Exercise 2

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1 EEECS 531: Computer Vision Assignment 4

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4/13/18

2 Exercise 2

In this notebook we will again be computing the optical flow field of two images in a sequence. This time we will be doing this by using a gradient based method.

2.1 Background

To compute the optical flow field we will first compute the discrete derivative components in the motion gradient constraint equation for a single pixel location of a moving image. We will then compute the matrix **A** and the vector **b** representing the motion gradient equations over a small $n \times n$ patch centered on pixel (x, y). Then we will solve the constraint equations using a least squares solver to estimate the motion vectors $(\mathbf{u}, \mathbf{v})$ at point (x, y). We will use that function to cover a grid of pixel points on the image, but first we should cover some theory. ### Gradient Constraint Equation If we make the assumption that the intensity of a moving point in an image doesn't change over time such that,

$$I(x, y, t) = I(x + \partial x, y + \partial y, t + \partial t)$$

we have:

$$\frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \frac{\partial I}{\partial t} = 0$$

This is the motion gradient constraint equation.

Partial Derivatives At any given point (x,y,t) we can compute the partial derivatives:

$$I_x = \frac{I(x+1, y) - I(x-1, y)}{2}$$
$$I_y = \frac{I(x, y+1) - I(x, y-1)}{2}$$
$$I_t = \frac{I(x, t+1) - I(x, t-1)}{2}$$

However, with only two frames we often use the estimate $I_t = I(x, t+1) - I(x, t)$.

Matrix Form The motion gradient constraint equation can also be represented in matrix form:

$$\mathbf{A}\begin{bmatrix} u \\ v \end{bmatrix} = \mathbf{b}t$$

where:

$$\mathbf{A} = \begin{bmatrix} I_x(x_1) & I_y(x_1) \\ \dots & \\ I_x(x_n) & I_y(x_n) \end{bmatrix}$$
$$\mathbf{b} = -\begin{bmatrix} I_t(x_1) \\ \dots \\ I_t(x_n) \end{bmatrix}$$

This form makes it easy to use regression to solve the motion constraint equation for the motion vectors at a specific point.

2.2 Imports and Preprocessing

```
In [5]: import numpy as np
        import matplotlib.pyplot as plt
        from skimage import io

In [6]: def plotflow(I, X, Y, U, V, scale=1, threshold=-1):
            fig, ax = plt.subplots(figsize=(10, 10), dpi=80)
            if threshold > 0:
                mask = np.abs(U + 1j*V) > threshold
                X = X[mask]
                Y = Y[mask]
                U = U[mask]
                V = V[mask]

            ax.imshow(I, cmap='gray')
            ax.quiver(X, Y, U*scale, V*scale, color='red', angles='xy', scale_units='xy', scal
            ax.set_aspect('equal')
            plt.show()

        seq1 = {'I1': io.imread('data/image/seq1/frame1.png', as_grey=True),
                'I2': io.imread('data/image/seq1/frame3.png', as_grey=True),
                'U' : np.loadtxt('data/flow/seq1/flow3.u', dtype='double', delimiter=','),
                'V' : np.loadtxt('data/flow/seq1/flow3.v', dtype='double', delimiter=',')}

        rubic = {'I1':io.imread('data/rubic/rubic.0.png', as_grey=True),
                 'I2':io.imread('data/rubic/rubic.5.png', as_grey=True)}

        sphere= {'I1': io.imread('data/sphere/sphere.1.png', as_grey=True),
                 'I2': io.imread('data/sphere/sphere.3.png', as_grey=True)}


        I = np.random.rand(128, 128)
        synth = {'I1': I[0:100, 0:100],
                 'I2': I[2:102, 2:102]}
```

2.3 Compute Partial Derivatives for a Single Pixel Location

```
In [3]: def gradient_xyt(I1, I2, x, y):
            h, w = I1.shape
            x = int(x)
            y = int(y)

            Ix = (x>0 and x<(w-1) and y>=0 and y<h ) and (I1[y, x+1] - I1[y, x-1])/2 or 0
            Iy = (x>=0 and x<w and y>0 and y< (h-1) ) and (I1[y+1, x] - I1[y-1, x])/2 or 0
            It = (x>=0 and x<w and y>=0 and y<h) and I2[y,x] - I1[y,x] or 0
            return (Ix, Iy, It)
```

