

# Unsupervised learning

Inteligencia Artificial en los Sistemas de Control Autónomo  
Máster en Ciencia y Tecnología desde el Espacio

Departamento de Automática

## Objectives

1. Extend unsupervised learning algorithms
2. Apply unsupervised learning to real-world problems

## Bibliography

- Géron, Aurélien. *Hands-On Machine Learning with Scikit-Learn, Keras & TensorFlow*. O'Reilly. 2020
- Müller, Andreas C., Guido, Sarah. *Introduction to Machine Learning with Python*. O'Reilly. 2016

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## Clustering

Set of unsupervised techniques that identify groups of data (named **clusters**)

- No universal definition of cluster: Centroid, medoid, dense regions, etc

## Applications

- Customer segmentation, data analysis, dimensionality reduction, anomaly detection, semi-supervised learning, search engines, image segmentation

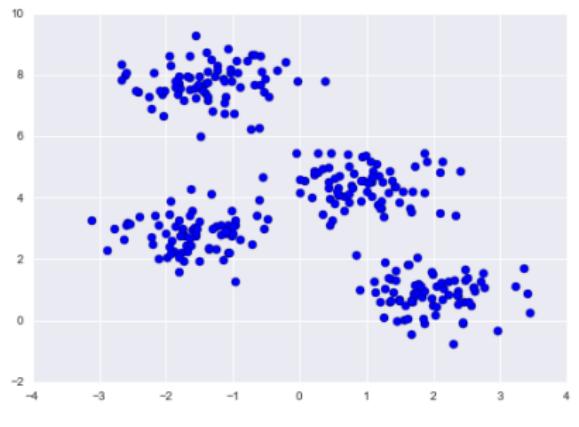
## Main algorithms

- K-means, DBScan, GMM, hierarchical clustering, ...

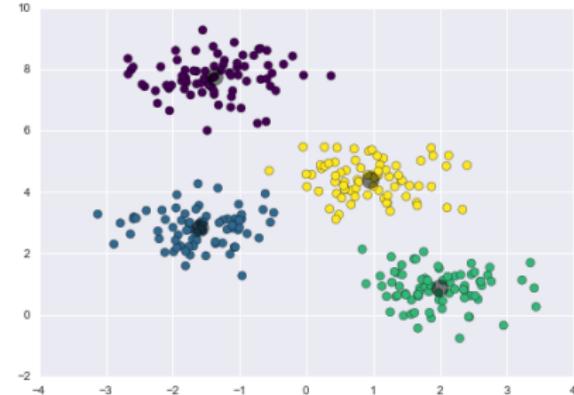
# K-means

## Overview

Original data



Clustered data



In k-means, clusters are identified by a **centroid**

## K-means

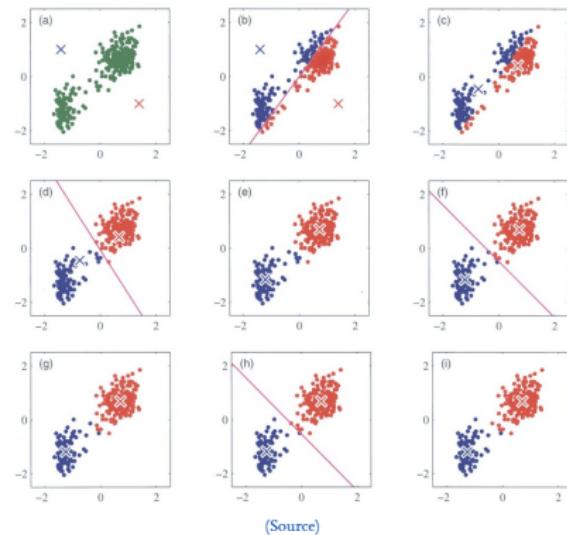
## K-means algorithm (I)

## K-means algorithm

1. Set  $k$  random centroids
  2. Assign each data point to its closest centroid
  3. Recompute centroids
  4. Go to 2 until no point reassignment

