3D Scanning & Motion Capture

Exercise - 3

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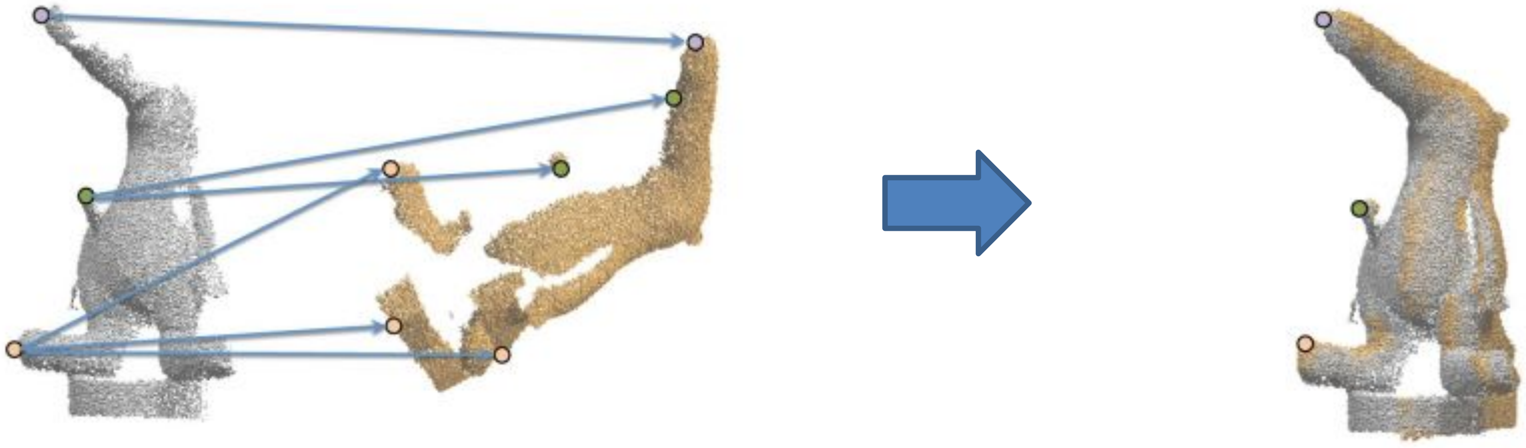
Exercises – Overview

1. Exercise Camera Intrinsics, Back-projection, Meshes
2. Exercise Surface Representations
3. **Exercise Coarse Alignment (Procrustes)**
4. Exercise Optimization
5. Exercise Object Alignment, ICP

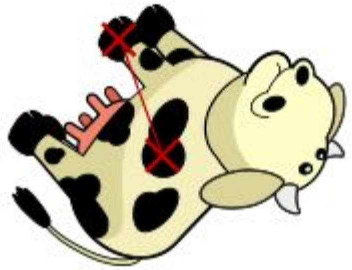
Procrustes

- Problem: Align two objects using known correspondences
  - scaling, translation, rotation



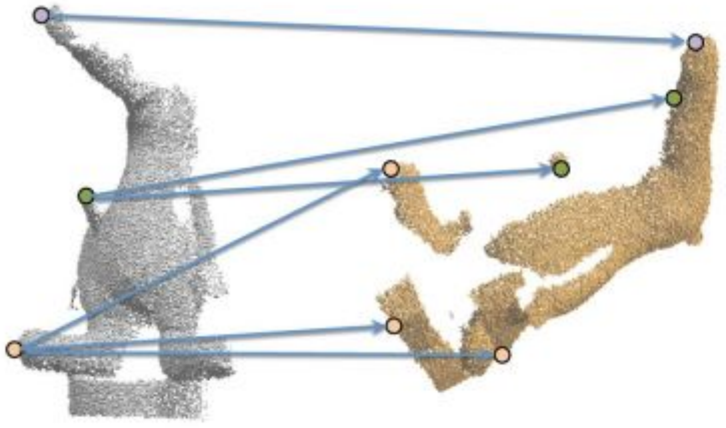
Procrustes

- Problem: Align two objects using known correspondences
  - scaling, translation, rotation
    - Compute center of gravity of both objects
    - Scale one object to match the avg. distance from all vertices to the center of gravity



Procrustes

- Problem: Align two objects using known correspondences
  - scaling, translation, rotation



$$\boxed{X = R\hat{X} + t}$$

Procrustes

- Problem: Align two objects using known correspondences
  - scaling, translation, rotation
    - The translation vector will depend on the rotation, so we will analyse it later

Procrustes

- Problem: Align two objects using known correspondences

  → scaling, translation, rotation

  - Assume objects that are zero-centered
    - Target object: $\{x_0, .. x_{n-1}\}$
    - Moving object: $\{\hat{x}_0\ , .. \hat{x}_{n-1}\}$

$$\sum_i \|x_i - R \cdot \hat{x}_i\|_2^2 \ \rightarrow min$$

$$\left\|X - \hat{X}R^T\right\|_F^2 \rightarrow min$$

Procrustes

- Problem: Align two objects using known correspondences
  - scaling, translation, rotation

$$\left\|X - \hat{X}R^T\right\|_F^2 \rightarrow min$$

$\|A\|_F^2 = trace(A^T A)$

Cyclic invariance of trace:

$trace(ABC) = trace(CAB)$

$$\left\|X - \hat{X}R^T\right\|_F^2 = trace(X^T X - X^T \hat{X}R^T - (\hat{X}R^T)^T X + (\hat{X}R^T)^T(\hat{X}R^T)) \rightarrow min$$

$$trace(-X^T \hat{X}R^T - (\hat{X}R^T)^T X + (\hat{X}R^T)^T(\hat{X}R^T)) \rightarrow min$$

$$-2 \cdot trace(X^T \hat{X}R^T) \rightarrow min$$

$$trace(X^T \hat{X}R^T) \rightarrow max$$

$SVD\colon X^T \hat{X} = USV^T$

$$trace(USV^T R^T) \rightarrow max$$

$$trace(SV^T R^T U) \rightarrow max$$

Singular values
positive

Product of orthogonal matrices
max if equal to Identity

Procrustes

- Problem: Align two objects using known correspondences
   - scaling, translation, rotation

$$\left\|X - \hat{X}R^T\right\|_F^2 \rightarrow min$$

- Compute SVD of the Cross-Covariance Matrix

$$X^T\hat{X} = USV^T$$

- Compute the rotation

$$R = UV^T$$

Procrustes

- Problem: Align two objects using known correspondences

  →scaling, translation, rotation

    - The computed rotation might be a mirroring!
    - The determinant of a rotation matrix must be 1
    - If $\det(UV^T) = -1$, compute the rotation as:

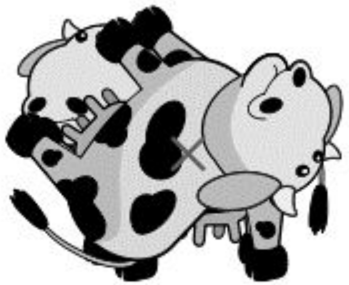
$$R = U \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} V^T$$

Procrustes

- Problem: Align two objects using known correspondences

  → scaling, translation, rotation

  - Assume objects that are zero-centered
    - Target object: $\{x_0, .. x_{n-1}\}$
    - Moving object: $\{\hat{x}_0, .. \hat{x}_{n-1}\}$



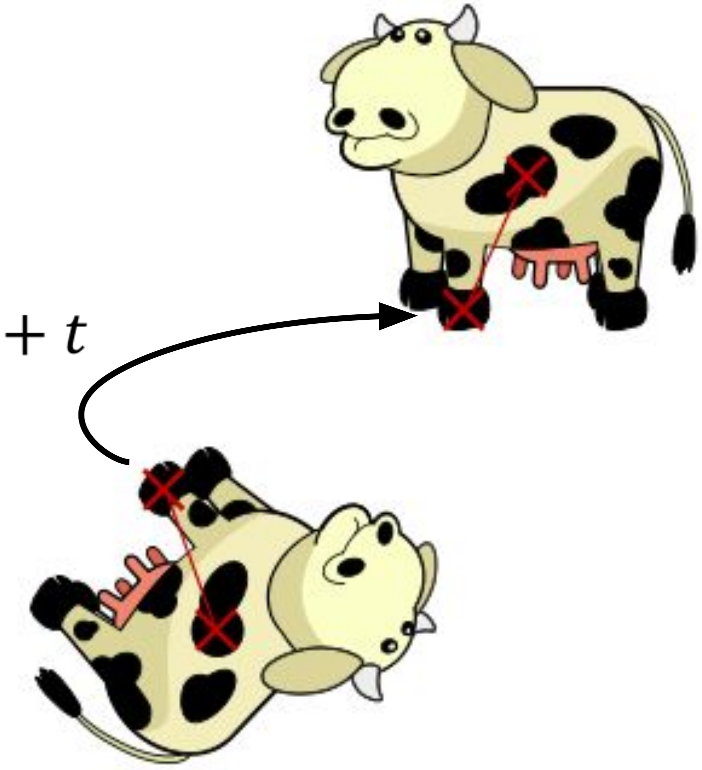
$$\sum_i \|x_i - R \cdot \hat{x}_i\|_2^2 \rightarrow min$$

