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MAT 370 100473993

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A) $a_n = \dfrac{4n^2+2}{n^2+3n-1} \Rightarrow \left(2, 2, \dfrac{38}{17}, \dfrac{22}{9}, \dfrac{34}{13}\right)$

$n=1 = \dfrac{4(1)^2+2}{(1)^2+3(1)-1} = 2$; $n=2 = \dfrac{4(2)^2+2}{(2)^2+3(2)-1} = 2$

$n=3 = \dfrac{4(3)^2+2}{(3)^2+3(3)-1} = \dfrac{38}{17}$; $n=4 = \dfrac{4(4)^2+2}{(4)^2+3(4)-1} = \dfrac{22}{9}$

$n=5 = \dfrac{4(5)^2+2}{(5)^2+3(5)-1} = \dfrac{34}{13}$

B) $a_n = (-1)^{n+1}\left(\dfrac{2}{n}\right) \Rightarrow 2, -1, \dfrac{2}{3}, -\dfrac{1}{2}, \dfrac{2}{5}$

$n=1 = (-1)^{(1)+1}\left(\dfrac{2}{1}\right) = \boxed{2}$, $n=2 = (-1)^{2+1}\left(\dfrac{2}{2}\right) = \boxed{-1}$

$n=3 \Rightarrow (-1)^{3+1}\left(\dfrac{2}{3}\right) = \boxed{\dfrac{2}{3}}$ $n=4 = (-1)^{4+1}\left(\dfrac{2}{4}\right) = \boxed{-\dfrac{1}{2}}$

$n=5 \Rightarrow (-1)^{5+1}\left(\dfrac{2}{5}\right) = \boxed{\dfrac{2}{5}}$

c) $a_n = \frac{\cos(n\pi)}{n} = -1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, -\frac{1}{5}$

$n=1 = \frac{\cos((1)\pi)}{(1)} = -1$ $\quad$ $n=2 = \frac{\cos((2)\pi)}{2} = \frac{1}{2}$

$n=3 = \frac{\cos((3)\pi)}{3} = -\frac{1}{3}$ $\quad$ $n=4 = \frac{\cos((4)\pi)}{2} = \frac{1}{4}$

$n=5 = \frac{\cos((5)\pi)}{5} = -\frac{1}{5}$

D) $a_n = \frac{\ln n}{\sqrt{n}} = 0, 0.49, 0.63, 0.69, 0.72$

$n=1 \Rightarrow \frac{\ln(1)}{\sqrt{1}} = 0$ $\quad$ $n=4 \Rightarrow \frac{\ln(4)}{\sqrt{4}} = 0.69$

$n=2 \Rightarrow \frac{\ln(2)}{\sqrt{2}} = 0.49$ $\quad$ $n=5 \Rightarrow \frac{\ln(5)}{\sqrt{5}} = 0.72$

$n=3 = \frac{\ln(3)}{\sqrt{3}}$ 0.63

e) $a_n = \frac{\sqrt{3n^2+2}}{2n+1}$

$n=1 \rightarrow \frac{\sqrt{3(1)^2+2}}{2(1)+1} = \frac{\sqrt{5}}{3}$ $\quad$ $n=4 = \frac{\sqrt{3(4)^2+2}}{2(4)+1}$ $\frac{5\sqrt{2}}{9}$

$n=2 = \frac{\sqrt{3(2)^2+2}}{2(2)+1} = \frac{\sqrt{14}}{5}$ $\quad$ $n=5 = \frac{\sqrt{3(5)^2+2}}{2(5)+1}$ $\frac{\sqrt{77}}{11}$

$n=3 = \frac{\sqrt{3(3)^2+2}}{2(3)+1} =$ $\frac{\sqrt{29}}{7}$

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F) $a_1 = 3,\ a_{k+1} = 2(a_k - 1) \Rightarrow (3, 4, 6, 10, 18)$

$a_1 = 3$
$a_2 = 2(a_1 - 1) = 2(3-1) = 2(2) = 4$
$a_3 = 2(a_2 - 1) = 2(4-1) = 2(3) = 6$
$a_4 = 2(a_3 - 1) = 2(6-1) = 2(5) = 10$
$a_5 = 2(a_4 - 1) = 2(10-1) = 2(9) = 18$

G) $a_1 = 6,\ a_{k+1} = \frac{1}{3}(a_k)^2 \rightarrow (6, 12, 48, 768, 196{,}608)$

$a_1 = 6$
$a_2 = \frac{1}{3}(6)^2 = \frac{36}{3} = 12$
$a_3 = 1/3\,(12)^2 = 144/3 = 48$
$a_4 = 1/3\,(48)^2 = 2304/3 = 768$
$a_5 = 1/3\,(768)^2 = 589{,}824/3 = 196{,}608$

