

Homework 2

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①

$$a) Y_1 = [A \cdot B' (C + B \cdot D) + A' \cdot B'] C$$

$$= [A \cdot B' \cdot C + A \cdot B' \cdot B \cdot D + A' \cdot B'] \cdot C$$

$$= A \cdot B' \cdot C \cdot C + A \cdot \underbrace{B' \cdot B}_0 \cdot C \cdot D + A' \cdot B' \cdot C$$

$$= A \cdot B' \cdot C + A' \cdot B' \cdot C = B' \cdot C (A + A') = \boxed{B' \cdot C}$$

$$b) Y_2 = (A + B') \cdot (A' + B' + D) \cdot (B' + C + D')$$

$$= (A \cdot A' + A \cdot B' + A \cdot D + B' \cdot A' + B' \cdot B' + B' \cdot D) \cdot (B' + C + D')$$

$$= (0 + A \cdot B' + A \cdot D + A' \cdot B' + B' + B' \cdot D) \cdot (B' + C + D')$$

$$= (B' \cdot (A + A') + A \cdot D + B' + B' \cdot D) \cdot (B' + C + D')$$

$$= (B' + A \cdot D + B' + B' \cdot D) \cdot (B' + C + D')$$

$$= (A \cdot D + B' (D + 1 + 1)) \cdot (B' + C + D')$$

$$\begin{aligned}
 &= (A \cdot D + B') \cdot (\underbrace{B' + C + D'}_{=1}) = A \cdot B' \cdot D + A \cdot C \cdot D + \underbrace{A \cdot D \cdot D'}_{=0} + \underbrace{B' \cdot B'}_{=0} + B' \cdot C + B' \cdot D' \\
 &= A \cdot C \cdot D + B' (A \cdot D + C + D' + 1) = \boxed{ADC + B'}
 \end{aligned}$$

$$c) Y_3 = AB \overline{C} \overline{D}' + A' B' \overline{C} D + \overline{C} D'$$

$$= A' B' \overline{C} D + \overline{C} D' (AB + 1) = A' B' \overline{C} D + \overline{C} D'$$

$$= \overline{C} (A' B' D + D') = \overline{C} (A' B' + D')$$

$$= \boxed{A' B' \overline{C} + \overline{C} D'}$$

②

$$f = \overline{((a \oplus b) + (c \oplus d))}$$

$$\rightarrow f = \overline{((a \odot b) \cdot (c \odot d))} = \overline{(a \odot b)} \cdot \overline{(c \odot d)}$$

$$\rightarrow \boxed{f = (a \oplus b) \cdot (c \oplus d)}$$

a)

$$f = (a \oplus b) \cdot (c \oplus d)$$

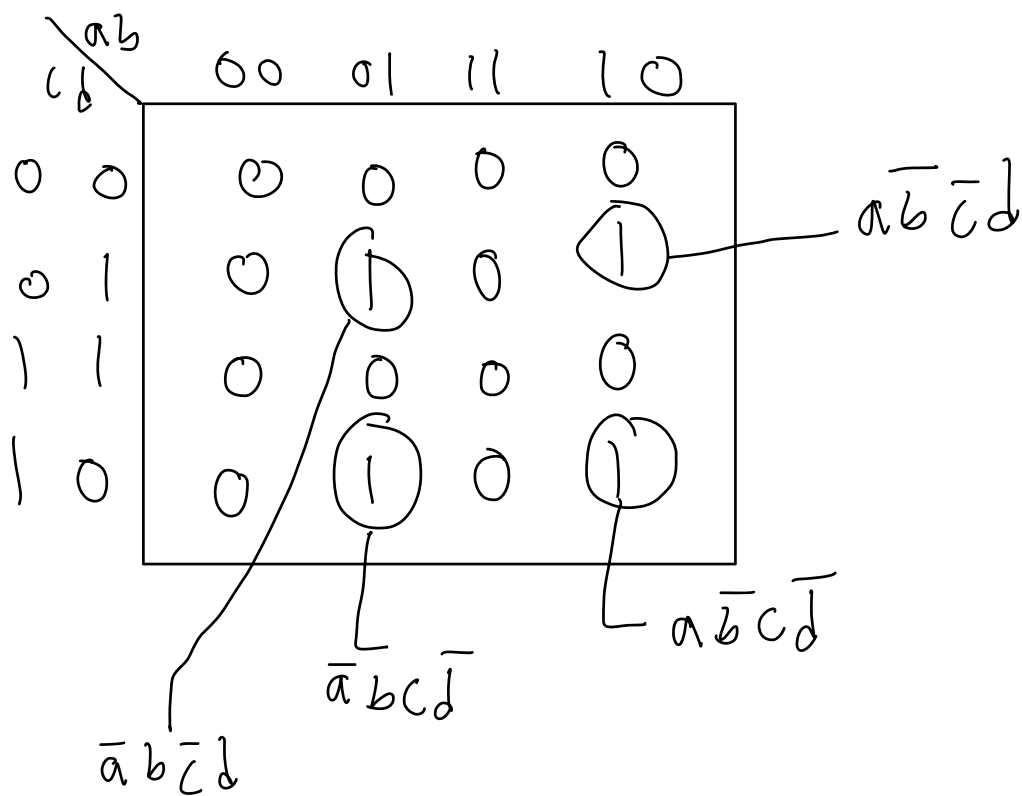
	<u>a</u>	<u>b</u>	<u>c</u>	<u>d</u>	<u>f</u>	
<u>0</u>	0	0	0	0	0	$(0 \oplus 0) \cdot (0 \oplus 0) = 0 \cdot 0 = 0$
<u>1</u>	0	0	0	1	0	$(0 \oplus 0) \cdot (0 \oplus 1) = 0 \cdot 1 = 0$
<u>2</u>	0	0	1	0	0	$(0 \oplus 0) \cdot (1 \oplus 0) = 0 \cdot 1 = 0$
<u>3</u>	0	0	1	1	0	$(0 \oplus 0) \cdot (1 \oplus 1) = 0 \cdot 0 = 0$
<u>4</u>	0	1	0	0	0	$(0 \oplus 1) \cdot (0 \oplus 0) = 1 \cdot 0 = 0$
<u>5</u>	0	1	0	1	1	$(0 \oplus 1) \cdot (0 \oplus 1) = 1 \cdot 1 = 1$
<u>6</u>	0	1	1	0	1	$(0 \oplus 1) \cdot (1 \oplus 0) = 1 \cdot 1 = 1$
<u>7</u>	0	1	1	1	0	$(0 \oplus 1) \cdot (1 \oplus 1) = 1 \cdot 0 = 0$
<u>8</u>	1	0	0	0	0	$(1 \oplus 0) \cdot (0 \oplus 0) = 1 \cdot 0 = 0$
<u>9</u>	1	0	0	1	1	$(1 \oplus 0) \cdot (0 \oplus 1) = 1 \cdot 1 = 1$
<u>10</u>	1	0	1	0	1	$(1 \oplus 0) \cdot (1 \oplus 0) = 1 \cdot 1 = 1$
<u>11</u>	1	0	1	1	0	$(1 \oplus 0) \cdot (1 \oplus 1) = 1 \cdot 0 = 0$
<u>12</u>	1	1	0	0	0	$(1 \oplus 1) \cdot (0 \oplus 0) = 0 \cdot 0 = 0$
<u>13</u>	1	1	0	1	0	$(1 \oplus 1) \cdot (0 \oplus 1) = 0 \cdot 1 = 0$
<u>14</u>	1	1	1	0	0	$(1 \oplus 1) \cdot (1 \oplus 0) = 0 \cdot 1 = 0$
<u>15</u>	1	1	1	1	0	$(1 \oplus 1) \cdot (1 \oplus 1) = 0 \cdot 0 = 0$

