Chapter 7. Multicarrier systems and multiuser detection

In this chapter, the fundamentals of multicarrier transmission, OFDM, MIMO and multiuser detection are introduced. In the case of MIMO, SIMO, and MISO are developed in order to establish the base for MIMO.

7.1 Multicarrier transmission

Multicarrier transmission is the technique where information is transmitted through several carriers in parallel. A set of N carriers with frequencies $f_0, f_1, f_2, \ldots, f_{N-1}$ is used to modulate the N information bits in parallel. The information bit duration is T_b and N of these bits are put together to produce the symbols with duration $T=NT_b$, as shown in Figure 7.1

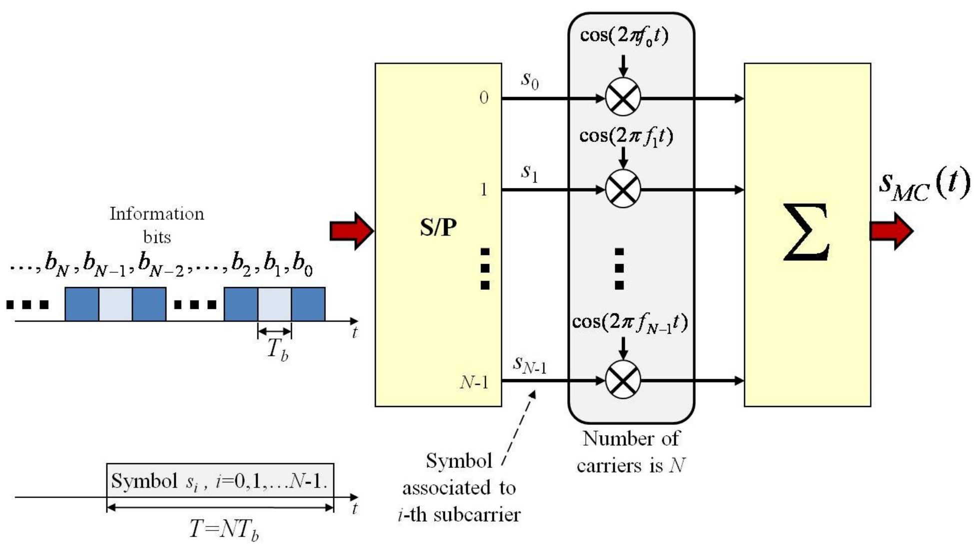

Figure 7.1 Multicarrier transmission system

Figure 7.1 shows the basic multicarrier system, see how the S/P block takes as input a stream of information bits in series of individual duration T_b, from which N bits are taken to produce N parallel symbols where each symbol has a duration of $T=N\,T_b$, while the S/P block is producing this output, it is at the same time putting together the following block of N information bits from the input stream. The multicarrier signal is given by

$$s_{MC}(t) = \sum_{i=0}^{N-1} s_i \cos(2\pi f_i t). \tag{7.1}$$

The frequencies in Equation (7.1) can be chosen as needed, but for convenience of reducing spectral occupancy, frequencies are chosen to have orthogonal carriers, this case is discussed in Section 7.4 in this Chapter. One of the motivations of using multicarrier instead of the classical single carrier modulation is in the time domain, because symbols have a larger duration which produces robustness against Inter-Symbol-Interference (ISI) due to delay spread. Recall from Chapter 2 that delay spread is the time that the receiver keeps getting delayed copies of a symbol with significant power levels due to the multipath effect. In the case of a symbol with larger duration, the delay spread will cause ISI with the next symbol instead of with several shorter symbols. The bandwidth of each symbol is given by $1/T$, and since T is larger than T_b, the bandwidth is reduced N times. Each of the N channels used in parallel will have a bandwidth which is the N-th part of $1/T_b$, and for each of these channels, it is recommended that its bandwidth be ten times less than the coherence bandwidth. Recall from Chapter 2 that the coherence bandwidth is the range of frequencies over which the frequency response of the channel is flat. The second motivation for multicarrier is in the frequency domain, since by making symbols with larger duration spectral occupancy is reduced for that channel in particular, thus satisfying the coherence bandwidth criterion. Both motivations to use multicarrier translate into a reduced effort for equalization.

7.2 MC-CDMA

MC-CDMA is a transmission system based on multicarrier and DS-CDMA. Given a chip sequence such as a Walsh-Hadamard of length or period N chips for every user in the system, the k-th user will use one bit and spread it over the entire set of carriers. Each chip of the sequence will multiply the information bit, but the first chip will do it in the first carrier, the second chip in the second carrier and so on. The symbol time duration is exactly the bit duration so individual carrier spectral occupancy is the same as that of the original information bit sequence, see Hara, S. and Prasad, R. (1997).

The transmitter for MC-CDMA is shown in Figure 7.2. The transmitter does not have a serial to parallel block converter as the system in Figure 7.2, instead, it has a block that copies the input information bit N times and the output has the same duration as the original bit. Note that the chip sequence has N chips, as many as carrier frequencies the system has.

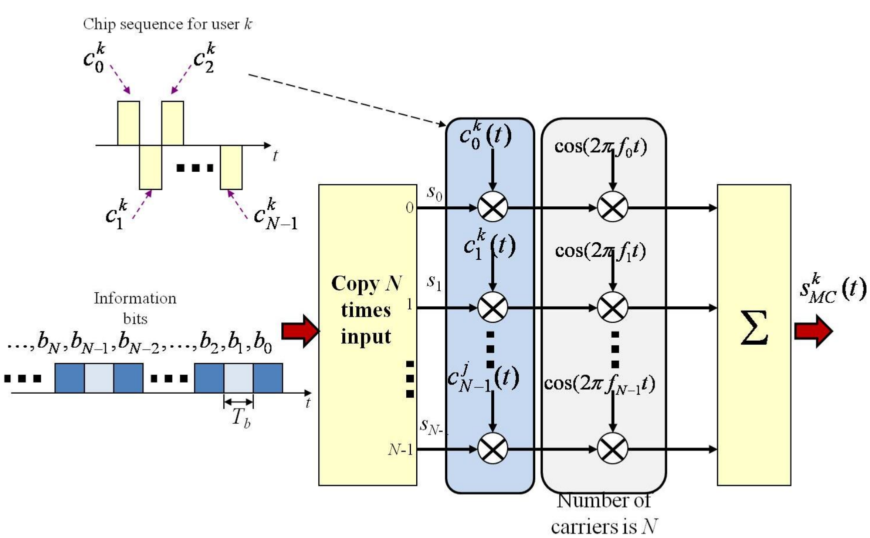

Figure 7.2 MC-CDMA system

Note that all the symbols s_j are identical to the bit being transmitted *b*. The system in Figure 7.2, produces for each MC-CDMA symbol the signal

$$\begin{aligned} s_{MC}^{k}(t) &= \sum_{j=0}^{N-1} s_j c_j^k(t)\cos\left(2\pi f_j t\right) \\ &= b\sum_{j=0}^{N-1} c_j^k(t)\cos\left(2\pi f_j t\right). \end{aligned} \tag{7.2}$$

A variation of the MC-CDMA system in Figure 7.2 that takes advantage of the simultaneous transmission of one bit in several carriers puts together the parallelism of the system in Figure 7.1 by modifying every branch in the transmitter of Figure 7.1 with an implementation of the transmitter of Figure 7.2. In this case the system will have *N* carriers for each branch in the transmitter and it will be transmitting in parallel *N* different information bits, see Hara, S. and Prasad, R. (1997).