2) A) $\Rightarrow 2, 8, 14, 20 \ldots \rightarrow 26, 32, 38,\ a_n = \boxed{6n-4}$

$8-2 = \boxed{6},\ 14-8 = \boxed{6},\ 20-14 = \boxed{6}$

$a_n = a_1 + (n-1)d = 2 + (n-1)d = 2 + (n-1)(6) = 6n-4$

B) $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots \frac{6}{7}, \frac{7}{8}, \frac{8}{9}$ $\quad a_n = \frac{n+1}{n+2}$

C) $\frac{1}{2\cdot 3}, \frac{2}{3\cdot 4}, \frac{3}{4\cdot 5} \ldots \frac{4}{5\cdot 6}, \frac{5}{6\cdot 7}, \frac{6}{7\cdot 8}$

$a_n = \frac{n}{(n+1)(n-1)}$

d) $\text{sen}(1), 2, 2\,\text{sen}(1/2), 3\,\text{sen}(1/3), \ldots$

$4\sin(1/4),\ 5\sin(1/5),\ 6\sin(1/6)$

$a_n = n\sin(1/n)$

e) $1, \frac{2}{(2)^2-1^2}, \frac{3}{3^2-2^2}, \frac{4}{4^2-3^2}, \ldots \frac{5}{9}, \frac{6}{11}, \frac{7}{13}$

$a_n = \frac{n}{2n-1} \Rightarrow a_n = \frac{n}{(n)^2-(n-1)^2}$

③ $a_n = \frac{5}{n+2} = \lim_{n\to\infty} \frac{5}{n+2} = 0$ "La sucesion converge a 0"

④ $a_n = (-1)^n \frac{n}{n+2} = \lim_{n\to\infty} (-1)^n \frac{n}{n+2}$ dividir entre n

$= \frac{1}{1+\cancel{\frac{2}{n}}^{0}} = \frac{1}{1}$ $\lim_{n\to\infty} \frac{n}{n+2} = \boxed{1}$ $(-1)^n$ esta expresion alterna el signo

" La sucesion diverge (oscilante en (-1, 1))

⑤ $a_n = (-1)^n \frac{n^2}{n^3+2n^2+2} = \lim_{n\to\infty} \frac{n^2}{n^3+2n^2+2} = \frac{n^2}{n^3(1+\frac{2}{n}+\frac{2}{n^3})} =$

$\frac{1}{n(1+\cancel{\frac{2}{n}}+\cancel{\frac{2}{n^3}})} = \frac{1}{n} \to 0$ La sucesion converge en 0

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⑥ $bn = \{0, 1, 0, 1 \ldots\}$ La sucesion solo toma dos valores (0, 1) y no se aproxima a un unico numero
"La sucesion Diverge! (oscilante).

⑦ $\frac{\ln n}{\sqrt{n}}$: $\lim_{n\to\infty} \frac{\ln n}{\sqrt{n}}$ S.h $\Rightarrow$ $\frac{\frac{1}{n}}{\frac{1}{2\sqrt{n}}} = \lim_{n\to\infty} \frac{2\sqrt{n}}{n} =$

$\lim_{n\to\infty} \frac{2}{\sqrt{n}} = 0$ La sucesión converge a cero (0)

⑧ $bn\{ \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4} \ldots$ $bn = \frac{1}{2^{n+1}}$ $\lim_{n\to\infty} \frac{1}{2^n} = 0$

"La sucesión converge en (0) .."

⑨ $bn = \left\{ \frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots \right\} = bn = \frac{1}{n}$ $\lim_{n\to\infty} \frac{1}{n} = 0$

La sucesión converge a 0

⑩ $an = \sqrt{\frac{n+1}{9n+n}} = \lim_{n\to\infty} \sqrt{\frac{n+1}{9n+n}} = \frac{n/n + \frac{1}{n}}{9n/n + n/n} = \sqrt{\frac{1+\cancel{\frac{1}{n}}}{9+\cancel{\frac{1}{n}}}}$

$\lim_{n\to\infty} \rightarrow \sqrt{\frac{1}{9}} = \boxed{\frac{1}{3}}$ La sucesion converge en $\boxed{\frac{1}{3}}$

⑪ $bn = \left\{ 1, \frac{2}{2^2 - 1^2}, \frac{2}{3^2 - 2^2}, \frac{2}{4^2 - 2^2}, \ldots \right\}$ $bn = \frac{n}{n^2 - (n-1)^2} = \frac{n}{n^2 - (n^2 - 2n + 1)} =$

$\frac{n}{2n-1} = \lim_{n\to\infty} \frac{n}{2n-1} = \frac{n/n}{2n/n - \frac{1}{n}} = \frac{1}{2 - \cancel{\frac{1}{n}}}\, 0 = \frac{1}{2}$

(converge en $\frac{1}{2}$)

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⑫ $\sum_{n=0}^{\infty} 5\left(\frac{2}{3}\right)^n$ Serie geométrica → $a=5$ $r=2/3$

$2/3 < 1$, La serie converge.

