Homework 1

1

a) $A'B + B'C + C'A = AB' + BC' + CA'$

A	B	C	A'	B'	C'	$A'B + B'C + C'A$	
0	0	0	1	1	1	$0+0+0=$	0
0	0	1	1	1	0	$0+1+0=$	1
0	1	0	1	0	1	$1+0+0=$	1
0	1	1	1	0	0	$1+0+0=$	1
1	0	0	0	1	1	$0+0+1=$	1
1	0	1	0	1	0	$0+1+0=$	1
1	1	0	0	0	1	$0+0+1=$	1
1	1	1	0	0	0	$0+0+0=$	0

A	B	C	A'	B'	C'	$AB' + BC' + CA'$	
0	0	0	1	1	1	$0+0+0=$	0
0	0	1	1	1	0	$0+0+1=$	1
0	1	0	1	0	1	$0+1+0=$	1
0	1	1	1	0	0	$0+0+1=$	1
1	0	0	0	1	1	$1+0+0=$	1
1	0	1	0	1	0	$1+0+0=$	1
1	1	0	0	0	1	$0+1+0=$	1
1	1	1	0	0	0	$0+0+0=$	0

* Equality is valid for both sides of the equation.

b) $X(Y \oplus Z) = XY \oplus XZ$

X	Y	Z	$X(Y \oplus Z)$	
0	0	0	$0(0) =$	0
0	0	1	$0(1) =$	0
0	1	0	$0(1) =$	0
0	1	1	$0(0) =$	0
1	0	0	$1(0) =$	0
1	0	1	$1(1) =$	1
1	1	0	$1(1) =$	1
1	1	1	$1(0) =$	0

X	Y	Z	$XY \oplus XZ$	
0	0	0	$0 \oplus 0 =$	0
0	0	1	$0 \oplus 0 =$	0
0	1	0	$0 \oplus 0 =$	0
0	1	1	$0 \oplus 0 =$	0
1	0	0	$0 \oplus 0 =$	0
1	0	1	$0 \oplus 1 =$	1
1	1	0	$1 \oplus 0 =$	1
1	1	1	$1 \oplus 1 =$	0

$1 \oplus 1 =$

* Equality is valid for both sides of the equation.

2

a)

$$
\begin{array}{r}
\overset{1}{2}34 \\
1221 \\
+\,2205 \\
\hline
3663
\end{array}
$$

$4+5+1 = 10$

3 left over

must be

$\boxed{\text{base } 7}$ ← Answer

$10 - 3 = 7$

sum remaining

b)

$6 \cdot 6 = 36$

multiple of 36
must be base
9, 4, 6, 3, 12, 18

$116 \times 76 = 10070$

10070

$(7a+6)(a+a^2+6) = a^4+7a$

$a=6$, $(7(6)+6)\cdot(6+6^2+6) = 48 \cdot 48$, $a^4+7a = 6^4+63=1$

wrong base!

$a=9$, $(7(9)+6)(9+81+6) = (69)(96) = \underline{6624}$, $9^4+7a = \underline{6624}$

a=9

holds true,

$(7a+6)(a+a^2+6) = a^4+7a$ for

$\boxed{\text{base } 9}$

(c)

$123 \rightarrow a^2+2a+3$

$456 \rightarrow 4a^2+5a+6$

$123 \times 456 = 56088 \rightarrow 5a^4+6a^3+8a+8$

$3 \times 6 = 18$ after carry out is $\underline{8}$

Must be $18 - 8$ which 10 then base is 10's multiple

$a = 2, 5$

$\times$ $a = 2$, $(\underbrace{2^2 + 2(2) + 3}_{11}) \cdot (\underbrace{4 \cdot 2^2 + 5(2) + 6}_{32}) = 352$

verify: $5(2^4) + 6(2^3) + 8(2) + 8 = 152$

base is not 2

$\times$ $a = 5$, $(\underbrace{5^2 + 2(5) + 3}_{38}) \cdot (\underbrace{4 \cdot 5^2 + 5(5) + 6}_{131}) = 169$

