APPLIED FINANCE PROJECT: Convertible Bonds Pricing Strategies

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OUTLINE

- Project Description
- Literature Review
  - Goldman Sachs Model
  - Tsiveriotis & Fernandez Model
  - Stochastic Volatility Model
- Data
- Analysis and Results

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PROJECT DESCRIPTION

MARKET OVERVIEW

- Convertible bonds are corporate debt securities that provide the holder the right to forgo future coupon and/or principal payments and convert to a specified number of shares of common stock instead.
- As of 12/31/2016 U.S. convertible market has $207.5 billion market capitalization, and convertible bonds make up 74%.
- Reasons for issuance:
    - Broaden investor base
    - Have flexible capital structure
    - Reduced interest rates
    - Reduce/defer tax liabilities

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PROJECT GOALS

Build multiple convertible bond pricing models and compare the results of the models to market prices.

» All the models are coded in R
» By company request: Excel spreadsheet contains all inputs, calls R script for valuation, and records the values in excel spreadsheet

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LITERATURE REVIEW

GOLDMAN SACHS MODEL

- Goldman Sachs Quantitative Research Notes (1994): binomial trees
- Assumptions:
  - Fixed interest rates
  - Fixed volatility of the underlying stock
  - Stock prices are lognormal with volatility
  - Default spread reflects credit risk

GOLDMAN SACHS MODEL

- Build stock tree over relevant period
- Start at the last period and calculate backwards
- Calculate payoff by comparing stock price with bond value
- Discount to the previous period with risk-free rate(if conversion is optimal) or risk-free rate plus credit spread (if bond value is optimal)
- Proceed to the previous period with these values
- Optional: Callability/Puttability of the bond are optional parameters into the model

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TSIVERIOTIS & FERNANDEZ MODEL

» Value convertible bond by separating it into equity portion and bond portion

Equity Portion

$$\text{CB}: \quad \frac{\partial u}{\partial t} + \frac{\sigma^2 S^2}{2}\frac{\partial^2 u}{\partial S^2} + r_g S \frac{\partial u}{\partial S} - r(u - v) - (r + r_c)v + f(t) = 0$$

Bond Portion

$$\text{COCB}: \frac{\partial v}{\partial t} + \frac{\sigma^2 S^2}{2}\frac{\partial^2 v}{\partial S^2} + r_g S \frac{\partial v}{\partial S} - (r + r_c)v + f(t) = 0$$

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STOCHASTIC VOLATILITY MODEL

- Convertible Bond
- Stock Price
- Path Dependent
- Monte Carlo Simulation
- Models

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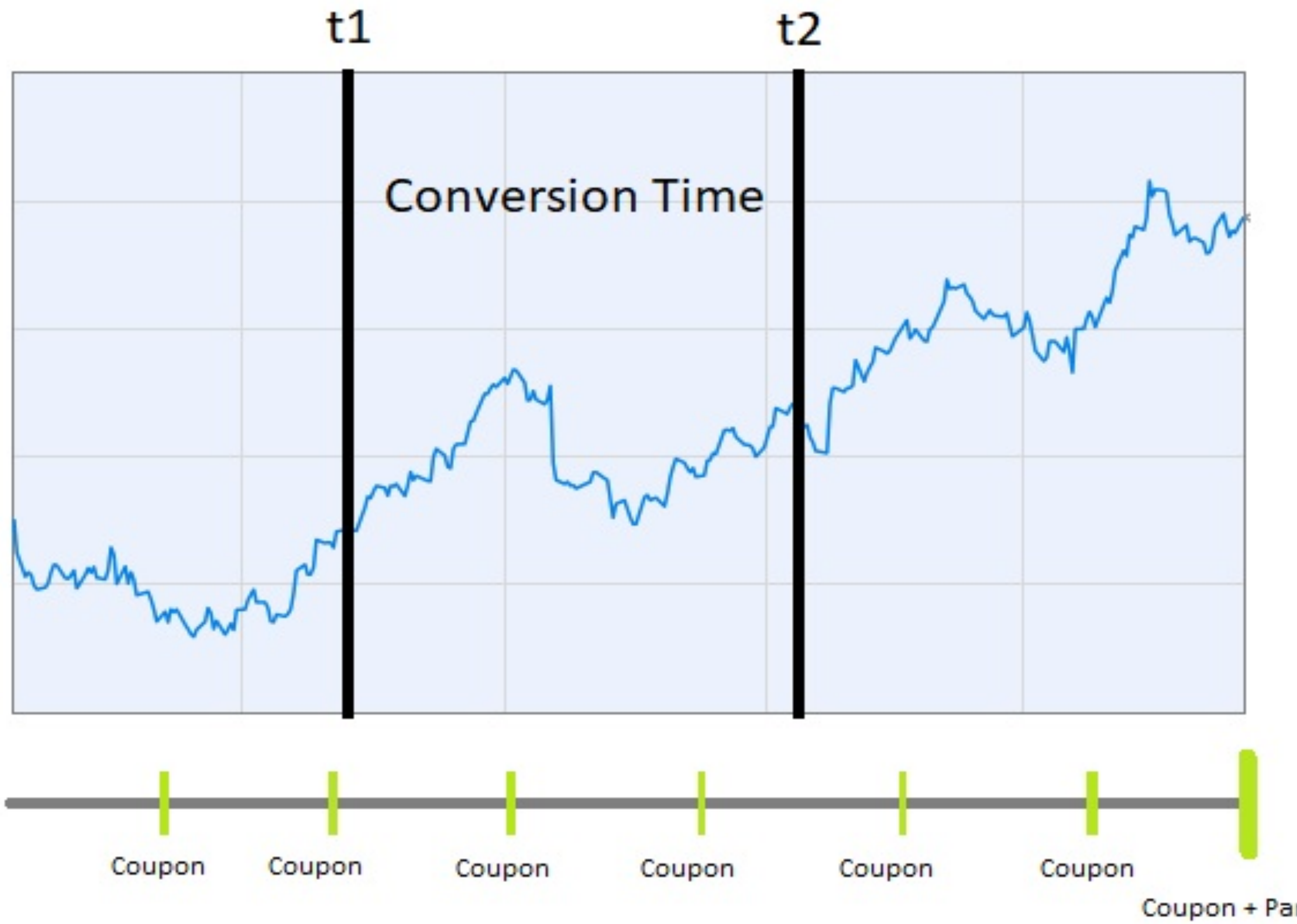
HOW TO PRICE USING MODELS ?