$S = \frac{a}{1-r} = \frac{5}{1-2/3} = \frac{5}{\frac{1}{3}} = 5 \cdot 3 = \boxed{15}$

⑬ $\sum_{n=0}^{\infty} \frac{4}{n(n+2)} = \frac{A}{n} + \frac{B}{n+2}$

$n=0$, $n=-2$

$4 = A(n+2) + Bn$

$4 = A(0+2) + \cancel{B(0)}$

$4 = 2A$ $A = \boxed{2}$

$n=-2$

$4 = A\cancel{(-2+2)} + B(-2)$

$4 = -2B$

$B = \boxed{-2}$

$\frac{2}{n} - \frac{2}{n+2}$ $\sum_{n=2}^{\infty}\left(\frac{2}{n} - \frac{2}{n+2}\right) =$

$\left(\frac{2}{2} - \cancel{\frac{2}{4}}\right) + \left(\frac{2}{3} - \cancel{\frac{2}{5}}\right) + \left(\cancel{\frac{2}{4}} - \frac{2}{6}\right) + \left(\cancel{\frac{2}{5}} - \frac{2}{7}\right) + \ldots$

$\lim_{k\to\infty}\left(2 + 1 - \frac{2}{k+1} - \frac{2}{k+2}\right) = \boxed{3}$ "converge"

⑮ $\sum_{n=0}^{\infty}\left(\frac{1}{2^n} - \frac{1}{3^n}\right) = \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n - \sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n$

$a=1$ $r=1/2$

$a=1$ $r=1/3$

$1/2 < 1$

$1/3 < 1$

$S = \frac{a}{1-r} = \frac{1}{1-\frac{1}{2}} - \frac{1}{1-1/3} = 2 - \frac{3}{2} =$

$\frac{4}{2} - \frac{3}{2} = \frac{1}{2}$ $\boxed{\frac{1}{2}}$ "converge"

⑤ $\sum_{n=0}^{\infty} (\sin 1)^n = \quad a = 1 \quad r = (\sin(1))$

$S = \frac{a}{1-r} = \frac{1}{1-\sin(1)} = \approx \boxed{6.30}$

⑱ a) $\sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+2} \right) = Sk = \frac{1}{2} - \frac{1}{k+2} = \boxed{\frac{1}{2}}$

$n=1 = \left(\frac{1}{1+2} - \frac{1}{1+2} \right) = \left(\frac{1}{2} - \frac{1}{3} \right)$ $\quad Sk = \left(\frac{1}{2} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{4} \right) + \dots \left(\frac{1}{k+1} - \frac{1}{k+2} \right)$

$n=2 = \left(\frac{1}{3} - \frac{1}{4} \right) = \left(\frac{1}{3} - \frac{1}{4} \right)$

$\lim_{k \to \infty} Sk = \frac{1}{2} - 0 = \boxed{\frac{1}{2}}$

$n=3 = \left(\frac{1}{4} - \frac{1}{5} \right)$

"La serie converge y su suma es $\frac{1}{2}$"

B) $\sum_{n=1}^{\infty} \frac{4}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$

$n=-1 \quad , \quad n=0$

$4 = A(n+1) + Bn$

$4 = A\cancel{(-1+1)} + -B$

$\boxed{B = -4}$

$4 = A + B\cancel{(0)}^{0}$

$A = \boxed{4}$

$\sum_{n=1}^{\infty} \left(\frac{4}{n} - \frac{4}{n+1} \right) = \left(\frac{4}{1} - \frac{4}{2} \right) \left(\frac{4}{2} - \frac{4}{3} \right)$

$\left(\frac{4}{3} - \frac{4}{4} \right) \dots \quad n = k \dots \left(\frac{4}{k} - \frac{4}{k+1} \right) = Sk = 4 - \frac{4}{k+1}$

$\lim_{k \to \infty} Sk = \lim_{k \to \infty} 4\left(1 - \frac{1}{k+1}\right) = 4\left(1 - \lim_{k \to \infty} \frac{1}{k+1}\right) = 4(1-0) = 4$

"La serie converge y la suma es (4)"

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$n=-2 \quad n=0$

c) $\sum_{n=0}^{\infty} \frac{4}{n(n+2)} = \frac{A}{n} + \frac{B}{n+2}$

$4 = A(n+2) + Bn$

$4 = A(\cancel{-2+2})^{0} + -2B$

$-2B = 4 \rightarrow \boxed{B=-2}$

$4 = A(0+2) + \cancel{B(0)}$

$4 = 2A \quad \boxed{A=2}$

$\sum_{n=0}^{\infty} \left(\frac{2}{n} - \frac{2}{n+2}\right) = \left(\frac{2}{1} - \frac{2}{3}\right)\left(\frac{2}{2} - \frac{2}{4}\right)$