verify: $5(5^4) + 6(5^3) + 8(5) + 8 \neq 169$

$\star$ $a = 10$, $(\underbrace{10^2 + 2(10) + 3}_{123}) \cdot (\underbrace{4 \cdot 10^2 + 5(10) + 6}_{456}) = \underline{56088}$

verify: $5(10^4) + 6(10^3) + 8(10) + 8 = \underline{56088}$

$\boxed{\text{Base} = 10 \text{ holds true for multiple of } 10 \text{ after carry}}$

d)

578	2	0
289	2	1
144	2	0
72	2	0
36	2	0
18	2	0
9	2	1
4	2	0
2	2	0
1	2	1

$578_{10} = (1001000010)_2$

$(-578)_{10} = (\underline{1}1001000010)_2$ ← sign

1's compl: $(\underline{1}0110111101)_2$ ← sign

+1

$\boxed{\text{2's compl: } (-578)_{10} : (10110111110)_2}$

$$
\begin{array}{r}
{}^{1} \\
0110111101 \\
+1 \\
\hline
\underline{1}\,0110111110
\end{array}
$$

↑ sign

1023	2	1
511	2	1
255	2	1
127	2	1
63	2	1
31	2	1
15	2	1
7	2	1
3	2	1
1	2	1

$$1023_{10} = (1111111111)_2$$

$$-1023_{10} = (\underline{1}\,1111111111)_2$$

↑sign

1's compl: $(\underline{1}\,0000000000)_2$

↑sign

+1

$$\boxed{\text{2's compl: } (\underline{1}\,0000000001)_2 \quad (-1023)_{10}}$$

↑sign

$(2048)_{10}$

2048	2	0
1024	2	0
512	2	0
256	2	0
128	2	0
64	2	0
32	2	0
16	2	0
8	2	0
4	2	0
2	2	0
1	2	1

Assuming 12 bit word

$(2048)_{10} = (100000000000)_2$

1's compl: $(011111111111)_2$

$$\begin{array}{r} 011111111111 \\ +\qquad\qquad 1 \\ \hline 100000000000 \end{array}$$

$\boxed{\text{2's compl: } (100000000000)_2 \quad (2048)}$

e)

$$2+10+16+\frac{1}{2}+\frac{1}{10}+\frac{1}{16}=28.6625$$

$(28.6625)_{10}$ to $(?)_{16}$

28	16	C
1	16	1

$\uparrow$ C MSB

1C

$$\begin{array}{r} 0.6625 \\ \times \quad 16 \\ \hline 3.9750 \\ 06625- \\ \hline 10.60000 \end{array}$$