- **Key Concept: Discount to time Zero and Compare**

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WHAT MODEL TO CHOOSE?

- A model to discount
- A model to create a path
- Volatility is not constant
- Interest rate is not constant
- Positive interest rate
- Not all convertible bonds are American or European
- Flexible for changes

Heston Model & CIR Model

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CIR MODEL

$$dr(t) = k(\theta - r(t))dt + \sigma\sqrt{r(t)}dW.$$

k is the mean reversion speed, θ is the long term mean, σ is the volatility and r_0 is the starting value for the short rate CIR process.

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)}$$

$$A(t,T) = \left[\frac{2he^{(k+h)(T-t)/2}}{2h + (k+h)(e^{(T-t)h} - 1)}\right]^{\frac{2k\theta}{\sigma^2}}$$

$$B(t,T) = \frac{2(e^{(T-t)h} - 1)}{2h + (k+h)(e^{(T-t)h} - 1)}$$

$$h = \sqrt{k^2 + 2\sigma^2}.$$

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HESTON MODEL

$$dS_t = rS_t dt + \sqrt{V_t} S_t dW_t^1$$

$$dV_t = a(\bar{V} - V_t)dt + \eta\sqrt{V_t} dW_t^2$$

$$dW_t^1 dW_t^2 = \rho dt$$

S_t is the price of the underlying asset at time t

r is the risk free rate

V_t is the variance at time t

$\bar{V}$ is the long-term variance

a is the variance mean-reversion speed

η is the volatility of the variance process

dW_t^1, dW_t^2 are two correlated Weiner processes, with correlation coefficient ρ

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HESTON CHARACTERISTIC FUNCTION TO PRICE CALL OPTIONS

$$\psi_{\ln(S_t)}^{Heston}(w) = e^{[C(t,w)\bar{V} + D(t,w)V_0 + iw\ln(S_0 e^{rt})]}$$

$$C(t,w) = a\left[r_- \cdot t - \frac{2}{\eta^2} ln\left(\frac{1 - ge^{-ht}}{1-g}\right)\right]$$

$$D(t,w) = r_- \frac{1 - e^{-ht}}{1 - ge^{-ht}}$$

$$r_\pm = \frac{\beta \pm h}{\eta^2}; \quad h = \sqrt{\beta^2 - 4\alpha\gamma}$$

$$g = \frac{r_-}{r_+}$$

$$\alpha = -\frac{w^2}{2} - \frac{iw}{2}; \ \beta = a - \rho\eta i w; \ \gamma = \frac{\eta^2}{2}$$

$$C_0 = S_0 \, \Pi_1 - e^{-rT} K \, \Pi_2$$

$$\Pi_1 = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{e^{-iw\ln(K)}\psi_{\ln S_T}(w-i)}{iw\psi_{\ln S_T}(-i)}\right] dw$$

$$\Pi_2 = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{e^{-iw\ln(K)}\psi_{\ln S_T}(w)}{iw}\right] dw$$