b)

$$(1's) \text{ SOP: } f(a,b,c,d) = \sum m(5,6,9,10)$$

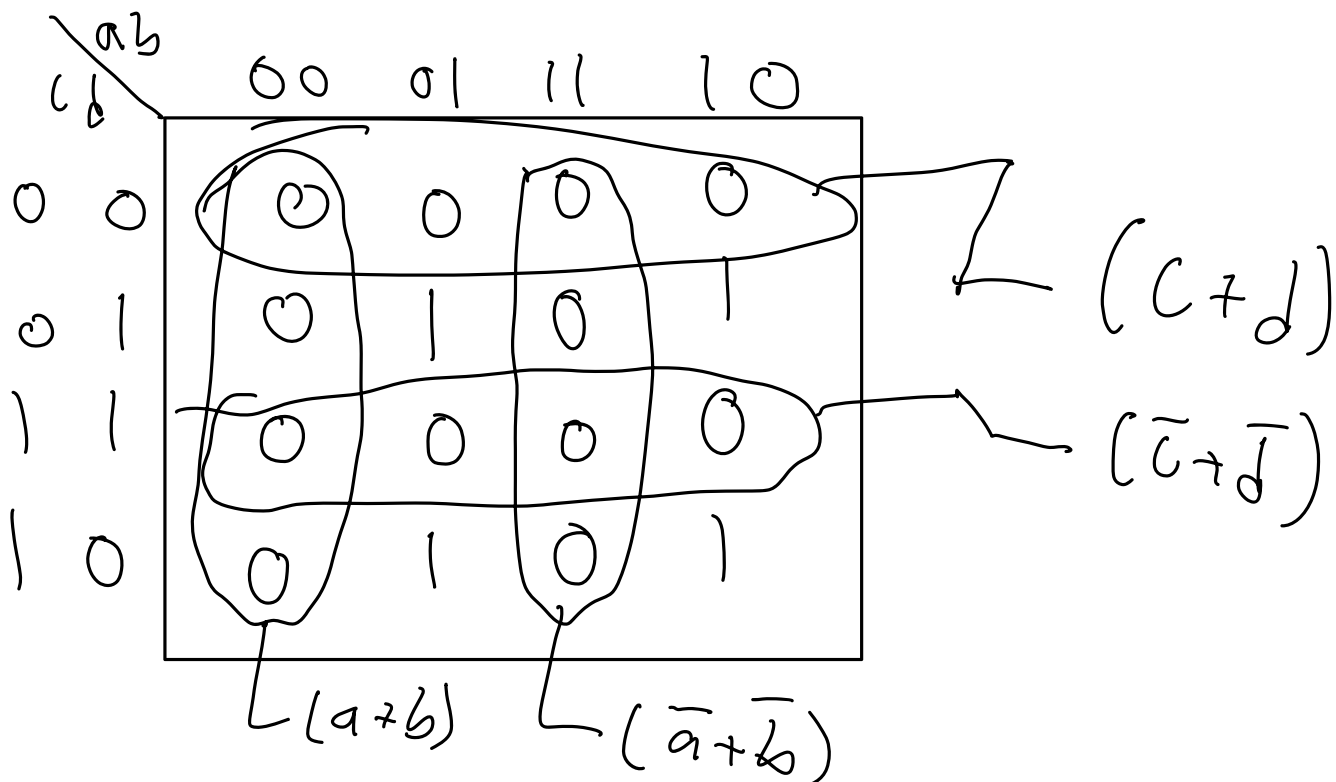
$$(0's) \text{ POS: } f(a,b,c,d) = \prod M(1,2,3,4,7,8,11,12,13,14,15)$$