Figure 7.3 shows the receiver for the MC-CDMA transmitter of Figure 7.2. It can be seen also the general form of the spectrum of the signal being received using this transmission technique. In each branch, the carrier multiplication produces a baseband signal that is multiplied by the individual chip from the Walsh-Hadamard sequence. At this point the signal must have been synchronized already in order to have a +1 when the chip multiplication is performed. Afterwards the product is delivered to a bank of correlators that will detect the information bit, such block puts together the information bit from the *N* branches in the system.

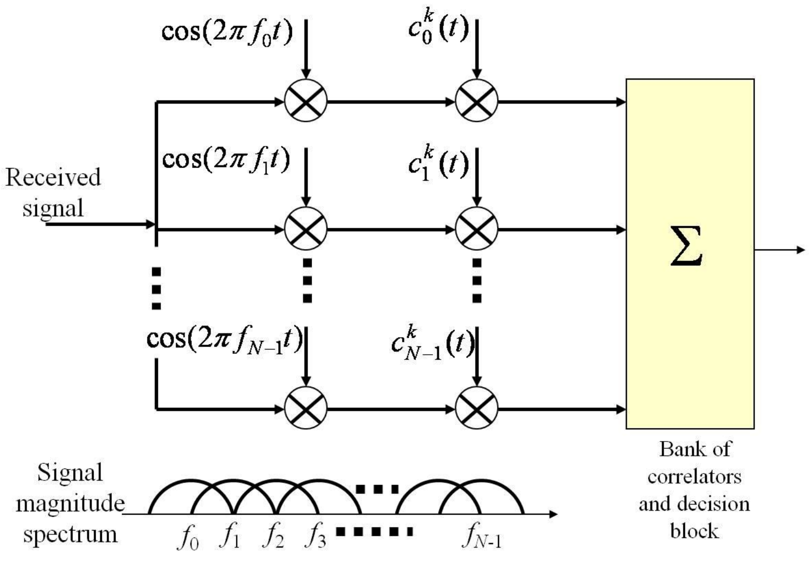

Figure 7.3 MC-CDMA receiver

The signal being received in the absence of noise coming from *K* users is given by

$$r_{MC}(t)=\sum_{k=1}^{K}\sum_{j=0}^{N-1}s_j^k c_j^k(t)\cos(2\pi f_j t). \qquad (7.3)$$

From Equation (7.3), and looking into branch zero of the receiver for user 1 of Figure 7.3, the signal received is processed as follows

$$\begin{aligned} r_{MC,0}^{1}(t) &= \sum_{k=1}^{K}\sum_{j=0}^{N-1}s_j^k c_j^k(t)\cos(2\pi f_j t)\cos(2\pi f_0 t)c_0^1(t) \\ &= s_0^1 c_0^1(t)\cos(2\pi f_0 t)\cos(2\pi f_0 t)c_0^1(t)+\sum_{k=2}^{K}\sum_{j=1}^{N-1}s_j^k c_j^k(t)\cos(2\pi f_j t)\cos(2\pi f_0 t)c_0^1(t) \qquad (7.4) \\ &= \frac{s_0^1}{2}+\frac{s_0^1}{2}\cos(4\pi f_0 t)+\sum_{k=2}^{K}\sum_{j=1}^{N-1}s_j^k c_j^k(t)\cos(2\pi f_j t)\cos(2\pi f_0 t)c_0^1(t). \end{aligned}$$

The last expression in Equation (7.4) will go through the bank of correlators and the summation over the N branches or chips of the sequence. The first term will provide the information symbol required, the second term will be eliminated by the correlator, the third term will be eliminated by orthogonality of the sequences once the N branches are put together.

7.3 MC-DS-CDMA

In this transmission system, there is a serial to parallel block at the beginning as that in Figure 7.1, and instead of using one chip per carrier as in MC-CDMA, the entire sequence of chips is used in every carrier of the system. This is shown in Figure 7.4

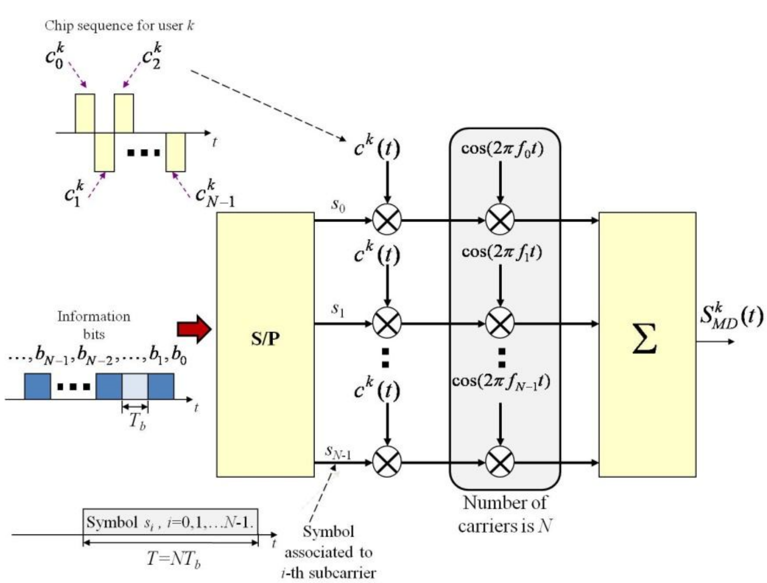

Figure 7.4 MC-DS-CDMA transmitter

The receiver for MC-DS-CDMA is seen in Figure 7.5. This system is basically formed by the transmission and reception of N parallel DS-CDMA transmitter-receiver pairs with different frequencies for the carriers. Due to the extension in time duration of the bits when pass through the S/P converter in the transmitter, the spectral occupancy is not the spread as if it were spread the information bit directly with the chip sequence.

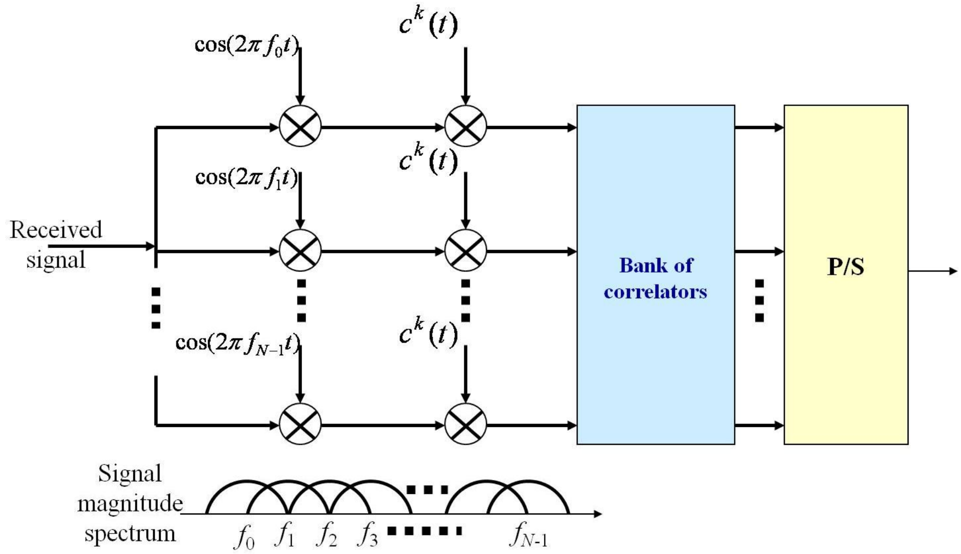

Figure 7.5 MC-DS-CDMA receiver

7.4 Orthogonal frequency division multiplexing (OFDM)

OFDM is a multicarrier transmission technique that has the property of orthogonality for each and every one of its carriers. Since the system is basically as that shown in Figure 7.1, the symbol time duration is T and the carrier separation will be given in terms of T.