$$\|X - \hat{X}R^T\|_F^2 \rightarrow min$$

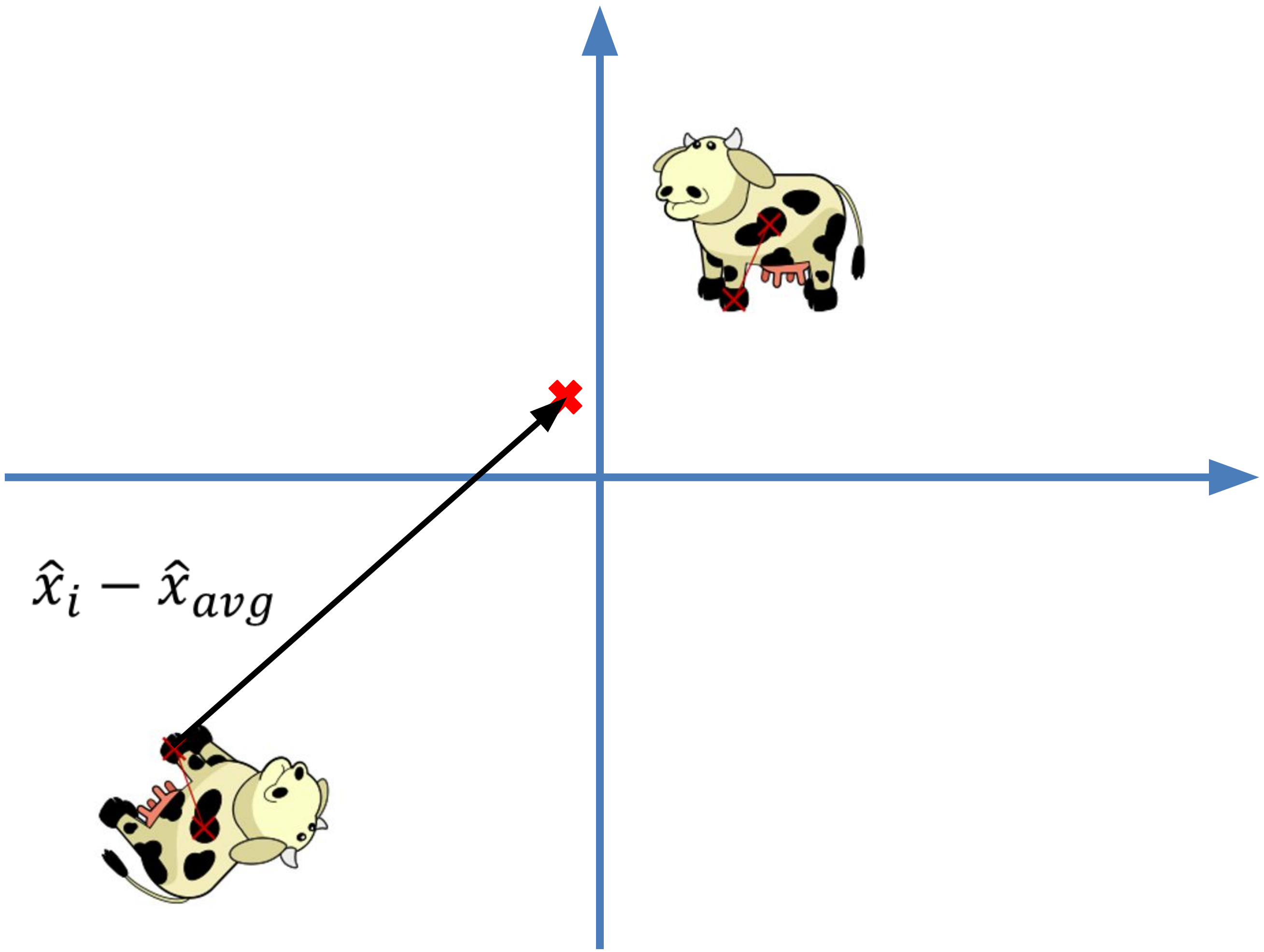
Procrustes

- Problem: Align two objects using known correspondences

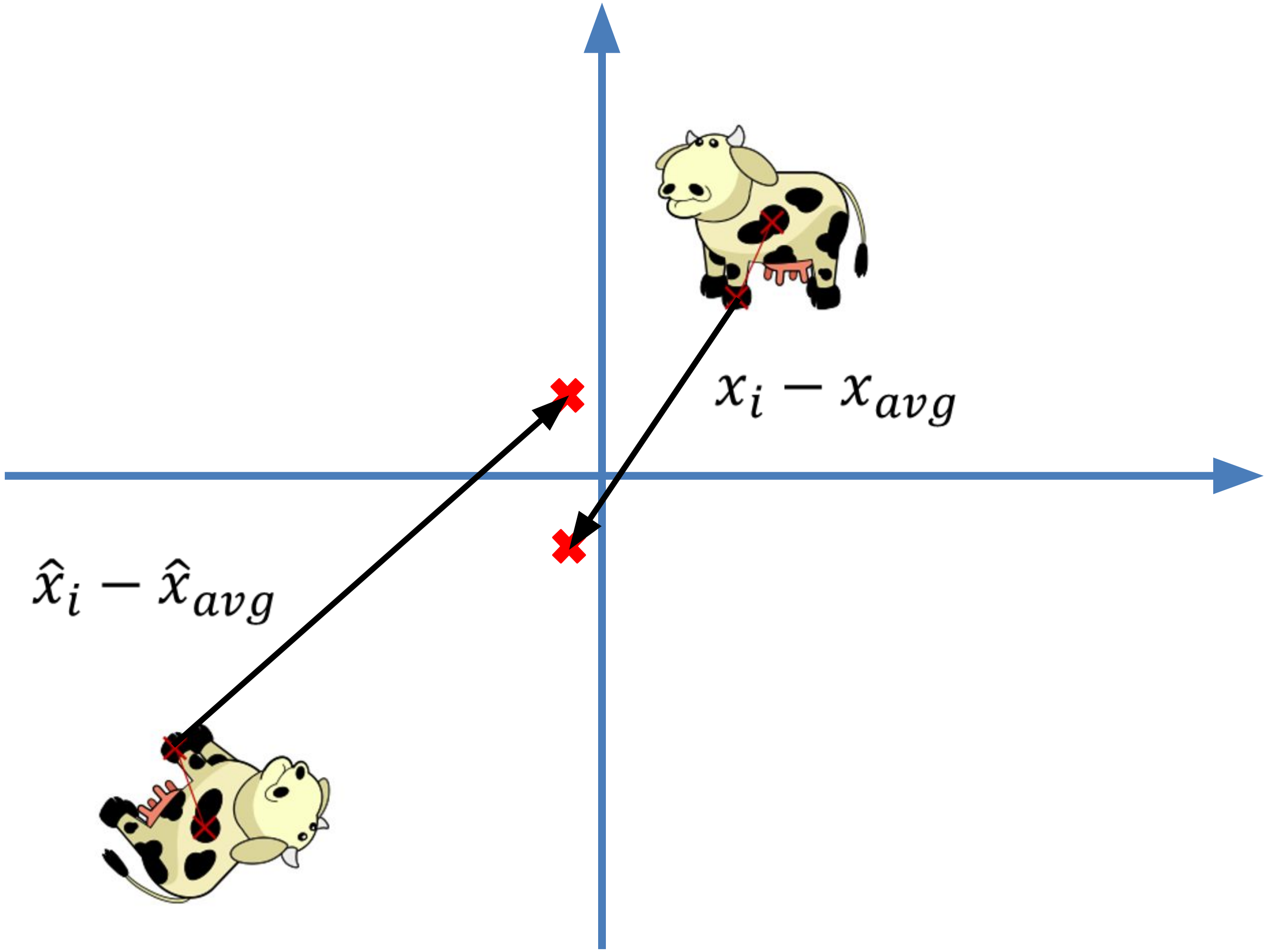
$$x_i = R\hat{x}_i + t$$