SOP: 0101, 0110, 1001, 1010



SOP: $\bar{a}\bar{b}\bar{c}d + \bar{a}b\bar{c}d + \bar{a}b\bar{c}\bar{d} + a\bar{b}\bar{c}d$

Answer

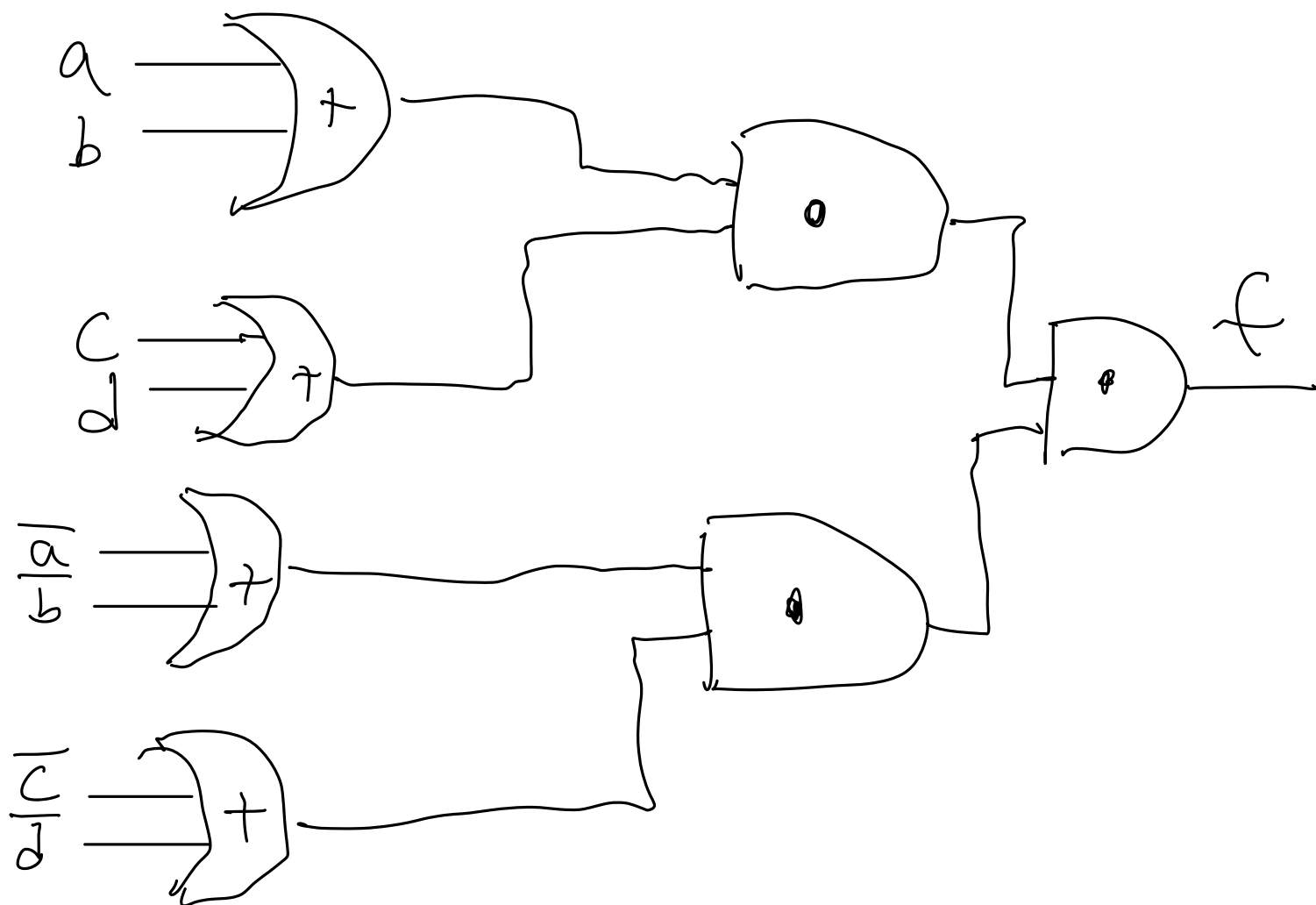