In order to see how OFDM is implemented, first consider the orthogonality of the sinusoidal signals, followed by a procedure of discretizing the symbols and using the complex envelope representation for passband signals. The basic multicarrier system is shown in Figure 7.6, where the symbols are seen as functions of continuous time t. These symbols are then multiplied by the carriers and this multiplication can be seen through the complex envelope representation as follows

$$s_i(t)\cos(2\pi f_i t) = \mathbb{R}\mathrm{e}\left\{s_i(t)e^{j2\pi f_i t}\right\}. \tag{7.5}$$

Equation (7.5) is the canonical representation or complex envelope representation of the i-th carrier modulated by the i-th symbol, see Haykin, S. (2010). In general, eachs cosenoidal function in the system is denoted as *subcarrier*. Now consider in general a continuous-time signal $s_i(t)$ that is going to be discretized as shown in Figure 7.7. This figure also shows the sampled or discrete-time signal and the samples taken every T_s seconds that produce the discrete-time sequence to be used in the following development of the OFDM signal.

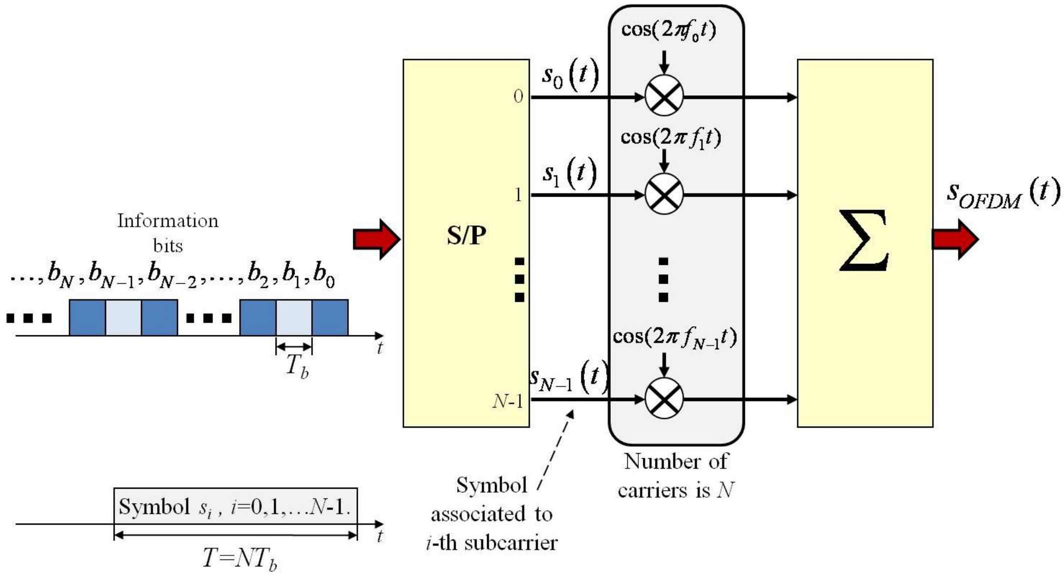

Figure 7.6 Multicarrier system

Since the signal or symbol to be discretized is an energy signal, then it can be chosen to have a fixed number of samples N in the duration of the signal. Once a signal $s_i(t)$ is discretized, the discrete-time sequence to consider is given by

$$s_i(t) \quad \Rightarrow \quad s_i(kT_s) = s_i[k], \quad k = 0,1,\ldots,N-1. \tag{7.6}$$

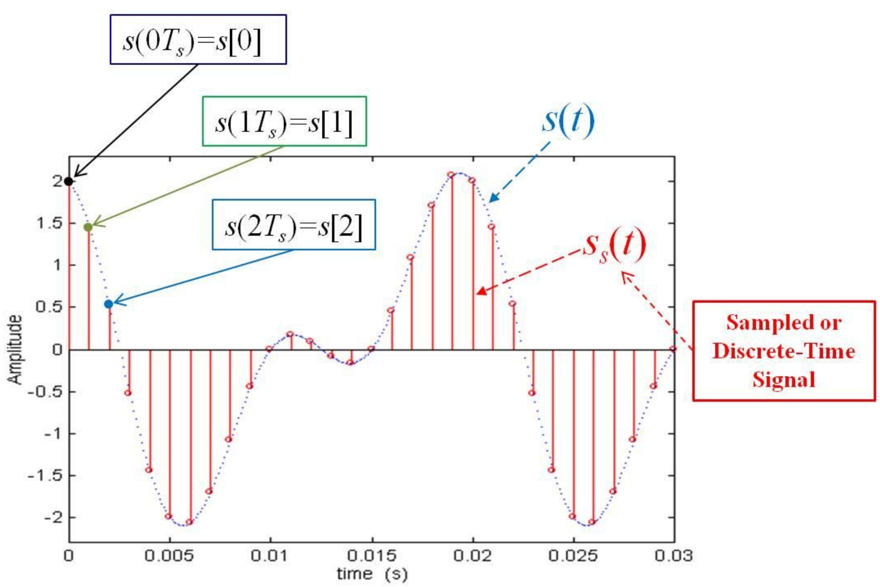

Figure 7.7 Discretization of continuous-time signal

For this example in particular, it is of interest to have the sampling time T_s equal to the N-th part of the symbol duration T, in other words, $T_s = T / N$, and to have the number of points N equal to the number of subcarrier frequencies.

It is known that the minimum separation required for subcarriers to remain orthogonal over the interval $[0,T]$ is $1/T$, see Fazel, K. and Kaiser, S. (2003), and Schulze, H., and Luders, C. (2005). Thus, it is necessary to fix a value of frequency, say f_0, and from there on the i-th subcarrier will have frequency given by

$$f_i = f_0 + \frac{i}{T}, \quad i = 0,1,\ldots,N-1. \tag{7.7}$$

Now, substituting Equation (7.7) into Equation (7.5), results in

$$\begin{aligned} s_i(t)\cos(2\pi f_i t) &= s_i(t)\cos\left(2\pi\left(f_0 + \frac{i}{T}\right)t\right) \\ &= \mathbb{R}\mathrm{e}\left\{ s_i(t) e^{j2\pi\left(f_0 + \frac{i}{T}\right)t} \right\} \\ &= \mathbb{R}\mathrm{e}\left\{ s_i(t) e^{j2\pi f_0 t} e^{j2\pi \frac{i}{T} t} \right\}, \quad i = 0,1,2,\ldots,N-1. \end{aligned} \tag{7.8}$$

Now, Equation (7.8) can be discretized by making the change of variable to discrete-time $t = kT_s = kT / N$, which results in

$$s_i(t) e^{j2\pi f_0 t} e^{j2\pi \frac{i}{T} t} = s_i[k] e^{j2\pi f_0 k \frac{T}{N}} e^{j2\pi \frac{i}{T} k \frac{T}{N}}, \quad i = 0,1,2,\ldots,N-1. \tag{7.9}$$

The OFDM symbol results from the summation over the N subcarriers as shown in Figure 7.6, hence carrying out this, the following is obtained

$$s_{OFDM}(t) = \sum_{i=0}^{N-1} s_i(t) e^{j2\pi f_0 t} e^{j2\pi \frac{i}{T} t}. \tag{7.10}$$

The OFDM symbol in Equation (7.10) is in continuous-time form, advantage for implementation will be to obtain the discretized version of it. Hence we need to discretized it by using the expression in Equation (7.9), which gives the following

$$
\begin{aligned}
s_{OFDM}\left(kT_s\right) &= s_{OFDM}\left[k\right] \\
&= \sum_{i=0}^{N-1} s_i\left[k\right] e^{j2\pi f_0 k\frac{T}{N}} e^{j2\pi \frac{i}{T} k \frac{T}{N}} \\
&= \underbrace{e^{j2\pi f_0 T\frac{k}{N}}}_{\text{carrier}} \underbrace{\sum_{i=0}^{N-1} s_i\left[k\right] e^{j\frac{2\pi}{N}ki}}_{\text{OFDM baseband symbol is IDFT}} \quad , \quad k = 0,1,\ldots,N-1.
\end{aligned}
\tag{7.11}
$$

From Equation (7.11), the first term is the base carrier with frequency f_0, and the second term, the OFDM baseband symbol, is given by the inverse discrete Fourier transform (IDFT), which can be implemented using the algorithm IFFT. With this OFDM symbol, the transmitter can be implemented as shown in Figure 7.8.