$k$  is an hyperparameter

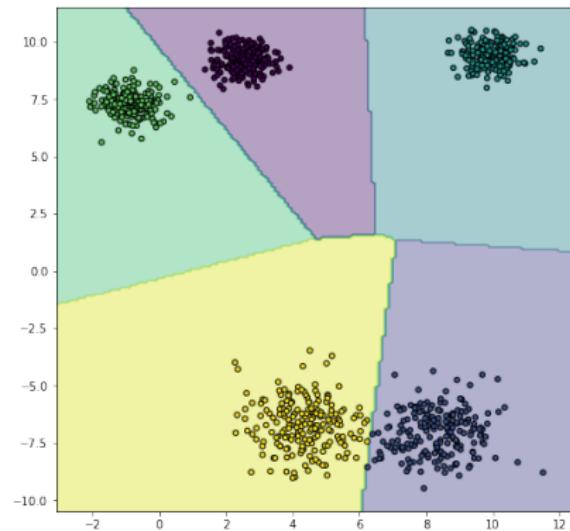
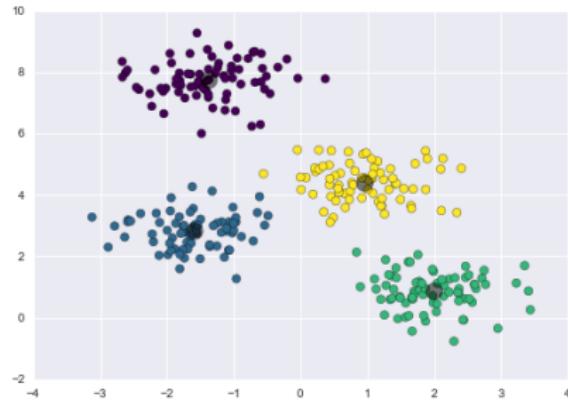
- Number of clusters



# K-means

## K-means algorithm (II)

New data points are assigned to its closest centroid

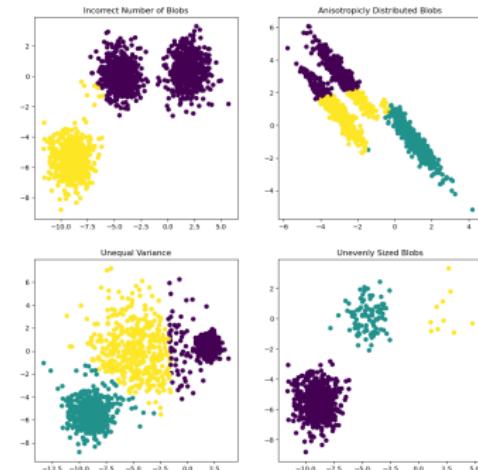


# K-means

## K-means limitations

K-means can fail in several conditions

- Incorrect number of clusters
- Different clusters variance
- Non-spheric clusters ⇒ normalization



(Source)

# K-means

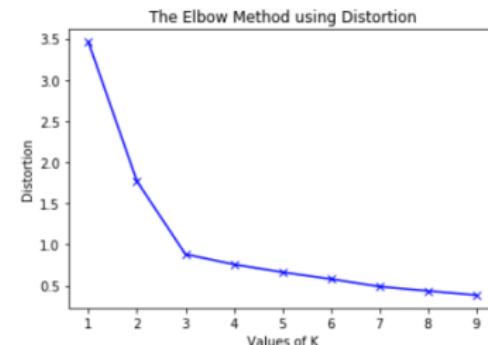
## Elbow's method

### Election of k

- Not a problem when domain information is available
- ... that is rarely the case

### Elbow's method

1. Select  $K = 1, \dots, n$
2. Visualize performance for each  $k$
3. Choose  $K$  where metric stabilizes



### Performance measures

- Inertia: mean squared error between each instance and its closest centroid
- Silhouette:  $(b - a) / \max(a, b)$ , where  $a$  mean intra-cluster distance, and  $b$  is the mean nearest-cluster distance

# K-means

## Application: Image segmentation



(Source)



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# K-means

## Application: Clustering for semi-supervised learning

Semi-supervised learning: Only a subset of the dataset is labeled

- Supervised and unsupervised learning
- Quite common in real-world applications (labels used to be expensive)

$f_1$	$f_2$	$\dots$	$f_n$	$y$
$a_{1,1}$	$a_{2,1}$	$\dots$	$a_{n,1}$	$y_1$
$a_{1,2}$	$a_{2,2}$	$\dots$	$a_{n,2}$	
$a_{1,3}$	$a_{2,3}$	$\dots$	$a_{n,3}$	
$a_{1,4}$	$a_{2,4}$	$\dots$	$a_{n,4}$	$y_4$
$a_{1,5}$	$a_{2,5}$	$\dots$	$a_{n,5}$	

## Label propagation

1. Obtain  $k$  clusters
2. Get a representative instance of each cluster (**medoid**) measuring the distance to the centroid
3. Label the members of each cluster with its medoid's label

