# Exercise_4

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## 1 EECS 531: Computer Vision Assignment 2

**David Fan**
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## 2 Exercise 4

### 2.1 Problem definition

In this exercise we will compute the principal components from the MNIST dataset. We will then show that individual digits can be approximated by the sum of the first $k$ principal components.

### 2.2 Setup

First we will load the MNIST dataset and do some preprocessing to get it into a workable state:

```
In [2]: import matplotlib.pyplot as plt
        import skimage.io as io
        import numpy as np
        from tensorflow.examples.tutorials.mnist import input_data
        from sklearn.decomposition import PCA
        from matplotlib.colors import Normalize
```

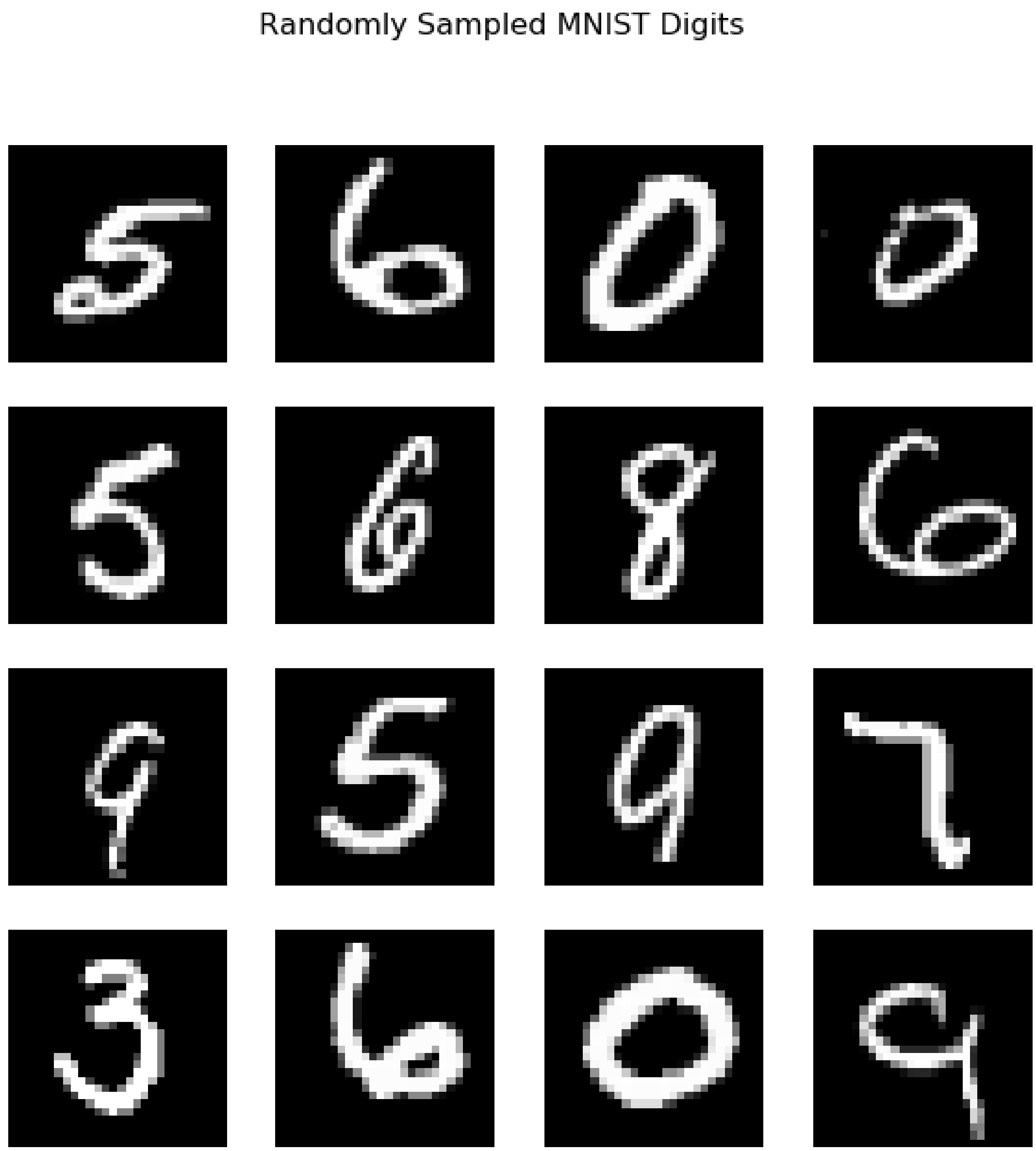
```
In [3]: mnist = input_data.read_data_sets("MNIST_data/", one_hot=True)
```

```
Extracting MNIST_data/train-images-idx3-ubyte.gz
Extracting MNIST_data/train-labels-idx1-ubyte.gz
Extracting MNIST_data/t10k-images-idx3-ubyte.gz
Extracting MNIST_data/t10k-labels-idx1-ubyte.gz
```

```
In [4]: imgs = mnist.train.images
        imgs = np.reshape(imgs, (55000, 28,28))
```

```
In [5]: import random
        samples = random.sample(range(0, np.shape(imgs)[0]), 16)
        figure, axes = plt.subplots(figsize=(10,10), nrows=4, ncols=4)
        figure.suptitle("Randomly Sampled MNIST Digits", fontsize=16)
```

```python
count = 0
for row in axes:
    for ax in row:
        ax.imshow(imgs[samples[count]], cmap='gray')
        ax.axis('off')
        count += 1
```




