Homework 1

1

a) $A'B + B'C + C'A = AB' + BC' + CA'$

A	B	C	A'	B'	C'	$A'B + B'C + C'A$
0	0	1	1	1	0	0 + 1 + 0 = 1
0	1	0	1	0	1	1 + 0 + 0 = 1
0	1	1	1	0	0	1 + 0 + 0 = 1
1	0	0	0	1	1	0 + 0 + 1 = 1
1	0	1	0	1	0	0 + 1 + 0 = 1
1	1	0	0	0	1	0 + 0 + 1 = 1
1	1	1	0	0	0	0 + 0 + 0 = 0

A	B	C	A'	B'	C'	$AB' + BC' + CA'$
0	0	1	1	1	0	0 + 0 + 1 = 1
0	1	0	1	0	1	0 + 1 + 0 = 1
0	1	1	1	0	0	0 + 0 + 1 = 1
1	0	0	0	1	1	1 + 0 + 0 = 1
1	0	1	0	1	0	1 + 0 + 0 = 1
1	1	0	0	0	1	0 + 1 + 0 = 1
1	1	1	0	0	0	0 + 0 + 0 = 0

* Equality is valid for both sides of the equation.

b) $x(y \oplus z) = xy \oplus xz$

X	Y	Z	X(Y⊕Z)
0	0	1	0(1) = 0
0	1	0	0(1) = 0
0	1	1	0(0) = 0
1	0	0	1(0) = 0
1	0	1	1(1) = 1
1	1	0	1(1) = 1
1	1	1	1(0) = 0

X	Y	Z	XY⊕XZ
0	0	1	0⊕0 = 0
0	1	0	0⊕0 = 0
0	1	1	0⊕0 = 0
1	0	0	0⊕0 = 0
1	0	1	0⊕1 = 1
1	1	0	1⊕0 = 1
1	1	1	1⊕1 = 0

* Equality is valid for both sides of the equation.

(2)

a)
$$\begin{array}{r} {\scriptstyle 1}\\ 234\\ 1221\\ +2205\\ \hline 3663 \end{array}$$

$4+5+1=10$

3 left over

must be

base 7 ← Answer

$10-3=7$

↑ sum ↑ remaining

b) $6 \cdot 6 = 36$

multiple of 36 must be base 9, 4, 6, 3, 12, 18

$116 \times 76 = 10070$

10070

$(7a+6)(a+a^2+6) = a^4+7a$

$a=6,\ (7(6)+6)\cdot(6+6^2+6) = 48 \cdot 48,$ $a^4+7a = 6^4+6^2=1$

wrong base!

$a=9,\ (7(9)+6)(9+81+6) = (69)(96) = \underline{6624},\ 9^4+7a = \underline{6624}$

$a=9$

holds true,

$(7a+6)(a+a^2+6) = a^4+7a$ for

base 9

c) $123 \rightarrow a^2+2a+3$

$456 \rightarrow 4a^2+5a+6$

$123 \times 456 = 56088 \rightarrow 5a^4+6a^3+8a+8$

$3 \times 6 = 18$ after carry out is $\underline{8}$

Must be $18 - 8$ which 10 then base is 10's multiple

$a = 2, 5$

✗ $a = 2, (\underbrace{2^2 + 2(2) + 3}_{11}) \cdot (\underbrace{4 \cdot 2^2 + 5(2) + 6}_{32}) = 352$

verify: $5(2^4) + 6(2^3) + 8(2) + 8 = 152$

base is not 2

✗ $a = 5, (\underbrace{5^2 + 2(5) + 3}_{38}) \cdot (\underbrace{4 \cdot 5^2 + 5(5) + 6}_{131}) = 169$

verify: $5(5^4) + 6(5^3) + 8(5) + 8 \neq 169$

✗ $a = 10, (\underbrace{10^2 + 2(10) + 3}_{123}) \cdot (\underbrace{4 \cdot 10^2 + 5(10) + 6}_{456}) = \underline{56088}$

verify: $5(10^4) + 6(10^3) + 8(10) + 8 = \underline{56088}$

Base = 10 holds true for multiple of 10 after carry

d)