$\left(\frac{2}{3} - \frac{2}{5}\right) \quad n = k: \left(\frac{2}{k} - \frac{2}{(k+2)}\right) \quad Sk = 2\left(1 + \frac{1}{2}\right) - 2\left(\frac{1}{k+1} + \frac{1}{k+2}\right)$

$Sk = 2\left(\frac{3}{2} - \frac{1}{k+1} - \frac{1}{k+2}\right) = Sk\,3 - \frac{2}{k+1} - \frac{2}{k+2}$

$\lim_{k \to \infty} 2\left(1 + \frac{1}{2} - \frac{1}{k+1} - \frac{1}{k+2}\right) = 2\left(1 + \frac{1}{2} - 0 - 0\right) = 2\left(\frac{3}{2}\right)$

$= \boxed{3}$ La seri converge y su suma es $\boxed{3}$

17 a) $\sum_{n=0}^{\infty} \frac{3}{5^n} = 3 \cdot \left(\frac{1}{5}\right)^n \qquad a = 3 \quad r = \frac{1}{5} < 1 \Rightarrow$ converge

$S = \frac{a}{1-r} = \frac{3}{1-\frac{1}{5}} = \frac{3}{4/5} = 3 \cdot \frac{5}{4} = \frac{15}{4} = \boxed{3.75}$

B) $\sum_{n=1}^{\infty} 2\left(-\frac{1}{2}\right)^n \qquad a = -1 \quad r = -\frac{1}{2} < 1 \rightarrow$ converge

$S = \frac{a}{1-r} = \frac{-1}{1-(-\frac{1}{2})} = \boxed{-\frac{2}{3}}$ "La seri converge"

$\boxed{-\frac{2}{3}}$

C) $\sum_{n=0}^{\infty} (0.9)^n$ $a=1$ $r=0.9<1 \Rightarrow$ converge

$$S=\frac{a}{1-r}=\frac{1}{1-0.9}=\boxed{10}$$

18 A) $\sum_{n=}^{\infty} \frac{n^2}{n^2+1}$ $\lim_{n\to\infty} \frac{n^2}{n^2+1} = \frac{n^2/n^2}{n^2/n^2+1/n^2} = \frac{1}{1+\cancel{1/n^2}^{\,0}}$

$\lim_{n\to\infty} = \frac{1}{1+0} = 1 \neq 0$ "La serie diverge"

18 B) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}} = \lim_{n\to\infty} = \frac{n}{\sqrt{n^2(1+1/n^2)}} = \lim_{n\to\infty} \frac{n}{n\sqrt{1+1/n^2}} =$

$\frac{1}{\sqrt{1+0}} = 1 \neq 0$ "La serie diverge"

18 C) $\sum_{n=0}^{\infty} \ln\left(\frac{n+1}{n}\right)$ $\lim_{n\to 0} \ln\left(\frac{1+1}{n}\right) = =$

$\ln(1+0) \to \ln(1)=0$

$SK = \ln(k+1) - \ln(1) = \ln(k+1)$ $k\to\infty$

$\ln(k+1) \to \infty$ "Diverge"

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19) A) $\frac{1}{\sqrt{1}(\sqrt{1}+1)} = f(x) = \frac{1}{x^{1/2}(x^{1/2}+1)} = \int_1^{\infty} \frac{1}{\sqrt{x}(\sqrt{x}+1)}\,dx$

$u = \sqrt{x}$
$x = u^2$
$dx = 2u\,du$

$\int_1^{\infty} \frac{1}{u(u+1)} \cdot 2u\,du = \int_1^{\infty} \frac{2}{u+1}\,du$

$\int_1^{\infty} \frac{2}{u+1}\,du = 2\ln(u+1)\Big|_1^{\infty} \to \infty$ La serie diverge

19) B) $\sum_{n=2}^{\infty} \frac{\ln(n)}{n^3} = Fx = \frac{\ln(x)}{x^3} = \int_2^{\infty} \frac{\ln(x)}{x^2}\,dx$

$u = \ln x$
$dv = x^{-3}\,dx$
$du = \frac{1}{x}\,dx$
$v = -1/2x^2$

$\int \frac{\ln(x)}{x^3}\,dx = \ln(x)\left(-\frac{1}{2x^2}\right) - \int\left(-\frac{1}{2x^2} \cdot \frac{1}{x}\right)dx$

$= \frac{\ln(x)}{2x^2} + \frac{1}{2}\int x^{-3}\,dx = \frac{\ln(X)}{2x^2} - \frac{1}{4x^2}$