### 2.3 PCA Analysis

Now that we've gone through the difficult process of loading MNIST in python (it's not trivial at first like it was with Matlab... trust me) we can do our PCA analysis:

```
In [6]: n, h, w = np.shape(imgs)
        X = np.resize(imgs, (n, h*w))
```

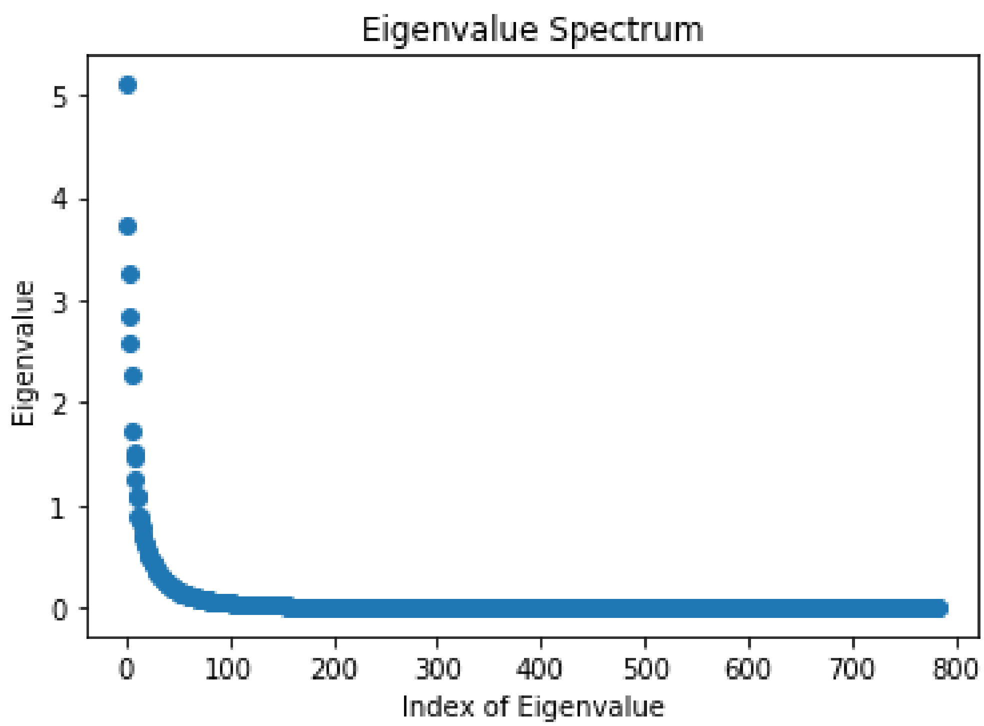
```
In [9]: mu = np.mean(X, axis=0)
        X = X - np.matlib.repmat(mu, n ,1)
```

```
In [11]: pca = PCA()
         pca.fit(X)
         coeff = pca.components_
         score = pca.transform(X)
         latent = pca.explained_variance_
```

First let us look at the eigenvalues that our PCA analysis has produced:

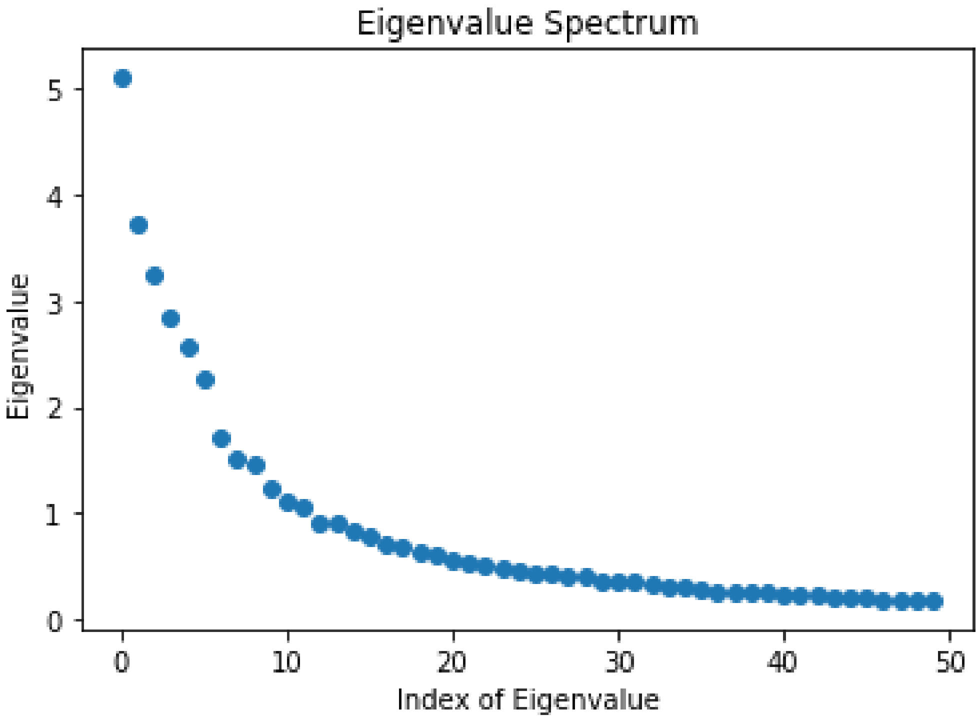
```
In [12]: fig, ax = plt.subplots()
         ax.set_title('Eigenvalue Spectrum')
         ax.set_xlabel('Index of Eigenvalue')
         ax.set_ylabel('Eigenvalue')
         ax.scatter(np.arange(len(latent)), latent)
```

```
Out[12]: <matplotlib.collections.PathCollection at 0x1c34707470>
```




```
In [13]: fig, ax = plt.subplots()
         ax.set_title('Eigenvalue Spectrum')
         ax.set_xlabel('Index of Eigenvalue')
         ax.set_ylabel('Eigenvalue')
         ax.scatter(np.arange(50), latent[0:50])
```