# K-means

## K-means: Summary

Hyperparameters	Advantages	Disadvantages
$k$	Fast Few hyperparameters Scalable	Simple shapes Determine $k$ Random initialization

# Other clustering algorithms

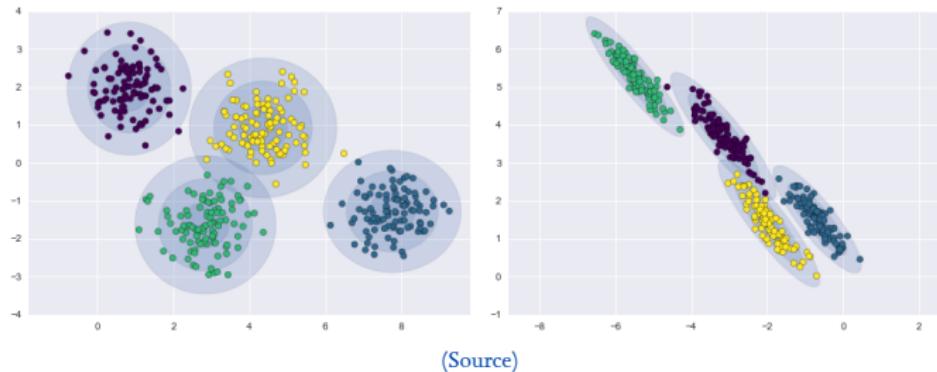
## Gaussian Mixure Model (GMM) (I)

GMM is a generative clustering algorithm

- Assumes data coming from a set of multidimensional gaussian distributions

GMM fits a set  $\{(\phi_i, \mu_i, \sigma_i)\}_{i=1,\dots,k}$ , where  $k$  is the number of clusters and

- $\phi$  is a weight
- $\mu$  is a multidimensional mean
- $\sigma$  is a covariance matrix



Unsupervised learning

# Other clustering algorithms

## Gaussian Mixure Model (GMM) (II)

Gaussian parameters are fit with the Expectation-Maximization (E-M) algorithm

- E-M is a generalization of K-means

### Expectation-Maximization algorithm

1. Init parameters randomly
2. Expectation step: Assign each instance to a cluster
  - Assignment is probabilistic
3. Maximization step: Update cluster parameters
  - Each cluster is updated using all the data
  - Instances contribution to a cluster parameters is weighted by the probability that it belongs to it
4. Go to 2

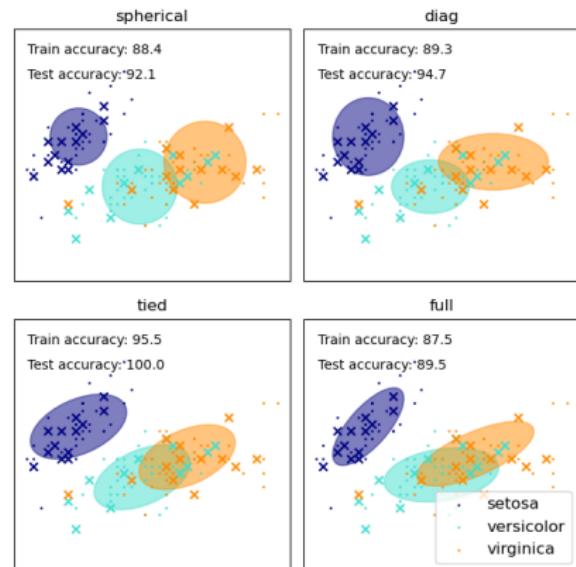
GMM can be seen as a fuzzy clustering algorithm

# Other clustering algorithms

## Gaussian Mixure Model (GMM) (III)

Covariance matrices can be constrained

- Full: No restriction
- Spherical: Sheprical shapes, different diameters
- Diag: Ellipsoidal shapes, axes parallel to the coordinate system
- Tied: Same ellipsoidal shapes, size and orientation

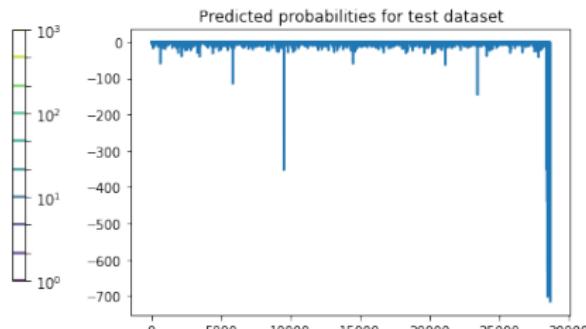
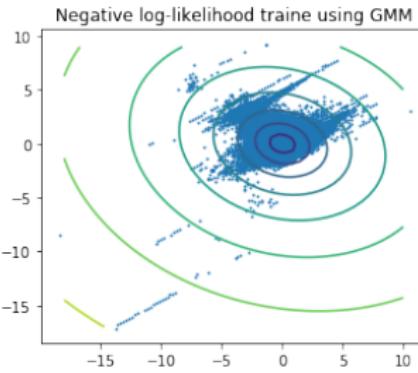