578	2	0
289	2	1
144	2	0
72	2	0
36	2	0
18	2	0
9	2	1
4	2	0
2	2	0
1	2	1

$578_{10} = (1001000010)_2$

$(-578)_{10} = (1\,1001000010)_2$ ← sign

1's compl: $(1\,0110111101)_2$ ← sign

↓ +1

2's compl: $(1\,0110111110)_2$ $(-578)_{10}$

0 1 1 0 1 1 1 1 0 1

\+ 1

1 0 1 1 0 1 1 1 1 0

↑ sign

1023	2	1
511	2	1
255	2	1
127	2	1
63	2	1
31	2	1
15	2	1
7	2	1
3	2	1
1	2	1

$$1023_{10} = (1111111111)_2$$

$$-1023_{10} = (\underbrace{1}_{\text{sign}}\,1111111111)_2$$

1's compl: $(\underbrace{1}_{\text{sign}}\,0000000000)_2$

+1

2's compl: $(\underbrace{1}_{\text{sign}}\,0000000001)_2$

$(-1023)_{10}$

$(2048)_{10}$

2048	2	0
1024	2	0
512	2	0
256	2	0
128	2	0
64	2	0
32	2	0
16	2	0
8	2	0
4	2	0
2	2	0
1	2	1

$(2048)_{10} = (100000000000)_2$

1's compl: $(011111111111)_2$

$$\begin{array}{r} 011111111111 \\ + \qquad\qquad 1 \\ \hline 100000000000 \end{array}$$

2's compl: $(100000000000)_2$
(2048)

e)