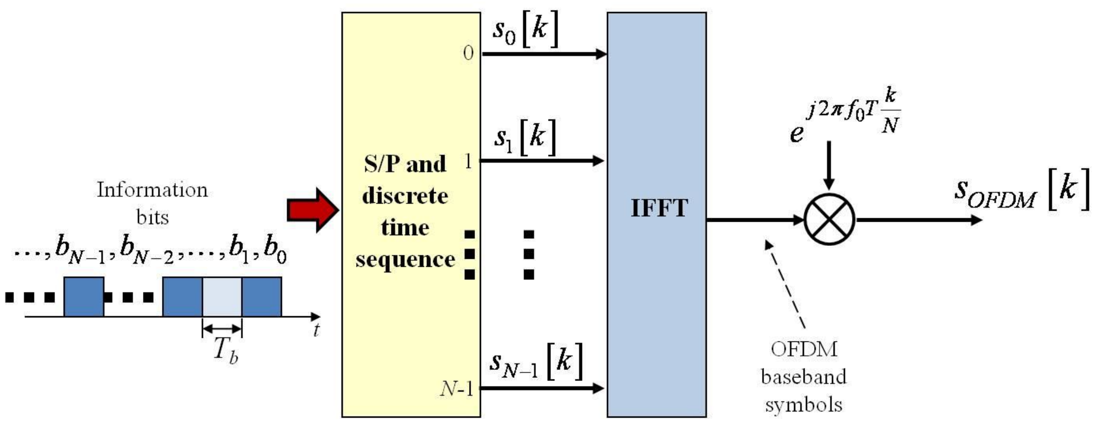

Figure 7.8 OFDM transmitter

Assuming that the symbols are rectangular pulses of duration T, and choosing the subcarrier frequencies to be orthogonal with minimum separation as in Equation (7.7), the amplitude spectrum of the OFDM signal will be given by a set of *sinc* functions centered at the subcarrier frequencies, where the maximum of each *sinc* pulse coincides with the nulls of all the other *sinc* pulses. This is shown in Figure 7.9 for the case of 16 subcarriers with a frequency f_0=20 kHz and symbol duration T=250 $\mu\,\text{sec}$, hence the subcarrier separation is of 1/T=4 kHz.

Figure 7.9 also shows the resultant OFDM spectrum by the superposition of such *sinc* pulses for each subcarrier. This resultant spectrum is what can be found in the screens of spectrum analyzers when an OFDM signal is fed to the analyzer.

Assume that the channel for transmission is characterized by a discrete-time impulse response $h[k]$ with finite length $M+1<N$, where N is the number of subcarriers or points in the IDFT block in the transmitter, also assume that the OFDM symbols are defined by a discrete-time sequence $s[k]$, k=0, 1, …, N-1. Then, the signal received will be the result of the convolution of the OFDM sequence and the channel impulse response.

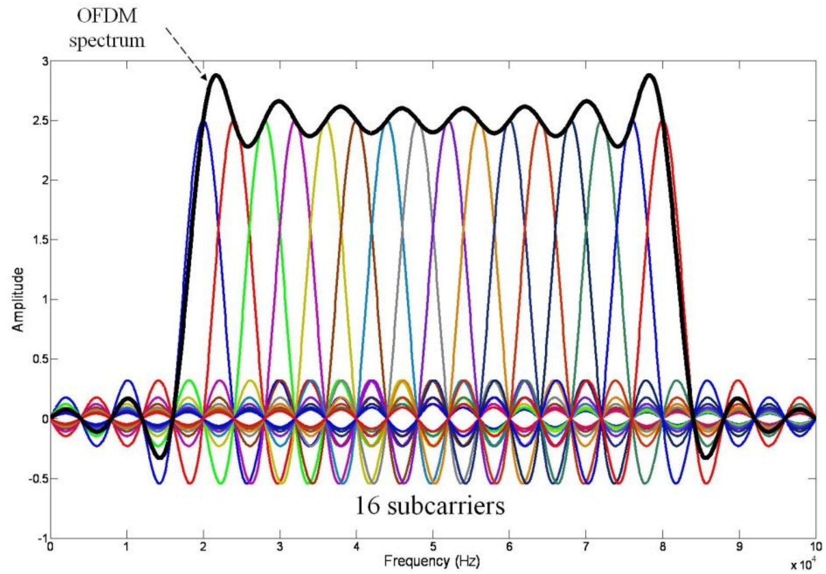

Figure 7.9 OFDM spectrum

Consider that the transmitter produces OFDM symbols indefinitely, and then at the receiver input the convolution of both sequences is present. Since the impulse response exists for times k=0,1,…,M+1 and the OFDM sequence for times k=0,1,…,N-1, the convolution will exists for times k=0,1,2,…,$M+N$. Now, recall that the transmitter produces symbols in a series fashion; hence the result of the convolution will be affecting the reception of the next OFDM symbol since M samples of the sequence resulting from the convolution will be overlap with the first M samples of the next OFDM symbol. This is an ISI phenomenon that is produced by the channel impulse response. This can also be seen as delay spread affecting reception of the following OFDM symbol.

In order to get rid of this ISI effect, the OFDM symbol originally made out of N samples is modified to include M extra samples that are called *cyclic prefix* (CP). The CP prevents ISI by having a set of samples affected by the ISI and keeping intact the N OFDM samples of the symbol. The CP procedure is only to copy from the original N-sample OFDM symbol the samples from N-M up to sample N-1 and placing them in front of the OFDM symbol, see Goldsmith, A. (2005), and Schulze, H., and Luders, C. (2005).

Another advantage of using CP is that induces circular convolution as it will be seen to generate the corresponding matrices for the transmission of OFDM symbols. The samples that are affected by ISI, the receiver will remove them.

Figure 7.10 shows the OFDM transmitter and receiver together with QAM modulation and CP block. This system is used in several standards such as IEEE 802.11a for WLANs. Recall that due to the ISI phenomenon, the output OFDM symbol has N+M samples. Also the received signal has length N+M and after the CP S/P block in the receiver, the sequence will be of length N. The QAM modulation generates the symbols to be transmitted and then OFDM is carried out, in the receiver OFDM is performed and then the QAM demodulation.