# Other clustering algorithms

## GMM for anomaly detection

GMM provides a probability of an instance to belong to a cluster

- This can be used to detect anomalies
- Just assign a probability threshold



# Other clustering algorithms

## GMM: Summary

Hyperparameters	Advantages	Disadvantages
Number of clusters	Probabilistic clustering	Number of clusters
Covariance matrix type	Generative model	Gaussian data
	Anomaly detection	Sensitive to outliers

# Other clustering algorithms

## DBSCAN (I)

DBSCAN: Density-Based Spatial Clustering of Applications with Noise

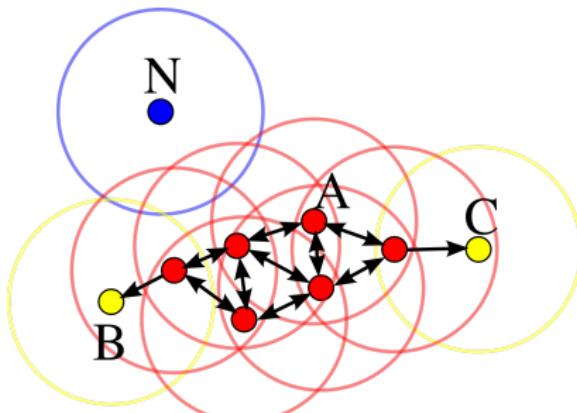
- Identifies high density regions (dense regions) in feature space
- Assumption: Clusters form dense regions separated by empty areas

Hyperparameters

- `min_samples`: Minimum cluster size
- $\epsilon$ : Radius of a neighborhood

Type of points

- Core instance
- Frontier instance
- Outliers

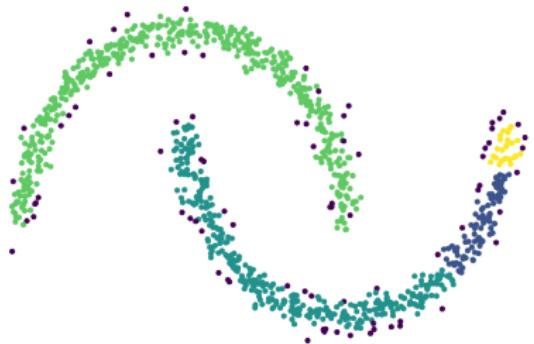


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# Other clustering algorithms

## DBSCAN (II)

$\epsilon=0.05, \text{min\_samples} = 5$



$\epsilon=0.2, \text{min\_samples} = 5$



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# Other clustering algorithms

## DBSCAN: Summary

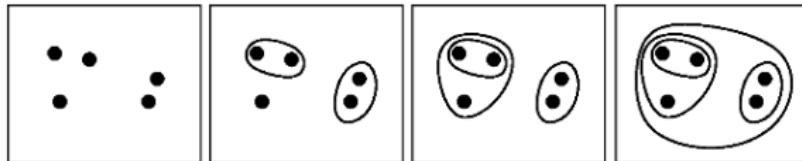
Hyperparameters	Advantages	Disadvantages
$\epsilon$	Unknown number of clusters	Slower than K-means
min_samples	Scales relatively well Almost deterministic  Robust to outliers Anomaly detection	No model Clusters with different densities

# Other clustering algorithms

## Agglomerative clustering (I)

### Agglomerative clustering

1. Initially, each instance forms a cluster
2. Merge the two most similar clusters according to a metric
3. Repeat 2 until a stop criterion is satisfied