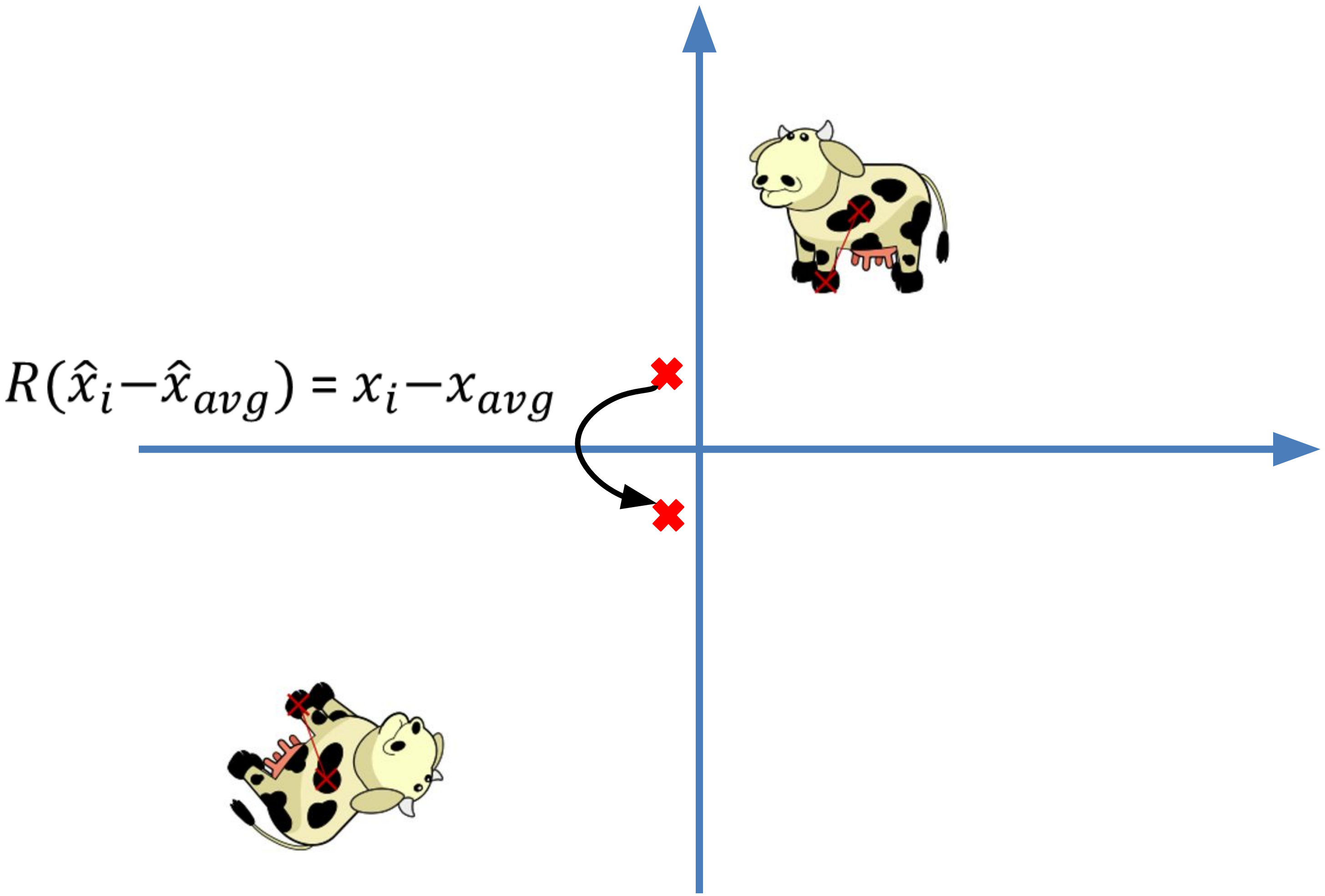
Procrustes



Procrustes



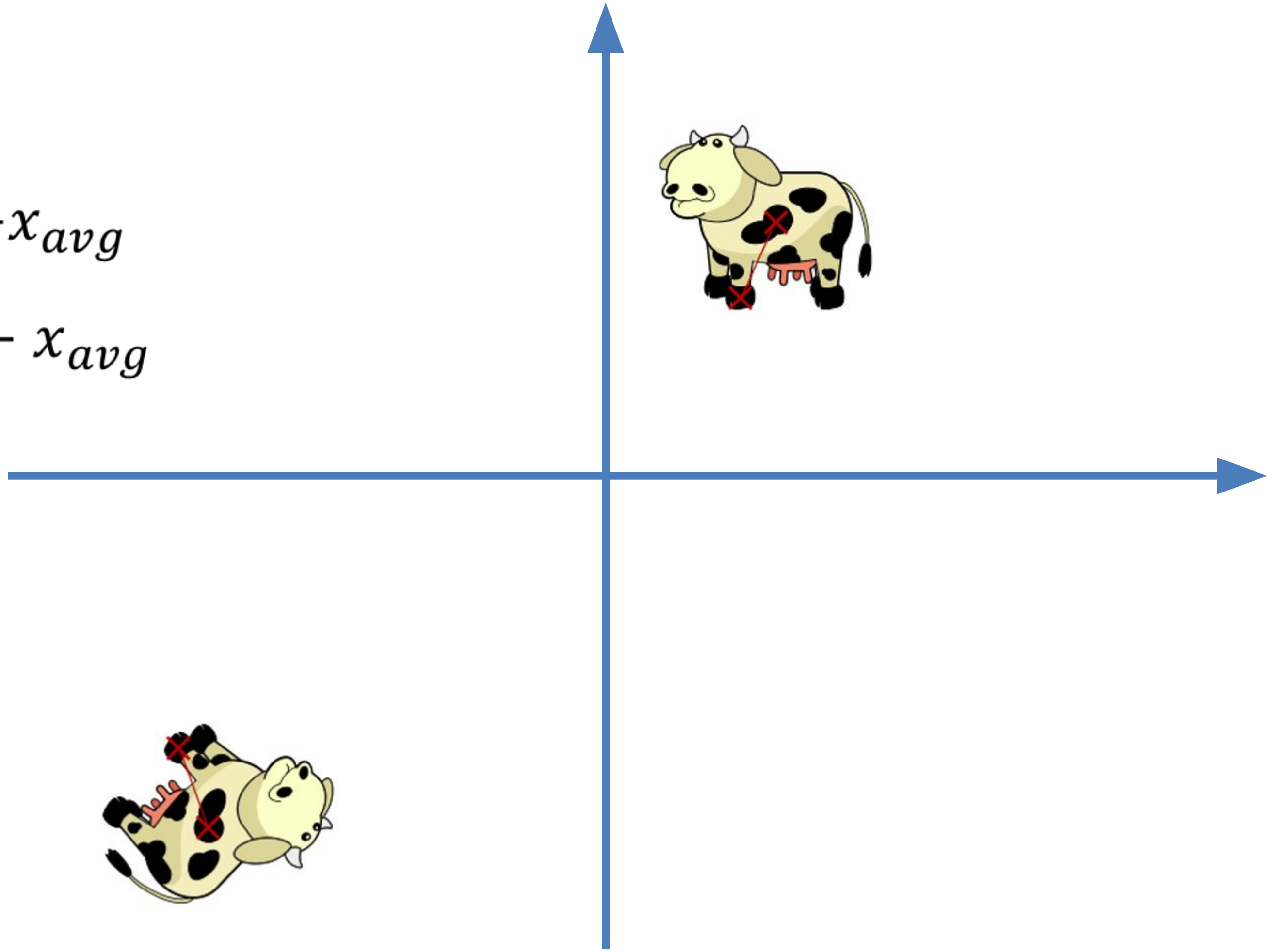
Procrustes



Procrustes

$$R(\hat{x}_i - \hat{x}_{avg}) = x_i - x_{avg}$$

$$x_i = R(\hat{x}_i - \hat{x}_{avg}) + x_{avg}$$