$\lim_{x\to\infty}\left(-\frac{\ln(x)}{2x^2} - \frac{1}{4x^2}\right) = 0$ La serie converge

19) C) $\sum \frac{n}{n^4+2n^2+1} =$ ~~…~~ $n^4+2n^2+1 = (n^2+1)^2$

$a_n = \frac{n}{(n^2+1)^2} - f(x)\ \frac{x}{(x^2+1)^2} = \int_1^{\infty} \frac{x}{(x^2+1)^2} = \int \frac{1}{2u^2}\,du$

$u = x^2+1$
$du = 2x\,dx$

$-\frac{1}{2u} = -\frac{1}{2(x^2+1)}$ $\lim_{x\to\infty} -\frac{1}{2(x^2+1)} = 0,\ en\ x = 1:\ \frac{1}{4}$

La serie converge.

20) A) $\sum_{n=1}^{\infty} \frac{1}{n^3} = \quad p=3 \rightarrow 3>1$ "La serie converge"

20) B) $\sum_{n=1}^{\infty} \frac{1}{n^{1/4}} = \quad p=1/4 \rightarrow 1/4<1$ La Serie Diverge"

20) C) $\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+\frac{1}{4\sqrt{4}}+\cdots \sum_{n=2}^{\infty} \frac{1}{n\sqrt{n}} = \sum_{n=2} \frac{1}{n^{3/2}} \quad p=3/2$

$3/2>1$ La serie converge

21) $\sum_{n=1}^{\infty} \frac{1}{5^n} = \quad a_n=\frac{1}{5^n} \rightarrow \frac{a_{n+1}}{a_n} = \frac{1/5^{n+1}}{\frac{1}{5^n}} = \frac{5^n}{5^{n+1}} = \frac{1}{5}$

$L = \frac{1}{5} < 1$ "La serie converge"

21) B) $\sum_{n=1}^{\infty} \frac{n^3}{3^n} = \frac{a_{n+1}}{a_n} = \frac{\frac{(n+1)^3}{3^{n+1}}}{\frac{n^3}{3^n}} = \frac{(n+1)^3}{3^{n+1}} \cdot \frac{3^n}{n^3} = \frac{(n+1)^3}{3n^3}$

$L = \lim_{n\to\infty} \left| \frac{(n+1)^3}{3n^3} \right| = \lim_{n\to\infty} \frac{(n+1)^3}{3n^3} = \lim_{x\to\infty} \frac{n^3+3n^2+3n+1}{3n^3} =$

$\lim_{n\to\infty} \frac{1+3/n+3/n^2+1/n^3}{3} = \frac{1}{3}$ $\quad L = 1/3 < 1$ "La serie converge"

21) C) $\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{n!}, \quad \frac{a_{n+1}}{a_n} = \frac{\frac{(-1)^{n+1} 2^{n+1}}{(n+1)!}}{\frac{(-1)^n 2^n}{n!}} = \frac{(-1)^{n+1} 2^{n+1}}{(n+1)!} \cdot \frac{n!}{(-1)^n 2^n} = \frac{-2}{n+1}$

$L = \lim_{n\to\infty} \left| \frac{-2}{n+1} \right| = \lim_{n\to\infty} \frac{2}{n+1} = 0 < 1$ (La serie converge)

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(22) A) $\sum_{n=1}^{\infty} \frac{1}{5^n} = \quad a_n = \frac{1}{5^n} = \sqrt[n]{|a_n|} = \sqrt[n]{\frac{1}{5^n}} =$

$$L = \lim_{n\to\infty} \sqrt[n]{\left|\frac{1}{5^n}\right|} = \lim_{n\to\infty} \sqrt[n]{\frac{1}{5^n}} = \lim_{n\to\infty} \frac{1}{5} = \frac{1}{5}$$

"$L = \frac{1}{5} < 1$, La serie converge"

B) $\sum_{n=1}^{\infty} \left(\frac{3n+2}{n+3}\right)^n \quad \lim_{n\to\infty} \sqrt[n]{\left|\left(\frac{3n+2}{n+3}\right)^n\right|} = \lim_{n\to\infty} \sqrt[n]{\left(\frac{3n+2}{n+3}\right)} = \lim_{n\to\infty} \frac{3n+2}{n+3}$

$\lim_{n\to\infty} \frac{3+\frac{2}{n}}{1+\frac{3}{n}} = \frac{3}{1} = 3 \quad L = 3 > 1$, "La serie diverge"

c) $\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^n} = L = \lim_{n\to\infty} \sqrt[n]{\left|\frac{(-1)^n}{(\ln n)^n}\right|} = \lim_{n\to\infty} \sqrt[n]{\frac{1}{(\ln n)^n}} =$

$\lim_{n\to\infty} \frac{1}{\ln n} = 0 \quad L = 0 < 1$ La serie converge.

