Exercise_1

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1 EECS 531: Computer Vision Assignment 2

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2 Exercise 1

2.1 Background

The two-dimensional discrete cosine transform (DCT) represents an image as a sum of sinusoids. The two-dimensional DCT of an M-by-N matrix A is defined as follows:

$$B_{pq} = \alpha_p \alpha_q \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} A_{mn} cos \frac{\pi(2m+1)p}{2M} cos \frac{\pi(2n+1)q}{2N}, \quad \begin{matrix} 0 \leq p \leq M-1 \\ 0 \leq q \leq N-1 \end{matrix}$$

$$\alpha_p = \begin{cases} 1/\sqrt{M}, & p = 0 \\ \sqrt{2/M}, & 1 \leq p \leq M-1 \end{cases} \quad \alpha_q = \begin{cases} 1/\sqrt{N}, & q = 0 \\ \sqrt{2/N}, & 1 \leq q \leq N-1 \end{cases}$$

The DCT can be inverted to give:

$$A_{mn} = \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} \alpha_p \alpha_q \beta_{pq} cos \frac{\pi(2m+1)p}{2M} cos \frac{\pi(2n+1)q}{2N}, \quad \begin{matrix} 0 \leq m \leq M-1 \\ 0 \leq n \leq N-1 \end{matrix}$$

$$\alpha_p = \begin{cases} 1/\sqrt{M}, & p = 0 \\ \sqrt{2/M}, & 1 \leq p \leq M-1 \end{cases} \quad \alpha_q = \begin{cases} 1/\sqrt{N}, & q = 0 \\ \sqrt{2/N}, & 1 \leq q \leq N-1 \end{cases}$$

This can be interpreted as meaning that any M-by-N matrix A can be written as a sum of MN basis functions defined by:

$$\alpha_p \alpha_q cos \frac{\pi(2m+1)p}{2M} cos \frac{\pi(2n+1)q}{2N}, \quad \begin{matrix} 0 \leq p \leq M-1 \\ 0 \leq q \leq N-1 \end{matrix}$$

2.2 Problem Definition

In this exercise I will plot the basis functions of a 16x16 DCT.

(Note to readers: This took forever trying to interpret the matlab example and convert the concepts directly into python... matlab does so many things different than python that by the time I got it working I was done with this problem)

2.3 Basis Functions

We will first create a function that returns a basis function given inputs:

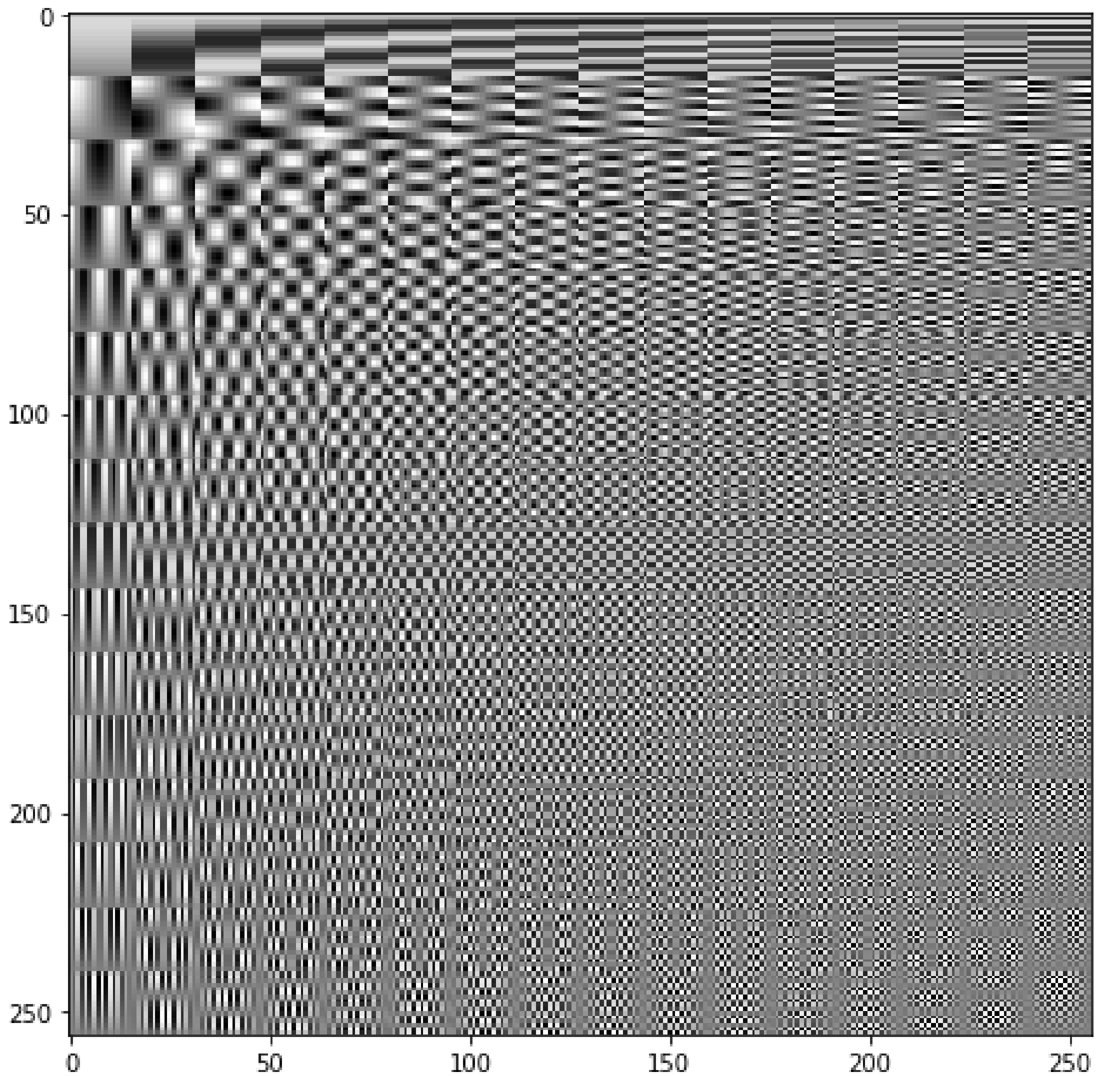
```
In [11]: import numpy as np
         from math import pi
         import matplotlib.pyplot as plt
```

```
In [79]: def dctBasis(u, v, M, N):
             cu = np.sqrt(2.0/M)
             if u == 0:
                 cu = 1/np.sqrt(M)

             cv = np.sqrt(2.0/N)
             if v == 0:
                 cv = 1/np.sqrt(N)

             J, I = np.meshgrid(np.arange(N), np.arange(M))
             T1 = np.cos(np.multiply(np.multiply(pi, (u/2.0/M)), (2*I+1)))
             T2 = np.cos(np.multiply(np.multiply(pi, (v/2.0/N)), (2*J+1)))
             B = np.multiply(np.multiply(np.multiply(cu, cv), T1), T2)
             return B
```

We will now print out the basis functions for 16x16:

```
In [85]: M = 16
         N = 16
         K = 0
         Adct = np.zeros([M*N, M*N])
         for u in range(M):
             for v in range(N):
                 B = dctBasis(u,v,M,N)
                 Adct[K, :] = B.flatten('F')
                 K = K+1
```

```
In [86]: fig = plt.figure(figsize=(8,8))
         ax = plt.subplot(111)
         ax.imshow(Adct, cmap='gray');
```



2.4 References

- *Discrete Cosine Transform*, MathWorks