2.4 Compute A and b for a Patch Centered on a Pixel

```
In [16]: def getAb(I1, I2, x, y, n):
             A = np.zeros((n*n, 2))
             b = np.zeros(n*n)

             offset = np.arange(0, n) - np.floor(n/2);
             dx, dy = np.meshgrid(offset, offset);
             dx = dx.reshape(n*n, 1);
             dy = dy.reshape(n*n, 1);

             for i in range(0, n*n):
                 Ix, Iy, It = gradient_xyt(I1, I2, x+dx[i], y+dy[i])
                 A[i, 0] = Ix
                 A[i, 1] = Iy
                 b[i] = -It

             return (A, b)
```

2.5 Solve for the Motion Vectors

```
In [17]: def estimate_flow_at_xy(I1, I2, x, y, n):
             A, b = getAb(I1, I2, x, y, n)
             result = np.linalg.lstsq(np.matmul(A.T, A), np.matmul(A.T, b))
             v = result[0]
             return v
```

2.6 Find the Motion Vectors at a Grid of Points on the Image

```
In [18]: def estimate_flow(I1, I2, gridX, gridY, n):
             H, W = gridX.shape
             U = np.zeros((H, W))
             V = np.zeros((H, W))
```

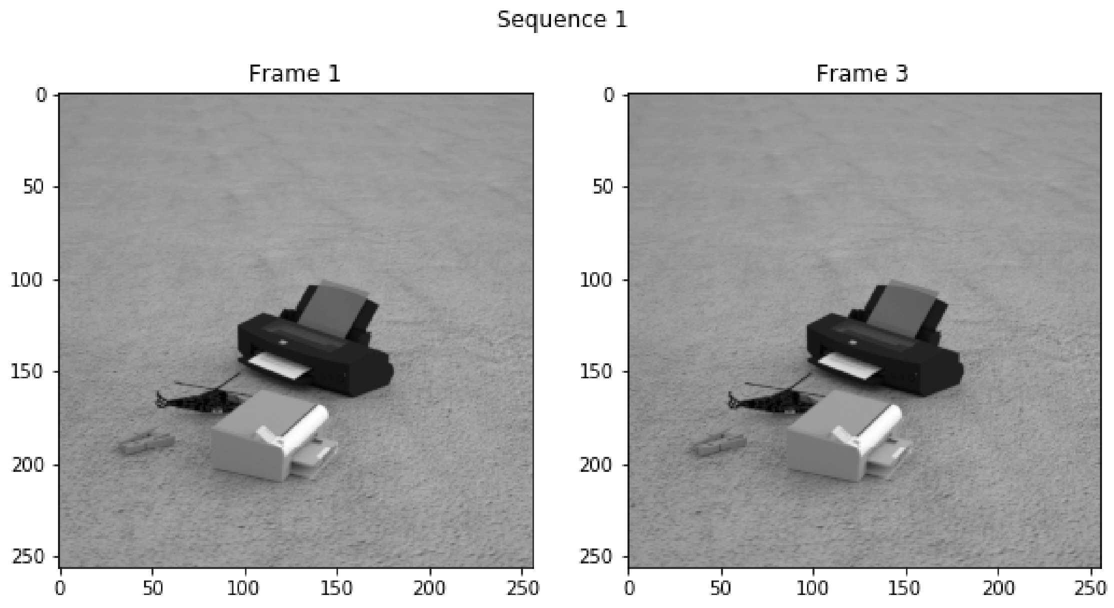
```
        for i in range(0, H):
            for j in range(0, W):
                v =  estimate_flow_at_xy(I1, I2,gridX[i, j], gridY[i, j], n)
                U[i, j] = v[0]
                V[i, j] = v[1]
        return (U, V)
```