$n \geq 1$

(23) A) $\sum_{n=1}^{\infty} = \sum \frac{1}{n} \quad \frac{1}{2n-1} > \frac{1}{2n}$ "La serie diverge por comparacion armonica"

(23) B) $\sum_{n=0}^{\infty} \frac{4^n}{5^n+3} = \sum_{n=0}^{\infty} \frac{4^n}{5^n} = \sum_{n=0}^{\infty} \left(\frac{4}{5}\right)^n \quad n \geq 0,\ 5^n + 3 > 5^n,$

$\frac{4^n}{5^n+3} < \frac{4^n}{5^n}; \sum_{n=0}^{\infty} \left(\frac{4}{5}\right)^n \quad r = \frac{4}{5};\ \frac{4}{5} < 1,$

$\frac{4^n}{5^n+3} < \frac{4^n}{5^n}, \sum^{\infty} \left(\frac{4}{5}\right)^n;$ La serie geometrica converge

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c) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}} = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ $\quad n>1,\ n^3+1>1>n^3$ $\quad \sqrt{n^3+1} > \sqrt{n^3}$

$\frac{1}{\sqrt{n^3+1}} < \frac{1}{\sqrt{n^3}}$; $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$; $p = \frac{3}{2}$ $\quad$ $p>1$ La serie converge

$\frac{1}{\sqrt{n^3+1}} < \frac{1}{n^{3/2}}$ y $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$;

24) $\sum_{n=1}^{\infty} \frac{n}{n^2+1} = a_n = \frac{n}{n^2+1}$, $a_n \sim \frac{n}{n^2} = \frac{1}{n}$

$b_n = \frac{1}{n}$ $\quad \lim_{n\to\infty} \frac{\frac{n}{n^2+1}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^2}{n^2+1} = \lim_{n\to\infty} \frac{1}{1+\frac{1}{n^2}} = 1$

$L=1$, La serie Diverge

B) $\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+1}} = \lim_{n\to\infty} \frac{\frac{1}{\sqrt{n^2+1}}}{1/n} = \lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n^2}}} = \boxed{1}$

$L=1$, La Serie Diverge.

c) $\sum_{n=1}^{\infty} \frac{2n^2-1}{3n^5+2n+1} = \lim_{n\to\infty} \frac{\frac{2n^2-1}{3n^5+2n+1}}{\frac{1}{n^3}} = \lim_{n\to\infty} \frac{n^3(2n^2-1)}{3n^5+2n+1} = \lim_{n\to\infty}$

$\frac{2n^5-n^3}{3n^5+2n+1} = \lim_{n\to\infty} \frac{2-\frac{1}{n^2}}{3+2/n^4+\frac{1}{n^5}} = \frac{2}{3} \Rightarrow L = \frac{2}{3}$ La serie converge.

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25) $\sum_{n=1}^{\infty} \frac{(-1)^n}{3^n}$ $a_n = \frac{1}{3^n}$; $a_{n+1} < a_n$

$\boxed{r = \frac{1}{3}}$

$a_{n+} = \frac{1}{3^n+1}$, $a_n = \frac{1}{3^n}$, $3^{n+1} > 3^n$ $n \geq 1$

$\lim_{n\to\infty} \frac{1}{3^n} = 0$ "La serie Converge"

B) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+1} =$ $a_n = \frac{1}{n+1}$, $a_{n+1} = \frac{1}{(n+1)+1} = \frac{1}{n+2}$

$a_n = \frac{1}{n+1}$, $n+2 > n+1$, $\frac{1}{n+2} < \frac{1}{n+1}$, $a_{n+1} < a_n$

$\lim_{n\to\infty} \frac{1}{n+1} = 0$ La serie converge pero no absolutamente

c) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{\ln(n+1)} =$ $a_n = \frac{n}{\ln(n+1)}$ $a_{n+1} \leq a_n$

$F(x) = \frac{x}{\ln(x+1)}$, $F'(x) = \frac{1 \cdot \ln(x+1) - x \cdot \frac{1}{x+1}}{(\ln(x+1))^2} = \frac{\ln(x+1) - \frac{x}{x+1}}{(\ln(x+1))^2}$

$\ln(x+1) - \frac{x}{x+1}$ $x = 1$ $\ln(2) - \frac{1}{2} > 0$

$x = 2$ $\ln(3) = -\frac{2}{3} > 0$

"La serie diverge"

copán

26 $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^2}{(n+1)^2} = \sum_{n=1}^{\infty} \left| \frac{(-1)^{n+1} n^2}{(n+1)^2} \right| = \sum_{n=1}^{\infty} \frac{n^2}{(n+1)^2}$

$\lim_{n \to \infty} \frac{n^2}{(n+1)^2} = \frac{n^2}{n^2+2n+1} = \lim_{n \to \infty} \frac{1}{1+\frac{2}{n}+\frac{1}{n^2}} = 1 \neq 0$