Out[13]: <matplotlib.collections.PathCollection at 0x1c38826470>

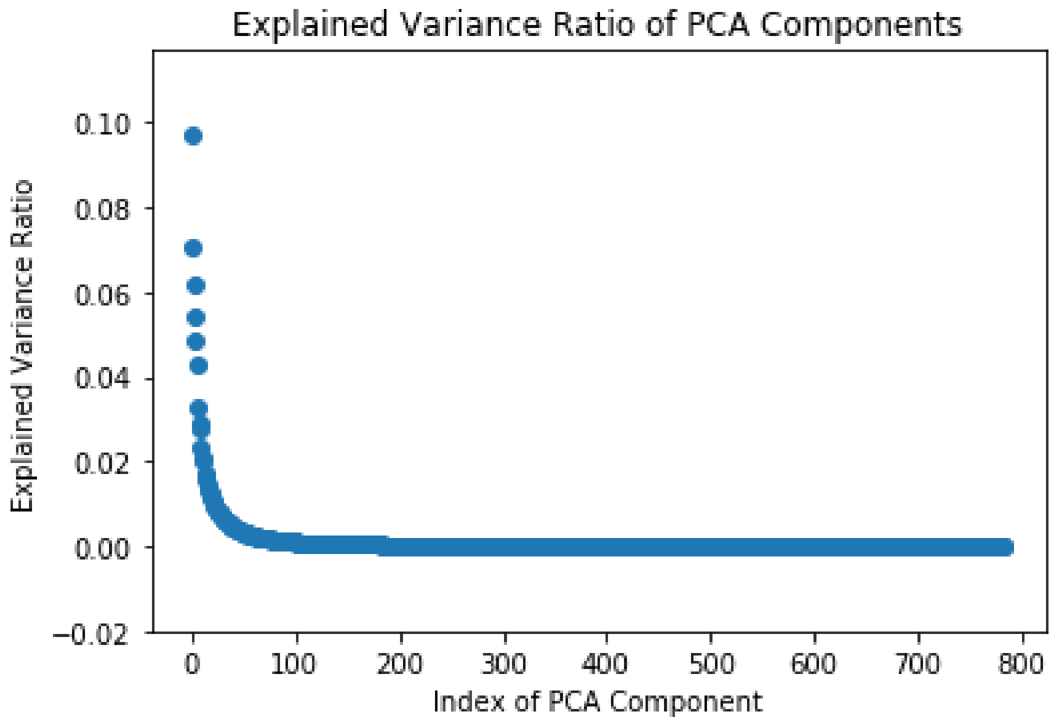



Now let us look at the variance ratio:

In [14]:
```
explained_variance_ratio = pca.explained_variance_ratio_
```

In [15]:
```
fig, ax = plt.subplots()
ax.set_title('Explained Variance Ratio of PCA Components')
ax.set_xlabel('Index of PCA Component')
ax.set_ylabel('Explained Variance Ratio')
ax.scatter(np.arange(len(explained_variance_ratio)), explained_variance_ratio)
```

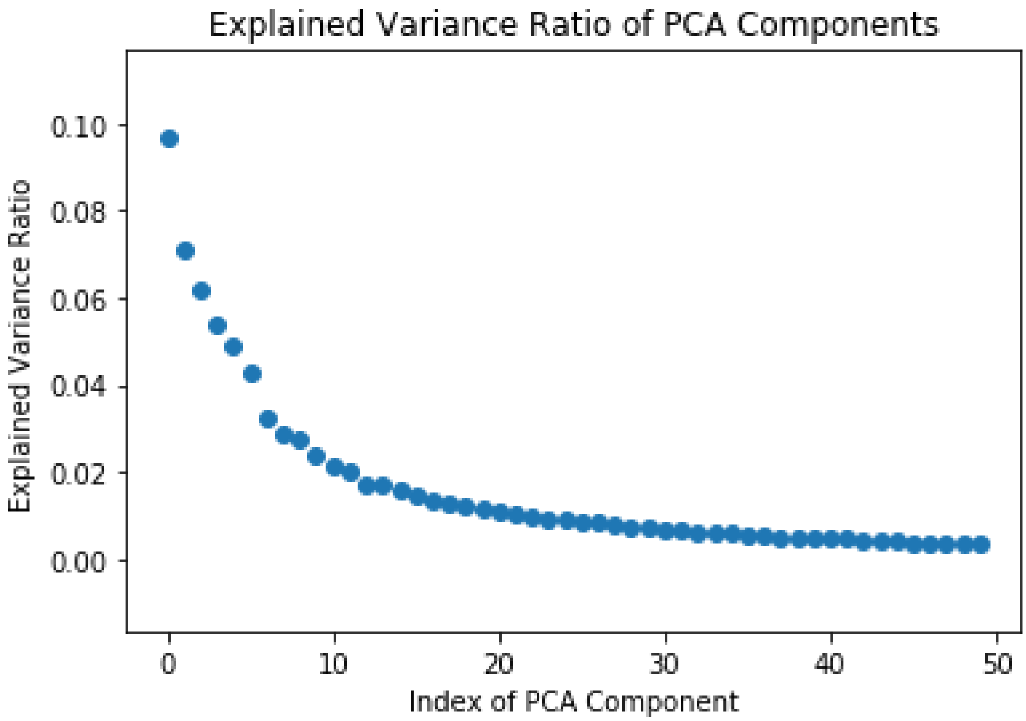
Out[15]: <matplotlib.collections.PathCollection at 0x1c388c2400>