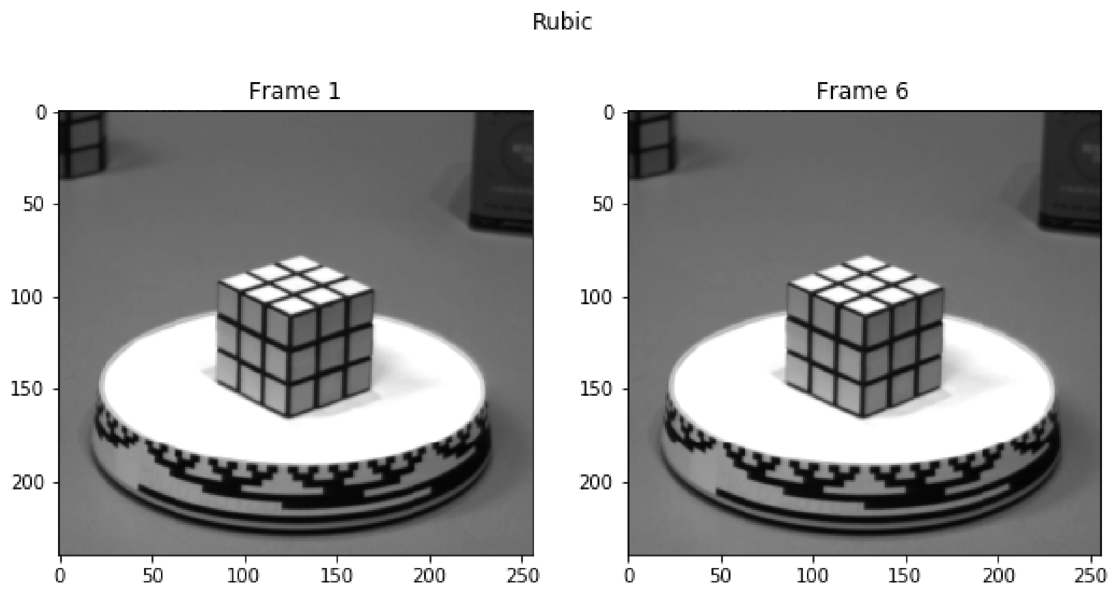
2.7 Compute Optical Flow Field for all Test Images

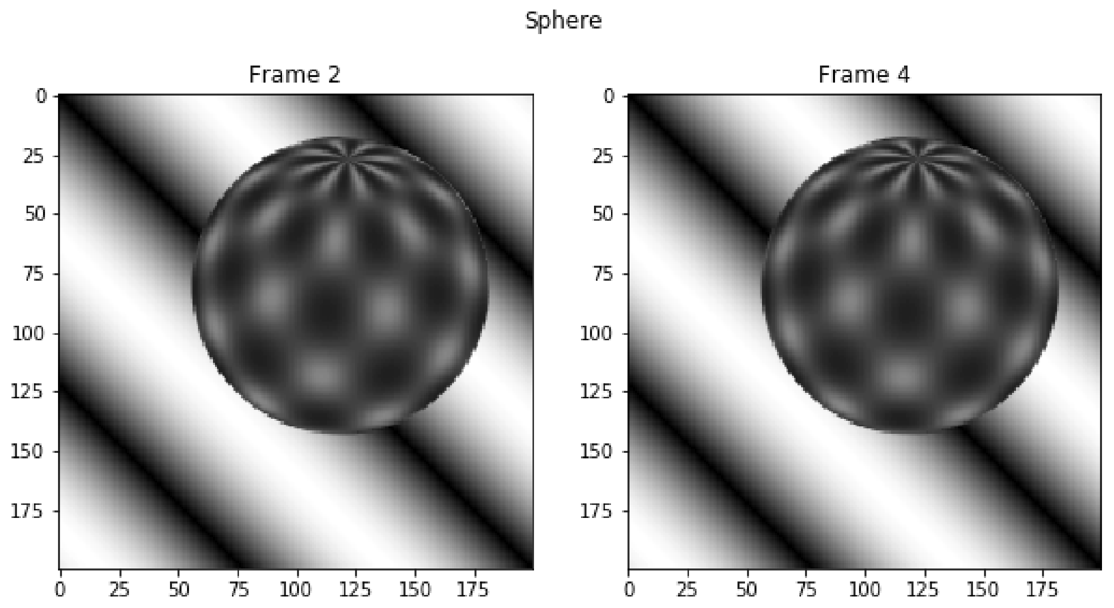
Similar to what we did in Exercise 1, we will be testing our algorithm on our test image sequences. More detail can be found in Exercise 1.

```
In [19]: def compare_frames(seq, name, I1_title, I2_title):
             fig, axes = plt.subplots(1, 2, figsize=(10,5))
             fig.suptitle(name)
             axes[0].set_title(I1_title)
             axes[0].imshow(seq["I1"], cmap='gray')
             axes[1].set_title(I2_title)
             axes[1].imshow(seq["I2"], cmap='gray')

         compare_frames(seq1, "Sequence 1", "Frame 1", "Frame 3")
         compare_frames(rubic, "Rubic", "Frame 1", "Frame 6")
         compare_frames(sphere, "Sphere", "Frame 2", "Frame 4")
```





