Practical Implementation of Advanced Process Control for Linear Processes

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Introduction

We have been working with development of linear LQR and MPC controllers for for the windows platform, using the C#/.NET based library $MPCMath$.
During the work we had to address a a number of practical issues, which are described in this presentation

- Terminal effects of finite control and evaluation horizons
- Noise models for Offset-Free operation
- Optimization algorithms and their CPU usage

The issues are illustrated on a simulated Four Tank Process

The Control Problem

The LQR and MPC controllers have the objective:

$$\min_{\{y,\Delta u\}} \Phi \quad = \quad \sum_{k=0}^{eh} \frac{1}{2}(y_k \ - \ r_k)'\theta_k(y_k \ - \ r_k) \ + \ \sum_{k=0}^{ch-1} \frac{1}{2}\Delta u_k' \rho_k \Delta u_k$$

Subject to process dynamics equality constraints:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k & \qquad k &= 0, \dots.., eh-1 \\ y_k &= Cx_k & \qquad k &= 0, \dots.., eh \end{aligned}$$

k	Time or controller step
ch	Control Horizon
eh	Evaluation Horizon ($eh \geq ch$)
y_k	Measured plant outputs
r_k	References
Δu_k	Movements of process inputs, defined as $\Delta u_k = u_k - u_{k-1}$.

Process Constraints for MPC controllers

Hard constraints:

$$\begin{aligned} u_{\min} \leq u_k \leq u_{max} \quad & k = 0, \dots, ch-1 \\ \Delta u_{\min} \leq \Delta u_k \leq \Delta u_{\max} \quad & k = 0, \dots, ch-1 \end{aligned}$$

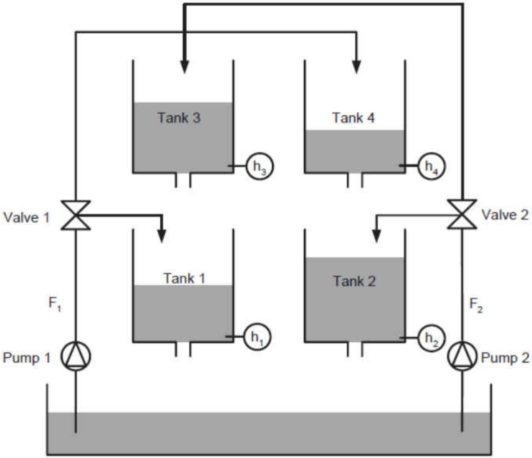
Soft constraints

$$\begin{aligned} y_{\min} + \eta_{Lk} \leq y_k \leq y_{\max} - \eta_{Lk} \quad & k = 0, \dots, eh \\ dy_{\min} + \eta_{Dk} \leq y_k - r_k \leq dy_{\max} - \eta_{Dk} \quad & k = 0, \dots, eh \\ 0 \leq \eta_{Lk} \quad & k = 0, \dots, eh \\ 0 \leq \eta_{Dk} \quad & k = 0, \dots, eh \end{aligned}$$

The two soft constraints requires an expansion of objective to:

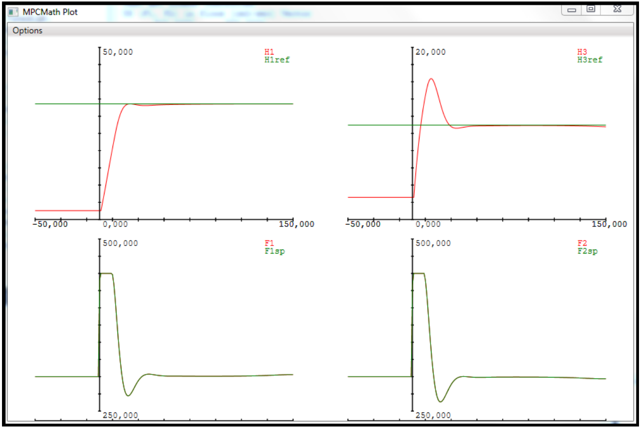
$$\begin{aligned} \min_{\{y, \eta_L, \eta_D, \Delta u\}} \Phi = & \sum_{k=0}^{eh} \frac{1}{2} (y_k - r_k)' \theta_k (y_k - r_k) \\ & + \sum_{k=0}^{ch-1} \frac{1}{2} \Delta u_k' \rho_k \Delta u_k + \sum_{k=0}^{eh} \frac{1}{2} \eta_{Lk}' \mu_{Lk} \eta_{Lk} + \sum_{k=0}^{eh} \frac{1}{2} \eta_{Dk}' \mu_{Dk} \eta_{Dk} \end{aligned}$$