We need a similarity measure between two clusters

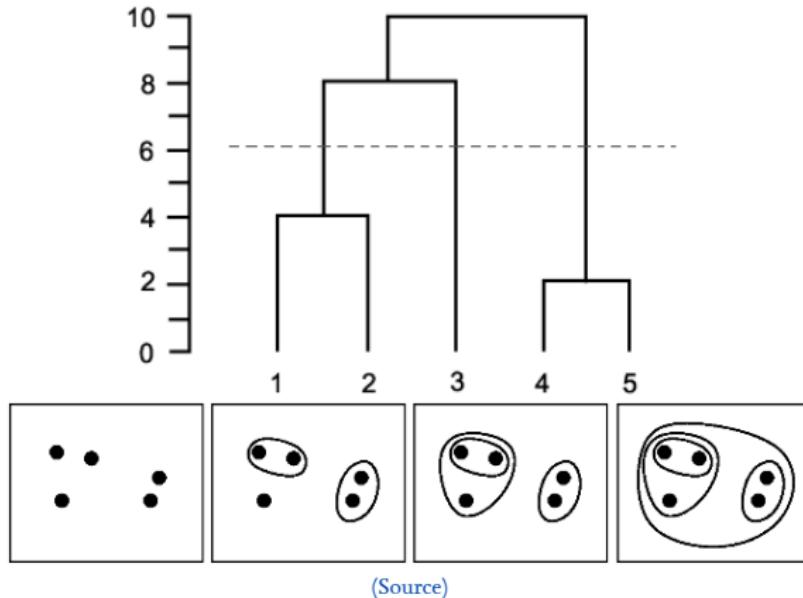
- Ward: Variance within merged clusters. Leads to equally sized clusters
- Average: Average distances
- Complete: Maximum distance
- Single: Minimum distance

# Other clustering algorithms

## Agglomerative clustering (II)

Agglomerative clustering is a special case of hierarchical clustering

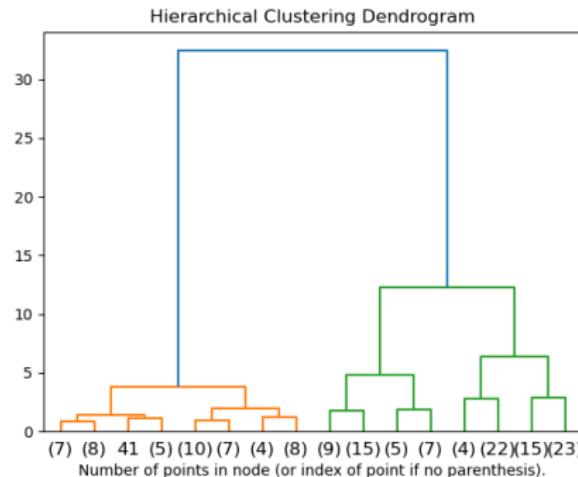
Dendrogram



# Other clustering algorithms

## Agglomerative clustering (III)

Iris dataset dendrogram



# Other clustering algorithms

## Agglomerative clustering: Summary

Hyperparameters	Advantages	Disadvantages
Similarity	Complex shapes Hierarchical clustering	Slow No model

# Anomaly detection

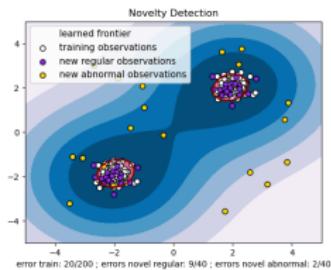
Two related concepts

- Outlayer detection and novelty detection

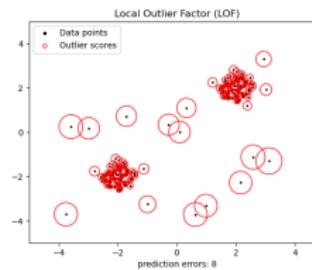
Adaptation of clustering and classification algorithms

- PCA, GMM, autoencoders, etc

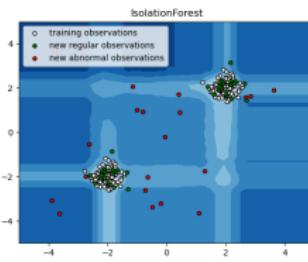
One-Class SVM



LOF



Isolation Forest



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# Dimensionality reduction

## Main approaches (I)

Two main approaches to dimensionality reduction: Projection and manifold learning