POS: $(a+b) \cdot (c+d) \cdot (\bar{a}+\bar{b}) \cdot (\bar{c}+\bar{d})$

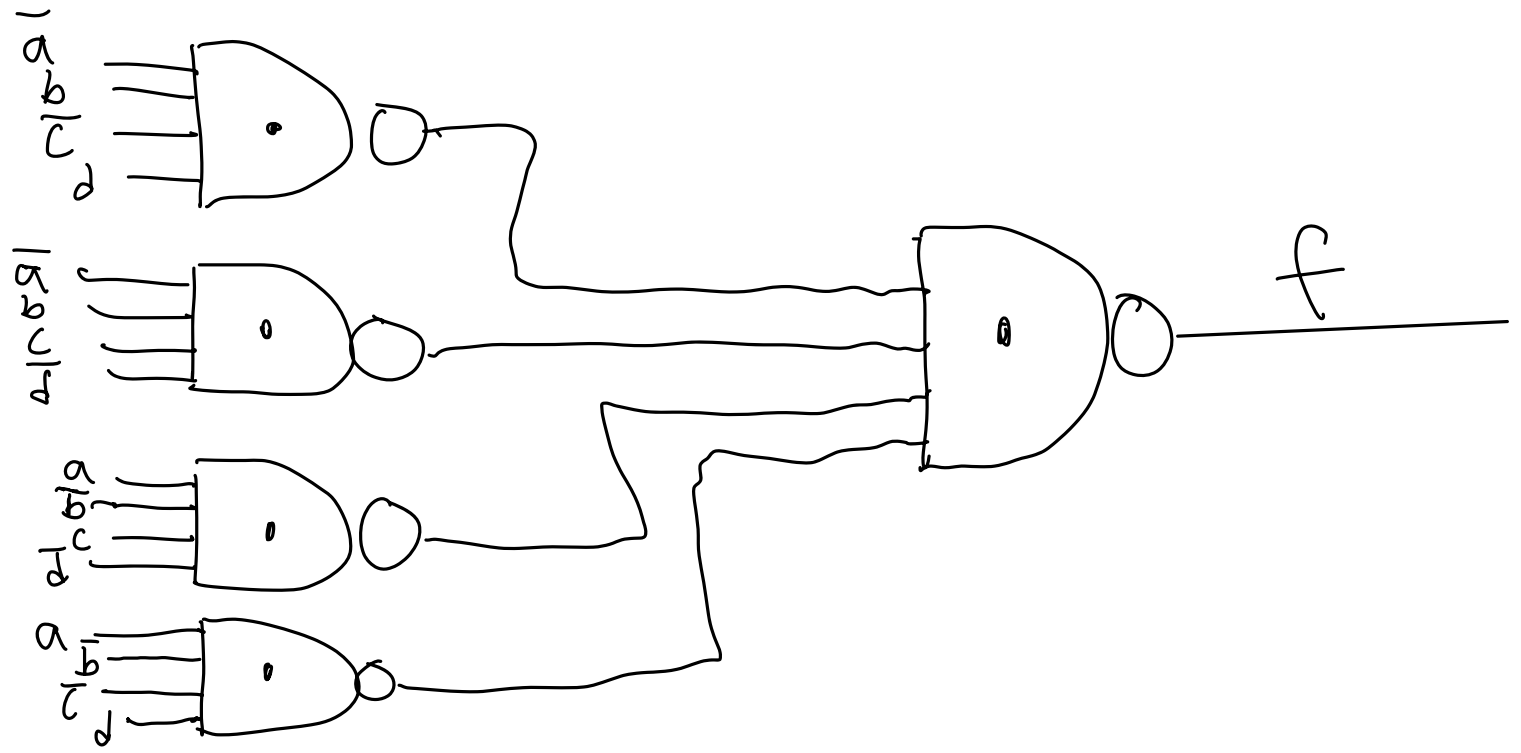
Answer

3

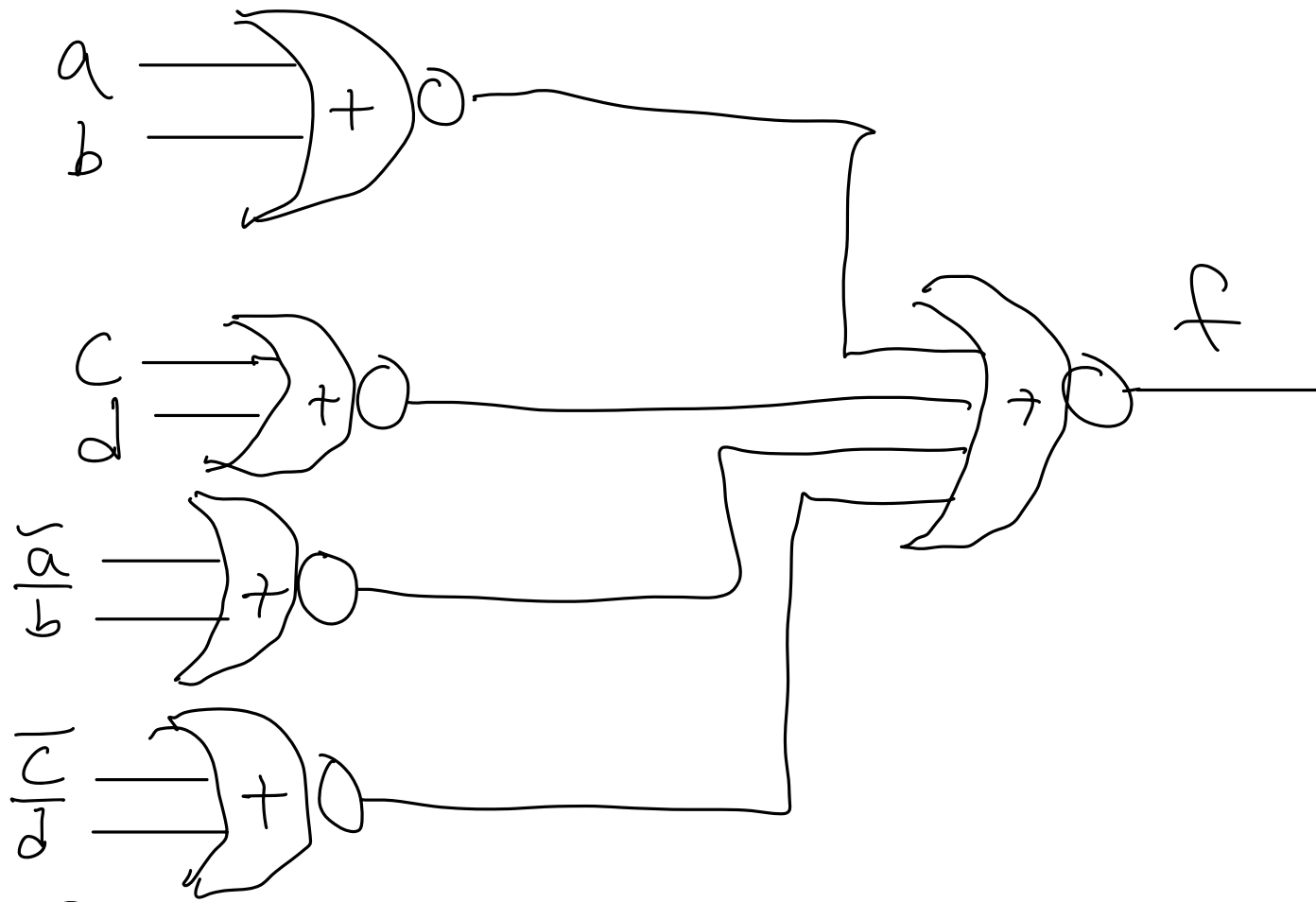
c)