Four Tank Process



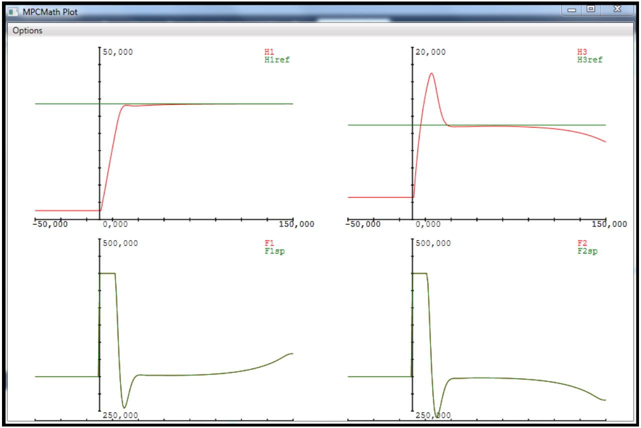
Four tank process used to illustrate problems

MPC controller response



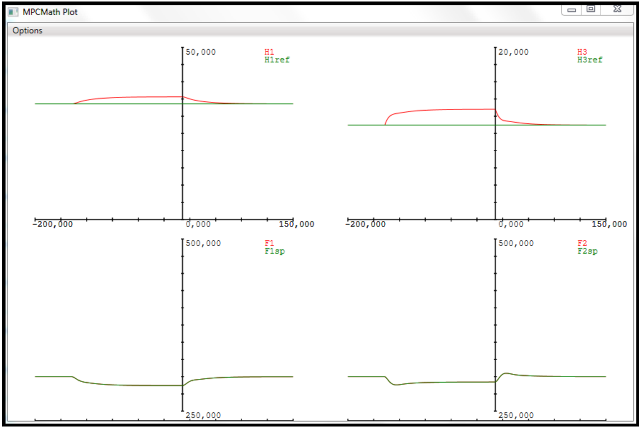
MPC controller. $horizon = 150$, $\theta = (100, 100, 1, 1)$, $\rho = (1, 1)$

Terminal effect



Terminal effect from finite control and prediction horizons.
$horizon = 150,\ \theta = (100, 100, 1, 1)\ ,\ \rho = (0.1, 0.1)$
Increasing the evaluation horizon to 300 removes the deficiency.

Set-point changes and unmeasured disturbances gives stationary offsets



An unmeasured disturbance is simulated by adding extra water into tank $H3$. A stationary offset is achieved, but the controller thinks it can remedy the offset in the future.

Noise Model

The plant model is expanded with a vector of unmeasured disturbances d_k.

$$x_{k+1} = Ax_k + Bu_k + B_d d_k$$
$$y_k = Cx_k + C_d d_k$$

The prediction equations are:

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k} + Bu_k + B_d\hat{d}_{k|k}$$
$$\hat{d}_{k+1|k} = \hat{d}_{k|k}$$

The innovation and Kalman filtering equations

$$\hat{\epsilon} = y_k - C\hat{x}_{k|k-1} - C_d\hat{d}_{k|k-1}$$
$$\hat{x}_{k|k-1} = \hat{x}_{k|k} + L_x\hat{\epsilon}_k$$
$$\hat{d}_{k|k-1} = \hat{d}_{k|k-1} + L_d\hat{\epsilon}_k$$

A pragmatic solution is to specify L_d as diagonal matrix, with integration factors $I_{fac} \geq 0.0$

Common Noise Models

The different noise models used in practice are described by substituting B_d and C_d with:

Noise model	B_d	C_d
Input noise	B	0
Output noise	0	I
ARX process	K	I

where the ARX process is described by the model:

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k} + Bu_k + K\hat{\epsilon}_{k|k}$$
$$\hat{y}_{k|k} = CX_{k|k} + \hat{\epsilon}_{k|k}$$

The output noise model cannot be applied for plants with pure integrators because the resulting model is non detectable.

