

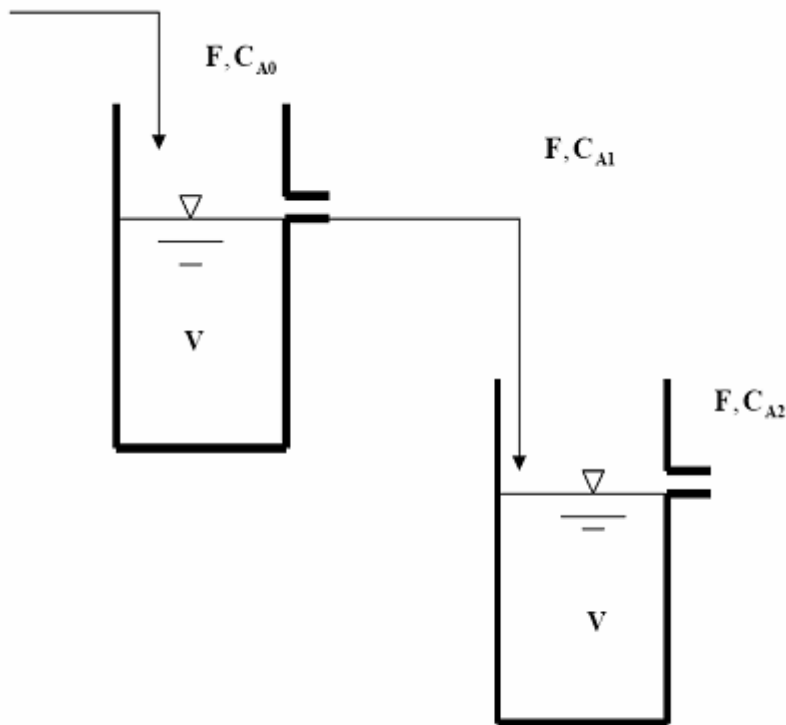
## Internal Model Control Design for a Chemical Reactor Plant

In process control applications, model-based control systems are often used to track setpoints and reject load disturbances. This example illustrates how to design a compensator in a IMC structure for series chemical reactors, using the IMC tuning feature available in SISO Design Tool.

### Contents

- [Mathematical Models for the Series Chemical Reactors](#)
- [Linear Plant Models](#)
- [IMC Design with Automatic Tuning](#)
- [Control Performance with Model Mismatch](#)

### Mathematical Models for the Series Chemical Reactors



### PLANT DESCRIPTION

The chemical reactor system, comprised of two well mixed tanks, is shown in the above figure. The reactors are isothermal and the reaction in each reactor is first order on

component A:

$$r_A = -kC_A$$

Material balance is applied to the system to generate the dynamic model for the system. The tank levels are assumed to stay constant because of the overflow nozzle and hence there is no level control involved.

For details about this plant, see Example 3.3 in Chapter 3 of "Process Control: Design Processes and Control systems for Dynamic Performance" by Thomas E. Marlin.

## EQUATIONS

We have the following differential equations to describe component balances:

$$V \frac{dC_{A1}}{dt} = F(C_{A0} - C_{A1}) - V k C_{A1}$$

$$V \frac{dC_{A2}}{dt} = F(C_{A1} - C_{A2}) - V k C_{A2}$$

At steady state, from

$$\frac{dC_{A1}}{dt} = 0$$

$$\frac{dC_{A2}}{dt} = 0$$

we have the following material balances:

$$F^*(C_{A0}^* - C_{A1}^*) - V k C_{A1}^* = 0$$

$$F^*(C_{A1}^* - C_{A2}^*) - V k C_{A2}^* = 0$$

where variables with \* denote steady state values.

By substituting the following design specifications and reactor parameters,

$$F^* = 0.085 \text{ mole/min}$$

$$C_{A0}^* = 0.925 \text{ mol/min}$$

$$V = 1.05 \text{ m}^3$$

$$k = 0.04 \text{ min}^{-1}$$

we obtain the steady state values of the concentrations in two reactors:

$$C_{A1}^* = K C_{A0}^* = 0.6191 \text{ mol/m}^3$$

$$C_{A2}^* = K^2 C_{A0}^* = 0.4144 \text{ mol/m}^3$$

where

$$K = \frac{F^*}{F^* + V k} = 0.6693$$

## CONTROL OBJECTIVE

The outlet concentration of reactant from the second reactor  $C_{A2}$  should be maintained by the molar flowrate of the reactant  $F$  entering the first reactor in the presence of disturbance

in feed concentration  $C_{A0}$ .

In this control design problem, the plant model is

$$\frac{C_{A2}(s)}{F(s)}$$

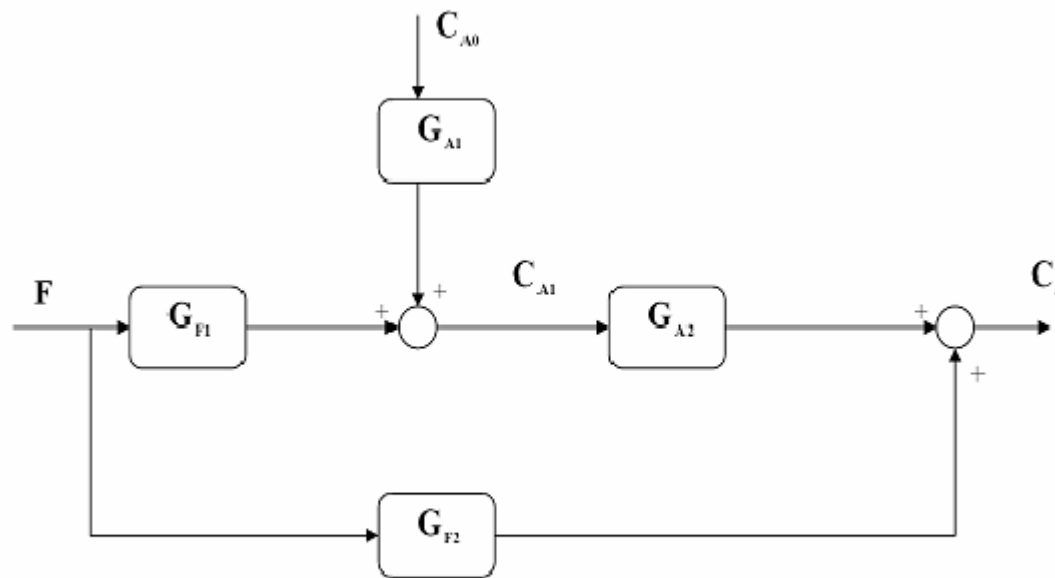
and the disturbance model is

$$\frac{C_{A0}(s)}{C_{A2}(s)}$$

In the next section we will discuss how these two models are obtained.