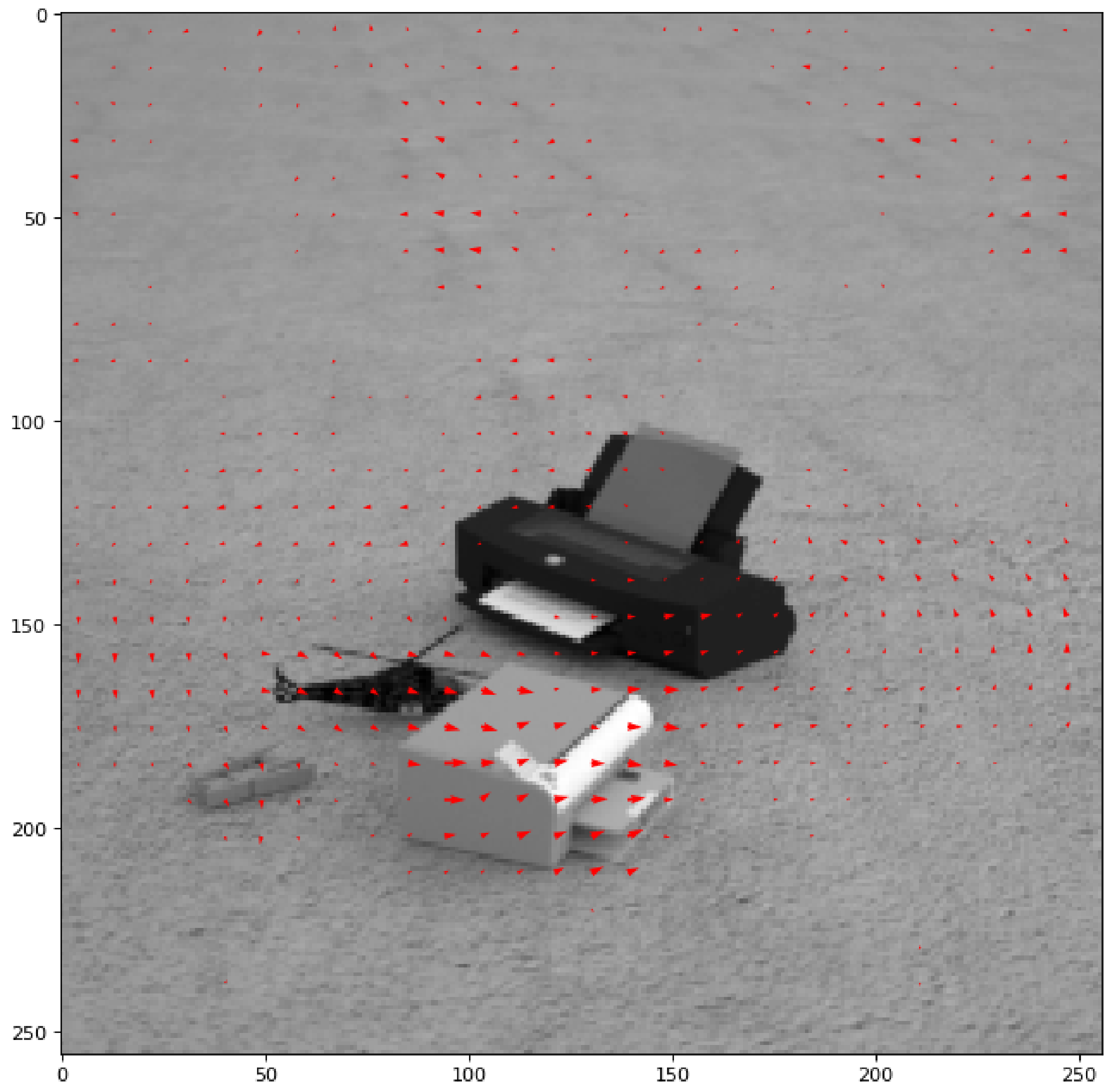


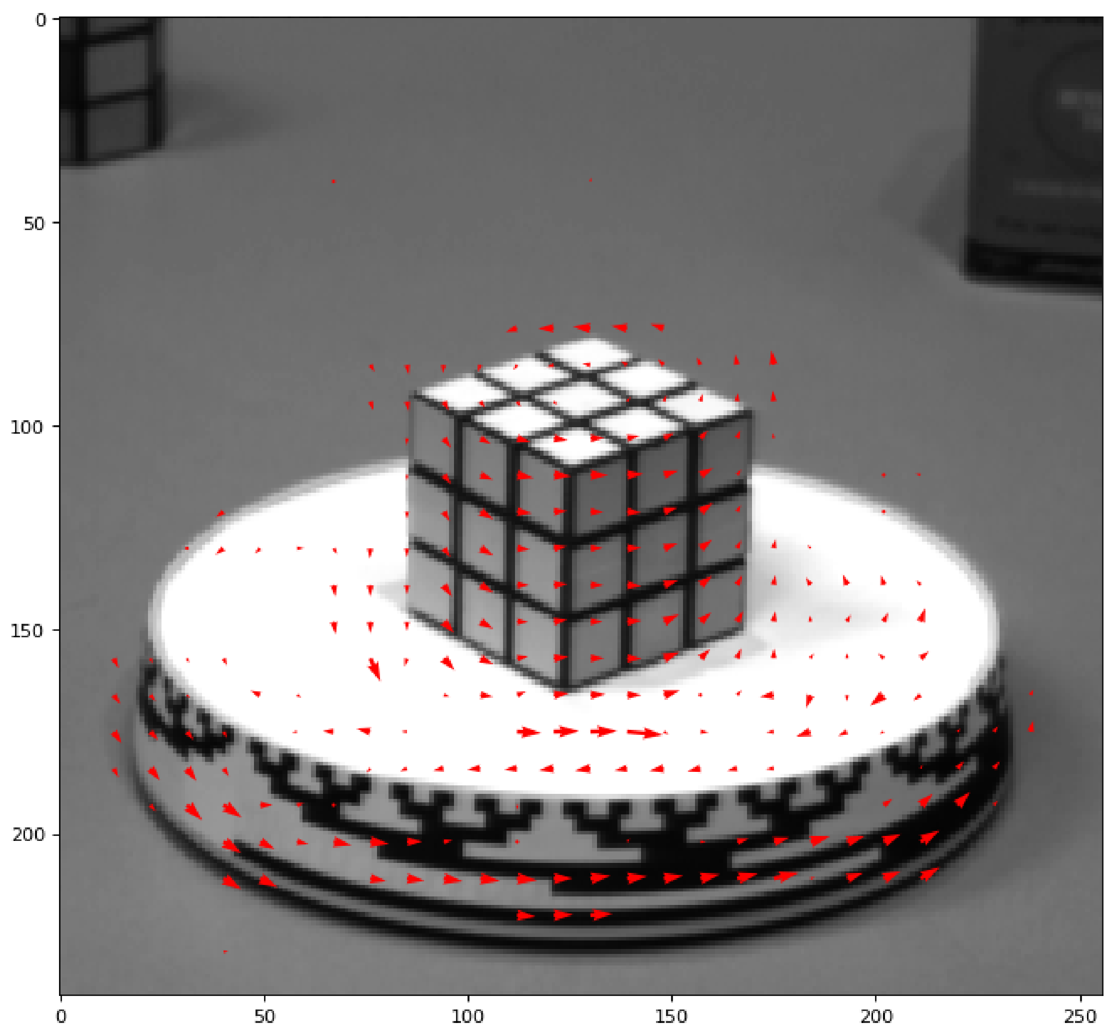
```
In [20]: def test_estimate_flow(seq):
             H, W = seq['I1'].shape
             gridsz = 9
             wsz  = 21
             x = np.arange(0, W-gridsz, gridsz) + np.floor(gridsz/2);
             y = np.arange(0, H-gridsz, gridsz) + np.floor(gridsz/2);
             gridX, gridY = np.meshgrid(x,y);
```

