Homework # 1

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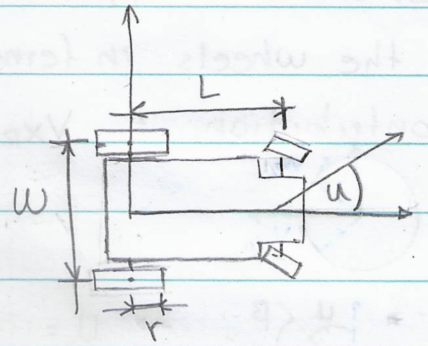
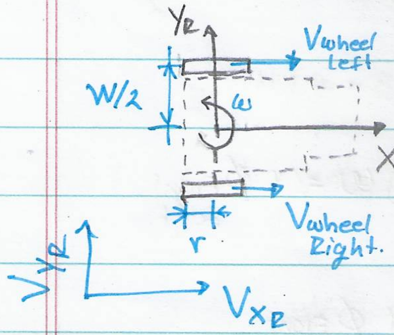


$r \to$ wheel radius
$u \to$ steering angle
$w \to$ distance between left and Right wheels
$L \to$ Distance between front and rear wheels.

$P_1 \to$ Validate that the kinematic constraints of the racecar are given as follows.

First take a look to the rear wheels configuration.



$\to$ to this configuration we can express the Rolling and sliding constraits. we wheel Project the velocity of the robot to the Velocity of Right and left rear wheels.

Constraints $V_{XR}\sin(\alpha+\beta) - V_{YR}\cos(\alpha+\beta) - L\cos(\beta)\omega = r\dot\phi$ / $V_{XR}\cos(\alpha+\beta) + V_{YR}\sin(\alpha+\beta) + L\sin\beta\,\omega = 0$

Rolling constraints $\to$ $V_{Xr} - \omega . \frac{W}{2} = r\,\dot\phi_{\substack{rear\\left}} = V_{\substack{wheel\\left}}$ (1)

$\alpha = \pi/2$
$\beta = 0$

$\alpha = -\pi/2$
$\beta = \pi$

Similarly : $V_{Xr} + \omega . \frac{W}{2} = r\,\dot\phi_{\substack{rear\\Right}} = V_{\substack{wheel\\Right}}$ (2)

For V_{YR} Projection we can find that $V_{YR} = 0$ (3)

(1) ∧ (2) ∧ (3) lead:

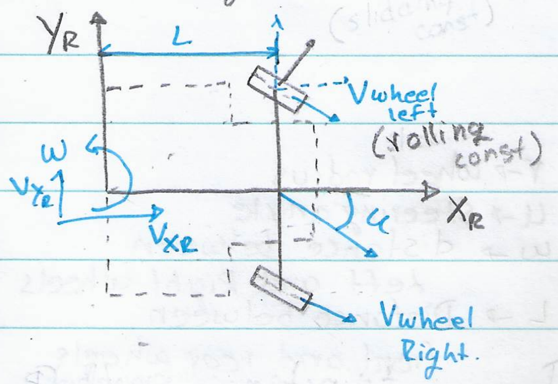
* For the rear left wheel $\to$ $(1 \quad 0 \quad -W/2) . (V_{XR} \quad V_{YR} \quad \omega) = r\,\dot\phi_{\substack{Rear\\left}}$

$(0 \quad 1 \quad 0) . (V_{XR} \quad V_{YR} \quad \omega) = 0$

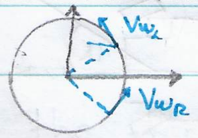
* For the rear Right wheel $\to$ $(1 \quad 0 \quad W/2) . (V_{XR} \quad V_{YR} \quad \omega) = r\,\dot\phi_{\substack{rear\\Right}}$

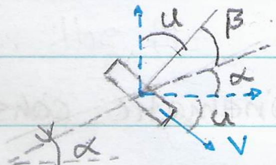
$(0 \quad 1 \quad 0) . (V_{XR} \quad V_{YR} \quad \omega) = 0$

For the front wheels we have the following configuration



Now, we can express the velocity of the wheels in terms of the contribution f V_{X_R}, V_{Y_R} and ω

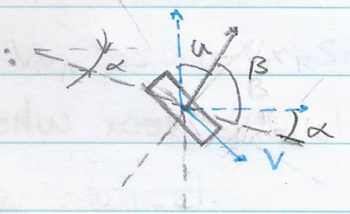




$\rightarrow u = \frac{\pi}{2} - \alpha - \beta$

For the Front Left wheel: $\boxed{\beta = \frac{\pi}{2} - \alpha + u}$

For the Front Right wheel:



$\beta = \frac{\pi}{2} - \alpha - u$

Rolling constraint Left wheel.

$$V_{X_R} \sin(\cancel{\alpha} + \tfrac{\pi}{2} - \cancel{\alpha} + u) - V_{Y_R} \cos(\cancel{\alpha} + \tfrac{\pi}{2} - \cancel{\alpha} + u) - \ell \cos(\tfrac{\pi}{2} - \alpha + u)\,\omega = r\,\dot{\phi}_{Front\ Left}$$

$$V_{X_R} \cos u + V_{Y_R} \sin u + \ell \sin(u - \alpha)\,\omega = r\,\dot{\phi}_{Front\ left}$$

$$V_{X_R} \cos u + V_{Y_R} \sin u + (\ell \sin u \cos\alpha - \ell \cos u \sin\alpha)\,\omega = r\,\dot{\phi}_{Front\ Left}$$

we have $\rightarrow \sin\alpha = \frac{W/2}{\ell}$ and $\cos\alpha = \frac{L}{\ell}$ $\rightarrow \ell = \frac{W/2}{\sin\alpha}$ $\ell = \frac{L}{\cos\alpha}$

then: $\ell \sin u \cos\alpha = \frac{L}{\cos\alpha} \sin u \cancel{\cos\alpha} = L \sin u$

$-\ell \cos u \sin\alpha = \frac{W/2}{\sin\alpha} \cos u \cancel{\sin\alpha} = W/2 \cos u$

finally $\rightarrow$ $\boxed{V_{X_R} \cos u + V_{Y_R} \sin u + (L \sin u - W/2 \cos u)\,\omega = r\,\dot{\phi}_{Front\ Left}}$ $\rightarrow$ Rolling constraint Front-left wheel.

Sliding constraint: $V_{X_R} \cos(\cancel{\alpha} + \frac{\pi}{2} - \cancel{\alpha} + u) + V_{Y_R} \sin(\cancel{\alpha} + \frac{\pi}{2} - \cancel{\alpha} + u) + \ell \sin(\frac{\pi}{2} - \alpha + u)\,\omega = 0$

$$-V_{X_R} \sin u + V_{Y_R} \cos u + \ell \cos(u - \alpha)\,\omega = 0$$

$$-V_{X_R} \sin u + V_{Y_R} \cos u + (\ell \cos u \cos\alpha + \ell \sin u \sin\alpha)\,\omega = 0$$

$$-V_{X_R} \sin u + V_{Y_R} \cos u + \left(\frac{L}{\cancel{\cos\alpha}} \cos u \cancel{\cos\alpha} + \frac{W/2}{\cancel{\sin\alpha}} \sin u \cancel{\sin\alpha}\right)\omega = 0$$

$\boxed{-V_{X_R} \sin u + V_{Y_R} \cos u + \left(L \cos u + \frac{W}{2} \sin u\right)\omega = 0}$ $\rightarrow$ Sliding constraint Front Left wheel

Now for the right-Front wheel:

Rolling constraint →

$$V_{x_R} \sin(\cancel{\alpha} + \tfrac{\pi}{2} - \cancel{\alpha} - u) - V_{y_R} \cos(\cancel{\alpha} + \tfrac{\pi}{2} - \cancel{\alpha} - u) - \ell \cos(\tfrac{\pi}{2} - \alpha - u)\,\omega = r\dot{\phi}_{\text{Front Right}}$$

$$V_{x_R} \cos u + V_{y_R} \sin u + \ell \sin(\alpha + u)\,\omega = r\dot{\phi}_{\text{Front Right}}$$

$$V_{x_R} \cos u + V_{y_R} \sin u + (\ell \sin u \cos\alpha + \ell \cos u \sin\alpha)\,\omega = r\dot{\phi}_{\text{Front Right}}$$

$$\rightarrow \ell = \frac{W/2}{\sin\alpha} \qquad \ell = \frac{L}{\cos\alpha}$$

$$V_{x_R} \cos u - V_{y_R} \sin u + \left(\frac{L}{\cancel{\cos\alpha}} \sin u \cancel{\cos\alpha} + \frac{W/2}{\cancel{\sin\alpha}} \cos u \cancel{\sin\alpha}\right)\omega = r\dot{\phi}_{\text{Front Right}}$$

$$\boxed{V_{x_R} \cos u + V_{y_R} \sin u + L \sin u\,\omega + \frac{W}{2} \cos u\,\omega = r\dot{\phi}_{\text{Front Right}}}$$ → Rolling constraint Front Right wheel