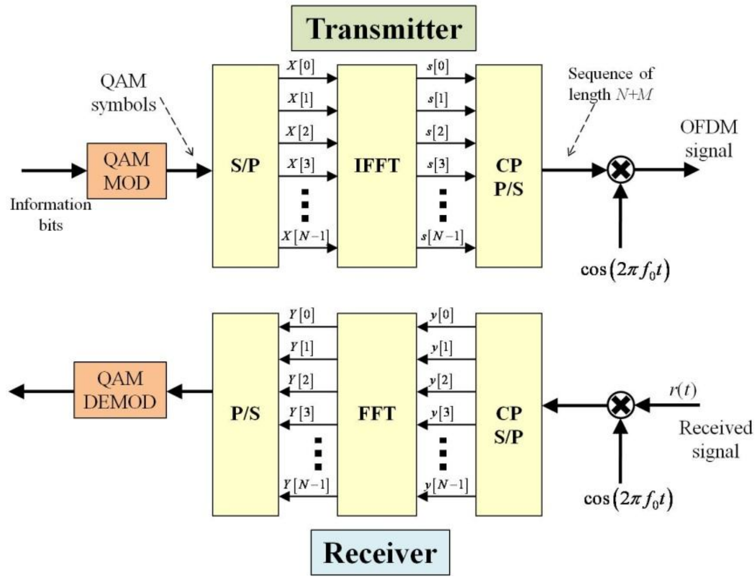

Figure 7.10 OFDM transmitter and receiver

The OFDM symbol that is transmitted together with the CP according to Figure 7.10, is given by

$$\underbrace{s[N-M],s[N-M+1],\ldots,s[N-1]}_{\text{CP}},\underbrace{s[0],s[1],s[2],\ldots,s[N-1]}_{\text{OFDM baseband symbol}}. \tag{7.12}$$

The advantage of using the format in (7.12) is that avoids ISI, as well as having fadings only in some subcarriers and no need of pulse shaping. The disadvantages are the computational load of the FFT algorithm, the Doppler effect on the subcarriers and a high power with linear amplifiers required, see Goldsmith, A. (2005).

Recall that the received signal is the convolution of the OFDM symbol with CP and the channel impulse response. In order to simplify notation, let the sequence of OFDM symbols $s[k]$ be represented by s_k and similarly for the channel impulse response and the received sequence, then the received signal in the absence of noise can be described in terms of a matrix equation as follows

$$\mathbf{y} = \widetilde{\mathbf{H}}\tilde{\mathbf{s}}. \tag{7.13}$$

In Equation (7.13) the matrix $\widetilde{\mathbf{H}}$ is the channel matrix given by the impulse response, and the vectors $\mathbf{y}$ and $\mathbf{s}$ are the received signal and OFDM symbol, respectively. These elements are given by the following

$$
\underbrace{\begin{bmatrix} y_{N-1} \\ y_{N-2} \\ y_{N-3} \\ \vdots \\ y_1 \\ y_0 \end{bmatrix}}_{N\text{x}1} = \underbrace{\begin{bmatrix} h_0 & h_1 & h_2 & h_3 & \cdots & h_{M-1} & h_M & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & h_0 & h_1 & h_2 & \cdots & h_{M-2} & h_{M-1} & h_M & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & h_0 & h_1 & \cdots & h_{M-3} & h_{M-2} & h_{M-1} & h_M & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & h_0 & h_1 & h_2 & h_3 & h_4 & \cdots & h_M & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & h_0 & h_1 & h_2 & h_3 & \cdots & h_{M-1} & h_M \end{bmatrix}}_{N\text{x}(N+M)} \underbrace{\begin{bmatrix} s_{N-1} \\ s_{N-2} \\ s_{N-3} \\ \vdots \\ s_1 \\ s_0 \\ s_{N-1} \\ s_{N-2} \\ \vdots \\ s_{N-M+1} \\ s_{N-M} \end{bmatrix}}_{(N+M)\text{x}1} \tag{7.14}
$$

Equation (7.14) is given by the linear convolution in discrete-time, but the channel matrix can be rearranged and the equation will be given in an equivalent interpretation by using the circular convolution as follows

$$
\underbrace{\begin{bmatrix} y_{N-1} \\ y_{N-2} \\ y_{N-3} \\ \vdots \\ y_1 \\ y_0 \end{bmatrix}}_{N\text{x}1} = \underbrace{\begin{bmatrix} h_0 & h_1 & h_2 & h_3 & \cdots & h_{M-1} & h_M & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & h_0 & h_1 & h_2 & \cdots & h_{M-2} & h_{M-1} & h_M & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & h_0 & h_1 & \cdots & h_{M-3} & h_{M-2} & h_{M-1} & h_M & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ h_4 & h_5 & h_6 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & h_2 & h_3 \\ h_3 & h_4 & h_5 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & h_1 & h_2 \\ h_2 & h_3 & h_4 & \cdots & h_M & 0 & 0 & 0 & 0 & 0 & \cdots & h_0 & h_1 \\ h_1 & h_2 & h_3 & \cdots & h_{M-1} & h_M & 0 & 0 & 0 & 0 & \cdots & 0 & h_0 \end{bmatrix}}_{N\text{x}N} \underbrace{\begin{bmatrix} s_{N-1} \\ s_{N-2} \\ s_{N-3} \\ \vdots \\ s_1 \\ s_0 \end{bmatrix}}_{N\text{x}1} \tag{7.15}
$$

Now Equation (7.15) has the matrix equation form

$$
\mathbf{y} = \mathbf{Hs}. \tag{7.16}
$$

Matrix **H** is the circulant convolution matrix over N samples of interest, and it has the properties of being a *normal* matrix, in other words, a square matrix is a normal matrix if the product of its Hermitian $\mathbf{H}^H$ with itself is commutative. Recall that the Hermitian of a matrix is its conjugate transpose.

7.5 Multiple-input multiple-output (MIMO)

In this section, we follow up what was introduced in Section 2.5 of Chapter 2, the Single-Input Single-Output (SISO) modeling. The SIMO, MISO and MIMO systems are developed in the same way throughout this section, see Tse, D. and Viswanath, P. (2005) and Biglieri, E., Calderbank, R., Constantinides, A., Goldsmith, A., Paulraj, A., and Vincent Poor, H. (2007).

In Section 2.5, it was remarked the procedure of discretizing the signal being transmitted, then sampling the signal received which helps in having a discrete-time convolution, and then the channel was vectorized by looking into several samples of the received signal at the same time. The signal received was given by

$$y[m]=\sqrt{E_s}\sum_k s[k]h[m-k]+n[m]. \quad m=0,1,2,\ldots \tag{7.17}$$

In Chapter 2, it was also assumed that the channel sequence length was of L samples, which also changing the time reference to the subindex for all the sequences in the equations, the following was obtained for the frequency selective fading channel

$$y_m=\sqrt{E_s}\left[h_{L-1}\, h_{L-2}\cdots h_2\, h_1\, h_0\right]\begin{bmatrix} s_{m-L+1} \\ s_{m-L+2} \\ \vdots \\ s_2 \\ s_1 \\ s_0 \end{bmatrix}+n_m \tag{7.18}$$

Now, vectorizing the channel, and considering the output sequence for a group of M discrete times, it was obtained

$$\left[y_m\, y_{m+1}\cdots y_{m+M-1}\right]=\sqrt{E_s}\left[h_{L-1}\, h_{L-2}\cdots h_0\right]S+\left[n_m\, n_{m+1}\cdots n_{m+M-1}\right]. \tag{7.19}$$

In Equation (7.19) the matrix S is given by the Hankel matrix

$$S=\begin{bmatrix} s_{m-L+1} & s_{m-L+2} & \cdots & s_{m-L+M} \\ s_{m-L+2} & s_{m-L+3} & \cdots & s_{m-L+M+1} \\ \vdots & \vdots & \ddots & \vdots \\ s_{m-1} & s_m & \cdots & s_{m+M-2} \\ s_m & s_{m+1} & \cdots & s_{m+M-1} \end{bmatrix}_{LxM} . \tag{7.20}$$

Equation (7.19) can also be rewritten so that the output vector of length M becomes a column vector. In order to do this, first the Hankel matrix needs to be *vectorized* in a column vector as follows

$$\underline{S}^T = \left[s_{m-L+1}, s_{m-L+2}, s_{m-L+3}, \ldots, s_{m-1}, s_m, s_{m+1}, \ldots, s_{m+M-2}, s_{m+M-1}\right]_{(M+L-1)\text{x}1}. \tag{7.21}$$