d) $f = \bar{a}b\bar{c}d + \bar{a}bc\bar{d} + a\bar{b}c\bar{d} + a\bar{b}\bar{c}d$



e)



4) $f(a,b,c,d) = acd + bd' + a'c'd + ab'cd + a'b'cd'$

	a	b	c	d	f	f'
0	0	0	0	0	0	1
1	0	0	0	1	1	0
2	0	0	1	0	1	0
3	0	0	1	1	0	1
4	0	1	0	0	1	0
5	0	1	0	1	1	0
6	0	1	1	0	1	0
7	0	1	1	1	0	1
8	1	0	0	0	0	1
9	1	0	0	1	0	1
10	1	0	1	0	0	1
11	1	0	1	1	1	0
12	1	1	0	0	1	0
13	1	1	0	1	0	1
14	1	1	1	0	1	0
15	1	1	1	1	1	0

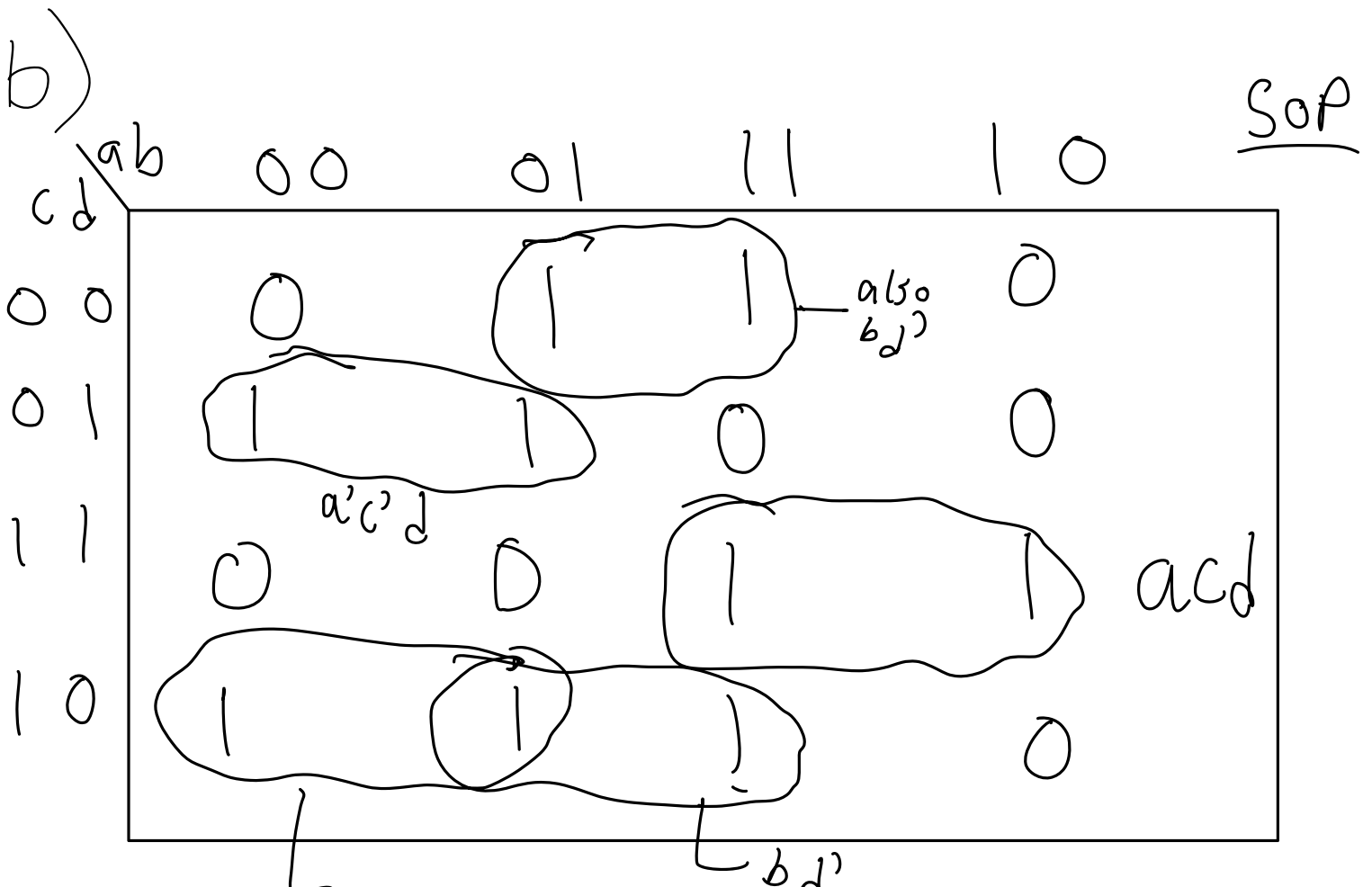
a)