### Linear Plant Models

This chemical process can be represented in the following diagram with LTI blocks.



where

$$G_{A1} = \frac{C_{A1}(s)}{C_{A0}(s)} = \frac{0.6693}{8.2677s + 1}$$

$$G_{F1} = \frac{C_{A1}(s)}{F(s)} = \frac{2.4087}{8.2677s + 1}$$

$$G_{A2} = \frac{C_{A2}(s)}{C_{A1}(s)} = \frac{0.6693}{8.2677s + 1}$$

$$G_{F2} = \frac{C_{A2}(s)}{F(s)} = \frac{1.6118}{8.2677s + 1}$$

Based on the block diagram, the plant and disturbance models are obtained as follows:

$$\frac{C_{A2}(s)}{F(s)} = G_{F1}G_{A2} + G_{F2} = \frac{13.3259s + 3.2239}{(8.2677s + 1)^2}$$

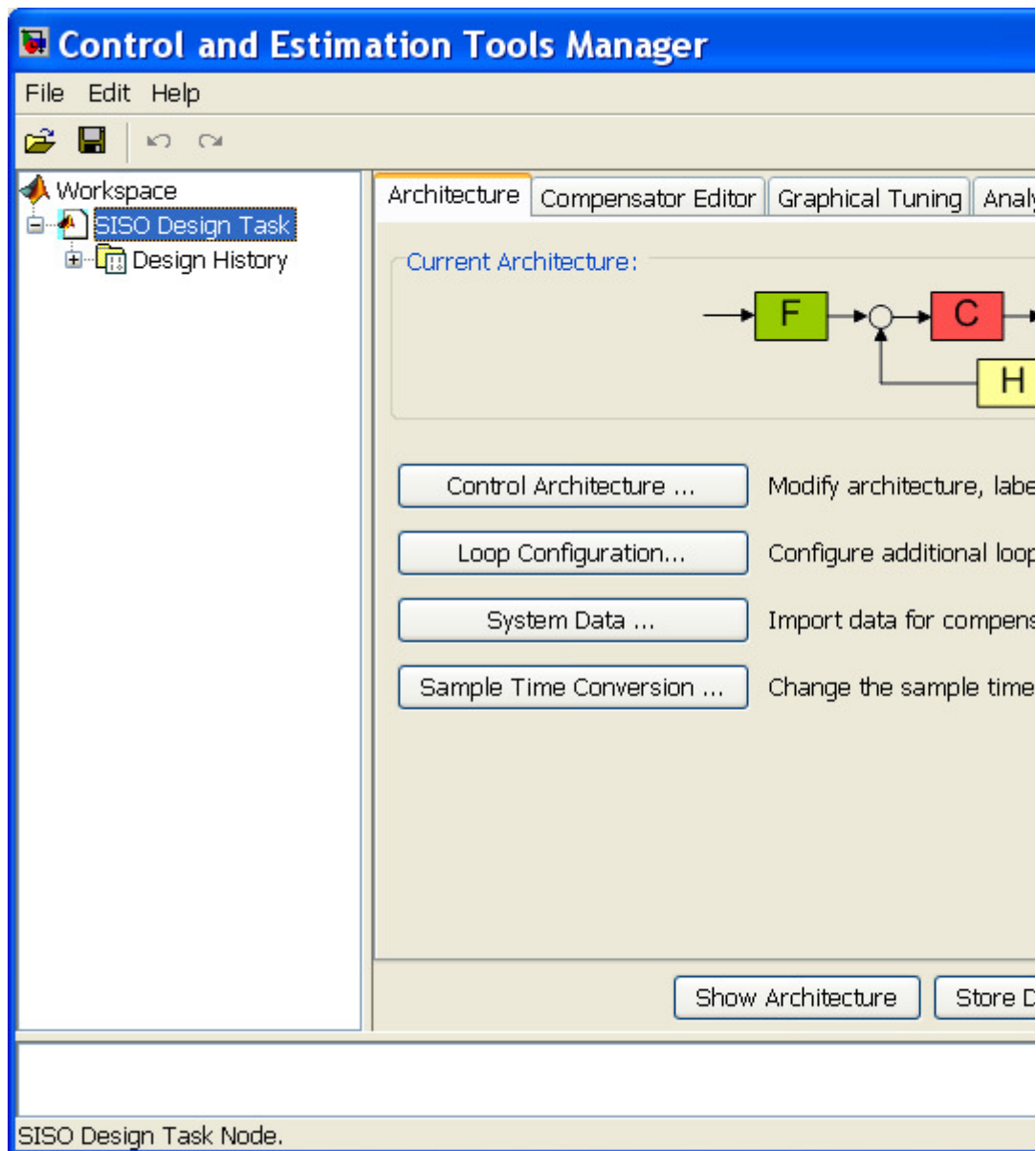
$$\frac{C_{A2}}{C_{A0}} = G_{A1}G_{A2} = \frac{0.4480}{(8.2677s + 1)^2}$$

### **IMC Design with Automatic Tuning**

We will now design the compensator in an IMC structure in SISO Design Tool.