$$2+10+16+\frac{1}{2}+\frac{1}{10}+\frac{1}{16}=28.6625$$

$(28.6625)_{10}$ to $(?)_{16}$

28	16	C
1	16	1

C MSB ↑ 1C

$$0.6625 \times 16 = 13.9750 - 0.6625 = 10.0000$$

A

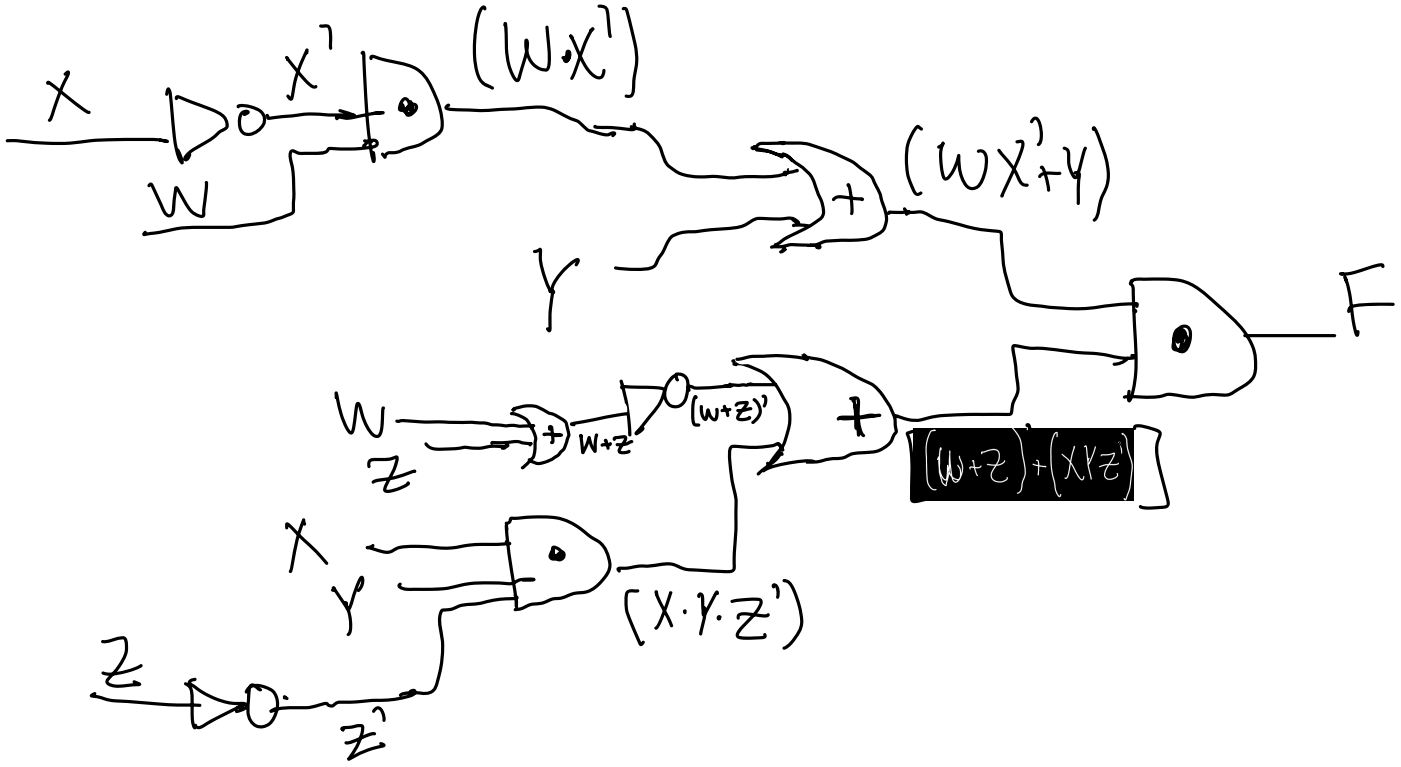
$$0.6 \times 16 = 9.6 \quad (9)$$

$$0.6 \times 16 = 9.6 \quad (9)$$

0.A99

$$(28.6625)_{10} = (1C.A99)_{16}$$

3 $F=(WX'+Y)\left[(W+Z)'+(XYZ')\right]$

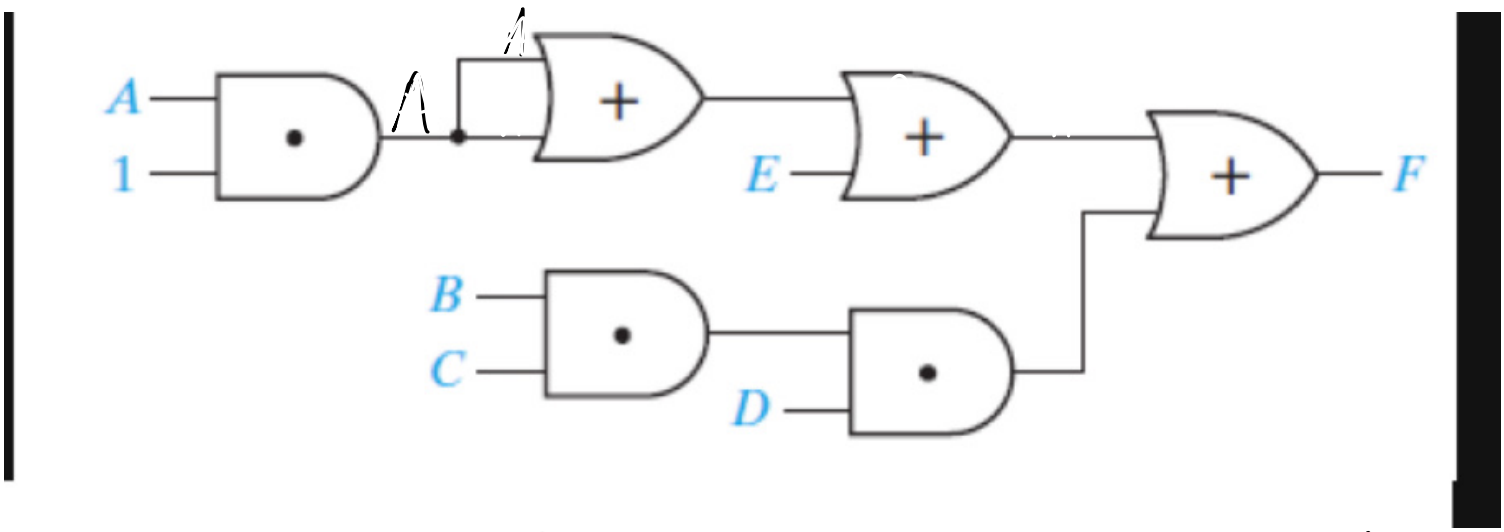