f min term: $\sum m(1, 2, 4, 5, 6, 11, 12, 14, 15)$

f' min term: $\sum m(0, 3, 7, 8, 9, 10, 13)$

f max term: $\prod M(0, 3, 7, 8, 9, 10, 13)$

f' max term: $\prod M(1, 2, 4, 5, 6, 11, 12, 14, 15)$



$$a'c'd'$$

FOR $f \rightarrow$

$$\text{SOP} \rightarrow bd' + a'c'd + acd + a'cd'$$

$$(b+c+d)$$

		<u>POS</u>			
cd \ ab		00	01	11	10
00		0	1	1	0
01		1	1	0	0
11		0	0	1	1
10		1	1	1	0

$$(a+c+d)$$

FOR $f \rightarrow$

$$(\bar{a}+c+\bar{d}) \quad (\bar{a}+b+d)$$

$$\text{POS} \rightarrow (b+c+d) \cdot (a+\bar{c}+\bar{d}) \cdot (\bar{a}+c+\bar{d}) + (\bar{a}+b+d)$$

For f' ↓

$$\text{SOP: } \bar{b}\bar{c}\bar{d} + \bar{a}cd + a\bar{c}d + a\bar{b}\bar{d}$$

For f' ↓

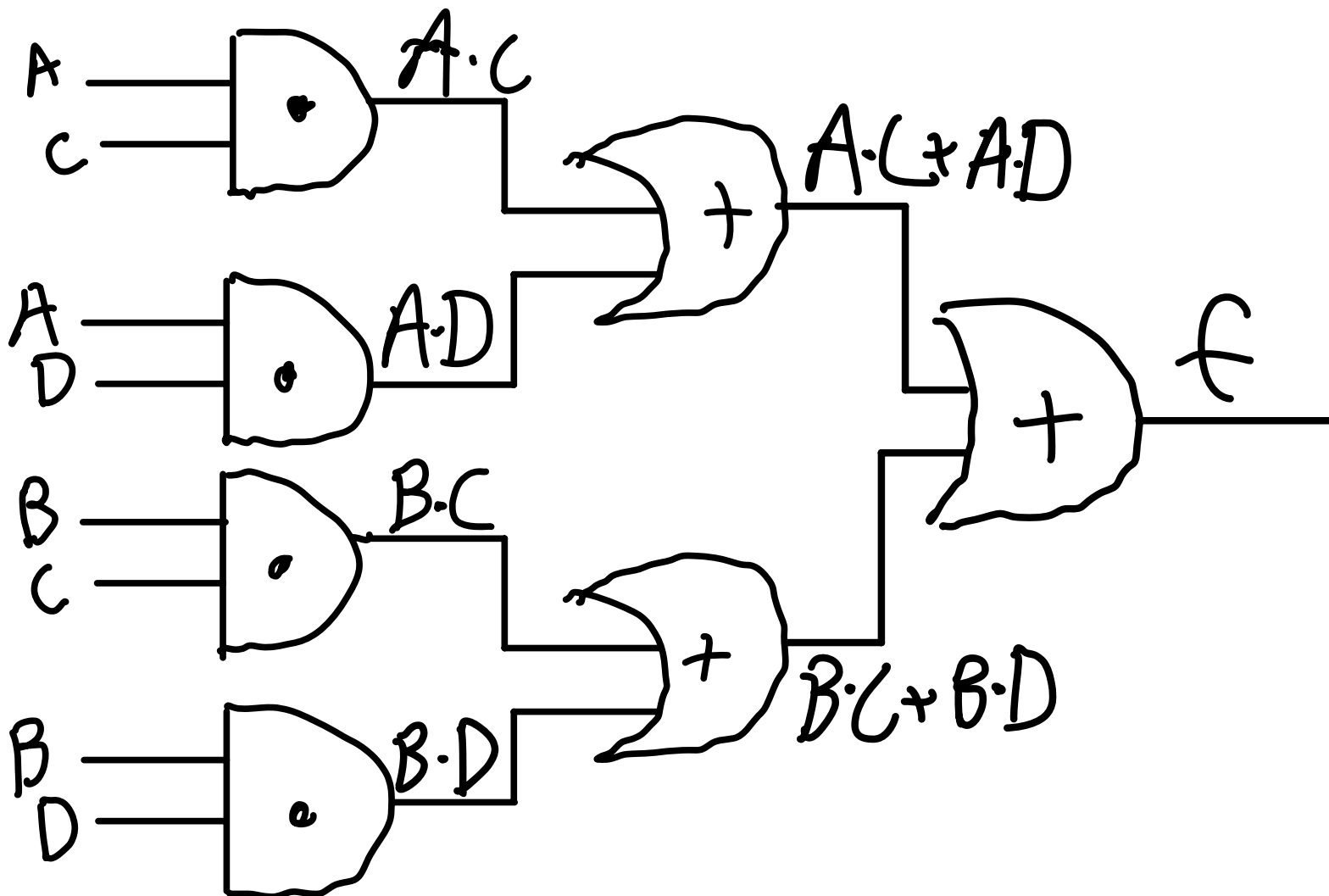
$$\text{POS: } (\bar{b} + d) \cdot (a + c + \bar{d}) \cdot (\bar{a} + \bar{c} + \bar{d}) \cdot (a + \bar{c} + d)$$

