.411050
       3
                  Unknown-Unknown
                                         494
                                                    7.407047
                                                                  0.030465
       4
              Payload-Rocket body
                                         293
                                                   14.086566
                                                                  0.265819
       5
                    Debris-Debris
                                         193
                                                   12.881185
                                                                  1.183780
       6
               Debris-Rocket body
                                          43
                                                   14.850467
                                                                  4.545728
       7
                   Debris-Unknown
                                          37
                                                   12.814541
                                                                  1.015493
       8
              Rocket body-Unknown
                                          19
                                                   13.422841
                                                                  1.979328
         Rocket body-Rocket body
                                          10
                                                   18.328455
                                                                  2.595006
          n_events_below_1km
       0
                        5249
                          32
       1
       2
                           1
                          42
       3
       4
                           2
       5
                           0
                           0
       6
       7
                           0
       8
                           0
```

9 0

4.2.2 By weighted risk

We want close approaches to count more heavily than distant ones.

- $0.1 \text{ km miss} \rightarrow \text{weight} = 10$
- $1.0 \text{ km miss} \rightarrow \text{weight} = 1$
- 10 km miss \rightarrow weight = 0.1

```
[117]: # map to buckets
      bucket_map = {"Payload": "Payload", "Rocket body": "Debris", "Debris": "Debris", __
        dfw2 = df_conj.copy()
      dfw2["bucket1"] = dfw2["type1"].map(bucket_map).fillna("Other/Unknown")
      dfw2["bucket2"] = dfw2["type2"].map(bucket_map).fillna("Other/Unknown")
      # stack both sides so each event counts for both participants
      tall = pd.concat([
          dfw2[["d min km", "bucket1"]].rename(columns={"bucket1": "bucket"}),
          dfw2[["d_min_km", "bucket2"]].rename(columns={"bucket2": "bucket"})
      ], ignore index=True)
      # recompute weights on the stacked view
      tall["weight"] = 1.0 / (tall["d_min_km"] + 1e-6)
      bucket_summary = (
          tall.groupby("bucket")
               .agg(
                   weighted_risk=("weight", "sum"),
                   count=("bucket", "size"),
                   q1_miss_km=("d_min_km", lambda x: x.quantile(0.25)),
                   median_miss_km=("d_min_km", "median"),
                   q3_miss_km=("d_min_km", lambda x: x.quantile(0.75))
               )
               .sort_values("weighted_risk", ascending=False)
               .rename_axis("object_type").reset_index()
               .assign(
                   weighted_risk=lambda df: df["weighted_risk"].round(0).astype(int),
                   q1_miss_km=lambda df: df["q1_miss_km"].round(2),
                   median_miss_km=lambda df: df["median_miss_km"].round(2),
                   q3_miss_km=lambda df: df["q3_miss_km"].round(2),
                   risk_share=lambda df: (
                       (100 * df["weighted_risk"] / df["weighted_risk"].sum())
                       .round(1).astype(str) + "%"
                   )
              )
```

```
bucket_summary
[117]:
         object_type
                       weighted risk
                                        count
                                                q1 miss km median miss km
                                                                              q3 miss km
       0
              Payload
                                60168
                                        79587
                                                      3.07
                                                                       10.22
                                                                                    14.96
       1
                                                      4.51
                                                                        8.85
              Unknown
                                   678
                                         2002
                                                                                    13.58
       2
               Debris
                                         1751
                                                      9.65
                                                                       14.11
                                                                                    16.98
                                   187
         risk_share
               98.6%
       0
       1
                1.1%
       2
                0.3%
```

4.3 Summary and Recommendations

Our analysis confirms that close approaches in the thousands can occur daily in low-Earth orbit.

Median miss distances are typically in the 8-14 km range. For Payload-Payload conjunctions, 5249 events (about 14%) have miss distances below 1 km. Following with 74 events with miss distances below 1 km, conjunctions involving Unknown objects should not be overlooked. Although debris and unknown objects contribute marginally, their presence still complicates risk management.

Weighted risk is dominated by payloads at 98.6%. With Q1 miss distances lower than those of other objects, conjunctions between operational spacecraft are both more frequent and riskier.

Recommendations

Space Traffic Management - Establish binding international standards for conjunction assessment and collision avoidance - Mandate maneuver protocols and thresholds, e.g. automated avoidance systems - Classify "Unknown" objects to better assess risk - Promote international agreements on information sharing, debris mitigation, and maneuver coordination

```
[122]: # export
FINAL_DIR = Path.cwd().parents[1] / "data" / "02_final" / "conjunction_risk"

events_10.to_csv(FINAL_DIR / "conjunctions_df.csv", index=False)
pair_summary.to_csv(FINAL_DIR / "pair_summary.csv", index=False)
bucket_summary.to_csv(FINAL_DIR / "bucket_summary.csv", index=False)
```