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DATA

BLOOMBERG DATA

- 160 currently traded convertible bonds
- Bond price, underlying equity price, conversion ratio, maturity, Bloomberg credit spread to maturity, coupon rate, risk-free rate
- Data cleaning: must have valid price and credit spread, must have sufficient bond volume(above ~100 weekly)

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PREDICT VOLATILITY

- Difficulty: Bonds in general have longer maturity than equities' implied volatilities from options
- Need to predict implied volatility for the underlying based on bond's time to maturity
  - Back solve for volatilities using the model and other known parameters
  - Regression:
    - Dependent variables: volatilities
    - Independent variables: time to maturity, historical volatilities, companies' market capitalizations
  - Now we have the correct term volatility

OPTIMIZING CIR MODEL

CMT:

Select type of Interest Rate Data

Daily Treasury Yield Curve Rates ▼ Go

Select Time Period

Current Month ▼ Go

Date	1 Mo	3 Mo	6 Mo	1 Yr	2 Yr	3 Yr	5 Yr	7 Yr	10 Yr	20 Yr	30 Yr
11/01/17	1.06	1.18	1.30	1.46	1.61	1.74	2.01	2.22	2.37	2.63	2.85
11/02/17	1.02	1.17	1.29	1.46	1.61	1.73	2.00	2.21	2.35	2.61	2.83
11/03/17	1.02	1.18	1.31	1.49	1.63	1.74	1.99	2.19	2.34	2.59	2.82
11/06/17	1.03	1.19	1.30	1.50	1.61	1.73	1.99	2.17	2.32	2.58	2.80
11/07/17	1.05	1.22	1.33	1.49	1.63	1.75	1.99	2.17	2.32	2.56	2.77
11/08/17	1.05	1.23	1.35	1.53	1.65	1.77	2.01	2.19	2.32	2.57	2.79
11/09/17	1.07	1.24	1.36	1.53	1.63	1.75	2.01	2.20	2.33	2.59	2.81
11/10/17	1.06	1.23	1.37	1.54	1.67	1.79	2.06	2.27	2.40	2.67	2.88
11/13/17	1.07	1.24	1.37	1.55	1.70	1.82	2.08	2.27	2.40	2.67	2.87
11/14/17	1.06	1.26	1.40	1.55	1.68	1.81	2.06	2.26	2.38	2.64	2.84
11/15/17	1.08	1.25	1.39	1.55	1.68	1.79	2.04	2.21	2.33	2.58	2.77
11/16/17	1.08	1.27	1.42	1.59	1.72	1.83	2.07	2.25	2.37	2.62	2.81
11/17/17	1.08	1.29	1.42	1.60	1.73	1.83	2.06	2.23	2.35	2.59	2.78
11/20/17	1.09	1.30	1.46	1.62	1.77	1.86	2.09	2.26	2.37	2.60	2.78
11/21/17	1.15	1.30	1.45	1.62	1.77	1.88	2.11	2.27	2.36	2.58	2.76
11/22/17	1.16	1.29	1.45	1.61	1.74	1.84	2.05	2.22	2.32	2.57	2.75

Wednesday Nov 22, 2017

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OPTIMIZING HESTON MODEL

Call Prices:

Spot	Maturity	Strike	Interest rate	Mid	Bid	Ask
308.77	2.150793651	300	0.01595632	81.4	79.8	83
308.77	2.150793651	305	0.01595632	79.25	77	81.5
308.77	2.150793651	310	0.01595632	77.1	76	78.2
308.77	2.150793651	315	0.01595632	75.25	73	77.5
308.77	2.150793651	320	0.01595632	73.25	71	75.5
308.77	0.242063492	300	0.01132163	29.55	28.5	30.6
308.77	0.242063492	305	0.01132163	26.675	26.2	27.15
308.77	0.242063492	310	0.01132163	24	23.65	24.35
308.77	0.242063492	315	0.01132163	21.65	21.2	22.1
308.77	0.242063492	320	0.01132163	19.525	19.15	19.9
308.77	1.158730159	290	0.01364671	65.4	63.6	67.2
308.77	1.158730159	310	0.01364671	56.15	54.7	57.6
308.77	1.158730159	330	0.01364671	47.625	46.75	48.5
308.77	0.571428571	300	0.01217852	42.975	42.05	43.9
308.77	0.571428571	305	0.01217852	40.8	40.25	41.35
308.77	0.571428571	310	0.01217852	38.4	37.9	38.9
308.77	0.571428571	315	0.01217852	36.125	35.55	36.7
308.77	0.571428571	320	0.01217852	33.85	33.35	34.35
308.77	0.099206349	305	0.01094242	13.75	13.5	14
308.77	0.099206349	307	0.01094242	12.4	12.3	12.5
308.77	0.099206349	310	0.01094242	11.15	11	11.3
308.77	0.099206349	312	0.01094242	9.9	9.7	10.1
308.77	0.099206349	315	0.01094242	8.9	8.8	9