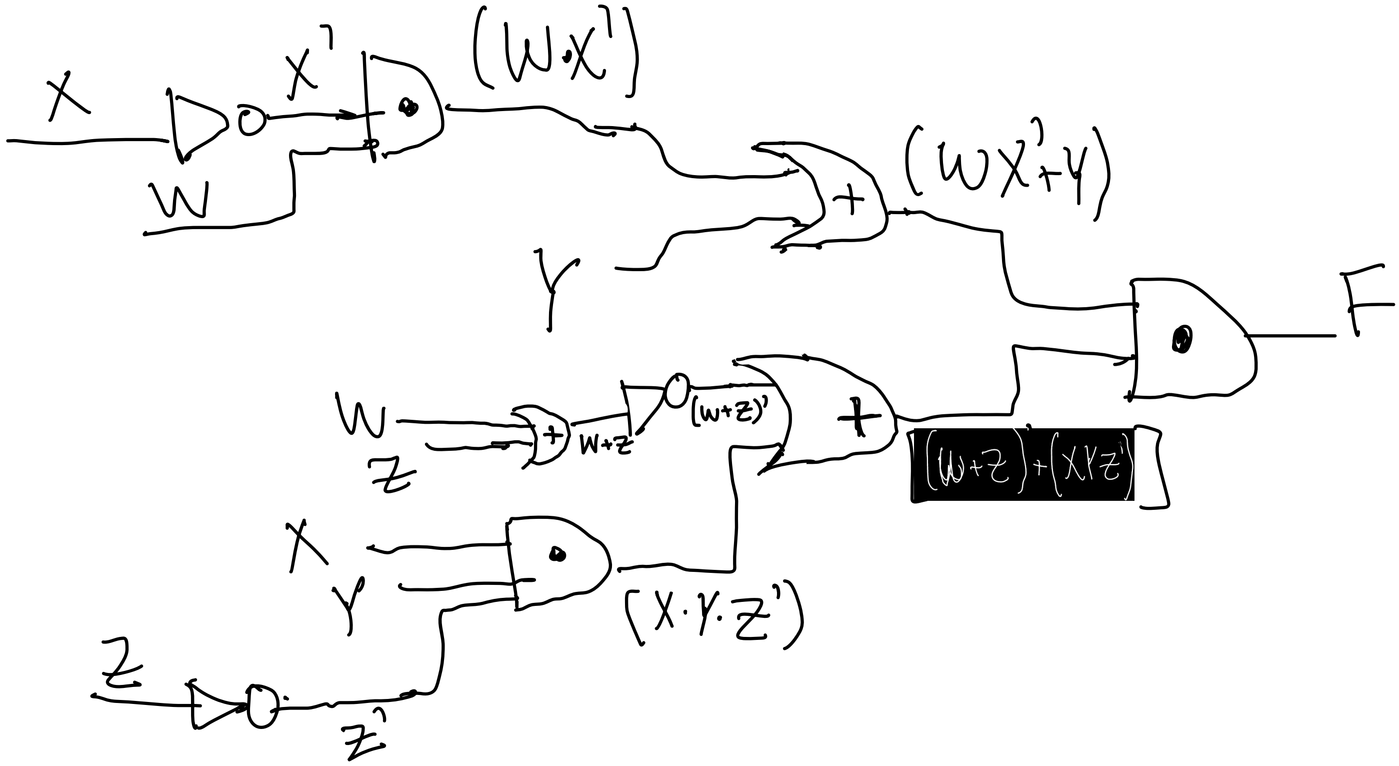
A

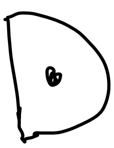
$$\begin{array}{r} 0.6 \\ 16 \\ \hline 36 \\ 06 \\ \hline 09.6 \end{array}
\qquad
\begin{array}{r} 0.6 \\ 16 \\ \hline 36 \\ 06 \\ \hline 09.6 \end{array}$$