Now, with Equation (7.21), the expression in (7.19) can be rewritten as follows

$$\begin{bmatrix} y_m \\ y_{m+1} \\ \vdots \\ y_{m+M-1} \end{bmatrix} = \sqrt{E_s}\mathbf{H}\underline{S} + \begin{bmatrix} n_m \\ n_{m+1} \\ \vdots \\ n_{m+M-1} \end{bmatrix}, \tag{7.22}$$
$$\mathbf{y} = \sqrt{E_s}\mathbf{H}\underline{S} + \mathbf{n}.$$

In (7.22) the matrix **H** is the channel matrix and has the form of a Toeplitz matrixand is known as a *fat* matrix, which is given by the following

$$\mathbf{H} = \begin{bmatrix} h_{L-1} & h_{L-2} & \cdots & h_0 & 0 & 0 & \cdots & 0 \\ 0 & h_{L-1} & \cdots & h_1 & h_0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & h_{L-1} & h_{L-2} & \cdots & h_0 \end{bmatrix}_{M\text{x}(M+L-1)} . \tag{7.23}$$

Single-Input, Multiple-Output (SIMO). In the previous paragraphs of this section, the frequency selective fading channel in the SISO case was introduced. In this part, the SIMO system, the transmitter has one antenna and the receiver has M_r receiving antennas as shown in Figure 7.11, where each path between transmitting and receiving antenna has a channel coefficient associated with such path.

As in the SISO case, the analysis is based on the sampled signal model with a receiver with M_r antennas. Since the channel consists of M_r components, the impulse response will be given by a column vector for time k as follows

$$\underline{h}[k] = \begin{bmatrix} h_1[k] \\ h_2[k] \\ \vdots \\ h_{M_r}[k] \end{bmatrix}_{M_r\text{x}1} \tag{7.24}$$

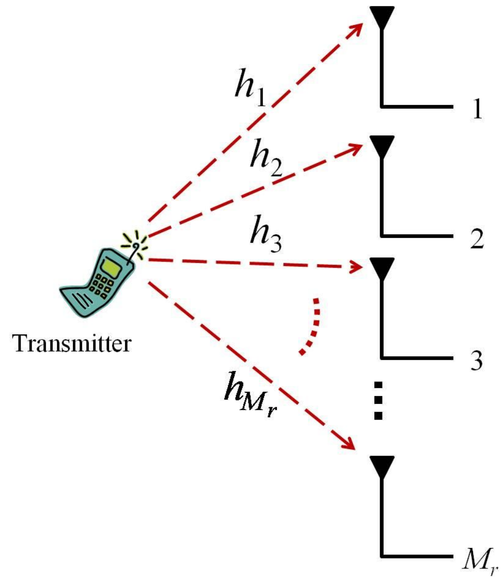

Figure 7.11 SIMO system

In Equation (7.24) $h_i[k]$ is the channel transfer function in the SISO case between transmitting antenna and the i-th receiving antenna at discrete-time k.

For the **flat fading channel**, the impulse response $h_i[k]$ consists only of the component at discrete-time k=0, for all the M_r SISO paths. In this case the impulse response vector is constant and will be denoted as $\underline{h}^T = \begin{bmatrix} h_1 & h_2 & \cdots & h_{M_T} \end{bmatrix}$. Also, the received signal will be a column vector of dimensions M_rx1, and the symbol will be a scalar at time k, $s[k]$, as follows

$$\begin{bmatrix} y_1[k] \\ y_2[k] \\ \vdots \\ y_{M_r}[k] \end{bmatrix} = \sqrt{E_s}\,\underline{h}\,s[k] + \begin{bmatrix} n_1[k] \\ n_2[k] \\ \vdots \\ n_{M_r}[k] \end{bmatrix}, \tag{7.25}$$

$$\underline{y}[k] = \sqrt{E_s}\,\underline{h}\,s[k] + \underline{n}[k].$$

Consider I_{M_r} as the square identity matrix of dimension M_r, then the noise vector $\underline{n}[k]$ is such that its autocorrelation matrix is given by

$$E\left[\underline{n}[k]\underline{n}^H[m]\right] = N_0 I_{M_r} \delta[k-m]. \tag{7.26}$$

For the **frequency selective fading channel**, the channel impulse response in (7.24) exists for discrete-times k=0,1,…,L-1. In this case, use the definition of the received signal vector

$\underline{y}[k]$ and the channel noise vector $\underline{n}[k]$ found in Equation (7.25). Define the channel matrix as follows

$$\mathbf{H}=\begin{bmatrix} h_1[L-1] & h_1[L-2] & \cdots & h_1[1] & h_1[0] \\ h_2[L-1] & h_2[L-2] & \cdots & h_2[1] & h_2[0] \\ h_3[L-1] & h_3[L-2] & \cdots & h_3[1] & h_3[0] \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ h_{M_r}[L-1] & h_{M_r}[L-2] & \cdots & h_{M_r}[1] & h_{M_r}[0] \end{bmatrix}_{M_r \mathrm{x} L} \tag{7.27}$$

Also, define the signal vector as follows

$$\underline{s}=\begin{bmatrix} s[k-L+1] \\ s[k-L+2] \\ \vdots \\ s[k-2] \\ s[k-1] \\ s[k] \end{bmatrix}_{L\mathrm{x}1} \tag{7.28}$$

Then putting together the equations in order to obtain the received signal for the frequency selective fading channel, the following is obtained

$$\underline{y}[k]=\sqrt{E_s}\mathbf{H}\underline{s}+\underline{n}[k]. \tag{7.29}$$

Now, define the received signal matrix for M different discrete-time samples as follows

$$Y=\begin{bmatrix} y_1[k] & y_1[k+1] & \cdots & y_1[k+M-1] \\ y_2[k] & y_2[k+1] & \cdots & y_2[k+M-1] \\ \vdots & \vdots & \ddots & \vdots \\ y_{M_r}[k] & y_{M_r}[k+1] & \cdots & y_{M_r}[k+M-1] \end{bmatrix}_{M_r \mathrm{x} M} \tag{7.30}$$

And define the noise channel matrix as follows

$$N = \begin{bmatrix} n_1[k] & n_1[k+1] & \cdots & n_1[k+M-1] \\ n_2[k] & n_2[k+1] & \cdots & n_2[k+M-1] \\ \vdots & \vdots & \ddots & \vdots \\ n_{M_r}[k] & n_{M_r}[k+1] & \cdots & n_{M_r}[k+M-1] \end{bmatrix}_{M_r \mathrm{x} M} \tag{7.31}$$

Using the Hankel matrix in (7.20) together with equations (7.27), (7.30) and (7.31), for the same frequency selective fading channel, the received signal for M different samples, k, k+1, k+2, …, k+M-1 is given by the following matrix equation

$$Y = \sqrt{E_s}\mathbf{H}S + N. \tag{7.32}$$

Multiple-Input, Single-Output (MISO). The MISO system consists of M_T transmitter antennas and one receiving antenna as shown in Figure 7.12, where each SISO path between transmitting and receiving antenna has a channel coefficient associated with such path.