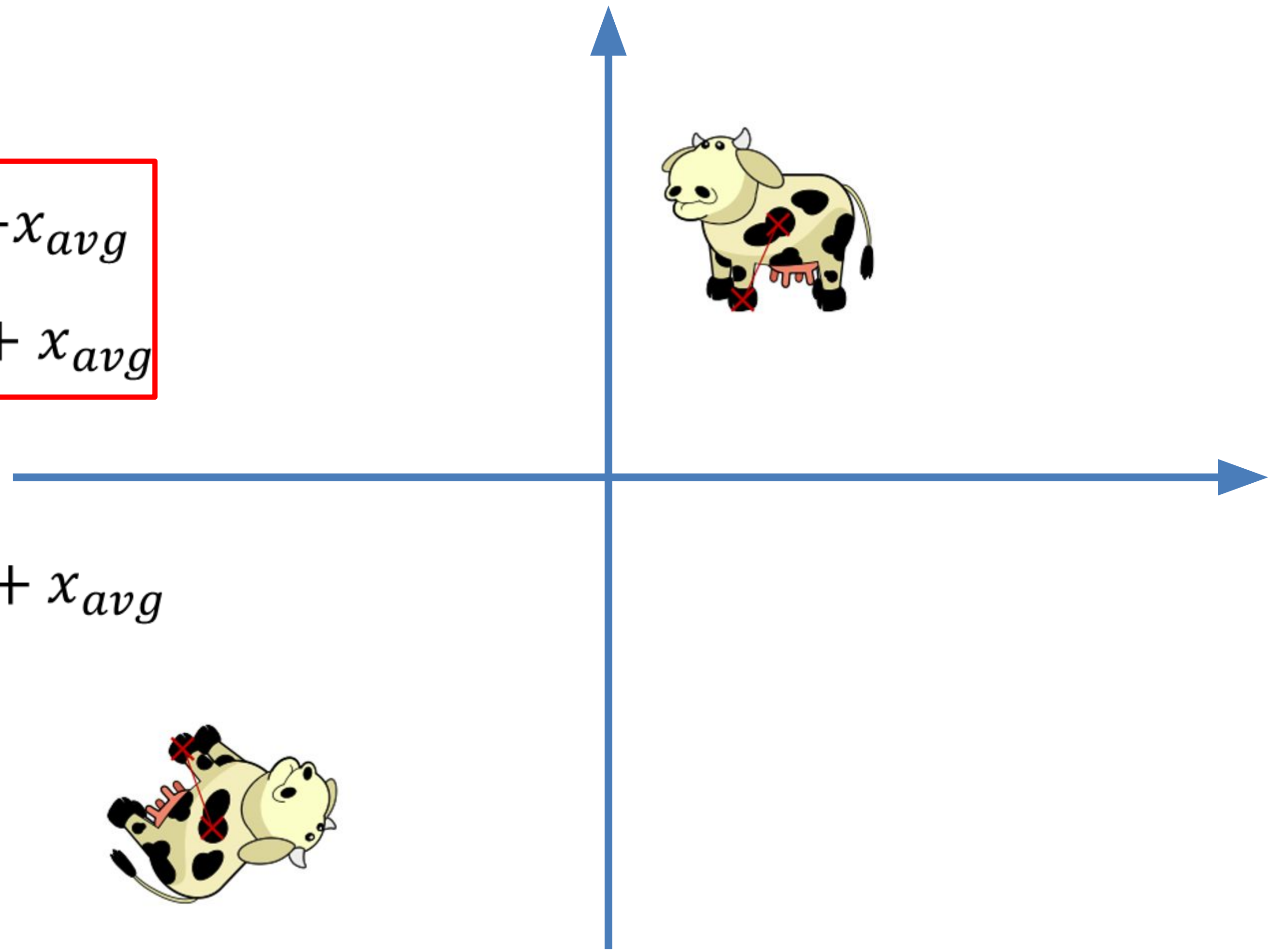


Procrustes

$$R(\hat{x}_i - \hat{x}_{avg}) = x_i - x_{avg}$$

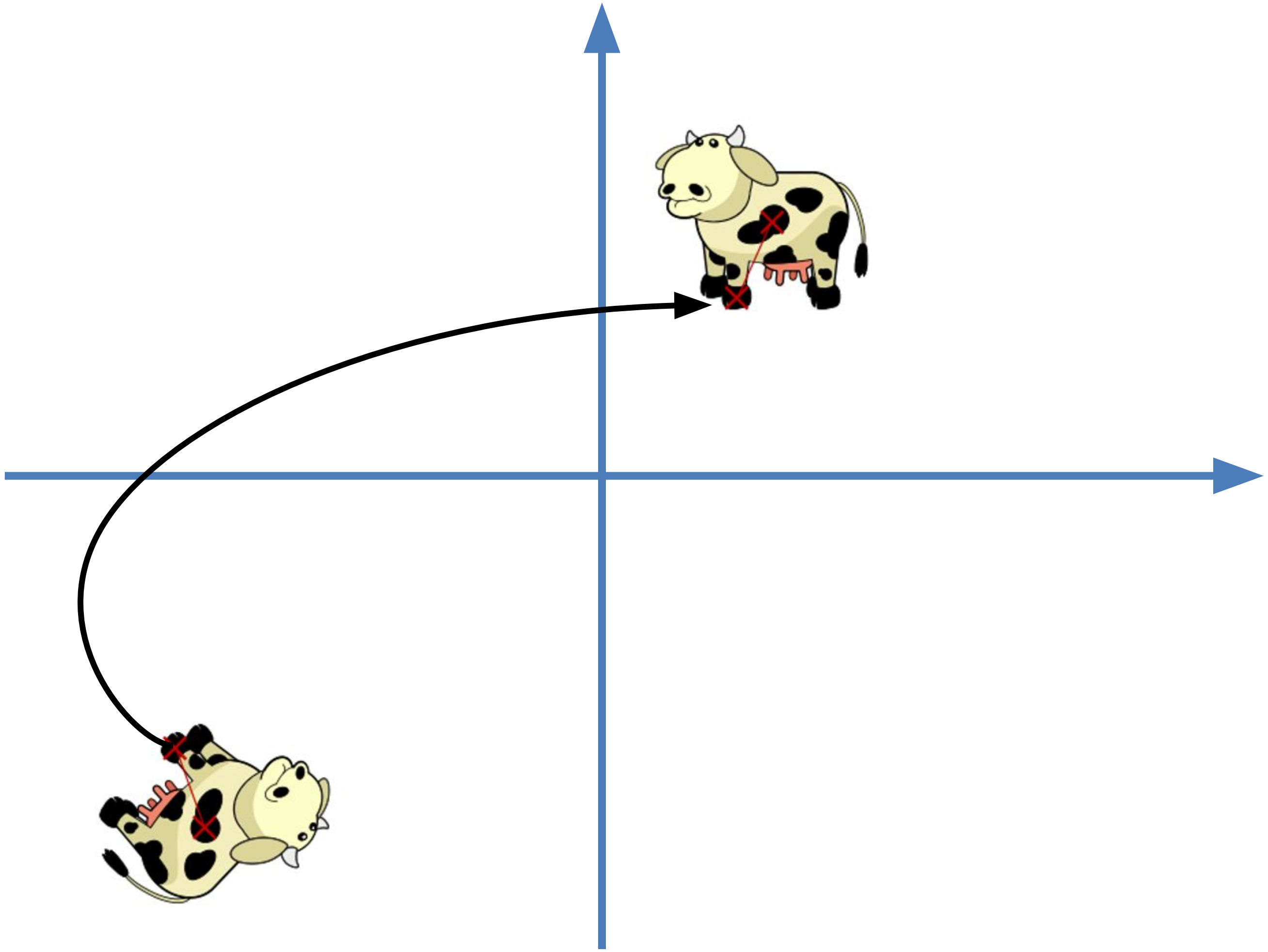
$$x_i = R(\hat{x}_i - \hat{x}_{avg}) + x_{avg}$$