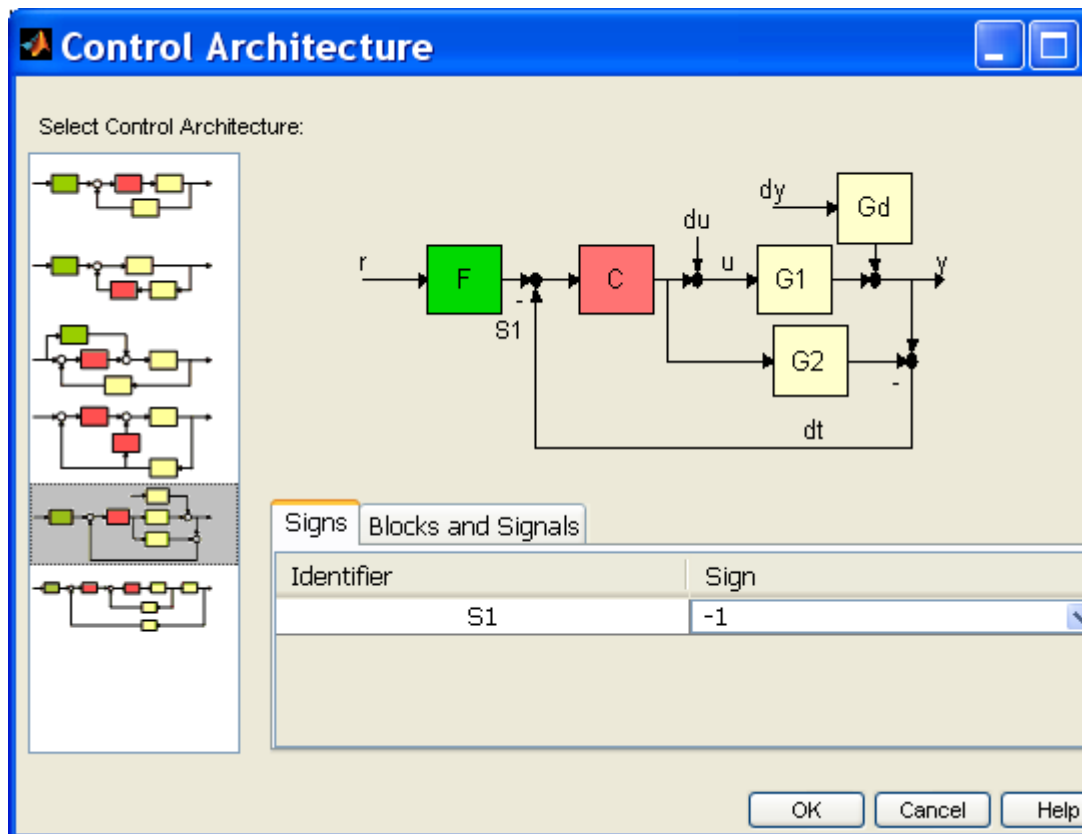
#### **Step 1: Open SISO Design Tool**

At the MATLAB® command prompt, type `sisotool` and the Controls and Estimation Tools Manager opens.

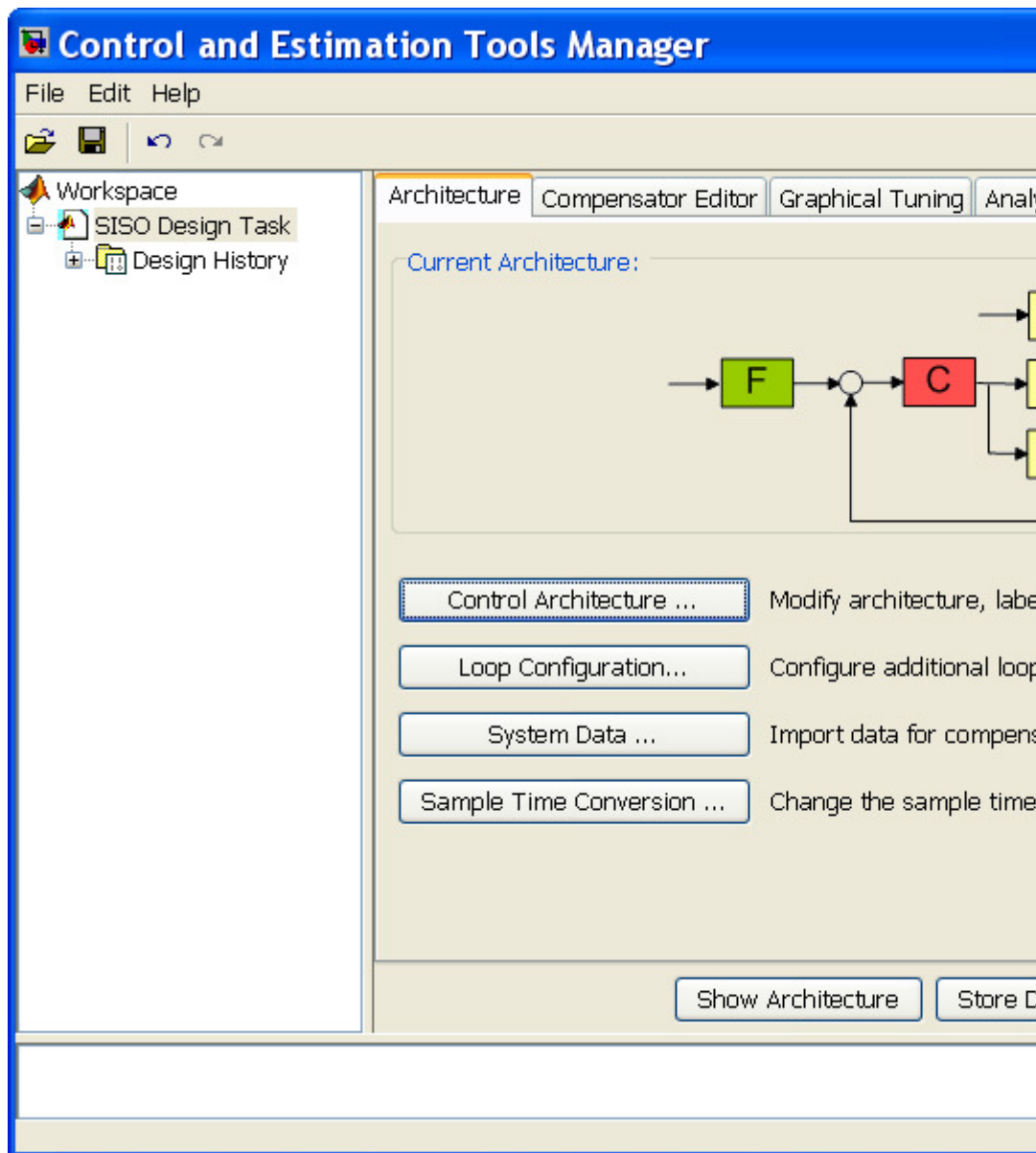


### Step 2: Select IMC as the Control Architecture

- Click on the Control Architecture... button
- Select Configuration 5 for IMC structure from the left panel in the Control Architecture dialog



- Click OK to select this configuration. The Controls and Estimation Tools Manager should look like the following figure



### Step 3: Load System Data into SISO Design Tool

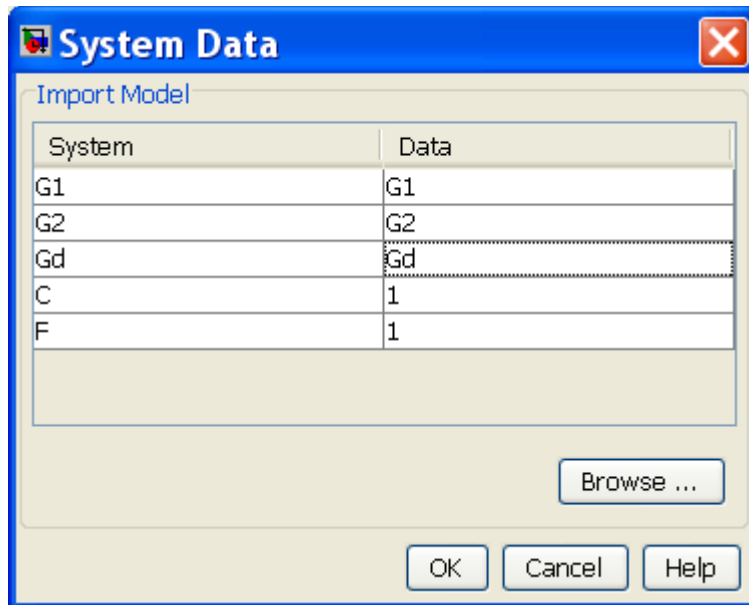
First we create the following LTI models in MATLAB command prompt:

```
s = tf('s');
G1 = (13.3259*s+3.2239)/(8.2677*s+1)^2;
G2 = G1;
Gd = 0.4480/(8.2677*s+1)^2;
```

Note: G1 is the real plant used in controller evaluation; G2 is an approximation of the real

plant and it is used as the predictive model in the IMC structure.  $G1 = G2$  means that there is no model mismatch.  $Gd$  is the disturbance model.