As in the SISO case, the analysis is based on the sampled signal model with a transmitter with M_T antennas. Since the channel consists of M_T components, the impulse response will be given by a row vector for time k as follows

$$\underline{h}[k] = \begin{bmatrix} h_1[k] & h_2[k] & \cdots & h_{M_T}[k] \end{bmatrix}_{1\mathrm{x}M_T} \tag{7.33}$$

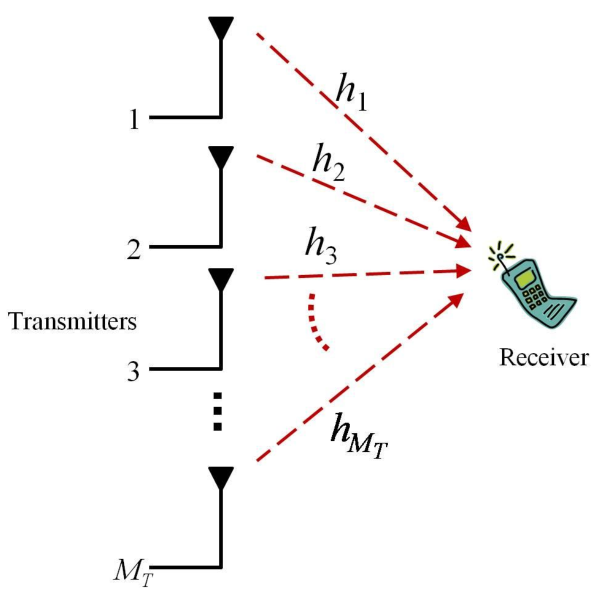

Figure 7.12 SIMO system

In Equation (7.33) $h_i[k]$ is the channel transfer function in the SISO case between the i-th transmitting antenna and the receiving antenna at discrete-time k.

For the **flat fading channel**, the impulse response $h_i[k]$ consists only of the component at discrete-time k=0, call it h_i, for all the M_T SISO paths. In this case the impulse response vector is constant and will be denoted as $\underline{h} = \begin{bmatrix} h_1 & h_2 & \cdots & h_{M_T} \end{bmatrix}$. Also, the transmitted signal will be a column vector of dimensions M_Tx1, and the receiving signal will be a scalar at time k, $y[k]$, as follows

$$
\begin{aligned}
y[k] &= \sqrt{\frac{E_s}{M_T}} \underline{h} \begin{bmatrix} s_1[k] \\ s_2[k] \\ \vdots \\ s_{M_T}[k] \end{bmatrix} + n[k], \\
y[k] &= \sqrt{\frac{E_s}{M_T}} \underline{h}\,\underline{s}[k] + n[k].
\end{aligned}
\tag{7.34}
$$

Note in Equation (7.34) that the symbol energy is divided equally among all the transmitting antennas so that the total energy provided to the channel is the same as that in the SISO and SIMO case. Note also in (7.34) that the noise component is a scalar as the received signal since the receiver has only one component.

For the **frequency selective fading channel**, the channel impulse response in (7.33) exists for discrete-times k=0,1,…,L-1. Define the following matrices

$$
\begin{aligned}
\underline{h}_j &= \begin{bmatrix} h_j[L-1] & h_j[L-2] & \cdots & h_j[1] & h_j[0] \end{bmatrix}_{1\text{x}L}, \\
\underline{s}_j[k] &= \begin{bmatrix} s_j[k-L+1] \\ s_j[k-L+2] \\ \vdots \\ s_j[k-2] \\ s_j[k-1] \\ s_j[k] \end{bmatrix}_{L\text{x}1}, \quad j = 1, 2, \ldots, M_T.
\end{aligned}
\tag{7.35}
$$

Then, the received signal for the frequency selective fading channel for discrete-time k is given by

$$
y[k] = \sqrt{\frac{E_s}{M_T}} \begin{bmatrix} \underline{h}_1 & \underline{h}_2 & \cdots & \underline{h}_{M_T} \end{bmatrix} \begin{bmatrix} \underline{s}_1[k] \\ \underline{s}_2[k] \\ \vdots \\ \underline{s}_{M_T}[k] \end{bmatrix} + n[k]. \tag{7.36}
$$

See that in (7.36), both vectors have LM_T elements and produce a scalar. If a set of M samples of the received signal are desired, then define first the received signal vector and the noise vector as follows

$$\begin{aligned}\underline{y}[k] &= \begin{bmatrix} y[k] & y[k+1] & \cdots & y[k+M-1] \end{bmatrix}_{1\mathrm{x}M} \\ \underline{n}[k] &= \begin{bmatrix} n[k] & n[k+1] & \cdots & n[k+M-1] \end{bmatrix}_{1\mathrm{x}M}\end{aligned} \tag{7.37}$$

Then putting together the equations in order to obtain the received signal for the frequency selective fading channel for M samples, the following is obtained

$$\underline{y}[k] = \sqrt{\frac{E_s}{M_T}} \begin{bmatrix} \underline{h}_1 & \underline{h}_2 & \cdots & \underline{h}_{M_T} \end{bmatrix} \begin{bmatrix} S_1[k] \\ S_2[k] \\ \vdots \\ S_{M_T}[k] \end{bmatrix} + \underline{n}[k]. \tag{7.38}$$

In (7.38), the signal vector contains $S_j[k]$, which is the Hankel matrix for $\underline{s}_j[k]$.

Multiple-Input, Multiple Output (MIMO). The MIMO case is straightforward once the SISO, SIMO, and MISO scenarios have been defined. The scenario is illustrated in Figure 7.13

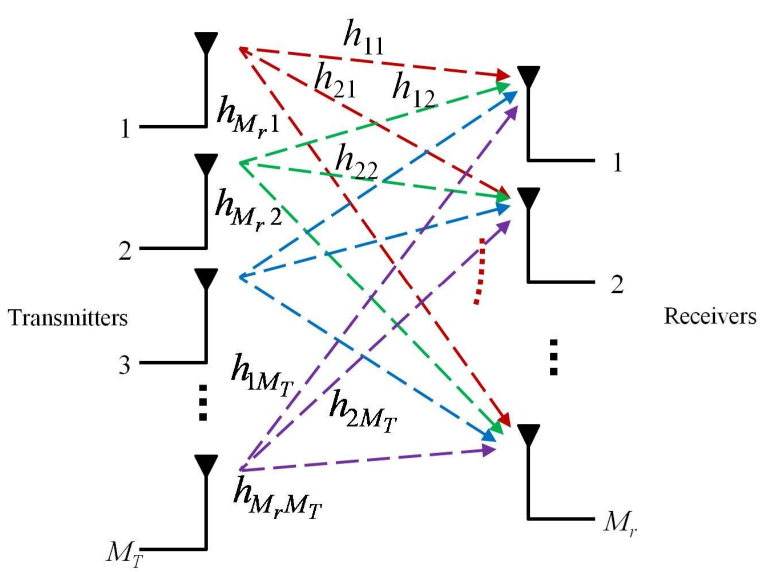

Figure 7.13 MIMO scenario

The MIMO scenario with a **flat fading channel**, has a channel matrix which is constant and is given by

$$\mathbf{H}=\begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1M_T} \\ h_{21} & h_{22} & \cdots & h_{2M_T} \\ \vdots & \vdots & \ddots & \vdots \\ h_{M_r 1} & h_{M_r 2} & \cdots & h_{M_r M_T} \end{bmatrix} \tag{7.39}$$

Then, the following equation is the expression for the received signal in the flat fading channel

$$\underbrace{\underline{y}[k]}_{M_r\mathrm{x}1}=\sqrt{\frac{E_s}{M_T}}\underbrace{\mathbf{H}}_{M_r\mathrm{x}M_T}\underbrace{\underline{s}[k]}_{M_T\mathrm{x}1}+\underbrace{\underline{n}[k]}_{M_r\mathrm{x}1}. \tag{7.40}$$

Now, for the **frequency selective fading channel**, consider the following definitions

$$\begin{gathered} \underline{h}_{ij}=\begin{bmatrix} h_{ij}[L-1] & h_{ij}[L-2] & \cdots & h_{ij}[1] & h_{ij}[0] \end{bmatrix}_{1\mathrm{x}L}, \\ \underline{s}_j[k]=\begin{bmatrix} s_j[k-L+1] \\ s_j[k-L+2] \\ \vdots \\ s_j[k-2] \\ s_j[k-1] \\ s_j[k] \end{bmatrix}_{L\mathrm{x}1}, \quad i=1,2,\ldots,M_r; \quad j=1,2,\ldots,M_T. \end{gathered} \tag{7.41}$$