⑤

a) And-or S o p

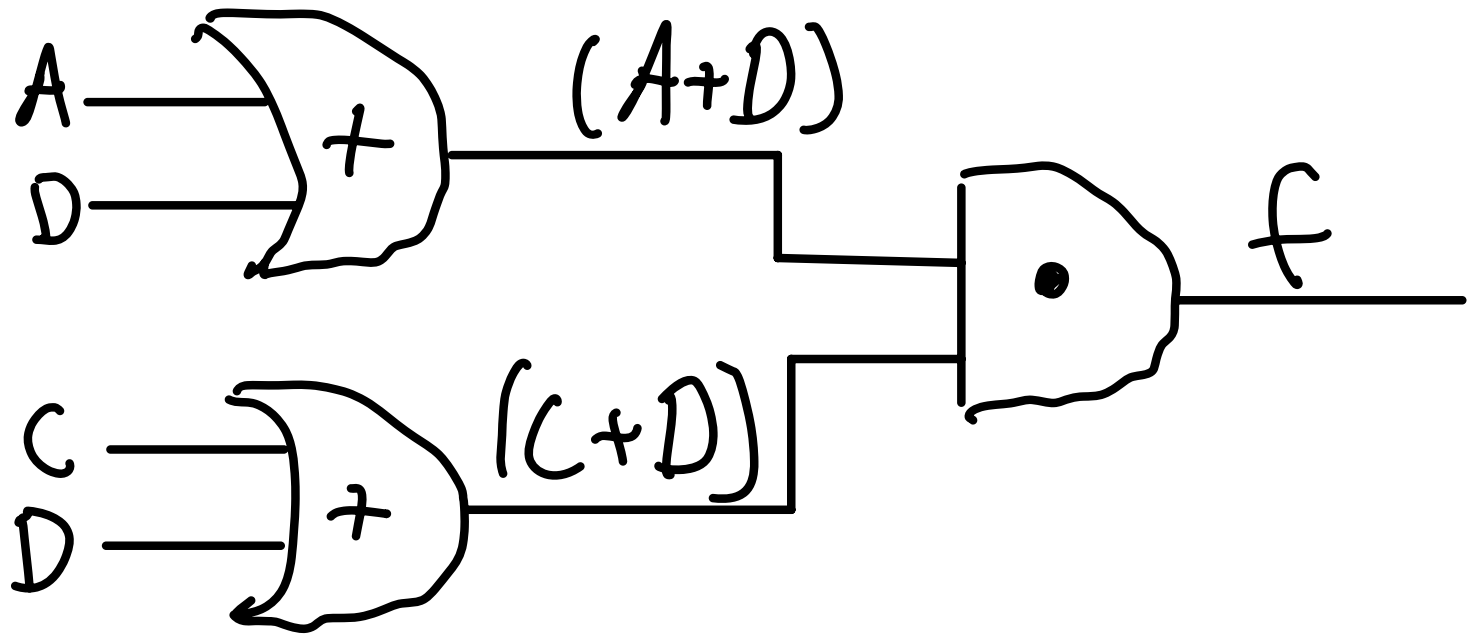
$$f = (A+B)(C+D)$$

$$= AC + AD + BC + BD$$

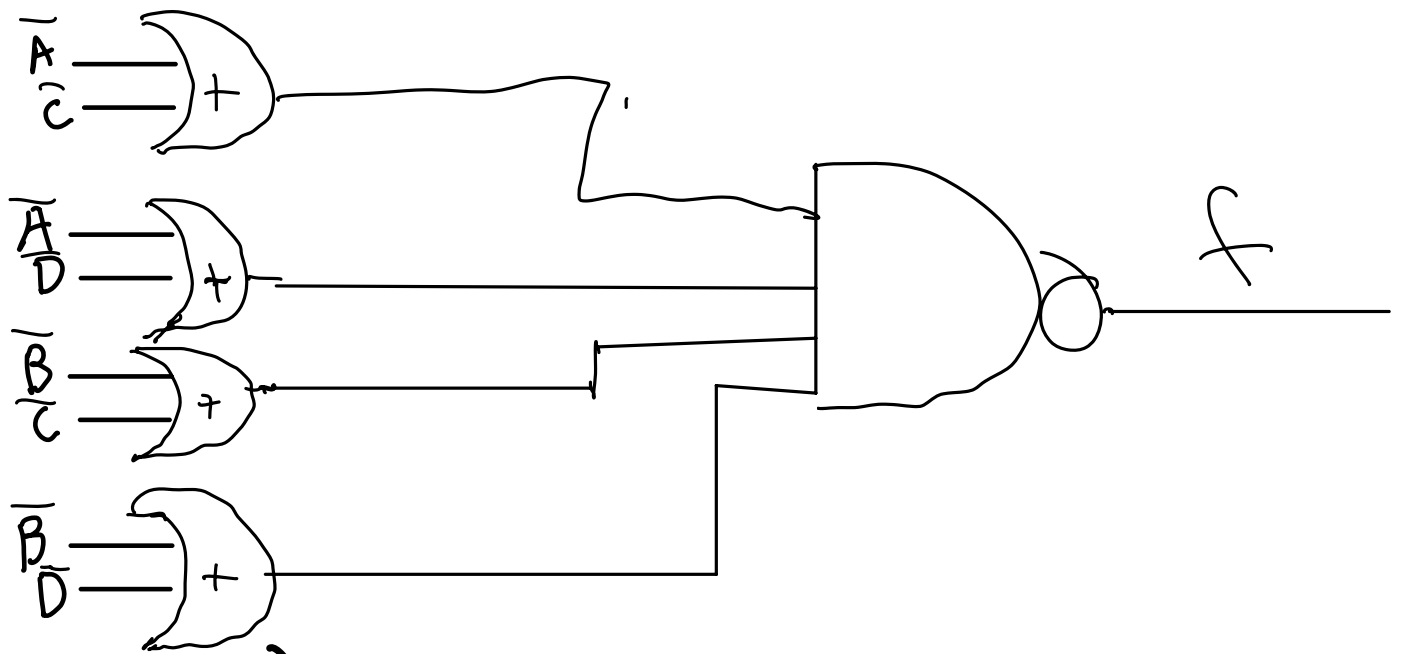