As expected, it seems the first few components account for much more of the variance than the rest of the components. Let's look closer at the first few components:

```
In [16]: fig, ax = plt.subplots()
         ax.set_title('Explained Variance Ratio of PCA Components')
         ax.set_xlabel('Index of PCA Component')
         ax.set_ylabel('Explained Variance Ratio')
         ax.scatter(np.arange(50), explained_variance_ratio[0:50])

Out[16]: <matplotlib.collections.PathCollection at 0x1c389bc080>
```

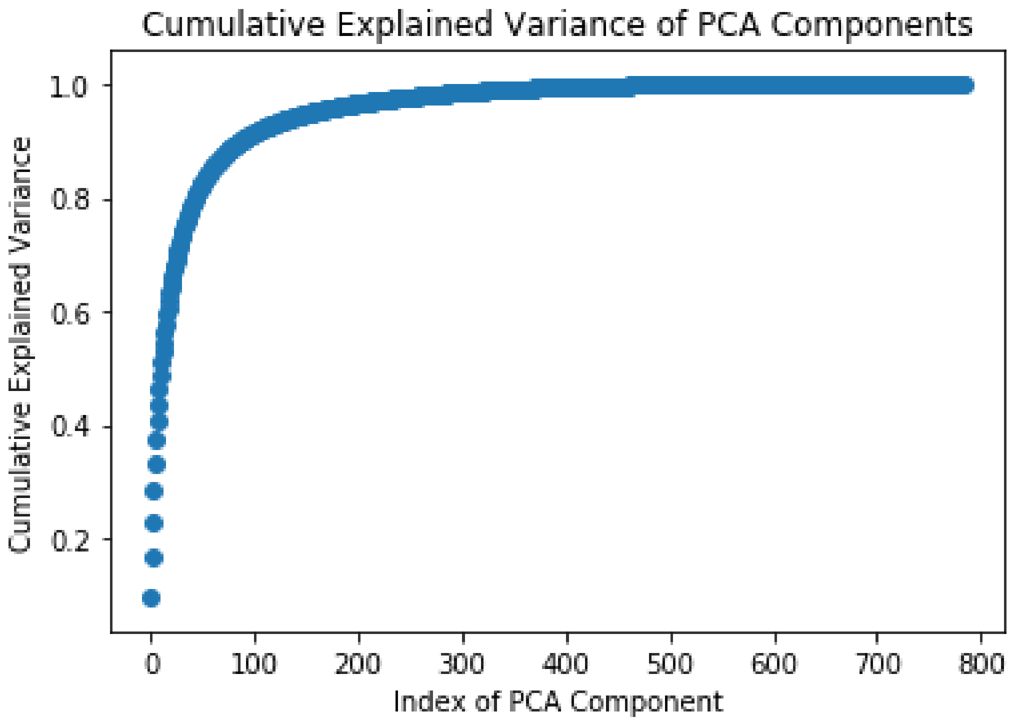



Interesting result. Let's visualize this with a cumulative graph:

```
In [17]: cumulative_evr = np.cumsum(explained_variance_ratio)
```

```
In [18]: fig, ax = plt.subplots()
         ax.set_title('Cumulative Explained Variance of PCA Components')
         ax.set_xlabel('Index of PCA Component')
         ax.set_ylabel('Cumulative Explained Variance')
         ax.scatter(np.arange(len(cumulative_evr)), cumulative_evr)
```

```
Out[18]: <matplotlib.collections.PathCollection at 0x1c4d39bb70>
```