La serie diverge

B) $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n} = \sum_{n=1}^{\infty} \left| \frac{(-1)^n}{2^n} \right| = \sum_{n=1}^{\infty} \frac{1}{2^n}$ $\quad r = \frac{1}{2} < 1$

"La serie converge" "absolutamente"

c) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}} = \sum_{n=1}^{\infty} \left| \frac{(-1)^{n+1}}{\sqrt{n}} \right| = \frac{1}{\sqrt{n}} = \frac{1}{n^{1/2}}$ $\quad p = \frac{1}{2} < 1$

"La serie Diverge Condicionalmente"

$a_n = \frac{1}{\sqrt{n}}$, $\lim_{n \to \infty} a_n = 0$

$\lim_{n \to \infty} \frac{1}{\sqrt{n}} = 0$

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(27) $\sum_{n=0}^{\infty} \frac{(x-1)^n}{(n+1)^2} = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{\frac{(x-1)^{n+1}}{((n+1)+1)^2}}{\frac{(x-1)^n}{(n+1)^2}} \right| = \left| \frac{(x-1)^{n+1}(n+1)^2}{(x-1)^n (n+2)^2} \right|$

$\left| (x-1) \frac{(n+1)^2}{(n+2)^2} \right| = |x-1| \left(\frac{n+1}{n+2}\right)^2 = |x-1| \cdot 1^2 = |x-1|$

$|x-1| < 1$, $R = 1$, $-1 < x-1 < 1 \Rightarrow 0 < x < 2$

$x=0$, $\sum_{n=0}^{\infty} \frac{(0-1)^n}{(n+1)^2} = \frac{(-1)^n}{(n+1)^2}$

$p = 2 > 1$

$x=2$ $\sum_{n=2}^{\infty} \frac{(2-1)^n}{(n+1)^2} = \frac{1}{(n+1)^2}$

$R = 1$ y $[0, 2]$

(28) $\sum_{n=0}^{\infty} \frac{(x-3)^{n+1}}{(n+1)4^{n+1}} = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{\frac{(x-3)^{n+2}}{(n+2)4^{n+2}}}{\frac{(x-3)^{n+1}}{(n+1)4^{n+1}}}$

$\left|\frac{x-3}{4}\right| \cdot \frac{n+1}{n+2} = \left|\frac{x-3}{4}\right| \cdot 1 = \left|\frac{x-3}{4}\right| < 1$

$|x-3| < 4$ $R=4$, $-4 < x-3 < 4 \Rightarrow -1 < x < 7$

$x=-1$ $\frac{(-4)^{n+1}}{(n+1)4^{n+1}} = \frac{(-1)^{n+1}}{n+1}$

$R = 4$, $Ic = [-1, 7]$

$x=7$ $\frac{(7-3)^{n+1}}{(n+1)4^{n+1}} = \frac{4^{n+1}}{(n+1)4^{n+1}} = \frac{1}{n+1}$

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(29) $\sum_{n=0}^{\infty} \frac{(3x)^n}{(2n)!} = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \left| \frac{\frac{(3x)^{n+1}}{(2(n+1))!}}{\frac{(3x)^n}{(2n)!}} \right| \left| \frac{(3x)^{n+1}(2n)!}{(3x)^n(2n+2)!} \right|$

$\frac{1}{(2n+2)(2n+1)} = |3x| \cdot 0 = 0$

$R = \infty \quad [-\infty, \infty]$

(30) $\sum_{n=0}^{\infty} \frac{n}{n+1}(-2x)^{2n-1} = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{\frac{n+1}{n+2}(-2x)^{2(n+1)-1}}{\frac{n}{n+1}(2x)^{2n-1}} \right| =$

$\left| \frac{(n+1)^2(-2x)^{2n+1}}{n(n+2)(-2x)^{2n-1}} \right| \left| \frac{(n+1)^2}{n(n+2)}(-2x)^2 \right| =$

$4x^2 \cdot 1 = 4x^2 < 1, \quad x^2 < 1/4 \quad |x| < \frac{1}{2}$

$R = \frac{1}{2}, \quad -\frac{1}{2} < x < \frac{1}{2}$

(31) $\sum_{n=1}^{\infty} \frac{2}{n^2+4n+3} = \frac{A}{n+1} + \frac{B}{n+3} =$

$n^2+4n+3 \to (n+1)(n+3)$

$N=-1, \quad N=-3$

$2 = A(n+3) + B(n+1)$

$2 = A(-1+3) + \cancel{B(-1+1)}\ 0$

$2 = 2A \to \boxed{A=1}$

$2 = \cancel{A(-3+3)}\ 0 + B(-3+1)$

$2 = -2B \to B = -1$

$\sum_{n=1}^{\infty}\left(\frac{1}{n+1} - \frac{1}{n+3}\right) = \left(\frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \left(\frac{1}{4} - \frac{1}{6}\right) + \cdots \left(\frac{1}{n+1} - \frac{1}{n+3}\right)$