$$b) f = (A + D) \cdot (C + D)$$

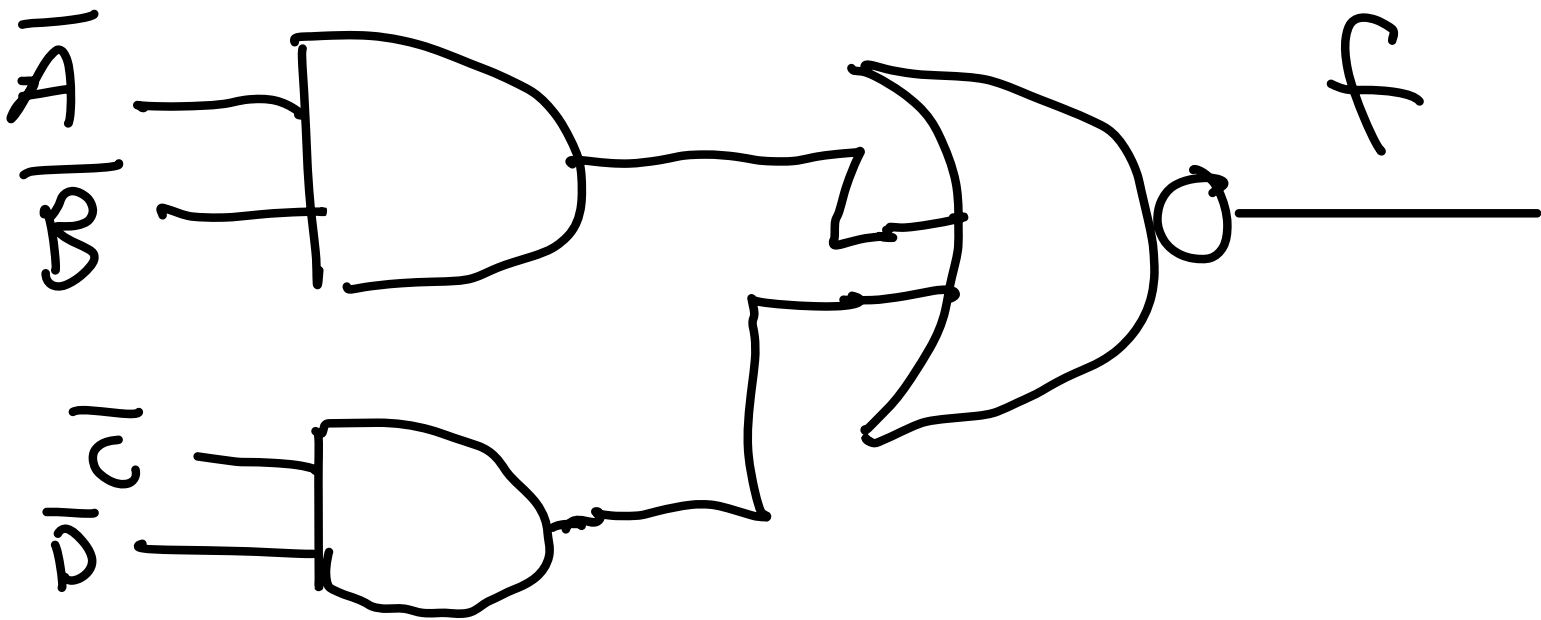
OR-AND



c) OR-NAND



d) AND-NOR



e) NOR-OR