Then we load the system data into the Controls and Estimation Tools Manager by clicking on the `System Data...` button. The `System Data` Dialog should look like what is shown below after  $G1$ ,  $G2$  and  $Gd$  are specified in the Data column:

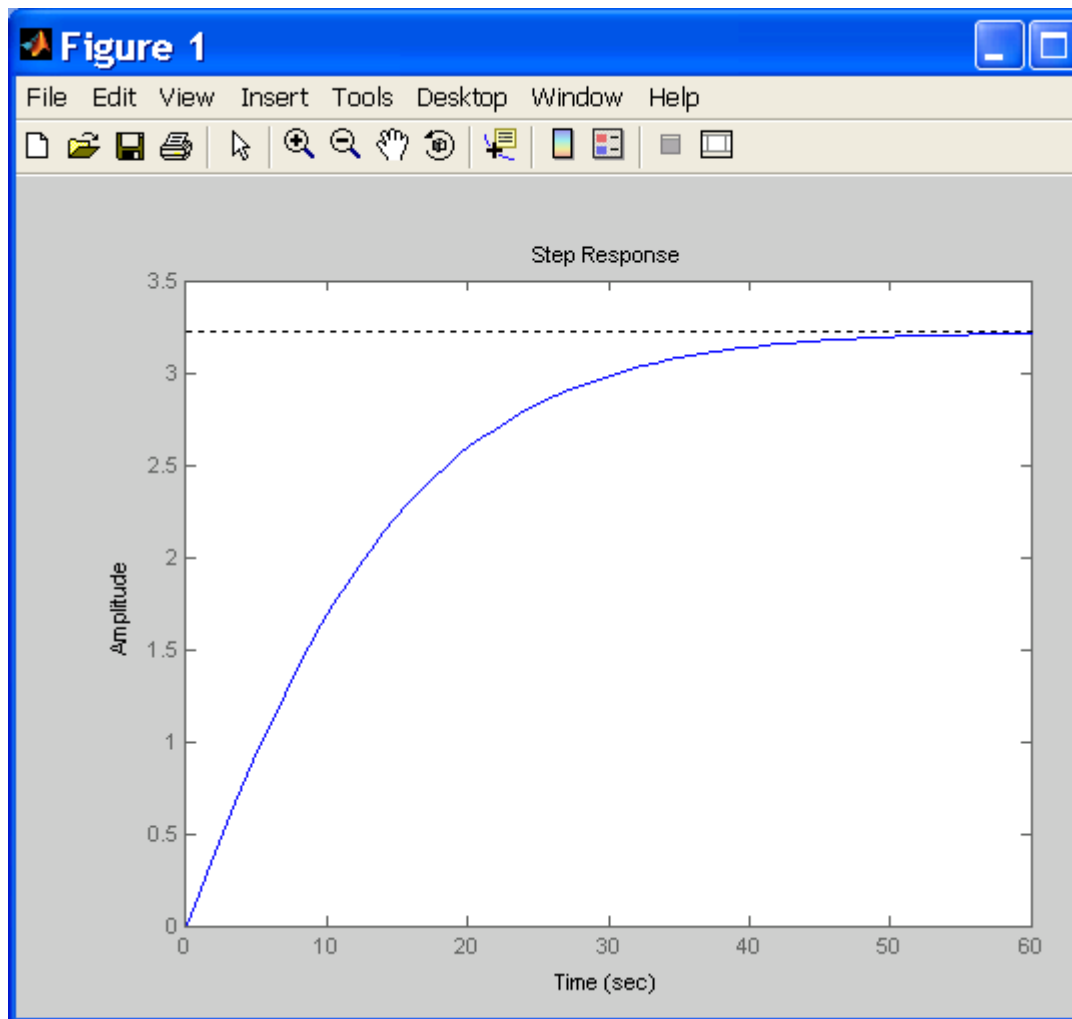


#### Step 4: Tune the IMC Compensator C

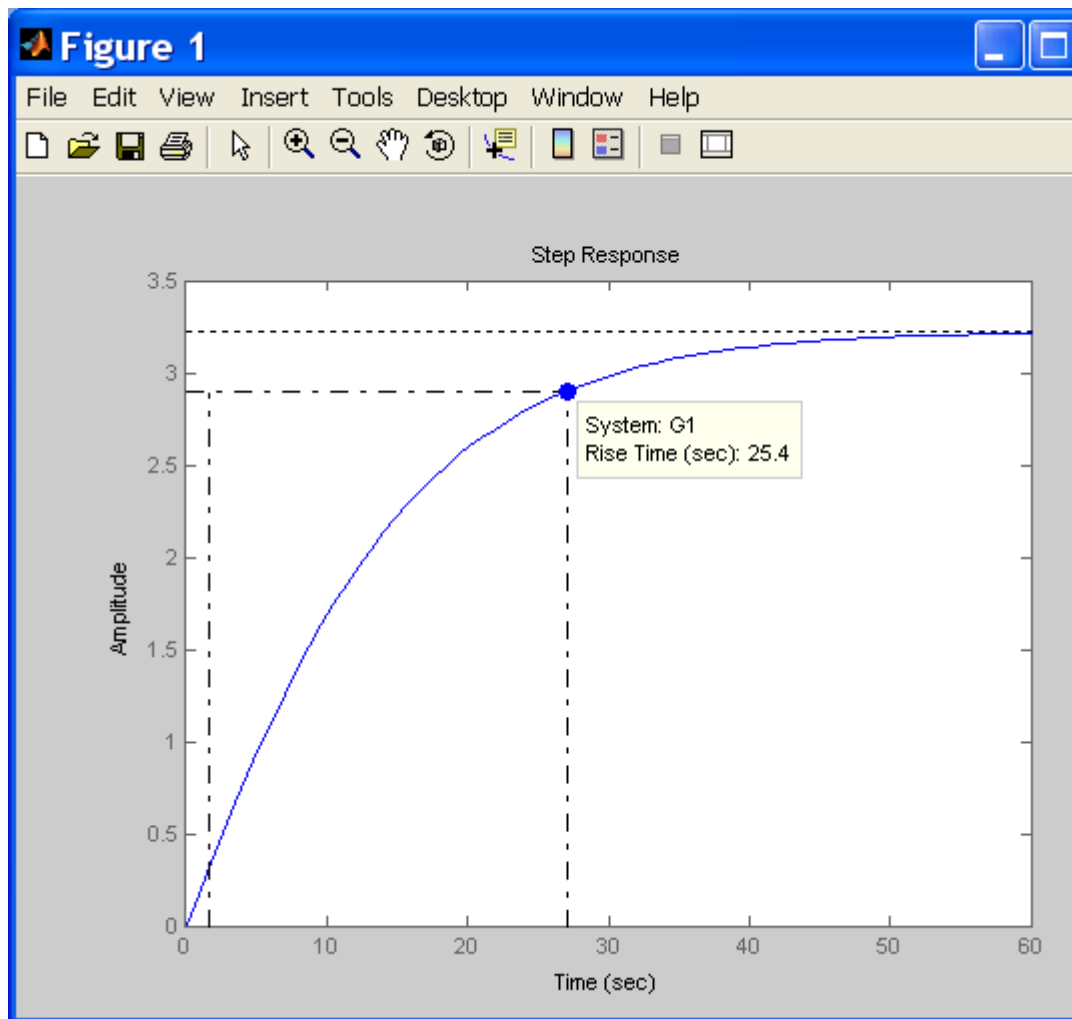
The open loop step response of  $G1$  is shown below:

`step(G1)`



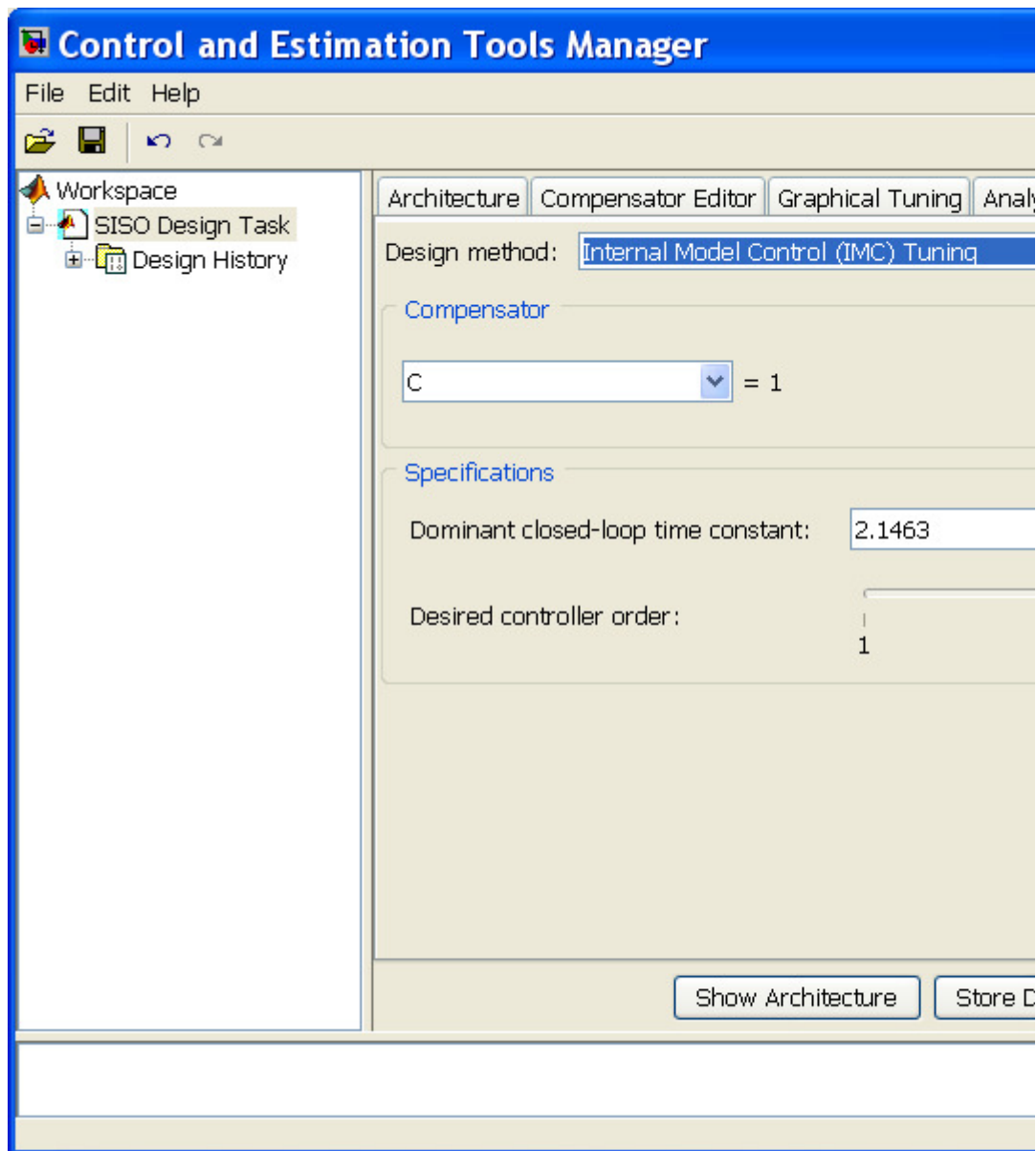


Right-click on the plot and select the `Characteristics -> Rise Time` submenu. Finally, click on the blue dot marker. The resulting plot is shown below:

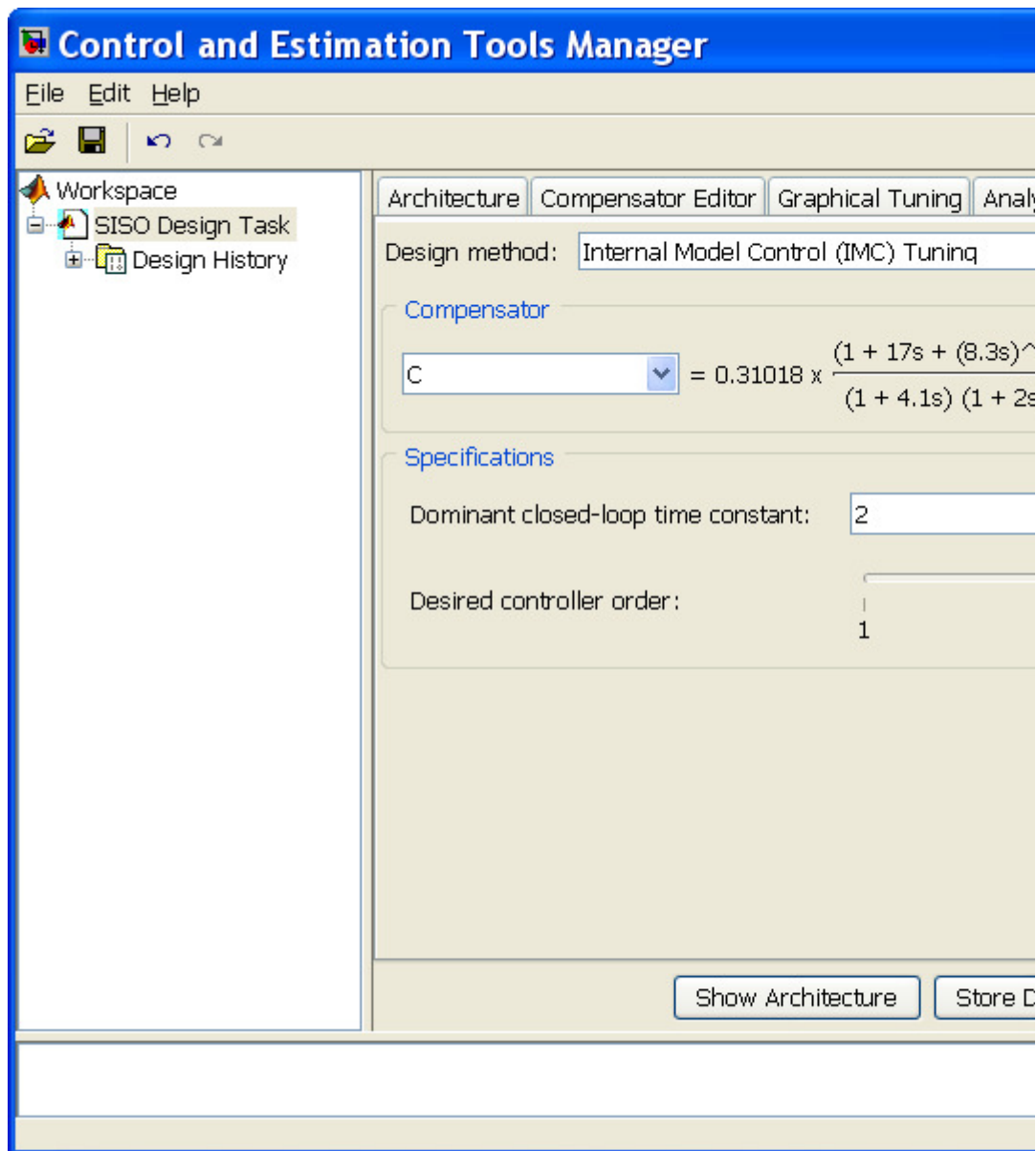


The step plot shows the rise time is about 25 seconds and we want the closed loop response becomes faster after the IMC compensator is tuned.

To tune the IMC compensator, click on the Automated Tuning on the Controls and Estimation Tools Manager and select **Internal Model Control (IMC) Tuning** as the design method. It should appear like the diagram shown below:

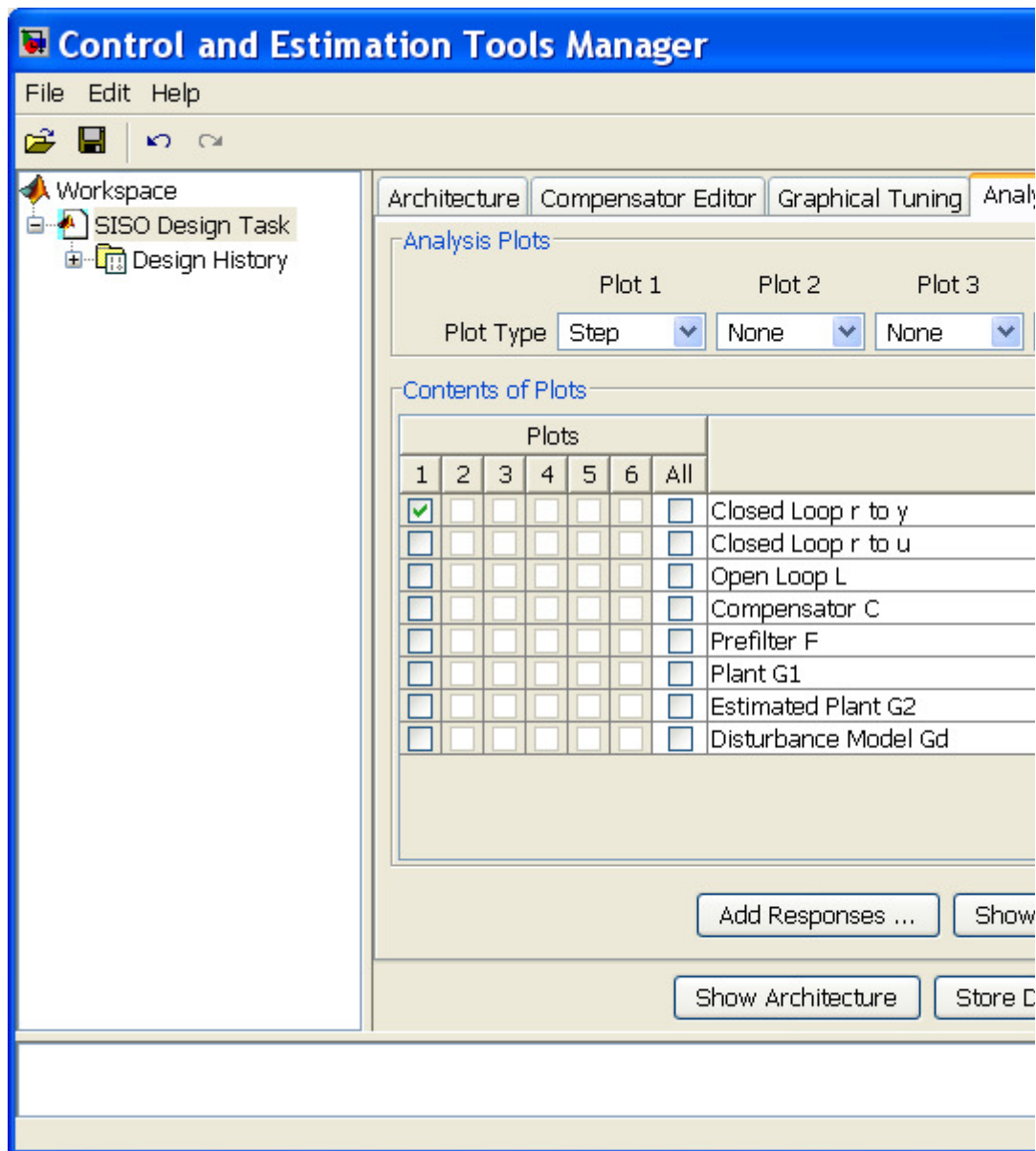


Select a closed-loop time constant of 2 and specify 2 as the desired compensator order. Click on the Update Compensator button to obtain the IMC compensator C.