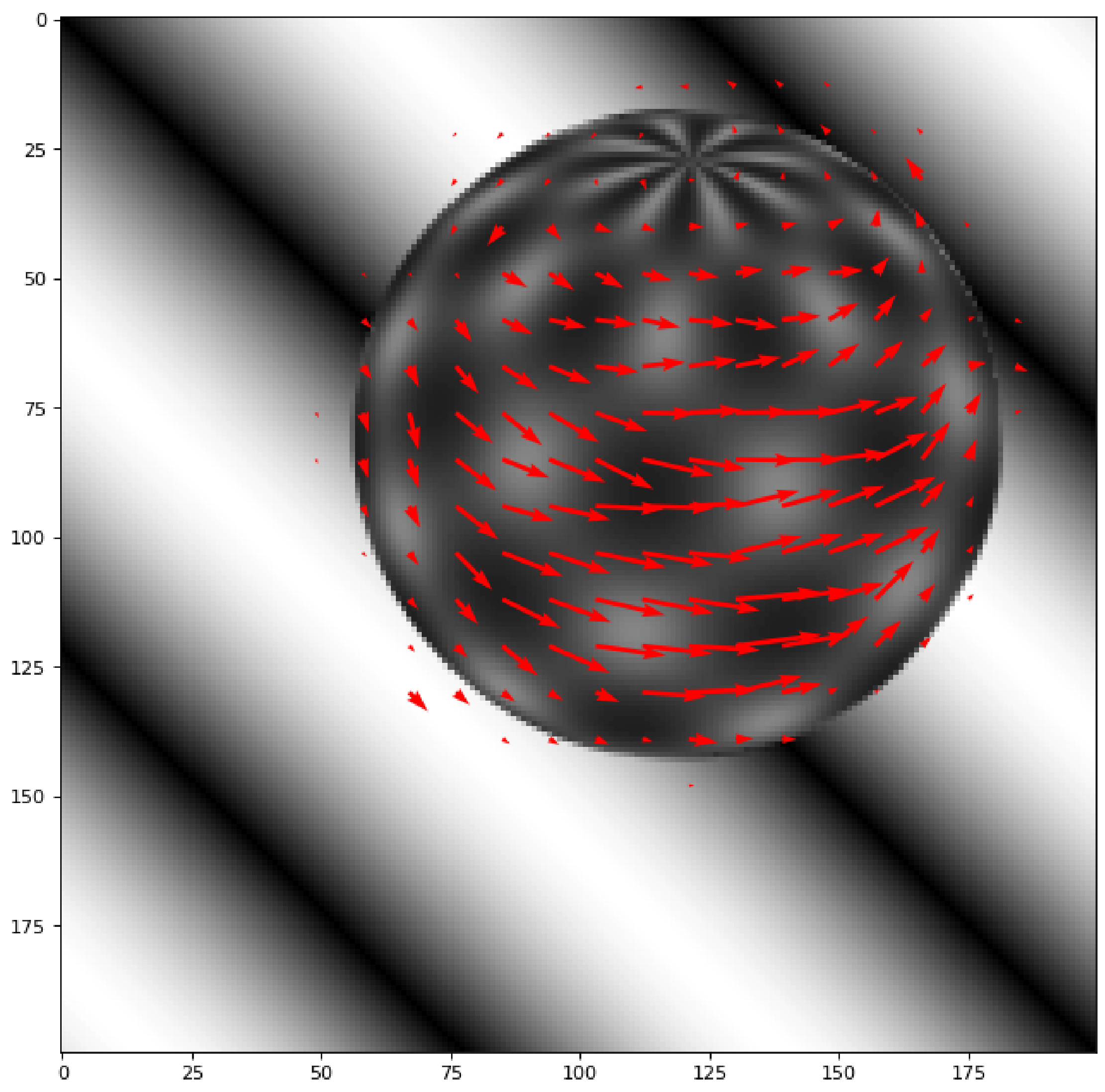
```
        U, V = estimate_flow(seq["I1"], seq["I2"], gridX, gridY, wsz)
        plotflow(seq["I1"], gridX, gridY, U, V, 3, 0.3)

    test_estimate_flow(seq1)
    test_estimate_flow(rubic)
    test_estimate_flow(sphere)
```

```
/Users/david/anaconda3/envs/531/lib/python3.6/site-packages/ipykernel_launcher.py:3: FutureWarn
To use the future default and silence this warning we advise to pass `rcond=None`, to keep usin
  This is separate from the ipykernel package so we can avoid doing imports until
```




0
50
100
150
200
0
50
100
150
200
250



The results are quite good! Perhaps a little over sensitive at points, but overall well performing.