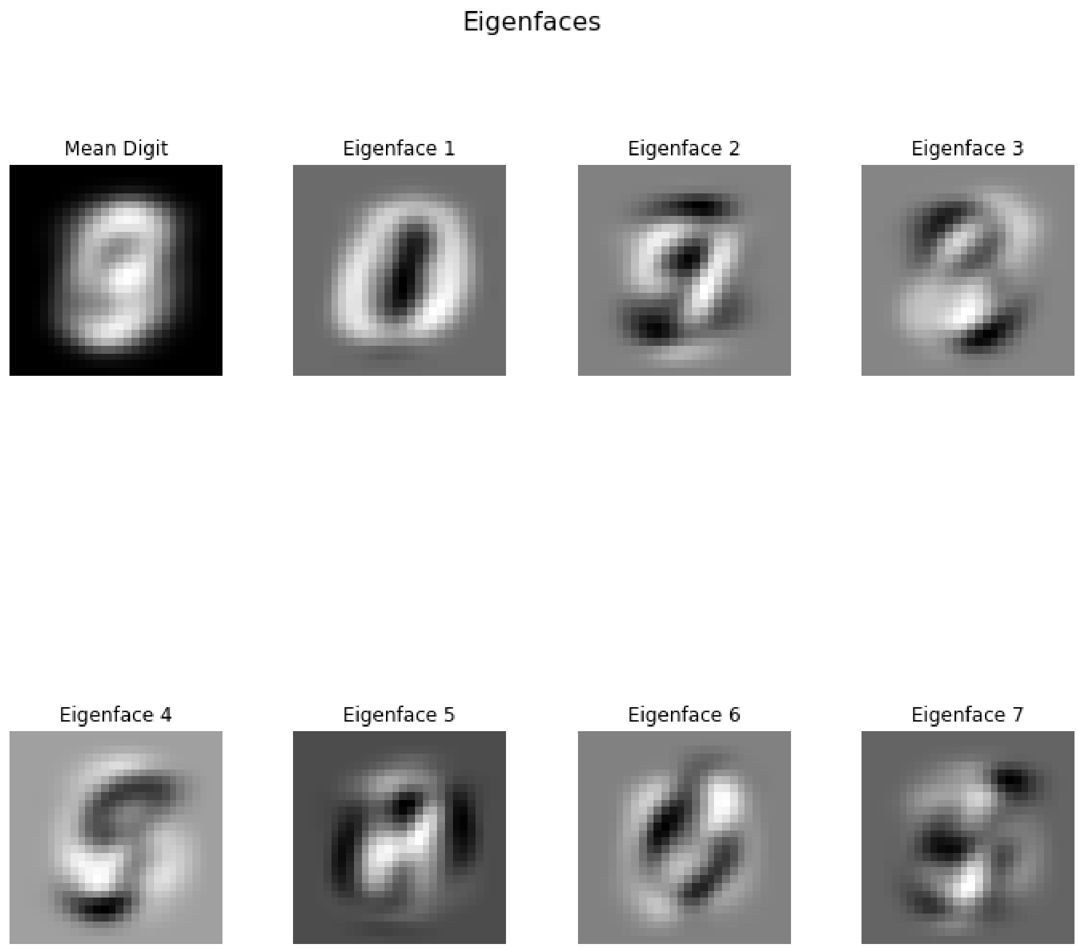



## 2.4 PCA Components

Here we will examine the eigenfaces.

In [19]:
```
components = []
for i in range(7):
    components.append(coeff[i, :].reshape((h, w)))
```

In [20]:
```
figure, axes = plt.subplots(figsize=(10,10), nrows=2, ncols=4)
figure.suptitle("Eigenfaces", fontsize=16)
plt.tight_layout()
count = 0
for row in axes:
    for ax in row:
        if count == 0:
            ax.imshow(np.resize(mu, (h, w)), cmap='gray')
            ax.set_title('Mean Digit')
        else:
            ax.imshow(components[count - 1], cmap='gray', interpolation='nearest')
            ax.set_title("Eigenface %i" % int(count))
        ax.axis('off')
        count += 1
```




## 2.5 Reconstruct Images

Now we shall attempt to reconstruct the images using their components

In [57]:
```
id = 255
org = imgs[id, :, :]
mse = np.zeros(4)
```

In [63]:
```
figure, axes = plt.subplots(figsize=(10,10), nrows=2, ncols=5)
plt.tight_layout()

axes[0,0].imshow(org, cmap='gray')
axes[0,0].set_title('Original')
axes[1,0].axis('off')

for i in range(1,5):
    rec_x = np.dot(score[id, 0:i*25].T, coeff[0:i*25, :])
```

```
rec = np.resize(rec_x+mu, (h,w))
err = rec - org
mse[i - 1] = np.mean(np.square(err[:]))
axes[0, i].imshow(rec, cmap='gray')
axes[0, i].set_title("i=%i" % (i*25))
axes[0, i].axis('off')
axes[1, i].imshow(np.abs(err), cmap='gray')
axes[1, i].set_title("mse=%f" % mse[i - 1])
axes[1, i].axis('off')
```