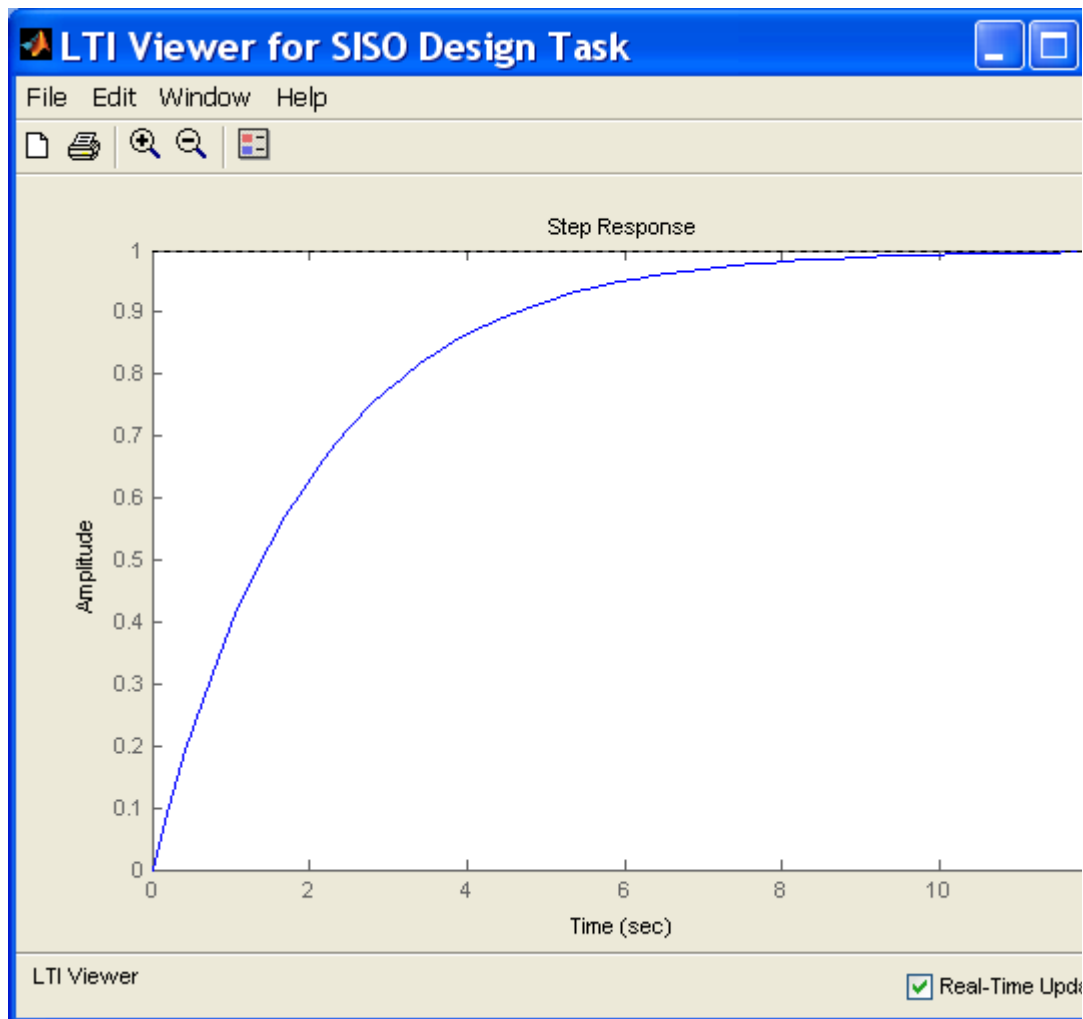


### Step 5: Check Closed Loop Step Response

To look at the closed loop step response, click on the Analysis Plots on the Controls and Estimation Tools Manager, select *Step* as the plot type for Plot 1 and make *Closed Loop r to y* as the content of Plot 1:



The step response plot looks like:



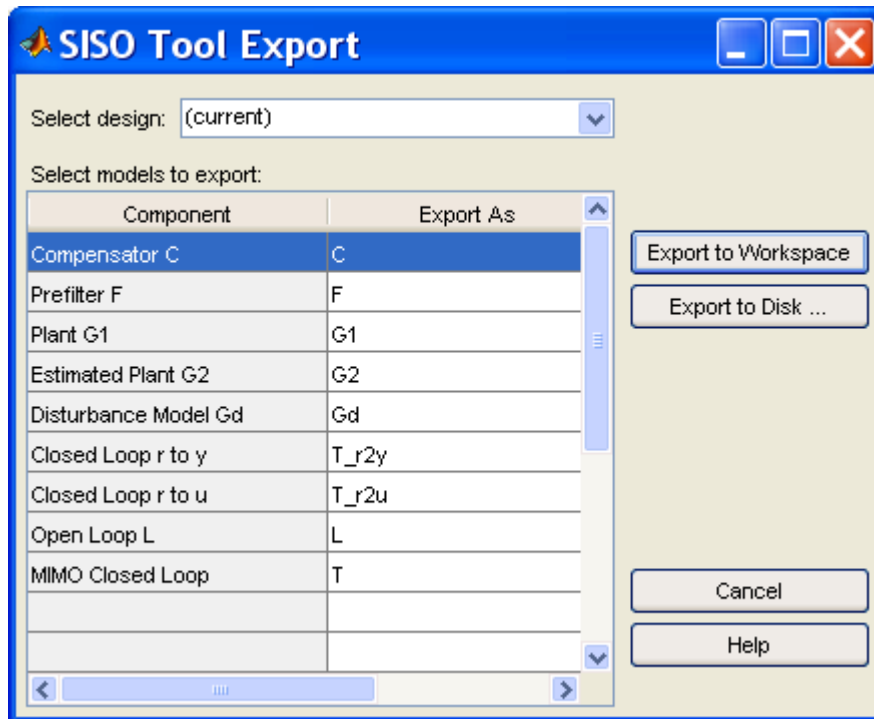
### Control Performance with Model Mismatch

In the previous section, we assume  $G_1$  is equal to  $G_2$ . In practice they are often different and the controller needs to be robust enough to track setpoints and reject disturbances. In this section we will change the real plant  $G_1$  but keep the predictive model  $G_2$  and IMC compensator  $C$  untouched.

We will create model mismatches between  $G_1$  and  $G_2$  and re-examine control performance in MATLAB command prompt with the presence of both set point change and load disturbance.

#### Step 1: Export IMC Compensator $C$ from SISO Design Tool to MATLAB Workspace

Go to the **File** menu of the Controls and Estimation Tools Manager and select **Export...** menu item. It opens the **SISO Tool Export** dialog:



Select Compensator C and click on the Export to Workspace button. An LTI object C is shown up in MATLAB workspace afterwards.