$$x_i = R\hat{x}_i - R\hat{x}_{avg} + x_{avg}$$



Procrustes

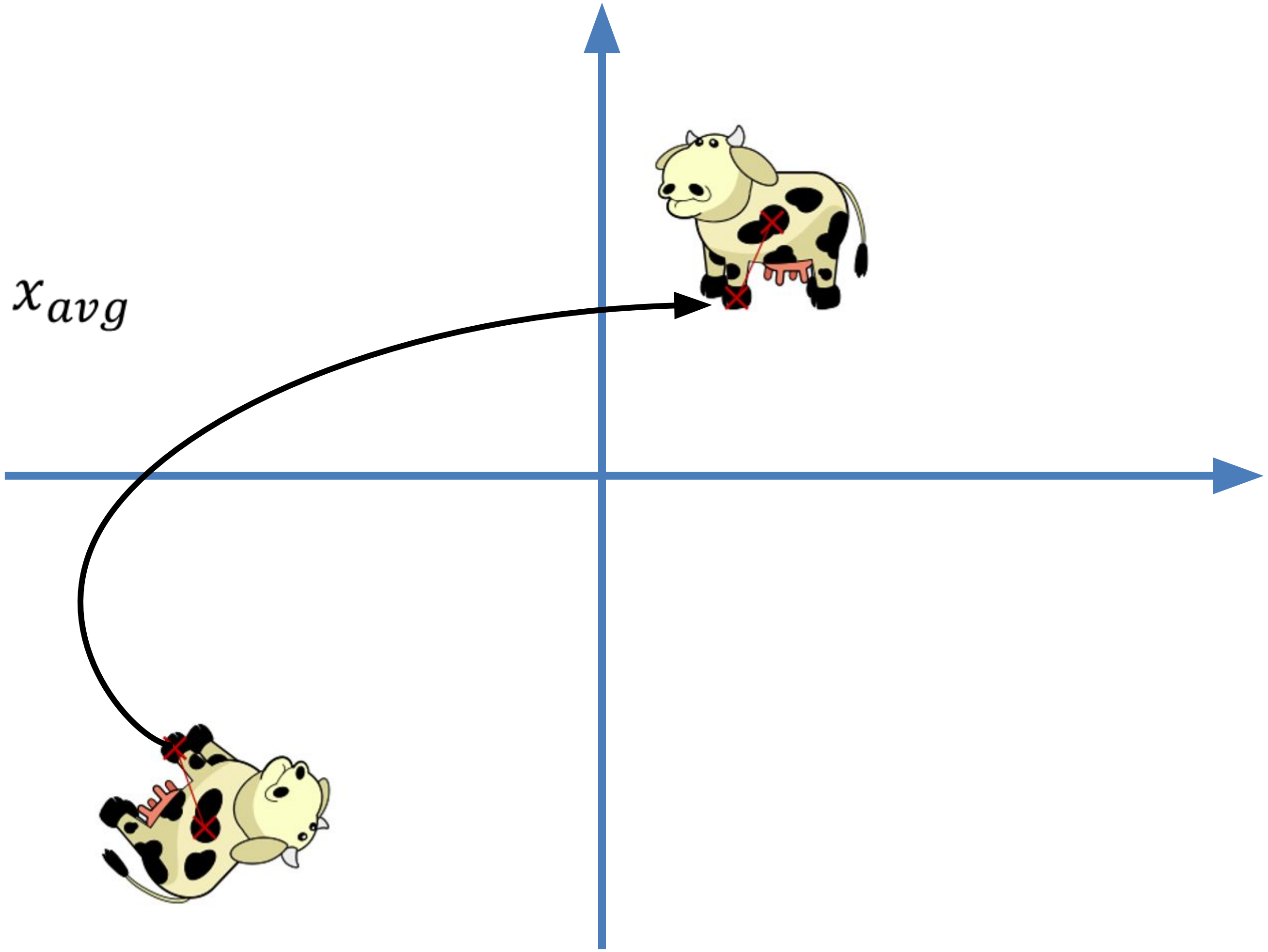
$$x_i = R\hat{x}_i + t$$



Procrustes

$$x_i = R\hat{x}_i + t$$

$$x_i = R\hat{x}_i - R\hat{x}_{avg} + x_{avg}$$

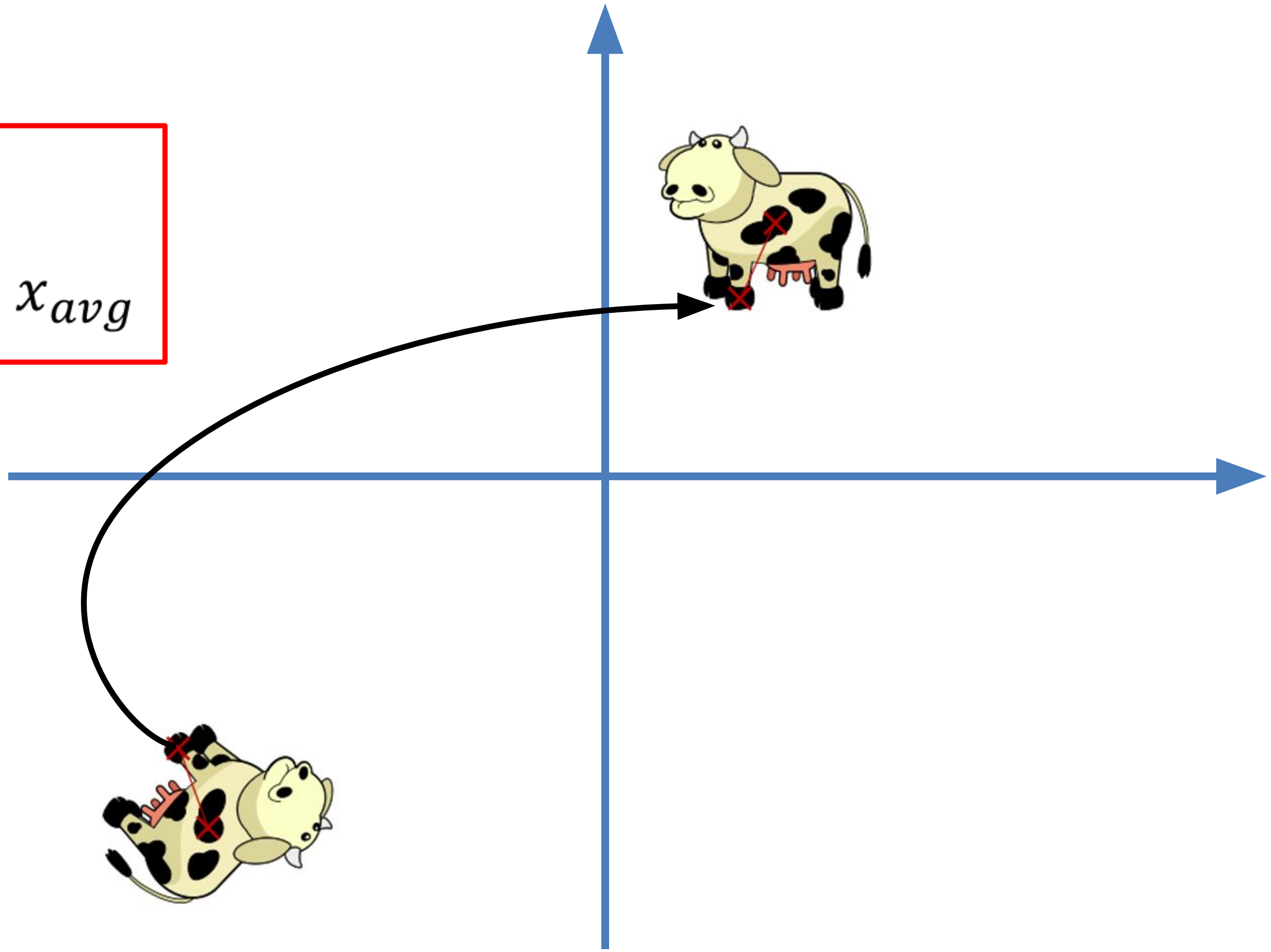


Procrustes

$$\boxed{\begin{aligned} x_i &= R\hat{x}_i + t \\ x_i &= R\hat{x}_i - R\hat{x}_{avg} + x_{avg} \end{aligned}}$$