### Projection

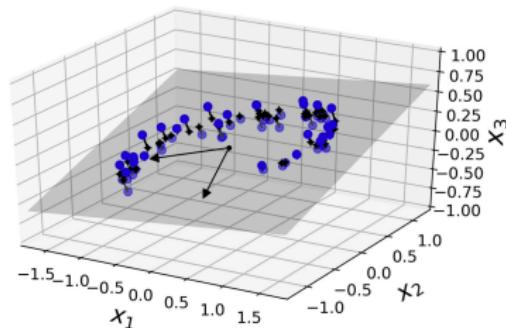


Figure 8-2. A 3D dataset lying close to a 2D subspace

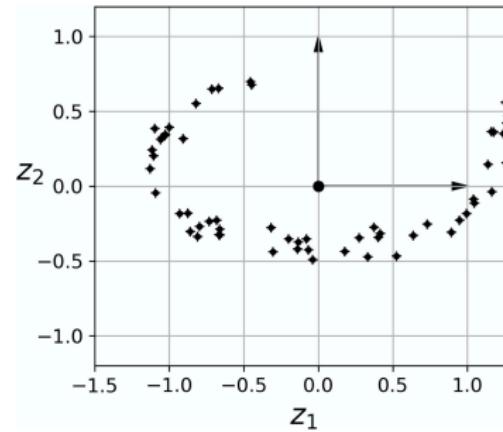


Figure 8-3. The new 2D dataset after projection

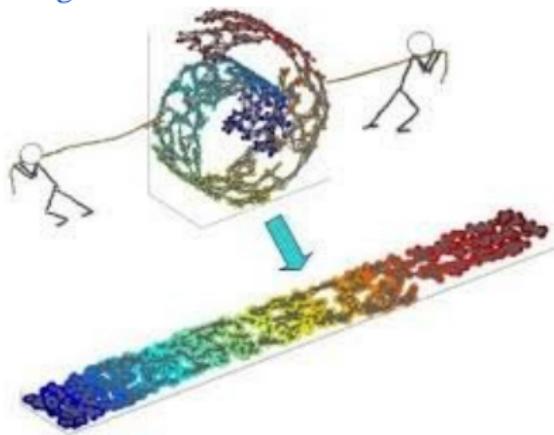
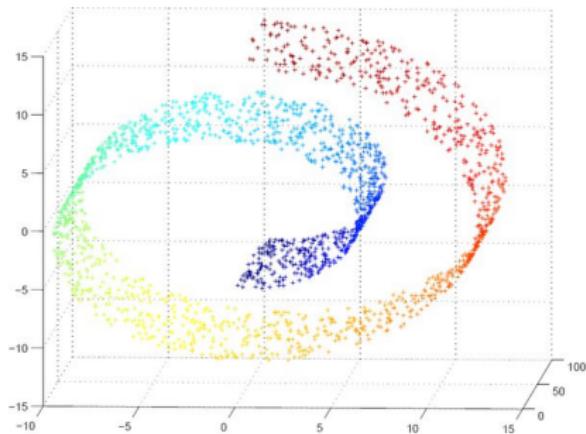
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Algorithms: Principal Components Analysis (PCA), kernelized PCA, ...

## Dimensionality reduction

## Main approaches (II)

Manifold learning



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# Manifold learning algorithms

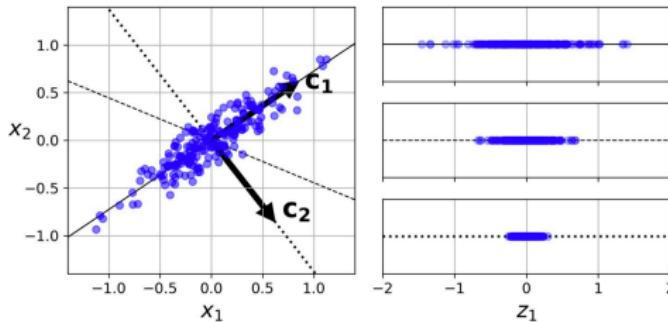
- Isomap, T-distributed Stochastic Neighbor Embedding (t-SNE), Multi-dimensional Scaling (MDS), Locally Linear Embedding (LLE), ...

# Dimensionality reduction

## Principal Components Analysis

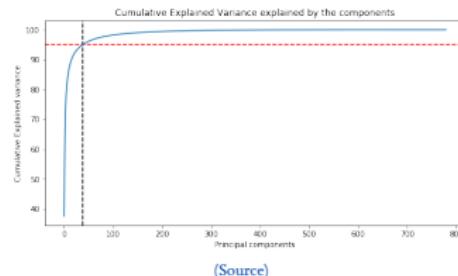
PCA create a new coordinate system

- New axes capture maximum variance and are orthogonal
    - They are named **principal components**
  - The amount of variance captured by each principal component is captured
  - PCA does not change the original dimensionality