### Step 2: Convert IMC Structure to Classic Feedback Control Structure

IMC structure can be converted into a classic feedback control structure with the controller in the feedforward path and unit feedback. The new controller C\_new is obtained as follows:

```
C = zpk([-0.121 -0.121], [-0.242, -0.466], 2.39);
C_new = feedback(C,G2,+1)
```

Zero/pole/gain:

$$\frac{2.39 (s+0.121)^4}{(s-0.0001594) (s+0.121) (s+0.1213) (s+0.2419)}$$

### Step 3: Define G1 That Differs From G2

So far we assume that G2 was a perfect model of the real plant G1. Now let us consider two possible ways G1 can differ from G2 due to imperfect modeling.

No Model Mismatch (G1 is the same as G2):

```
G1p = (13.3259*s+3.2239)/(8.2677*s+1)^2;
```

G1's time constant is changed by 5%:

```
G1t = (13.3259*s+3.2239)/(8.7*s+1)^2;
```

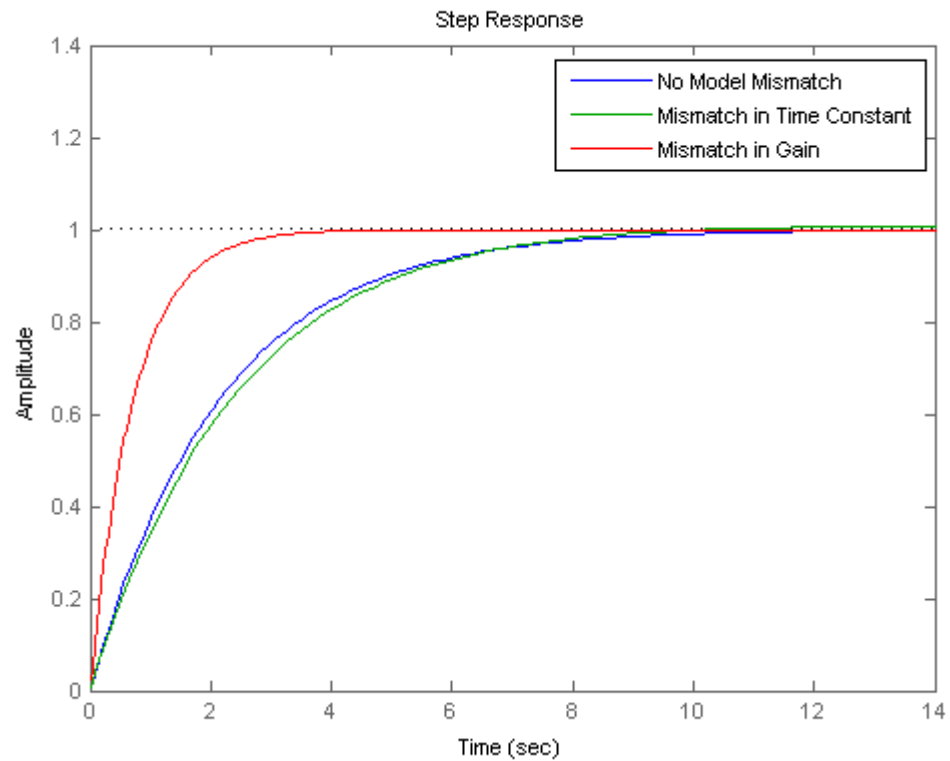
G1's gain is increased by 3 times:

```
G1g = 3*(13.3259*s+3.2239)/(8.2677*s+1)^2;
```

#### Step 4: Evaluate Performance of Set-Point Tracking and Load Disturbance Rejection

- Set Point Tracking

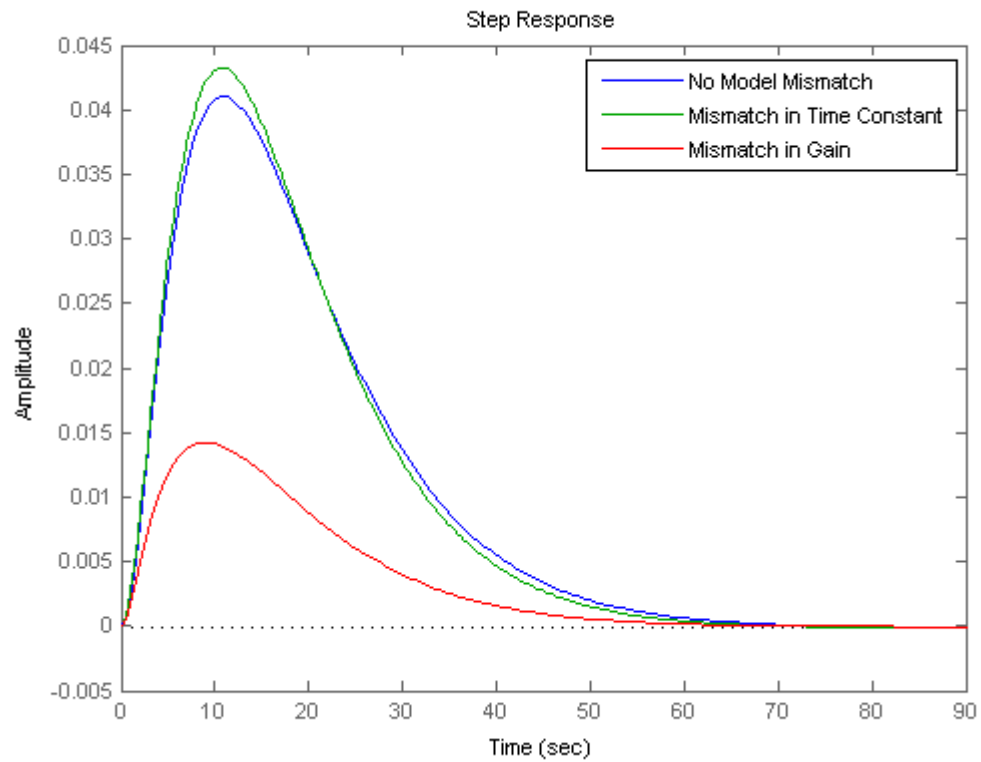
```
step(feedback(G1p*C_new,1),feedback(G1t*C_new,1),feedback(G1g*C_new,  
legend('No Model Mismatch','Mismatch in Time Constant','Mismatch in Gain'
```



- Load Disturbance Rejection



```
step(Gd*feedback(1,Glp*C_new),Gd*feedback(1,Glt*C_new),Gd*feedback(1,
legend('No Model Mismatch','Mismatch in Time Constant','Mismatch in
```



The above figures show that our controller is fairly robust to uncertainties in the plant parameters.

Copyright 1986-2009 The MathWorks, Inc.  
Published with the MATLAB R2009a Software

MATLAB and Simulink are registered trademarks of The MathWorks, Inc. Please visit [www.mathworks.com/trademarks](http://www.mathworks.com/trademarks) for a list of other trademarks owned by The MathWorks, Inc. Other product or brand names are trademarks or registered trademarks of their respective owners.