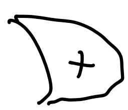
0.A99

$$\boxed{(28.6625)_{10}=(1C.A9\bar{9})_{16}}$$

③ $F = (WX' + Y)\left[(W+Z)' + (XYZ')\right]$

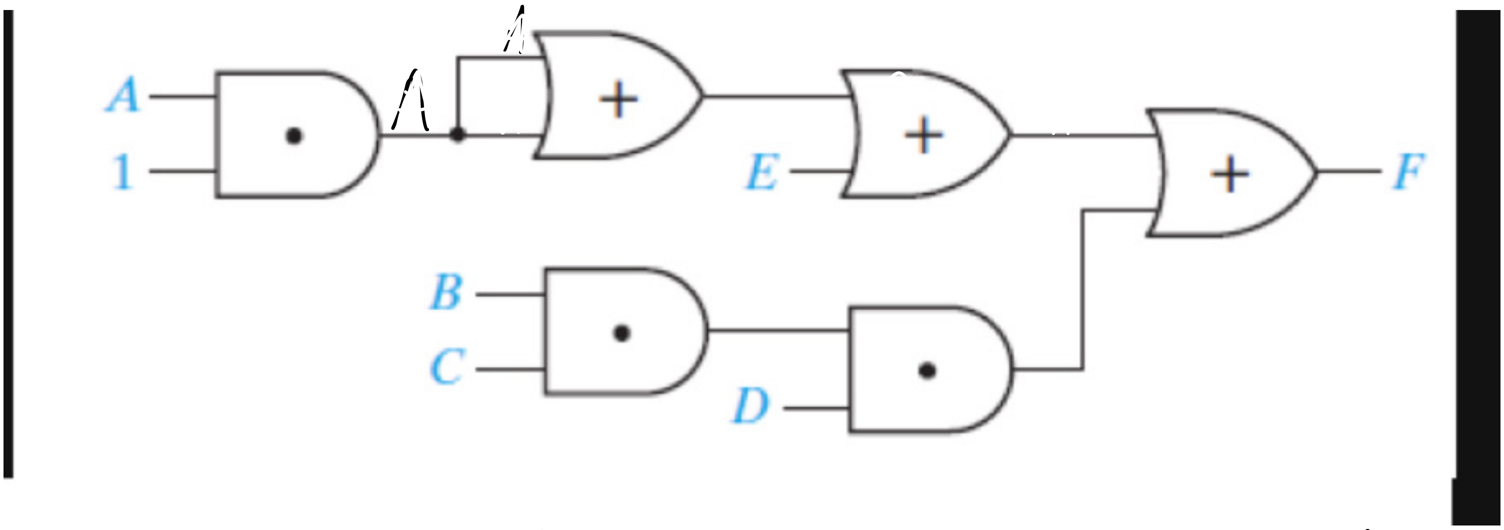


AND : 

OR : 

NOT : 