Target Calculation

The optimal achievable target for the controller can be calculated solving the small quadratic problem:

$$\min_{\{x_{tg},u_{tg}\}} (y_{tg} - y_{sp})' Q_s (y_{tg} - y_{sp}) + (u_{tg} - u_{sp})' R_s (u_{tg} - u_{sp})$$

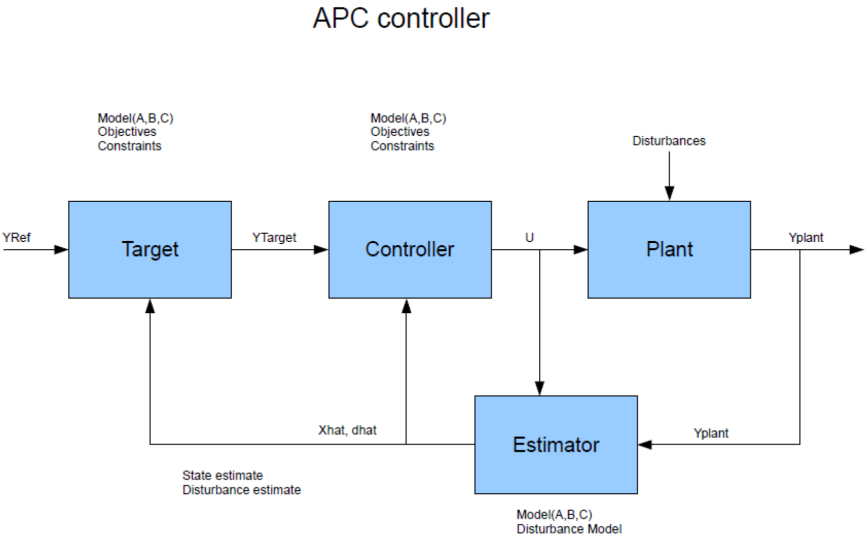
subject to

$$(I - A)x_{tg} - Bu_{tg} = B_d \hat{d}_{k|k}$$

and the process constraints given in the initial formulation of the LQR or MPC problem.

Offset-free operation off course has to obey the rules given by the degrees of freedom. With two process inputs its only possible to achieve offset-free operation for two of the controlled variables for the four tank process, i.e. $H1$ and $H2$ in this case

APC Structure



Optimization algorithms

In this work we have implemented three optimization algorithms

- **LQR Riccati**

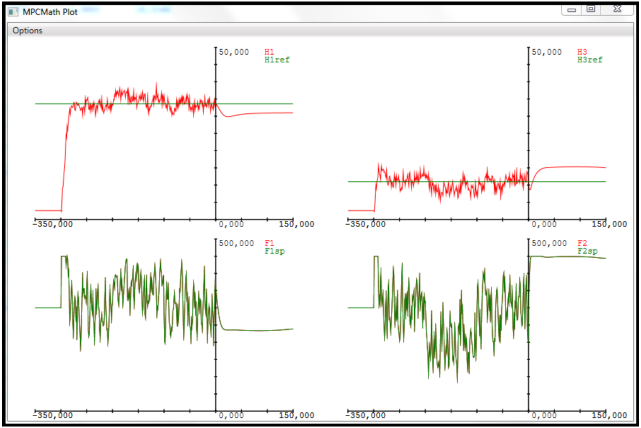
  Process dynamics implemented as equality constraints.
  Execution time $\approx o(ch\ n_x^3)$
- **MPC Riccati**

  Process dynamics implemented as equality constraints.
  Execution time $\approx o(ch\ n_x^3)$
- **MPC Condensed**