Slidding constraint

$$V_{x_r} \cos(\cancel{\alpha} + \tfrac{\pi}{2} - \cancel{\alpha} - u) + V_{y_R} \sin(\cancel{\alpha} + \tfrac{\pi}{2} - \cancel{\alpha} - u) + \ell \sin(\tfrac{\pi}{2} - \alpha - u)\,\omega = 0$$

$$-V_{x_R} \sin u + V_{y_R} \cos u + \ell \cos(\alpha + u)\,\omega = 0$$

$$-V_{x_R} \sin u + V_{y_R} \cos u + (\ell \cos\alpha \cos u - \ell \sin\alpha \sin u)\,\omega = 0$$

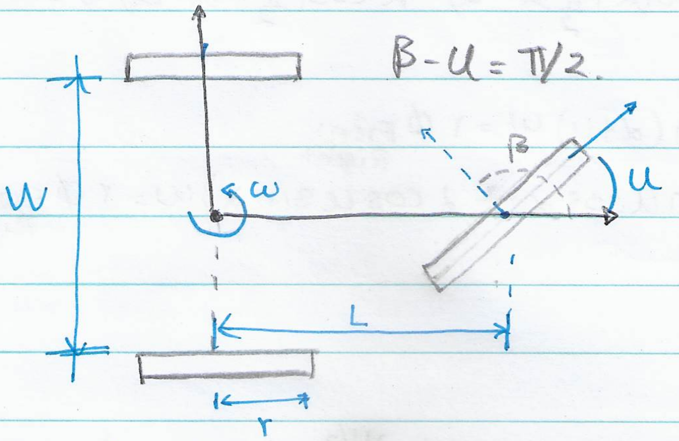
$$-V_{x_R} \sin u + V_{y_R} \cos u + \left(\frac{L}{\cancel{\cos\alpha}} \cos u \cancel{\cos\alpha} - \frac{W/2}{\cancel{\sin\alpha}} \sin u \cancel{\sin\alpha}\right)\omega = 0$$

$$\boxed{-V_{x_R} \sin u + V_{y_R} \cos u + (L \cos u - W/2 \sin u)\,\omega = 0}$$ → slidding constraint Front Left wheel.

P2. Considere the tricycle in Figure 2. Derive the Kinematic model of the tricycle.

$\dot{x} = V\cos\theta \quad \dot{\theta} = \frac{V}{L}\tan u$

$\dot{y} = V\sin\theta$



Rear wheels:

Rolling constraint → $V_{x_R}\sin(\alpha+\beta) - V_{y_R}\cos(\alpha+\beta) - \ell\cos\beta\,\omega = r\,\dot\phi_{\substack{rear\\left}}$ ($\alpha+\beta = \pi/2$, $\cos\beta = 0$, $\ell = w/2$)

Left wheel

$$\boxed{V_{x_R} - \frac{W}{2}\omega = r\,\dot\phi_{\substack{rear\\left}}}$$

Slidding constraint left wheel $V_{x_R}\cos(\alpha+\beta) + V_{y_R}\sin(\alpha+\beta) + \ell\sin\beta\,\omega = 0$

$$\boxed{V_{y_R} = 0}$$

Similarly

Rolling Constraint → $V_{x_R}\sin(\alpha+\beta) - V_{y_R}\cos(\alpha+\beta) - \ell\cos\beta\,\omega = r\,\dot\phi_{\substack{rear\\Right}}$ → $\alpha = -\pi/2$, $\beta = \pi$.

Right wheel

$$\boxed{V_{x_R} + W/2\,\omega = r\,\dot\phi_{\substack{rear\\Right}}}$$

slidding constraint → $\boxed{V_{y_R} = 0}$ same as rear left wheel

Front wheel

: $V_{x_R}\sin(\alpha+\beta) - V_{y_R}\cos(\alpha+\beta) - \ell\cos\beta\,\omega = r\,\dot\phi_{\substack{Front\\left}}$ $\quad \alpha = 0$, $\beta = u + \pi/2$.

→ $V_{x_R}\sin(\pi/2+u) - V_{y_R}\cos(\pi/2+u) - \ell\cos(\pi/2+u)\,\omega = r\,\dot\phi_{\substack{Front\\left}}$

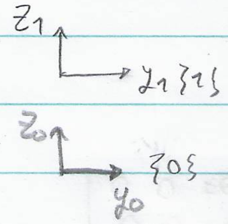
→ $\boxed{V_{x_R}\cos u + V_{y_R}\sin u + L\sin u\,\omega = r\,\dot\phi_{\substack{Front\\left}}}$ → Rolling constraint. Front wheel.