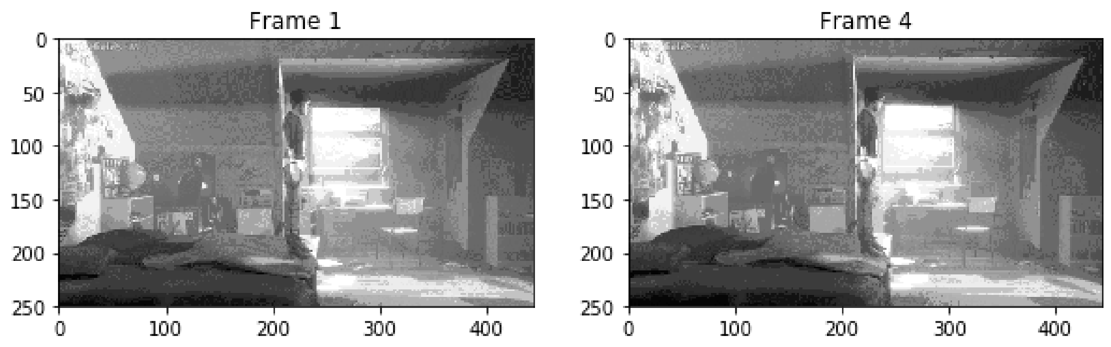
2.8 Extension to a random GIF

With this exercise and the past one we were able to test our algorithms on the provided image data. However, for curiousity sake let's download a random gif from the internet (found from reddit and stored in the data folder) and try to calculate the optical flow field between two frames of the GIF.

```
In [30]: gif_raw = io.imread('data/test.gif', as_grey=True)
         gif = {'I1':gif_raw[0],
                  'I2':gif_raw[3]}
```

```
In [31]: compare_frames(gif, "Random GIF", "Frame 1", "Frame 4")
```