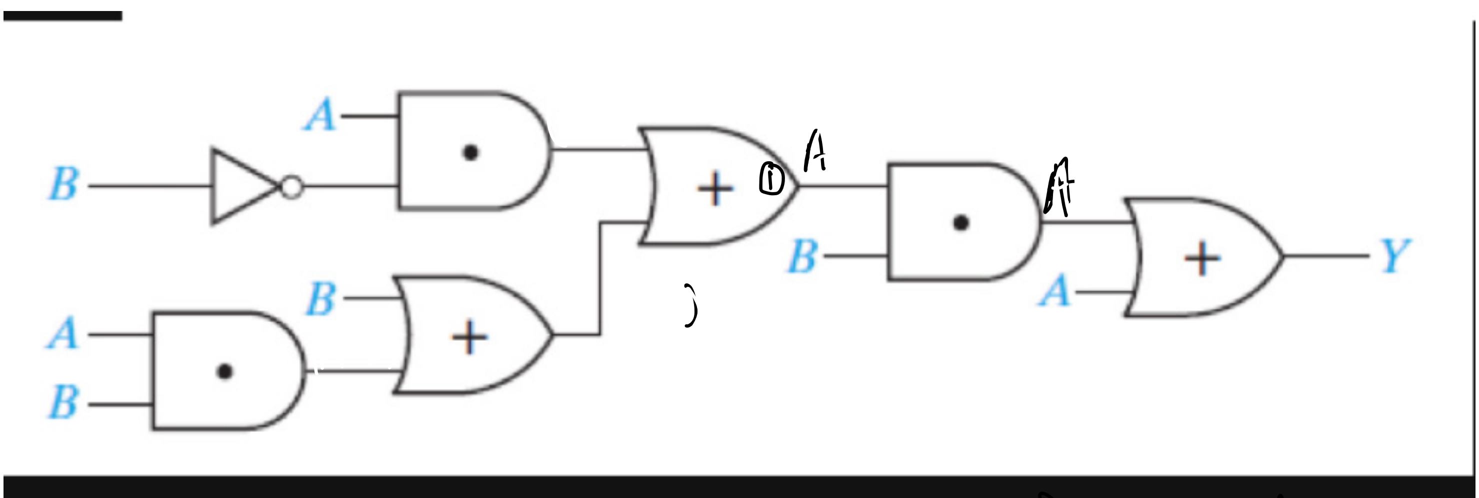
4



$$\boxed{F = (A + E) + (B \cdot C \cdot D)}$$

Answer

P_1: $A + A + E$
$= A + E$



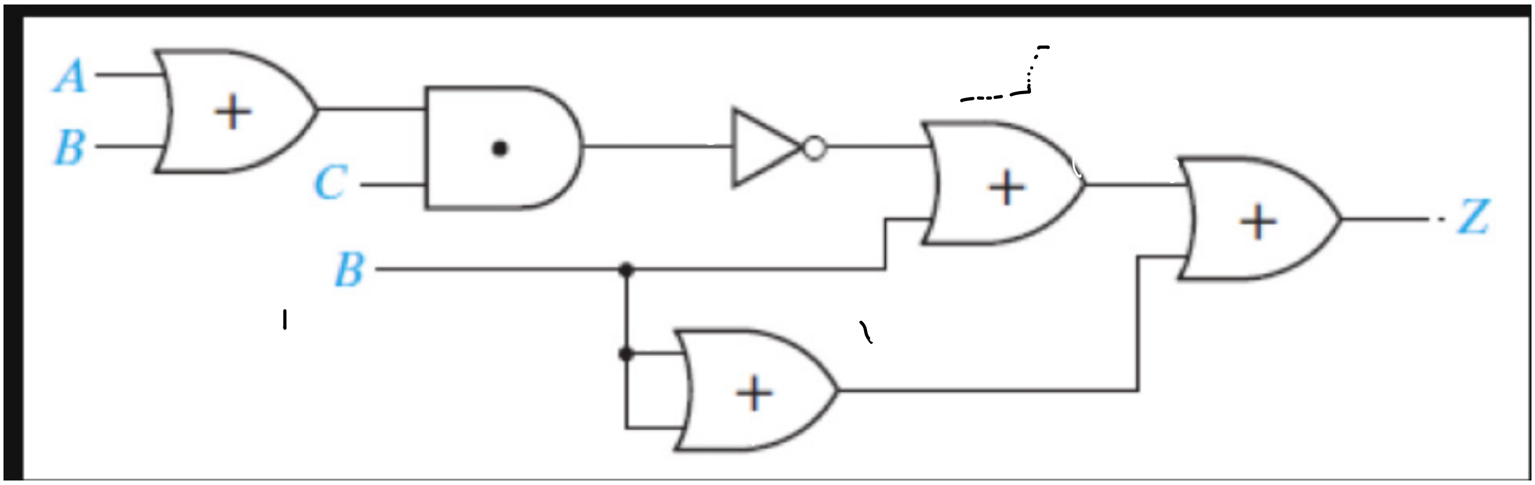
P_1: $(A \cdot B') + (A \cdot B) + B = A(B + B') + B = A + B$

P_2: $B(A + B) = AB + BB = AB + B$

$Y = AB + B + A = A(B + 1) + B = B(A + 1) + A$

$$\boxed{Y = A + B}$$

Answer



P_1: $B+B=B$

$$Z=(AC+BC)'+B.$$

$$Z=(\bar{A}+\bar{C})\cdot(\bar{B}+\bar{C})+B$$

$$Z=\bar{A}\bar{B}+\bar{A}\bar{C}+\bar{B}\bar{C}+\bar{C}+B$$

$$Z=\bar{A}\bar{B}+\bar{C}(\bar{A}+\bar{B}+1)+B=\bar{A}\bar{B}+\bar{C}+B$$

$$=\bar{A}\bar{B}+B+\bar{C}=\bar{A}+B+\bar{C}$$

$$\boxed{Z=B+A'+C'}$$ Answer