*Figure 8-7. Selecting the subspace to project onto*

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# Dimensionality reduction

## Principal Components Analysis: Applications (I)

### PCA application: Image compression

Original image



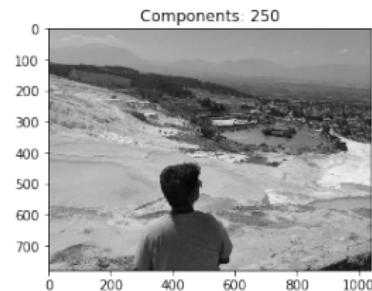
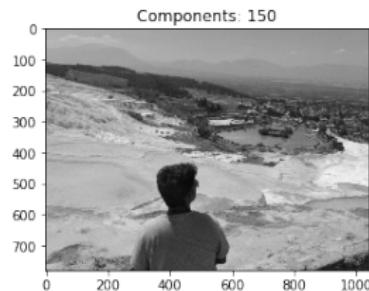
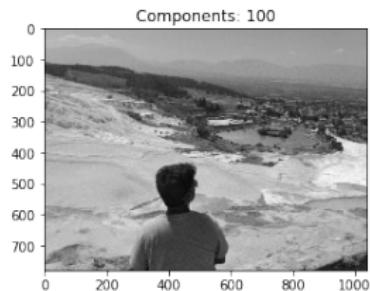
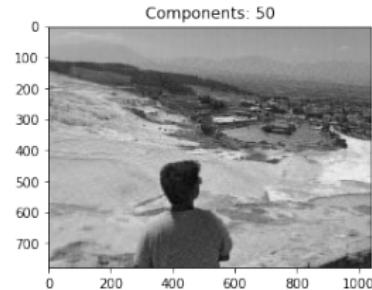
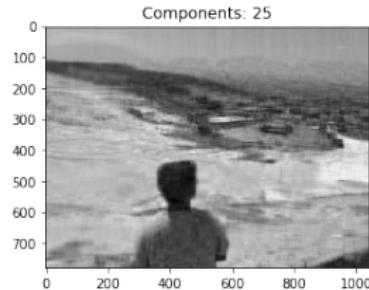
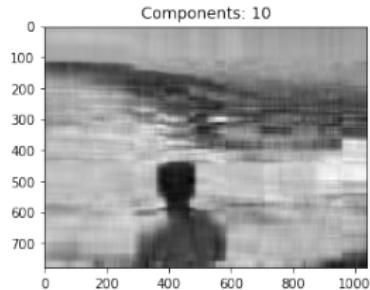
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Compressed image (38 dimensions)



# Dimensionality reduction

## Principal Components Analysis: Applications (II)



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# Dimensionality reduction

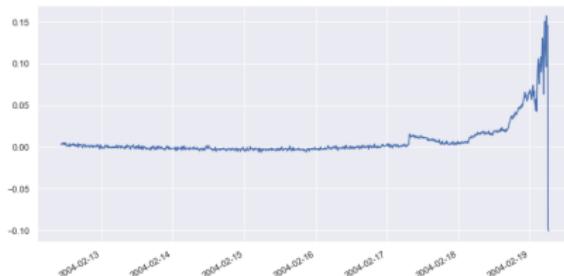
## Principal Components Analysis: Applications (III)

Application: Anomaly detection to predict bearing failure, vibrations of four bearings