Slidding constraint : $V_{x_R}\cos(\alpha+\beta) + V_y\sin(\alpha+\beta) + \ell\sin\beta\,\omega = 0$

$V_{x_R}\cos(u+\pi/2) + V_{y_R}\sin(u+\pi/2) + L\sin(u+\pi/2)\,\omega = 0$

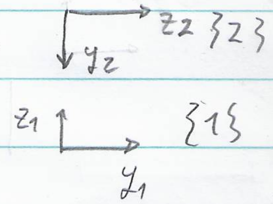
$\boxed{-V_{x_R}\sin u + V_{y_R}\cos u + L\cos u\,\omega = 0}$ → slidding constraint Front wheel.

P3 → Consider Kuka iiwa, as illustrated in Figure 3. Compute all the transformations matrices ${}^{2}_{1}T, {}^{3}_{2}T \cdots {}^{8}_{7}T$ between two adjacent frames.



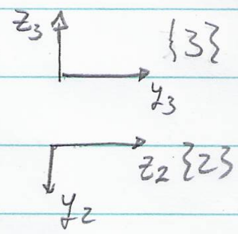
→ $\{0\} \to \{1\}$: $\begin{pmatrix} I & \begin{matrix}0\\0\\d_{01}\end{matrix} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$${}^{1}_{0}T = \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



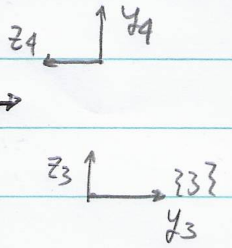
→ $\{1\} \to \{2\}$: $\begin{pmatrix} I & \begin{matrix}0\\0\\d_{12}\end{matrix} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$${}^{2}_{1}T = \begin{pmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\theta_2 & -\cos\theta_2 & 0 & d_{12} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



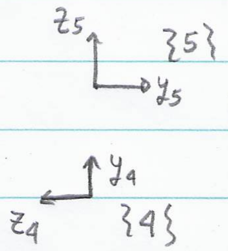
→ $\{2\} \to \{3\}$: $\begin{pmatrix} I & \begin{matrix}0\\-d_{23}\\0\end{matrix} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$${}^{3}_{2}T = \begin{pmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & 0 \\ 0 & 0 & -1 & -d_{23} \\ \sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



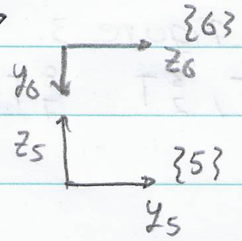
→ $\{3\} \to \{4\}$: $\begin{pmatrix} I & \begin{matrix}0\\0\\d_{34}\end{matrix} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & 0 \\ \sin\theta_4 & \cos\theta_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$${}^{4}_{3}T = \begin{pmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin\theta_4 & \cos\theta_4 & 0 & d_{34} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



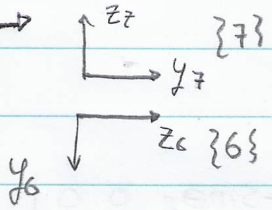
→ $\{4\} \to \{5\}$: $\begin{pmatrix} I & \begin{matrix}0\\d_{45}\\0\end{matrix} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \cos\theta_5 & -\sin\theta_5 & 0 & 0 \\ \sin\theta_5 & \cos\theta_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$${}^{5}_{4}T = \begin{pmatrix} \cos\theta_5 & -\sin\theta_5 & 0 & 0 \\ 0 & 0 & 1 & d_{45} \\ -\sin\theta_5 & -\cos\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



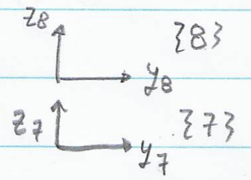
$\{5\} \rightarrow \{6\}:$ $\begin{pmatrix} I & \begin{matrix} 0 \\ 0 \\ d_{56} \end{matrix} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$$ {}^{6}_{5}T = \begin{pmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\theta_6 & -\cos\theta_6 & 0 & d_{56} \\ 0 & 0 & 0 & 1 \end{pmatrix} $$



$\{6\} \rightarrow \{7\}:$ $\begin{pmatrix} I & \begin{matrix} 0 \\ -d_{67} \\ 0 \end{matrix} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta_7 & -\sin\theta_7 & 0 & 0 \\ \sin\theta_7 & \cos\theta_7 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$$ {}^{7}_{6}T = \begin{pmatrix} \cos\theta_7 & -\sin\theta_7 & 0 & 0 \\ 0 & 0 & -1 & -d_{67} \\ \sin\theta_7 & \cos\theta_7 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$



$\{7\} \rightarrow \{8\}:$ $ {}^{8}_{7}T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_{78} \\ 0 & 0 & 0 & 1 \end{pmatrix} $