$S_n = \frac{1}{2} + \frac{1}{3} - \frac{1}{n+2} - \frac{1}{n+3} \quad n\to\infty$

$\lim_{n\to\infty} S_n = \lim_{N\to\infty}\left(\frac{1}{2} + \frac{1}{3} - \frac{1}{n+2} - \frac{1}{n+3}\right) = \frac{1}{2} + \frac{1}{3} - 0 - 0 = \frac{5}{6}$

La serie converge a $\frac{5}{6}$

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32 $\sum_{n=1}^{\infty} \frac{(-1)^n}{3^n n!} = \quad a_n = \frac{(-1)^n}{3^n n!} \quad \left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(-1)^{n+1}}{3^{n+1}(n+1)!} \cdot \frac{3^n n!}{(-1)^n}\right| =$

$\frac{3^n n!}{3^{n+1}(n+1)!} = \frac{1}{3(n+1)} \quad n \to \infty \quad \lim_{n\to\infty} \frac{1}{3(n+1)} = 0$

La serie converge a 0

33 $\sum_{K=1}^{\infty} \frac{\tan^{-1} K}{1+K^2} = \quad F(x) = \frac{\tan^{-1} x}{1+x^2} \quad x \geq 1$

$\int_1^{\infty} \frac{\tan^{-1}}{1+x^2} dx = \int_{\pi/4}^{\pi/2} u \, du = \left[\frac{1}{2} u^2\right]_{\pi/4}^{\pi/2} =$

$u = \tan^{-1}(x)$

$du = \frac{1}{1+x^2} dx$

$\frac{1}{2}\left(\frac{\pi}{2}\right)^2 - \left(\frac{\pi}{4}\right)^2 = \frac{1}{2}\left(\frac{\pi^2}{4} - \frac{\pi^2}{16}\right) = \frac{1}{2}\left(\frac{3\pi^2}{16}\right) = \boxed{\frac{3\pi^2}{32}}$

$x = 1 \quad u = \tan^{-1}(1) = \frac{\pi}{4}$

$x \to \infty, u = \frac{\pi}{2}$

"La serie converge"

34 $\sum_{n=1}^{\infty} \frac{(-1)^{n+3} 3^n}{n!} = \left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(-1)^{n+2} 3^{n+1}}{(n+1)!} \cdot \frac{n!}{(-1)^{n+3} 3^n}\right| = \frac{3^{n+1} n!}{3^n (n+1)!} =$

$\frac{3}{n+1} \quad n \to \infty \quad \lim_{n\to\infty} \frac{3}{n+1} = 0$

"La serie converge"

35 $\sum_{n=1}^{\infty} \left(\frac{4n+5}{9n-3}\right)^n = \lim_{n\to\infty} \sqrt[n]{|a_n|} = L = \lim_{n\to\infty} \sqrt[n]{\left|\left(\frac{4n+5}{9n-3}\right)^n\right|} = \frac{4n+5}{9n-3} =$

$\frac{4 + \cancel{\frac{5}{n}}}{9 - \cancel{\frac{3}{n}}} = \frac{4}{9} \quad L = \frac{4}{9} < 1$ "La serie converge"

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36) $\sum_{n=1}^{\infty} \frac{n}{5n^2-4} = \sum \frac{1}{n} \rightarrow \lim_{n\to\infty} \frac{\frac{n}{5n^2-4}}{1/n} = \frac{n^2}{5n^2-4} = \frac{1}{5-\frac{4}{n^2}} = \frac{1}{5}$

$0 < L = \frac{1}{5} < \infty$ "La serie diverge"

37) $\sum_{n=1}^{\infty} \frac{n+3}{n^2\sqrt{n}} = \frac{n+3}{n^{5/2}} = \frac{3}{n^{5/2}} + \frac{3}{n^{5/2}} = \frac{1}{n^{3/2}} + \frac{3}{n^{5/2}}$

$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}} + 3\sum_{n=1}^{\infty} \frac{1}{n^{5/2}}$ $\quad p_1 = \frac{3}{2} > 1$ y $p_2 = \frac{5}{2} > 1$

"La serie converge"

38) $\sum_{k=1}^{\infty} \frac{1}{1+4k^2} = L = \lim_{k\to\infty} \frac{\frac{1}{1+4k^2}}{\frac{1}{k^2}} = \frac{k^2}{1+4k^2} = \frac{1}{\frac{1}{k^2}+4} = \boxed{\frac{1}{4}}$ $\quad k^2$

$0 < L = \frac{1}{4} < \infty$ "La serie converge"

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