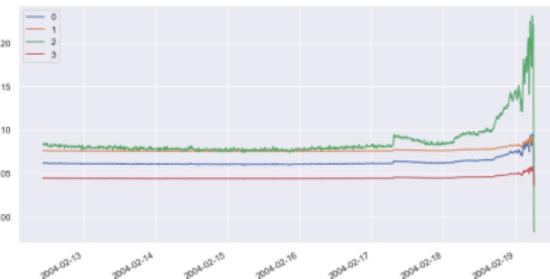
Vibration time series



First component



Reconstructed time series



Reconstruction error



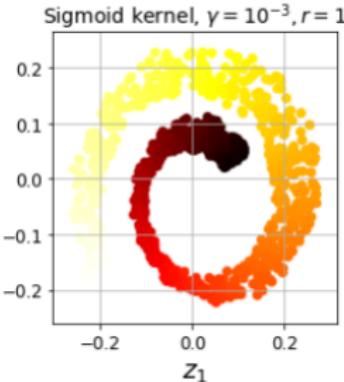
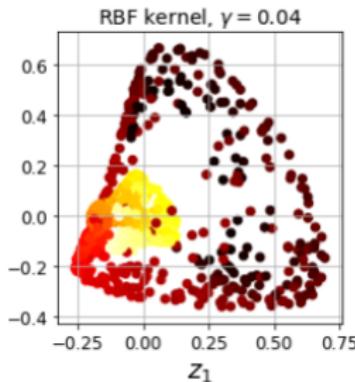
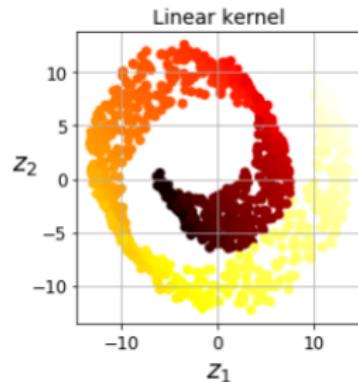
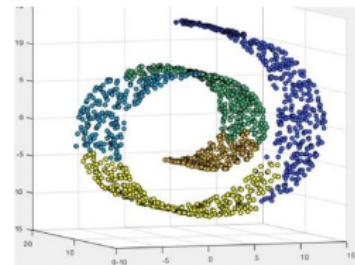
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## Dimensionality reduction

## Kernel PCA

The kernel trick applies to PCA

- kPCA captures non-linear structures



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# Dimensionality reduction

## Manifold learning

### Locally Linear Embedding (LLE)

- Measures how much each training instance linearly relates to its closest neighbors preserving local relations

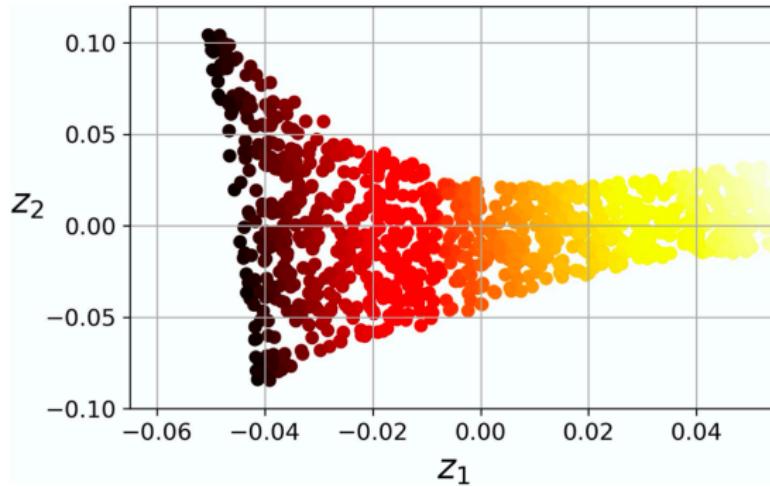


Figure 8-12. Unrolled Swiss roll using LLE

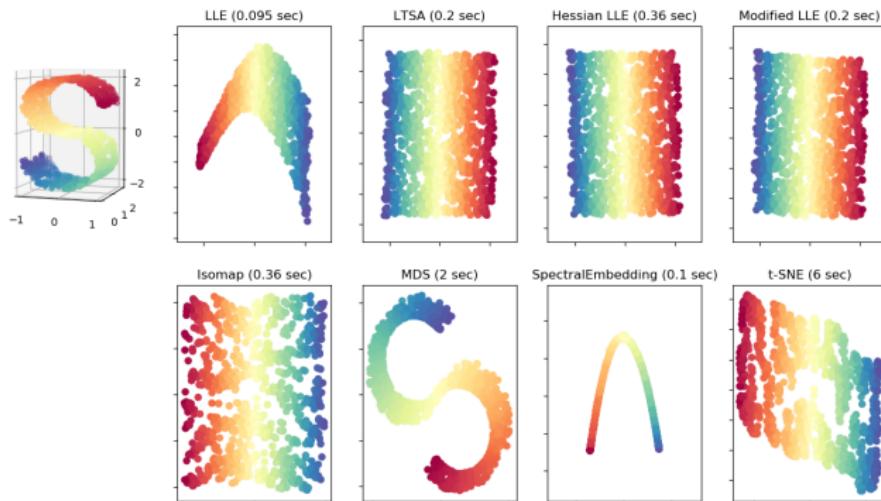
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# Dimensionality reduction

## Other manifold learning techniques

- Multidimensional Scaling (MDS): Preserves distances
- Isomap: Preserves geodesic distance
- t-Distributed Stochastic Neighbor Embedding (t-SNE): Preserves local distances and keep dissimilar instances apart

Manifold Learning with 1000 points, 10 neighbors



Clustering



K-means



Other clustering algorithms



Anomaly detection



Dimensionality reduction



# Dimensionality reduction

## Manifold learning demo

(Interactive demo)