Thus, the received signal for all the antennas at time *k* is given by

$$\underline{y}[k]=\sqrt{\frac{E_s}{M_T}}\begin{bmatrix} \underline{h}_{11} & \underline{h}_{12} & \cdots & \underline{h}_{1M_T} \\ \underline{h}_{21} & \underline{h}_{22} & \cdots & \underline{h}_{2M_T} \\ \vdots & \vdots & \ddots & \vdots \\ \underline{h}_{M_r 1} & \underline{h}_{M_r 2} & \cdots & \underline{h}_{M_r M_T} \end{bmatrix}\begin{bmatrix} \underline{s}_1[k] \\ \underline{s}_2[k] \\ \vdots \\ \underline{s}_{M_T}[k] \end{bmatrix}+\underline{n}[k]. \tag{7.42}$$

7.6 Multi-User Detection (MUD)

Multiuser detection is a technique that mitigates Multiple Access Interference (MAI) or Multi-User Interference (MUI) by subtracting the undesired signals from the received signal which is the superposition of the desired signal and the transmission from other users sharing the same frequency band. The technology has been evolving in general, or for such technologies as DS-

CDMA, MIMO and OFDM, see Duel-Hallen, A., Holtzman, J., and Zvonar, Z. (1995), Jiang, M. and Hanzo, L. (2007), Moshavi, S. (1996), and Verdu, S. (1998).

A single user conventional receiver tries to separate users (MAI) and noise from the desired signal. The interference process for a DS-CDMA system contains not only the MAI signals, but also the multipath effect for the desired signal, which has random time offsets. The CDMA sequences are not completely orthogonal to eliminate these effects; hence MAI is a capacity limiting factor that needs to be mitigated.

The technology of multiuser detection is also referred as *interference cancellation or joint detection*. The strategy is to detect each user individually by using jointly information of multiple users in the communication.

Assume a system with K users transmitting in DS-CDMA with BPSK modulation with single-path real channel. Assume that all users are synchronized; hence the signal received with noise is given by

$$r(t) = \sum_{k=1}^{K} \alpha_k(t) c_k(t) b_k(t) + n(t). \tag{7.43}$$

In Equation (7.43) $\alpha_k(t)$ is the amplitude with which the signal from user k is received, $c_k(t)$ is the chip sequence for user k, $b_k(t)$ is the information bit of user k, and $n(t)$ is the white noise in the channel with $\sigma^2 = N_0/2$. Information bits and chip sequence are constructed in polar form, in other words using +1 and -1. The conventional detector is that detector with multiple branches with correlators. The receiver will have as many branches as active users in the transmission. Each branch processes the signal through each of the chip sequences, where the decision variable in the j-th branch is obtained as follows

$$\begin{aligned} r_k &= \frac{1}{T}\int_0^T r(t) c_j(t)\,dt \\ &= \frac{1}{T}\int_0^T \alpha_j c_j(t) b_j c_j(t)\,dt + \sum_{\substack{i=1 \\ i\neq j}}^{K} \alpha_i b_i \frac{1}{T}\int_0^T c_i(t) c_j(t)\,dt + \frac{1}{T}\int_0^T n(t) c_j(t)\,dt \\ &= \underbrace{\alpha_j b_j}_{\text{Information}} + \underbrace{\sum_{\substack{i=1 \\ i\neq j}}^{K} \alpha_i b_i \rho_{i,j}}_{\text{MAI}} + \underbrace{\frac{1}{T}\int_0^T n(t) c_j(t)\,dt}_{\text{noise correlated with chip sequence}} . \end{aligned} \tag{7.44}$$

As seen in (7.44), the second term in the final result (MAI) will be present in the decision variable for detection, and depends on the cross-correlation of the chip sequences with the sequence for the j-th branch. In (7.44), the MAI contains the cross-correlations of the chip sequences given by

$$\rho_{i,j} = \frac{1}{T}\int_0^T c_i(t)c_j(t)dt \tag{7.45}$$

Optimal multiuser detection has been introduced, see Verdu, S. (1998), and has also been known as *maximum likelihood sequence detector*. The system proposed has a high computational complexity that is not convenient to be implemented in mobile devices yet. Thus, several suboptimal solutions have been proposed. These solutions are divided in two groups

- **Linear multiuser detectors**: these detectors use a transformation at the correlator outputs to produce outputs that help the detector to have better performance
- **Subtractive interference cancellation detectors**: these detectors estimate the MAI and generate such at the receiver to subtract out it from the incoming signal and remain with the desired signal.

With known sequences, these detectors might be possible to implement, the limitation is in a multicell environment that the receiver experiments MAI not only from its own cell, but from neighboring cells, and it might not be possible to know the chip sequences for the users in other cells interfering.

From Equation (7.44), it can be seen that the set of decision variables can be expressed in a matrix form as follows

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ \vdots \\ r_K \end{bmatrix} = \begin{bmatrix} 1 & \rho_{2,1} & \rho_{3,1} & \cdots & \rho_{K,1} \\ \rho_{1,2} & 1 & \rho_{3,2} & \cdots & \rho_{K,2} \\ \rho_{1,3} & \rho_{2,3} & 1 & \cdots & \rho_{K,3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho_{1,K} & \rho_{2,K} & \rho_{3,K} & \cdots & 1 \end{bmatrix} \begin{bmatrix} \alpha_1 & 0 & 0 & \cdots & 0 \\ 0 & \alpha_2 & 0 & \cdots & 0 \\ 0 & 0 & \alpha_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \alpha_K \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_K \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ \vdots \\ n_K \end{bmatrix} \tag{7.46}$$

In (7.46), n_j refers to the noise correlated with the chip sequence in the j-th branch of the receiver. Equation (7.46) can be written as follows

$$\mathbf{r} = \mathbf{RAb} + \mathbf{n}. \tag{7.47}$$

In (7.47), the **R** matrix is a correlation matrix, it is square and symmetric, and can be expressed as **R=I+Q**, where **I** is the identity matrix and **Q** is the correlation matrix with the main diagonal with zeros. Substituting this matrix into (7.47) gives the following

$$\begin{aligned} \mathbf{r} &= (\mathbf{I} + \mathbf{Q})\mathbf{Ab} + \mathbf{n} \\ &= \mathbf{Ab} + \mathbf{QAb} + \mathbf{n}. \end{aligned} \tag{7.48}$$

Within the group of linear multiuser detectors, there are the decorrelating detectors, the MMSE detector and the polynomial expansion detectors. The decorrelating detector apply a transformation to the decision variables **r** before the decision block, the transformation is simply the inverse of the matrix **R,** in other words, $\mathbf{L}=\mathbf{R}^{-1}$. Applying this transformation to Equation (7.47) gives the following

$$\begin{aligned}\mathbf{Lr} &= \mathbf{R}^{-1}\mathbf{r} \\ &= \mathbf{R}^{-1}\mathbf{RAb} + \mathbf{R}^{-1}\mathbf{n} \\ &= \mathbf{Ab} + \mathbf{n}_{dec}.\end{aligned} \tag{7.49}$$

As seen in Equation (7.49), the decision variable will contain the information and the noise correlated with the chip sequences, but the MAI term is completely eliminated. The advantage is that the receiver does not need to perform estimation of amplitudes; it only needs to know the chip sequences of all the users. The disadvantages are that it causes enhancement to the correlated noise **n**, and that the inverse of a matrix need to be computed in real-time since its dimensions depend on the number of active users for the bit to be detected.

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