AND : D

OR : +

NOT : —▷○—

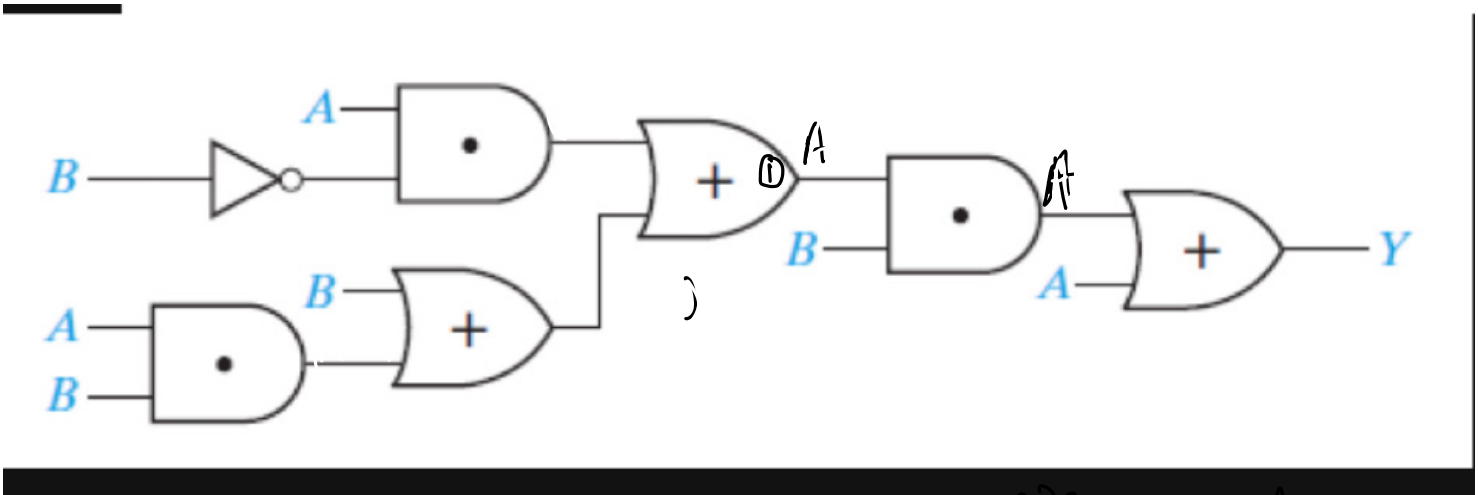
4



$F = (A + E) + (B \cdot C \cdot D)$

$P_1: A + A + E = A + E$

Answer

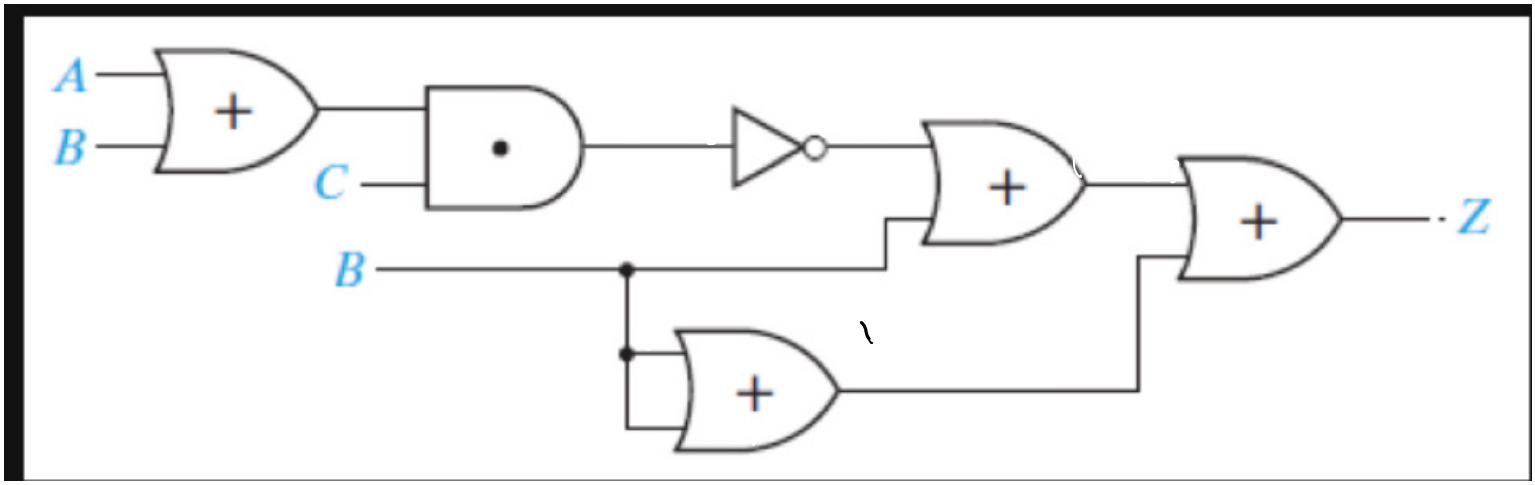


$P_1: (A \cdot B') + (A \cdot B) + B = A(B + B') + B = A + B$

$P_2: B(A + B) = AB + BB = AB + B$

$Y = AB + B + A = A(B + 1) + B = B(A + 1) + A$

$Y = A + B$ Answer



P1: $B + B = B$

$$Z = (AC + BC)' + B$$

Answer