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ANALYSIS AND RESULTS

GOLDMAN SACHS MODEL

Here are some convertible bonds priced by our Goldman Sachs model.
(Model is still under calibration.)

Bond Price	Fitted Price	SolvedGS Vol	Fitted Vol	Price Diff%
162.00	163.00	0.31	0.34	0.62%
116.28	118.71	0.24	0.27	2.09%
142.00	144.11	0.08	0.09	1.48%
93.48	92.68	0.42	0.40	-0.85%
142.03	140.93	0.19	0.14	-0.77%
93.42	91.89	0.40	0.36	-1.64%
90.63	91.95	0.11	0.15	1.46%
96.43	95.19	0.25	0.23	-1.28%
127.91	129.73	0.27	0.30	1.43%
188.67	187.34	0.33	0.11	-0.70%
139.61	139.35	0.30	-0.29	-0.19%
101.00	100.21	0.29	0.28	-0.78%
95.29	93.79	0.30	0.24	-1.58%
99.38	98.39	0.20	0.08	-1.00%
114.25	116.08	0.05	0.06	1.60%
215.75	214.47	0.29	0.08	-0.59%
107.11	105.53	0.40	0.37	-1.48%
146.53	143.06	0.25	0.15	-2.37%
155.04	151.23	0.34	0.27	-2.46%
187.83	187.24	0.23	0.22	-0.31%

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STOCHASTIC VOLATILITY MODEL: OPTIMIZATION RESULT

» CIR Model: $dr(t) = k(\theta - r(t))dt + \sigma\sqrt{r(t)}dW.$

Filter

kk	sigma	rbar
0.09291284	0.127362	0.0686392

Filter

	Date	1 Mo	3 Mo	6 Mo	1 Yr	2 Yr	3 Yr	5 Yr	7 Yr
actual	11/20/17	0.0109	0.013	0.0146	0.0162	0.0177	0.0186	0.0209	0.0226
CIR Estimate	NA	0.0109	0.0113425597042397	0.0119947446354736	0.0132574067316529	0.01561879298689	0.0177693615356185	0.0214876001306962	0.0245247397747119
% error	NA	0	12.7495407366177	17.8442148255233	18.1641559774512	11.7582317124859	4.46579819559946	2.81148387892919	8.51654767571637

	1 Yr	2 Yr	3 Yr	5 Yr
	0.0162	0.0177	0.0186	0.0209
354736	0.0132574067316529	0.01561879298689	0.0177693615356185	0.021487
5233	18.1641559774512	11.7582317124859	4.46579819559946	2.811483

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STOCHASTIC VOLATILITY MODEL: OPTIMIZATION RESULT

» HESTON MODEL:

$$dS_t = rS_t dt + \sqrt{V_t} S_t dW_t^1$$

$$dV_t = a(\bar{V} - V_t)dt + \eta\sqrt{V_t} dW_t^2$$

$$dW_t^1 dW_t^2 = \rho dt$$

v0	vbar	vvol	rho	a
0.1100836	0.1888277	1.141432	-0.4564487	6.882538

	Mid	Bid	Ask	Calc	Within-Bid-Ask?	% Error
1	81.4	79.8	83	82.13	Yes	0
2	79.25	77	81.5	80.03	Yes	0
3	77.1	76	78.2	77.98	Yes	0
4	75.25	73	77.5	75.98	Yes	0
5	73.25	71	75.5	74.02	Yes	0
6	29.55	28.5	30.6	27.76	No	6.06
7	26.675	26.2	27.15	25.01	No	6.24
8	24	23.65	24.35	22.43	No	6.54
9	21.65	21.2	22.1	20.04	No	7.44
10	19.525	19.15	19.9	17.82	No	8.73
11	65.4	63.6	67.2	64.99	Yes	0
12	56.15	54.7	57.6	55.55	Yes	0

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