  State variables are eliminated using FIR models.
  Execution time $\approx o((ch\ n_u)^3)$

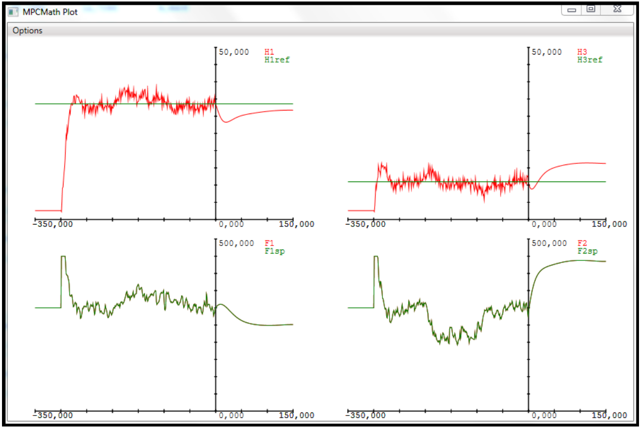
The condensed method requires a stable plant. If not a stabilizing proportional controller must be included.

Test scenario, LQR and standard MPC

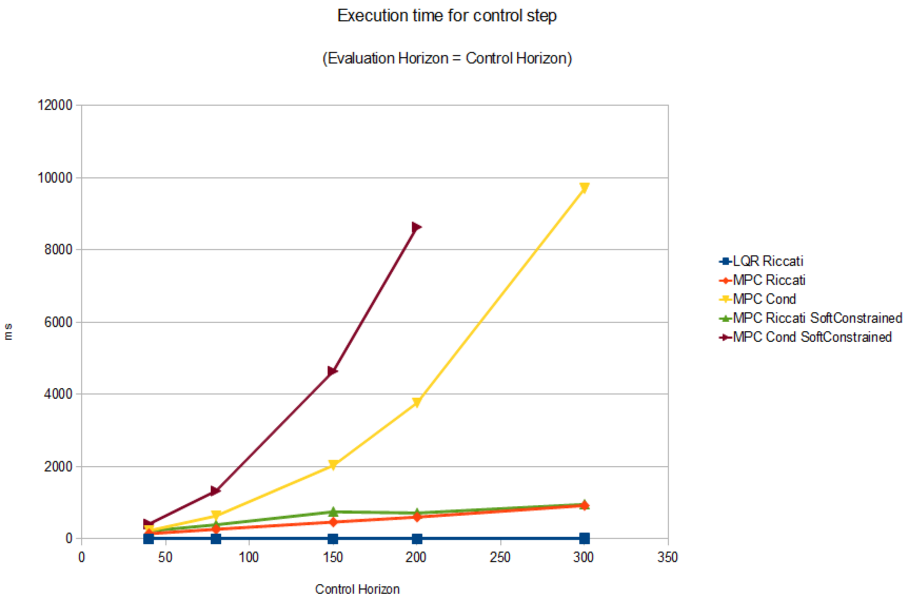


MPC controller performance with measurement and process noise

Test scenario, LQR and standard MPC



MPC controller performance with soft constraints on deviations from references


Execution time for control step
(Evaluation Horizon = Control Horizon)
12000
10000
8000
6000
4000
2000
0
ms
0
50
100
150
200
250
300
350
Control Horizon
LQR Riccati
MPC Riccati
MPC Cond
MPC Riccati SoftConstrained
MPC Cond SoftConstrained

Conclusions

- Terminal effects can be eliminated by separate control and evaluation horizons.
- Offset-free control can be obtained by use of noise models and integrators
- Soft limits on process outputs guarantees feasible solutions to Control problem
- Soft limits on deviation from references reduces control actions
- The Riccati based LQR controller is extremely cpu efficient, but lacks treatment of process constraints
- The Riccati based MPC algorithm is much more cpu efficient, than the condensed MPC for long control horizons. The Condensed MPC might be optimal for plant with high state dimension
- The controller is implemented in an industrial environment using C#/.NET and *MPCMath*

Questions and Comments

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