Random GIF



```
In [32]: test_estimate_flow(gif)
```

```
/Users/david/anaconda3/envs/531/lib/python3.6/site-packages/ipykernel_launcher.py:3: FutureWarn
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```
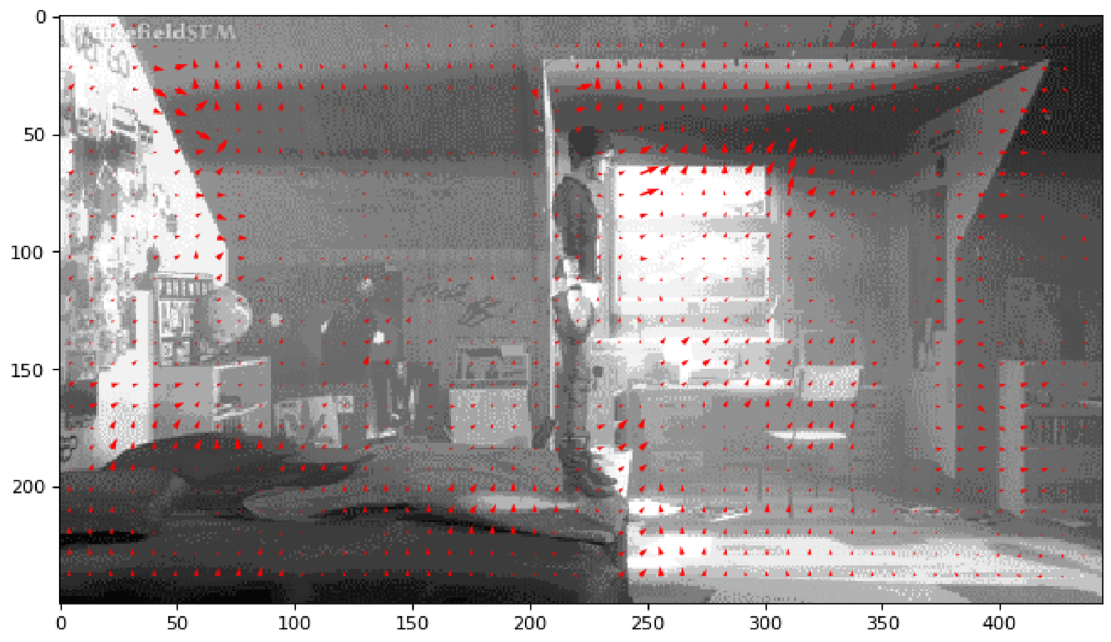
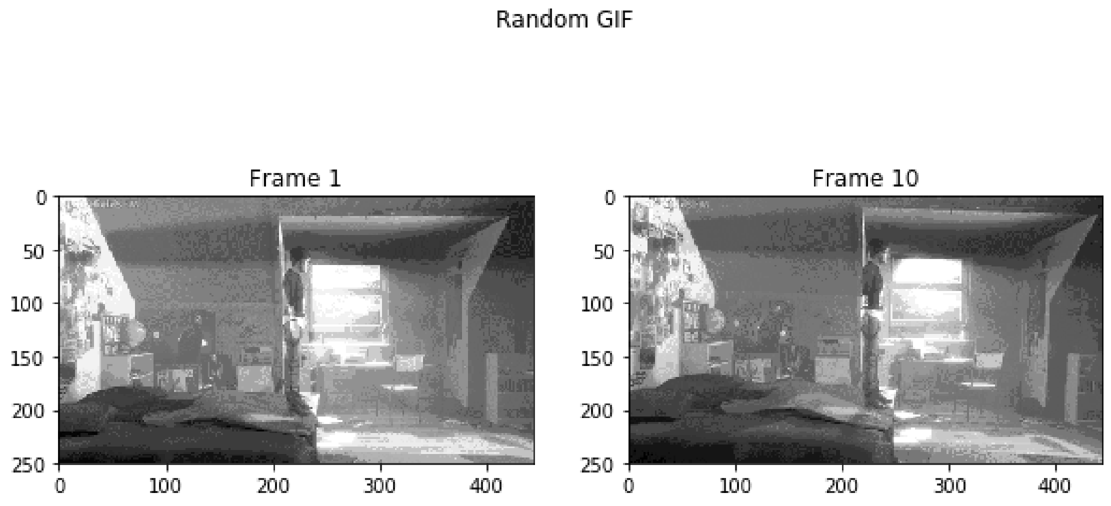



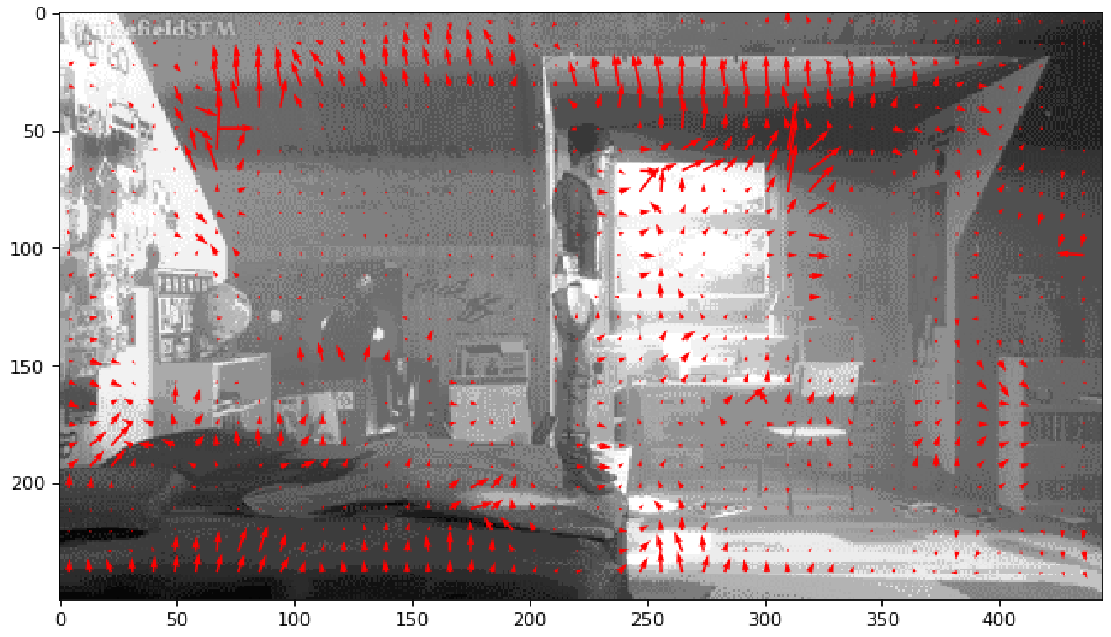
Let's try a bigger Δt:

```
In [59]: gif2 = {'I1':gif_raw[0],
                 'I2':gif_raw[9]}
```

```
In [60]: compare_frames(gif2, "Random GIF", "Frame 1", "Frame 10")
```



```
In [61]: test_estimate_flow(gif2)

/Users/david/anaconda3/envs/531/lib/python3.6/site-packages/ipykernel_launcher.py:3: FutureWar
To use the future default and silence this warning we advise to pass `rcond=None`, to keep usi
  This is separate from the ipykernel package so we can avoid doing imports until
```



Cool!