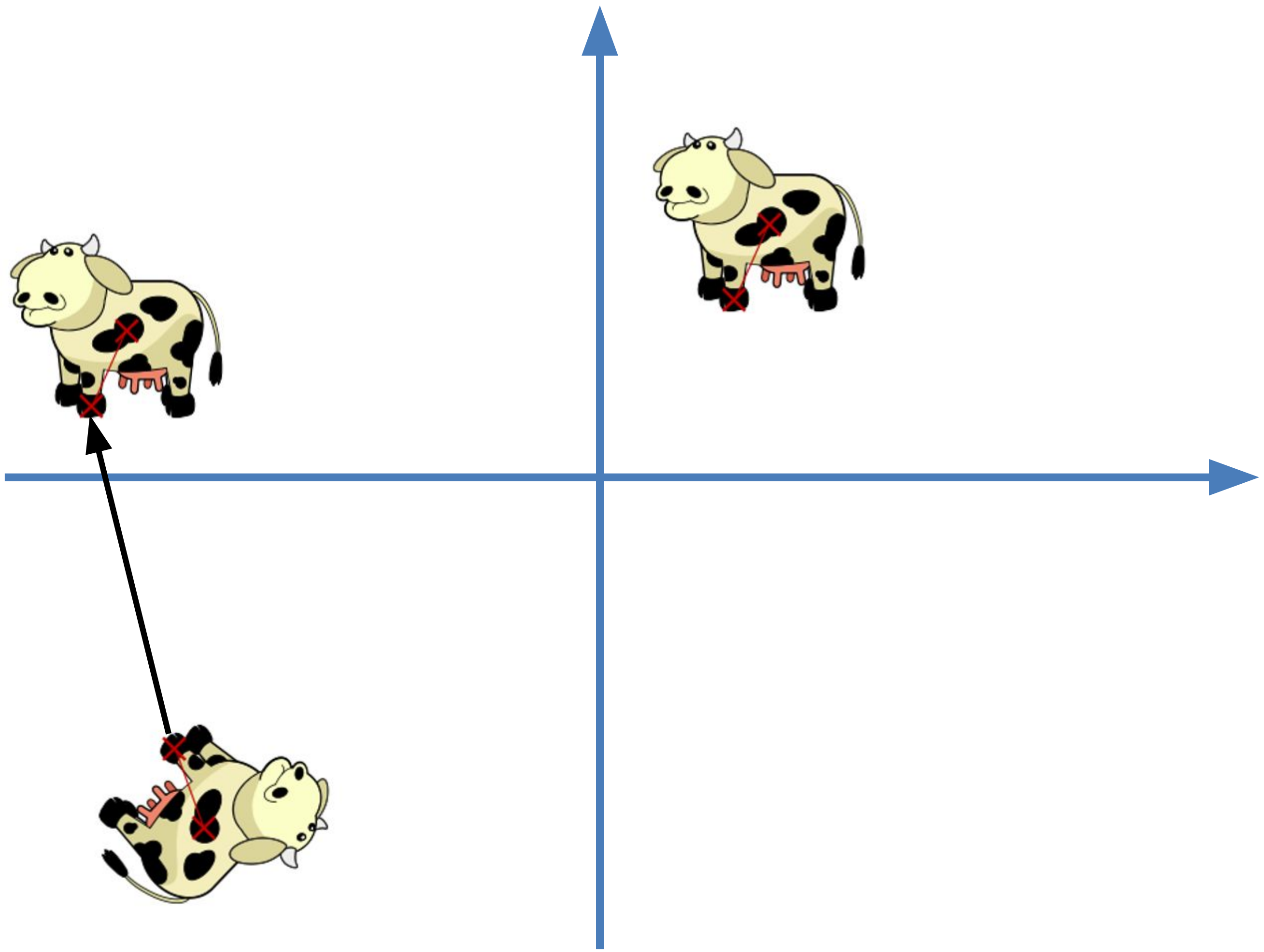
$$\Downarrow$$

$$t = -R\hat{x}_{avg} + x_{avg}$$



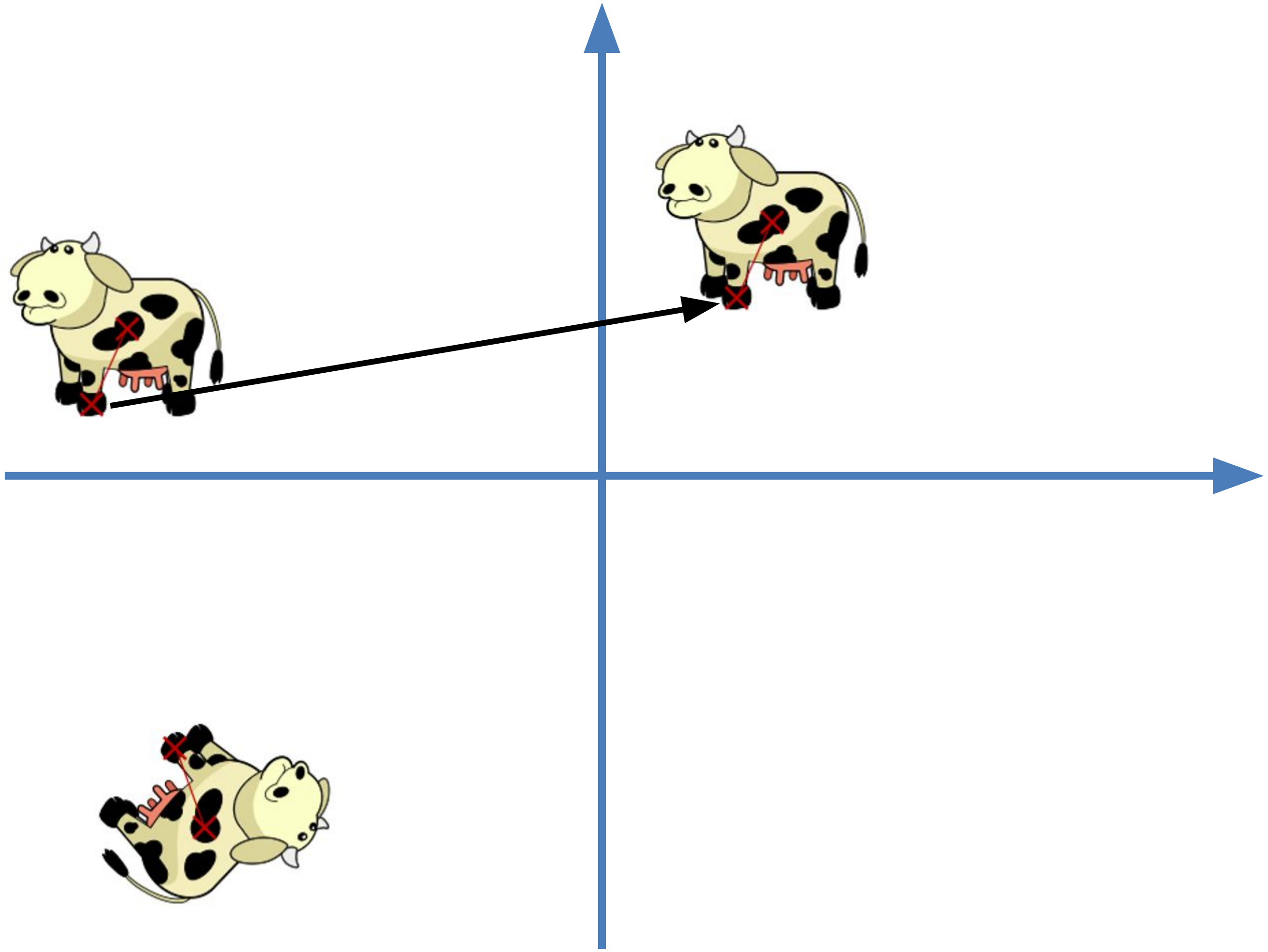
Procrustes

$x_i = R\hat{x}_i + t$



Procrustes

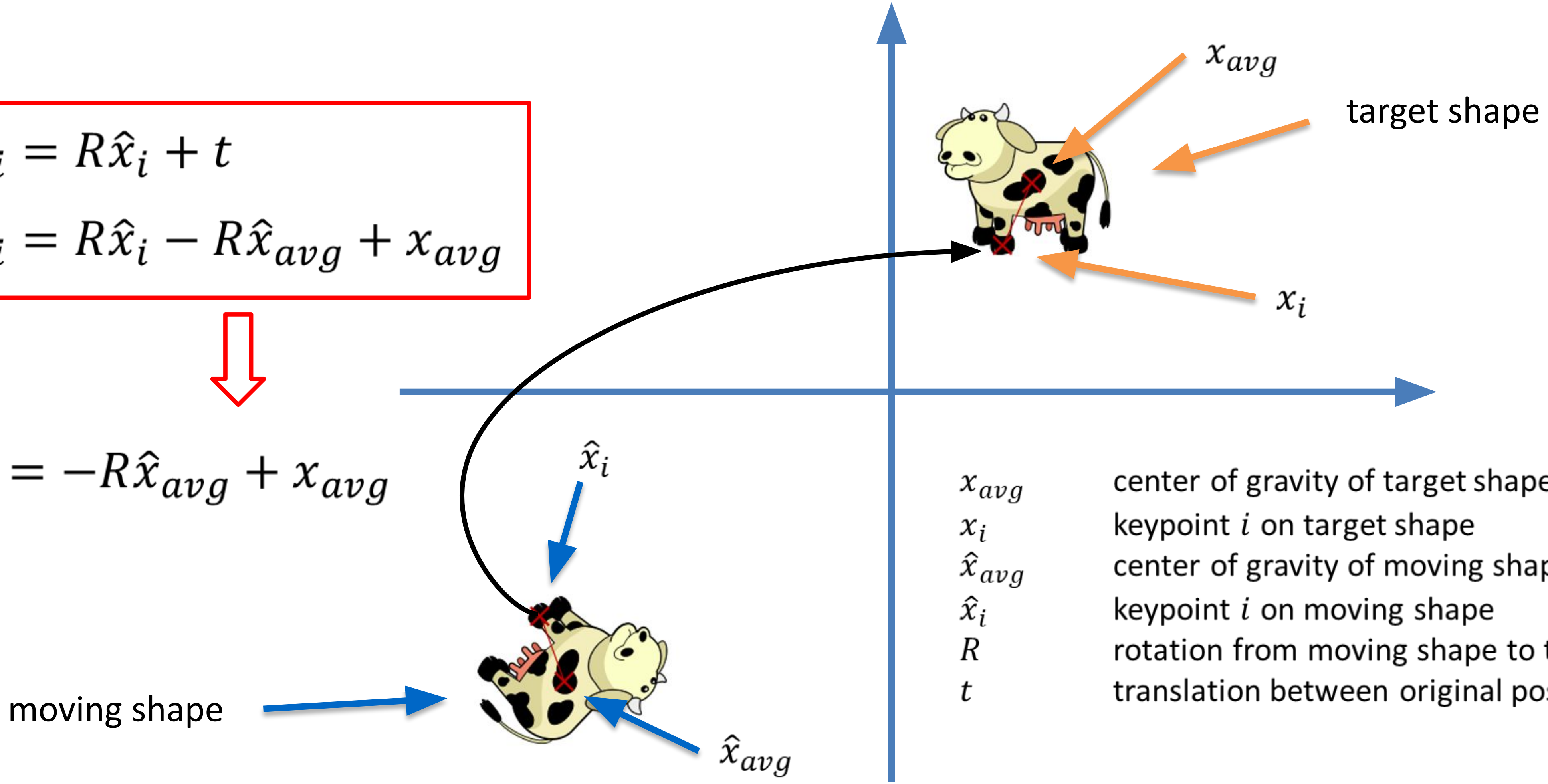
$x_i = R\hat{x}_i + t$



Procrustes

$$x_i = R\hat{x}_i + t$$
$$x_i = R\hat{x}_i - R\hat{x}_{avg} + x_{avg}$$

$$t = -R\hat{x}_{avg} + x_{avg}$$



x_{avg}	center of gravity of target shape
x_i	keypoint i on target shape
$\hat{x}_{avg}$	center of gravity of moving shape
$\hat{x}_i$	keypoint i on moving shape
R	rotation from moving shape to target shape
t	translation between original poses after rotation

See you next time!